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ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electrical Circuits + Systems & Signal Processing

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	34
Q.2	26
Q.3	
Q.4	46
Section-B	
Q.5	38
Q.6	
Q.7	46
Q.8	
Total Marks Obtained	190

Signature of Evaluator: Sourabh kumar

Cross Checked by: ....

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

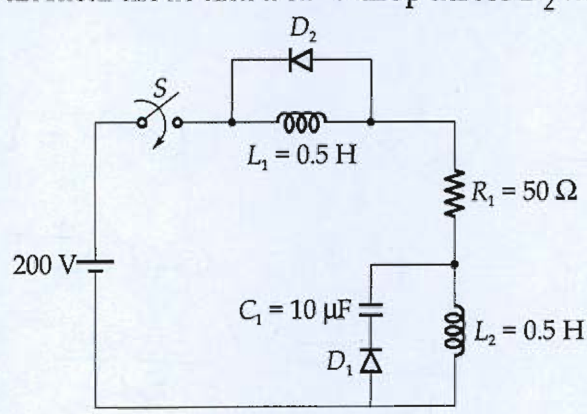
DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Do not write in this margin

Section A : Electrical Circuits

(a) In the circuit shown below the switch '*S*' is closed at $t = 0$, and is opened after 10 ms. What will be the currents in R_1, L_1 and L_2, and voltage across C_1, 8 ms after switch '*S*' opens? Assume D_1 to be an ideal diode and a 0.7 V drop across D_2 whenever it conducts.



[12 marks]

Step 1 : at $t = 0^-$
switch S is open.

$\Rightarrow i_{L1}(0^-) = 0$

$i_{L2}(0^-) = 0$

and $V_{C1}(0^-) = 0$

Step 2 : at $t = 0^+$
L does not allow sudden change in current and capacitor does not allow sudden change in voltage.

$\Rightarrow i_{L1}(0^+) = i_{L1}(0^-) = 0$

$i_{L2}(0^+) = i_{L2}(0^-) = 0$

and $V_{C1}(0^+) = V_{C1}(0^-) = 0$

at $t = \infty$ ~~switch~~ with voltage source so
$L \rightarrow$ short circuit and
$C \rightarrow$ Open circuit.

so $i(\infty) = \frac{200}{50} = 4A$

diode D_1 and D_2 will be off only.

$\Rightarrow i_L(t) = i(\infty) + (i(0^+) - i(\infty))e^{-t/\tau}$

where $\tau \rightarrow$ time const.

$\tau = \frac{L}{R} = \frac{0.5+0.5}{50} = \frac{1}{50}$

$\Rightarrow i_{L1}(t) = 4(1 - e^{-t/50})$ A

$i_{L2}(t) = 4(1 - e^{-t/50})$ A

at $t = 8$ ms

$i_{L1}(t) = i_{L2}(t) = 4(1 - e^{-8\times10^{-3}/50})$

$= \underline{6.4 \text{ mA}}$

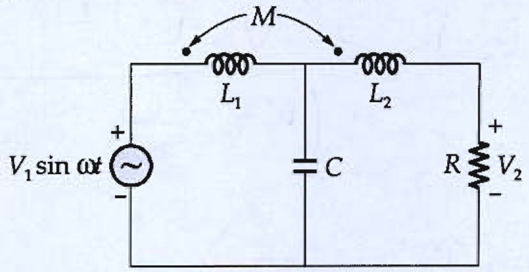
and V_c (at $t = 8$ ms) $= 0$ V

also $i_{R1}(t) = i_{L1}(t) = i_{L2}(t) = \underline{6.4 \text{ mA}}$

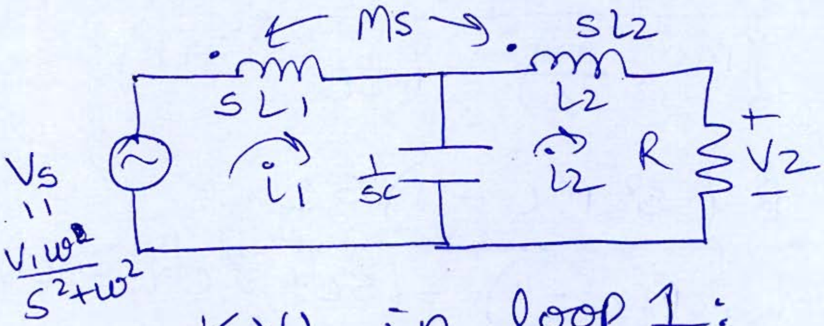
5

Wrong value calculated

(b) Find the voltage transfer function V_2/V_1 for the network given below:



[12 marks]



converting to s-domain

KVL in loop 1:

$$V_s - i_1(sL_1) - i_2 Ms - (i_1 - i_2)\frac{1}{sC} = 0$$

$$\Rightarrow V_s = i_1\left(sL_1 + \frac{1}{sC}\right) + i_2\left(Ms - \frac{1}{sC}\right) \quad -(1)$$

KVL in loop 2:

$$i_2\left(\frac{1}{sC} + sL_2 + R\right) - i_1 \times \frac{1}{sC} + i_1 Ms = 0$$

$$\Rightarrow i_1\left(Ms - \frac{1}{sC}\right) + i_2\left(\frac{1}{sC} + sL_2 + R\right) = 0$$

$$\Rightarrow i_1 = \frac{-i_2\left(\frac{1}{sC} + sL_2 + R\right)}{Ms - \frac{1}{sC}} \quad -(2)$$

using (2) in (1)

$$V_s = \frac{-i_2\left(\frac{1}{sC} + sL_2 + R\right)}{Ms - \frac{1}{sC}} + i_2\left(Ms - \frac{1}{sC}\right)$$

$$\Rightarrow i_2 = \frac{V_s\left(Ms - \frac{1}{sC}\right)}{\left(Ms - \frac{1}{sC}\right)^2 - \left(\frac{1}{sC} + sL_2 + R\right)} \quad -(3)$$

now by Ohm's law

$$V_2 = i_2 R = \frac{V_s R\left(ms - \frac{1}{sC}\right)}{\left(ms - \frac{1}{sC}\right)^2 - \left(\frac{1}{sC} + sL_2 + R\right)}$$

$$\Rightarrow \frac{V_2}{V_s} = \frac{R\left(ms - \frac{1}{sC}\right)}{\left(ms - \frac{1}{sC}\right)^2 - \left(\frac{1}{sC} + sL_2 + R\right)}$$

$$\frac{V_2}{V_s} = \frac{R(s^2 mC - 1)sC}{(s^2 mC - 1) - (sC + s^3 C^2 L_2 + s^2 C^2 R)}$$

where $V_s = \frac{V_1 . \omega}{s^2 + \omega^2}$

$$\Rightarrow \boxed{\frac{V_2}{V_1} = \frac{(s^2+\omega^2) R (s^2 mC - 1) sC}{\omega \{ (s^2 mC - 1) - (sC + s^3 C^2 L_2 + s^2 C^2 R)}}$$

is the required transfer function.

Simplify

(7)

$$\frac{SRC\,(1 - s^2 mC)}{(s^2 L_1 C + 1)(s^2 L_2 C + SRC) - (}$$

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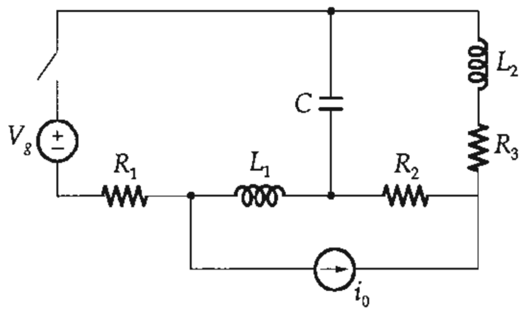
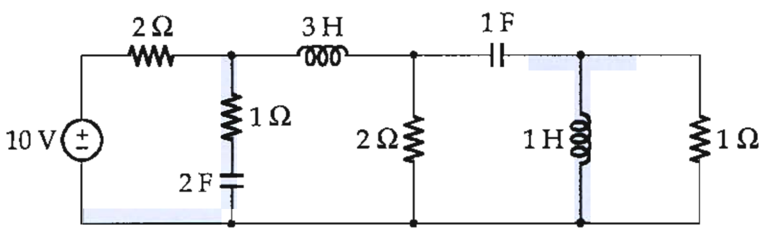
(c) Draw the dual of the circuit shown in figure.

1.

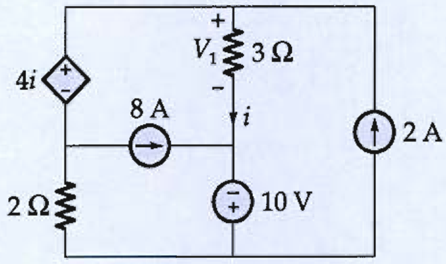


2.



[12 marks]

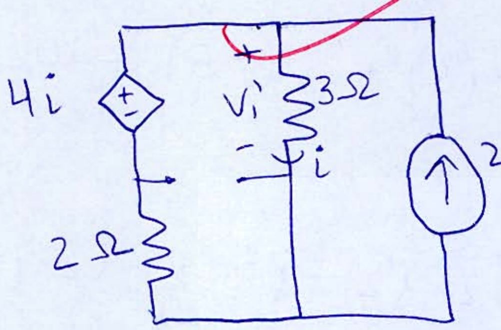
(d) Using the superposition theorem determine V_1, the voltage across the 3 Ω resistor in figure below,



[12 marks]

Three independent sources are present so $V_1 = V_1' + V_1'' + V_1'''$

using 2 A current source: deactivating other sources.



by KCL

$$\frac{V_1'}{3} + \frac{V_1' - 4i}{2} = 2$$

Good Approach

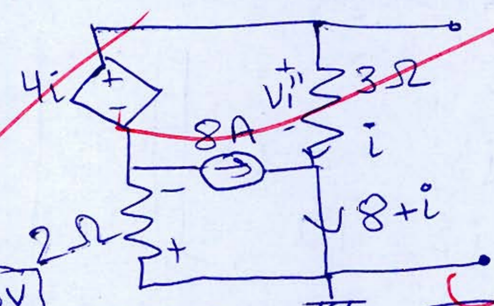
$$\frac{5V_1'}{6} = 2 + 2i \quad \text{where } i = \frac{V_1'}{3}$$

$$V_1'\left(\frac{5}{6} - \frac{2}{3}\right) = 2$$

$$\Rightarrow \boxed{V_1' = 12\,V}$$

using 8 A current source: other sources deactivates

so ckt becomes



using KVL:

$$-2(8+i) + 4i - 3i = 0$$

$$-16 - 2i + i = 0$$

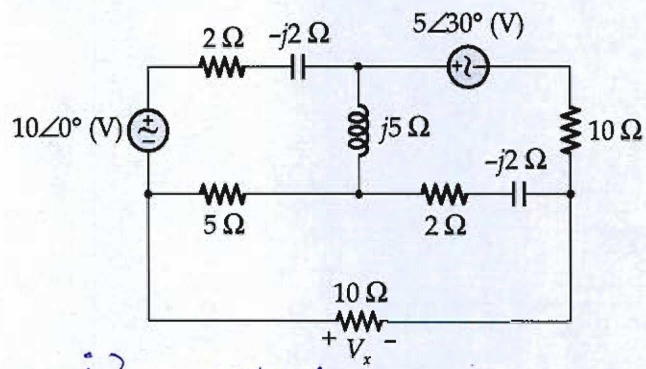
$$\Rightarrow i = -16\,A$$

$$\Rightarrow \boxed{V_1'' = -48V}$$

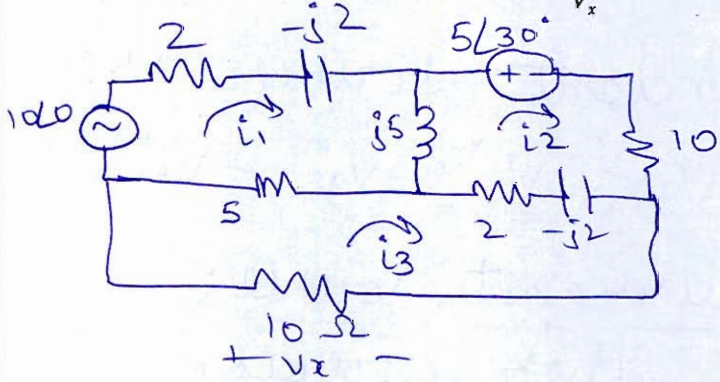
and $\boxed{V_3'' = 30V}$

$\Rightarrow \boxed{V = +12 - 48 + 30 = -6V}$

Q.1 (e) Write the loop equations of the circuit and find the voltage V_x.



[12 marks]



KVL in loop 1:

$$i_1(7+j3) + i_2(-j5) + i_3(-5) = 10\angle 0 \quad \text{—(1)}$$

KVL in loop 2:

$$-5\angle 30 - 10 i_2 - i_2(2-j2) - j5\, i_2 + (2-j2) i_3 + j5\, i_1 = 0$$

$$\Rightarrow i_1(-j5) + i_2(12+j3) + i_3(-2+j2) = -5\angle 30 \quad \text{—(2)}$$

KVL in loop 3:

$$i_3(17) - 5 i_1 - i_2(2-j2) = 0$$

$$\Rightarrow i_1(-5) + i_2(-2+j2) + i_3(17) = 0 \quad \text{—(3)}$$

in Matrix form:

$$\begin{bmatrix} 7+j3 & -j5 & -5 \\ -j5 & 12+j3 & -2+j2 \\ -5 & -2+j2 & 17 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ -5\angle 30 \\ 0 \end{bmatrix}$$

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by Kramer's rule: $i_3 = \frac{\Delta_3}{\Delta}$

$$\Delta_3 = \begin{bmatrix} 7+j3 & -j5 & 10 \\ -j5 & 12+j3 & -5\angle 30 \\ -5 & -2+j2 & 0 \end{bmatrix}$$

$$= -5(+j5 \times 5\angle 30 - 10(12+j3)]$$
$$- (-2+j2)[-5\angle 30(7+j3) + j50]$$

$$\Rightarrow \Delta_3 = 667.96\angle 10.9^\circ$$

&

$$\Delta = (7+j3)\{17(12+j3) - (-2+j2)^2\}$$
$$+ j5\{-85j + (-10+j10)\}$$
$$- 5(-j5(-2+j2) + 5(12+j3)\}$$

$$\Delta = 1533.19\angle 33.67^\circ$$

$$\Rightarrow i_3 = \frac{667.96\angle 10.9}{1533.19\angle 33.67}$$

$$= 0.435\angle -22.7^\circ$$

$$\Rightarrow V_x = -i_3 \times 10$$

$$\Rightarrow \boxed{V_x = 4.35\angle 157.3^\circ}$$

$$\Rightarrow \boxed{V_x = 4.35\ V}$$

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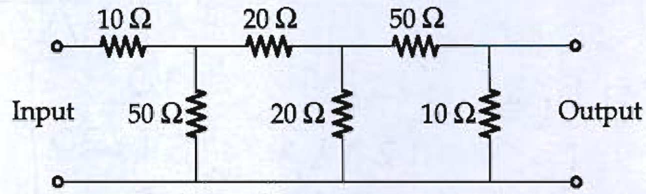
Q.2 (a) (i) Obtain the $ABCD$ parameters for the network shown in figure below.

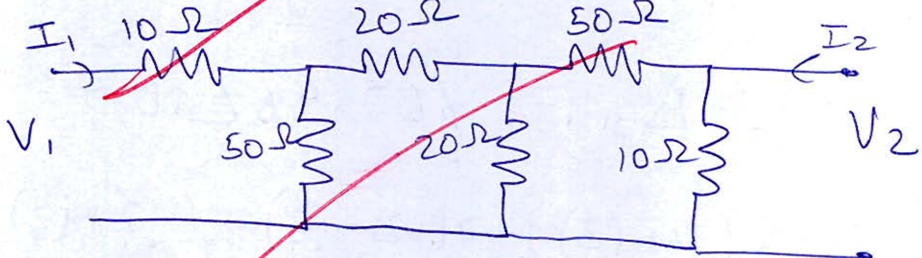


[12 marks]

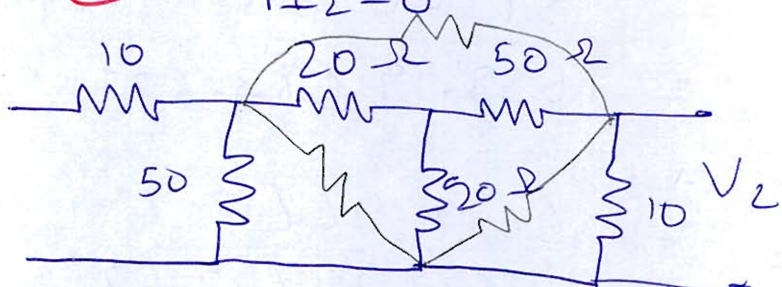
ABCD parameters are given by

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$



convert Y to Δ

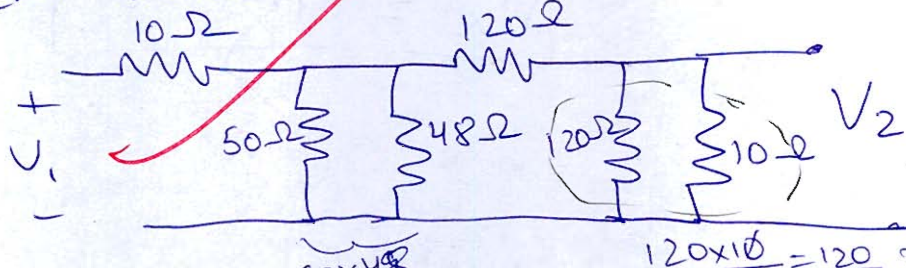
$R_a = 20 + 20 + \frac{20 \times 20}{50}$

$= 48\Omega$

$R_b = 20 + 50 + \frac{20 \times 50}{20}$

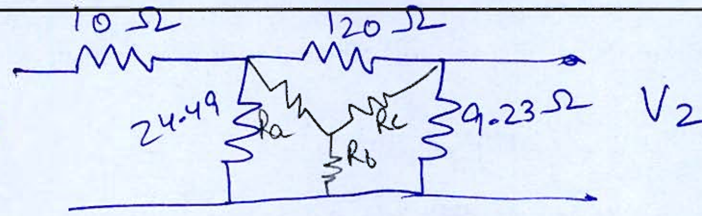
$R_b = R_c = 120\Omega$

so ckt becomes



$\frac{50 \times 48}{98} = 24.49$

$\frac{120 \times 10}{130} = \frac{120}{13} = 9.23$

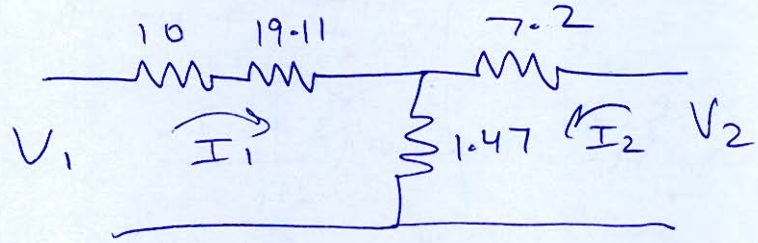


Δ to Y : $R_a = \dfrac{24.49 * 120}{24.49+120+9.23} = 19.11\,\Omega$

$R_b = \dfrac{24.49 \times 9.23}{24.49+120+9.23} = 1.47\,\Omega$

$R_c = \dfrac{120 \times 9.23}{24.49+120+9.23} = 7.2\,\Omega$

so ckt



$\Rightarrow V_1 = 30.58 I_1 + 1.47 I_2$ — ①

& $V_2 = 1.47 I_1 + 8.67 I_2$ — ②

rearranging

$I_1 = \dfrac{1}{1.47} V_2 - 8.67 I_2$ — ③

using ③ in ①

$V_1 = \dfrac{30.58}{1.47} V_2 + I_2\left(1.47 - 30.58 \times 8.67\right)$

$V_1 = 20.8 V_2 - 263.65 I_2$ — ④

comparing with std

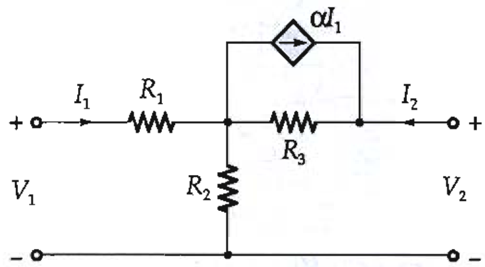
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 20.8 & -263.65 \\ 0.68 & -8.67 \end{bmatrix}$$

Ans

4

Use T and π Network

Q.2 (a) (ii) Find the hybrid parameters for the network shown in figure below.

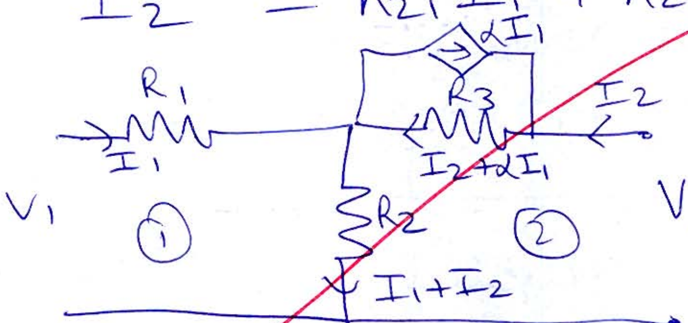


[8 marks]

hybrid parameters are given as

$V_1 = h_{11} I_1 + h_{12} V_2$ V_1, V_2, I_1, I_2

$I_2 = h_{21} I_1 + h_{22} V_2$



KVL in loop ①

$\Rightarrow V_1 = I_1(R_1+R_2) + R_2 I_2$ — ①

KVL in loop ②:

$V_2 = I_2 R_3 + \alpha I_1 R_3 + I_1 R_2 + I_2 R_2$

$= I_1(\alpha R_3 + R_2) + I_2(R_3+R_2)$

$\Rightarrow I_2 = \frac{(-\alpha R_3 - R_2)}{R_3+R_2} I_1 + \frac{1}{R_3+R_2} V_2$ — ②

using ② in ①

$V_1 = I_1\left(R_1+R_2 - \frac{R_2(R_2+\alpha R_3)}{R_2+R_3}\right) + \frac{R_2}{R_2+R_3} V_2$

comparing with std equations

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$$h_{11} = R_1 + R_2 - \frac{R_2(R_2+\alpha R_3)}{R_2+R_3}$$

$$h_{12} = \frac{R_2}{R_2+R_3}$$

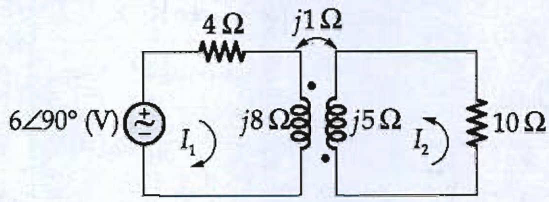
$$h_{21} = -\frac{(R_2+\alpha R_3)}{R_2+R_3}$$

$$h_{22} = \frac{1}{R_3+R_2}$$

are the required hybrid parameters.

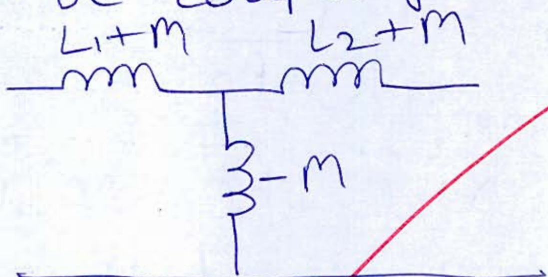
7

Q.2 (b) (i) Determine the current I_1 and I_2 in the circuit shown in figure below, using T-equivalent circuit for the linear transformer.

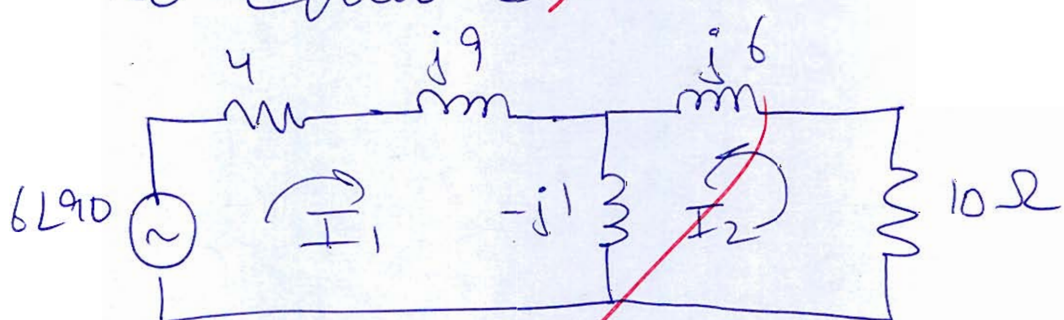


[8 marks]

in Loop 1 I_1 enters dot
& in loop 2 I_2 leaves dot
so −ve coupling so equiv. T-N/W



so equiv. ckt :



KVL in loop 1 :

$I_1(4+j9-j1)+I_2(-j1) = 6\angle 90$

$\Rightarrow I_1(4+j8)+I_2(-j1) = 6\angle 90$ — ①

KVL in loop 2 :

$-10I_2 - j6I_2 + j1I_1 + j_1I_2 = 0$

$\Rightarrow I_2(10+j5) = jI_1$

$\Rightarrow \boxed{I_1 = (5-j10)I_2}$ — ②

Do not write in this margin

using eq^n ② in ①

$((5-j10)(4+j8) - j1) I_2 = 6\angle 90$

$\Rightarrow I_2 = \dfrac{6\angle 90}{(5-j10)(4+j8) - j1}$

$I_2 = 0.06\angle 90.57°$

$\Rightarrow I_1 = 0.67\angle 27.13°$

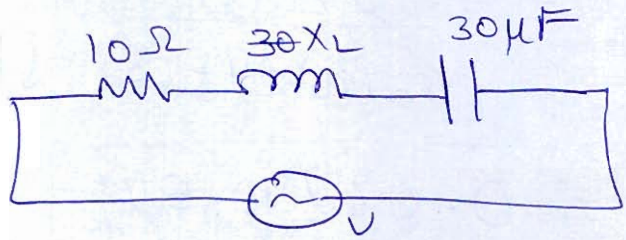
8

Good Approach

Q.2 (b) **(ii)** A voltage of $v = (2000 \sin \omega t + 400 \sin 3\omega t + 100 \sin 5\omega t)$ volts is applied to a series circuit having $R = 10\ \Omega$ and $C = 30\ \mu F$ and a variable inductance.

1. Find the value of inductance so as to give resonance at 3rd harmonic frequency.
2. What are the rms values of voltage and current with this inductance in circuit? (Take $\omega = 300$ rad/sec).

[12 marks]



equiv imp. $Z = R + j(X_L - X_C)$

i) resonance at 3rd harmonic freq.

so $X_L = 3\omega L$ & $X_C = \frac{1}{3\omega C}$

$\Rightarrow Z_3 = R + j\left(3\omega L - \frac{1}{3\omega C}\right)$

at resonance img part = 0

$\Rightarrow 3\omega L = \frac{1}{3\omega C}$

substituting values

$L = \frac{1}{9 \times 300 \times 300 \times 30 \times 10^{-6}}$

$L = 41.15\ mH$

2) Rms value of voltage

$V_{Rms} = \sqrt{\left(\frac{2000}{\sqrt{2}}\right)^2 + \left(\frac{400}{\sqrt{2}}\right)^2 + \left(\frac{100}{\sqrt{2}}\right)^2}$

$V_{Rms} = 1443.95\ V$

$$Z_1 = 10 + j\left(300 \times 41.15 \times 10^{-3} - \frac{1}{300 \times 30 \times 10^{-6}}\right)$$

$$Z_3 = 10\,\Omega \quad \because \text{resonance}$$

$$Z_5 = 10 + j\left(5 \times 300 \times 41.15 \times 10^{-3} - \frac{1}{5 \times 300 \times 30 \times 10^{-6}}\right)$$

$$Z_1 = 10 - 98.76j = 44.27\angle 63.14$$

$$Z_3 = 10$$

$$Z_5 = 10 + 39.502j = 40.74\angle 75.8$$

$$\Rightarrow i = \frac{v_1}{Z_1} + \frac{v_3}{Z_3} + \frac{v_5}{Z_5}$$

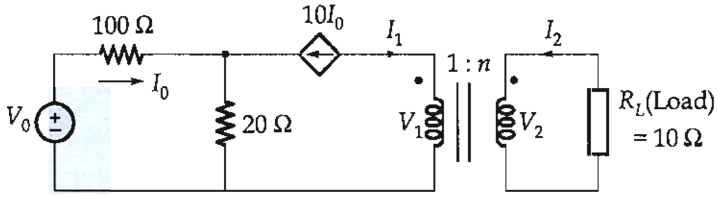
$$\Rightarrow i = 45.177\sin(\omega t - 63.14) + 40\sin 3\omega t + 2.45\sin(\omega t - 75.8^\circ)$$

$$\Rightarrow i_{Rms} = \sqrt{\left(\frac{45.177}{\sqrt{2}}\right)^2 + \left(\frac{40}{\sqrt{2}}\right)^2 + \left(\frac{2.45}{\sqrt{2}}\right)^2}$$

$$\boxed{i_{Rms} = 42.698\,A}$$

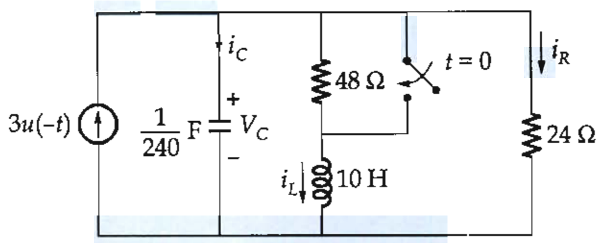
7

Q.2 (c) (i) What is the voltage and power gain of the circuit shown in figure? Assume $n = \frac{1}{10}$.



[10 marks]

Q.2 (c) **(ii)** Consider the circuit shown below:



After being open for a long time, the switch is closed at $t = 0$. Find

1. $i_L(0^-)$
2. $V_C(0^-)$
3. $i_R(0^+)$
4. $i_C(0^+)$
5. $V_C(0.2)$ using Laplace transform approach.

[10 marks]

Q.3 (a) (i) A 415-V, 50-Hz, three-phase voltage is applied to three star-connected identical impedances. Each impedance consists of a resistance of 15 Ω, a capacitance of 177 μF and an inductance of 0.1 henry in series.

Find:

1. the phase current,
2. the line current,
3. the power factor,
4. the active power,
5. the reactive power and
6. the total VA.

Draw a neat phasor diagram. If the same impedances are connected in delta, find the (i) line current and (ii) power consumed.

[10 marks]

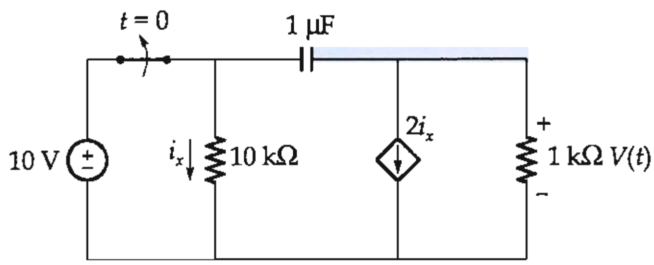
(a) (ii) A coil having a resistance of 20 Ω and an inductance of 200 μH is connected in parallel with a variable capacitor. This parallel combination is connected in series with a resistance of 8000 Ω. A voltage of 230 V at a frequency of 10^6 Hz is applied across the circuit.

Calculate:

1. the value of capacitance at resonance,
2. *Q*-factor of the circuit,
3. dynamic impedance of the circuit, and
4. total circuit current.

[10 marks]

(b) (i) For the circuit shown in figure:

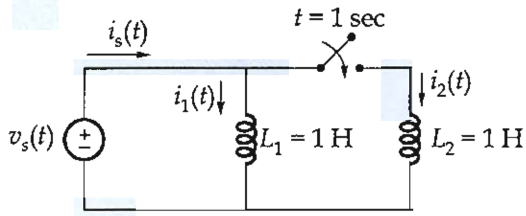


1. Find the expression of $V(t)$, the voltage across 1 kΩ resistor when the switch is opened at time, $t = 0$.
2. Sketch $V(t)$ with respect to time (t) and mark the time constant t.

[10 marks]

(b) (ii) For the parallel inductive circuit shown below with switch closed at t = 1s, $v_s(t) = \cos(t)$ V for $t \geq 0$ and 0 otherwise, **find:**

1. the input current $i_s(t)$ for $t \geq 0$ sec.
2. the energy stored in each of the inductors for the intervals $[0, t]$ for $0 \leq t \leq 1$ and for $1 \leq t$.



[10 marks]

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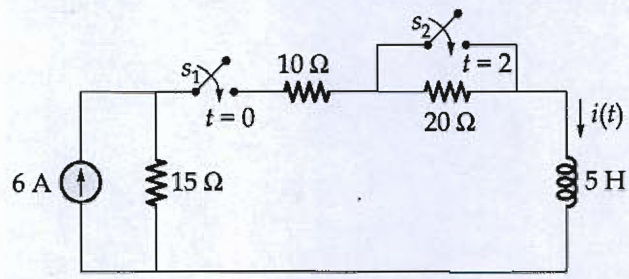
Q.3 (c) Consider the network shown below:



Switch S_1 is closed at $t = 0$, and switch S_2 is closed at $t = 2$ sec. Calculate current $i(t)$ for all t, and also find $i(t)|_{t = 1\ sec}$ and $i(t)|_{t = 3\ sec}$.

[20 marks]

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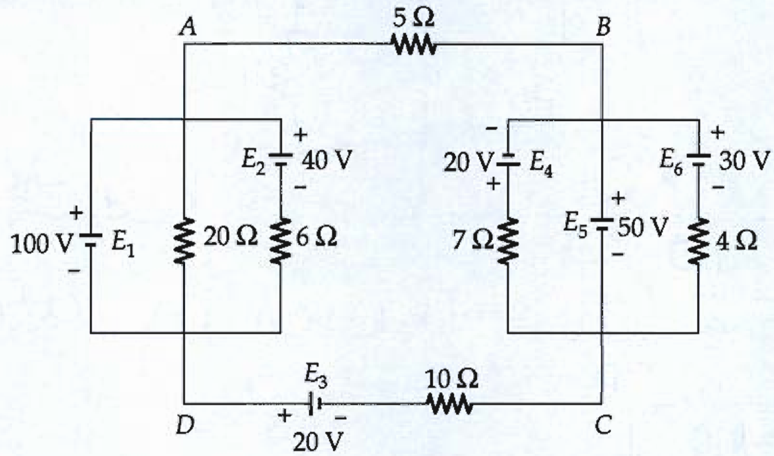
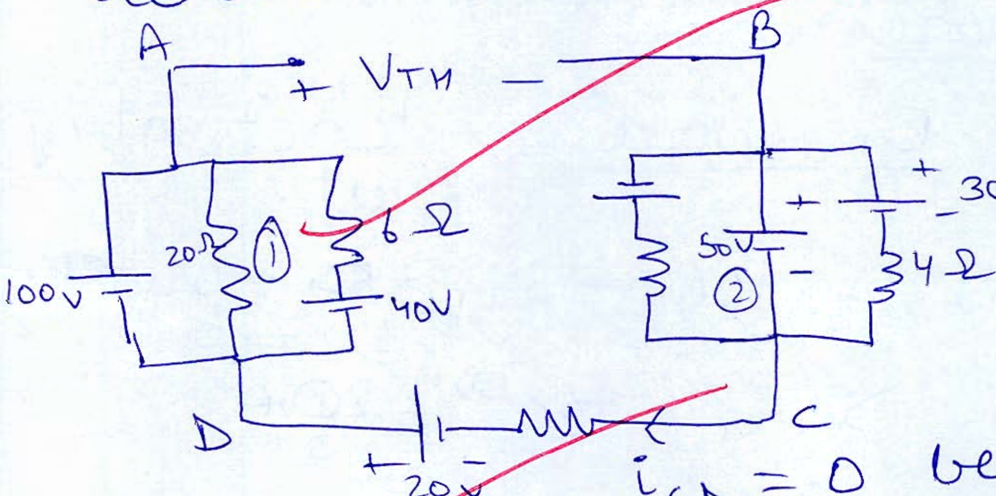
(a) For the circuit shown in figure, find the current through 5 Ω resistor by using Thevenin's theorem and verify the same by using superposition theorem.



[20 marks]

Case 1: Disconnect the 5 Ω resistor to find open circuit voltage across A B terminals.



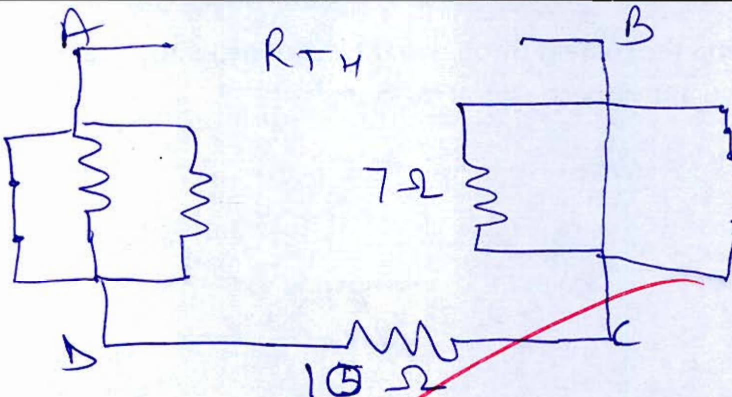
$i_{CD} = 0$ because no return path exist.

by KVL:

$100 - V_{TH} - 50 + 20 = 0$

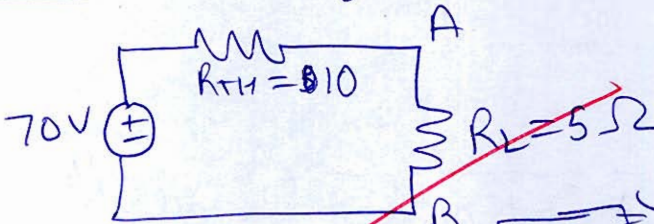
$\Rightarrow \boxed{V_{TH} = 70V}$

Case 2: Deactivate the independent sources to find the equivalent resistange w.r.t terminals AB



$\Rightarrow R_{TH} = 10\Omega$

Case 3: Equiv. Thevenin circuit:



$\Rightarrow i_{5\Omega} = \frac{70}{5+10}$

$\Rightarrow$ ~~$i_{5\Omega} = 7A$~~

$i_{5\Omega} = \frac{14}{3} A$

Now verifying using Superposition theorem:

using E_1 source: deactivating rest.

by KVL: $100 - 5i_1 - \text{[illegible]} - 10i_1 + \text{[illegible]} = 0$

$\Rightarrow i_1 = \frac{100}{15} = \frac{20}{3} A$

with $E_2 = 40V$: deactivating rest sources.

$\Rightarrow i_{2AB} = 0A$

with $E_3 = 20V$:

$i_{3AB} = \frac{20}{15} = \frac{4}{3} A$

with $E_4 = 20V$: deactivating rest.

~~KVL: $+20 - 10i - 5i = 0$~~ $i_{4AB} = 0A$

Do not write in this margin

with $E_5 = 50V$: deactivating rest

$$50 - 5 i_{BA} - 10 i_{CD} = 0 \quad [i_{BA} = i_{CD}]$$

$$\Rightarrow i_{BA} = \frac{50}{15} = \frac{10}{3} A$$

$$\Rightarrow i_{AB5} = -\frac{10}{3} A$$

with $E_6 = 30V$: $i_{AB} = 0$

so

$$i_{AB\,net} = i_1 + i_2 + i_3 + i_4 + i_5 + i_6$$

$$= \frac{20}{3} + \frac{4}{3} - \frac{10}{3}$$

$$\Rightarrow \boxed{i_{AB} = \frac{14}{3} A = i_{5\Omega}}$$

Hence verified.

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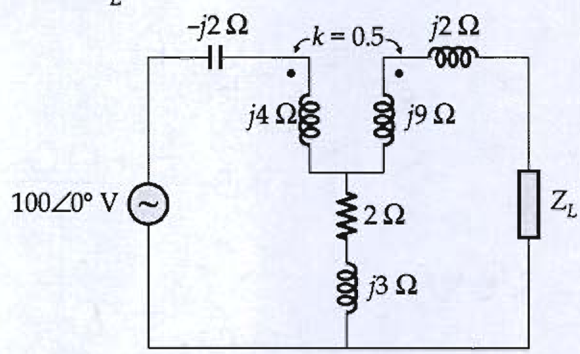
Good Approach

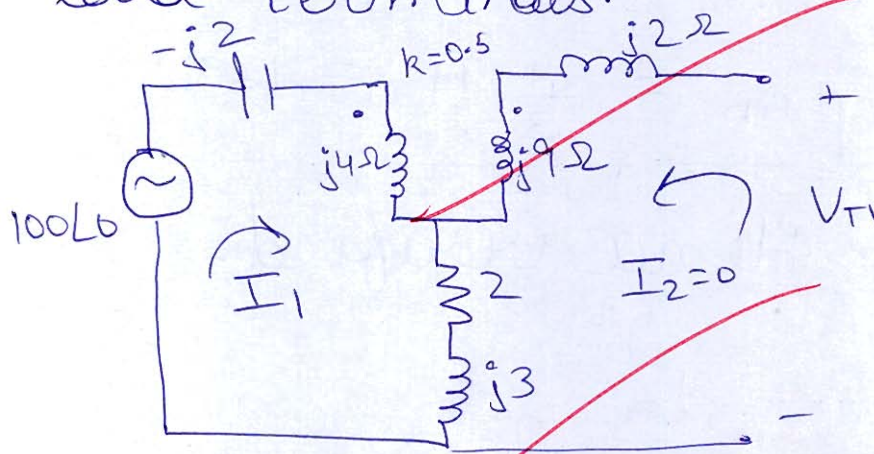
Q.4 (b) (i) Find the Thevenin's equivalent of the circuit shown in figure below as seen from the load impedance Z_L.

(ii) Find the value of Z_L for maximum power transfer and also the maximum power transfer to the load Z_L.



[10 + 10 marks]

i) Case 1: Disconnect Z_L to find equiv. open ckt voltage wrt load terminals.



we know $X_m = K\sqrt{X_{L1}X_{L2}} = j3\,\Omega$

KVL in loop 1:

$I_1(-j2 + j4 + 2 + j3) + j3 I_2 = 100\angle 0°$
$+ I_2(2 + j3)$

$\Rightarrow I_1 = \dfrac{100\angle 0}{2 + j5}$

j3 I_1

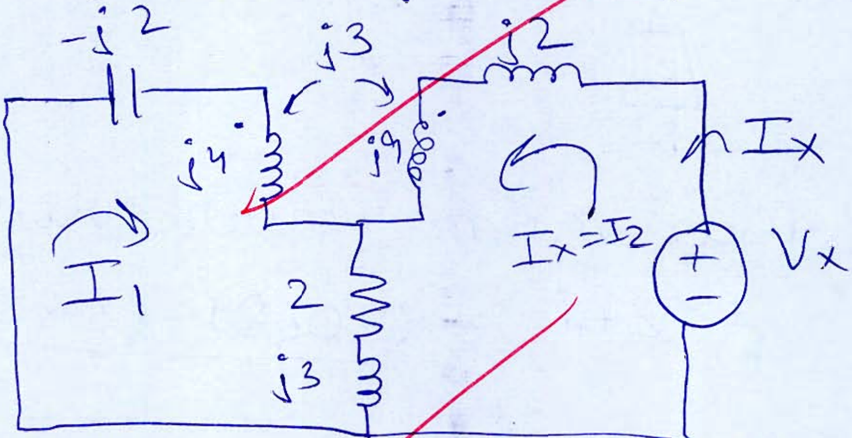
KVL in loop 2:

$\Rightarrow V_{TH} = (2 + j3) I_1 = \left(\dfrac{2+j3}{2+j5}\right) \times 100$

$\Rightarrow \boxed{V_{TH} = 66.95\angle -11.88°}$

Do not write in this margin

Case 2 : Deactivate independent sources and connect V_x across load terminals to find equivalent impedance given by $Z = \frac{V_x}{I_x}$



KVL in loop 1 : $I_1(-j2 + j4 + 2 + j3) + j3 I_x + (2+j3) I_x$

$\Rightarrow I_1(2+j5) + I_x(2+j6) = 0$

$I_1 = -I_x \frac{(2+j6)}{2+j5}$ — (1)

KVL in loop 2 :

$V_x = I_x(j2 + j9 + 2 + j3) + j3 I_1 + I_1(2+j3)$

$V_x = I_x(2+j14) + I_1(2+j6)$ — (2)

using (1) in (2)

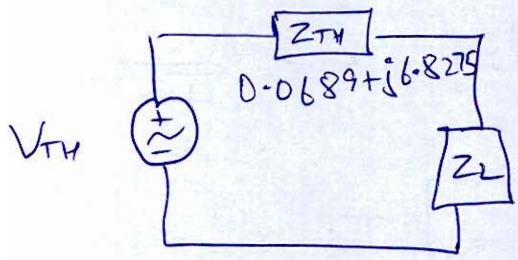
$V_x = I_x(2+j14) - I_x \frac{(2+j6)^2}{2+j5}$

$\Rightarrow Z_{TH} = \frac{V_x}{I_x} = 2 + j14 - \frac{(2+j6)^2}{2+j5}$

$\Rightarrow \boxed{Z_{TH} = 6.827 \angle 89.42^\circ}$

or $Z_{TH} = 0.0689 + j6.8275$

Case 3: Thevenin equivalent ckt.



ii) For maximum power transfer

$$Z_L = Z_{TH}^* = 0.0689 - j6.8275\ \Omega$$

and

$$P_{max} = \frac{V_{TH}^2}{4R_L}$$

$$= \frac{(66.95)^2}{4 \times 0.0689}$$

$$= 16263.79\ W$$

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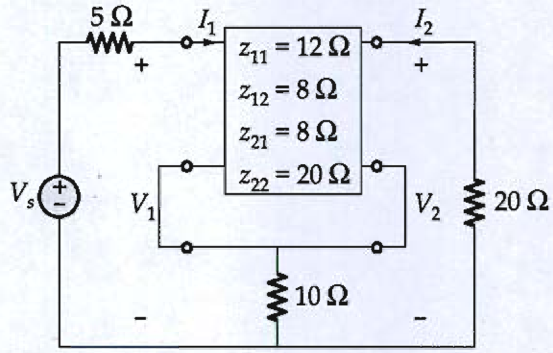
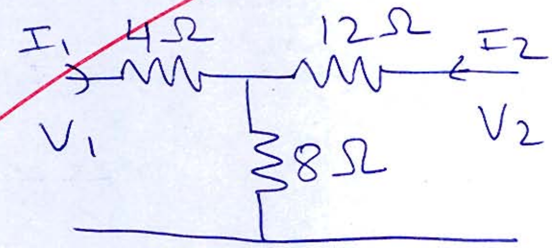
Q.4 (c) (i) Evaluate the ratio V_2/V_s in the circuit shown below.

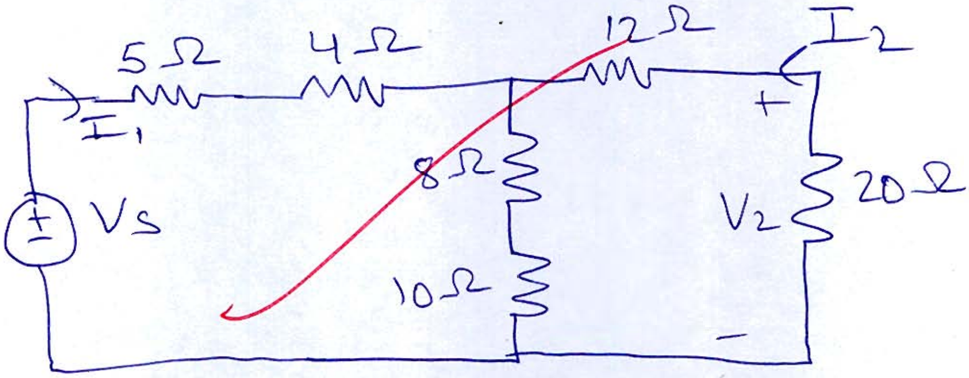


[10 marks]

For given Z-network
as $Z_{12} = Z_{21} = 8\ \Omega$
$\Rightarrow$ Network is Reciprocal
and equivalent T-network could be drawn.



so circuit becomes:



by KVL in loop 1:

$$V_s = 26 I_1 + 18 I_2 \quad -(1)$$

KVL in loop 2:

$$V_2 = -20 I_2 = 30 I_2 + 18 I_1$$

$$\Rightarrow I_1 = -\frac{50}{18} I_2 \quad -(2)$$

using equation (2) in (1)

$$\Rightarrow V_s = 26\left(-\frac{50}{18}\right) I_2 + 18 I_2$$

$$V_s = I_2\left(\frac{18\times18 - 26\times50}{18}\right)$$

$$\boxed{V_s = -\frac{976}{18} I_2}$$

and $V_2 = -20 I_2$

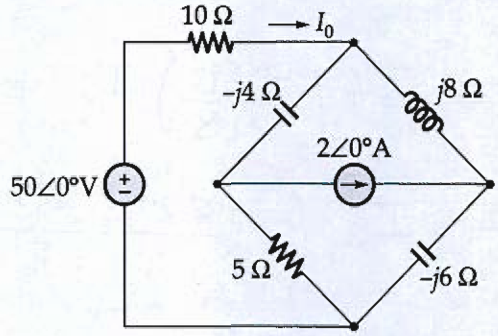
$$\Rightarrow \frac{V_2}{V_s} = \frac{+20 I_2}{+\frac{976}{18} I_2}$$

$$\boxed{\frac{V_2}{V_s} = 0.368}$$

(8)

Good Approach

Q.4 (c) (ii) By using superposition theorem, find current I_0 in the circuit shown below:

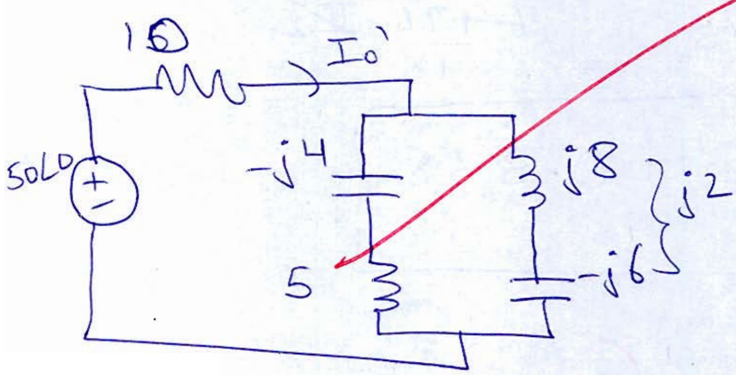


[10 marks]

By Superposition:

$I_o = I_o' + I_o''$

Case 1 : using 50V voltage source, current source deactivated



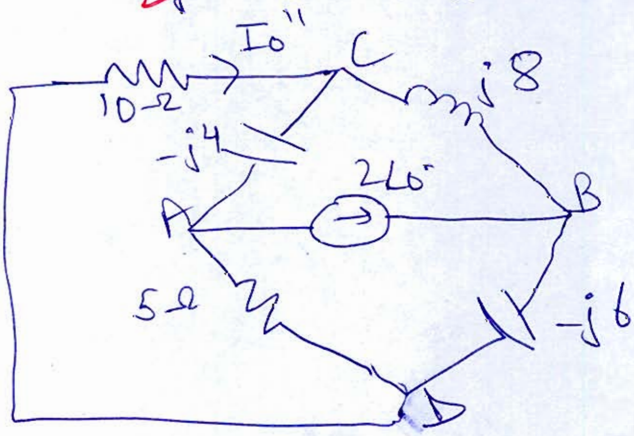
$(5-j4) \| j2$

$= 0.689 + 2.27j$

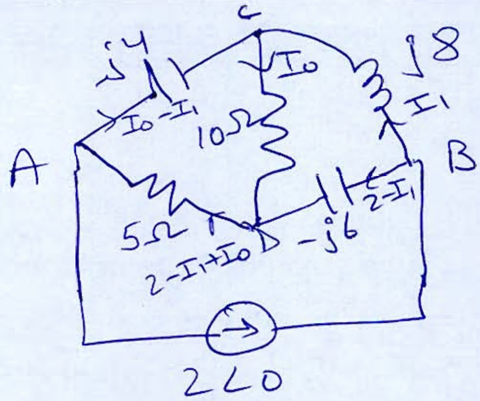
$\Rightarrow I_o' = \dfrac{50}{10 + 0.689 + 2.27j}$

$\Rightarrow \boxed{I_o' = 4.574\angle{-12.02^\circ}}$

Case 2 : using 2 A current source. Voltage source deactivated.



redrawing



KVL in ADB:

$I_1(j8) + j6(2-I_1) + 10 I_o = 0$

$\Rightarrow I_1(j2) + 10 I_o = -j12$ — (1)

KVL in ACD:

$+j4(I_o - I_1) - 10 I_o'' - 5(2 - I_1 + I_o'') = 0$

$I_o''(j4 - 10 - 5) + I_1(-j4 + 5) = 10$

$\Rightarrow I_o''(-15 + j4) + I_1(5 - j4) = 10$ — (2)

in Matrix form

$$\begin{bmatrix} j2 & 10 \\ 5-j4 & -15+j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_o'' \end{bmatrix} = \begin{bmatrix} -j12 \\ 10 \end{bmatrix}$$

$$\Rightarrow I_o'' = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} j2 & -j12 \\ 5-j4 & 10 \end{vmatrix}}{\begin{vmatrix} j2 & 10 \\ 5-j4 & -15+j4 \end{vmatrix}}$$

$$I_o'' = \frac{48 + 80j}{-58 + 10j} = 1.585\angle -111.18°$$

7

$\Rightarrow I_o = I_o' + I_o'' = 4.574\angle -12.02 + 1.585\angle -111.18$

$\Rightarrow \boxed{I_o = 4.6\angle -31.92}$

$\Rightarrow I_o = 4.6 A$

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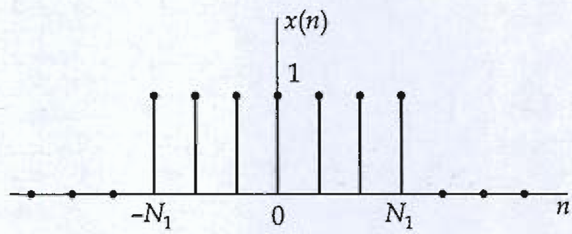
Section B : Systems & Signal Processing

Q.5 (a) Find the Fourier transform of the rectangular pulse

$$x(n) = u(n+N_1) - u(n-N_1-1) = \begin{cases} 1, & |n| \le N_1 \\ 0, & |n| > N_1 \end{cases}$$

which is illustrated in figure below. Also draw the magnitude and phase spectrum for $N_1 = 2$.



[12 marks]

Discrete time Fourier transform is given by:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

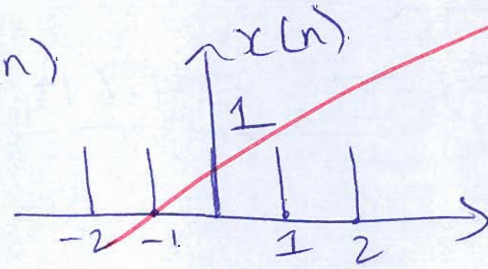
$$= \sum_{-N_1}^{N_1} 1 \cdot e^{-j\omega n}$$

$$= e^{+j\omega N_1} + e^{-j\omega(-N_1+1)} + \dots 1 + \dots + e^{-j\omega N_1}$$

$$\Rightarrow X(e^{j\omega}) = \frac{\sin\left((2N+1)\frac{\omega}{2}\right)}{\sin\frac{\omega}{2}}$$

for $N_1 = 2$

x(n)



$$X(e^{j\omega}) = \sum_{-2}^{2} e^{-j\omega n}$$

$$= e^{j\omega 2} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j\omega 2}$$

$$X(e^{j\omega}) = 2\left\{\frac{e^{j\omega 2} + e^{-j\omega 2}}{2}\right\} + 1 + 2\left\{\frac{e^{j\omega} + e^{-j\omega}}{2}\right\}$$

$$X(e^{j\omega}) = 2\cos 2\omega + 1 + 2\sin\omega$$

✗

7

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Q.5 (b) Consider the continuous-time signal $x(t) = \cos(100\pi t)$:

(i) Determine the minimum sampling rate required to avoid aliasing.

(ii) Suppose that the signal is sampled at the rate $f_s = 200$ Hz. What is the discrete-time signal obtained after sampling?

(iii) Suppose that the signal is sampled at the rate $f_s = 75$ Hz. What is the discrete-time signal obtained after sampling?

(iv) What is the frequency $0 < f < f_s/2$ of a sinusoid that yields samples identical to those obtained in part (iii)?

[12 marks]

$x(t) = \cos(100\pi t)$

$\omega_m = 100\pi$

$f_m = \frac{\omega}{2\pi} = 50\text{Hz}$

i) to avoid aliasing it should be sampled at its Nyquist rate

$f_s = 2f_m$

$= 2 \times 50 = \underline{100\text{Hz}}$

ii) signal sampled at $f_s = 200\text{Hz}$

$x(t) = \cos(100\pi t)$

replace $t \rightarrow nT_s = \frac{n}{f_s}$

$x(n) = \cos\left(100\pi \times \frac{n}{200}\right)$

$\boxed{x(n) = \cos\left(n\frac{\pi}{2}\right)}$

is the required discrete time signal.

iii) at $f_s = 75\text{Hz}$

$x(t)$ $t \rightarrow nT_s = \frac{n}{f_s} = \frac{n}{75}$

$\Rightarrow x(n) = \cos\left(\frac{100^4 \pi n}{753}\right)$

$$\boxed{x(n) = \cos\left(\frac{4\pi}{3} n\right)}$$

(iv) $0 < f < f_s/2$

yields samples identical to as in (iii)

6

Q.5 (c) Determine the signal $x(n)$ whose z-transform is given by
$X(z) = \log(1 + az^{-1}),\ |z| > |a|$

[12 marks]

$$X(z) = \log(1+az^{-1}),\ |z|>|a|$$

ie right sided signal.

let $$Y(z) = \frac{d}{dz}X(z) = \frac{1}{1+az^{-1}} \times a(-z^{-2})$$

$$= -\frac{az^{-2}}{1+az^{-1}}$$

Multiply by $-z$ with both sides

$$-z\frac{d}{dz}X(z) = \frac{az^{-1}}{1+az^{-1}}$$

by Differentiation in z property

$$x(n) \overset{ZT}{\rightleftharpoons} X(z)$$

$$n\,x(n) \overset{ZT}{\rightleftharpoons} -z\frac{d}{dz}X(z)$$

as $$u(n) \rightleftharpoons \frac{1}{1-z^{-1}}$$

$$a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}}$$

$\downarrow n = -n$

$$a^{-n}u(-n) \rightleftharpoons \frac{1}{1+az^{-1}}$$

$\downarrow$

$$a \cdot a^{-n}u(-n) \rightleftharpoons \frac{a}{1+az^{-1}}$$

$\downarrow n \to n+1$

$$a \cdot a^{-n-1}u(-n-1) \rightleftharpoons \frac{az^{-1}}{1+az^{-1}}$$

$$\text{''}\ \ a^{-n}u(-n-1)$$

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$\Rightarrow y(n) = n(x(n)) = a^{-n}u(-n-1)$

$\Rightarrow \boxed{x(n) = \frac{1}{n} a^{-n}u(-n-1)}$

(5)

Go Through the made easy solution

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Q.5 (d) Discuss the Dirichlet conditions for the existence of the Continuous-Time Fourier Transform. Are these conditions mandatory for a signal to possess a Fourier transform? Explain with examples.

[12 marks]

Dirichlet conditions for existence of continuous time Fourier transform are:

1) Signal should have finite no. of discontinuties and size of discontinuity should be finite.

2) Signal should have finite number of maxima's and minima's within the time range.

3) Signal should be absolutely integrable.

→ No these conditions are not mandatory for existence of Fourier Transform.

eg: $\delta(t) \xrightarrow{FT} 1$

10

Go Through the made easy solution

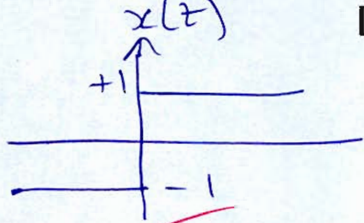
(e) Find the Fourier transform of the signum function $x(t) = \text{sgn}(t)$.

$$\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$

Also draw the magnitude and phase spectrum of $X(\omega)$.

[12 marks]

$x(t) = \text{sgn}(t)$



~~by using~~ Fourier transform of a signal is given by: $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$$= \int_{-\infty}^{0} -e^{-j\omega t} dt + \int_{0}^{\infty} e^{-j\omega t} dt$$

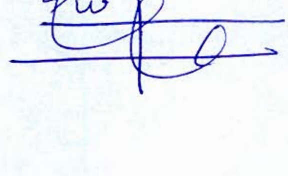
$$= + \frac{e^{-j\omega t}}{+j\omega}\Big|_{-\infty}^{0} + \frac{e^{-j\omega t}}{-j\omega}\Big|_{0}^{\infty}$$

$$= \frac{1}{j\omega}[-0 + 1] - \frac{1}{j\omega}[0 - 1]$$

$$= \frac{1}{j\omega} + \frac{1}{j\omega} = \frac{2}{j\omega}$$

$$\Rightarrow x(t) = \text{sgn}(t) \xrightarrow{FT} \frac{2}{j\omega}$$

$$|X(\omega)| = \frac{2}{\omega} \text{ and } \angle X(\omega) = -90^\circ$$



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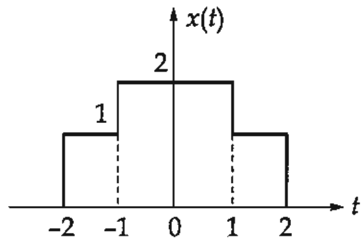
a) (i) An LTI system has a unit step response given by $s(t) = (1 - e^{-t} - te^{-t})u(t)$. For a certain input $x(t)$, the output is observed to be equal to $y(t) = (2 - 3e^{t} + e^{-3t})u(t)$. What is $x(t)$?

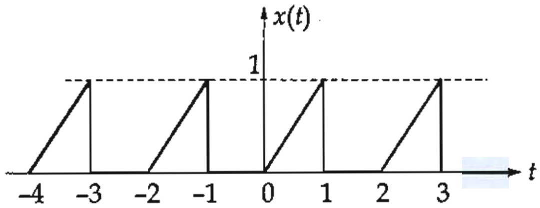
[12 marks]

(ii) Determine the Fourier transform of the signal shown in the following figure.



[8 marks]

)| (i) Find the trigonometric Fourier series for the waveforms shown in figure below and sketch the line spectrum.



[12 marks]

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Q.6 (b) (ii) A causal and stable LTI system " S" has the property that when we apply the input;

$\left(\frac{4}{5}\right)^n u(n)$, it gives the output $n\left[\frac{4}{5}\right]^n u(n)$.

Determine the transfer function $H(e^{j\omega})$ for the system.

[8 marks]

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(i) Determine the transient response and steady-state response of the system characterized by the difference equation, $y(n) = 0.5y(n-1) + x(n)$, when the input signal is $x(n) = 10\cos(n\pi/4)u(n)$. The system is initially at rest (i.e., it is relaxed).

[10 marks]

(ii) Solve the difference equation using the one-sided z-transform $y(n) = x(n) + by(n-1)$ with initial condition $y(-1) = P$. Assume input be $x(n) = e^{j\omega_0 n}u(n)$.

[10 marks]

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) Determine the values of power and energy for each of the following signals. Also find the nature of signals.

(i) $x_1(t) = e^{-2t} u(t)$.

(ii) $x_2(t) = e^{j(2t + \pi/4)}$.

(iii) $x_3(n) = \cos\left(\left(\frac{\pi}{4}\right)n\right)$.

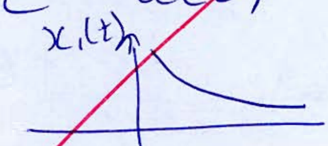
[20 marks]

Energy of a signal $= \int_{-\infty}^{\infty} |x(t)|^2 dt$

Power of a signal $= \frac{1}{T}\int_T |x(t)|^2 dt$ if periodic

else $\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$

1) $x_1(t) = e^{-2t}u(t)$



is an Energy signal

Energy $= \int_0^{\infty} |e^{-2t}|^2 dt$

$= \int_0^{\infty} e^{-4t} dt = \frac{e^{-4t}}{-4}\Big|_0^{\infty}$

Energy $= \frac{1}{4}$ J

Power $= 0$

ii) $x_2(t) = e^{j2t} \cdot e^{j\pi/4}$

$\omega = 2$

$T = \frac{2\pi}{\omega} = \pi$

Periodic signal are Power signal

$\Rightarrow$ Power $= \frac{|e^{j\frac{\pi}{4}}|}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{j4t} dt$

$\frac{1}{\pi} \quad \frac{e^{j4t}}{j4}\Big|_{-\pi/2}^{\pi/2}$

$$= \frac{1}{4\pi j}\{e^{j2\pi t} - e^{-j2\pi t}\}$$

Power = 0 W

iii) $x_3(n) = \cos\left(\frac{\pi}{4}n\right)$

$\omega_0 = \frac{\pi}{4} \Rightarrow N_0 = \frac{2\pi}{\omega_0} = 8$

is a Periodic signal

ie Power Signal

Improve presentation

$$\text{Power} = \frac{1}{N}\sum_{n=-\frac{N_0}{2}}^{N_0/2} |x(n)|^2$$

$$= \frac{1}{8}\sum_{-4}^{4} \left|\cos\frac{\pi}{4}n\right|^2$$

or $x_3(n) = \cos\left(\frac{\pi}{4}n\right) = \frac{e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}}{2}$

$$= \frac{1}{2}e^{j\frac{\pi}{4}n} + \frac{1}{2}e^{-j\frac{\pi}{4}n}$$

so ~~complex~~ discrete fourier series coefficients are: $C_1 = C_{-1} = \frac{1}{2}$

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and

$$\text{Power} = \sum C_n^2 = \frac{1}{4} + \frac{1}{4}$$

$$\text{Power of } x(n) = \frac{1}{2}\text{ W}$$

$$= 0.5\text{ W}$$

Q.7 (b) Find the inverse Laplace transform of

$$X(s) = \frac{-5s-7}{(s+1)(s-1)(s+2)}$$

if the ROC is

(i) $\Re\{s\} > 1$

(ii) $\Re\{s\} < -2$

(iii) $-1 < \Re\{s\} < 1$

(iv) $-2 < \Re\{s\} < -1$

[20 marks]

$$X(s) = \frac{-5s-7}{(s+1)(s-1)(s+2)}$$

By Partial fraction

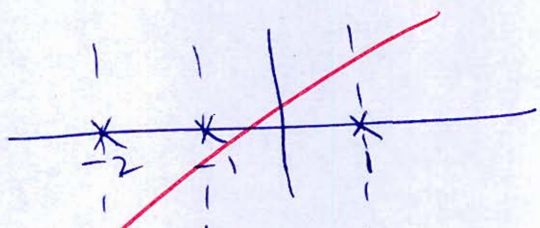
$$X(s) = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s+2}$$

$$A\Big|_{s=-1} = \frac{+5-7}{-2\times1} = 1$$

$$B\Big|_{s=1} = \frac{-5-7}{2\times3} = -2$$

$$C\Big|_{s=-2} = \frac{10-7}{-1\times(-3)} = 1$$

$$\Rightarrow \quad X(s) = \frac{1}{s+1} + \frac{-2}{s-1} + \frac{1}{s+2}$$



i) ROC is $\sigma > 1$

ie Right sided signal

$\Rightarrow$ By inverse Laplace transform

$x(t) = (e^{-t} - 2e^{t} + e^{-2t})u(t)$

$\because e^{-at}u(t) \rightleftharpoons \frac{1}{s+a}$

ii) $Re\{s\} < -2$ ie $\sigma < -2$

left sided signal

$$\Rightarrow \boxed{x(t) = (-e^{-t} + 2e^{t} - e^{-2t})u(-t)}$$

$\because u(-t) \rightleftharpoons \frac{-1}{s}$

and $e^{-at}u(-t) \rightleftharpoons \frac{-1}{s+a}$

iii) $-1 < \sigma < 1$, ie ROC is a strip in s-plane.

Right sided inverse [RSS] — left sided inverse [LSS]

$$X(s) = \underbrace{\frac{1}{s+1}}_{RSS} - \underbrace{\frac{2}{s-1}}_{LSS} + \underbrace{\frac{1}{s+2}}_{LSS}$$

$$\boxed{x(t) = e^{-t}u(t) + 2e^{t}u(-t) - e^{-2t}u(-t)}$$

IV) $-2 < \sigma < -1$, ie strip ROC

Right sided inverse — Left sided inverse.

$$X(s) = \underbrace{\frac{1}{s+1}}_{RSS} - \underbrace{\frac{2}{s-1}}_{LSS} + \underbrace{\frac{1}{s+2}}_{RSS}$$

$$\boxed{x(t) = e^{-t}u(t) + 2e^{t}u(-t) + e^{-2t}u(t)}$$

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An analog filter has the transfer function:

$$H_a(s) = \frac{10}{s^2 + 7s + 10}$$

Design a digital filter $H(z)$ using the Bilinear Transformation method with a sampling period of $T = 0.2$ seconds.

(i) Determine the discrete-time transfer function $H(z)$.

(ii) Find the difference equation of the system relating $x(n)$ and $y(n)$.

(iii) Determine the poles of $H(z)$ and comment on the stability of the digital filter.

[20 marks]

$$H_a(s) = \frac{10}{s^2+7s+10} = \frac{10}{(s+3.5)^2 - 2.25}$$

analog filter freq $\Omega = \sqrt{2.25} = 1.5$

Bilinear transformation is given by

$$H(z) = H(s)\Big|_{s = \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

given $T_s = 0.2$

$$\Rightarrow \frac{2}{T_s} = \frac{2}{0.2} = 10$$

$$\Rightarrow H(z) = \frac{10}{\left(\frac{10z-10}{z+1} + 3.5\right)^2 - 2.25}$$

$$= \frac{10}{\left(\frac{10z-10+3.5z+3.5}{z+1}\right)^2 - 2.25}$$

$$= \frac{10(z+1)^2}{(13.5z-6.5)^2 - 2.25(z+1)^2}$$

$$= \frac{10(z^2+2z+1)}{182.25z^2 - 175.5z + 42.25 - (2.25z^2+4.5z+2.25)}$$

Simplify

$$\Rightarrow H(z) = \frac{10z^2 + 20z + 10}{180z^2 - 180z + 40}$$

ii) as $H(z) = \frac{Y(z)}{X(z)} = \frac{10z^2+20z+10}{180z^2-180z+40}$

$\Rightarrow 180z^2Y(z) - 180zY(z) + 40Y(z)$
$= 10z^2X(z) + 20zX(z) + 10X(z)$

taking inverse z-transform

Simplify

$$180y(n+2) - 180y(n+1) + 40y(n) = 10x(n+2) + 20x(n+1) + 10x(n)$$

is the required difference equation.

iii) $H(z) = \frac{10z^2 + 20z + 10}{180z^2 - 180z + 40}$

Poles of $H(z) = 180z^2 - 180z + 40 = 0$
are $z = 0.67, 0.33$

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as $|z| < 1$ and lie within unit circle.
$\Rightarrow$ system is stable.

Q.8 (a) Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

(i) $x(n) = (0.8)^n\, u(n)$ and $h(n) = (0.4)^n\, u(n)$.

(ii) $x(n) = u(n-1)$ and $h(n) = \alpha^n u(n-1)$.

(iii) $x(n) = r(n) = nu(n)$ and $h(n) = -\alpha^{-n}u(n-1)$, where $a < 1$.

[20 marks]

Q.8 (b) Consider a periodic square wave $x(t)$ with amplitude A, period T centered at $t = 0$, and duty cycle 50%:

(i) Derive the expression for the Trigonometric Fourier Series coefficients.

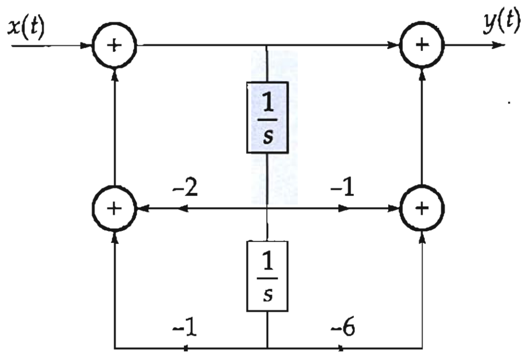
(ii) Explain the Gibbs Phenomenon in the context of reconstructing this signal using a finite number of harmonics.

(iii) Use the duality property of the Fourier Transform to find the transform of

$$g(t) = \frac{\sin(at)}{\pi t}.$$

[20 marks]

(i) The input $x(t)$ and output $y(t)$ of a causal LTI system are related through the block diagram representation shown in the below figure.



1. Determine a differential equation relating $y(t)$ and $x(t)$.
2. Is this system stable?

(ii) The input and output of a causal LTI system is related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

What is the response of this system, if $x(t) = te^{-2t}u(t)$?

[10 + 10 marks]

OOOO

Space for Rough Work

Space for Rough Work

10 - 2i + 4i - 3i = 0

$\Rightarrow$ i = 10

v₃ = 30V

662-5 + j 41-74