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ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electrical Circuits + Systems & Signal Processing

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	25
Q.2	
Q.3	
Q.4	50
Section-B	
Q.5	26
Q.6	33
Q.7	50
Q.8	[illegible]
Total Marks Obtained	184

Signature of Evaluator ....

Cross Checked by ....

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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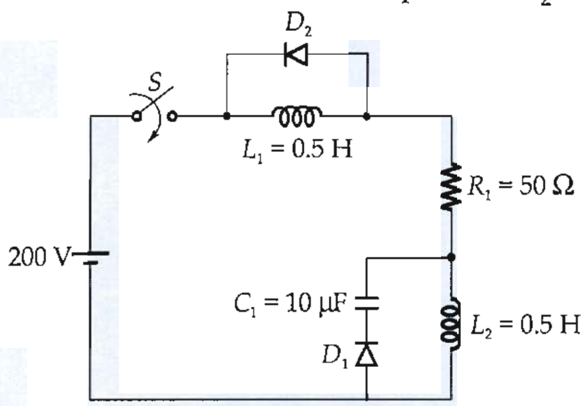
Section A : Electrical Circuits

.1 (a) In the circuit shown below the switch 'S' is closed at $t = 0$, and is opened after 10 ms. What will be the currents in R_1, L_1 and L_2, and voltage across C_1, 8 ms after switch 'S' opens? Assume D_1 to be an ideal diode and a 0.7 V drop across D_2 whenever it conducts.



[12 marks]

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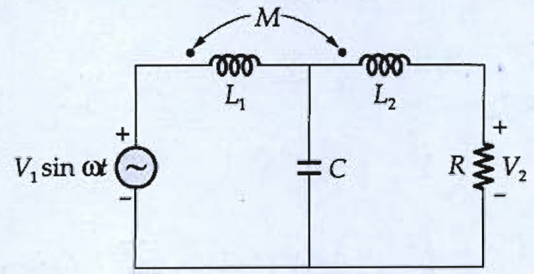
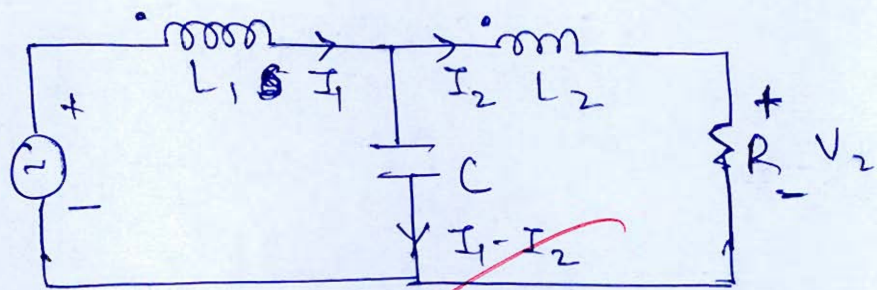
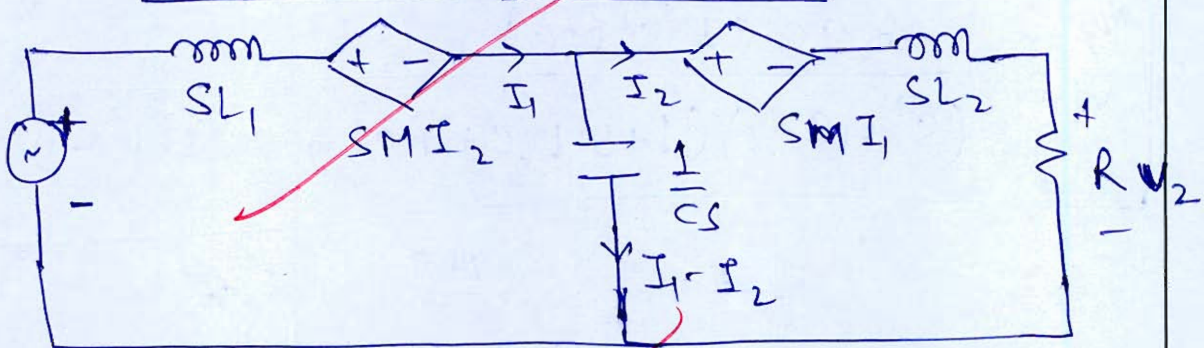
Q.1 (b) Find the voltage transfer function V_2/V_1 for the network given below:



[12 marks]

Solⁿ:-





$$V_1 = sL_1 I_1 + sMI_2 + \frac{1}{Cs}(I_1 - I_2)$$

$$V_1 = \left(sL_1 + \frac{1}{Cs}\right) I_1 + \left(sM - \frac{1}{Cs}\right) I_2 \quad \text{—(1)}$$

Also

$$\frac{1}{Cs}(I_1 - I_2) = sMI_1 + sL_2 I_2 + RI_2$$

$$\frac{I_1}{Cs} - \frac{I_2}{Cs} = sM I_1 + (sL_2 + R) I_2$$

$$I_1\left[\frac{1}{Cs} - sM\right] = \left(\frac{1}{Cs} + sL_2 + R\right) I_2$$

$$I_1 = \frac{(1 + s^2 L_2 C + RCs)}{1 - s^2 MC} I_2 \quad \text{—(2)}$$

Substituting I_1 from eqn (2) → eqn (1)

$$V_1 = \left(\frac{S^2L_1C + 1}{CS}\right)\frac{(1+S^2L_2C + RCS)}{(1-S^2MC)} I_2 + \left(\frac{S^2MC-1}{CS}\right)I$$

$$V_1 = \left[\frac{(S^2L_1C+1)(1+S^2L_2C+RCS) - (S^2MC-1)^2}{SC\ (1-S^2MC)}\right] I_2$$

$$V_2 = R I_2$$

$$\boxed{\frac{V_2}{V_1} = \frac{RSC(1-S^2MC)}{(S^2L_1C+1)(1+S^2L_2C+RCS) - (S^2MC-1)^2}}$$

Ans

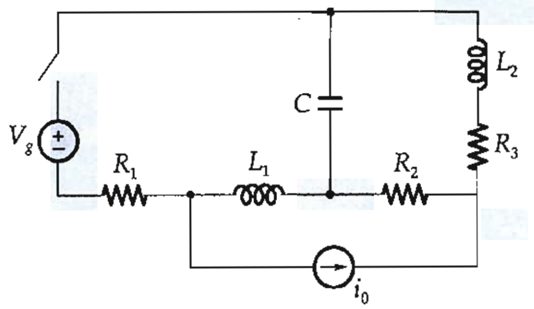
Good Approach

11

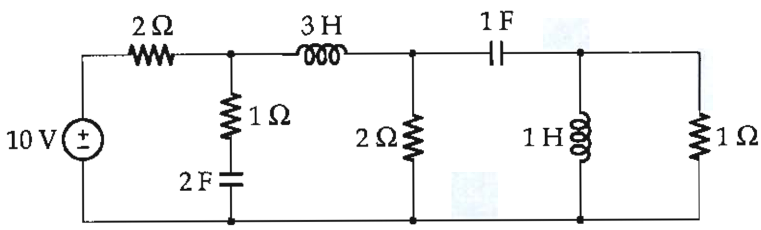
Do not write in this margin

.1 (c) Draw the dual of the circuit shown in figure.

1.

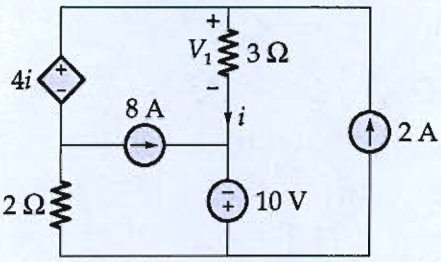


2.



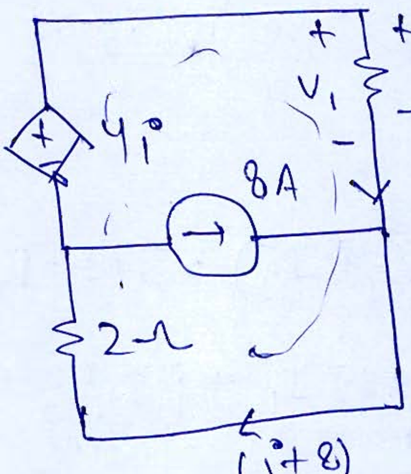
[12 marks]

.1 (d) Using the superposition theorem determine V_1, the voltage across the 3 Ω resistor in figure below,



[12 marks]

Solⁿ :- due to 8 A current source alone -



$4i = 3i + 2(i+8)$

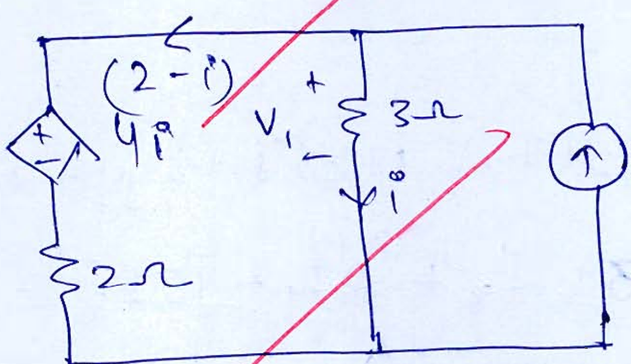
$4i = 3i + 2i + 16$

$-i = 16$

$i = -16$ Amp

$V_1' = 3i = -48$ V

Due to 2A current source alone



11

Good Approach

$4i = 3i - 2(2-i) = 3i - 4 + 2i$

$-i = -4 \Rightarrow i = 4A$

$V_1'' = 3i = 12$ V

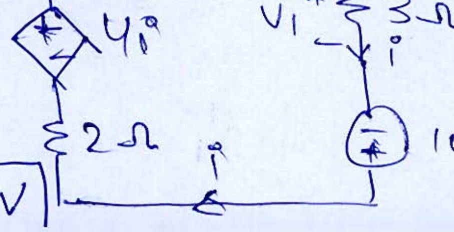
Due to 10V voltage source alone -

$10 = 2i - 4i + 3i$

$= 2i - i = i$

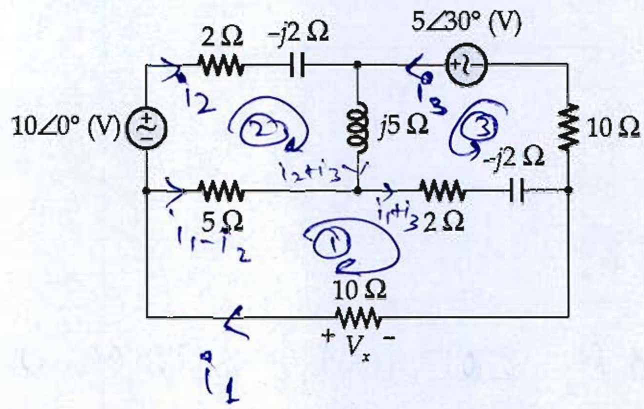
$i = 10$ A

$V_1''' = 3i = 30$V



Ans $V_1 = -48 + 12 + 30 = -6$ V

Q.1 (e) Write the loop equations of the circuit and find the voltage V_x.



[12 marks]

Soln.: Let currents in loops be i_1, i_2, i_3

Loop ②

$$10 = 2I_2 - j2I_2 + j5(I_2 + I_3) - 5(I_1 - I_2)$$
$$= (2 - j2 + j5 + 5)I_2 - 5I_1 + j5I_3$$
$$10 = (7 + j3)I_2 - 5I_1 + j5I_3 \quad \text{— eqn ①}$$

Loop ③

$$5\angle 30° = j5(I_2 + I_3) + (2-j2)(I_1 + I_3) + 10I_3$$
$$5\angle 30° = (12 + j3)I_3 + (2-j2)I_1 + j5I_2 \quad \text{— eqn ②}$$

Loop ①

$$0 = 5(I_1 - I_2) + (2-j2)(I_1 + I_3) - 10I_1$$
$$0 = (5 + 2 - j2 - 10)I_1 - 5I_2 + (2-j2)I_3$$
$$0 = (-3 - j2)I_1 - 5I_2 + (2-j2)I_3 \quad \text{— eqn ③}$$

Loop Eqn. –

$-(3+j2)I_1 - 5I_2 + (2-j2)I_3 = 0$

$(2-j2)I_1 + j5I_2 + (12+j3)I_3 = 5\angle 30^\circ$

$-5I_1 + (7+j3)I_2 + j5I_3 = 10$

$$I_1 = \frac{\begin{vmatrix} 0 & -5 & (2-j2) \\ 5\angle 30^\circ & j5 & (12+j3) \\ 10 & (7+j3) & j5 \end{vmatrix}}{\det \begin{bmatrix} -(3+j2) & -5 & (2-j2) \\ (2-j2) & j5 & (12+j3) \\ -5 & (7+j3) & j5 \end{bmatrix}}$$

$$= \frac{5[25\angle 120^\circ - 123.693\angle 14.036] + (2-j2)[30.016\angle -40.539^\circ]}{-(3+j2)[115.104\angle -150.31^\circ] + 5[74.33\angle 19.65^\circ] + (2-j2)[26.248\angle 40.364]}$$

3

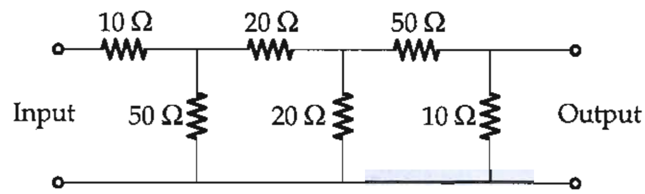
Wrong value calculated

$$= \frac{667.962\angle -169.09}{782.398\angle 38.79^\circ} = 0.853\angle 152.13^\circ \text{ Amp}$$

$V_x = -10I_1 = 8.537\angle -27.865^\circ$ Volts

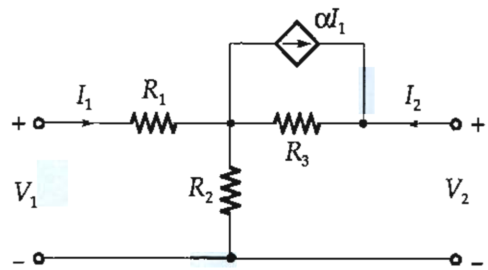
Ans

Q.2 (a) (i) Obtain the *ABCD* parameters for the network shown in figure below.



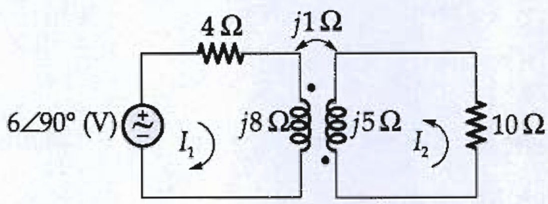
[12 marks]

Q.2 (a) (ii) Find the hybrid parameters for the network shown in figure below.



[8 marks]

Q.2 (b) **(i)** Determine the current I_1 and I_2 in the circuit shown in figure below, using T-equivalent circuit for the linear transformer.



[8 marks]

Q.2 (b) **(ii)** A voltage of $v = (2000 \sin \omega t + 400 \sin 3\omega t + 100 \sin 5\omega t)$ volts is applied to a series circuit having $R = 10\ \Omega$ and $C = 30\ \mu F$ and a variable inductance.

1. Find the value of inductance so as to give resonance at 3rd harmonic frequency.
2. What are the rms values of voltage and current with this inductance in circuit? (Take $\omega = 300$ rad/sec).

[12 marks]

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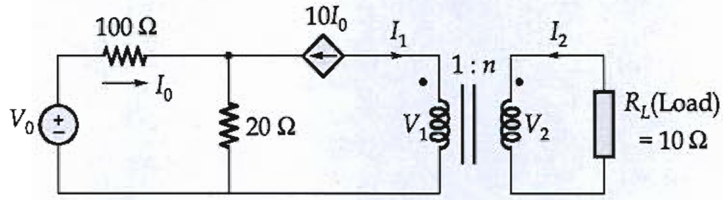
Q.2 (c) (i) What is the voltage and power gain of the circuit shown in figure? Assume $n = \frac{1}{10}$.



[10 marks]

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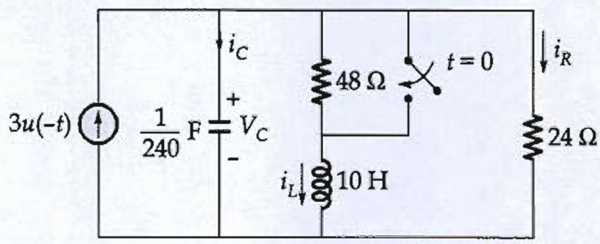
Q.2 (c) (ii) Consider the circuit shown below:



After being open for a long time, the switch is closed at $t = 0$. Find

1. $i_L(0^-)$ 2. $V_C(0^-)$ 3. $i_R(0^+)$
4. $i_C(0^+)$ 5. $V_C(0.2)$ using Laplace transform approach.

[10 marks]

Q.3 (a) (i) A 415-V, 50-Hz, three-phase voltage is applied to three star-connected identical impedances. Each impedance consists of a resistance of 15 Ω, a capacitance of 177 μF and an inductance of 0.1 henry in series.

Find:

1. the phase current,
2. the line current,
3. the power factor,
4. the active power,
5. the reactive power and
6. the total VA.

Draw a neat phasor diagram. If the same impedances are connected in delta, find the (i) line current and (ii) power consumed.

[10 marks]

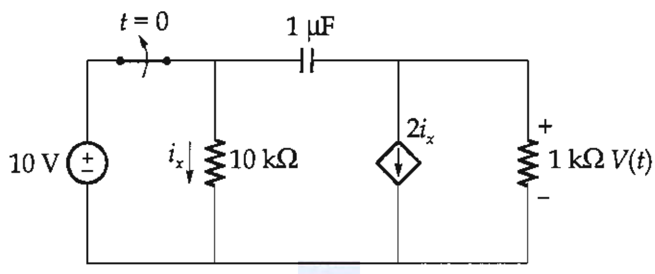
.3 (a) **(ii)** A coil having a resistance of 20 Ω and an inductance of 200 μH is connected in parallel with a variable capacitor. This parallel combination is connected in series with a resistance of 8000 Ω. A voltage of 230 V at a frequency of 10^6 Hz is applied across the circuit.

Calculate:

1. the value of capacitance at resonance,
2. *Q*-factor of the circuit,
3. dynamic impedance of the circuit, and
4. total circuit current.

[10 marks]

.3 (b) (i) For the circuit shown in figure:

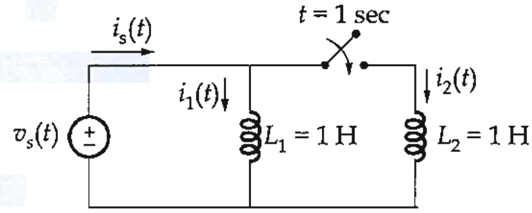


1. Find the expression of $V(t)$, the voltage across 1 kΩ resistor when the switch is opened at time, $t = 0$.
2. Sketch $V(t)$ with respect to time (t) and mark the time constant t.

[10 marks]

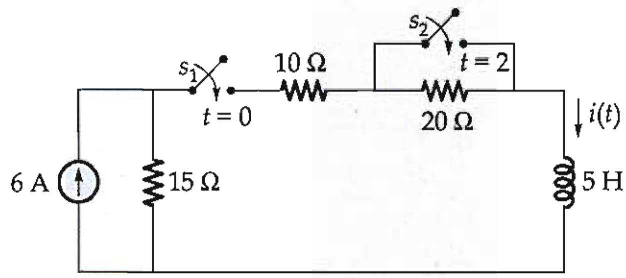
Q.3 (b) (ii) For the parallel inductive circuit shown below with switch closed at $t = 1$s, $v_s(t) = \cos(t)$ V for $t \geq 0$ and 0 otherwise, **find:**

1. the input current $i_s(t)$ for $t \geq 0$ sec.
2. the energy stored in each of the inductors for the intervals $[0, t]$ for $0 \leq t \leq 1$ and for $1 \leq t$.



[10 marks]

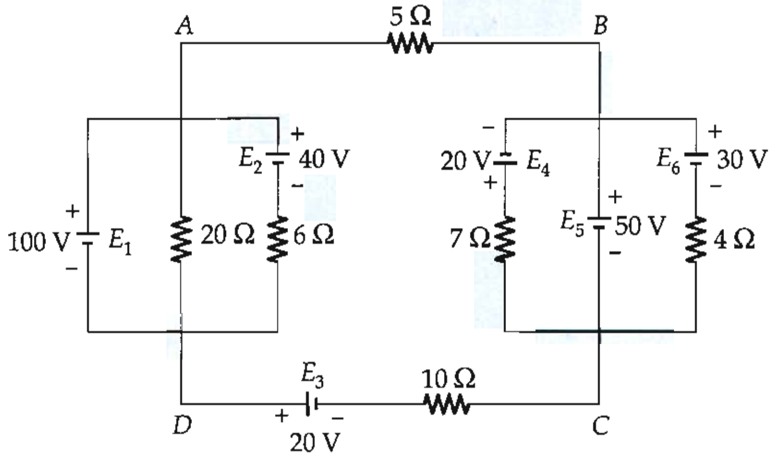
Q.3 (c) Consider the network shown below:



Switch S_1 is closed at $t = 0$, and switch S_2 is closed at $t = 2$ sec. Calculate current $i(t)$ for all t, and also find $i(t)\,|_{t=1\text{ sec}}$ and $i(t)\,|_{t=3\text{ sec}}$.

[20 marks]

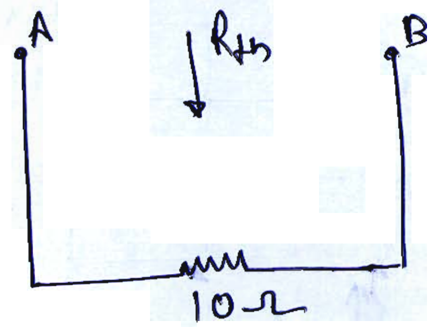
Q.4 (a) For the circuit shown in figure, find the current through 5 Ω resistor by using Thevenin's theorem and verify the same by using superposition theorem.



[20 marks]

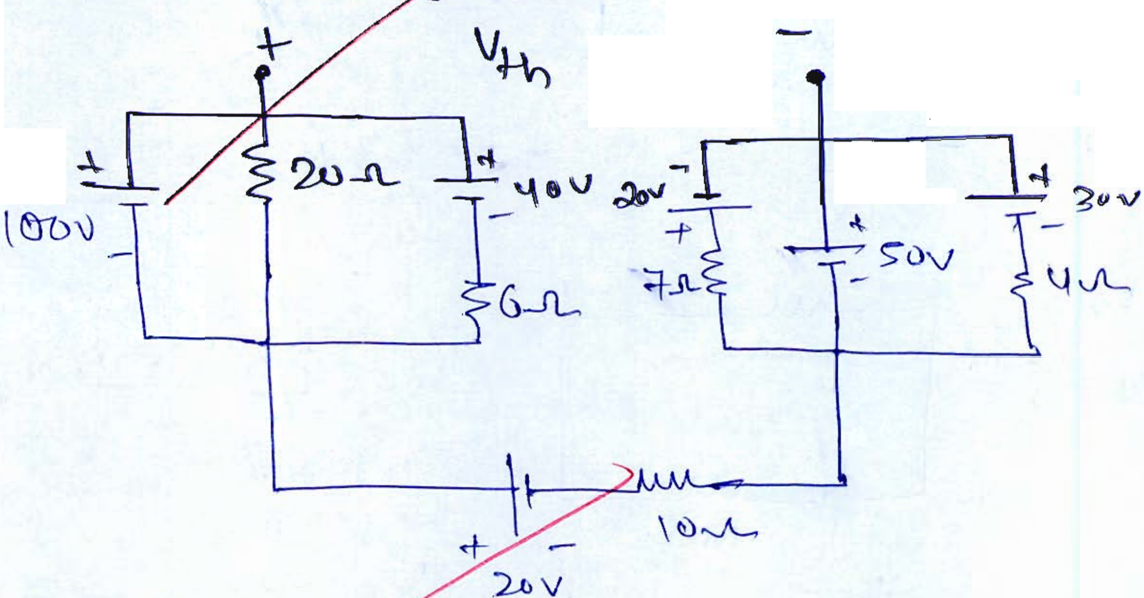
Soln:-

R_{th} →



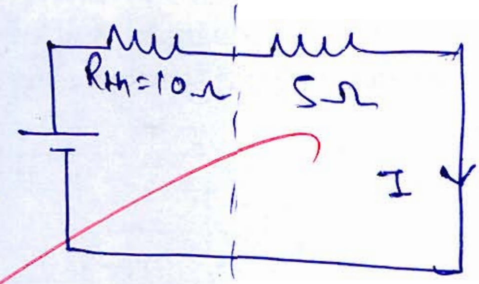
$R_{th} = 10\ \Omega$

Thevenin's voltage V_{th} →



$V_{th} = 100V + 20V - 50V = 70$ Volts

Thevenin's equivalent –



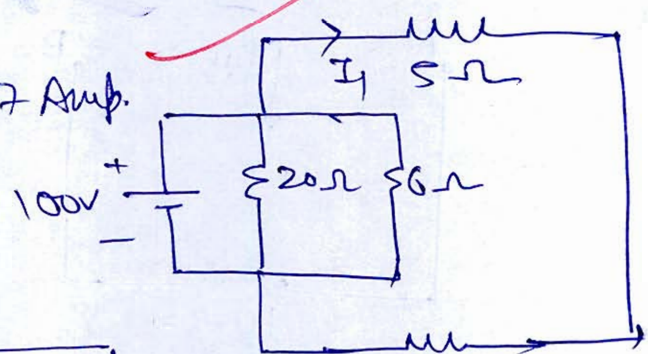
$$I_{5\Omega} = \frac{V_{th}}{R_{th}+R_L} = \frac{70}{10+5}$$

$$\boxed{I = 4.67 \text{ Amp}}$$

Ans.

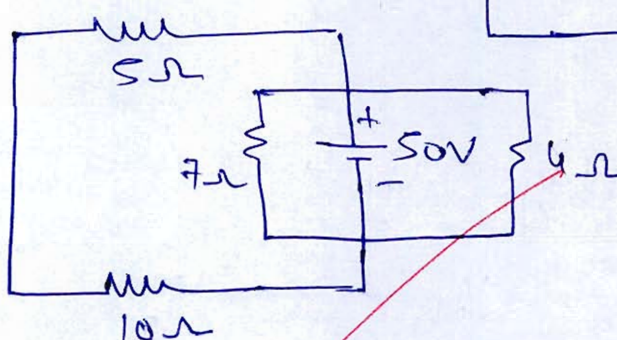
Superposition theorem –

case-1: $E_1 \rightarrow$ energised, rest other are de-energised.



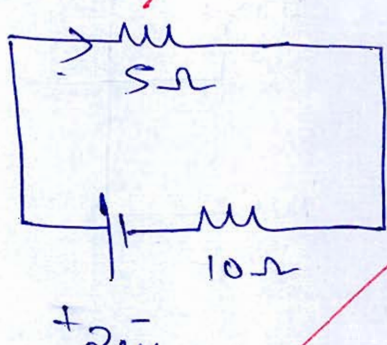
$$I_1 = \frac{100}{15} = 6.67 \text{ Amp.}$$

E_2 case-2



$$I_2 = -\frac{50}{15} = -\frac{10}{3} = -3.33 \text{ Amp.}$$

E_3 case-3



$$I_3 = \frac{20}{15} = 1.33A$$

In other cases, the current through 5Ω
20

Total current through 5Ω

$= 6.67 - 3.33 + 1.33$

$I_{5\Omega} = 4.67A$

Hence, verified

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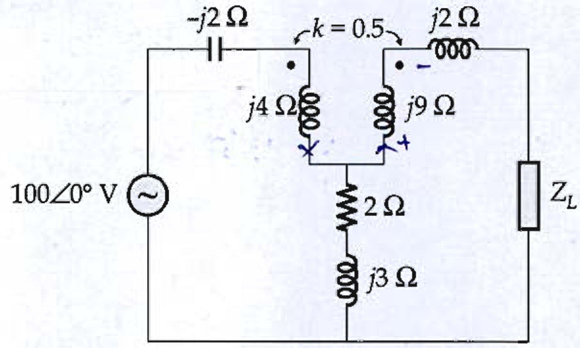
Good Approach

Q.4 (b) **(i)** Find the Thevenin's equivalent of the circuit shown in figure below as seen from the load impedance Z_L.

(ii) Find the value of Z_L for maximum power transfer and also the maximum power transfer to the load Z_L.

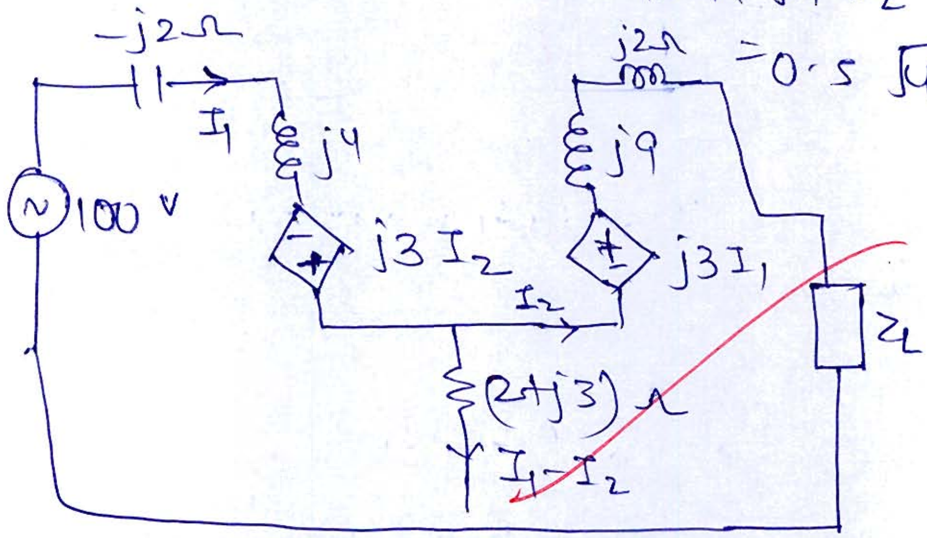


[10 + 10 marks]

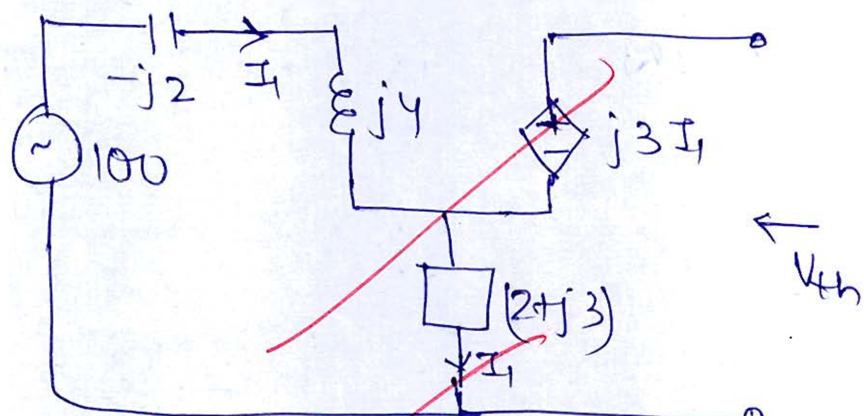
Soln.:

Mutual inductance $M = K\sqrt{L_1 \cdot L_2}$

$= 0.5\sqrt{4 \times 9} = j3\,\Omega$



$V_{th} \rightarrow$



$100 = (-j2 + j4 + 2 + j3)\, I_1$

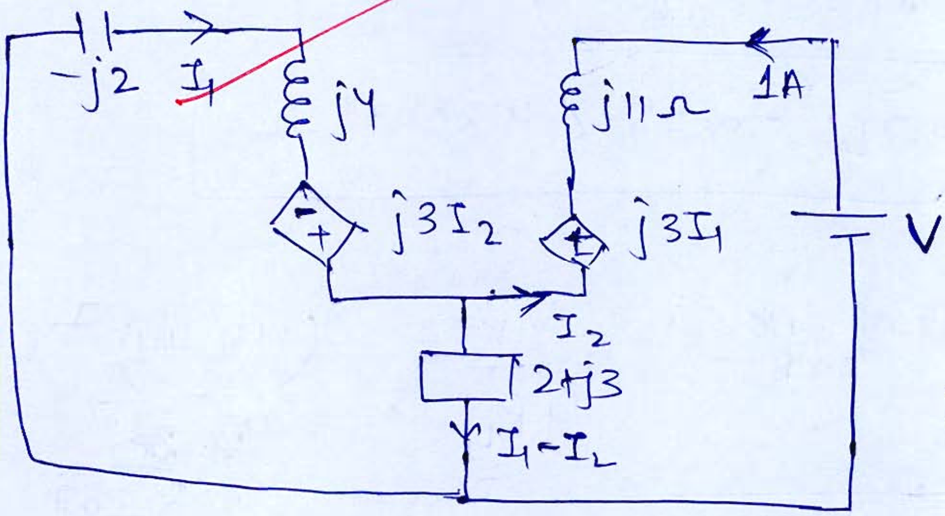
$I_1 = \left(\frac{100}{2+j5}\right)$

$V_{th} = (2+j6)\, I_1$

$= \frac{100}{(2+j5)} \times 2+j6$

$$V_{th} = 117.44 \angle 3.366^\circ \text{ Volts}$$

Z_{th}



$I_2 = -1$

$$V = j11 + j3I_1 + (2+j3)(I_1+1)$$

$$= (j11 + 2 + j3) + (2+j6)I_1$$

$$0 = j2 - j3I_2 + (2+j3)(I_1+1)$$

$$0 = j2 + j3 + (2+j3) + (2+j3)I_1$$

$$0 = (2+j8) + (2+j3)I_1$$

$$\Rightarrow I_1 = -\frac{(2+j8)}{(2+j3)} \text{ Amp.}$$

$$V = (2+j14) + (2+j6) \times -\frac{(2+j8)}{2+j3}$$

$$V = 2.35 \angle -11.309^\circ$$

$$Z_{th} = 2.35 \angle -11.309^\circ \ \Omega$$

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(ii) for maximum ~~Pa~~ power –

$Z_L = Z^*_{th}$

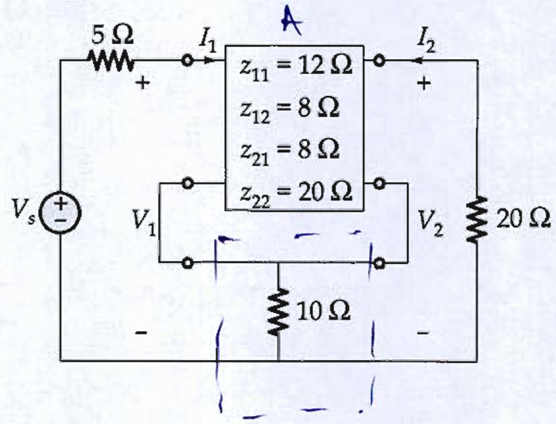
$Z_L = 2.35 \angle 11.3099^\circ \Omega$

Maximum power $= \frac{V_{th}^2}{4R_L} = \frac{(117.44)^2}{4 \times 2.307}$

$P_{max} = 1494.598 W$

Wrong value calculated

Q.4 (c) (i) Evaluate the ratio V_2/V_s in the circuit shown below.

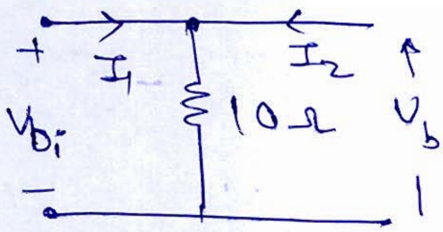


[10 marks]

Soln:-

$V_{bi} = 10(I_1 + I_2)$

$V_{bo} = 10(I_1 + I_2)$



Hence Z-parameter of system 'B'–

$$[Z_B] = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} \Omega$$

Since,
System A & System B are connected in series. Hence, there Z-parameters are added.

$$Z = [Z_A] + [Z_B]$$

$$= \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

$V_1 = 22 I_1 + 18 I_2$

$V_2 = 18 I_1 + 30 I_2$

Also $V_2 = -20I_2$

So, $-20I_2 = 18I_1 + 30I_2$

$-50I_2 = 18I_1$

$I_2 = \frac{-18I_1}{50} = \frac{-9}{25}I_1$

$I_1 = -\frac{25}{9}I_2$

$V_s = 5I_1 + V_1 = 27I_1 + 18I_2$

$V_2 = -20I_2$

$\frac{V_2}{V_s} = \frac{-20I_2}{27I_1 + 18I_2} = \frac{-20I_2}{27 \times -\frac{25}{9}I_2 + 18I_2}$

$= \frac{-20I_2}{-57I_2}$

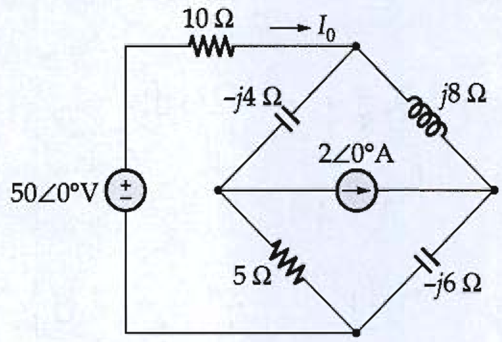
$\boxed{\frac{V_2}{V_s} = \frac{20}{57}}$ Ans

9

Good Approach

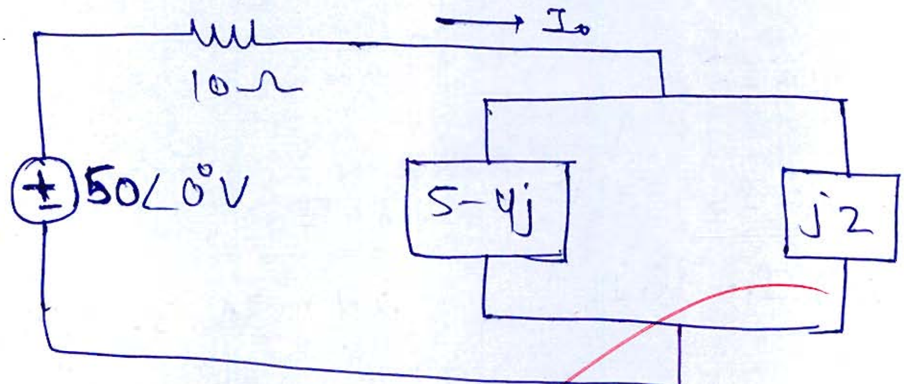
Do not write in this margin

Q.4 (c) (ii) By using superposition theorem, find current I_0 in the circuit shown below:



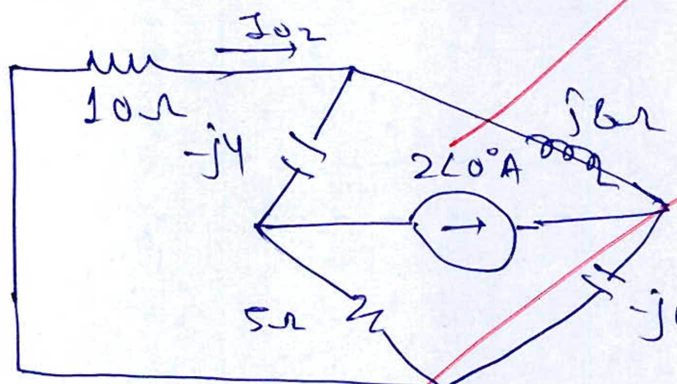
[10 marks]

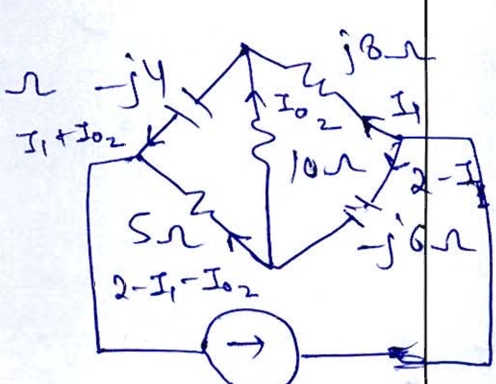
Soln:- Due to voltage source alone -



$$I_{o_1} = \frac{50}{10 + [(j2) \parallel (5-4j)]} = 4.57\angle -12.019° \text{ Amps}$$

Current I_{o_2} due to current source alone.





$5(2-I_1-I_{o_2}) = 10I_{o_2} - j4(I_1+I_{o_2})$

$10 - 5(I_1+I_{o_2}) = 10I_{o_2} - j4(I_1+I_{o_2})$

$10 = 10I_{o_2} + (5-j4)(I_1+I_{o_2})$

$10 = (15-j4)I_{o_2} + (5-j4)I_1$ — ①

$10I_{o_2} - j6(2-I_1) = j8I_1$

$10I_{o_2} - 12j + j6I_1 = j8I_1$

$-12j = -10I_{o_2} + j2I_1$ — ②

$$I_{o_2} = \frac{\begin{bmatrix} 10 & (5-j4) \\ -12j & j2 \end{bmatrix}}{\begin{bmatrix} 15-j4 & 5-j4 \\ -10 & j2 \end{bmatrix}} = \frac{93.295\angle 59.036}{58.855\angle 9.78}$$

$= 1.585\angle 68.818°$ Amp

Current $I_o = I_{o_1} + I_{o_2}$

$= 4.57\angle -12.019° + 1.585\angle 68.816°$

$I_o = 5.07\angle 5.96°$ Amp.

9

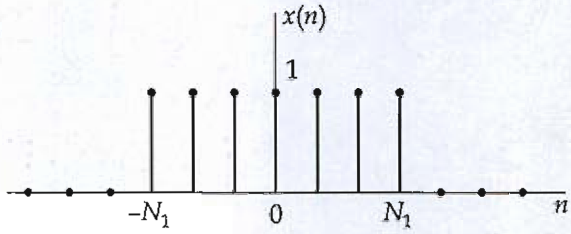
Good Approach

Section B : Systems & Signal Processing

Q.5 (a) Find the Fourier transform of the rectangular pulse

$$x(n) = u(n+N_1) - u(n-N_1-1) = \begin{cases} 1, & |n| \le N_1 \\ 0, & |n| > N_1 \end{cases}$$

which is illustrated in figure below. Also draw the magnitude and phase spectrum for $N_1 = 2$.



[12 marks]

Soln:-

$$X(e^{j\omega}) = \int_{-2}^{2} x(n)\, e^{\dots}$$

$x(n) = u(n+N_1) - u(n-(N_1+1))$

Taking z-txf on both sides -

$$X(z) = \frac{z^{N_1}}{1-z^{-1}} - \frac{z^{-(N_1+1)}}{1-z^{-1}}$$

$$X(z) = \frac{z^{N_1} - z^{-(N_1+1)}}{(1-z^{-1})}$$

Substitute $x = e^{j\omega}$

$$X(e^{j\omega}) = \frac{e^{j\omega N_1} - e^{-j\omega(N_1+1)}}{(1-e^{-j\omega})}$$

↳ Fourier txf.

for $N_1 = 2$

$$X(e^{j\omega}) = \frac{e^{j2\omega} - e^{-j\omega 3}}{1 - e^{-j\omega}} = \frac{e^{2j\omega} - \frac{1}{e^{3j\omega}}}{\frac{e^{j\omega} - 1}{e^{j\omega}}}$$

$$X(e^{j\omega}) = \frac{e^{5j\omega} - 1}{e^{j\omega} - 1} \cdot e^{j\omega - 3j\omega} \quad \left[= \frac{\frac{e^{5j\omega} - 1}{e^{3j\omega}}}{\frac{e^{j\omega} - 1}{e^{j\omega}}} \right]$$

$$= \frac{e^{5j\omega} - 1}{e^{j\omega} - 1} \cdot e^{-2j\omega}$$

$$= \left[\frac{e^{3j\omega} - e^{-2j\omega}}{e^{j\omega} - 1}\right] \times \frac{e^{-j\omega} + 1}{e^{-j\omega} + 1}$$

$$= \frac{e^{j2\omega} + e^{j3\omega} - e^{-j3\omega} - e^{-j2\omega}}{(e^{j\omega} - 1)(e^{-j\omega} + 1)}$$

$$= \frac{2\cos 2\omega + 2j\sin 3\omega}{1 + e^{j\omega} - e^{-j\omega} - 1}$$

$$X(e^{j\omega}) = \frac{2\cos 2\omega + 2j\sin 3\omega}{2j\sin\omega}$$

$$= \frac{\sin 3\omega}{\sin\omega} - j\frac{\cos 2\omega}{\sin\omega}$$

(2)

Go through the made easy solution

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Q.5 (b) Consider the continuous-time signal $x(t) = \cos(100\pi t)$:

(i) Determine the minimum sampling rate required to avoid aliasing.

(ii) Suppose that the signal is sampled at the rate $f_s = 200$ Hz. What is the discrete-time signal obtained after sampling?

(iii) Suppose that the signal is sampled at the rate $f_s = 75$ Hz. What is the discrete-time signal obtained after sampling?

(iv) What is the frequency $0 < f < f_s/2$ of a sinusoid that yields samples identical to those obtained in part (iii)?

[12 marks]

Soln:

$x(t) = \cos(100\pi t)$

$\hookrightarrow \omega_m = 100\pi$ rad/sec

$f_m = \frac{\omega_m}{2\pi} = \frac{100\pi}{2\pi} = 50$ Hz

(i) Minimum sampling rate to avoid aliasing →

$f_s = 2f_m = 2 \times 50 = 100$ Hz

$f_s = 100$ Hz Ans.

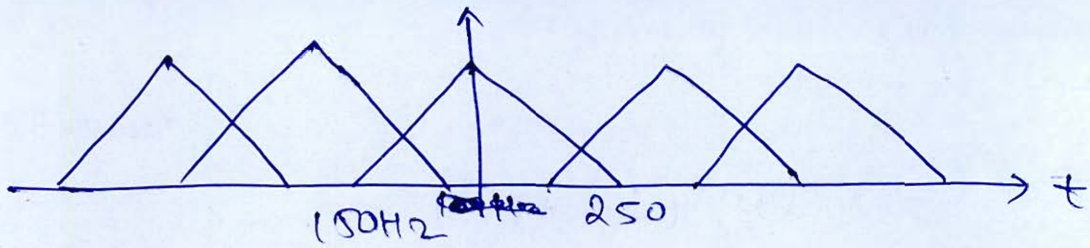
(ii) $x(t) = \cos(100\pi t)$

$f_s - f_m = 200 - 50 = 150$ Hz

$f_s + f_m = 200 + 50 = 250$ Hz

$f_s - 2f_m = 200 - 100 = 100$ Hz

$f_s + 2f_m = 200 + 100 = 300$ Hz



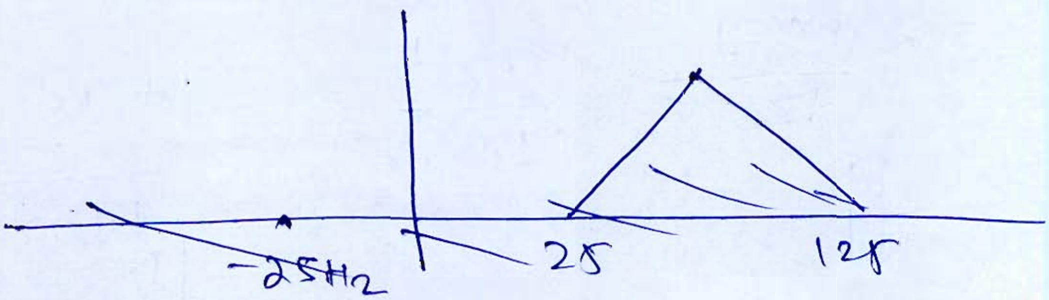
iii) when $f_s = 75$ Hz

$f_s - f_m = 75 - 50 = 25$ Hz

$f_s + f_m = 125$ Hz

$f_s - 2f_m = 75 - 100 = -25$ Hz

$f_s + 2f_m = 175$ Hz



G.B = 125 - 25 = 100 Hz

the pulses are separated by a guard band distance of 100 Hz.

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Q.5 (c) Determine the signal $x(n)$ whose z-transform is given by
$X(z) = \log(1 + az^{-1}),\ |z| > |a|$

[12 marks]

Sol^n. $X(z) = \log(1 + az^{-1})$

differentiating both sides w.r.t z

$$\frac{dX(z)}{dz} = \frac{1}{1+az^{-1}}(0 + a \times -1 z^{-2})$$

$$\frac{dX(z)}{dz} = \frac{-az^{-2}}{(1+az^{-1})}$$

$$-z\frac{dX(z)}{dz} = \frac{az^{-1}}{(1+az^{-1})} = \frac{1+az^{-1}-1}{1+az^{-1}}$$

$$nx(n) = \text{I.Z.T}\left[1 - \frac{1}{1+az^{-1}}\right]$$

10

$$nx(n) = \delta(n) - (-a)^n u(n)$$

Good Approach

$$x(n) = \frac{1}{n}\left[\delta(n) - (-a)^n u(n)\right]$$

Ans

Q.5 (d) Discuss the Dirichlet conditions for the existence of the Continuous-Time Fourier Transform. Are these conditions mandatory for a signal to possess a Fourier transform? Explain with examples.

[12 marks]

Soln:- Dirichlet conditions—

1.) Signal should have finite number of maxima & minima.

2.) Signal should ~~have~~ finite no. of discontinuities.

3.) Signal should be absolutely integrable.

Dirichlet conditions ~~have~~ are sufficient but not necessary. This is because there are numerous no. of signal which are not following these conditions but Fourier transform of the signals are existing.

for eg:- $\delta(t) \rightleftharpoons 1$

But $\delta(t)$ has ∞ amount/length of discontinuity.

10

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5 (e) Find the Fourier transform of the signum function $x(t) = \text{sgn}(t)$.

$$\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$

Also draw the magnitude and phase spectrum of $X(\omega)$.

[12 marks]

Soln:- $\text{sgn}(t) = u(t) - u(-t)$

we know that

$$u(t) \rightleftharpoons \frac{1}{j\omega} + \pi\delta(\omega)$$

$$x(-t) \rightleftharpoons X(-\omega)$$

So $u(-t) \rightleftharpoons -\frac{1}{j\omega} + \pi\delta(\omega)$

$F(\text{sgn}(t))$ = fourier txf of $u(t)$ − Fourier txf. of $u(-t)$

$$= \frac{1}{j\omega} + \pi\delta(\omega) - \left[-\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

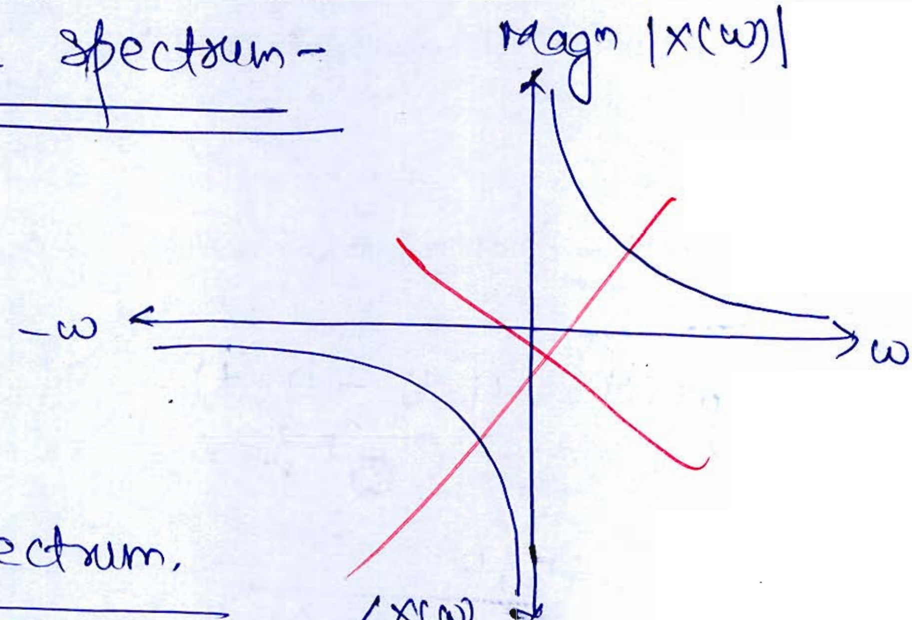
$$\boxed{F(\text{sgn}(t)) = \frac{2}{j\omega}} \quad X(\omega) \qquad = \frac{2}{\omega}\angle -90^\circ$$

|| Ans.

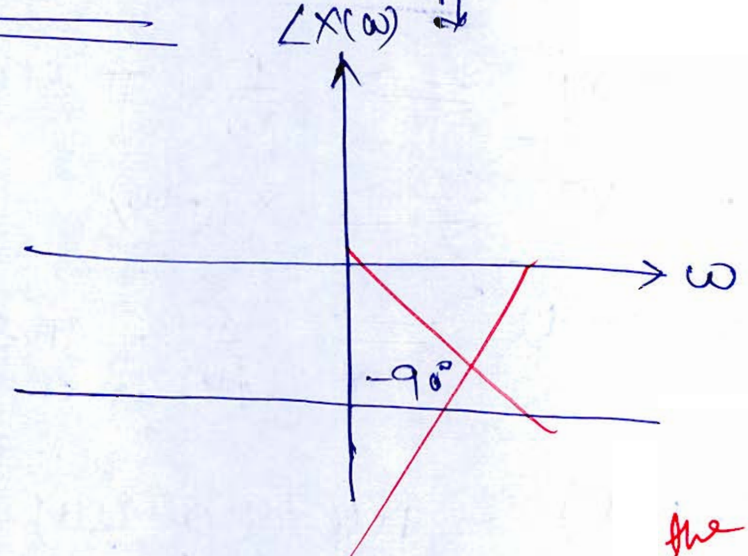
10

Incomplete solution

Magnitude spectrum-



Phase spectrum.



Go through the made easy solution

.6 (a) (i) An LTI system has a unit step response given by $s(t) = (1 - e^{-t} - te^{-t})u(t)$. For a certain input $x(t)$, the output is observed to be equal to $y(t) = (2 - 3e^{t} + e^{-3t})u(t)$. What is $x(t)$?

[12 marks]

Sol^n:- Given $s(t) = (1 - e^{-t} - te^{-t})\,u(t)$

for $x(t) = u(t)$

System transfer function $= \frac{S(s)}{X(s)}$

$$= \frac{\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}}{\frac{1}{s}}$$

$$= \frac{\frac{(s+1)^2 - s(s+1) - s}{s(s+1)^2}}{1/s} = \frac{s^2 + 2s + 1 - s^2 - s - s}{(s+1)^2}$$

$$H(s) = \frac{1}{(s+1)^2}$$

for i/p $y(t) = (2 - 3e^{t} + e^{-3t})\,u(t)$

$$Y(s) = \frac{2}{s} - \frac{3}{s-1} + \frac{1}{s+3}$$

$$H(s) = \frac{Y(s)}{X(s)} \Rightarrow X(s) = \frac{Y(s)}{H(s)}$$

$$= \left[\frac{2(s-1)(s+3) - 3s(s+3) + s(s-1)}{(s)(s-1)(s+3)}\right](s+1)^2$$

$$= \left[\frac{2[s^2 + 3s - s - 3] - 3s^2 - 9s + s^2 - s}{s(s-1)(s+3)}\right](s+1)^2$$

$$= \left[\frac{2s^2 + 4s - 6 - 2s^2 - 10s}{s(s-1)(s+3)}\right](s+1)^2$$

$$= \left[\frac{-6s-6}{s(s-1)(s+3)}\right](s+1)^2$$

$$= -6\frac{[s+1]^3}{s[s-1][s+3]} = -\frac{6[s+1]^3}{s^3+2s^2-3s}$$

$$= -6\left[1 + \frac{s^2+6s+1}{s(s-1)(s+3)}\right]$$

$$X(s) = \frac{s^2+6s+1}{s(s-1)(s+3)} = \frac{A}{s} + \frac{B}{(s-1)} + \frac{C}{s+3} \quad X(s)$$

$$s^2+6s+1 = A(s-1)(s+3) + B[s^2+3s] + C[s^2-s]$$

$$\left.\begin{aligned} A+B+C &= 1 \\ +2A+3B-C &= 6 \\ -3A &= 1 \end{aligned}\right\} \rightarrow A = \frac{-1}{3},\ B = 2,\ C = \frac{-2}{3}$$

$$X(s) = -6\left[1 - \frac{1}{3s} + \frac{2}{s-1} - \frac{2}{3(s+3)}\right]$$

8

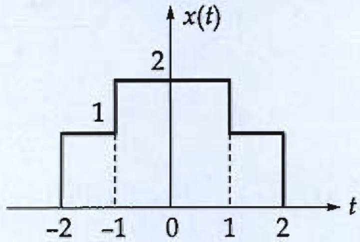
Taking Inverse laplace trf --

$$x(t) = -6\left[\delta(t) - \frac{1}{3}u(t) + 2e^{t}u(t) - \frac{2}{3}e^{-3t}u(t)\right]$$

Ans.

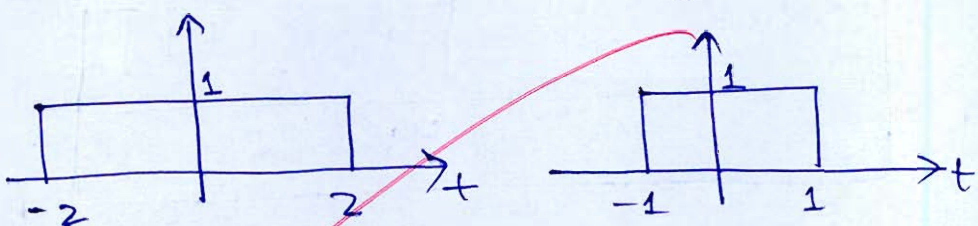
$$x(t) = \left(2 - 12e^{t} + 4e^{-3t}\right)u(t)$$

Q6 (a) **(ii)** Determine the Fourier transform of the signal shown in the following figure.



[8 marks]

Soln:- $x(t) = x_1(t) + x_2(t)$



$x_1(t) = A\ rect(t/\tau) = 1\ rect(t/4)$

$= rect(t/4)$

$x_2(t) = rect(t/2)$

we know that according to property –

$A\ rect(t/\tau) \rightleftharpoons A\tau\ Sa\left(\frac{\omega\tau}{2}\right)$

So, $rect(t/4) \rightleftharpoons 4\ Sa(2\omega)$

$rect(t/2) \rightleftharpoons 2\ Sa(\omega)$

6

$x(t) = x_1(t) + x_2(t)$

So, $X(\omega) = X_1(\omega) + X_2(\omega)$

$X(\omega) = 4\ Sa(2\omega) + 2\ Sa(\omega)$

$$X(\omega) = \frac{4 \sin 2\omega}{2\omega} + \frac{2 \sin \omega}{\omega}$$

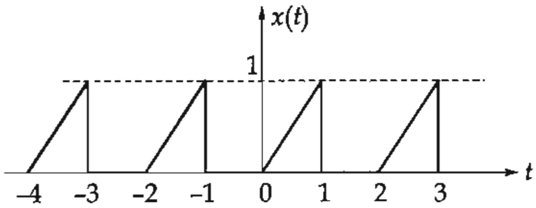
$$X(\omega) = \frac{2 \sin 2\omega + 2 \sin \omega}{\omega}$$

$$= \frac{4 \sin \omega \cdot \cos \omega + \sin \omega}{\omega}$$

$$X(\omega) = \frac{\sin \omega \left[4 \cos \omega + 1\right]}{\omega}$$

Ans

Q.6 (b) (i) Find the trigonometric Fourier series for the waveforms shown in figure below and sketch the line spectrum.



[12 marks]

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Q.6 (b) (ii) A causal and stable LTI system "S" has the property that when we apply the input; $\left(\frac{4}{5}\right)^n u(n)$, it gives the output $n\left[\frac{4}{5}\right]^n u(n)$.

Determine the transfer function $H(e^{j\omega})$ for the system.

[8 marks]

Given:-

$x(n) = \left(4/5\right)^n u(n)$

$y(n) = n\left[\frac{4}{5}\right]^n u(n)$

Applying z-trf $X(z) = \frac{1}{1-\frac{4}{5}z^{-1}}$

$Y(z) = \frac{\frac{4}{5}z^{-1}}{\left(1-\frac{4}{5}z^{-1}\right)^2}$

Transfer function

$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{\frac{4}{5}z^{-1}}{\left(1-\frac{4}{5}z^{-1}\right)^2}}{\frac{1}{\left(1-4/5\, z^{-1}\right)}} = \frac{\frac{4}{5}z^{-1}}{1-\frac{4}{5}z^{-1}}\ ;\ |z|>\frac{4}{5}$

7

Since $|z|>\frac{4}{5}$; Hence ROC includes unit circle. Hence, absolutely summable.

Substitute → $z = e^{j\omega}$

$$H(e^{j\omega}) = \frac{\frac{4}{5}e^{-j\omega}}{1-\frac{4}{5}e^{-j\omega}}$$

Ans

Good Approach

5 (c) (i) Determine the transient response and steady-state response of the system characterized by the difference equation, $y(n) = 0.5y(n-1) + x(n)$, when the input signal is $x(n) = 10\cos(n\pi/4)u(n)$. The system is initially at rest (i.e., it is relaxed).

[10 marks]

Soln:-

$$y(n) = 0.5y(n-1) + x(n) \quad —①$$

$$x(n) = 10\cos(n\pi/4)\,u(n)$$

put n=0 in eqn -①

$$y(0) = 0.5\,y(-1) + x(0)$$

$$0 = 0.5\,y(-1) + 10 \Rightarrow y(-1) = \frac{-10}{0.5} = -20$$

Zero i/p response (transient Response):-

$$Y(z) = 0.5\left[z^{-1}Y(z) + y(-1)\right] + X(z)^{\to 0}$$

$$= 0.5\left[z^{-1}Y(z) + (-20)\right]$$

$$Y(z)\left[1-0.5z^{-1}\right] = -20\times 0.5 = -10$$

$$Y(z) = \frac{-10}{1-0.5z^{-1}} \Rightarrow \boxed{y(n)_{ZIR} = -10\,(0.5)^n u(n)}$$

Zero state response (steady state response)

$$Y(z) = 0.5z^{-1}Y(z) + X(z)$$

$$Y(z)\left[1-0.5z^{-1}\right] = X(z)$$

$$x(n) = 10\cos(n\pi/4)\,u(n) \quad ; \quad \omega_0 = \pi/4$$

$$\Rightarrow X(z) = \left[\frac{z^2 - z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}\right]\times 10$$

$$= 10\left[\frac{z^2 - \frac{z}{\sqrt{2}}}{z^2 - \frac{2z}{\sqrt{2}} + 1}\right] = \frac{10\left[z^2 - \frac{z}{\sqrt{2}}\right]}{z^2 - \sqrt{2}z + 1}$$

$$Y(z) = \frac{X(z)}{1 - 0.5z^{-1}} = \frac{10z^2\left[1 - \frac{z^{-1}}{\sqrt{2}}\right]}{z^2\left[1 - \sqrt{2}z^{-1} + z^{-2}\right]\left[1 - 0.5z^{-1}\right]}$$

$$= \frac{10\left[1 - \frac{z^{-1}}{\sqrt{2}}\right]}{(1 - 0.5z^{-1})\left[1 - \sqrt{2}z^{-1} + z^{-2}\right]}$$

$$= \frac{A}{(1 - 0.5z^{-1})} + \frac{Bz^{-1} + C}{(1 - \sqrt{2}z^{-1} + z^{-2}]}$$

4

$$= \frac{A\left[1 - \sqrt{2}z^{-1} + z^{-2}\right] + (1 - 0.5z^{-1})(Bz^{-1} + C)}{(1 - 0.5z^{-1})(1 - \sqrt{2}z^{-1} + z^{-2})}$$

$$10 = A + C$$

$$\frac{-10}{\sqrt{2}} = -\sqrt{2}A + B - 0.5C$$

$$0 = A - 0.5B$$

$A = -1.907$

$B = -3.81$

$C = 11.9$

$$Y(z) = \frac{-1.907}{1 - 0.5z^{-1}} + \frac{-3.81z^{-1} + 11.9}{1 - \sqrt{2}z^{-1} + z^{-2}}$$

↓ Inverse z-trf

$y(n)_{ZSR}$

Do not write in this margin

6 (c) **(ii)** Solve the difference equation using the one-sided z-transform $y(n) = x(n) + by(n-1)$ with initial condition $y(-1) = P$. Assume input be $x(n) = e^{j\omega_0 n}u(n)$.

[10 marks]

Soln:-

$$y(n) = x(n) + b\,y(n-1)$$

z-txf -

$$Y(z) = X(z) + b\left[z^{-1}Y(z) + y(-1)\right]$$

$$Y(z)\left[1 - bz^{-1}\right] = X(z) + bP$$

$$Y(z) = \frac{X(z)}{1-bz^{-1}} + \frac{bP}{1-bz^{-1}}$$

$$x(n) = e^{j\omega_0 n}u(n) \rightleftharpoons X(z) = \frac{1}{1 - e^{j\omega_0}z^{-1}}$$

$$Y(z) = \underbrace{\frac{1}{(1-bz^{-1})(1-e^{j\omega_0}z^{-1})}}_{Y_1(z)} + \underbrace{\frac{bP}{1-bz^{-1}}}_{Y_2(z)}$$

$$Y_1(z) = \frac{1}{(1-bz^{-1})(1-e^{j\omega_0}z^{-1})} = \frac{A}{1-bz^{-1}} + \frac{B}{1-e^{j\omega_0}z^{-1}}$$

$$1 = A - Ae^{j\omega_0}z^{-1} + B - Bbz^{-1}$$

$$A + B = 1$$

$$-Ae^{j\omega_0} - Bb = 0 \Rightarrow Bb = -Ae^{j\omega_0}$$

$$B = \frac{-Ae^{j\omega_0}}{b}$$

$$A - \frac{Ae^{j\omega_0}}{b} = 1$$

$$A = \frac{b}{b - e^{j\omega_0}}$$

So; $B = -\frac{e^{j\omega_0}}{b} \times \frac{b}{b - e^{j\omega_0}}$

$$= -e^{j\omega_0} / b - e^{j\omega_0}$$

So; $Y(z) = \frac{b}{b - e^{j\omega_0}} \cdot \frac{1}{1 - bz^{-1}} - \frac{e^{j\omega_0}}{b - e^{j\omega_0}} \cdot \frac{1}{1 - e^{j\omega_0} z^{-1}} + \frac{bp}{1 - bz^{-1}}$

Applying Inverse z-txf:-

$y(n) = \frac{b}{b - e^{j\omega_0}} (b)^n u(n) - \frac{e^{j\omega_0}}{b - e^{j\omega_0}} (e^{j\omega_0 n}) u(n) + bp\,(b)^n u(n)$

$$y(n) = \frac{1}{b - e^{j\omega_0}} \left[b^{n+1} u(n) - (e^{j\omega_0})^{n+1} u(n) \right] + bp\, b^n u(n)$$

Ans

6

Good Approach

(a) Determine the values of power and energy for each of the following signals. Also find the nature of signals.

(i) $x_1(t) = e^{-2t}u(t)$.

(ii) $x_2(t) = e^{j(2t+\pi/4)}$.

(iii) $x_3(n) = \cos\left(\left(\frac{\pi}{4}\right)n\right)$.

[20 marks]

Sol^n:- (i) $x_1(t) = e^{-2t}u(t)$

Energy of signal $= \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_0^{\infty} (e^{-2t})^2 dt = \int_0^{\infty} e^{-4t} dt$

$= \left[\frac{e^{-4t}}{-4}\right]_0^{\infty}$

$E = \frac{1}{4}$

Since, it is an energy signal. So, its power is zero.

$E_1 = \frac{1}{4}$

$P_1 = 0$

Ans

(ii) $x_2(t) = e^{j(2t+\pi/4)}$

$= \cos(2t+\pi/4) + j\sin(2t+\pi/4)$

Since, cosine & sine functions are periodic power signals. Hence, its energy is not defined.

Power $x_2(t) = \frac{1}{T}\int_T |x(t)|^2 dt$

$= (RMS)^2 = (1)^2 = 1$

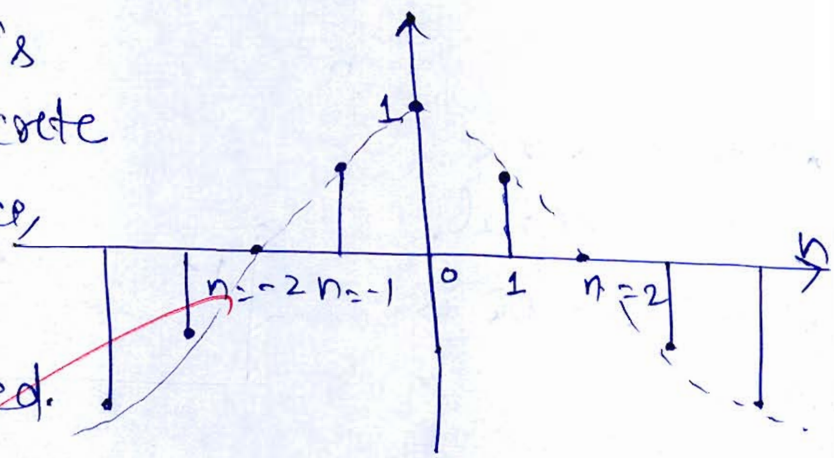
$E_{x_2(t)} = \infty$

$P_{x_2(t)} = 1$

Ans.

(iii) $x_3(n) = \cos\left[\frac{\pi}{4} n\right]$

Since, $x_3(n)$ is periodic discrete signal. Hence, its energy is not defined.



Power $= \dfrac{1^2 + 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 2\times 0}{5} = \dfrac{1 + 2\times\frac{1}{2} + 0}{5}$

$= \frac{2}{5} = 0.4$

$E_{x_3(n)} =$ not-defined.

$P_{x_3(n)} = 0.4$

14

Go through the made easy solution

Do not write this margin

Q.7 (b) Find the inverse Laplace transform of

$$X(s) = \frac{-5s-7}{(s+1)(s-1)(s+2)}$$

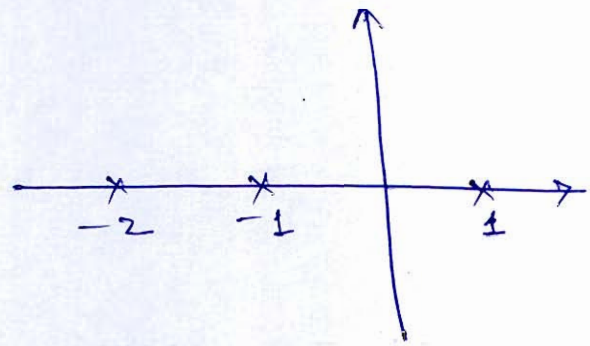
if the ROC is

(i) $\Re\{s\} > 1$

(ii) $\Re\{s\} < -2$

(iii) $-1 < \Re\{s\} < 1$

(iv) $-2 < \Re\{s\} < -1$



[20 marks]

Sol^n

$$X(s) = \frac{-5s-7}{(s+1)(s-1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s-1)} + \frac{C}{(s+2)}$$

$$-5s-7 = A[s-1][s+2] + B[s+1][s+2] + C[s^2-1]$$

$$-5s-7 = A[s^2+s-2] + B[s^2+3s+2] + C[s^2-1]$$

$$\left.\begin{aligned} A+B+C &= 0 \\ A+3B &= -5 \\ -2A+2B-C &= -7 \end{aligned}\right\} \rightarrow \begin{aligned} A &= 1 \\ B &= -2 \\ C &= 1 \end{aligned}$$

So; $X(s) = \frac{1}{s+1} - \frac{2}{s-1} + \frac{1}{s+2}$

i) For ROC : $Re\{s\} > 1$

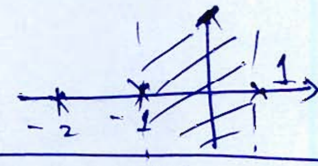
Inverse laplace txf –

$$\boxed{x(t) = e^{-t}u(t) - 2e^{t}u(t) + e^{-2t}u(t)}$$

i) Re{s} < -2 ; x(t) is left-sided.

$$x(t) = -e^{-t}u(-t) + 2e^{t}u(-t) - e^{-2t}u(-t)$$

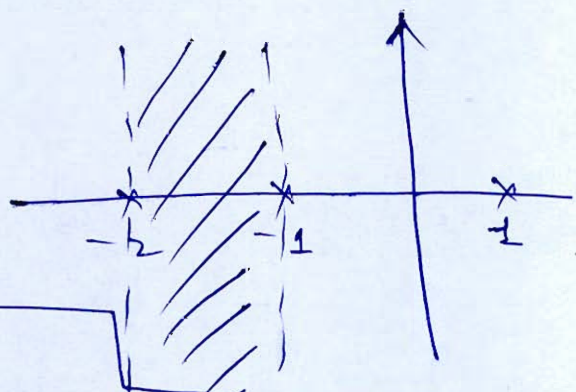
ii) For ROC: $-1 < Re\{s\} < 1$



$$x(t) = e^{-t}u(t) + 2e^{t}u(-t) + e^{-2t}u(t)$$

$\sigma > -1$ $\sigma < 1$ $\sigma > -2$

v) For ROC: $-2 < Re\{s\} < -1$



$$x(t) = -e^{-t}u(t) + 2e^{t}u(-t) + e^{-2t}u(t)$$

Ans

18

Good Approach

(c) An analog filter has the transfer function:

$$H_a(s) = \frac{10}{s^2 + 7s + 10}$$

Design a digital filter $H(z)$ using the Bilinear Transformation method with a sampling period of $T = 0.2$ seconds.

(i) Determine the discrete-time transfer function $H(z)$.

(ii) Find the difference equation of the system relating $x(n)$ and $y(n)$.

(iii) Determine the poles of $H(z)$ and comment on the stability of the digital filter.

[20 marks]

Soln:- Given: -

$$H_a(s) = \frac{10}{s^2+7s+10} = \frac{10}{s^2+5s+2s+10}$$

$$= \frac{10}{s(s+5)+2(s+5)} = \frac{10}{(s+2)(s+5)}$$

$$H_a(s) = \frac{10}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

$$= \frac{As+5A+Bs+2B}{(s+2)(s+5)} = \frac{10}{(s+2)(s+5)}$$

$$\left.\begin{array}{l} A+B=0 \\ 5A+2B=10 \end{array}\right\} \rightarrow A = 10/3, \quad B = \frac{-10}{3}$$

$$H_a(s) = \frac{10}{3(s+2)} - \frac{10}{3(s+5)}$$

$$h_a(t) = \left(\frac{10}{3}e^{-2t} - \frac{10}{3}e^{-5t}\right)u(t)$$

$\downarrow t = nT_s = 0.2n$

$$h_a(n) = \left(\frac{10}{3}e^{-2\times 0.2n} - \frac{10}{3}e^{-5\times 0.2n}\right)u(n)$$

$$h_a(n) = \left(\frac{10}{3}e^{-0.4n} - \frac{10}{3}e^{-n}\right)u(n)$$

~~On~~ applying ~~z-T×f~~

$H_a(s) = \frac{10}{3(s+2)} - \frac{10}{3(s+5)}$

~~$H_a(z) =$~~

In bilinear transformation -

$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = 10\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

So; $H_a(z) = \frac{10}{\left[10\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 70\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 10}$

$H_a(z) = \frac{10(1+z^{-1})^2}{100(1-z^{-1})^2 + 70(1-z^{-1})(1+z^{-1}) + 10(1+z^{-1})^2}$

$H_a(z) = \frac{10\left[1+z^{-2}+2z^{-1}\right]}{100\left[1+z^{-2}-2z^{-1}\right] + 70\left[1-z^{-2}\right] + 10\left[1+z^{-2}+2z^{-1}\right]}$

$$H_a(z) = \frac{10\left[z^{-2}+2z^{-1}+1\right]}{180+40z^{-2}-180z^{-1}}$$

Ans - (1)

$\frac{Y(z)}{X(z)} = \frac{10z^{-2}+20z^{-1}+10}{40z^{-2}-180z^{-1}+180}$

$$4y(n-2) - 18y(n-1) + 18y(n) = x(n-2) + 2x(n-1) + x(n)$$

Ans - (2)

$$\frac{Y(z)}{X(z)} = \frac{z^{-2} + 2z^{-1} + 1}{\left(1 - \frac{2}{3}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)}$$

Poles at $z = \frac{2}{3}$, $z = \frac{1}{3}$

Since, the poles are inside unit circle.
Hence the system is **Stable**.

Ans ③

18

Good Approach

Q.8 (a) Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

(i) $x(n) = (0.8)^n u(n)$ and $h(n) = (0.4)^n u(n)$.

(ii) $x(n) = u(n-1)$ and $h(n) = \alpha^n u(n-1)$.

(iii) $x(n) = r(n) = nu(n)$ and $h(n) = -\alpha^{-n} u(n-1)$, where $a < 1$.

[20 marks]

Do wr thi

Q.8 (b) Consider a periodic square wave $x(t)$ with amplitude A, period T centered at $t = 0$, and duty cycle 50%:

(i) Derive the expression for the Trigonometric Fourier Series coefficients.

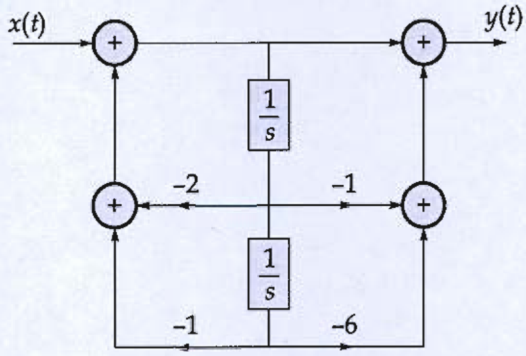
(ii) Explain the Gibbs Phenomenon in the context of reconstructing this signal using a finite number of harmonics.

(iii) Use the duality property of the Fourier Transform to find the transform of

$$g(t) = \frac{\sin(at)}{\pi t}.$$

[20 marks]

(i) The input $x(t)$ and output $y(t)$ of a causal LTI system are related through the block diagram representation shown in the below figure.



1. Determine a differential equation relating $y(t)$ and $x(t)$.
2. Is this system stable?

(ii) The input and output of a causal LTI system is related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

What is the response of this system, if $x(t) = te^{-2t}u(t)$?

[10 + 10 marks]

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Space for Rough Work

Space for Rough Work