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ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-1 : Network Theory + Signals and Systems [All Topics]

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Hyderabad ☐		

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	47
Q.2	—
Q.3	44
Q.4	20
Section-B	
Q.5	30
Q.6	
Q.7	24+1
Q.8	
Total Marks Obtained	166

Signature of Evaluator 25/03/26

Cross Checked by ....

• Your accuracy is good .. try to attempt more questions.

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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Section A : Network Theory

(a) Briefly explain sources used in network (independent and dependent sources) and their types.

[12 marks]

In → Sources used in the electrical networks are mainly of two types :

① Dependent sources ② Independent sources

Explaining the above two,

11

Good

① Dependent sources ⇒

→ Dependent sources are the sources which depend upon other branch current or terminal voltage.

→ Then don't have their own power dissipation or absorption but depends on other independent sources in the network.

→ while solving problems in case of dependent sources we can't open circuit or short circuit them for disabling.

→ A circuit having no independent source and have only dependent source have thevenin's equivalent voltage equals to zero.

→ In case of finding equivalent Resistance in a network we can't short circuit or open circuit them.

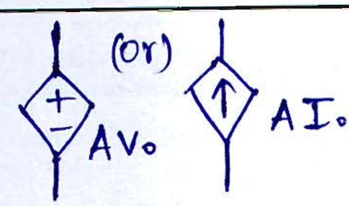
→ Types of depend sources are

(i) Current control current source.

(ii) Voltage control current source.

(iii) Voltage control voltage source.

(iv) ~~Voltage~~ current control voltage source.

→ They are reprensented by 

where V_o and I_o are independent branch current or voltages.

② <u>Independent sources</u> →

→ They can dissipate or absorb power in the circuit

→ They are independent of any branch current or voltage across any element.

→ While find equivalent resistance in a network they are disabled by replacing them with their internal resistances.

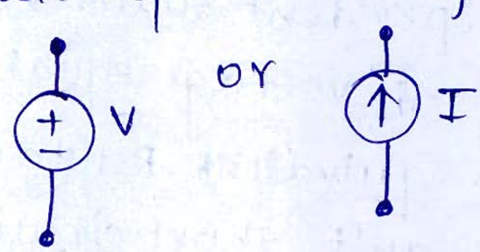
→ They are externally connected to the circuit.

→ They are of two types

(i) Voltage source.

(ii) current source.

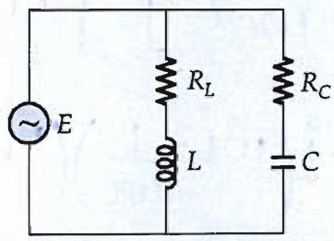
→ They are represented by



~~→ In care of Indepent s~~

(b) Draw the phasor diagram for the circuit shown and prove that the condition for resonance is

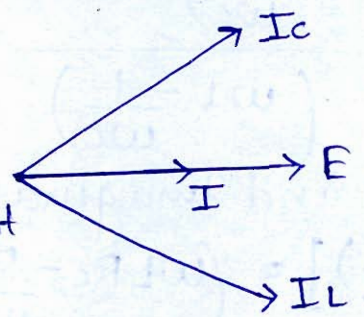
$$\frac{L}{C}=\frac{R_L^2+\omega^2L^2}{1+R_C^2\omega^2C^2}$$

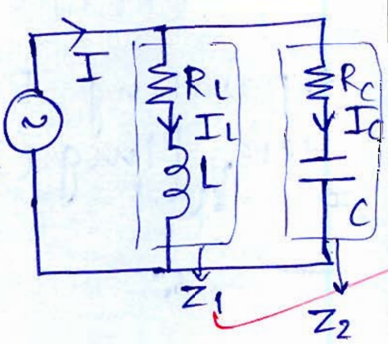


Also, calculate the value of ω (rad/sec) at resonance.

[12 marks]

Phasor ⇒





let 'I' be the current from independent source 'E' and

I_L be current from inductor

I_C be current from capacitor.

Condition for Resonance ⇒

Here, $Z_1 = [R_L + j\omega L]$ and $Z_2 = R_C + \frac{1}{j\omega C}$

$= \left[R_C - \frac{j}{\omega C}\right]$

$$Z_{eq} = \left[\frac{(Z_1)(Z_2)}{(Z_1+Z_2)}\right]$$

$$= \left[\frac{(R_L+j\omega L)\left[R_C - \frac{j}{\omega C}\right]}{(R_L+R_C)+j\left(\omega L - \frac{1}{\omega C}\right)}\right]$$

$$= \left[\frac{R_LR_C - j\frac{R_L}{\omega C} + j\omega L R_C + \frac{L}{C}}{(R_L+R_C)+j\left(\omega L - \frac{1}{\omega C}\right)}\right]$$

Separating Real & Imaginary terms

$$= \frac{\left(R_LR_C + \frac{L}{C}\right) + j\left(\omega L R_C - \frac{R_L}{\omega C}\right)}{(R_L+R_C) + j(\omega L - 1/\omega C)}$$

Rationalising ⇒

$$\left[\frac{\left(R_L R_C + \frac{L}{C}\right) + j\left(\omega L R_C - \frac{R_L}{\omega C}\right)}{(R_L + R_C) + j\left(\omega L - \frac{1}{\omega C}\right)}\right] \times \left[\frac{(R_L + R_C) - j\left(\omega L - \frac{1}{\omega C}\right)}{R_L + R_C - j\left(\omega L - \frac{1}{\omega C}\right)}\right]$$

$$= \frac{\left(R_L R_C + \frac{L}{C}\right)(R_L + R_C) - j\left(\omega L - \frac{1}{\omega C}\right)\left(R_L R_C + \frac{L}{C}\right) + j\left(\omega L R_C - \frac{R_L}{\omega C}\right)(R_L + R_C) + \left(\omega L R_C - \frac{R_L}{\omega C}\right)\left(\omega L - \frac{1}{\omega C}\right)}{(R_L + R_C)^2 + \left(\omega L - \frac{1}{\omega C}\right)}$$

Separating Real and Imaginary terms

Here, $[\text{Img}(Z_{eq})] = \left(\omega L R_C - \frac{R_L}{\omega C}\right)(R_L + R_C) - \left(\omega L - \frac{1}{\omega C}\right)\left(R_L R_C + \frac{L}{C}\right)$

For condition of Resonance

$$\text{Img}(Z_{eq}) = 0$$

$$\left(\omega L R_C - \frac{R_L}{\omega C}\right)(R_L + R_C) - \left(\omega L - \frac{1}{\omega C}\right)\left(R_L R_C + \frac{L}{C}\right) = 0$$

$$\left(\omega L R_C - \frac{R_L}{\omega C}\right)(R_L + R_C) = \left(\omega L - \frac{1}{\omega C}\right)\left(R_L R_C + \frac{L}{C}\right)$$

$$\omega L R_C R_L - \frac{R_L^2}{\omega C} + \omega L R_C^2 - \frac{R_L R_C}{\omega C} = \omega L R_L R_C + \frac{\omega L^2}{C} - \frac{R_L R_C}{\omega C} - \frac{L}{\omega C^2}$$

$$\omega L R_C^2 - \frac{R_L^2}{\omega C} = \frac{\omega L^2}{C} - \frac{L}{\omega C^2}$$

$$\therefore \frac{L}{C} = \left[\frac{R_L^2 + \omega^2 L^2}{1 + R_C^2 \omega^2 C^2}\right]$$

Here $L + L R_C^2 \omega^2 C^2 = R_L^2 C + \omega^2 L^2 C$

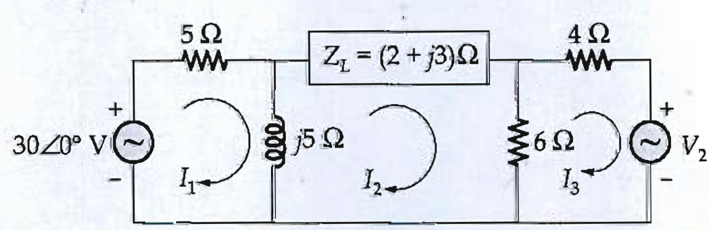
$$\omega^2 = \left[\frac{R_L^2 C - L}{L R_C^2 C^2 - L^2 C}\right] \Rightarrow \boxed{\omega = \sqrt{\frac{R_L^2 C - L}{L R_C^2 C^2 - L^2 C}}} \text{ rad/sec}$$

Do not write in this margin

(c) An iron-cored coil takes 4A at a power factor of 0.5 when connected to a 200-V, 50 Hz supply. When the iron core is removed and the voltage is reduced to 40 V, the current rises to 5 A at a pf of 0.8. Find the iron loss in the core and inductance in each case.

[12 marks]

Q.1 (d) In the network shown below, find the value of V_2 so that the current through $Z_L = (2 + j3)\ \Omega$ impedance is zero.



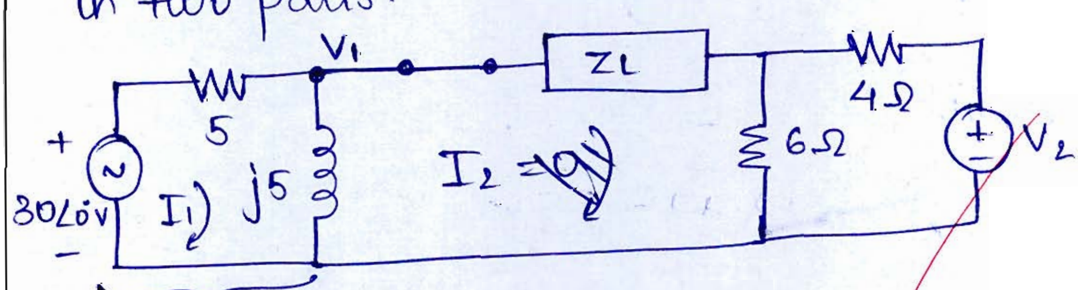
[12 marks]

Soln →

Here for current from Z_L to be zero

$I_2 = 0$ ⇒ current flowing from Z_L

This will separate the above given network in two parts.



KVL in I_1 loop ⇒

$$-30\angle 0^\circ + 5I_1 + j5(I_1 - I_2) = 0$$

$$I_1(5 + j5) - j5I_2 = 30\angle 0^\circ \quad \text{---(i)}$$

KVL in loop 2 ⇒

$$+(2+j3)I_2 + 6(I_2 - I_3) + j5(I_2 - I_1) = 0$$

$$-j5I_1 + (2+j3+6+j5)I_2 - 6I_3 = 0$$

$$-j5I_1 + (8+j8)I_2 - 6I_3 = 0 \quad \text{—(ii)}$$

KVL in loop 3 ⇒

$$4I_3 + V_2 + 6(I_3 - I_2) = 0$$

$$V_2 + 10I_3 - 6I_2 = 0 \quad \text{—(iii)}$$

Good

12

putting $I_2 = 0$ in eqn (i)

$$I_1 = \left[\frac{30\angle 0^\circ}{5+j5}\right] = \frac{30\angle 0^\circ}{7.07\angle 45^\circ}$$

$$I_1 = (4.24\angle -45^\circ)\,A$$

$$= (3 - j3)\,A \quad \text{—(iv)}$$

putting I_2 and I_1 value in (ii)

$$-j5(3-j3) + (8+j8)(0) - 6I_3 = 0$$

$$I_3 = \frac{-j5(3-j3)}{6}$$

$$I_3 = \left(\frac{-5}{2} - j\frac{5}{2}\right) \quad \text{—(iv)}$$

putting value of I_2 and I_3 in (iii)

$$V_2 + 10\left(-\frac{5}{2} - j\frac{5}{2}\right) - 6(0) = 0$$

$$V_2 - 25 - j25 = 0$$

$$V_2 = 25 + j25$$

$$\boxed{V_2 = 35.35\angle 45^\circ\ V}$$ Ans.

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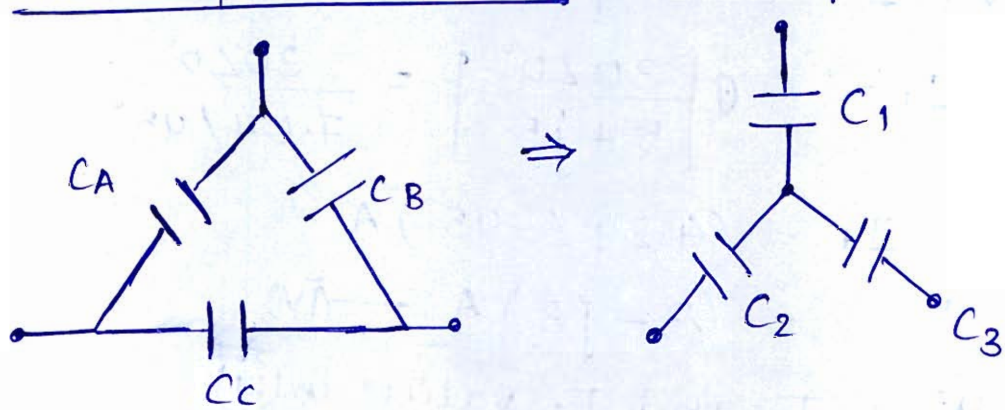
Q.1 (e) Represent a Delta connected capacitive circuit as an equivalent star connection and also express the star connected capacitive elements in terms of the delta connected capacitive elements.

[12 marks]

Soln, Delta capacitive circuit ⇒ Star equivalent



Here,

$$C_1 = C_A + C_B + \frac{C_A \cdot C_B}{C_C}$$

$$\boxed{C_1 = \frac{C_A C_C + C_B C_C + C_A C_B}{C_C}}$$

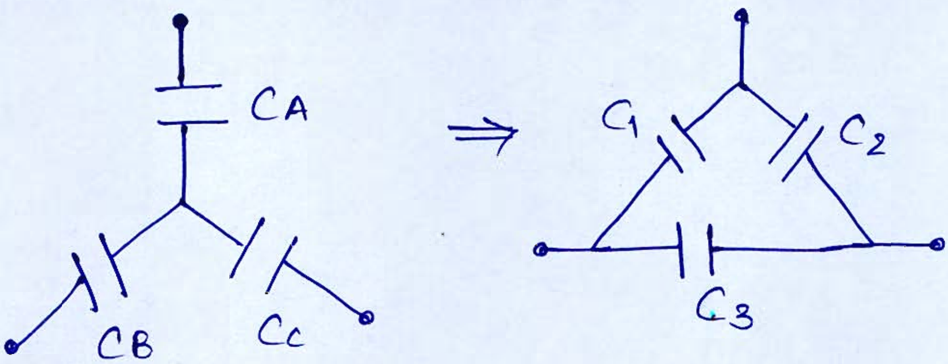
$$C_2 = C_A + C_C + \frac{C_A \cdot C_C}{C_B}$$

$$\boxed{C_2 = \frac{C_A \cdot C_B + C_B \cdot C_C + C_A C_C}{C_B}}$$

$$C_3 = C_B + C_C + \frac{C_B \cdot C_C}{C_A}$$

$$C_3 = \frac{C_A C_B + C_B \cdot C_C + C_C \cdot C_A}{C_A}$$

Star capacitive Network ⇒ Equivalent Delta



$$C_A = \frac{C_1 C_2}{C_1 + C_2 + C_3}$$

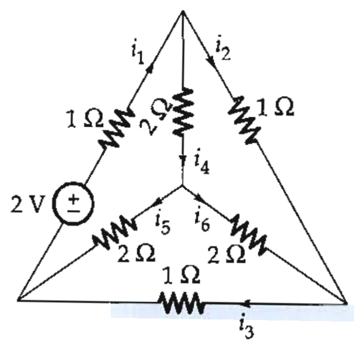
$$C_B = \frac{C_1 C_3}{C_1 + C_2 + C_3}$$

$$C_C = \frac{C_2 C_3}{C_1 + C_2 + C_3}$$

12 Good

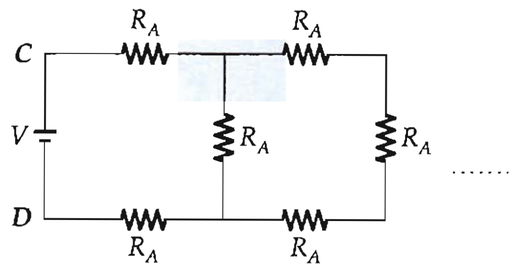
Do not write in this

Q.2 (a) For the network shown in figure below, write down the tieset matrix and obtain the network equilibrium equations in matrix form using KVL. Calculate the loop currents and branch currents.

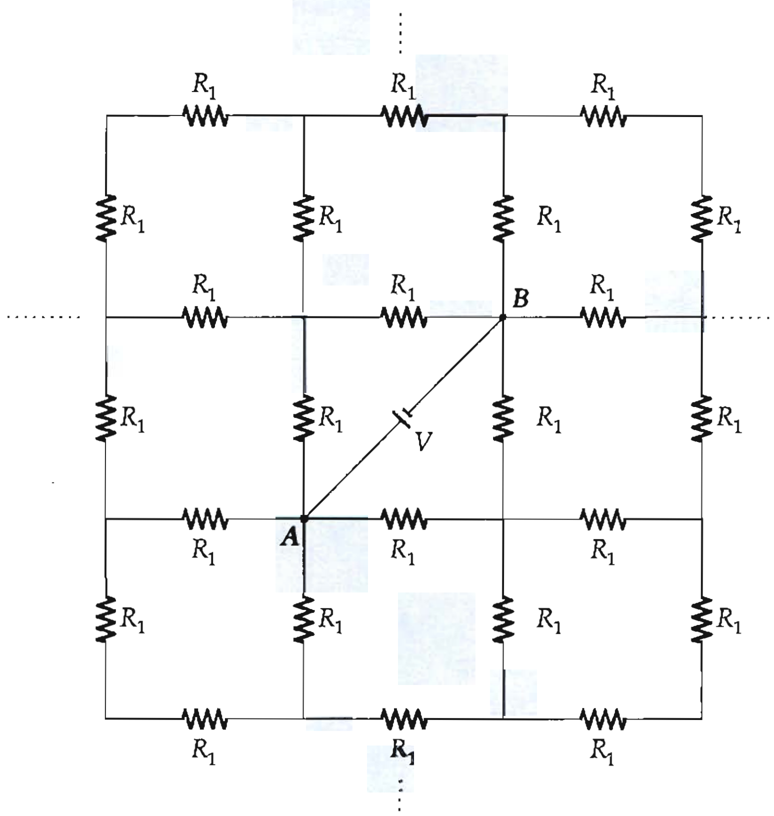


[20 marks]

(b) For the networks shown in figure below,



Network (i)

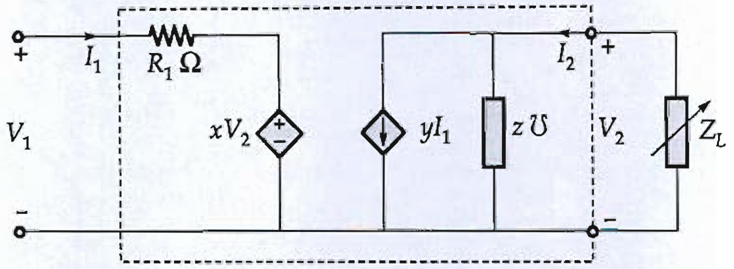


Network (ii)

On applying 10 V voltage source across terminal *CD* and *AB* respectively in network (i) and (ii), we get same power delivered by the source. Derive the relationship between the resistance present in network (i) and (ii). And also calculate the power delivered by the source for $R_A = 10\ \Omega$.

[20 marks]

Q.2 (c) Consider a two port network shown in figure below,



If transmission parameters matrix of the network is $\begin{bmatrix} 10^{-2} & 10^{2} \\ 0 & 10^{-1} \end{bmatrix}$.

Then, calculate:

(i) parameters of the circuit: R_1, x, y and z.

(ii) the value of load impedance (Z_L), for maximum power transfer.

(iii) maximum power transfer to load for $V_1 = 0.1$ volt.

[20 marks]

(a) (i) State 'Voltage to current source transformation' theorem. It is required to replace network N in figure (a) by a suitable equivalent network. Which of the networks of figure (b) could be valid equivalent network (s)?

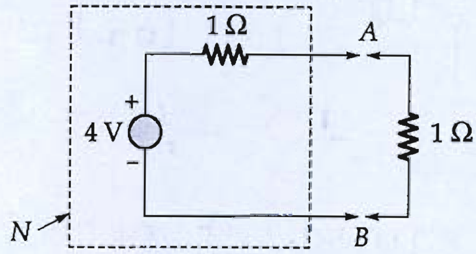


Fig. (a)

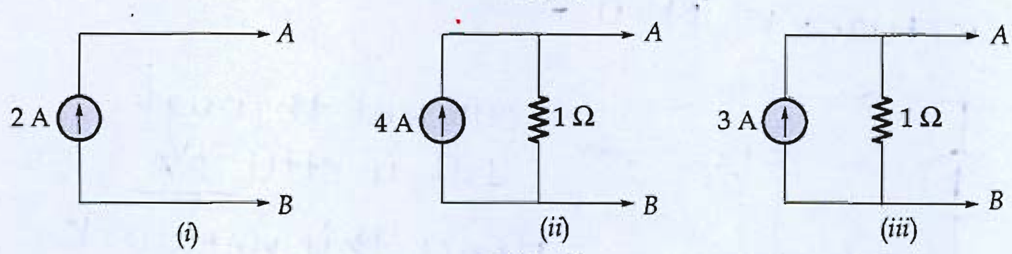
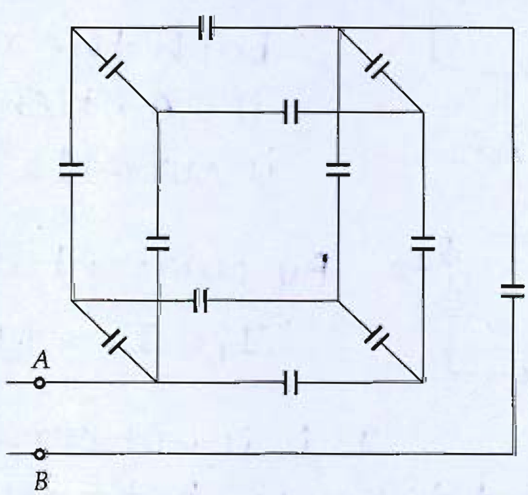


Fig. (b)

(ii) The network of capacitors in the figure below is composed of a 2F capacitor on each edge of a cube along with a capacitor of 2F connected to the vertices of the cube as shown. Find the $C_{equivalent}$ between the terminals A-B and also calculate the energy stored by the capacitive circuit if 5 V is applied across the terminal A-B.

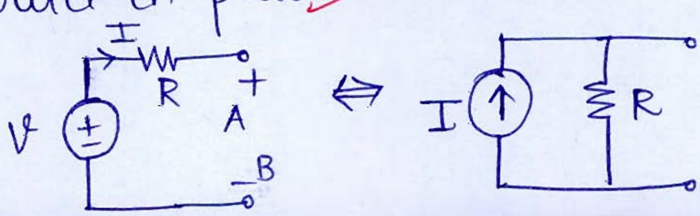


[10 + 10 marks]

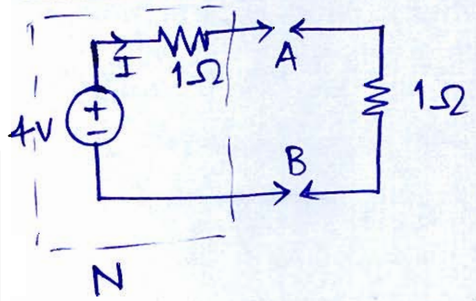
→ (i) Voltage to current source Transformation⇒

→ This theorem states that any network having a voltage source in series with a resistor can be transformed into a equivalent source in parallel with the Resistor. & vice-versa

10 Good



Given network ⇒

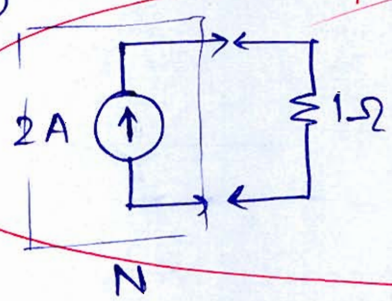


Since the current flowing in the network when 1Ω load is connected

$I = \frac{4}{2} = 2A$ → (current from both Resistor)

∴ Possible equivalent network in place of network 'N' is ⇒ *not required*

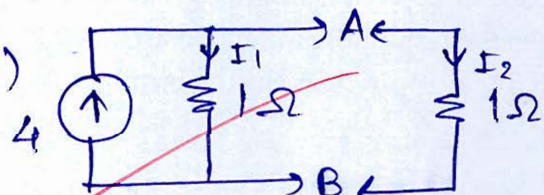
(i)



⇒ current through 1Ω is still 2A

Hence this network option is correct

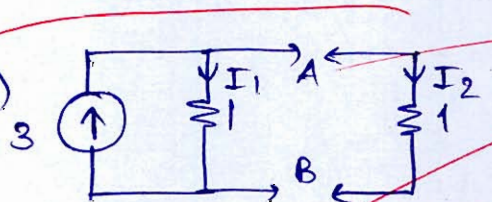
(ii)



$I_1 = I_2 = 2A.$

By current division rule current through both the resistor is 2A. Hence this option is correct

(iii)



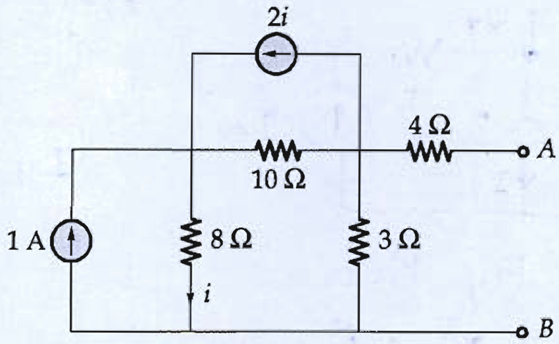
By current division

$I_1 = I_2 = 1.5$

This is not correct as current through both resistor should be 2A.

This option is not valid replacement for above network 'N'.

(b) For the circuit shown in figure below:



Calculate:

(i) Norton equivalent circuit across *A-B*.

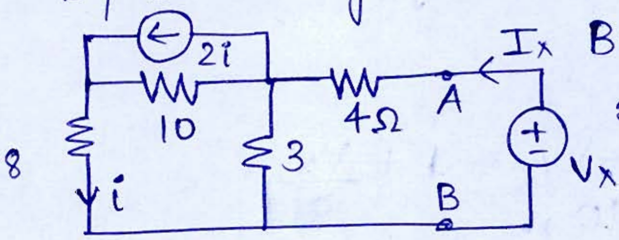
(ii) Load impedance across *AB* for Maximum Power Transfer.

(iii) Maximum Power Transfer to load, obtained in part (ii) and also comment on the result.

[20 marks]

(i) Norton's equivalent circuit across A-B

→ calculateq equivalent Norton's resistance by disabling all the independent sources ⇒

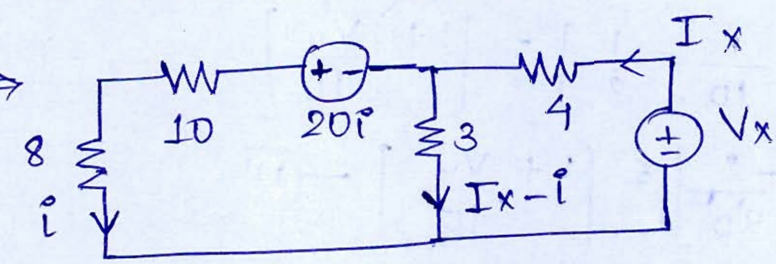


By voltage to current source transformation

Very Good

20

⇒



Applying KVL in above two loops ⇒

Ist loop ⇒ $-20i + 18i - 3(I_x - i) = 0$

$-20i + 18i - 3I_x + 3i = 0$

$i = 3I_x$ ——(i)

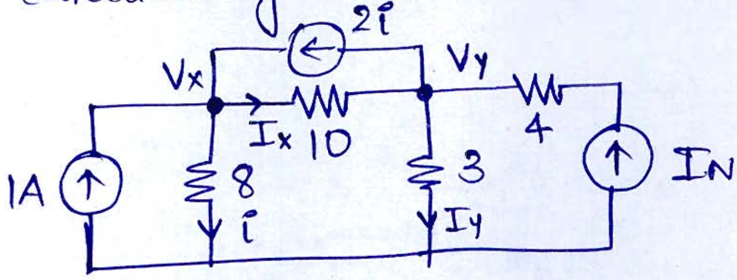
IInd loop ⇒ $-V_x + 4I_x + 3(I_x - i) = 0$

$-V_x + 7I_x - 3i = 0$

$V_x = -2I_x \Rightarrow \frac{V_x}{I_x} = R_n = -2$

$\boxed{R_n = -2\Omega}$

Calculating norton's current ⇒



Here $i = \frac{V_x}{8}$, $I_x = \frac{V_x - V_y}{10}$

$I_y = \frac{V_y}{3}$

At node Vx ⇒ By KCL

$$1 + 2i = I_x + i$$

$$[I_x = 1 + i] \text{ --- (i)}$$

At node Vy ⇒ By KCL

$$I_N + I_x = 2i + I_y$$

$$I_N + (1 + i) = 2i + I_y$$

$$[I_y = (I_N + 1 - i)] \text{ --- (ii)}$$

Here at node Vx ⇒

In eqn (i) $\frac{V_x - V_y}{10} = 1 + \frac{V_x}{8}$

$$V_x\left[\frac{1}{10} - \frac{1}{8}\right] = \left[1 + \frac{V_y}{10}\right]$$

$$V_x\left[\frac{-2}{80}\right] = \left[1 + \frac{V_y}{10}\right] \text{ --- (iii)}$$

At node Vy ⇒

$$\frac{V_y}{4} + \frac{V_y}{3} + \frac{V_y - V_x}{10} + 2i = 0$$

$$-\frac{V_x}{10} + \frac{V_x}{4} + V_y\left(\frac{1}{4} + \frac{1}{3} + \frac{1}{10}\right) = 0$$

$$\frac{3V_x}{20} + V_y\left(\frac{41}{60}\right) = 0$$

$$V_x = \left(\frac{-41}{9}\right)V_y \text{ --- (iv)}$$

put V_x in eqn (iii)

$$\left(\frac{-41}{9}\right)\left(\frac{-2}{80}\right) V_y = 1 + \frac{V_y}{10}$$

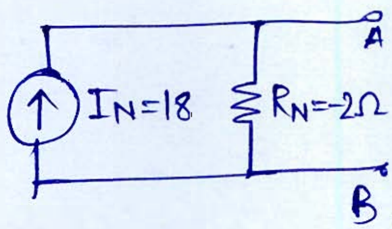
$$\left(\frac{41}{360}\right) V_y = 1 + \frac{V_y}{10}$$

$$\frac{V_y}{72} = 1$$

$$\boxed{V_y = 72}$$

Here $\boxed{I_N = \frac{V_y}{4} = 18\,A}$

Nortan's equivalent circuit ⇒

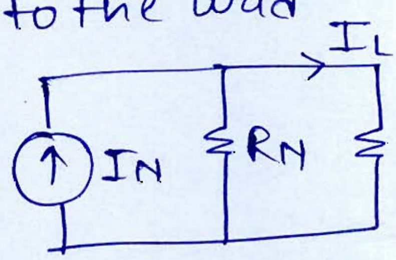


i) For maximum power transfer at load the value of load Resistance at load (R_L) should be equal to nortal equivalent Resistance

$$\boxed{R_L = R_N = -2\,\Omega}$$ from (i)

i) Maximum power transfer to the load

$$P_L = I_L^2 R_L$$



⇒ $I_L = \frac{I_N}{2}$ By current division rule as $R_N = R_L$

⇒ $\boxed{I_L = 9\,A}$

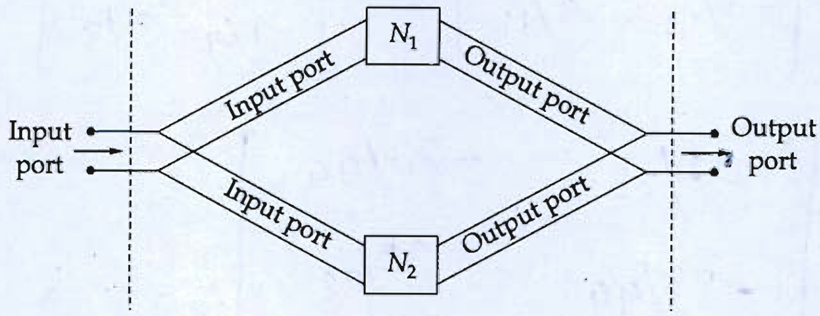
$$P_L = (9)^2(-2)$$

$$\boxed{P_L = -162\,W}$$

Power dissipated by the load is 162 W

Do not write in this margin

(c) Consider the two-port network 'N' given below:



N_1 and N_2 are two 2-port networks connected in parallel on both input port side as well as output port side, to form a composite 2-port network N as indicated.
N_1 and N_2 are defined by the z-parameters as below:

$$[Z_{n_1}] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix} \Omega, [Z_{n_2}] = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} \Omega$$

Obtain the transmission parameters for the composite 2-port network N. Also, express the transmission parameters in terms of short circuit parameters.

[20 marks]

Since, the above two port network 'N₁' and 'N₂' are in parallel, ∴ we can directly write that,

$$\rightarrow (Y_{eq})_N = [Y_{N1}] + [Y_{N2}]$$

Here $Z_{n1} = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix} \Omega$

Since $[Y_{N1}] = \frac{1}{[Z_{n1}]} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix}$

$$[Y_{N1}] = \begin{bmatrix} 5/11 & -3/11 \\ -3/11 & 4/11 \end{bmatrix} \mho \text{ --- (i)}$$

Similarly, $Z_{n2} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} \Omega$

$[Y_{n2}] = \frac{1}{[Z_{N2}]} = \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$

$$[Y_{N2}] = \begin{bmatrix} 1/2 & -1/4 \\ -1/4 & 3/8 \end{bmatrix} \mho \text{ --- (ii)}$$

$$(Y_N)_{eq} = \begin{bmatrix} 5/11 & -3/11 \\ -3/11 & 4/11 \end{bmatrix} + \begin{bmatrix} 1/2 & -1/4 \\ -1/4 & 3/8 \end{bmatrix}$$

$$(Y_N)_{eq} = \begin{bmatrix} 21/22 & -23/44 \\ -23/44 & 65/88 \end{bmatrix}$$

Since, writing above matrix in terms of equations ⇒

$$I_1 = \left(\frac{21}{22}\right)V_1 + \left(\frac{-23}{44}\right)V_2 \quad \text{—(ii)}$$

$$I_2 = \left(-\frac{23}{44}\right)V_1 + \left(\frac{65}{88}\right)V_2 \quad \text{—(iii)}$$

Here transmission parameters are given by

$$V_1 = AV_2 - BI_2 \quad \text{—(iv)}$$

$$V_2 = CV_2 - DI_2 \quad \text{—(V)}$$

In eqn (iii) by rearranging.

$$\left(\frac{23}{44}\right)V_1 = \left(\frac{65}{88}\right)V_2 - I_2$$

$$V_1 = \left(\frac{65}{46}\right)V_2 - \left(\frac{44}{23}\right)I_2 \quad \text{—(vi)}$$

put value of V_1 in (ii)

$$I_1 = \left(\frac{21}{22}\right)\left(\frac{65}{46}\right)V_2 - \left(\frac{21}{22}\right)\left(\frac{44}{23}\right)I_2 - \frac{23}{44}V_2$$

$$I_1 = ~~1.348V_2~~ \left(\frac{19}{23}\right)V_2 - \left(\frac{42}{23}\right)I_2 \quad \text{—(V)}$$

$\therefore \quad [T]_N = \begin{bmatrix} (65/46) & (44/23) \\ (19/23) & (42/23) \end{bmatrix}$ ← Ans

equivalent transmission parameter of the network.

→ Expression transmission parameter in terms of short circuit parameter ⇒

T-parameter	Short-circuit parameter
$V_1 = AV_2 - BI_2$ —(vi)	$I_1 = y_{11}V_1 + y_{12}V_2$ —(viii)
$I_1 = CV_2 - DI_2$ —(vii)	$I_2 = y_{21}V_1 + y_{22}V_2$ —(ix)

Rearrang eq^n (vi)

$$I_2 = \left(\frac{-1}{B}\right)V_1 + \left(\frac{A}{B}\right)V_2 \quad \text{—(x)}$$

Comparing above eq^n with (ix)

$(y_{21} = -1/B)$ and $(y_{22} = A/B)$

put I_2 of eq^n (x) in (vii)

$$I_1 = CV_2 - D\left(\frac{-V_1}{B} + \frac{A}{B}V_2\right)$$

$$= CV_2 + \frac{D}{B}V_1 - \frac{AD}{B}V_2$$

$$I_1 = \left(\frac{D}{B}\right)V_1 + V_2\left(C - \frac{AD}{B}\right) \quad \text{—(xi)}$$

Comparing above eqn with (viii)

$y_{11} = D/B$, $y_{12} = \left(\frac{CB - AD}{B}\right)$

Hence,

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} (D/B) & \left(\frac{CB-AD}{B}\right) \\ (-1/B) & (A/B) \end{bmatrix}$$

← Ans.

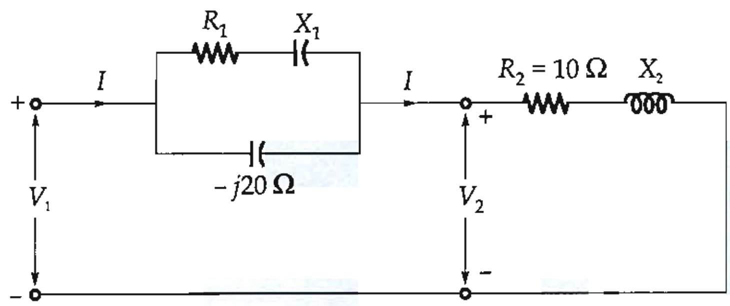
We have to write Transmission parame in terms of Y.

Q.4 (a) A series RC circuit has $R = 9\ k\Omega$, $C = 20\ \mu F$ and two voltage source in series given by $V_1 = 20u(-t)$ and $V_2 = 20u(t - t')$ V. Determine the complete expression for the voltage across the capacitor and plot it as a function of time, assuming t' as a positive quantity.

[20 marks]

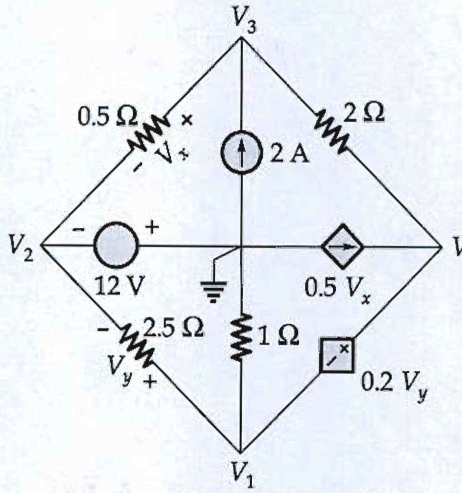
Do not write in this margin

(b) In the circuit shown in the figure below, $|V_1| = 200$ V, $V_2 = 200\angle 0°$ V and $|I| = 12$ A. The total power absorbed by the circuit is 1.8 kW. Find R_1, X_1 and X_2.



[20 marks]

Q.4 (c) For the circuit shown in figure, obtain the value of voltage across 0.5 Ω and 2.5 Ω resistors using nodal analysis.



[20 marks]

Sol^n → By supernode between V_2 and ground,

$V_2 - 0 = -12$

$\boxed{V_2 = -12\ V}$

Applying nodal analysis at Node V_3 ⇒

$$\left[\frac{V_3 - V_2}{0.5}\right] + \left[\frac{V_3 - V_4}{2}\right] = 2$$

$$2(V_3 + 12) + \left[\frac{V_3 - V_4}{2}\right] = 2$$

$$V_3(2.5) - \frac{V_4}{2} = -22 \quad \text{—(i)}$$

Apply ~~nodal~~ supernodal between V_4 and V_1 ⇒

$$V_4 - V_1 = 0.2 V_y \quad \text{—(ii)}$$

and
$$\frac{V_1}{1} + \frac{V_1 - V_2}{2.5} + \frac{V_4 - V_3}{2} = 0.5 V_x$$

$$V_1\left[\frac{7}{5}\right] - \left[\frac{2}{5}\right]V_2 + \frac{V_4}{2} - \frac{V_3}{2} = 0.5 V_x \quad \text{—(iii)}$$

Here, $V_x = V_3 - V_2$ —(iv)

put V_x in (iii)

$$\frac{7}{5}V_1 - \frac{2}{5}V_2 - \frac{V_3}{2} + \frac{V_4}{2} = \frac{V_3 - V_2}{2}$$

$$\frac{7}{5}V_1 - \frac{V_2}{10} - V_3 + \frac{V_4}{2} = 0$$

$$\frac{7}{5}V_1 + \frac{12}{10} - V_3 + \frac{V_4}{2} = 0$$

$$\frac{7}{5}V_1 - V_3 + \frac{V_4}{2} = \frac{-12}{10} \quad \text{—(v)}$$

In eqⁿ (ii)

$$V_4 - V_1 = 0.2(V_1 - V_2)$$

$$V_4 - V_1 = 0.2V_1 - 0.2V_2$$

$$1.2V_1 - 0.2V_2 - V_4 = 0$$

Excellent

$$1.2V_1 + 2.4 - V_4 = 0$$

$$1.2V_1 - V_4 = -2.4 \quad \text{—(vi)}$$

By eqⁿ (i) ⇒

$$V_4 = 2\left[\frac{5}{2}V_3 + 22\right]$$

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$$V_4 = 5V_3 + 44 \quad \text{—(vii)}$$

put V_4 in (vi)

$$1.2V_1 - 5V_3 - 44 = -2.4$$

$$1.2V_1 - 5V_3 = 41.6 \quad \text{—(viii)}$$

put (vii) in (v)

$$\frac{7}{5}V_1 - V_3 + \frac{5}{2}V_3 + 22 = \frac{-12}{10}$$

$$\frac{7}{5}V_1 - \frac{3}{2}V_3 = -\frac{116}{5} \quad \text{—(ix)}$$

Solving eqⁿ (viii) & (ix)

$V_1 = -6.83\,V$, $V_3 = -9.089\,V$, $V_4 = -1.45\,V$

Here voltage across 0.5Ω Resistor = V_x

$\therefore V_x = V_3 - V_2$

$= (-9.089) - (-12)$

$\boxed{V_x = 2.911\,V}$ ← Ans

Voltage across 2.5Ω Resistor = V_y

$V_y = V_1 - V_2$

$= -6.83 - (-12)$

$\boxed{V_y = 5.17\,V}$ ← Ans

Section B : Signals and Systems

; (a) (i) State and prove commutative property and distributive property of convolution in discrete time domain.

(ii) Sketch the spectrum of modulated signal $y(t) = x(t) \cdot m(t)$, if

1. $X(f) = \begin{cases} 1-|f| \; ; & |f| \le 1 \\ 0 \; ; & \text{otherwise} \end{cases}$ and $m(t) = \cos 15\pi t$.

2. $X(f) = \text{rect}\,(0.25 f)$ and $m(t) = \cos 2\pi t$.

3. $X(f) = \text{rect}\,(f)$ and $m(t) = \cos \pi t$.

[6 + 6 marks]

In (i) Commutative property of convolution ⇒

$$[x_1(n) \otimes x_2(n)] \otimes h(n) = [x_1(n) \otimes h(n)] \otimes x_2(n)$$

Distributive property of convolution ⇒

$$[x_1(n) + x_2(n)] \otimes h(n)$$

$$\Rightarrow [x_1(n) \otimes h(n)] + [x_2(n) \otimes h(n)].$$

2

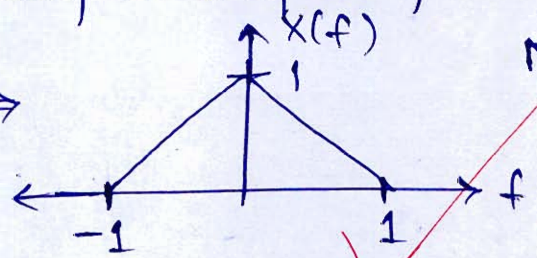
Proof??

(ii) Here, $y(t) = x(t) \cdot m(t)$

$$Y(f) = \frac{1}{2}\left[X_1(f) \circledast M(f) \right]$$

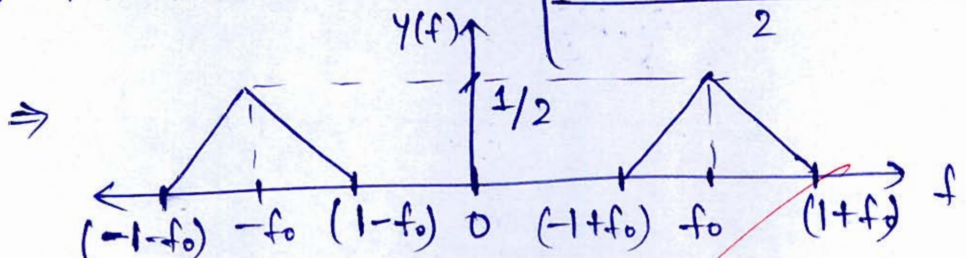
{ By ~~convo~~ multiplication property in time domain to convolution property in frequency domain }

① $X(f) \Rightarrow$

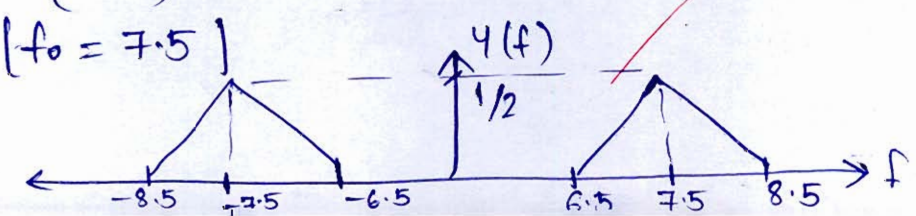


$Y(f) \Rightarrow \quad M(f) \circledast X(f) \Rightarrow \left[\dfrac{X(f-f_0) + X(f+f_0)}{2} \right]$

$\Rightarrow$

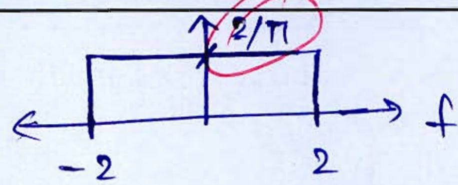


here $[f_0 = 7.5]$



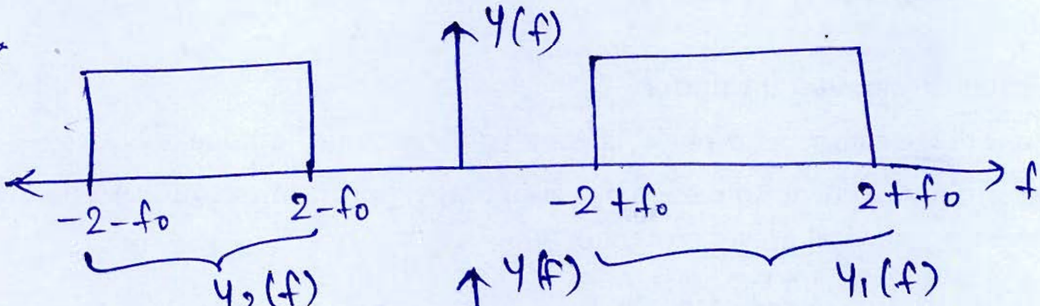
Ans

② $X(f) =$

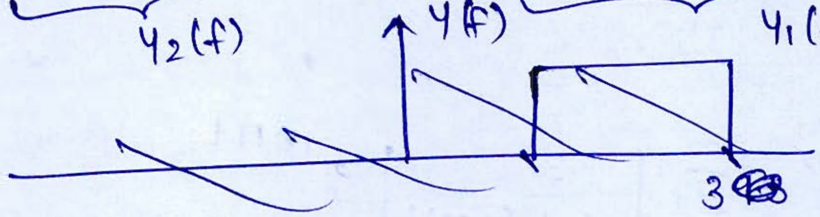


Here in $M(f)$ ↳ $[f_0 = 1]$

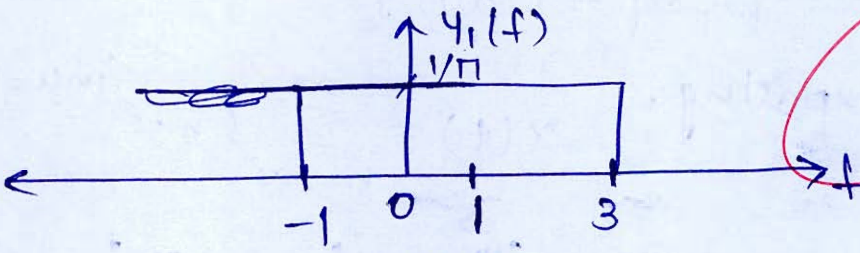
⇒



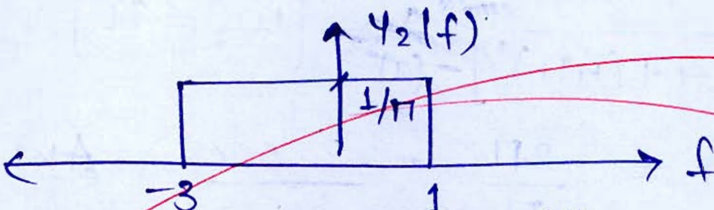
⇒



⇒

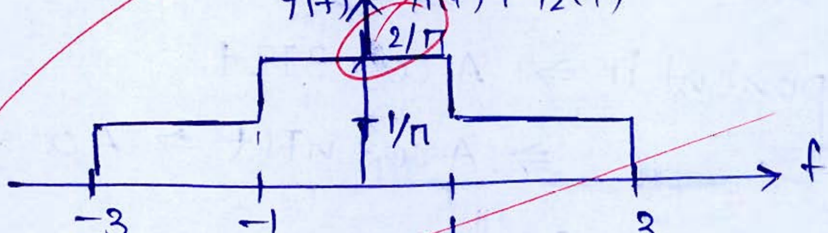


⇒



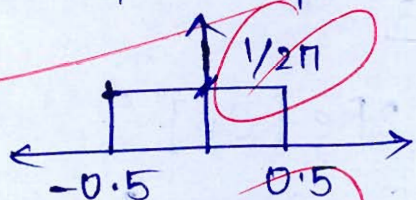
$Y(f) = Y_1(f) + Y_2(f)$

⇒

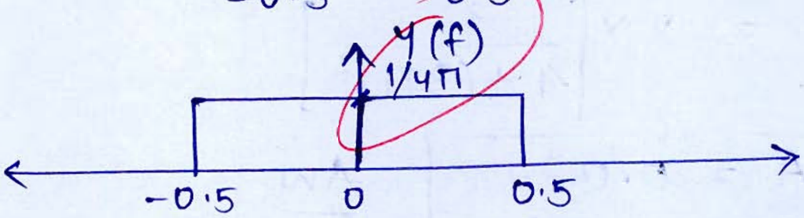


Ans ←

③ $X(f) \Rightarrow$



; $M(f)$ have $\left[f_0 = \frac{1}{2}\right]$



Ans ←

Q.5 (b) The complex exponential Fourier series representation of a signal $f(t)$ over the interval $(0, T)$ is

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(n\pi)^2} e^{jn\pi t}$$

(i) What is the numerical value of T?

(ii) If one of the components of $f(t)$ is $A\cos 3\pi t$, determine the value of A.

(iii) Determine the power contained by the signal $f(t)$ upto the first four harmonics as a percentage of total power of signal.

Note: $\sum_{n=-\infty}^{\infty} \left| \frac{3}{4+(n\pi)^2} \right| \approx 0.669$

[4 + 4 + 4 marks]

Soln →

$$f(t) = \sum_{n=-\infty}^{\infty} \left[\frac{3}{4+(n\pi)^2} \right] e^{jn\pi t}$$

By comparing, $x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}$

$$\left[C_n = \frac{3}{4+(n\pi)^2} \right] \text{—(a)} \quad \& \quad [\omega_0 = \pi]$$

(i) $T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = \underline{2 \text{ sec}}$ Ans

(ii) component is $\Rightarrow A\cos 3\pi t$

$\Rightarrow A\cos n\pi t \Rightarrow A\cos n\omega_0 t$

here $\underline{n = 3}$

~~$C_3 \Rightarrow$~~ $A = 2\,\text{Re}[C_n]$

$$= 2 \times \left[\frac{3}{4+(3\pi)^2} \right]$$

$$\boxed{A = 0.064}$$ Ans

(iii) Since, By parseval's power theorem.

$$\Rightarrow \frac{1}{T}\int_{-\infty}^{\infty} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |C_n|^2$$

Do not write in this margin

$P = \sum_{n=-4}^{4} |C_n|^2 \Rightarrow 2\left[C_1^2 + C_2^2 + C_3^2 + C_4^2\right] + C_0^2$

here put $n = 1, 2, 3, 4, 0$ in 'C_n'

$C_n = \left[\frac{3}{4+(n\pi)^2}\right]$

$C_0 = \frac{3}{4}$, $C_1 = 0.216$, $C_2 = 0.0689$

$C_3 = 0.823$, $C_4 = 0.018$

$\boxed{P' = 0.668\,W}$ Ans

Percentage of power in 4 harmonics

$\Rightarrow \left[\frac{P'}{P_T}\right] \times 100$

$\left\{ \text{here } P_T = \sum_{n=-\infty}^{\infty} |C_n|^2 = 0.669 \text{ (given)} \right\}$

$\Rightarrow \left(\frac{0.668}{0.669}\right) \times 100$

$\Rightarrow$ 99.8 % Ans

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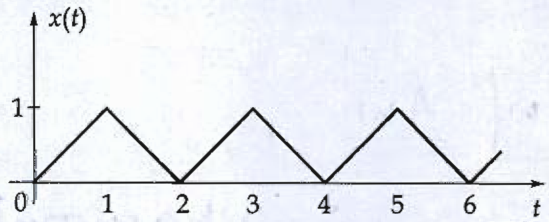
Good

Q.5 (c) (i) Find the Laplace transform of the triangular wave shown in figure.

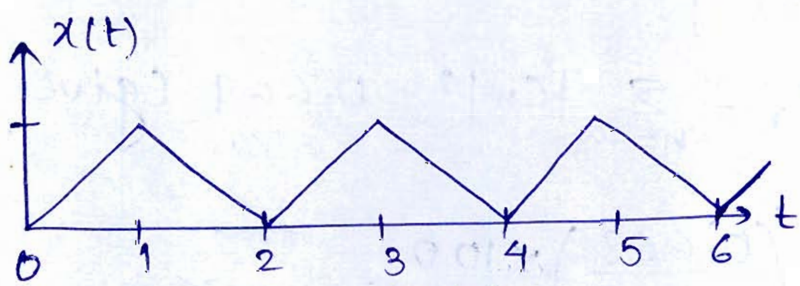


(ii) Determine whether the signal below is periodic or not and if periodic, determine the fundamental period of the signal.

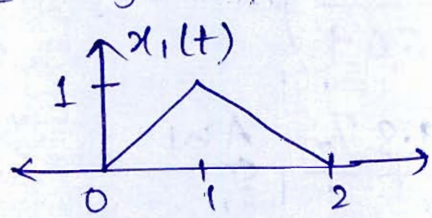
$$x(n) = \text{Re}\left[e^{jn\pi/12}\right] + \text{Im}\left[e^{jn\pi/18}\right]$$

[8 + 4 marks]

Soln → (i)



⇒ let $x_1(t)$ ⇒



$x_1(t) = r(t) - 2r(t-1) + r(t-2)$ ----

$x_1(t) = t\,u(t) - 2(t-1)u(t-1) + (t-2)u(t-2)$

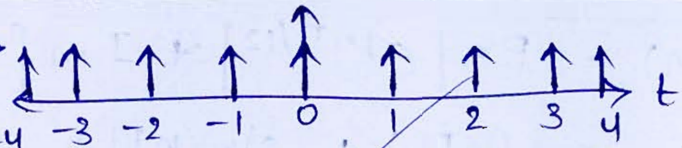
Taking laplace transform of $x_1(t) \longrightarrow X_1(s)$

~~$X_1(s) = \frac{1}{s^2}$~~

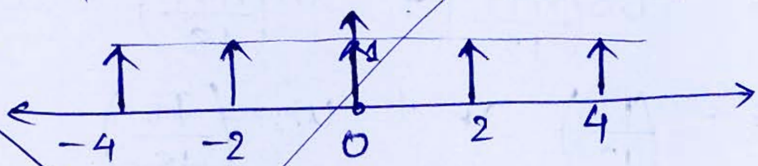
$x_1(t) = t\,u(t) - (2t-2)u(t-1) + (t-2)u(t-2)$

$X_1(s) = \frac{1}{s^2} - \left(\frac{2}{s^2} - \frac{2}{s}\right)e^{-s} + \left(\frac{1}{s^2} - \frac{2}{s}\right)e^{-2s}$

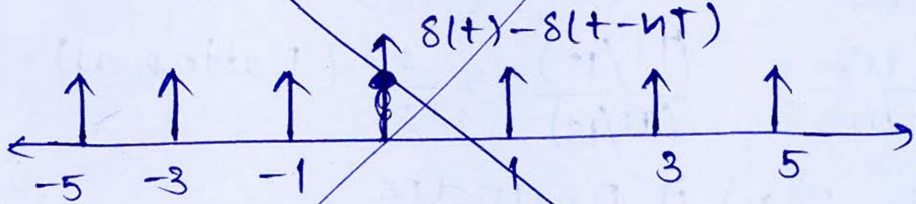
Here, $\delta(t) \Rightarrow$



$\delta(t-nT) \Rightarrow$



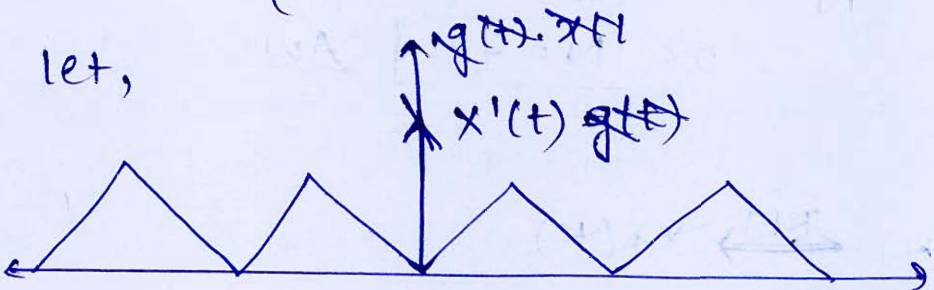
$\Rightarrow$



$\Rightarrow \quad g(t) = \delta(t) - \delta(t-nT)$

$$G(s) = \left[\frac{1-e^{-nTs}}{1}\right] = \left[1-e^{-nTs}\right]$$

$\Rightarrow$ let,



↳ The above sig is the repeating periodically $x_1(t)$

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$$x'(s) = \left[\frac{X_1(s)}{1-e^{-Ts}}\right] = \left[\frac{X_1(s)}{1-e^{-2s}}\right]$$

$$\Rightarrow X(s) = \sum_{n=0}^{\infty} x_1(nt) = x_1(nt) \otimes u(t)$$

$$X(s) \Rightarrow \left[\frac{X_1(s)}{s}\right]$$

$$\Rightarrow \boxed{X(s) = \frac{1}{s^3} - \left(\frac{2}{s^3} - \frac{1}{s^2}\right)e^{-s} + \left(\frac{1}{s^3} - \frac{2}{s^2}\right)e^{-2s}}$$

Ans

Need to modify more

(ii) $x(n) = Re[e^{jn\pi/12}] + Im[e^{jn\pi/18}]$

$x(n) = \cos\left(\frac{n\pi}{12}\right) + \sin\left(\frac{n\pi}{18}\right) = \cos(n\omega_1) + \sin(n\omega_2)$

$\Rightarrow \omega_1 = \left(\frac{\pi}{12}\right)$ and $\omega_2 = \left(\frac{\pi}{18}\right)$

Here $\frac{\omega_1}{\omega_2} = \frac{(\pi/12)}{(\pi/18)} = \frac{3}{2}$ (Rational)

Hence, $x(n)$ is periodic

$\omega_0 = HCF\left(\frac{\pi}{12}, \frac{\pi}{18}\right) = \left(\frac{\pi}{36}\right)$

$\omega_0 = \left(\frac{2\pi}{N}\right)K = \frac{\pi}{36} \Rightarrow \frac{N}{K} = 72$

or $\boxed{N = 72}$ Ans

4

Q.5 (d) Compute the circular convolution of the following sequences using DFT and IDFT:

$x_1(n) = \{1, 2, 1, 2\}$ and $x_2(n) = \{4, 3, 2, 1\}$

[12 marks]

Soln →

~~$x_1(n) \xleftrightarrow{DFT} X_1(K)$~~

~~$X_1(K) =$~~

$y(n) = x_1(n) \circledast x_2(n)$

By circular convolution using IDFT

$$y(n) = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$y(n) = [14, 16, 14, 16]$ Ans

Now, circular convolution of following sequence using their DFTs

$x_1(n) \Rightarrow X_1(K) = \sum_{n=-\infty}^{\infty} x_1(n) \cdot W_N^K$

$$X_1(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

$X_1(K) = \{6, -j4, -2, -j4\}$

similarly $x_2(n) \Rightarrow X_2(K) = \sum_{n=-\infty}^{\infty} x_2(n) \cdot W_N^K$

$$X_2(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$X_2(K) = \{10, 2-j4, 2, 2-j4\}$ ②

$y(K) = X_1(K) \circledast X_2(K)$

$$= \begin{bmatrix} 6 & -j4 & -2 & -j4 \\ -j4 & 6 & -j4 & -2 \\ -2 & -j4 & 6 & -j4 \\ -j4 & -2 & -j4 & 6 \end{bmatrix} \begin{bmatrix} 10 \\ 2-j4 \\ 2 \\ 2-j4 \end{bmatrix}$$

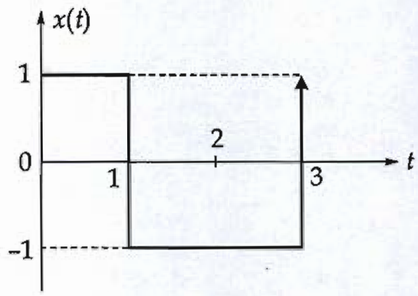
$y(K) = \{-24-j16, 8-j64, -40-j16, 8-j64\}$

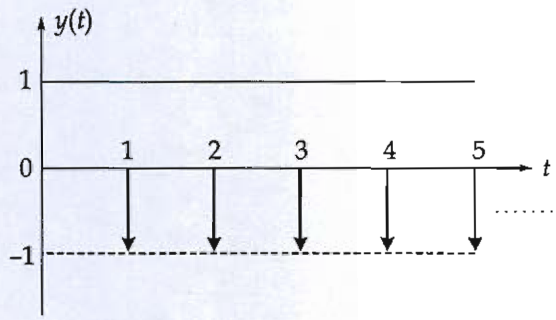
Ans

Go through soln

Q.5 (e) Derive the impulse response $h_d(n)$ of a highpass filter to meet the following specifications:
Cutoff frequency = 250 Hz
Sampling frequency f_s = 1 kHz and
Filter length = 7 **[12 marks]**

Q.6 (a) (i) For the signals $x(t)$ and $y(t)$ given below, determine and sketch $\int_{-\infty}^{t} x(\tau)d\tau$ and $\int_{-\infty}^{t} y(\tau)d\tau$:





(ii) For the second order FIR lattice filter with reflection coefficients $K_1 = \frac{1}{2}$, $K_2 = \frac{1}{4}$, draw the FIR lattice structure and find the transfer function and impulse response of the FIR system.

[8 + 12 marks]

(b) (i) Determine the time domain signals corresponding to the bilateral Laplace transforms given below. Specify the properties used:

1. $X(s) = \frac{1}{s^2} \cdot \frac{d}{ds}\left(\frac{e^{-3s}}{s}\right)$, ROC : Re(s) > 0

2. $X(s) = s\left(\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}\right)$, ROC : Re(s) > 0

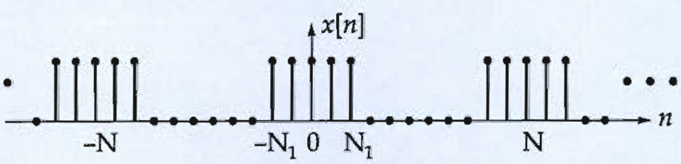
(ii) The impulse response of a relaxed linear time invariant system is $h(n) = \alpha^n u(n)$ with $|\alpha| < 1$. Determine the value of the step response as $n \to \infty$.

[5 + 5 + 10 marks]

(c) **(i)** Let $z = re^{j\omega}$ and $s = \sigma + j\Omega$. Use bilinear transformation to show that if $r < 1$, then $\sigma < 0$, and if $r > 1$, then $\sigma > 0$, and when $r = 1$, then $\sigma = 0$.

(ii) Consider a signal $x[n]$ such that

$$x[n] = \begin{cases} 1 & \text{for } -N_1 < n < N_1 \\ 0 & \text{for rest of time period} \end{cases}$$



Discrete-time periodic square wave

If time period of $x[n]$ is N where $(N > 2N_1 + 1)$, then determine Fourier series coefficient a_k for signal $x[n]$.

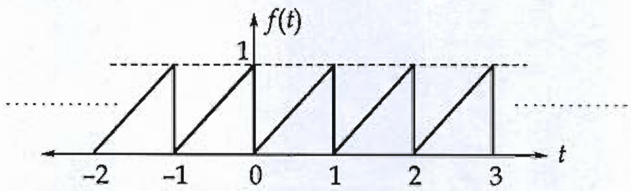
[10 + 10 marks]

Do not write in this

Q.7 (a) (i) Determine the signal $x[n]$ and rational z-transform $X(z)$ for the following cases:

1. $x[n]$ is right sided, $X(z)$ has a single pole, $x[0] = 4$, $x[2] = \frac{1}{4}$.
2. $X[z]$ has poles at $z = \frac{1}{4}$ and $z = -1$, ROC includes $|z| = \frac{1}{2}$, $x[1] = 1$, $x[-1] = 1$.

(ii) 1. Obtain exponential Fourier series representation of the periodic signal shown below.



2. Draw the magnitude and phase plot of the Fourier series coefficient of the above signal.

[10 + 10 marks]

Q.7 (b) (i) Suppose we are given the following facts about an LTI system S with impulse response $h[n]$ and frequency response $H(e^{j\omega})$:

1. $\left(\frac{1}{4}\right)^n u[n] \xrightarrow{S} g[n]$, where $g[n] = 0$ for $n \geq 2$ and $n < 0$.
2. $\text{Re}\{H(e^{j\pi/2})\} = 1$.
3. $H(e^{j\omega}) = H(e^{j(\omega - \pi)})$.

Determine $h[n]$.

(ii) 1. Mention any five properties of ROC of Laplace transform.

2. Find inverse LT of $H(s) = \ln\left(\frac{1}{3s+2}\right)$.

[12 + 8 marks]

(ii) ① Properties of ROC of laplace transform

→ For left sided signal ROC will be left to the leftmost pole.

→ for right sided signal ROC will be right side to the rightmost pole.

→ For both sided signal ROC will be a strip.

→ ROC does not include any pole.

→ for finite durated signal ROC will be entire s-plane excluding s = 0 and s = ∞

4

②

(i) ① $g(n) = \left(\frac{1}{4}\right)^n u(n)$

$$G(e^{j\omega}) = \left[\frac{1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)}\right].$$

incomplete

Do not write in this margin

(c) **(i)** Consider the 6-length sequence defined for $0 \le n < 6$ as $x(n) = \{\underset{\uparrow}{1}, -2, 3, 0, -1, 1\}$ with a 8-point DFT $X(k)$. Evaluate the following functions of $X(k)$ without computing DFT:

1. $X(0)$ **2.** $X(3)$

3. $\sum_{k=0}^{5} X(k)$ **4.** $\sum_{k=0}^{5} |X(k)|^2$

(ii) Suppose the following facts are given about the signal $x(t)$ with Laplace transform $X(s)$:

1. $x(t)$ is real and even.

2. $X(s)$ has four poles and no zeros in the finite s-plane.

3. $X(s)$ has a pole at $s = \frac{1}{2} e^{j\frac{\pi}{4}}$.

4. $\int_{-\infty}^{\infty} x(t)dt = 4$

Determine $X(s)$.

[10 + 10 marks]

(ii) ① $x(t) \Rightarrow$ Real and even.

$x(t) \xrightarrow[LT]{} X(s)$

10

② $X(s)$ has four poles ~~and~~ no zeros.

③ $X(s)$ has a pole at $s = \frac{1}{2} e^{j\pi/4}$.

Since, $x(t) \Rightarrow$ Real

$X(s)$ has poles in conjugate pair so

if $\left[s_1 = \frac{1}{2} e^{j\pi/4} \right]$ then, $s_2 = s_1^*$

$= \left[\frac{1}{2} e^{-j\pi/4} \right]$

and $x(t) \Rightarrow$ even, poles will exist with -ve pair

$s_3 = -s_1 = -\frac{1}{2} e^{j\pi/4}$

$s_4 = -s_2 = -\frac{1}{2} e^{-j\pi/4}$.

(4) $\Rightarrow \int_{-\infty}^{\infty} x(t)\,dt = X(s=0) = 4$

Here, $X(s) = \dfrac{K}{(s+s_1)(s+s_2)(s+s_3)(s+s_4)}$

$$\Rightarrow \frac{K}{\left(s+\frac{1}{2}e^{j\pi/4}\right)\left(s+\frac{1}{2}e^{-j\pi/4}\right)\left(s-\frac{1}{2}e^{j\pi/4}\right)\left(s-\frac{1}{2}e^{-j\pi/4}\right)}$$

$\Rightarrow$ put $s=0$, in $X(s)$

$$\Rightarrow \frac{K}{\left(\frac{1}{2}e^{j\pi/4}\right)\left(\frac{1}{2}e^{-j\pi/4}\right)\left(\frac{-1}{2}e^{j\pi/4}\right)\left(\frac{-1}{2}e^{-j\pi/4}\right)} = 4$$

$$\Rightarrow \frac{K}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)} = 4$$

$$K = \left[4 \times \frac{1}{16}\right]$$

$$\boxed{K = 1/4}$$

$$\therefore X(s) = \frac{(1/4)}{\left[\left(s+\frac{1}{2}e^{j\pi/4}\right)\left(s-\frac{1}{2}e^{j\pi/4}\right)\right]\left[\left(s+\frac{1}{2}e^{-j\pi/4}\right)\left(s-\frac{1}{2}e^{-j\pi/4}\right)\right]}$$

$$= \frac{(1/4)}{\left[s^2-\frac{1}{4}e^{j\pi/2}\right]\left[s^2-\frac{1}{4}e^{-j\pi/2}\right]}$$

$$= \frac{(1/4)}{(s^2-1/4)(s^2+1/4)}$$

$$\left[X(s) = \frac{(1/4)}{\left[s^4 + \left(\frac{1}{16}\right)\right]}\right] \text{ Ans.}$$

) ① $x(n) = \{\underset{\uparrow}{1}, -2, 3, 0, -1, 1\}$

here, $0 \le n < 6$

since, $x(n) \xrightarrow{DFT} X(K)$

$$X(K=0) = \sum_{n=0}^{5} x(n)$$

$$\boxed{X(0) = 2} \text{ Ans.}$$

② $X(3) = X\left(K = \frac{N}{2}\right) = X(K=3)$

$$= x(0) - x(1) + x(2) - x(3) + x(4) - x(5)$$

$$= 1 - (-2) + 3 - 0 + (-1) - 1$$

$$\boxed{X(3) = 4} \text{ Ans}$$

③ $\sum_{K=0}^{5} X(K) = N\,x(0)$

$$= 6[1]$$

$$= 6$$

10

④ $\sum_{K=0}^{5} |X(K)^2| = N \sum_{n=0}^{5} |x(n)|^2$

By parseval's power theorem

$$= 6[1 + 4 + 9 + 0 + 1 + 1]$$

$$= \underline{96} \text{ Ans}$$

End

Q.8 (a) Using DIT FFT and inverse DIT FFT, determine the output of the system if input $x(n)$ and impulse response $h(n)$ are given as $x(n) = \{2, 2, 4\}$ and $h(n) = \{1, 1\}$.

[20 marks]

) A discrete time system is described by the difference equation:

$$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)+x(n-1)$$

with $y(-1) = 0$ and $y(-2) = -1$.

Find:

(i) The zero input response of the system.

(ii) The zero state response of the system due to step input $u(n)$.

[10 + 10 marks]

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Obtain the direct form-I and direct form-II realizations of the LTI system governed by the equation

$$y(n) = -\frac{13}{12}y(n-1) - \frac{9}{24}y(n-2) - \frac{1}{24}y(n-3) + x(n) + 4x(n-1) + 3x(n-2)$$

[20 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work

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