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ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-1 : Network Theory + Signals and Systems [All Topics]

Name :

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Hyderabad ☐		

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE	
Question No.	Marks Obtained
Section-A	
Q.1	45
Q.2	
Q.3	35
Q.4	19
Section-B	
Q.5	37
Q.6	30
Q.7	
Q.8	
Total Marks Obtained	166

Signature of Evaluator

Cross Checked by

1. Revise signals
2. Avoid calculation error

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Network Theory

(a) Briefly explain sources used in network (independent and dependent sources) and their types.

[12 marks]

(a) In Network Theory, Sources are of two types: 1. dependent Sources

Here, value of the source depends upon an element.

2. Independent sources

The value of the source is independent of any element.

Some examples of independent Sources.

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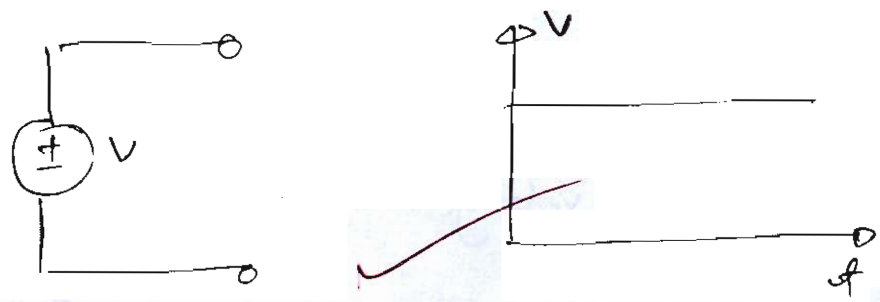
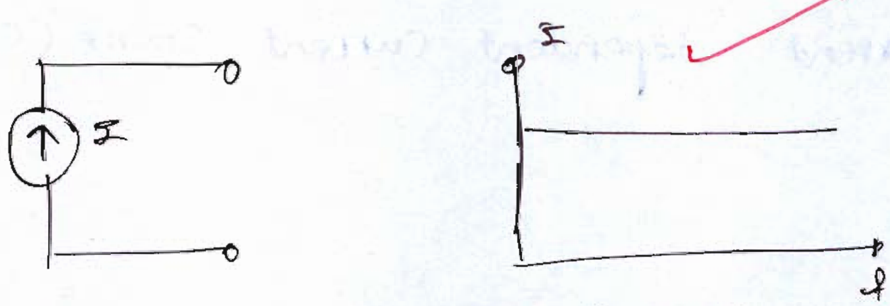
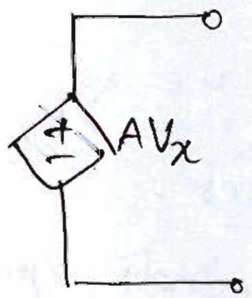


independent voltage source where value of voltage is constant/independent of time.

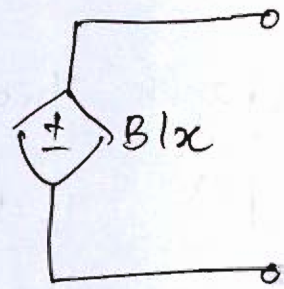


[independent Current Source]

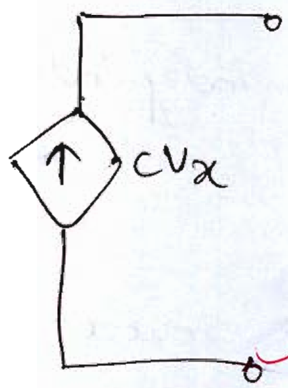
Dependent Sources : Define it



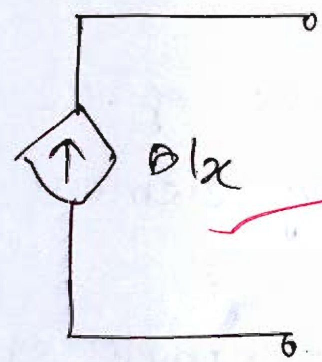
(i)



(ii)



(iii)



(iv)

(i) Voltage dependent voltage Source (VDVS)

(ii) Current dependent voltage Source (CDVS)

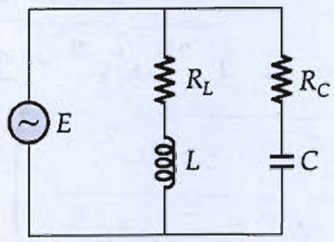
(iii) Voltage dependent current source (VDCS)

(iv) current dependent current Source (CDCS)

Do not write in this margin

(b) Draw the phasor diagram for the circuit shown and prove that the condition for resonance is

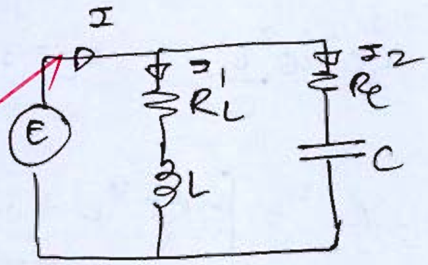
$$\frac{L}{C}=\frac{R_L^2+\omega^2L^2}{1+R_C^2\omega^2C^2}$$



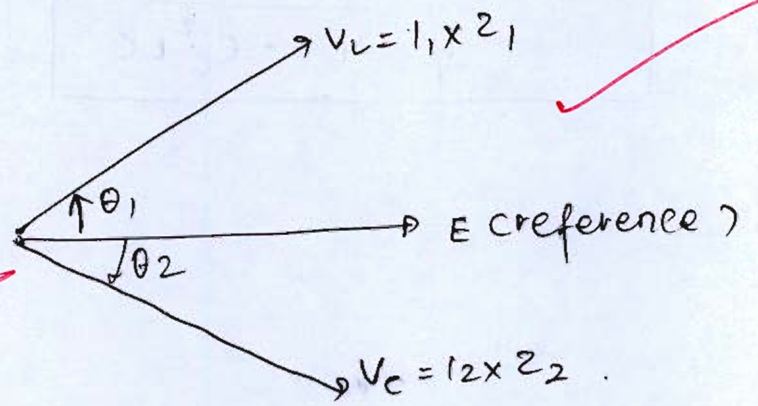
Also, calculate the value of ω (rad/sec) at resonance.

[12 marks]

(b) Given circuit :



Phasor representation :

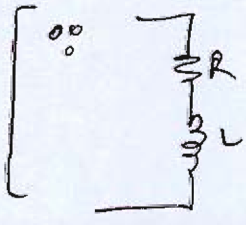


12

For resonance to occur, imaginary part of the admittances must be zero,

Hence $B_L = B_C$.

$$\frac{X_L}{R_L^2+X_L^2} = \frac{X_C}{R^2+X_C^2}.$$



$$Z = R + jX_L \; ; \; Y = \frac{1}{R+jX_L} = \frac{R}{R^2+X_L^2} - j\frac{X_L}{R^2+X_L^2}$$

$$= G - jB_L$$

$$\frac{\omega L}{R_L^2 + \omega^2 L^2} = \frac{1/\omega C}{R_C^2 + 1/\omega^2 C^2}$$

$$\frac{\cancel{\omega} L}{R_L^2 + \omega^2 L^2} = \frac{1 \times \cancel{\omega} C}{R_C^2 \omega^2 C^2 + 1}$$

Hence $\boxed{\frac{L}{C} = \frac{R_L^2 + \omega^2 L^2}{1 + R_C^2 \omega^2 C^2}}$

Now,

$$\frac{L}{C} + R_C^2 C^2 \omega^2 \frac{L}{C} = R_L^2 + \omega^2 L^2$$

$$\frac{L}{C} - R_L^2 = \left[-R_C^2 C L + L^2\right] \omega^2$$

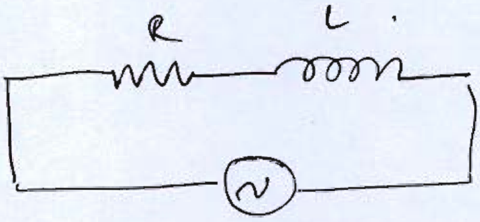
$$\boxed{\omega = \sqrt{\frac{\frac{L}{C} - R_L^2}{L^2 - R_C^2 LC}}} \text{ rad/sec.}$$

(c) An iron-cored coil takes 4A at a power factor of 0.5 when connected to a 200-V, 50 Hz supply. When the iron core is removed and the voltage is reduced to 40 V, the current rises to 5 A at a pf of 0.8. Find the iron loss in the core and inductance in each case.

[12 marks]

(c)

Given:



200, 50.

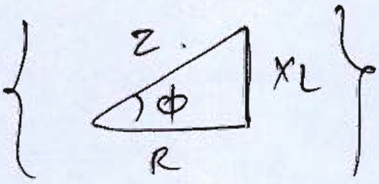
2

$4A, \cos\phi = 1/2$.

$I = \frac{V}{Z}$ $\therefore$ $4 = \frac{200}{Z}$; $|Z| = 50\,\Omega$. -(1)

where, $Z = R + jX_L$.

also, $\cos\phi = \frac{R}{Z}$.



$\frac{1}{2} = \frac{R}{Z}$.

$\therefore \boxed{Z = 2R}$

incomplete solution

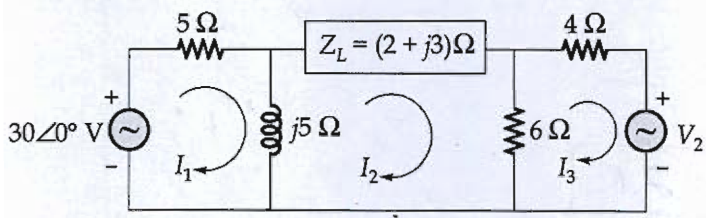
also, condⁿ - 2.

when, $160 = \frac{5}{Z}$ $Z = 32$.

$\cos\phi = \frac{4}{5} = \frac{R}{Z}$.

$$\boxed{Z = \frac{5}{4}R}$$

Q.1 (d) In the network shown below, find the value of V_2 so that the current through $Z_L = (2 + j3)\ \Omega$ impedance is zero.



[12 marks]

Q1 (d) from above Circuit,

Taking mesh equation,

$$(5+j5)\, I_1 - j5\, I_2 = 30\angle 0^\circ \quad -(1)$$

$$(2+j3+6+j5)\, I_2 - j5\, I_1 - 6\, I_3 = 0 \quad -(2)$$

$$10\, I_3 - 6\, I_2 = -V_2 \quad -(3)$$

we desire the current I_2 to be zero. (I_2 is current through Z_L]

Hence, writing equations according to conditions,

$$(5+j5)I_1 = 30\angle 0^\circ$$

$$I_1 = 3 - 3j \text{ A}$$

$$\boxed{I_1 = 4.24\angle -45^\circ \text{ A}}$$

from (2), $6I_3 = -j5\,I_1$

$$I_3 = \frac{-5}{2} - \frac{5}{2}j$$

$$\boxed{I_3 = 3.53\angle -135^\circ} \text{ A}$$

from (3), $10I_3 = -V_2$.

$$\therefore \boxed{V_2 = 25 + j25} \text{ V}$$

Hence, $\boxed{V_2 = 35.35\angle 45^\circ}$ V.

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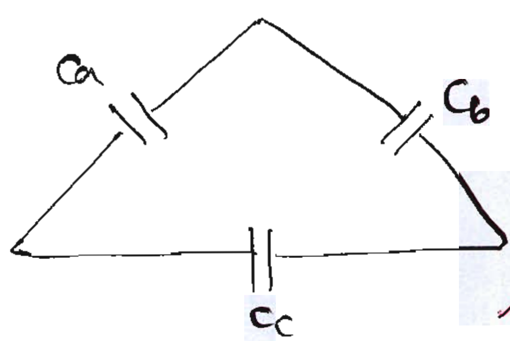
Good

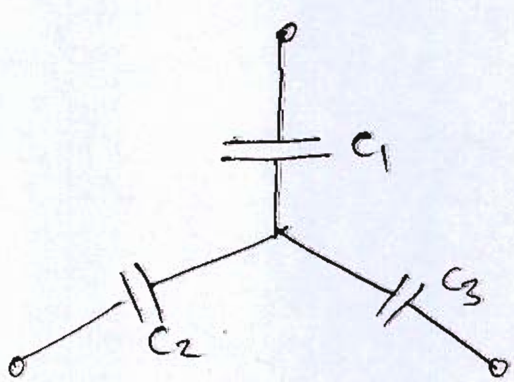
Q.1 (e) Represent a Delta connected capacitive circuit as an equivalent star connection and also express the star connected capacitive elements in terms of the delta connected capacitive elements.

[12 marks]

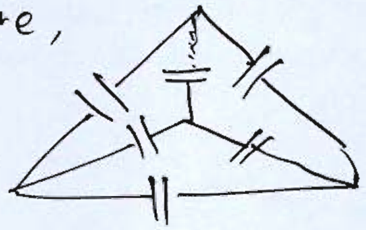
Q1 (e) A delta connected capacitive circuit is



Equivalent star representation:



where,



connections were as follows,

$$\frac{1}{C_a} = \frac{\frac{1}{C_1}\times\frac{1}{C_3} + \frac{1}{C_3}\times\frac{1}{C_2} + \frac{1}{C_2}\times\frac{1}{C_1}}{\frac{1}{C_3}}$$

Similarily for C_b & C_c,

$$\frac{1}{C_b} = \frac{\frac{1}{C_1 \cdot C_3} + \frac{1}{C_2 \cdot C_2} + \frac{1}{C_2 C_1}}{\frac{1}{C_2}}$$

$$\frac{1}{C_c} = \frac{\frac{1}{C_1 C_3} + \frac{1}{C_3 C_2} + \frac{1}{C_2 C_1}}{\frac{1}{C_1}}$$

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⟹ Try to write it in simflied way

for star representations,

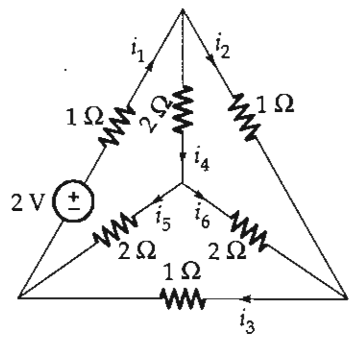
$$\frac{1}{C_1} = \frac{\frac{1}{C_a}\times\frac{1}{C_b}}{\frac{1}{C_a}+\frac{1}{C_b}+\frac{1}{C_c}} = \frac{\frac{1}{\cancel{C_aC_b}}}{\frac{C_bC_c + C_aC_c + C_aC_b}{\cancel{C_aC_bC_c}}} = \frac{C_c}{C_aC_b + C_bC_c + C_aC_c}$$

$$\frac{1}{C_2} = \frac{\frac{1}{C_b}\times\frac{1}{C_c}}{\frac{1}{C_a}+\frac{1}{C_b}+\frac{1}{C_c}} = \frac{C_a}{C_aC_b + C_bC_c + C_aC_c}$$

$$\frac{1}{C_3} = \frac{\frac{1}{C_c}\times\frac{1}{C_a}}{\frac{1}{C_a}+\frac{1}{C_b}+\frac{1}{C_c}} = \frac{C_b}{C_aC_b + C_bC_c + C_aC_c}.$$

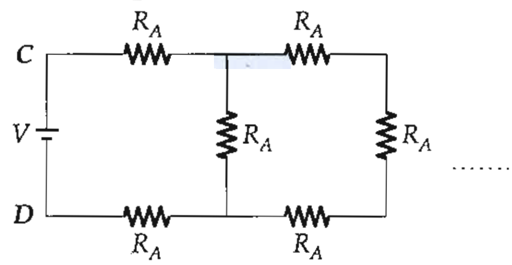
Do not write in this

Q.2 (a) For the network shown in figure below, write down the tieset matrix and obtain the network equilibrium equations in matrix form using KVL. Calculate the loop currents and branch currents.

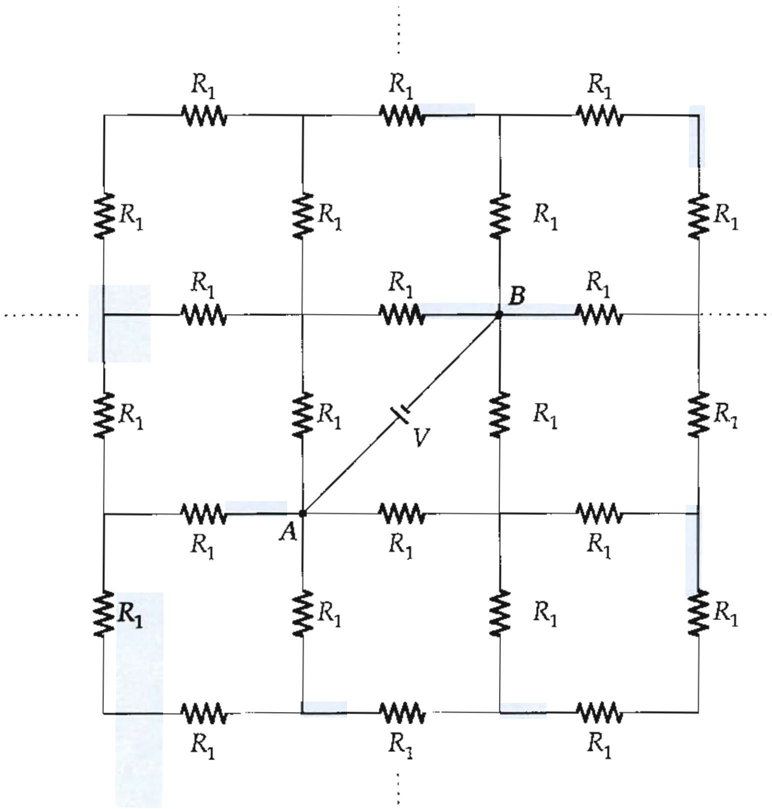


[20 marks]

(b) For the networks shown in figure below,



Network (i)



Network (ii)

On applying 10 V voltage source across terminal CD and AB respectively in network (i) and (ii), we get same power delivered by the source. Derive the relationship between the resistance present in network (i) and (ii). And also calculate the power delivered by the source for $R_A = 10\ \Omega$.

[20 marks]

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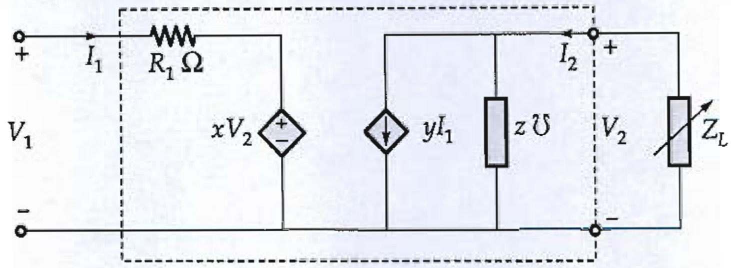
Q.2 (c) Consider a two port network shown in figure below,



If transmission parameters matrix of the network is $\begin{bmatrix} 10^{-2} & 10^{2} \\ 0 & 10^{-1} \end{bmatrix}$.

Then, calculate:

(i) parameters of the circuit: R_1, x, y and z.

(ii) the value of load impedance (Z_L), for maximum power transfer.

(iii) maximum power transfer to load for $V_1 = 0.1$ volt.

[20 marks]

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(a) **(i)** State 'Voltage to current source transformation' theorem. It is required to replace network N in figure (a) by a suitable equivalent network. Which of the networks of figure (b) could be valid equivalent network (s)?

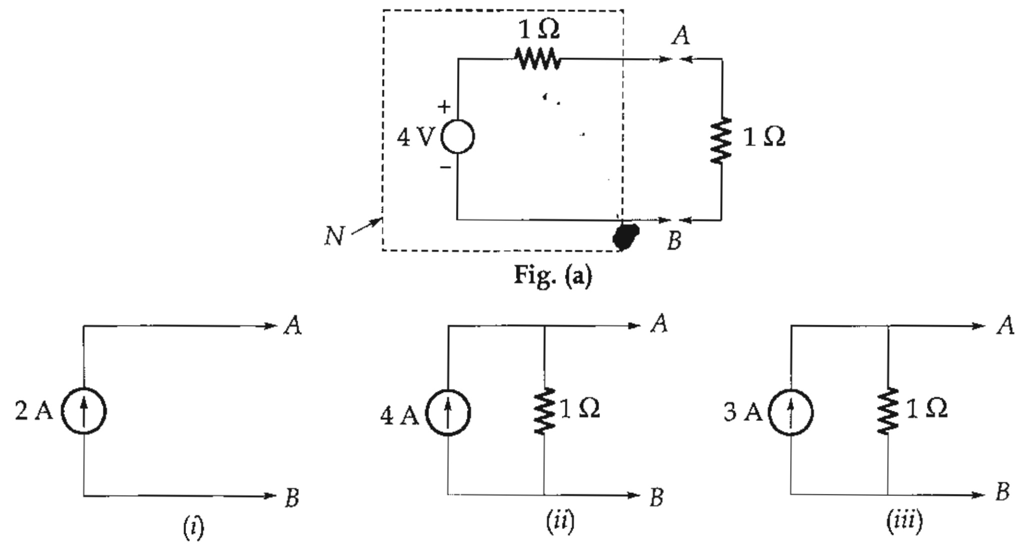
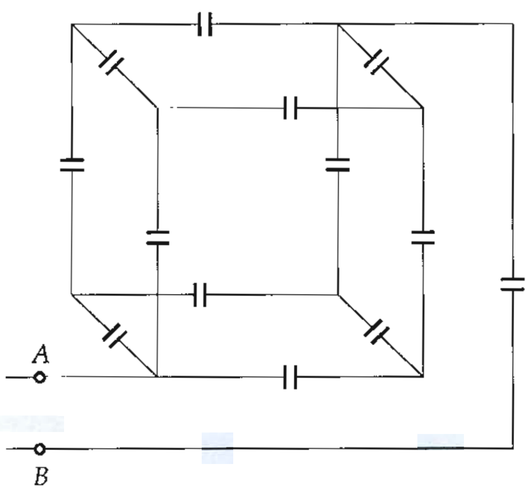


Fig. (a)

Fig. (b)

(ii) The network of capacitors in the figure below is composed of a 2F capacitor on each edge of a cube along with a capacitor of 2F connected to the vertices of the cube as shown. Find the $C_{equivalent}$ between the terminals A-B and also calculate the energy stored by the capacitive circuit if 5 V is applied across the terminal A-B.

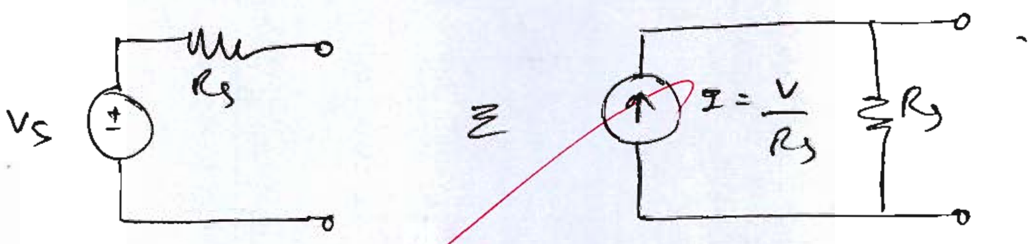


3 (a) (i).

[10 + 10 marks]

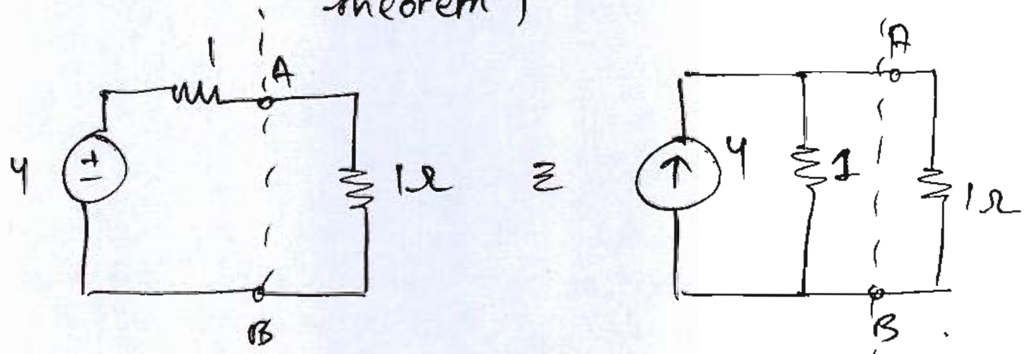
voltage to current source transformation theorem :

In a linear, bilateral circuit consisting of active and passive elements, the voltage source having internal impedance in series, can be replaced by an equivalent current source which has a resistance in parallel with it.



[Source transformation theorem]

given:



Hence from figure B, (ii) is the only equivalent option.

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Good

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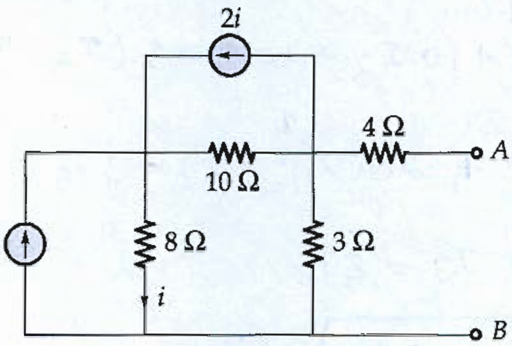
(b) For the circuit shown in figure below:



Calculate:

(i) Norton equivalent circuit across *A-B*.

(ii) Load impedance across *AB* for Maximum Power Transfer.

(iii) Maximum Power Transfer to load, obtained in part (ii) and also comment on the result.

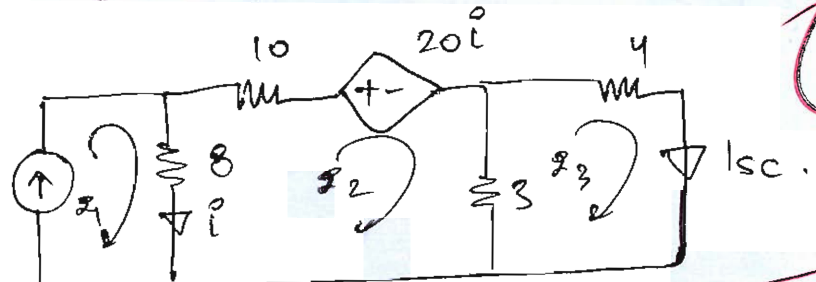
[20 marks]

13 (b) Norton's Theorem:

required

In a linear, bidirectional circuit having active & passive elements, the equivalent current & impedance can be calculated across an element. Equivalent current being the short circuit current across branch & resistance when all the sources are deactivated & replaced by their internal resistances.

(Isc)



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[simplified circuit]

Consider current is clockwise direction as shown,

$I_1 = 1A$ & $i = I_1 - I_2$.

$I_3 = I_{SC}$

By KVL, in loop (2),

$$8(I_2 - \cancel{I_1}^{\,1}) + 10 I_2 + 20 i + 3(I_2 - I_3) = 0 .$$

$$21 I_2 - 8 + 20(\cancel{I_1}^{\,1} - I_2) - 3 I_3 = 0 .$$

$$-8 + 20 + I_2 = 3 I_3 .$$

$$\boxed{12 + I_2 = 3 I_3} \quad - (1)$$

By KVL in loop (3),

$$\boxed{7 I_3 - 3 I_2 = 0} \quad - (2)$$

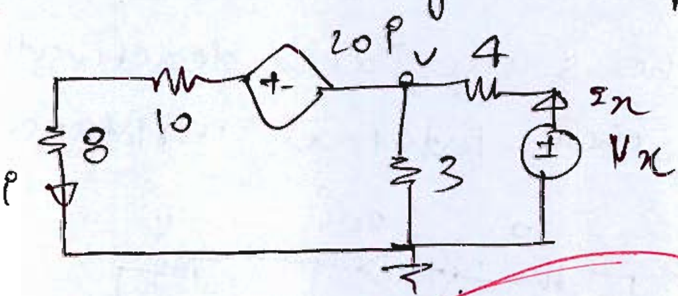
on solving (1) & (2),

$$I_3 = 18 \text{ A}$$

$$I_2 = 42 \text{ A}$$

Hence $\boxed{I_{sc} = 18 \text{ A}}$

R_{Th}/R_N : 1. deactivating all independent sources



Here, $R_{Th} = \dfrac{1}{I_x}$.

By KCL, and $\dfrac{V_x - V}{4} = I_x$.

$$i + \frac{V}{3} = I_x \quad - (1) .$$

~~$1 - 4I_x = V$~~ . (3)

$$V_x - V = 4 I_x$$

also,

$$-V - 20 i + 18 i = 0$$

$$V = -2 i \quad - (2)$$

~~Put ② in ① & ⑤~~,

~~$\left[1-\frac{2}{3}\right]i = I_x$~~ ~~$1+2i = 4I_x$~~

~~$\frac{i}{3} = I_x$~~ ~~$1 + 4I_x = -2i$~~

~~$1 = 4I_x - 2(3I_x)$~~

~~$\boxed{I_x = -1/2}$~~

on solving,

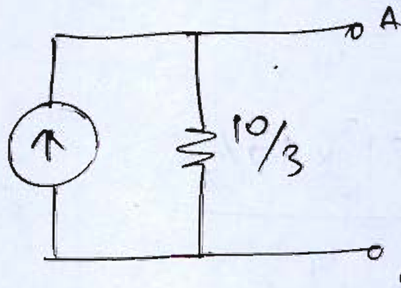
$$\frac{V_x}{I_x} = \frac{10}{3}\ \Omega.$$

$\therefore \boxed{R_{Th} = \frac{10}{3}\ \Omega}$.

$\Downarrow$

-2Ω

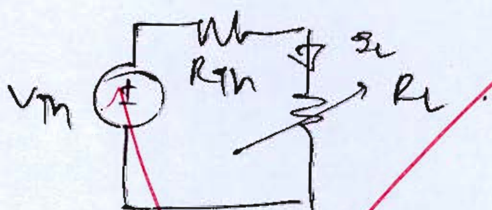
(i)



[Norton equivalent circuit]

(ii) for maximum power transfer :

$$R_L = R_{Th} = R_N,$$



$$I_L = \frac{V_{Th}}{R_{Th}+R_L}$$

$$P = I_L^2 \times R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \times R_L.$$

$\frac{dP}{dR_L} = 0$ {for maxima}.

on solving $\boxed{R_L = R_{Th}}$.

$\therefore \boxed{R_L = \frac{10}{3}\ \Omega}$

(iii) For maximum Power,

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}.$$

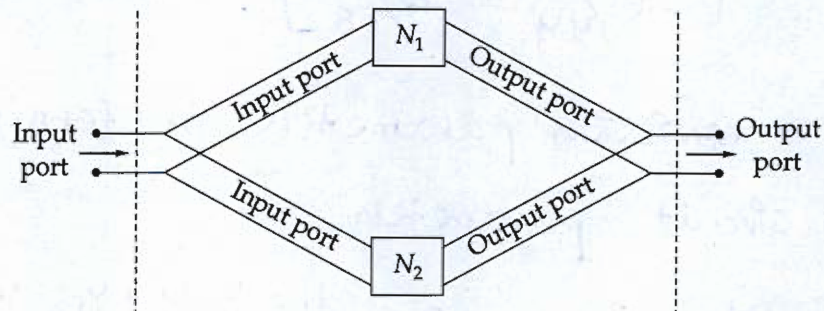
we know, $\frac{V_{Th}}{I_{SC}} = R_{Th}$.

$$\therefore P_{max} = \frac{I_{SC}^2 \times R_{Th}}{4}.$$

$$= \frac{(18)^2 \times 10/3}{4}.$$

$$\boxed{P_{max} = 270 \text{ watts}}.$$

(c) Consider the two-port network 'N' given below:



N_1 and N_2 are two 2-port networks connected in parallel on both input port side as well as output port side, to form a composite 2-port network N as indicated.

N_1 and N_2 are defined by the z-parameters as below:

$$[Z_{n_1}] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}\Omega, [Z_{n_2}] = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}\Omega$$

Obtain the transmission parameters for the composite 2-port network N. Also, express the transmission parameters in terms of short circuit parameters.

[20 marks]

.3 (c) Since N_1 & N_2 are connected in parallel,

we know, $[Y_{eq}] = [Y_1] + [Y_2]$

Given $[Z_1] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$.

$[Y_1] = [Z_1]^{-1} = \frac{1}{\Delta}\begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix}$

$[\Delta_1 = 20 - 9 = 11]$.

$[Y_2] = [Z_2]^{-1} = \frac{1}{\Delta_2}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$

$[\Delta_2 = 12 - 4 = 8]$

$\therefore [Y_{eq}] = \begin{bmatrix} 5/11 & -3/11 \\ -3/11 & 4/11 \end{bmatrix} + \begin{bmatrix} 1/2 & -1/4 \\ -1/4 & +3/8 \end{bmatrix}$.

$$[Y_{eq}] = \begin{bmatrix} 21/22 & -23/44 \\ -23/44 & 65/88 \end{bmatrix}.$$

Expressing transmission parameters in terms of short circuit parameters,

$$\left.\begin{array}{l} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{array}\right] [ABCD] \qquad \left.\begin{array}{l} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{array}\right] (Y)$$

Case-1 : $V_2 = 0$.

$$Y_{11} = \frac{I_1}{V_1} \quad \& \quad \frac{I_2}{V_1} = Y_{21}$$

on solving,

$V_1 = -BI_2$

$I_1 = -DI_2$

$$\therefore \frac{I_1}{V_1} = \boxed{Y_{11} = \frac{D}{B}}.$$

$$\frac{I_2}{V_1} = \boxed{Y_{21} = \frac{-1}{B}}$$

Case-2 : $V_1 = 0$.

$$\frac{I_1}{V_2} = Y_{12} \quad \& \quad \frac{I_2}{V_2} = Y_{22}.$$

$\therefore \quad AV_2 = BI_2.$

$$\frac{I_2}{V_2} = \boxed{Y_{22} = \frac{A}{B}}.$$

$I_1 = CV_2 - DI_2$

$$I_1 = CV_2 - \frac{DA}{B}V_2.$$

$$\frac{I_1}{V_2} = \boxed{Y_{12} = \frac{BC - DA}{B}}$$

$$\therefore \ [Y_{eq}] = \begin{bmatrix} D/B & \frac{BC-DA}{B} \\ -1/B & A/B \end{bmatrix}.$$

on solving using above relation,

$$-\frac{1}{B} = \frac{23}{44} \qquad \therefore \ B = \frac{44}{23}.$$

$$\frac{D}{B} = \frac{21}{22}. \qquad \therefore \ D = \frac{44}{23} \times \frac{21}{22} = \frac{42}{23}.$$

$$\frac{A}{B} = \frac{65}{88} \qquad \therefore \ A = \frac{44}{23} \times \frac{65}{88} = \frac{65}{46}.$$

$$\frac{BC-DA}{B} = \frac{-23}{44}. \qquad BC = -1 + DA.$$

$$\therefore \ C = \frac{19}{23}$$

$$\therefore \ BC - DA = -1.$$

-1.92

$$\text{Hence} \quad [T] = \begin{bmatrix} 65/46 & 44/23 \\ 19/23 & 42/23 \end{bmatrix}.$$

Ans

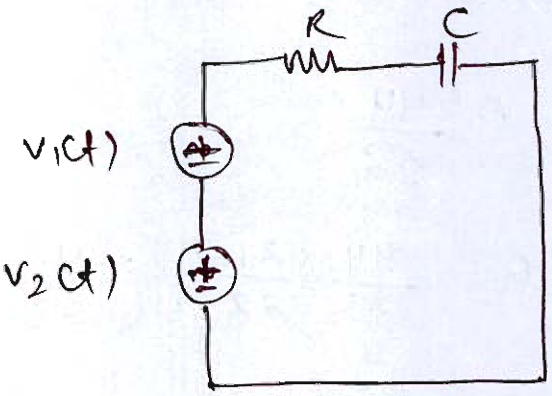
15

Do not write this r

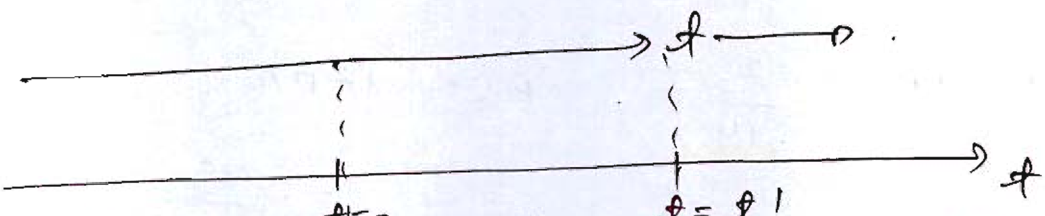
Q.4 (a) A series RC circuit has $R = 9\ k\Omega$, $C = 20\ \mu F$ and two voltage source in series given by $V_1 = 20u(-t)$ and $V_2 = 20u(t - t')$ V. Determine the complete expression for the voltage across the capacitor and plot it as a function of time, assuming t' as a positive quantity.

[20 marks]

Q4 (a) making a circuit using above parameters,



Above time parameters / Instances:-



- V_1 is active.
- capacitor is charging.
- V_2 is not active

- V_1 is inactive.
- V_2 is also inactive

- V_2 is active
- capacitor starts charging again.

for $t<0$: charging capacitor,

$$V_C(t) = V_0\left[1 - e^{-t/\tau}\right].$$

where $\tau = R \times C. = 9 \times 10^3 \times 20 \times 10^{-6} = \frac{9}{50}$ sec.

$$\boxed{V_C(t) = 20\left[1 - e^{50t/9}\right]}$$

Do not write in this margin

@ $t=0$, v_1 is inactive,

capacitor is discharging,

@ $t=t'$, v_2 is now active, (c) again charging,

$$v_c(t) = [v_c(0^+) - v_c(\infty)]\,e^{-t/\tau} + v_c(\infty).$$

But time instance is $t = t'$

$$v_c(t) = [v_c(t^+) - v_c(\infty)]\,e^{-(t-t')/\tau} + v_c(\infty)$$

TS

$v_c(t')$: $v_c(t') = 20\,[1 - e^{-50t'/9}]$.

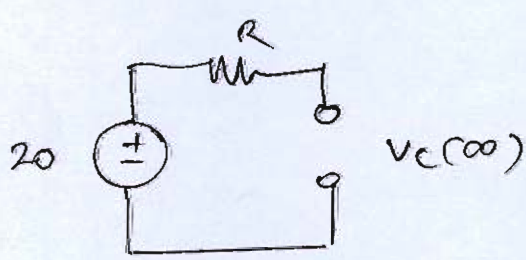
Good

substituting above,

$$\boxed{v_c(t) = [20\,[1 - e^{-50t'/9}] - 20]\,e^{-(t-t')\frac{50}{9}} + 20}.$$

Hence above is the final expression of the voltage across capacitor.

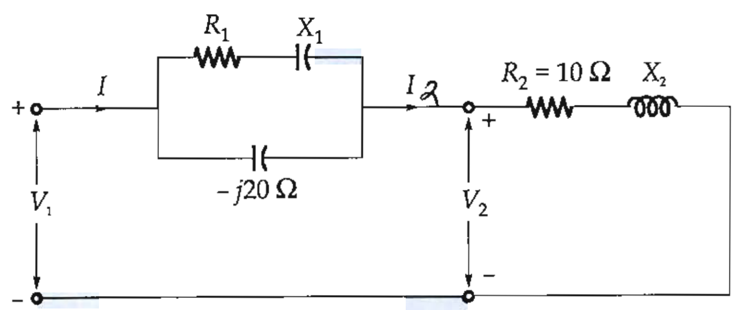
@ $t = \infty$



$\therefore$ $v_c(\infty) = 20V$.

Curve ??

(b) In the circuit shown in the figure below, $|V_1| = 200$ V, $V_2 = 200\angle 0°$ V and $|I| = 12$ A. The total power absorbed by the circuit is 1.8 kW. Find R_1, X_1 and X_2.



[20 marks]

(b) $|V_1| = 200V$. $P_T = 1.8\,kW$.

$V_2 = 200\angle 0°$.

$|I| = 12A$.

4

from above circuit,

$$V_2 = I_2 \times [R_2 + jX_2].$$

$$\text{So}\quad I_2 = \frac{V_2}{R_2 + jX_2} = \frac{200}{10 + jX_2}.$$

$$|I_2| = 12 = \frac{200}{\sqrt{10^2 + X_2^2}}.$$

on solving,

$$\boxed{X_2 = \frac{40}{3} = 13.33\,\Omega}$$

also,

$$I_2 = \frac{200\angle 0°}{10 + j13.33} = 7.202 - j9.6\ A.$$

$$\boxed{I_2 = 12\angle -53.12°}$$

here $[I = I_2$ only$]$.

from circuit;

power dissipation across R_2.

$P = |I_2|^2 \times R_2$

$= (12)^2 \times 10$.

$= 1440$ watts

$P_T = 1.8$ kW.

$\therefore$ power dissipation across R_1

$P_D = 1800 - 1440$

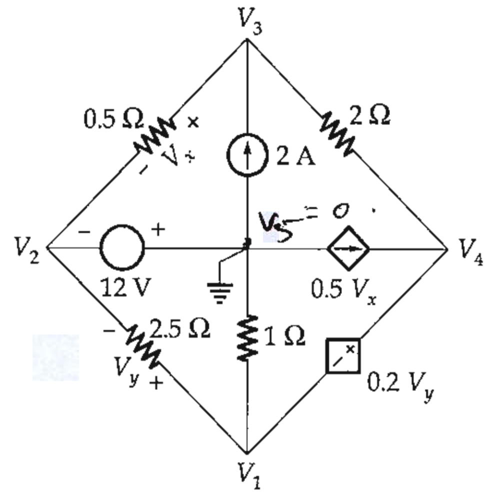
$= 360$ watts.

$P = I^2 \times R$.

$360 = |I_1|^2 \times R_1$.

incomplete soln

Q.4 (c) For the circuit shown in figure, obtain the value of voltage across 0.5 Ω and 2.5 Ω resistors using nodal analysis.



[20 marks]

Q4 (c)

using nodal analysis,

KCL @ V_3:

$$\frac{V_3 - V_2}{1/2} + \frac{V_3 - V_4}{2} = 2$$

$$2V_3 - 2V_2 + \frac{V_3 - V_4}{2} = 2$$

x^ly by 2

$$4V_3 - 4V_2 + V_3 - V_4 = 4.$$

$$\boxed{5V_3 - 4V_2 - V_4 = 4} \quad -(1)$$

KCL @ V_4 & V_1 [By supernodal analysis].

$$\frac{V_4 - V_3}{2} + \frac{V_1}{1} + \frac{V_1 - V_2}{5/2} = \frac{V_x}{2} \quad -(2)$$

from above diagram,

$V_y = V_1 - V_2$, & $\boxed{V_2 = -12}$

$V_x = V_3 - V_2$.

also, $V_4 - V_1 = 0.2 V_y$

$V_4 - V_1 = 0.2\,[V_1 - V_2]$.

$V_4 - V_1 - 0.2V_1 + 0.2V_2 = 0$.

$V_4 - 1.2V_1 + 0.2[-12] = 0$.

$\boxed{-1.2V_1 + V_4 = 2.4}$ — (3).

Put $V_2 = -12$ in (1).

$5V_3 - V_4 = 48+4$; $\boxed{5V_3 - V_4 = 52}$ (5)

solving (2),

$$\frac{V_4 - V_3}{2} + V_1 + \frac{2V_1 - 2V_2}{5} = \frac{V_3 - V_2}{2}.$$

(−12) (−12)

$$\frac{V_4 - V_3}{2} + V_1 + \frac{2V_1 + 24}{5} = \frac{V_3 + 12}{2}.$$

multiply by 10,

$\cancel{5V_4} - \cancel{5V_3} + \cancel{10V_1} + \cancel{4V_1} \cancel{+48} = \cancel{5V_3} + \cancel{60}$.

$\boxed{14V_1 - 10V_3 + 5V_4 = 12}$ — (4).

Solving (2), (3) & (4), (5)

$V_1 = 6.18\,V$.

$V_3 = 12.36\,V$.

$V_4 = 9.818\,V$.

Since $V_x = V_3 - V_2 = V_3 + 12$.

$\boxed{V_x = 24.36\,V}$ — (6)

& $V_y = V_1 - V_2$

$= V_1 + 12$

$$\boxed{V_y = 18.18\,V} \quad \text{—(7)}$$

Hence (6) & (7) are respective answers.

Section B : Signals and Systems

5 (a) (i) State and prove commutative property and distributive property of convolution in discrete time domain.

(ii) Sketch the spectrum of modulated signal $y(t) = x(t) \cdot m(t)$, if

1. $X(f) = \begin{cases} 1-|f| \ ; & |f| \le 1 \\ 0 \ ; & \text{otherwise} \end{cases}$ and $m(t) = \cos 15\pi t$.

2. $X(f) = \text{rect}\,(0.25 f)$ and $m(t) = \cos 2\pi t$.

3. $X(f) = \text{rect}\,(f)$ and $m(t) = \cos \pi t$.

[6 + 6 marks]

5 (a) (i) In discrete time domain,

commutative property :

$[x_1(n) * x_2(n)] * x_3(n) = x_1(n) * [x_2(n) * x_3(n)]$

let $x_1(n) = \delta(n)$

$x_2(n) = \delta(n+2)$

$x_3(n) = \delta(n+3)$.

3

LHS, $[\delta(n+2)] * \delta(n+3) = \delta(n) * [\delta(n+5)]$

$\because [\delta(n) * \delta(n+n_0) = \delta(n+n_0)]$.

$\boxed{\delta(n+5) = \delta(n+5)}$

Distributive property :

$[x_1(n) + \cancel{x_2(n)}] * x_3(n) = [x_3 + \cancel{x_3(n)}] * x(n)$

$[\delta(n) + \cancel{\delta(n+2)}] * \delta(n+3) = [\delta(n+2) + \cancel{\delta(n+3)}] * \delta(n)$

$\delta(n+3) + \cancel{\delta(n+5)} = \delta(n+3)$

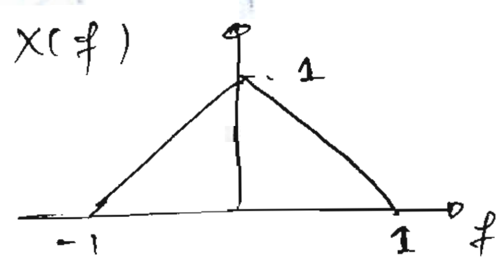
$\therefore \quad x_1(n) * x_2(n) = x_2(n) * x_1(n)$

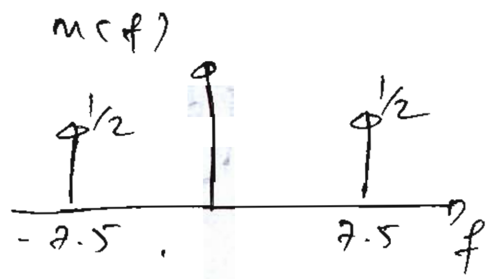
Hence Proved.

(Q5) (a)(ii) $y(t) = x(t) \cdot m(t)$

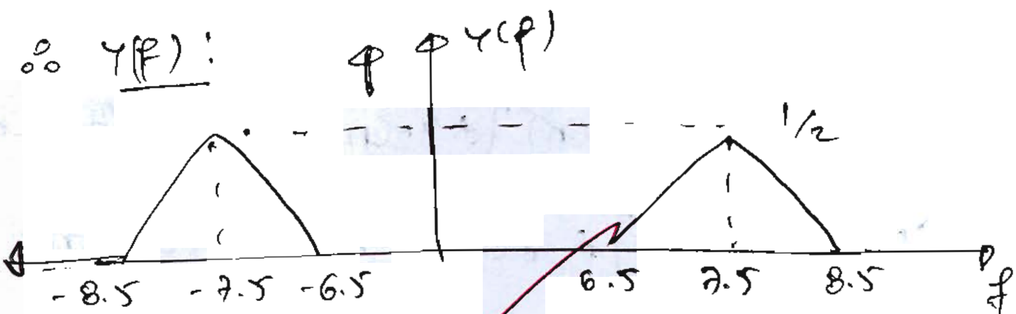
Taking Fourier Transform,

$$Y(f) = X(f) \cdot M(f)$$

1. 

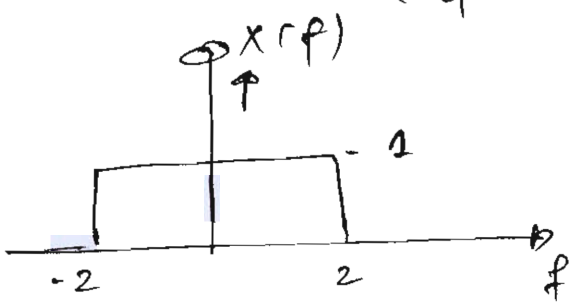


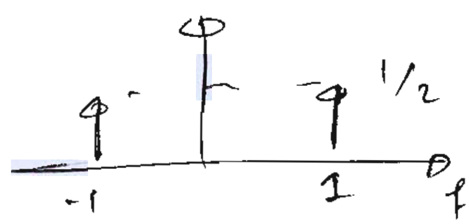
$\therefore$ Y(f) :

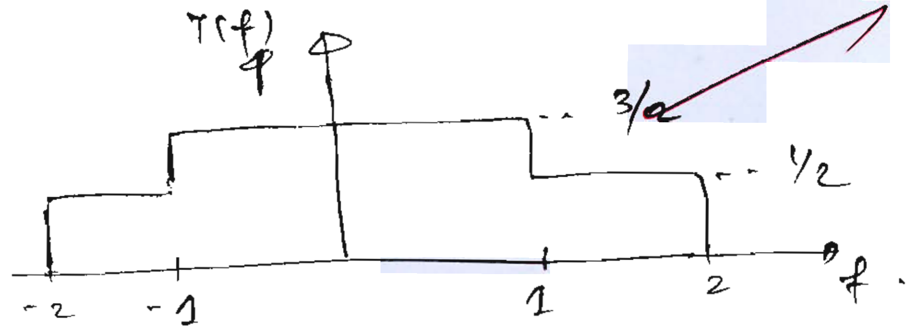


(2) $X(f) = rect(0.25f)$. $m(t) = \cos 2\pi f$

$\Rightarrow 1 \cdot rect\left(\frac{f}{4}\right)$.

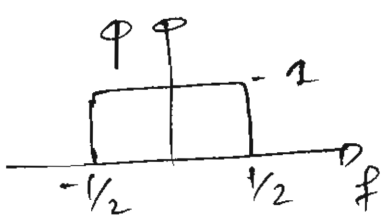


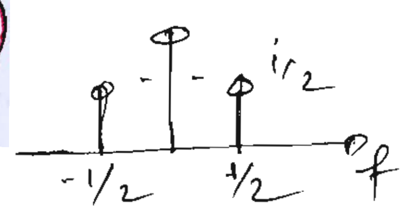


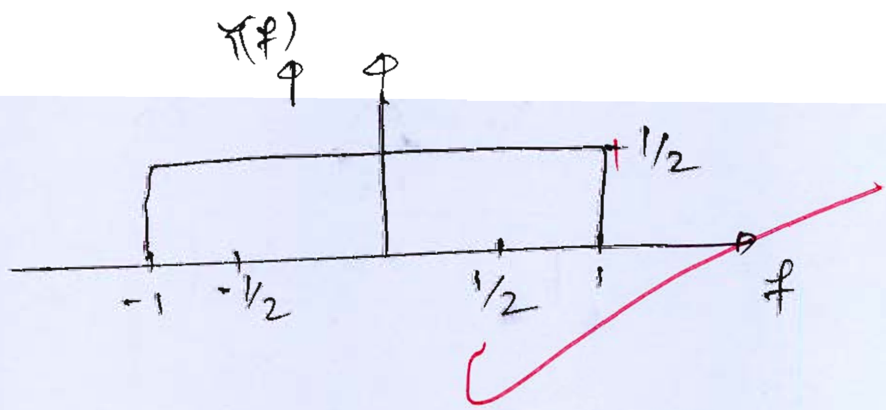


(3) $X(f) = rect(f)$ $m(t) = \cos \pi f$.

5







Q.5 (b) The complex exponential Fourier series representation of a signal $f(t)$ over the interval $(0, T)$ is

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(n\pi)^2} e^{jn\pi t}$$

(i) What is the numerical value of T?

(ii) If one of the components of $f(t)$ is $A\cos 3\pi t$, determine the value of A.

(iii) Determine the power contained by the signal $f(t)$ upto the first four harmonics as a percentage of total power of signal.

Note: $\sum_{n=-\infty}^{\infty} \left|\frac{3}{4+(n\pi)^2}\right| \approx 0.669$

[4 + 4 + 4 marks]

Q5 (b) Given : $f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(n\pi)^2} e^{jn\pi t}$. — ①

Statement : from fourier series complex exponential representation.

$f(t) = \sum_{n=-\infty}^{\infty} C_n . e^{jn\omega_0 t}$. — ②

on comparison, ① & ② .

(i) $\omega_0 = \pi$.

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi}$

$\boxed{T_0 = 2 \text{ seconds}}$

(ii) Given one component $A\cos 3\pi t$;

$3(\pi) = 3\omega_0$

Hence represents 3rd component . ∴ $\boxed{n = 3}$

$\therefore$ $C_n = C_{-n}$. { from above analysis } .

$\therefore$ $a_n = C_n + C_{-n} = 2C_n$.

$$a_3 = A = 2C_3 = \frac{2 \times 3}{4 + (3\pi)^2} .$$

$\therefore$ $\boxed{A = 0.0646}$

(iii) Total power as per parseval's power theorem would be :

$$P = \frac{1}{T} \int_T |x(t)|^2 . dt = \sum_{n=-\infty}^{\infty} |C_n|^2 .$$

8

from note, $P = [0.669]^2$.

$\boxed{P_T = 0.4475}$ watts .

Power from 1st (4) harmonics ,

$$P_K = \sum_{n=1}^{4} |C_n|^2 = C_1^2 + C_2^2 + C_3^2 + C_4^2$$

$$P_K = \left[\frac{3}{4+\pi^2}\right]^2 + \left[\frac{3}{4+(2\pi)^2}\right]^2 + \left[\frac{3}{4+(3\pi)^2}\right]^2 + \left[\frac{3}{4+(4\pi)^2}\right]^2 .$$

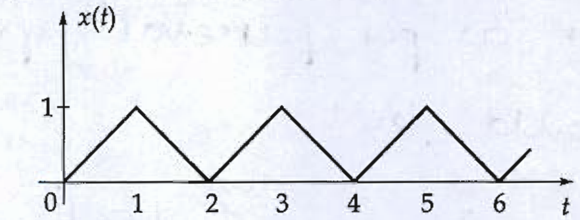
$P_K = 0.0467 + 4.76 \times 10^{-3} + \cdots \approx 0$.

$$\% = \frac{P_k}{P_T} \times 100$$

$$= 0.1149$$

$$\simeq 11.50\ \%.$$

Q.5 (c) (i) Find the Laplace transform of the triangular wave shown in figure.



(ii) Determine whether the signal below is periodic or not and if periodic, determine the fundamental period of the signal.

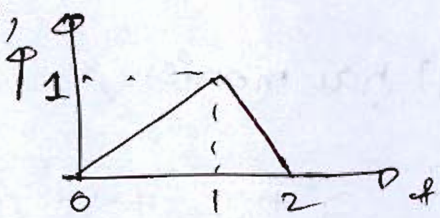
$$x(n) = \text{Re}\left[e^{jn\pi/12}\right] + \text{Im}\left[e^{jn\pi/18}\right]$$

[8 + 4 marks]

Q5 (c) (i)

Given: periodic triangular waveform

let $g(t)$,



$$g(t) = r(t) - r(t-1) - r(t-1) + r(t-2)$$

$$g(t) = r(t) - 2r(t-1) + r(t-2) \quad \text{(1)}$$

Taking laplace of $g(t)$,

$$G(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2}. \quad \text{(2)}$$

we can write f(t) in terms of g(t) as,

$$f(t) = g(t) + g(t-2) + g(t-4) + \dots$$

Laplace,

$$F(s) = G(s) + e^{-2s} G(s) + e^{-4s} G(s) + \dots$$

$$\therefore F(s) = \frac{G(s)}{1-e^{-2s}} \quad \text{--- (3)}$$

5

Substituting (2) in (3),

$$\boxed{F(s) = \frac{1-2e^{-s}+e^{-2s}}{s^2[1-e^{-2s}]}}$$ Ans.

5) (c)(ii)

Given: $x(t) = Re[e^{jn\pi/12}] + Img[e^{jn\pi/18}]$

$$e^{jn\pi/12} = \sin\frac{n\pi}{12} + j\cos\frac{n\pi}{12}$$

& Real part

$$= \sin\frac{n\pi}{12}.$$

4

$$\therefore x(t) = \sin\frac{n\pi}{12} + \cos\frac{n\pi}{18}.$$

$$\omega_1 = \frac{\pi}{12}. \qquad \omega_2 = \frac{\pi}{18}.$$

$$N_1 = \frac{2\pi}{\omega_1}.k \quad [K: \text{an Integer}].$$

$$N_1 = \frac{2\cancel{\pi}}{\cancel{\pi}} \times 12 \;.k \quad [\text{let } K=1].$$

$\boxed{N_1 = 24}$.

$N_2 = \frac{2\cancel{\pi}}{\cancel{\pi}} \times 18.k \quad [k=1]$

$\boxed{N_2 = 36}$

fundamental time period

$N_0 = \text{lcm}[N_1, N_2]$

$= \text{lcm}[24, 36]$.

$\boxed{N_0 = 12 \text{ sec}.}$

72 sec

Q.5 (d) Compute the circular convolution of the following sequences using DFT and IDFT:

$x_1(n) = \{1, 2, 1, 2\}$ and $x_2(n) = \{4, 3, 2, 1\}$

[12 marks]

Q5 (d).

$y(n) = x_1(n) \circledast x_2(n)$.

[$\circledast$: circular convolution operator]

In time domain,

$$y(n) = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} ..$$

$$y(n) = \begin{bmatrix} 4+6+2+2 \\ 8+3+4+1 \\ 4+6+2+2 \\ 8+3+4+1 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

from DFT property .

$$y(n) = x_1(n) \circledast x_2(n) \Longrightarrow X_1(k) \cdot X_2(k) .$$

$X_1(k)$:

$$X_1(k) = \frac{1}{N} [W_N]^* [x] .$$

$$X_1(k) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} .$$

$$X_1(k) = \frac{1}{4} \begin{bmatrix} 1 + 2 + 1 + 2 \\ 0 \\ 1 - 2 + 1 - 2 \\ 1 - 2j - 1 + 2j \end{bmatrix} = \begin{bmatrix} 3/2 \\ 0 \\ -1/2 \\ 0 \end{bmatrix} .$$

12

$X_2(k)$:

$$X_2(k) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$X_2(k) = \frac{1}{4} \begin{bmatrix} 4 + 3 + 2 + 1 \\ 4 + 3j - 2 - j \\ 4 - 3 + 2 - 1 \\ 4 - 3j - 2 + j \end{bmatrix} = \begin{bmatrix} 5/2 \\ 1/2 + j/2 \\ 1/2 \\ 1/2 - j/2 \end{bmatrix} .$$

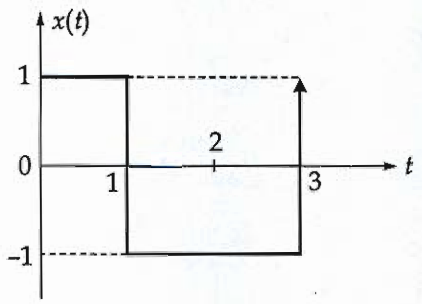
$$\therefore X_1(k) \cdot X_2(k) = Y(k)$$

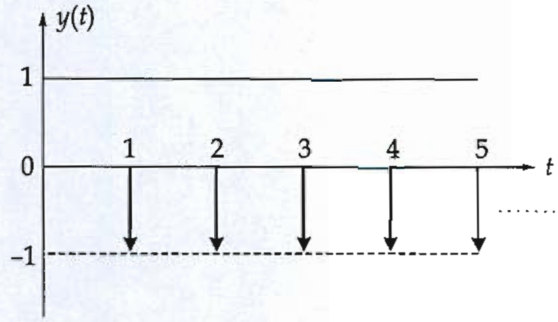
on solving, $Y(K) \xRightarrow{IDFT} y(n)$

$$y(n) = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix} \text{ ans.}$$

Q.5 (e) Derive the impulse response $h_d(n)$ of a highpass filter to meet the following specifications:
Cutoff frequency = 250 Hz
Sampling frequency f_s = 1 kHz and
Filter length = 7 **[12 marks]**

Q.6 (a) (i) For the signals $x(t)$ and $y(t)$ given below, determine and sketch $\int_{-\infty}^{t} x(\tau)d\tau$ and $\int_{-\infty}^{t} y(\tau)d\tau$:



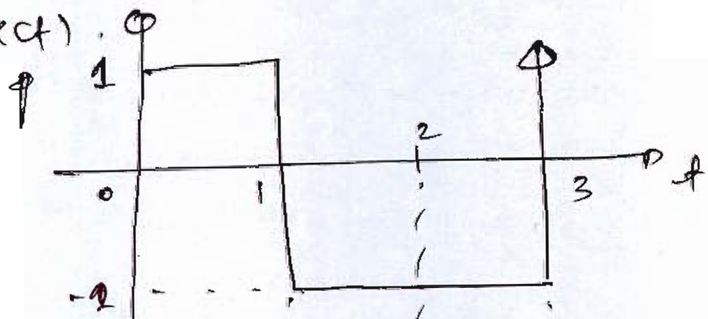


(ii) For the second order FIR lattice filter with reflection coefficients $K_1 = \frac{1}{2}, K_2 = \frac{1}{4}$, draw the FIR lattice structure and find the transfer function and impulse response of the FIR system.

[8 + 12 marks]

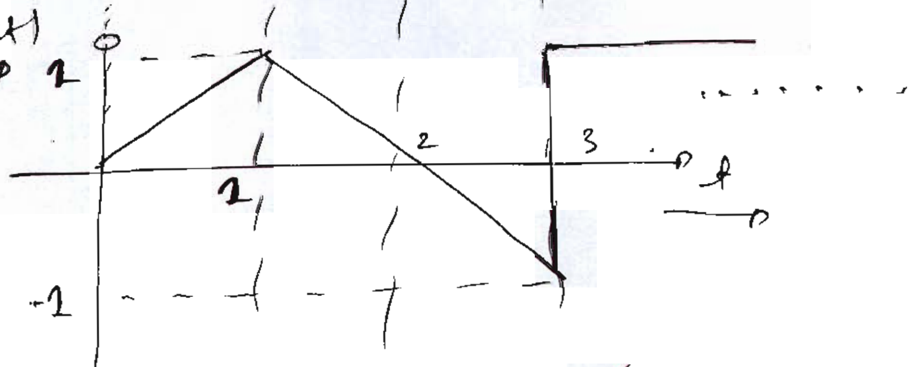
Q6 (a)(i)

Given $x(t)$.



let $p(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$.

$p(t)$



Given $y(t) = u(t) + \sum_{K=1}^{\infty}[-\delta(t-K)]$

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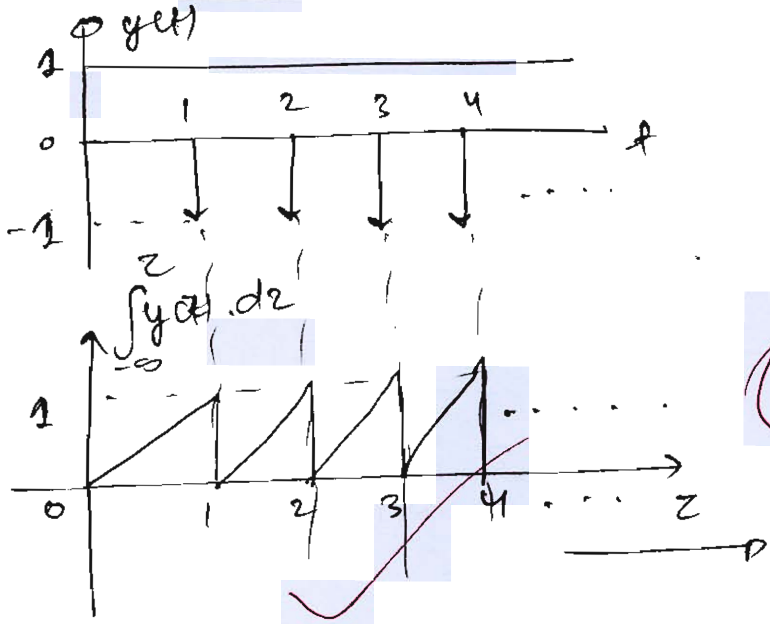

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(a) (iii) Given $k_1 = 1/2$, $k_2 = 1/4$.

from relation, $A_m(z) = A_{m-1}(z) + k_m z^{-1} B_{m-1}(z)$

$$A_1(z) = \cancel{A_0(z)} + k_1 z^{-1} \cancel{B_0(z)}$$

$$A_1(z) = 1 + \frac{1}{2} z^{-1}$$

$$\therefore B_1(z) = \frac{1}{2} + 1.z^{-1}$$

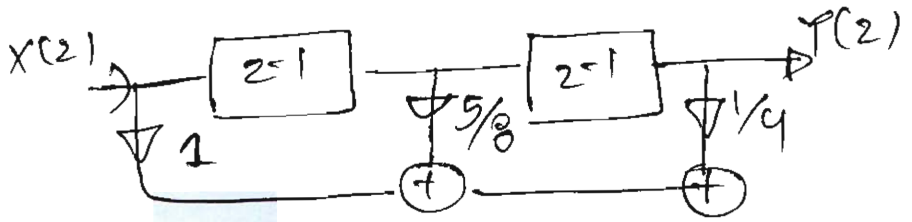
$$A_2(z) = A_1(z) + k_2 z^{-1} B_2(z)$$

$$A_2(z) = 1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-1} \left[\frac{1}{2} + z^{-1}\right].$$

$$A_2(z) = 1 + \frac{1}{2} z^{-1} + \frac{1}{8} z^{-1} + \frac{1}{4} z^{-2}.$$

$$\boxed{A_2(z) = 1 + \frac{5}{8} z^{-1} + \frac{1}{4} z^{-2}}$$

FIR structure :



$$\boxed{H(z) = 1 + \frac{5}{8} z^{-1} + \frac{1}{4} z^{-2}}$$

$$\frac{Y(z)}{X(z)} = 1 + \frac{5}{8} z^{-1} + \frac{1}{4} z^{-2}$$

$$\boxed{y(n) = x(n) + \frac{5}{8} x(n-1) + \frac{1}{4} x(n-2)}$$

difference equation.

$$\boxed{h(n) = 1 + \frac{5}{8} \delta(n-1) + \frac{1}{4} \delta(n-2)}$$

Impulse response.

12

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b) (i) Determine the time domain signals corresponding to the bilateral Laplace transforms given below. Specify the properties used:

1. $X(s) = \frac{1}{s^2} \cdot \frac{d}{ds}\left(\frac{e^{-3s}}{s}\right)$, ROC: $\text{Re}(s) > 0$

2. $X(s) = s\left(\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}\right)$, ROC: $\text{Re}(s) > 0$

(ii) The impulse response of a relaxed linear time invariant system is $h(n) = \alpha^n u(n)$ with $|\alpha| < 1$. Determine the value of the step response as $n \to \infty$.

[5 + 5 + 10 marks]

(b) (i)

1. $X(s) = \frac{1}{s^2} \frac{d}{ds}\left[\frac{e^{-3s}}{s}\right]$; $\sigma > 0$

$= \frac{1}{s^2} \frac{d}{ds}\left[\frac{e^{-3s}}{s}\right] = \frac{1}{s^2}\left[\frac{-3e^{-3s} \cdot s - e^{-3s} \cdot 1}{s^2}\right]$

$X(s) \Rightarrow \frac{-3\cancel{s}e^{-3s}}{s^{\cancel{2}}} - \frac{e^{-3s}}{s^2}$

Taking IFT,

$\boxed{x(t) = -3u(t-3) - r(t-3)}$

$\because \left[\frac{1}{s^2} \xRightarrow{IFT} r(t) \quad \frac{1}{s} \xRightarrow{IFT} u(t)\right]$

See the solution

(ii) $X(s) = \frac{1}{s} - \frac{e^{-s}}{s} - e^{-2s}$. $\sigma > 0$.

after simplification,

2

$\therefore \boxed{x(t) = u(t) - u(t-1) - \delta(t-2)}$

$\left[\because \text{from time shifting property}, \quad x(t-t_0) \rightleftharpoons e^{-st_0} \cdot X(s)\right]$

Q6 (b) (ii) given $h(n) = \alpha^n u(n)$. $|\alpha| < 1$.

Taking Z-T/F of above function,

$\therefore H(z) = \frac{1}{1-\alpha z^{-1}}$.

given $X(z) = \frac{1}{1-z^{-1}}$ $\left[\therefore u(n) \overset{Z.T.}{\rightleftharpoons} \frac{1}{1-z^{-1}}\right]$

$\therefore Y(z) = X(z) \cdot H(z)$

5

$Y(z) = \frac{1}{[1-\alpha z^{-1}]} \cdot \frac{1}{[1-z^{-1}]}$.

$Y(z) = \frac{\cancel{A}\ 1/1-\frac{1}{\alpha}}{1-\alpha z^{-1}} + \frac{\cancel{B}\ 1/1-\alpha}{1-z^{-1}}$.

$\left[\therefore Y(z) \overset{I.Z.T}{\rightleftharpoons} y(n)\right]$.

See the solution

Hence $y(n) = \frac{1}{\left[1-\frac{1}{\alpha}\right]} \cdot \alpha^n . u(n) + \frac{1}{1-\alpha} . u(n)$

$y(n) = \frac{\alpha}{\alpha-1} . \alpha^n u(n) + \frac{1}{1-\alpha} u(n)$.

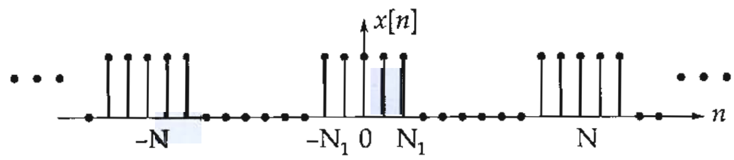
$$\therefore \boxed{y(n) = \frac{\alpha^{n+1} u(n)}{\alpha - 1} + \frac{1}{1-\alpha} u(n)}$$

Hence above is desired step response.

6 (c) (i) Let $z = re^{j\omega}$ and $s = \sigma + j\Omega$. Use bilinear transformation to show that if $r < 1$, then $\sigma < 0$, and if $r > 1$, then $\sigma > 0$, and when $r = 1$, then $\sigma = 0$.

(ii) Consider a signal $x[n]$ such that

$$x[n] = \begin{cases} 1 & \text{for } -N_1 < n < N_1 \\ 0 & \text{for rest of time period} \end{cases}$$



Discrete-time periodic square wave

If time period of $x[n]$ is N where ($N > 2N_1 + 1$), then determine Fourier series coefficient a_k for signal $x[n]$.

[10 + 10 marks]

6 (c) (i) $z = re^{j\omega}$.

$s = \sigma + j\Omega$.

By property of B.L.T,

$$H(s) \Longrightarrow H(z)$$

when $s = \frac{2}{T_s}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$.

or $s = \frac{2}{T_s}\left[\frac{z-1}{z+1}\right]$.

2

Do wri thi

we know, $z = e^{sT_s}$ ..

$$re^{j\omega} = e^{\frac{2}{\cancel{T_s}} \times \cancel{T_s}\left[\frac{z-1}{z+1}\right]}.$$

$$re^{j\omega} = e^{2[z-1]/[z+1]}.$$

see the solution.

Q 6 (c) (ii).

$$x(n) \rightleftharpoons a_k$$

D+P D+P

we know, $\left\{ a_k = \sum_{n=0}^{N} x(n).\, e^{-j\omega_0 n.k} \right\}$.

1

$$\omega_0 = \frac{2\pi}{N_0}.$$

$$\therefore a_k = \sum_{n=-N_1}^{N_1} (1)\, e^{-jk\omega_0 n}.$$

$a_k =$

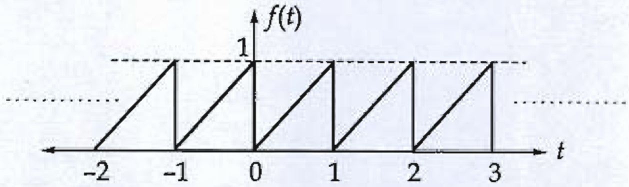
incomplete.

Q.7 (a) **(i)** Determine the signal $x[n]$ and rational z-transform $X(z)$ for the following cases:

1. $x[n]$ is right sided, $X(z)$ has a single pole, $x[0] = 4$, $x[2] = \frac{1}{4}$.

2. $X[z]$ has poles at $z = \frac{1}{4}$ and $z = -1$, ROC includes $|z| = \frac{1}{2}$, $x[1] = 1$, $x[-1] = 1$.

(ii) **1.** Obtain exponential Fourier series representation of the periodic signal shown below.



2. Draw the magnitude and phase plot of the Fourier series coefficient of the above signal.

[10 + 10 marks]

Q.7 (b) (i) Suppose we are given the following facts about an LTI system S with impulse response $h[n]$ and frequency response $H(e^{j\omega})$:

1. $\left(\frac{1}{4}\right)^n u[n] \xrightarrow{S} g[n]$, where $g[n] = 0$ for $n \geq 2$ and $n < 0$.
2. $\text{Re}\{H(e^{j\pi/2})\} = 1$.
3. $H(e^{j\omega}) = H(e^{j(\omega - \pi)})$.

Determine $h[n]$.

(ii) 1. Mention any five properties of ROC of Laplace transform.

2. Find inverse LT of $H(s) = \ln\left(\frac{1}{3s+2}\right)$.

[12 + 8 marks]

c) (i) Consider the 6-length sequence defined for $0 \le n < 6$ as $x(n) = \{\underset{\uparrow}{1}, -2, 3, 0, -1, 1\}$ with a 8-point DFT $X(k)$. Evaluate the following functions of $X(k)$ without computing DFT:

1. $X(0)$
2. $X(3)$
3. $\sum_{k=0}^{5} X(k)$
4. $\sum_{k=0}^{5} |X(k)|^2$

(ii) Suppose the following facts are given about the signal $x(t)$ with Laplace transform $X(s)$:

1. $x(t)$ is real and even.
2. $X(s)$ has four poles and no zeros in the finite s-plane.
3. $X(s)$ has a pole at $s = \frac{1}{2}e^{j\frac{\pi}{4}}$.
4. $\int_{-\infty}^{\infty} x(t)dt = 4$

Determine $X(s)$.

[10 + 10 marks]

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Q.8 (a) Using DIT FFT and inverse DIT FFT, determine the output of the system if input $x(n)$ and impulse response $h(n)$ are given as $x(n) = \{2, 2, 4\}$ and $h(n) = \{1, 1\}$.

[20 marks]

)) A discrete time system is described by the difference equation:

$$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)+x(n-1)$$

with $y(-1) = 0$ and $y(-2) = -1$.

Find:

(i) The zero input response of the system.

(ii) The zero state response of the system due to step input $u(n)$.

[10 + 10 marks]

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) Obtain the direct form-I and direct form-II realizations of the LTI system governed by the equation

$$y(n) = -\frac{13}{12}y(n-1) - \frac{9}{24}y(n-2) - \frac{1}{24}y(n-3) + x(n) + 4x(n-1) + 3x(n-2)$$

[20 marks]

OOOO

Space for Rough Work

Space for Rough Work

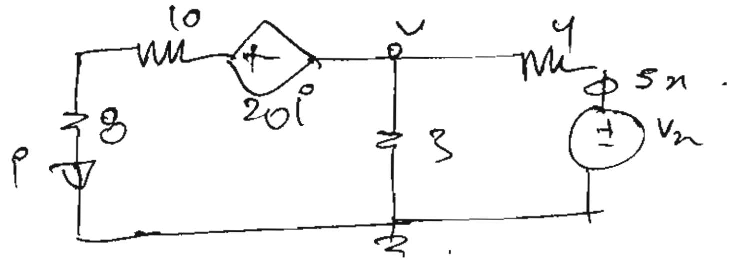
$$P = \frac{V_{th}^2}{4R_L} = \frac{V_{th}^2}{4 \times R_{th}}$$

$$= \frac{(I_{sc} \times R_{th})^2}{4R_{th}}$$

$$= \frac{I_{sc}^2 \times R_{th}^2}{4 \times \cancel{R_{th}}} = \frac{I_{sc}^2 \times R_{th}}{4}$$

Space for Rough Work

$A_n(z) = A_{n-1}(z) + K_n z^{-1} B_{n-1}(z)$.



$i + \frac{V}{3} = 5x$ — ①.

$\frac{V_x - V}{4} = 5x$ — ②.

$-V - 20i + 18i = 0$.

$V = -2i$ — ③.

$V_x - V = 4 I_x$.

$V_x - 4 I_x = V$.

$i - \frac{2}{3} i = 5x$.

$\boxed{\frac{1}{3} i = 5x}$.

$V_x + 2i = 45x$.

$V_x + \frac{2}{3} I_x = 45x$.

$V_x = \left(4 - \frac{2}{3}\right) 5x$

$V_x = \frac{(12 - 2)}{3} 5x$

$\left\{ \begin{array}{l} V_1 = A \\ V_3 = B \\ V_4 = C \end{array} \right\}$

$-1.2A + 0B + 1.C = 2.4$.

$0.A + 5B - C = 52$.

$14A - 10B + 5C = 12$

$V = 1 \times 2$.

$12^2 \times R_2 = 10$.

$\boxed{V}$

v_1

v_2

$I_A = \frac{I \times (R_1 + jX_1)}{R_1 + jX_1 - j20}$.

?

Space for Rough Work