• Try to avoid calculation mistake

• Improve presentation

MADE EASY

Leading Institute for ESE, GATE & PSUs

ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electrical Circuits + Systems & Signal Processing

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Hyderabad ☐		

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	24
Q.2	50
Q.3	
Q.4	33
Section-B	
Q.5	38
Q.6	
Q.7	53
Q.8	
Total Marks Obtained	198

Signature of Evaluator: Sourabh Kumar ....

Cross Checked by ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

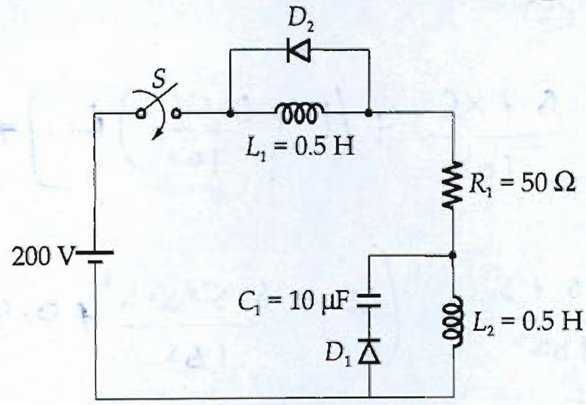
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Electrical Circuits

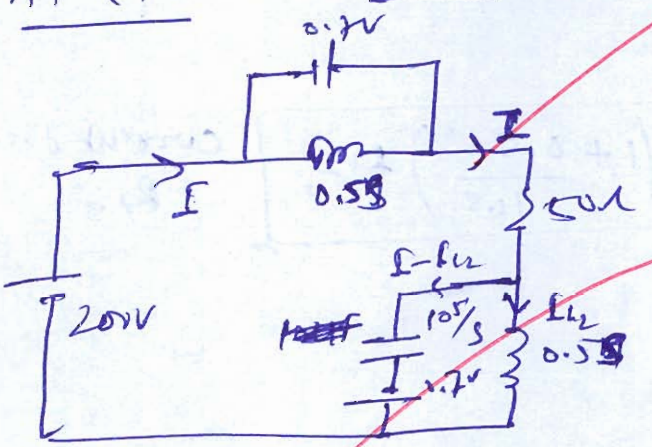
(a) In the circuit shown below the switch '*S*' is closed at $t = 0$, and is opened after 10 ms. What will be the currents in R_1, L_1 and L_2, and voltage across C_1, 8 ms after switch '*S*' opens? Assume D_1 to be an ideal diode and a 0.7 V drop across D_2 whenever it conducts.



[12 marks]

At $t < 0$, $i_{L_1}(0^-) = i_{L_2}(0^-) = 0$ Amp.

At $t > 0$ and $t = 8$ ms, switch closed



Apply KVL

$$-200 - 0.7 + 50I + 0.5S\, I_{L_2} = 0$$

$$200.7 = 50I + 0.5S\, I_{L_2} \quad \text{—(1)}$$

Now

$$\frac{10^5}{S}(I - I_{L_2}) + 0.7 - 0.5S\, I_{L_2} = 0$$

$$\frac{10^5}{S} I - \left(\frac{10^5}{S} + 0.5S\right) I_{L_2} = 0.7 \quad \text{—(2)}$$

from (1) and (2)

$$\frac{10^5}{s} I = 0.7 + \left(\frac{10^5}{s} + 0.5s\right) I_{L2} \quad \text{—— (3)}$$

from (1) and (3)

$$200.7 = 50\left[\frac{0.7 \times s}{10^5} + \left(1 + \frac{0.5s^2}{10^5}\right) I_{L2}\right] + 0.5s I_{L2}$$

$$200.7 = \frac{50 \times 0.7s}{10^5} + \left[50 + \frac{0.5 \times 50 s^2}{10^5} + 0.5s\right] I_{L2}$$

$$I_{L2} = \frac{200 - \frac{50 \times 0.7}{10^5} s}{50 + \frac{0.5 \times 50 s^2}{10^5} + 0.5s}$$

current from L_2

Now from (3)

$$I = \frac{0.7 \times s}{10^5} + \left(1 + \frac{0.5s^2}{10^5}\right) I_{L2}$$

current from R_1.

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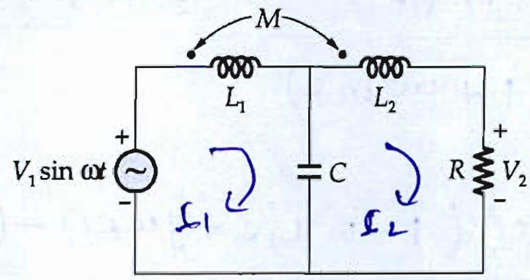
Go Through the made easy Solution.

(b) Find the voltage transfer function V_2 / V_1 for the network given below:

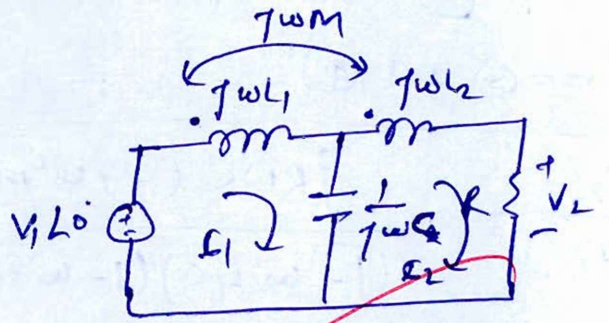


[12 marks]

Answer

By KVL in loop ①

$V_1 \sin \omega t = V_1 \angle 0$



$$-V_1 \angle 0 + \left(j\omega L_1 + \frac{1}{j\omega C}\right) I_1 - \frac{1}{j\omega C} I_2 + j\omega M I_2 = 0$$

$$V_1 = \left(j\omega L_1 + \frac{1}{j\omega C}\right) I_1 + \left(j\omega M - \frac{1}{j\omega C}\right) I_2 \quad \text{—①}$$

KVL in loop ②

$$\left(R + j\omega L_2 + \frac{1}{j\omega C}\right) I_2 - \frac{1}{j\omega C} I_1 + j\omega M I_1 = 0$$

$$\left(R + j\omega L_2 + \frac{1}{j\omega C}\right) I_2 = I_1 \left(\frac{1}{j\omega C} - j\omega M\right)$$

$$I_1 = \frac{\left(R + j\omega L_2 + \frac{1}{j\omega C}\right) I_2}{\left(\frac{1}{j\omega C} - j\omega M\right)} \quad \text{—②}$$

and $\quad V_2 = I_2 R \quad$ —③

from ① and ②

$$V_1 = \left(j\omega L_1 + \frac{1}{j\omega C}\right) \times \left[\frac{R + j\omega L_2 + \frac{1}{j\omega C}}{\frac{1}{j\omega C} - j\omega M}\right] I_2 + \left(j\omega M - \frac{1}{j\omega C}\right) I_2$$

$$V_1 = \frac{\left(j\omega L_1 + \frac{1}{j\omega C}\right)\left(R + j\omega L_2 + \frac{1}{j\omega C}\right) - \left(j\omega M - \frac{1}{j\omega C}\right)^2}{\left(\frac{1}{j\omega C} - j\omega M\right)} I_2$$

$$V_1 = \left[\frac{(-\omega^2 L_1 C + 1)(jR\omega C - \omega^2 L_2 C + 1) - (-\omega^2 CM - 1)^2}{j\omega C(1 + \omega^2 MC)}\right] I_2$$

$$V_1 = \left[\frac{(1 - \omega^2 L_1 C)(1 - \omega^2 L_2 C + j\omega RC) - (1 + \omega^2 CM)^2}{j\omega C(1 + \omega^2 MC)}\right] I_2 \quad \text{—— (4)}$$

from (3) and (4)

$$\frac{V_2}{V_1} = \frac{jR\omega C(1 + \omega^2 MC)}{(1 - \omega^2 L_1 C)(1 - \omega^2 L_2 C + j\omega RC) - (1 + \omega^2 CM)^2}$$

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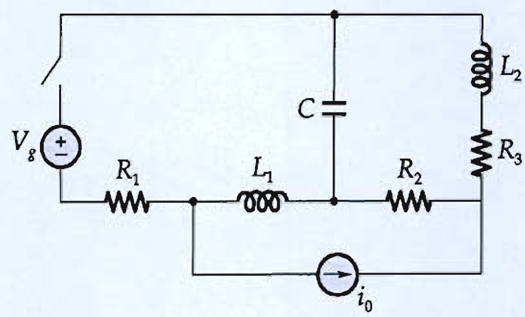
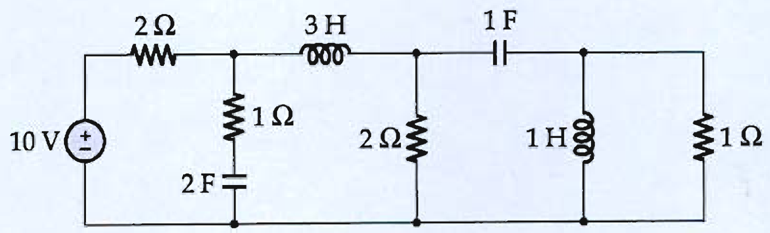
Improve presentation

(c) Draw the dual of the circuit shown in figure.

1.

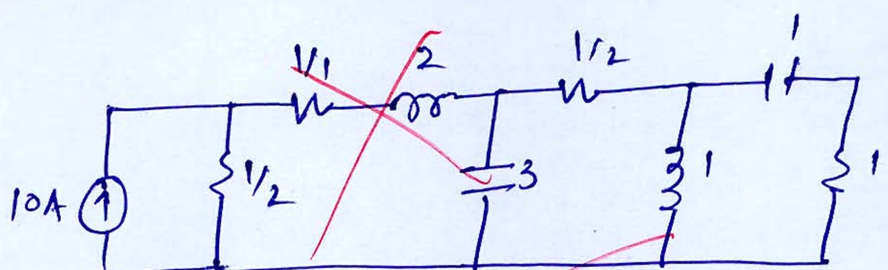


2.



[12 marks]

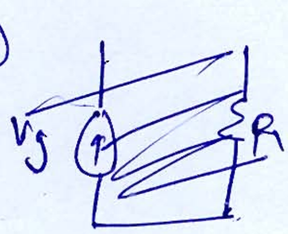
Ans. (2) Dual of above circuit



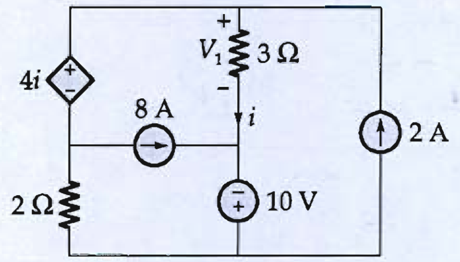
in above diagram we have converted Inductance into capacitance and vice-versa also, current source into voltage source, vice versa.

(2)

(1)



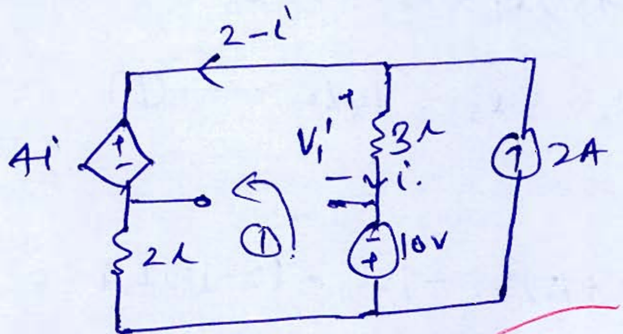
(d) Using the superposition theorem determine V_1, the voltage across the 3 Ω resistor in figure below,



[12 marks]

Answer

Case-1 taking 2A source, and 8A source as open circuit



(Let $V_1 = V_1'$ for 2A source)

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Applying KVL in loop ①

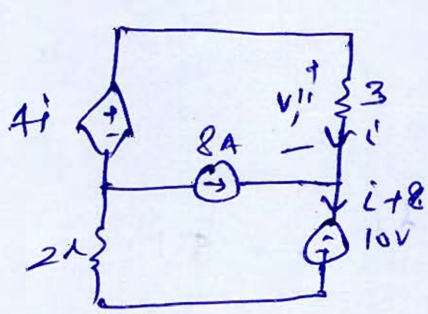
$4i + 2(2-i) + 10 - 3i = 0$

$4i + 4 - 2i + 10 - 3i = 0$

$-i = -14 \Rightarrow i = 14$ Amp.

So $V_1' = 3i = 3 \times 14 = 42$ Volt

Case-2 taking 8A source, then $V_1 = V_1''$



Applying KVL in Bigger loop

$3i - 10 + 2(i+8) - 4i = 0$

$3i - 10 + 2i + 16 - 4i = 0$

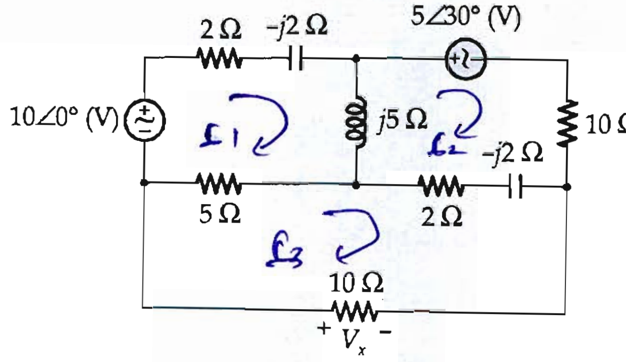
$i = -6$ Amp.

$V_1'' = 3i = 3 \times (-6) = -18$ Volt

So $V_1 = V_1' + V_1'' = 42 - 18$

$V_1 = 24$ Volt

Q.1 (e) Write the loop equations of the circuit and find the voltage V_x.



[12 marks]

KVL in loop ①

$-10\angle 0 + (2 - j2 + j5 + 5)I_1 - j5I_2 - 5I_3 = 0$

$(7 + j3)I_1 - j5I_2 - 5I_3 = 10\angle 0$ —— ①

KVL in loop ②

$5\angle 30 + (10 + 2 - j2 + j5)I_2 - j5I_1 - (2 - j2)I_3 = 0$

$(12 + j3)I_2 - j5I_1 - (2 - j2)I_3 = -5\angle 30°$

$j5I_1 - (12 + j3)I_2 + (2 - j2)I_3 = 5\angle 30°$ —— ②

KVL in loop ③

$(5 + 2 - j2 + 10)I_3 - 5I_1 - (2 - j2)I_2 = 0$

$5I_1 + (2 - j2)I_2 + (17 - j2)I_3 = 0$ —— ③

and $V_x = -10I_3$ —— ④

from ③

$I_1 = \frac{(17 - j2)I_3}{5} - \frac{(2 - j2)I_2}{5}$ —— ⑤

from ① and ⑤

$(7 + j3)\left[\frac{17 - j2}{5}I_3 - \frac{(2 - j2)}{5}I_2\right] - j5I_2 - 5I_3 = 10\angle 0$

$\left(20 + j\frac{37}{5}\right)I_3 - \left(4 + j\frac{17}{5}\right)I_2 = 10\angle 0$ —— ⑥

from (2) and (5)

$$j5\left[\frac{17-j2}{5}I_3-\frac{(2+j2)}{5}I_2\right]-(12+j3)I_2+(2-j2)I_3=5\angle 30$$

$$(4+j115)I_3+(14+5j)I_2=5\angle 30 \quad —(7)$$

from (7)

$$I_2=\frac{5\angle 30-(4+j115)I_3}{14+j5}$$

from (6)

$$\left(20+j\frac{37}{5}\right)I_3-\left(4+j\frac{17}{5}\right)\left[\frac{5\angle 30-(4+j115)I_3}{14+j5}\right]=10\angle 0$$

$$(23.312\angle 33.46)I_3=11.20\angle 7°$$

$$I_3=0.4805\angle -26.45°$$

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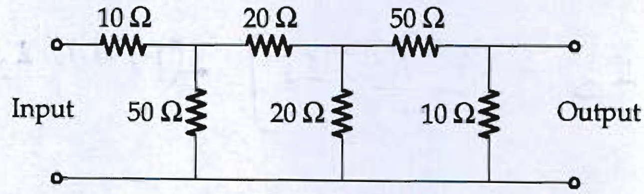
So

$$V_n=-10I_3$$

$$\boxed{V_n=4.805\angle 153.55}\ \text{Volt}$$

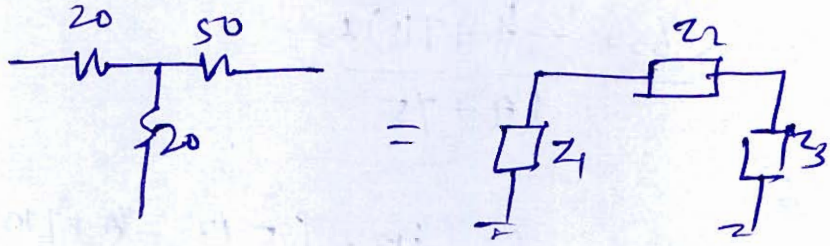
Q.2 (a) (i) Obtain the *ABCD* parameters for the network shown in figure below.



[12 marks]

Answer

By converting Star into Delta

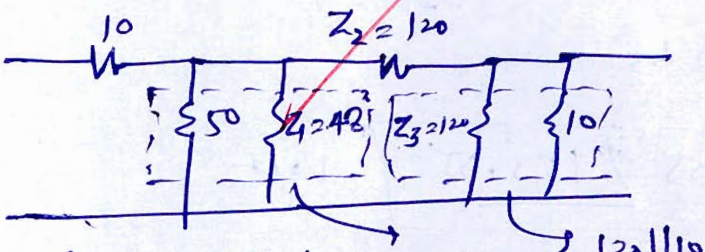


$Z_1 = 20 + 20 + \frac{20 \times 20}{50} = 48\,\Omega$

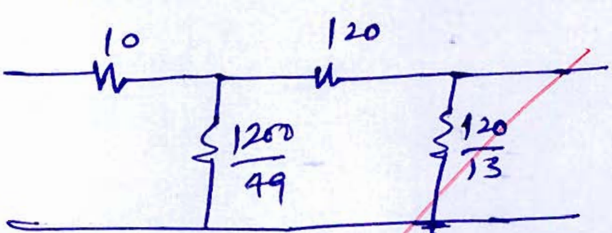
$Z_2 = 20 + 50 + \frac{20 \times 50}{20} = 120\,\Omega$

$Z_3 = 20 + 50 + \frac{20 \times 50}{20} = 120\,\Omega$

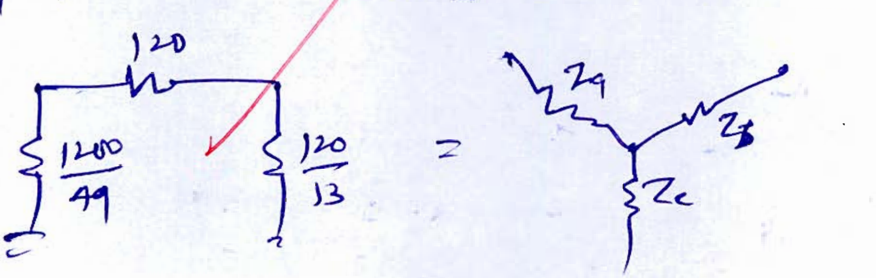
Now



Now further solving 48||50



By further converting Delta into star

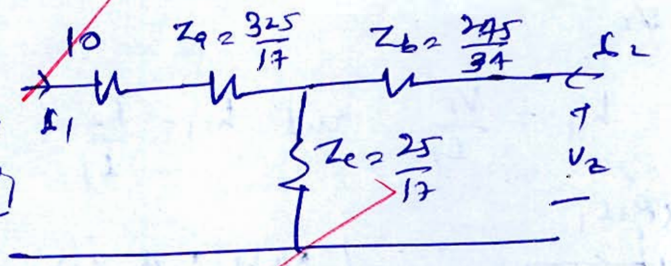


$$Z_a = \frac{120 \times \frac{1200}{49}}{120 + \frac{1200}{49} + \frac{120}{13}} = \frac{325}{17}$$

$$Z_b = \frac{120 \times \frac{120}{13}}{120 + \frac{1200}{49} + \frac{120}{13}} = \frac{245}{34}$$

$$Z_c = \frac{\frac{1200}{49} \times \frac{120}{13}}{120 + \frac{1200}{49} + \frac{120}{13}} = \frac{25}{17}$$

So



$$V_1 = \left(10 + \frac{325}{17} + \frac{25}{17}\right) I_1 + \frac{25}{17} I_2$$

$$V_1 = \frac{520}{17} I_1 + \frac{25}{17} I_2 \quad \text{--- (1)}$$

Now

$$V_2 = \left(\frac{245}{34} + \frac{25}{17}\right) I_2 + \frac{25}{17} I_1$$

$$V_2 = \frac{295}{34} I_2 + \frac{25}{17} I_1 \quad \text{--- (2)}$$

As we know from (2)

$$I_1 = 0.68 V_2 - 5.9 I_2 \quad \text{--- (3)}$$

from (1) and (3)

$$V_1 = 20.8 V_2 - 179 I_2 \quad \text{--- (4)}$$

So
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 20.8 & 179 \\ 0.68 & 5.9 \end{bmatrix}$$

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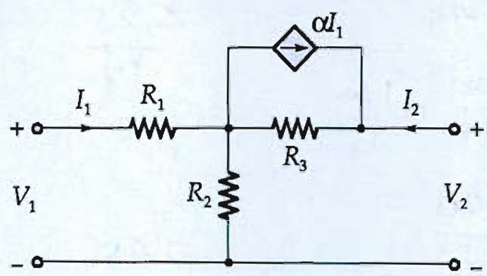
Good Approach

Q.2 (a) **(ii)** Find the hybrid parameters for the network shown in figure below.



[8 marks]

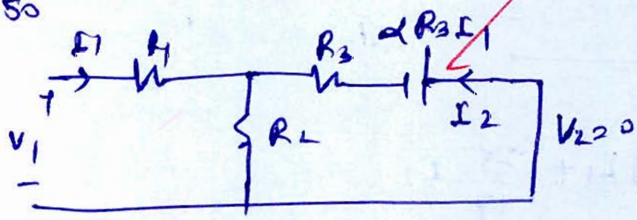
Answer hybrid parameter

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

if $V_2 = 0$ then $h_{11} = \frac{V_1}{I_1}$ and $h_{21} = \frac{I_2}{I_1}$

So



(Note: By converting current source into volt. source)

from KVL

$$V_1 = (R_1 + R_2) I_1 + R_2 I_2 \quad \text{—(1)}$$

and

$$\alpha R_3 I_1 + (R_2 + R_3) I_2 + R_2 I_1 = 0$$

$$(\alpha R_3 + R_2) I_1 = -(R_2 + R_3) I_2$$

So

$$\boxed{h_{21} = \frac{I_2}{I_1} = -\left(\frac{\alpha R_3 + R_2}{R_2 + R_3}\right)} \quad \text{—(2)}$$

from (1) and (2)

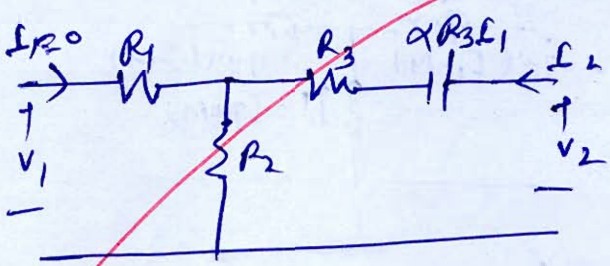
$$V_1 = (R_1 + R_2) I_1 + R_2 \left[\frac{-(\alpha R_3 + R_2)}{R_2 + R_3}\right] I_1$$

$$\frac{V_1}{I_1} = \frac{(R_1 + R_2)(R_2 + R_3) - R_2 (R_2 + \alpha R_3)}{(R_2 + R_3)}$$

$$\frac{V_1}{I_1} = \frac{R_1R_2 + R_2^2 + R_1R_3 + R_2R_3 - R_2^2 - R_2R_3\alpha}{R_2+R_3}$$

$$h_{11} = \frac{R_1R_2 + R_1R_3 + R_2R_3 - R_2R_3\alpha}{R_2+R_3} = \frac{R_1(R_2+R_3) + R_2R_3(1-\alpha)}{(R_2+R_3)}$$

Now $I_1 = 0$ then $\frac{V_1}{V_2} = h_{12}$, $h_{22} = \frac{I_2}{V_2}$



$$-V_1 + (R_2)I_2 = 0$$

$$V_1 = R_2 I_2 \quad \text{—(3)}$$

$$V_2 = \alpha R_3 I_1 + (R_2+R_3)I_2$$

as $I_1 = 0$

$$V_2 = (R_2+R_3)I_2 \quad \text{—(4)}$$

So from (4)

$$\frac{I_2}{V_2} = h_{22} = \frac{1}{R_2+R_3}$$

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from (3) and (4)

$$V_1 = R_2 \times \frac{V_2}{R_2+R_3}$$

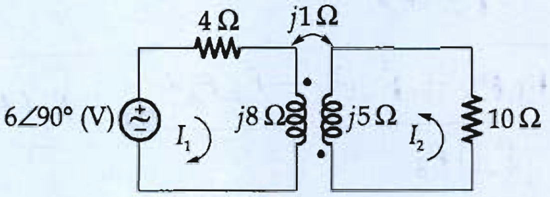
$$h_{12} = \frac{V_1}{V_2} = \frac{R_2}{R_2+R_3}$$

So,

$$h_{11} = R_1 + \frac{R_2R_3(1-\alpha)}{R_2+R_3}, \quad h_{12} = \frac{R_2}{R_2+R_3}$$

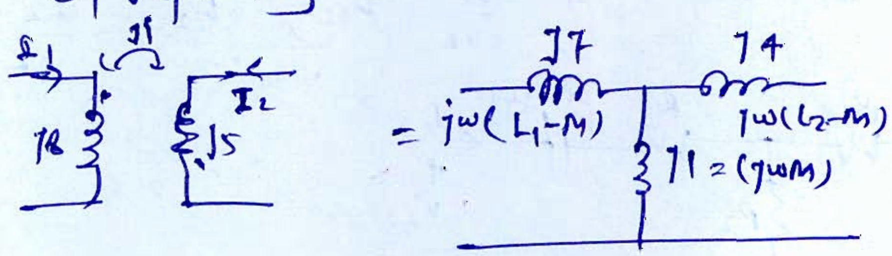
$$h_{21} = -\frac{(R_2+\alpha R_3)}{R_2+R_3}, \quad h_{22} = \frac{1}{R_2+R_3}$$

Q.2 (b) (i) Determine the current I_1 and I_2 in the circuit shown in figure below, using *T*-equivalent circuit for the linear transformer.

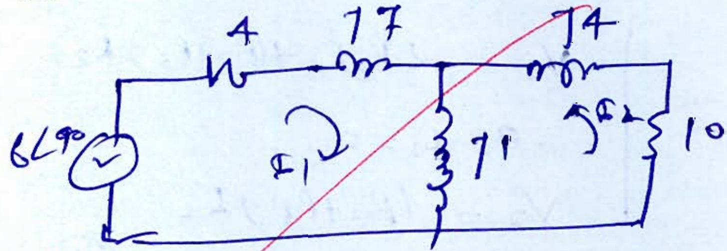


[8 marks]

Answer By Replacing



So



KVL in loop ①

$-6\angle 90 + (4 + j8)I_1 + jI_2 = 0$

$(4 + j8)I_1 + j1 I_2 = 6\angle 90$ —— ①

KVL in loop ②

$(10 + j5)I_2 + jI_1 = 0$

$I_1 = -\left(\frac{10 + j5}{j1}\right) I_2$ —— ②

from ① and ②

$\left[-(4 + j8)\left(\frac{10 + j5}{j1}\right) + j1\right] I_2 = 6\angle 90$

$I_2 = 0.06\angle{-89.42^\circ}$ Amp

from ②

$$I_1 = -\left(\frac{10+j5}{j1}\right) \times 0.06 \angle -89.42$$

$$\boxed{I_1 = 0.6708 \angle 27.14^\circ} \text{ Amp}$$

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Good Approach

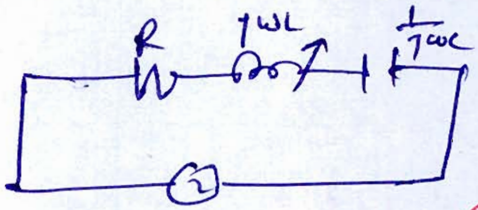
Q.2 (b) **(ii)** A voltage of $v = (2000 \sin \omega t + 400 \sin 3\omega t + 100 \sin 5\omega t)$ volts is applied to a series circuit having $R = 10\ \Omega$ and $C = 30\ \mu F$ and a variable inductance.

1. Find the value of inductance so as to give resonance at 3rd harmonic frequency.
2. What are the rms values of voltage and current with this inductance in circuit? (Take $\omega = 300$ rad/sec).

[12 marks]

Answer

(1) $v = 2000 \sin \omega t + 400 \sin 3\omega t + 100 \sin 5\omega t$



As $\omega_0' = 3\omega = 3 \times 3\omega = 9\omega$ rad/sec

so at resonance.

$$j\omega_0 L = \frac{1}{j\omega_0 C}$$

$$3\omega_0 L = \frac{1}{3\omega_0 C}$$

$$L = \frac{1}{9\omega_0^2 C} = \frac{1}{9 \times (300)^2 \times 30 \times 10^{-6}}$$

$$\boxed{L = 41.15\ mH}$$

11

Good Approach

(2) At this value of inductance, circuit become resistive.

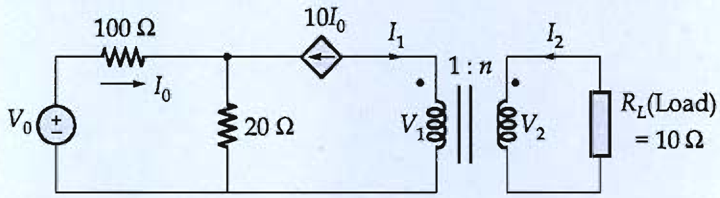
$$V_{rms} = \sqrt{\left(\frac{2000}{\sqrt{2}}\right)^2 + \left(\frac{400}{\sqrt{2}}\right)^2 + \left(\frac{100}{\sqrt{2}}\right)^2}$$

$$\boxed{V_{rms} = 1443.95\ \text{Volt}}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{1443.95}{10}$$

$$\boxed{I_{rms} = 144.39\ \text{Amp.}}$$

Q.2 (c) (i) What is the voltage and power gain of the circuit shown in figure? Assume $n = \frac{1}{10}$.

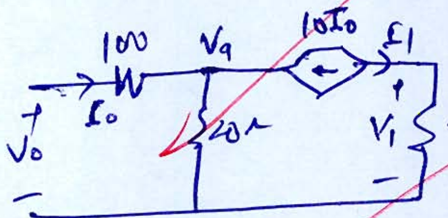


[10 marks]

Ans. As $\frac{V_1}{V_2} = \frac{1}{10}$, $\frac{I_1}{I_2} = -10$

So By referring R_L into primary side

$R_L' = R_L \times (n)^2 = 10 \times \frac{1}{100} = 0.1\Omega$



Nodal Analysis at Va

$\frac{V_a}{20} + \frac{V_a - V_0}{100} = 10 I_0$

$6V_a - V_0 = 1000 I_0$ — (1)

As $I_1 = -10 I_0$ — (2)

$I_0 = \frac{V_0 - V_a}{100}$ — (A)

$V_0 - V_a = 100 I_0$ — (3)

and $V_1 = 0.1 I_1 = 0.1(-10 I_0) = -I_0$ — (4)

from (1) and (3)

$6(V_0 - 100 I_0) - V_0 = 1000 I_0$

$5V_0 = 1600 I_0$

$\boxed{V_0 = 320 I_0}$ — (5)

from (4) and (5)

$V_0 = 320(-V_1)$

$$\boxed{\frac{V_o}{V_1} = -320}$$ —(6)

and $\frac{V_1}{V_2} = 0.1$

$V_1 = 0.1V_2$

So from (6)

$$\frac{V_o}{0.1V_2} = -320 \Rightarrow \boxed{\frac{V_o}{V_2} = -32}$$ Voltage gain

from (3) and (5)

$V_o = 320 I_o - V_a = 100 I_a$

$V_a = 220 I_o$

and $V_o = 320 I_o$

$$\frac{V_o}{V_a} = \frac{32}{22} = \frac{16}{11} \Rightarrow V_o = \frac{16}{11} V_a$$

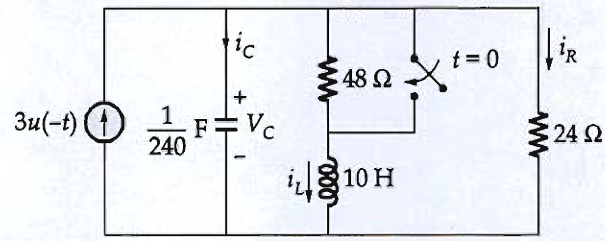
~~from (A)~~

~~$I_o =$~~

5

Do not write in this margin

Q.2 (c) (ii) Consider the circuit shown below:



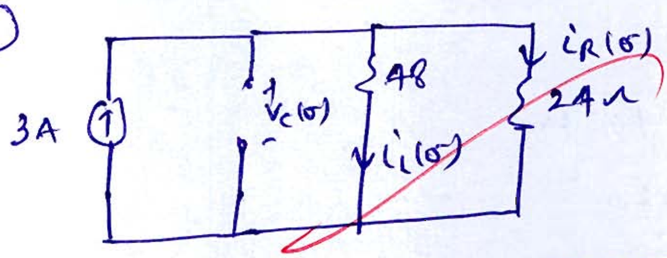
After being open for a long time, the switch is closed at $t = 0$. Find

1. $i_L(0^-)$ 2. $V_C(0^-)$ 3. $i_R(0^+)$
4. $i_C(0^+)$ 5. $V_C(0.2)$ using Laplace transform approach.

[10 marks]

Answer at $t < 0$, circuit become like,

(1)



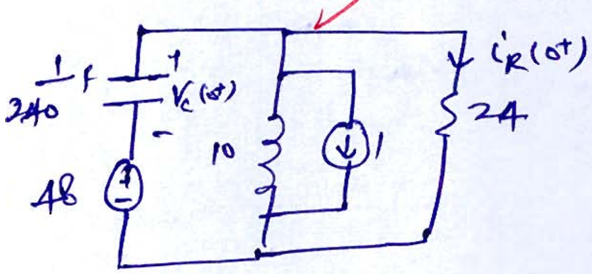
$$i_L(0^-) = \frac{3 \times 24}{24 + 48} \Rightarrow \boxed{i_L(0^-) = 1 \text{ Amp.}}$$

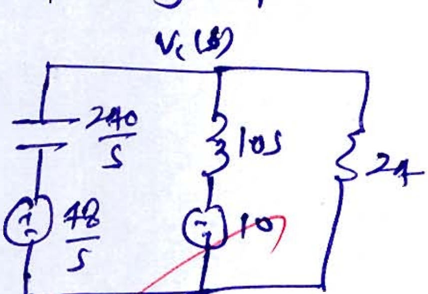
(2)

$$V_c(0^-) = 48\, i_L(0^-) = 48 \times 1$$

$$\boxed{V_c(0^-) = 48 \text{ Volt}}$$

At $t > 0$ or at $t = 0^+$, By using Laplace Transform





By Nodal Analysis

$$\frac{V_c(s) - 48/s}{240/s} + \frac{V_c(s) + 10}{10s} + \frac{V_c(s)}{24}$$

$$V_c(s)\left[\frac{s}{240} + \frac{1}{10s} + \frac{1}{24}\right] = -\frac{1}{s} + \frac{1}{5}$$

$$V_c(s) = \left[\frac{s^2 + 24s + 10s}{240s}\right] = \left(\frac{s-5}{5s}\right)$$

$$V_c(s) = \frac{48(s-5)}{s^2+24s+10} = \frac{A}{s+6} + \frac{B}{s+4}$$

$$V_c(s) = \frac{264}{s+6} - \frac{216}{s+4}$$

$$\boxed{V_c(t) = (264e^{-6t} - 216e^{-4t})u(t)} \quad \text{(By ILT)}.$$

Now

$$i_R(t) = \frac{V_c(t)}{24} = (11e^{-6t} - 9e^{-4t})u(t)$$

(3)

$$\boxed{i_R(0^+) = 2\ \text{Amp.}}$$

(4)

$$i_c(t) = C\frac{dV_c}{dt} = \frac{1}{240}\left[-6\times 264\, e^{-6t} + 216\times 4 e^{-4t}\right]u(t) + \frac{1}{240}\left[264e^{-6t} - 216e^{-4t}\right]\delta(t)$$

$$i_c(0^+) = \frac{1}{240}\left[-6\times 264 + 216\times 4\right] + \frac{1}{240}\left[264 - 216\right]$$

$$\boxed{i_c(0^+) = -2.8\ \text{Amp.}}$$

(5)

$$V_c(0.2) = \left[264\, e^{-6\times 0.2} - 216 e^{-4\times 0.2}\right]$$

$$\boxed{V_c(0.2) = -17.54\ \text{Volt}}$$

9

Good Approach

Do not write in this

Q.3 (a) **(i)** A 415-V, 50-Hz, three-phase voltage is applied to three star-connected identical impedances. Each impedance consists of a resistance of 15 Ω, a capacitance of 177 μF and an inductance of 0.1 henry in series.

Find:

1. the phase current,
2. the line current,
3. the power factor,
4. the active power,
5. the reactive power and
6. the total VA.

Draw a neat phasor diagram. If the same impedances are connected in delta, find the (i) line current and (ii) power consumed.

[10 marks]

Do not write in this margin

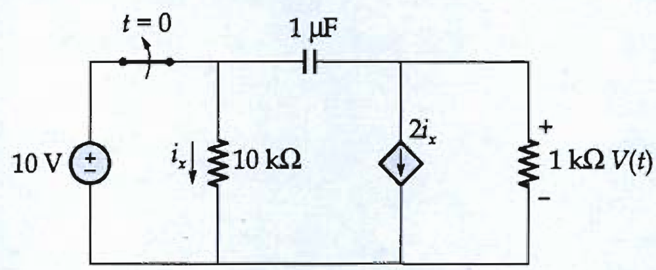
3 (a) (ii) A coil having a resistance of 20 Ω and an inductance of 200 μH is connected in parallel with a variable capacitor. This parallel combination is connected in series with a resistance of 8000 Ω. A voltage of 230 V at a frequency of 10^6 Hz is applied across the circuit.

Calculate:

1. the value of capacitance at resonance,
2. Q-factor of the circuit,
3. dynamic impedance of the circuit, and
4. total circuit current.

[10 marks]

3 (b) (i) For the circuit shown in figure:

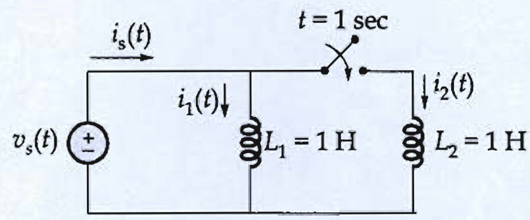


1. Find the expression of $V(t)$, the voltage across 1 kΩ resistor when the switch is opened at time, $t = 0$.
2. Sketch $V(t)$ with respect to time (t) and mark the time constant t.

[10 marks]

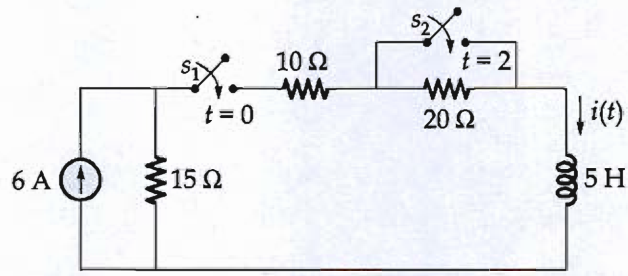
3 (b) **(ii)** For the parallel inductive circuit shown below with switch closed at t = 1s, $v_s(t) = \cos(t)$ V for $t \geq 0$ and 0 otherwise, **find:**

1. the input current $i_s(t)$ for $t \geq 0$ sec.
2. the energy stored in each of the inductors for the intervals [0, t] for $0 \leq t \leq 1$ and for $1 \leq t$.



[10 marks]

Q.3 (c) Consider the network shown below:

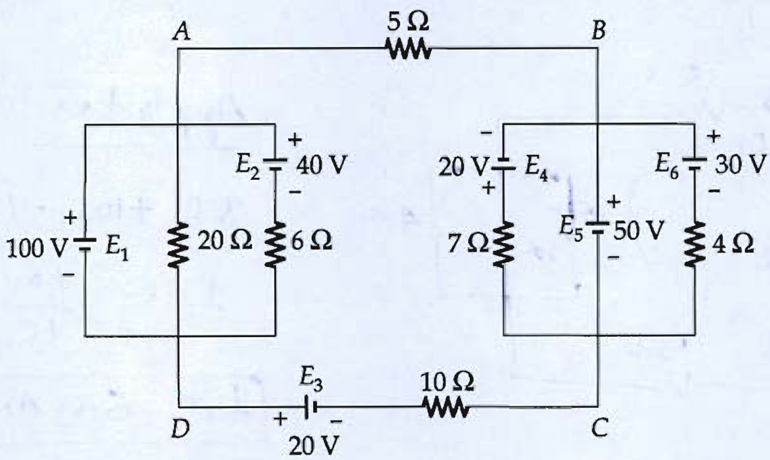


Switch S_1 is closed at $t = 0$, and switch S_2 is closed at $t = 2$ sec. Calculate current $i(t)$ for all t, and also find $i(t)\,|_{t = 1\text{ sec}}$ and $i(t)\,|_{t = 3\text{ sec}}$.

[20 marks]

Do not write in this margin

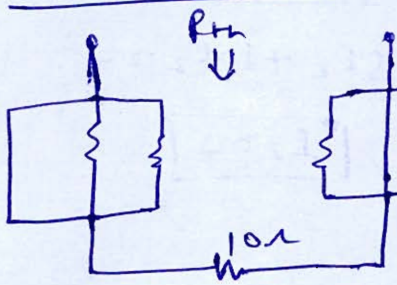
.4 (a) For the circuit shown in figure, find the current through 5 Ω resistor by using Thevenin's theorem and verify the same by using superposition theorem.



[20 marks]

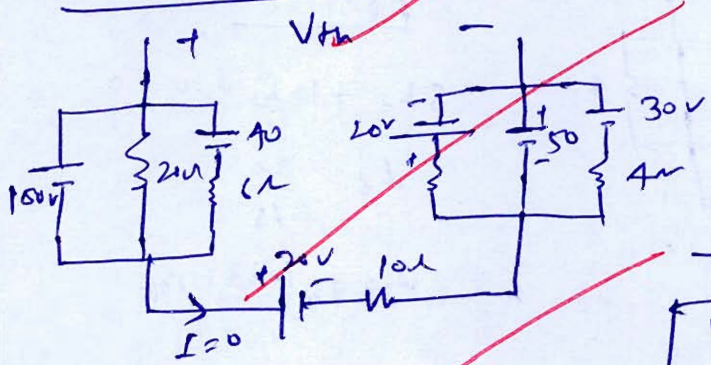
Answer Using Thevenin's Theorem

Rth Calculation



$R_{th} = 10\,\Omega$

Vth Calculation

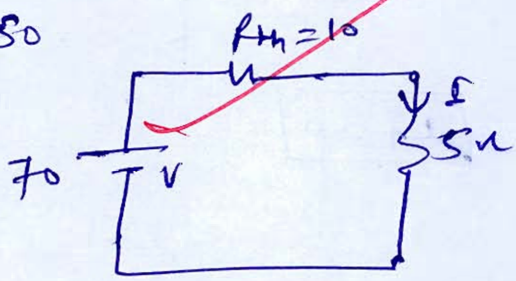


Applying KVL

$-V_{th} + 100 + 20 - 50 = 0$

$V_{th} = 70$ Volt

So



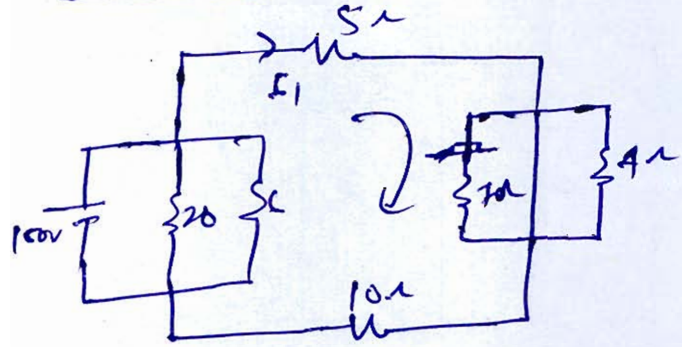
$I = \frac{70}{10+5}$

$I = 4.67$ Amp.

(1)

Using Superposition theorem

taking E_1



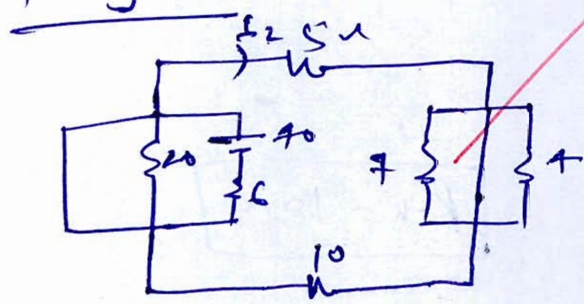
Apply KVL

$5I_1 + 10I_1 - 100 = 0$

$I_1 = \frac{100}{15}$

$I_1 = 6.66$ Amp

taking E_2

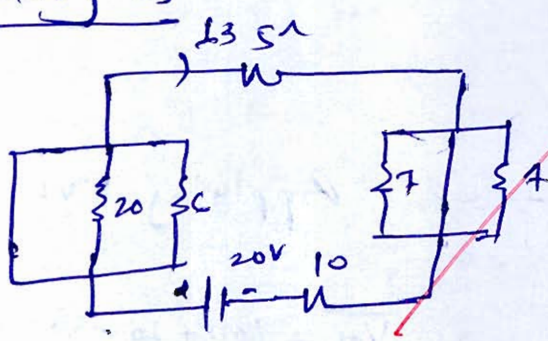


Apply KVL

$5I_2 + 10I_2 = 0$

$I_2 = 0$

taking E_3



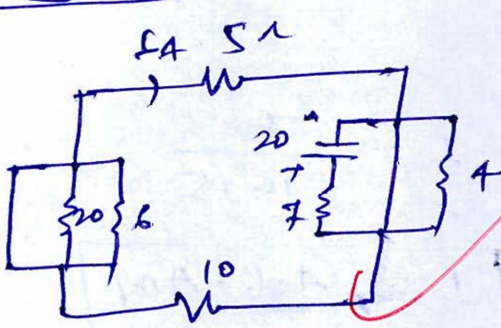
Apply KVL

$5I_3 + 10I_3 - 20 = 0$

$I_3 = \frac{20}{15}$

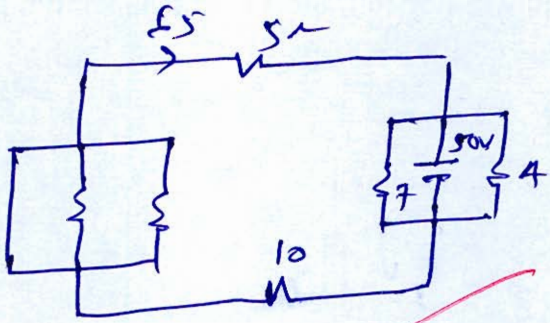
$I_3 = 1.33$ Amp

taking E_4



$I_4 = 0$

taking I_5



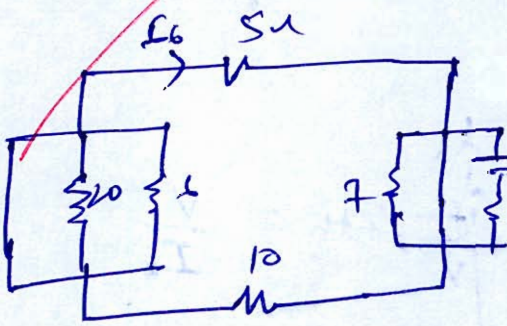
Apply KVL

$5I_5 + 50 + 10I_5 = 0$

$I_3 = -\frac{50}{15}$

$\boxed{I_5 = -3.33\,A}$

taking I_6



$5I_6 + 10I_6 = 0$

$\boxed{I_6 = 0}$

So

$I = I_1 + I_2 + I_3 + I_4 + I_5 + I_6$

$I = 6.66 + 0 + 1.33 + 0 + (-3.33) + 0$

$\boxed{I = 4.66\,Amp}$ ⟶ (2)

from (1) and (2)

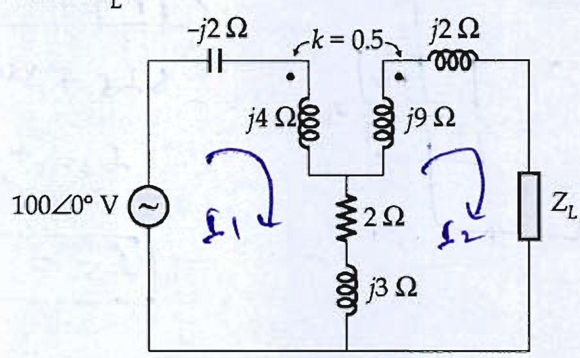
Both current are same.

18

Good Approach

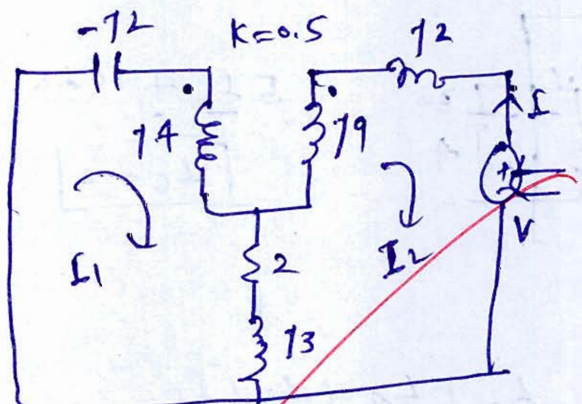
Q.4 (b) (i) Find the Thevenin's equivalent of the circuit shown in figure below as seen from the load impedance Z_L.

(ii) Find the value of Z_L for maximum power transfer and also the maximum power transfer to the load Z_L.



[10 + 10 marks]

Answer (i) for Thevenin's equivalent Impedence



$$Z_{th} = \frac{V}{I}$$

$$I = -I_2 \quad \text{—} ①$$

As $j\omega M = K\sqrt{j\omega L_1 \times j\omega L_2} = j0.5\sqrt{4\times 9} = j3$

KVL in loop ①

$$(-j2 + j4)I_1 + (2+j3)(I_1 - I_2) - j3I_2 = 0$$

$$(j2+2+j3)I_1 - (2+j3)I_2 - j3I_2 = 0$$

$$(2+j5)I_1 = (2+j6)I_2$$

$$I_1 = \left(\frac{2+j6}{2+j5}\right)I_2 \quad \text{—} ②$$

KVL in loop ②

$$(j2+j9+2+j3)I_2 - (2+j3)I_1 - j3I_1 + V = 0$$

$$(2+j14)I_2 - (2+j6)I_1 = -V \quad \text{—} ③$$

from ② and ③

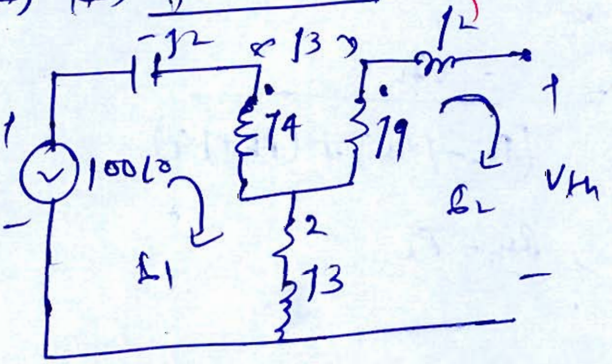
$$(2+j14)I_2 - (2+j6) \times \frac{(2+j6)}{(2+j5)} I_2 = -V$$

$$I_2\left(\frac{2}{29} + j\frac{198}{29}\right) = -V$$

$$-\frac{V}{I_2} = \frac{V}{I} = 0.146 \angle -89.42^\circ$$

$$\boxed{Z_{th} = 0.0015 - j0.146}$$

(iii) for V_{th}



As $I_2 = 0$

KVL in loop ①

$$100\angle 0 = (-j2 + j4 + 2 + j3) I_1$$

$$I_1 = \frac{100\angle 0}{2+j5} \quad — ④$$

KVL in loop ②

$$+(2+j3)I_1 - V_{th} - j3I_1 = 0$$

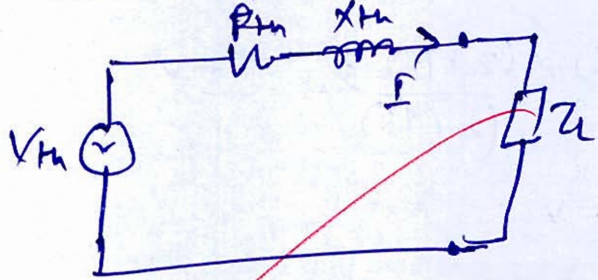
$$V_{th} = 2I_1 \quad — ⑤$$

from ④ and ⑤

$$V_{th} = \frac{200\angle 0}{2+j5}$$

$$\boxed{V_{th} = 37.139 \angle -68.19^\circ} \text{ Volt}$$

So Thevenin equivalent



$V_{th} = 37.14 \angle -68.19^\circ$

$Z_{th} = 0.0015 - j\,0.146$

(ii) for maximum power transfer Z_L will be

$$Z_L = Z_{th}^*$$

$$\boxed{Z_L = 0.0015 + j\,0.146}$$

$$I = \frac{V_{th}}{Z_{th} + Z_L} = \frac{V_{th}}{(R_{th} - jX_{th}) + (R_L + jX_L)}$$

As $X_L = X_{th}$, $R_{th} = R_L$

So

$$I_{max} = \frac{V_{th}}{2R_{th}}$$

fo maximum power

$$P_{max} = I_{max}^2 R_{th}$$

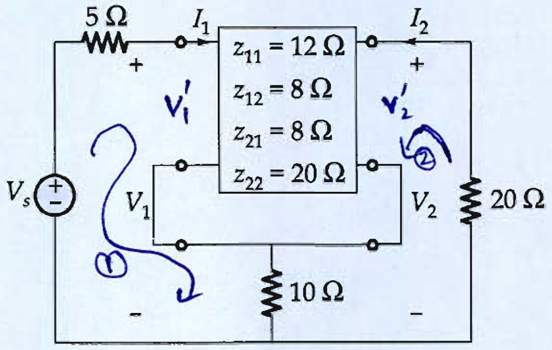
$$P_{max} = \left(\frac{V_{th}}{2R_{th}}\right)^2 R_{th} = \frac{V_{th}^2}{4R_{th}}$$

$$P_{max} = \frac{(37.14)^2}{4 \times 0.0015}$$

$$\boxed{P_{max} = 229.896\ \text{kW}}$$

5

Q.4 (c) (i) Evaluate the ratio V_2/V_s in the circuit shown below.



[10 marks]

given $V_1' = 12I_1 + 8I_2$ —— ①

$V_2' = 8I_1 + 20I_2$ —— ②

By Applying KVL in loop ①

$-V_s + 5I_1 + V_1' + 10(I_1 + I_2) = 0$

$V_s = 15I_1 + 10I_2 + V_1'$

from ①

$V_s = 15I_1 + 10I_2 + 12I_1 + 8I_2$

$V_s = 27I_1 + 18I_2$ —— ③

KVL in loop ②

$V_2' + 10(I_1 + I_2) + 20I_2 = 0$

from ②

$8I_1 + 20I_2 + 10I_1 + 10I_2 + 20I_2 = 0$

$18I_1 + 50I_2 = 0$

$I_1 = -\frac{25}{9} I_2$ —— ④

And

$V_2 = -20I_2$ —— ⑤

from ③ and ④

$V_s = 27\left(-\frac{25}{9}\right)I_2 + 18I_2$

$V_s = -57I_2$ —— ⑥

from (5) and (6)

$$\frac{V_2}{V_s} = \frac{-20 I_2}{-57 I_2}$$

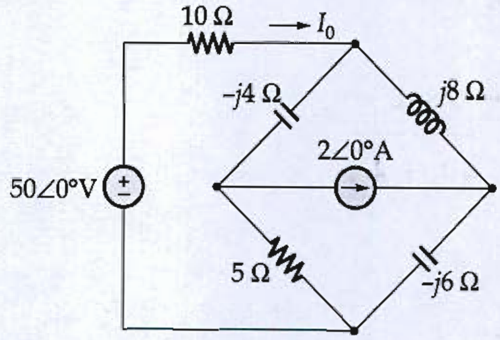
$$\frac{V_2}{V_s} = \frac{20}{57}$$

$$\boxed{\frac{V_2}{V_s} = 0.3507}$$

9

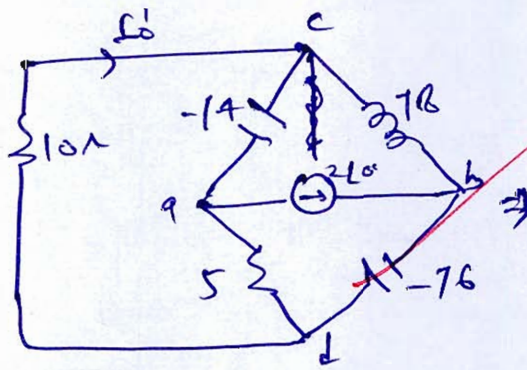
Good Approach

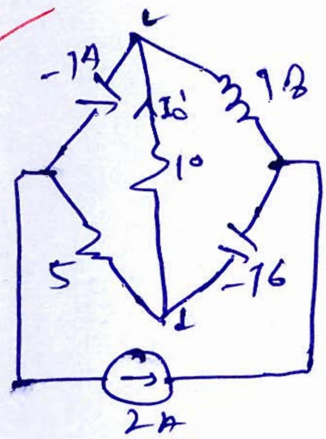
Q.4 (c) (ii) By using superposition theorem, find current I_0 in the circuit shown below:



[10 marks]

Answer Case-1 By using 2∠0° A source
then $I_0 = I_0'$
Voltage source will be short circuited.





1

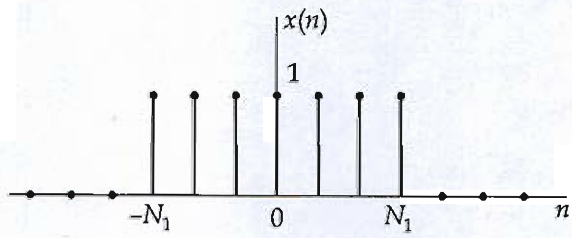
Incomplete Solution

Section B : Systems & Signal Processing

Q.5 (a) Find the Fourier transform of the rectangular pulse

$$x(n) = u(n+N_1) - u(n-N_1-1) = \begin{cases} 1, & |n| \le N_1 \\ 0, & |n| > N_1 \end{cases}$$

which is illustrated in figure below. Also draw the magnitude and phase spectrum for $N_1 = 2$.



[12 marks]

Answer

$$DFT = \sum_{N=-N}^{N} x(n) e^{-j\omega_0 kn}$$

$$DFT = X(k) = \sum_{-N_1}^{N_1} 1 \cdot e^{-j\omega_0 kn}$$

$\omega_0 = \frac{2\pi}{N}$

$$X(k) = \sum_{-N_1}^{N_1} e^{-j\frac{\pi}{N_1}kn}$$

$\omega_0 = \frac{2\pi}{2N_1} = \pi/N_1$

$$\boxed{X(k) = e^{-j\frac{\pi}{N_1}kn} \times (2N_1+1)}$$

1

Incomplete Solution

Do not write in this margin

Q.5 (b) Consider the continuous-time signal $x(t) = \cos(100\pi t)$:

(i) Determine the minimum sampling rate required to avoid aliasing.

(ii) Suppose that the signal is sampled at the rate $f_s = 200$ Hz. What is the discrete-time signal obtained after sampling?

(iii) Suppose that the signal is sampled at the rate $f_s = 75$ Hz. What is the discrete-time signal obtained after sampling?

(iv) What is the frequency $0 < f < f_s/2$ of a sinusoid that yields samples identical to those obtained in part (iii)?

[12 marks]

Answer (i) $x(t) = \cos(100\pi t)$

$2\pi f_m = 100\pi$

$f_m = 50$ Hz

Nyquist rate $f_N = 2f_m = 100$ Hz

$\boxed{f_N = 2f_m = 100 \text{ Hz}}$

(ii) given $f_s = 200$ Hz

$x(t) = \cos(100\pi t)$

$t = nT_s = n/f_s$

$x(n) = \cos\left(100\pi \times \frac{n}{200}\right)$

$\boxed{x(n) = \cos\left(\frac{\pi}{2} n\right)}$

(iii) $f = 75$ Hz

$x(t) = \cos(100\pi t)$

$x(n) = \cos\left(\frac{100\pi n}{75}\right)$

$\boxed{x(n) = \cos\left(\frac{4}{3}\pi n\right)}$

9

Incomplete solution

Do not write in this margin

Q.5 (c) Determine the signal $x(n)$ whose z-transform is given by

$X(z) = \log(1 + az^{-1}),\ |z| > |a|$

[12 marks]

Answer

$$X(z) = \log(1+az^{-1})$$

$$\frac{dX(z)}{dz} = \frac{1}{1+az^{-1}}(-az^{-2})$$

By multiplying both side with $-z$

$$-z\frac{dX(z)}{dz} = \frac{az^{-1}}{1+az^{-1}} \quad \text{—(1)}$$

as we know

$$n\,x(n) \rightleftharpoons -z\frac{dX(z)}{dz} \quad \text{—(2)}$$

Now

$$(-a)^n u(n) \overset{Z.T.}{\rightleftharpoons} \frac{1}{1+az^{-1}}$$

$$a.(-1)^n a^n u(n) \rightleftharpoons \frac{a}{1+az^{-1}}$$

Now

$$a(-1)^{n-1} a^{n-1} u(n-1) \rightleftharpoons \frac{az^{-1}}{(1+az^{-1})} \quad \text{—(3)}$$

[Note: $x(n) \rightleftharpoons X(z)$

then $x(n-1) \rightleftharpoons z^{-1}X(z)$]

from eq^n (1) and (3)

$$n\,x(n) = (-1)^{n-1} a^n u(n-1)$$

$$\boxed{x(n) = \frac{(-1)^{n-1} a^n u(n-1)}{n}}$$

10

Improve presentation

Ans: Dirichlet conditions

(1) For existence of Fourier transform, signal should have finite number of maxima & minima in a finite time period.

Example

(2) For existence of Fourier transform, signal must have finite number of discontinuities.

Example

(3) For existence of Fourier transform, signal must be absolutely integrable.

$\sum |x(n)| < \infty$

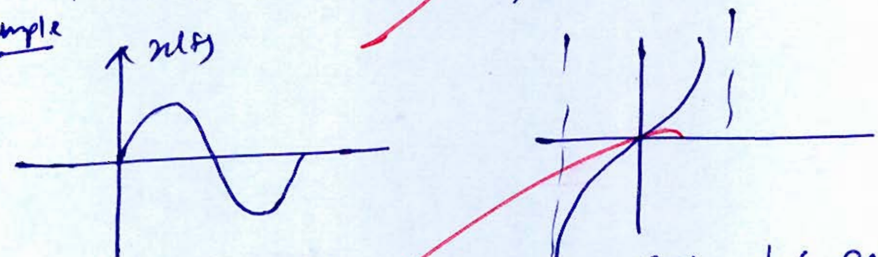
Q.5 (d) Discuss the Dirichlet conditions for the existence of the Continuous-Time Fourier Transform. Are these conditions mandatory for a signal to possess a Fourier transform? Explain with examples.

[12 marks]

Ans. Dirichlet conditions

(1) to exist fourier transform, signal must have finite number of maxima or minima in a definite time period.

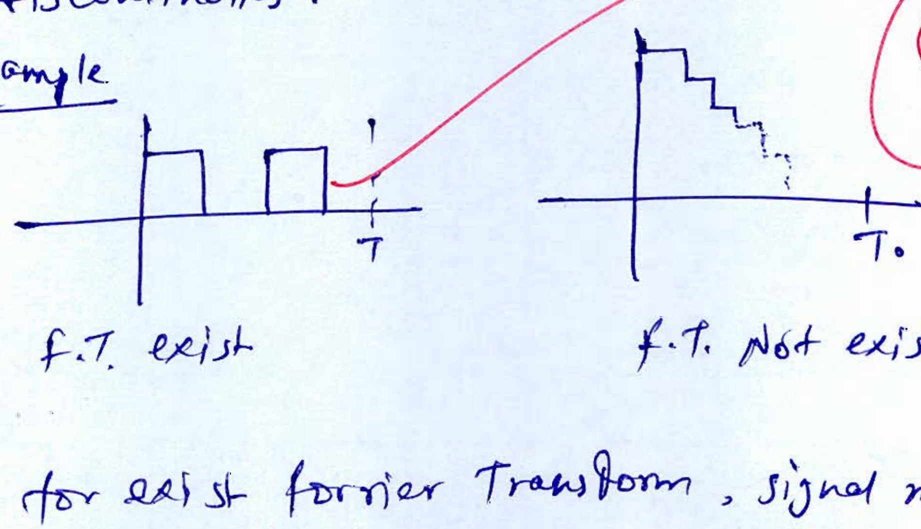
example



f.T. exist f.T. Not exist

(2) for exist fourier transform of a signal, signal must have a finite number of discontinuities.

example



f.T. exist f.T. Not exist

9

(3) for exist fourier Transform, signal must be stable for a bounded input signal. i.e. signal should satisfy BIBO criteria. or sum of signal in a period should be finite: i.e. Absolutely summable

$$\sum_{n=-\infty}^{\infty} |x(t)| < \infty$$

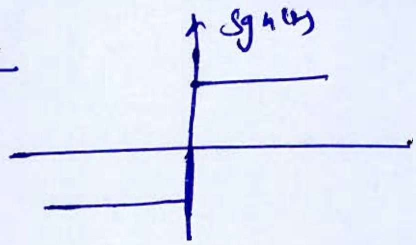
(e) Find the Fourier transform of the signum function $x(t) = \text{sgn}(t)$.

$$\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$

Also draw the magnitude and phase spectrum of $X(\omega)$.

[12 marks]

Answer



$\text{Sgn}(t) = u(t) - u(-t) = x(t)$ —①

As we know

$u(t) \rightleftharpoons \frac{1}{j\omega} + X(0)\delta(\omega)$ —②

$u(-t) \rightleftharpoons \frac{1}{-j\omega} + X(0)\delta(\omega)$ —③

As Avg. of above signal is zero.

So from ① & ②, and ③

$\boxed{\text{Sgn}(t) \rightleftharpoons \frac{2}{j\omega}} = X(\omega)$

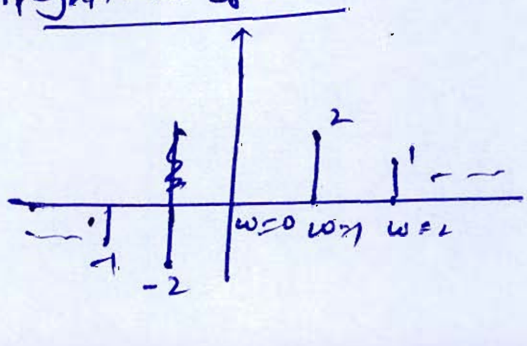
9

Good Approach

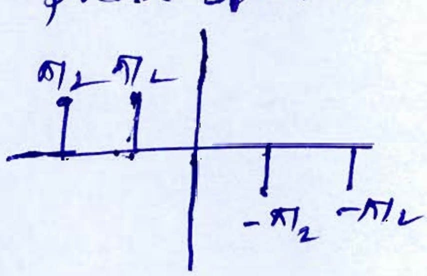
So $|X(\omega)| = 2/\omega$

$\angle X(\omega) = -90°$

Magnitude spectrum



phase spectrum



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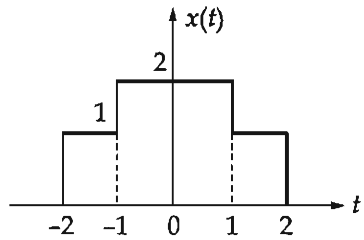
(a) (i) An LTI system has a unit step response given by $s(t) = (1 - e^{-t} - te^{-t})u(t)$. For a certain input $x(t)$, the output is observed to be equal to $y(t) = (2 - 3e^{t} + e^{-3t})u(t)$. What is $x(t)$?

[12 marks]

(a) (ii) Determine the Fourier transform of the signal shown in the following figure.



[8 marks]

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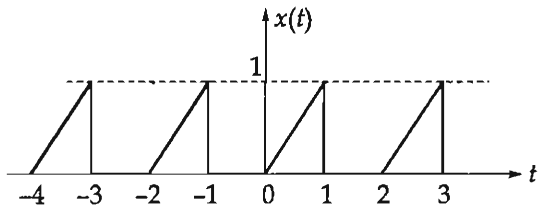
(b) (i) Find the trigonometric Fourier series for the waveforms shown in figure below and sketch the line spectrum.



[12 marks]

Q.6 (b) (ii) A causal and stable LTI system " S" has the property that when we apply the input;

$\left(\frac{4}{5}\right)^n u(n)$, it gives the output $n\left[\frac{4}{5}\right]^n u(n)$.

Determine the transfer function $H(e^{j\omega})$ for the system.

[8 marks]

(c) (i) Determine the transient response and steady-state response of the system characterized by the difference equation, $y(n) = 0.5y(n-1) + x(n)$, when the input signal is $x(n) = 10\cos(n\pi/4)u(n)$. The system is initially at rest (i.e., it is relaxed).

[10 marks]

(c) **(ii)** Solve the difference equation using the one-sided z-transform $y(n) = x(n) + by(n-1)$ with initial condition $y(-1) = P$. Assume input be $x(n) = e^{j\omega_0 n}u(n)$.

[10 marks]

(a) Determine the values of power and energy for each of the following signals. Also find the nature of signals.

(i) $x_1(t) = e^{-2t}u(t)$.

(ii) $x_2(t) = e^{j(2t+\pi/4)}$.

(iii) $x_3(n) = \cos\left(\left(\frac{\pi}{4}\right)n\right)$.

[20 marks]

Answer (i) $x(t) = e^{-2t}u(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{\infty} |e^{-2t}|^2 dt$$

$$E_x = \int_0^{\infty} e^{-4t} dt = \left[\frac{e^{-4t}}{-4}\right]_0^{\infty}$$

$$\boxed{E_x = \frac{1}{4}} \text{ J.}$$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{0}^{\infty} e^{-4t} dt = \lim_{T\to\infty} \frac{1}{T} \left[\frac{e^{-4t}}{-4}\right]_0^{\infty}$$

$$\boxed{P_x = 0 \text{ watt}}$$

Nature of signal = Energy signal.

(ii) $x_2(t) = e^{j(2t+\pi/4)}$

$$x_2(t) = \cos(2t+\pi/4) + j\sin(2t+\pi/4)$$

Rms value of Above signal

$$x_{rms} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

Power of $x_2(t)$ = $(Rms)^2$

$$\boxed{P_{x_2} = 1 \text{ watt}}$$

for Energy

$$E_{x_2(t)} = \int_{-\infty}^{\infty} |e^{j(2t+\pi/4)}|^2 dt$$

As $|e^{j(2t+\pi/4)}| = 1$

So $$\boxed{E_{x_2(t)} = \int_{-\infty}^{\infty} 1\, dt = \infty}$$

So Nature of signal = power signal.

(iii) $x_3(n) = \cos(\frac{\pi}{4}n)$

Rms value of above signal is = $1/\sqrt{2}$

So

Power of $x_3(n)$ = P_{x_3} = $(Rms)^2$

$$\boxed{P_{x_3} = \frac{1}{2}}$$

Now

$$\text{Energy} = \sum_{N=-\infty}^{\infty} |x_3(n)|^2$$

$$\text{Energy} = \sum_{N=-\infty}^{\infty} \cos(\frac{\pi}{4}n)$$

$$\boxed{E_{x_3(n)} = \infty}$$

Nature of signal = Power signal

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Good Approach

Q.7 (b) Find the inverse Laplace transform of

$$X(s) = \frac{-5s-7}{(s+1)(s-1)(s+2)}$$

if the ROC is

(i) $\Re\{s\} > 1$

(ii) $\Re\{s\} < -2$

(iii) $-1 < \Re\{s\} < 1$

(iv) $-2 < \Re\{s\} < -1$

[20 marks]

Anser

$$X(s) = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s+2}$$

$$X(s) = \frac{1}{s+1} - \frac{2}{s-1} + \frac{1}{s+2} \quad \text{—} (1)$$

(i) for $R(s) > 1$

As we $e^{-at}u(t) \rightleftharpoons \frac{1}{s+a}$. $R(s) > a$

and $-e^{-at}u(-t) \rightleftharpoons \frac{1}{s+a}$; $R(s) < a$

So taking inverse Laplace of eqn (1)

$$x(t) = e^{-t}u(t) - 2e^{t}u(t) + e^{-2t}u(t)$$

(ii) $R\{s\} < -2$

then taking inverse Laplace of (1)

$$x(t) = -e^{-t}u(-t) + 2e^{t}u(-t) - e^{-2t}u(-t)$$

$$x(t) = (-e^{-t} + 2e^{t} - 2e^{-2t})\,u(-t)$$

(iii) $-1 < R(s) < 1$

from ①

$$X(s) = \frac{1}{s+1} - \frac{2}{s-1} + \frac{1}{s+2}$$

taking ILT of ①

$$\boxed{x(t) = e^{-t}u(t) + 2e^{t}u(-t) + e^{-2t}u(t)}$$

(iv) $-2 < R(s) < -1$

$$X(s) = \frac{1}{s+1} - \frac{2}{s-1} + \frac{1}{s+2}$$

taking ILT of ①

$$x(t) = -e^{-t}u(-t) + 2e^{+t}u(-t) + e^{-2t}u(t)$$

$$\boxed{x(t) = \left(-e^{-t} + 2e^{t}\right)u(-t) + e^{-2t}u(t)}$$

18

Good Approach

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c) An analog filter has the transfer function:

$$H_a(s) = \frac{10}{s^2 + 7s + 10}$$

Design a digital filter $H(z)$ using the Bilinear Transformation method with a sampling period of $T = 0.2$ seconds.

(i) Determine the discrete-time transfer function $H(z)$.

(ii) Find the difference equation of the system relating $x(n)$ and $y(n)$.

(iii) Determine the poles of $H(z)$ and comment on the stability of the digital filter.

[20 marks]

Answer (i)

As we know $s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$ (Bilinear Transformation)

$$s = \frac{2}{0.2}\left(\frac{z-1}{z+1}\right) = 10\left(\frac{z-1}{z+1}\right)$$

$$H_a(s) = \frac{10}{s^2+7s+10}$$

$$H_a(z) = \frac{10}{\left[10\left(\frac{z-1}{z+1}\right)\right]^2 + 7\left[10\,\frac{z-1}{z+1}\right] + 10}$$

$$H_a(z) = \frac{10(z+1)^2}{100(z-1)^2 + 70(z^2-1) + 10(z+1)^2}$$

$$H_a(z) = \frac{10(z+1)^2}{100[z^2-2z+1] + 70z^2 - 70 + 10[z^2+2z+1]}$$

$$H_a(z) = \frac{10z^2+20z+10}{180z^2-180z+40}$$

$$\boxed{H_a(z) = \frac{z^2+2z+1}{18z^2-18z+4}}$$

(ii)

$$H_a(z) = \frac{Y(z)}{X(z)} = \frac{z^2+2z+1}{18z^2-18z+4} = \frac{1+2z^{-1}+z^{-2}}{18-18z^{-1}+4z^{-2}}$$

$$Y(z)\left[18-18z^{-1}+4z^{-2}\right] = X(z)\left[1+2z^{-1}+z^{-2}\right]$$

By Applying Z-Inverse Transform

$$18y(n)-18y(n-1)+4y(n-2) = x(n)+2x(n-1)+x(n-2)$$

$$\boxed{18y(n)-18y(n-1)+4y(n-2) = x(n)+2x(n-1)+x(n-2)}$$

(iii)

$$H_q(z) = \frac{1+2z^{-1}+z^{-2}}{18-18z^{-1}+4z^{-2}}$$

$$H_q(z) = \frac{z^2+2z+1}{18(z-2/3)(z-1/3)}$$

$$\boxed{\text{Poles: } z=2/3, \; z=1/3}$$

As

$$H_q(z) = \frac{z^2+2z+1}{18(z^2-z+2/9)}$$

$$H_q(z) = \frac{1}{18}\left[1+\frac{3z+7/9}{(z-2/3)(z-1/3)}\right]$$

$$H_q(z) = \frac{1}{18}\left[1+\frac{25/3}{z-2/3}+\frac{16/3}{z-1/3}\right]$$

$$H_q(z) = \frac{1}{18}+\frac{25/54}{z-2/3}+\frac{16/54}{z-1/3}$$

Do not write in this margin

$H_q(z)$ will be stable for

ROC : $|z| > 1/3$

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Q.8 (a) Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

(i) $x(n) = (0.8)^n\, u(n)$ and $h(n) = (0.4)^n\, u(n)$.

(ii) $x(n) = u(n - 1)$ and $h(n) = \alpha^n u(n - 1)$.

(iii) $x(n) = r(n) = nu(n)$ and $h(n) = -\alpha^{-n}u(n - 1)$, where $a < 1$.

[20 marks]

Q.8 (b) Consider a periodic square wave $x(t)$ with amplitude A, period T centered at $t = 0$, and duty cycle 50%:

(i) Derive the expression for the Trigonometric Fourier Series coefficients.

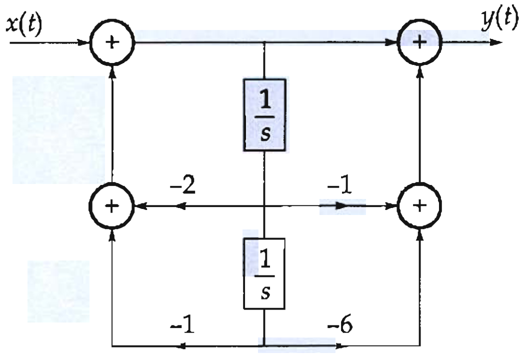
(ii) Explain the Gibbs Phenomenon in the context of reconstructing this signal using a finite number of harmonics.

(iii) Use the duality property of the Fourier Transform to find the transform of

$$g(t) = \frac{\sin(at)}{\pi t}.$$

[20 marks]

(i) The input $x(t)$ and output $y(t)$ of a causal LTI system are related through the block diagram representation shown in the below figure.



1. Determine a differential equation relating $y(t)$ and $x(t)$.
2. Is this system stable?

(ii) The input and output of a causal LTI system is related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

What is the response of this system, if $x(t) = te^{-2t}u(t)$?

[10 + 10 marks]

OOOO

Space for Rough Work

Space for Rough Work

$A(s+4) + B(s+1) = 48s - 240$

$-2A =$

$A = 264$

$B = -216$

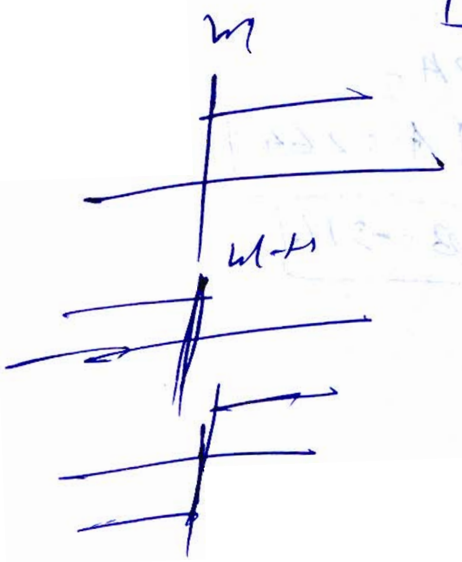
$E = \rho\, x t$

$\frac{\text{[illegible]}}{E} = \rho$

$A(s-1)(s+2) + B(s+1)(s+2) + C(s+1)(s-1) = -5s-7$

$-2 \times 1 \times A = 5-7 = -2$

$A = 1$ | $2 \times 3 \times B = -12$, $B = -2$ | $-1 \times -3 \times C$, $3C = +10-$, $C = 1$



$e^{-at} u(t) - \frac{1}{s+a}$

$e^{at} u(-t) = \frac{1}{-s+a}$

$-e^{at} u(-t) = \frac{1}{-s-a}$

$18(z^2 - z + 7/18)$

$1 - 2/9$

$\frac{1}{18} \left[z^2 - z + 7/9 \right) z^2 + 2z + 1 (1$

$z^2 - z + 7/9$

$- \quad + \quad -$

$3z + 2/9$

$A(z - 1/3) + B(z - 2/3) =$

$A \times 1/3 = 2 + 7/9 \quad \frac{25}{9}$

$A = 25/3$

$-1/3 = +1 + 7/9 = \frac{16}{9}$