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Detailed Solutions

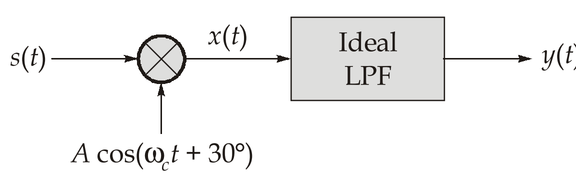
ESE-2026 Mains Test Series

E & T Engineering Test No : 3

Section A : Analog and Digital Communication Systems

Q.1 (a) Solution:

(i) 1. The given system can be represented as below:



$$x(t) = s(t)A\cos(\omega_c t + 30°)$$

$$= A^2[\cos(\omega_c t + 30°)\cos(\omega_c t)]m(t)$$

$$= \frac{A^2}{2}[\cos(2\omega_c t + 30°) + \cos 30°]m(t)$$

After passing through LPF with bandwidth equal to that of message signal, we get,

$$y(t) = \frac{A^2}{2}\cos(30°)\, m(t) = \frac{\sqrt{3}A^2}{4}m(t)$$

2. Average power of $y(t)$,

$$P_y = \left(\frac{\sqrt{3}}{4}A^2\right)^2 P_m = \frac{3A^4}{16}P_m$$

(ii) 1. Two signals are orthogonal over an interval T_b if their inner product is zero:

$$\int_0^{T_b} s_0(t)s_1(t)dt = 0$$

For coherent BFSK, to maintain orthogonality,

$$f_1 - f_0 = \frac{n}{2T_b}$$

For the minimum separation or maximum R_b, we take $n = 1$.

$$\Delta f_{min} = f_1 - f_0 = \frac{1}{2T_b} = \frac{(R_b)_{max}}{2} \quad ...(1)$$

Given $f_0 = 1000$ kHz

$f_1 = 1050$ kHz

We get, $\Delta f = 1050 - 1000 = 50$ kHz

Using the orthogonality condition, from equation (1),

$$\Delta f_{min} = \frac{(R_b)_{max}}{2}$$

$$50 \text{ kHz} = \frac{(R_b)_{max}}{2}$$

Maximum bit rate,

$$(R_b)_{max} = 100 \text{ kbps}$$

2. New frequency (f_1) = ? when $R_b = 200$ kbps

To maintain orthogonality,

$$\Delta f_{min} = \frac{R_b}{2} = \frac{200 \text{ kbps}}{2} = 100 \text{ kHz}$$

$$f_1 = f_0 + \Delta f_{min}$$

$$= 1000 + 100 = 1100 \text{ kHz}$$

Hence, new frequency $f_1 = 1100$ kHz

3. $$BW = (f_1 - f_0) + 2R_b$$

We have, $f_1 - f_0 = 50$ kHz and $R_b = 100$ kbps

$$\text{BW} = 50 \text{ kHz} + 2(100 \text{ kHz})$$

$$= 250 \text{ kHz}$$

Q.1 (b) Solution:

(i) The Shannon-Hartley theorem states that the capacity (C) of a Gaussian channel is given by:

$$C = B\log_2\left(1+\frac{S}{N}\right)bps$$

where B is the bandwidth

S is the average signal power ($S = E_b\, R_b$)

N is the total noise power (i.e., $N = N_0B$)

For Error-free transmission, the bit rate R_b must be less than or equal to the channel capacity (i.e., $R_b \le C$)

(ii) 1. Channel capacity for infinite bandwidth (C_∞):

The capacity C of a channel with bandwidth B (Hz) perturbed by Additive White Gaussian Noise (AWGN) is given by:

$$C = B\log_2\left(1+\frac{S}{N}\right) \text{ bits/sec}$$

Substituting $N = N_0B$ into the capacity equation:

$$C = B\log_2\left(1+\frac{S}{N_0B}\right)$$

To analyze the limit as $B \to \infty$, we manipulate the expression to match the standard calculus limit form $\lim\limits_{x\to 0}(1+x)^{1/x} = e$. We multiply and divide the exponent by $\frac{S}{N_0}$.

$$C = B\frac{\frac{S}{N_0}}{\frac{S}{N_0}}\log_2\left(1+\frac{S}{N_0B}\right)$$

$$C = \frac{S}{N_0}\cdot\frac{N_0B}{S}\cdot\log_2\left(1+\frac{S}{N_0B}\right)$$

Using the logarithmic property

$$n \log x = \log(x^n)$$

We get,
$$C = \frac{S}{N_0}\log_2\left(1+\frac{S}{N_0B}\right)^{\frac{N_0B}{S}}$$

As the bandwidth $B \to \infty$, the term $\frac{S}{N_0 B} \to 0$.

Let $x = \frac{S}{N_0 B}$. As $B \to \infty$, $x \to 0$. The expression becomes:

$$C_\infty = \lim_{x \to 0} \frac{S}{N_0} \log_2 (1+x)^{\frac{1}{x}}$$

Since $\lim_{x \to 0} (1+x)^{1/x} = e$

$$C_\infty = \frac{S}{N_0} \log_2 e \qquad \left[\log_2 e = \frac{1}{\ln 2}\right]$$

For ideal channel with infinite bandwidth,

$$C_\infty = \frac{S}{N_0} \log_2 (e) = \frac{E_b R_b}{N_0} \log_2 (e)$$

For error-free transmission, $R_b \le C$

So, $$R_b \le \frac{E_b R_b}{N_0} \log_2 (e)$$

$$E_b \ge \frac{N_0}{\log_2 (e)} = N_0 \ln(2) \quad \Rightarrow \quad E_{b(\min)} = N_0 \ln(2)$$

2. $$E_{b\,(\min)} = \left(\frac{N_0}{2}\right) 2\ln(2) = \left(\frac{N_0}{2}\right) \ln(4)$$

$$= (7)\ln(4) \text{ mW-sec}$$

$$= 9.7 \text{ mW-sec}$$

Q.1 (c) Solution:

(i) From the orthonormal axis $\phi_1(t)$ and $\phi_2(t)$, the signal vectors are:

- $s_1 = [1, 1]^T$
- $s_0 = [-1, -1]^T$

The distance (d) of a signal point (x, y) from the origin (0, 0) is calculated using the Euclidean normal.

- For s_1 : $d_1 = \sqrt{1^2 + 1^2} = \sqrt{2}$
- For s_0 : $d_0 = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

The distance of a signal point from the origin represents the square root of the energy of that signal. The energy E of a signal is the square of its distance from the origin:

i.e., $$E = |s|^2$$

In this specific case, both signals have an energy $E = (\sqrt{2})^2 = 2$ Joules.

(ii) From the provided 2D signal space (ϕ_1, ϕ_2)

- s_1 is located at (1, 1)
- s_0 is located at (–1, –1)

The Eucledian distance between two points (x_1, y_1) and (x_0, y_0) is given by the formula:

$$d_{\min} = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

Substituting the values:

$$d_{\min} = \sqrt{(1-(-1)^2 + (1-(-1))^2}$$

$$d_{\min} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$$

Hence, the minimum Euclidean distance is $d_{\min} = \sqrt{8}$ or $2\sqrt{2}$.

(iii) Given the two-sided PSD $\frac{N_0}{2} = 0.50$ W/ Hz

Hence, $N_0 = 1$ W/Hz

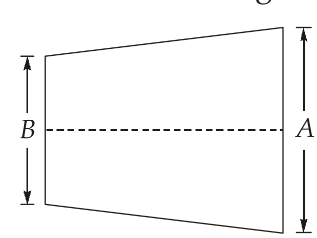
For a binary digital signalling scheme with optimum receiver having equiprobable symbols, the BER is:

$$P_e = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right) = Q\left(\sqrt{\frac{8}{2(1)}}\right)$$

$$P_e = Q(\sqrt{4}) = Q(2)$$

Q.1 (d) Solution:

(i) **1.** The standard X-Y plot of a CRO for a single tone AM signal is as follows:

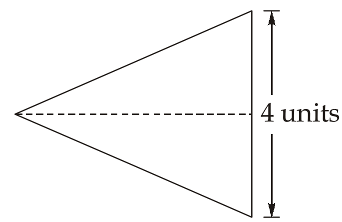


The modulation index of the AM signal in terms of A and B can be given as,

$$\mu = \frac{A-B}{A+B}$$

From the problem statement,

The pattern is a triangle, meaning it tapers to a point. Therefore, the minimum vertical height is zero.



From the CRO display (Trapezoidal Method)

- Maximum vertical height (A) = 4 units
- Minimum vertical height (B) = 0 units (since it is a triangle)

$$\mu = \frac{A-B}{A+B} = \frac{4-0}{4+0} = 1$$

This represents 100% modulation (i.e., critical modulation).

2. Given: Peak amplitude of the message signal = 10 V. Thus,

$$A_m = 10 \text{ V}$$

The modulation index is given as:

$$\mu = \frac{A_m}{A_c}$$

$$A_c = \frac{10}{1} = 10 \text{ V}$$

$$\text{Carrier Power } (P_c) = \frac{A_c^2}{2R} = \frac{(10)^2}{2\times 5} = \frac{100}{10} = 10 \text{ Watt}$$

$$\text{Total power } (P_t) = P_c\left(1+\frac{\mu^2}{2}\right)$$

$$= 10(1 + 0.5) = 15 \text{ Watt}$$

Total sideband power (P_{sb})

$$P_{sb} = P_t - P_c$$

$$= 15 - 10 = 5 \text{ Watt}$$

3. Transmission Efficiency:

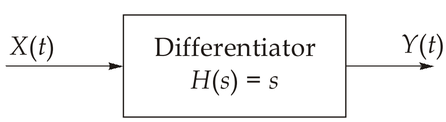
$$\eta = \frac{P_{sb}}{P_t} = \frac{P_c * \mu^2/2}{P_c\left(1+\frac{\mu^2}{2}\right)} = \frac{\mu^2}{2+\mu^2}$$

For the given case, $\mu = 1$. Thus,

$$\eta = \frac{1^2}{2+1^2} = \frac{1}{3} \approx 33.33\%$$

In standard AM, the maximum efficiency occurs at $\mu = 1$, which is 33.33%. This highlights the main disadvantage of AM i.e., at least 66.67% of the power is wasted in the carrier, which contains no information. Mathematically a higher μ would increase η, however it causes the envelope detector to cross the zero axis. This leads to phase reversals and envelope distortion, making the message unrecoverable by standard diode detectors. Therefore, 33.33% remains the practical upper limit of efficiency for AM wave without overmodulation.

(ii)



For a differentiator,
$$H(s) = s$$
$$H(j\omega) = j\omega$$
$$|H(j\omega)|^2 = \omega^2$$

PSD of $Y(t)$,
$$S_Y(\omega) = S_X(\omega)\,|H(\omega)|^2$$
$$= \omega^2 S_X(\omega)$$

The auto-correlation function and Power Spectral Density form a Fourier Transform pair. Thus,

$$R_X(\tau) \xleftrightarrow{CTFT} S_X(\omega)$$

$$\frac{dR_X(\tau)}{d\tau} \xleftrightarrow{CTFT} (j\omega)S_X(\omega)$$

$$\frac{d^2R_X(\tau)}{d\tau^2} \xleftrightarrow{CTFT} (j\omega)^2 S_X(\omega)$$

$$\frac{d^2R_X(\tau)}{d\tau^2} \xleftrightarrow{CTFT} -\omega^2 S_X(\omega)$$

$$-\frac{d^2R_X(\tau)}{d\tau^2} \xleftrightarrow{CTFT} \omega^2 S_X(\omega) = S_Y(\omega)$$

So, $$R_Y(\tau) = \frac{-d^2R_X(\tau)}{d\tau^2}$$

Hence, the auto-correlation function of the differentiated process $Y(t)$ is

$$R_Y(\tau) = \frac{-d^2R_X(\tau)}{d\tau^2}$$

Q.1 (e) Solution:

(i) 1. In any angle-modulated signal, the general form of the carrier is

$$s(t) = A_c \cos(2\pi f_c t + \phi(t))$$

where $\phi(t)$ is the time-varying phase.

The instantaneous frequency (in Hz) is

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}[2\pi f_c t + \phi(t)]$$

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t)$$

For FM: The phase deviation $[\phi(t)]$ is given as

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau$$

Differentiating,

$$\frac{d\phi(t)}{dt} = 2\pi k_f m(t)$$

$$\Rightarrow \quad f_i(t) = f_c + k_f m(t)$$

Therefore, the maximum frequency deviation is

$$(\Delta f)_{\text{max, FM}} = |f_i(t) - f_c|_{\text{max}} = k_f |m(t)|_{\text{max}} = k_f A_m$$

For PM: The phase deviation $[\phi(t)]$ is given as,

$$\phi(t) = k_p m(t)$$

Differentiating,

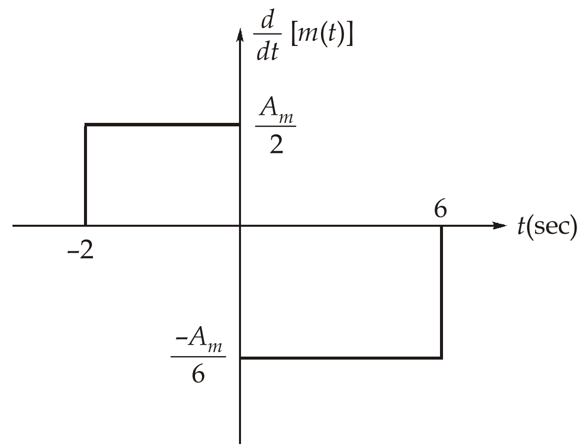
$$\frac{d\phi(t)}{dt} = k_p \frac{dm(t)}{dt}$$

$$\Rightarrow \quad f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$$

Therefore, the maximum frequency deviation is

$$(\Delta f)_{\text{max, PM}} = \frac{k_p}{2\pi}\left|\frac{dm(t)}{dt}\right|_{\text{max}}$$

2. Differentiating the given signal $m(t)$, we get



$$\left|\frac{d}{dt}m(t)\right|_{\text{max}} = \frac{A_m}{2}\text{ Volt}$$

Hence, its maximum absolute value is $\frac{A_m}{2}$ Volt

3. We have, $(\Delta f_{\text{max}})_{\text{FM}} = k_f A_m$

$$(\Delta f_{\text{max}})_{\text{PM}} = \frac{k_p}{2\pi}\left|\frac{dm(t)}{dt}\right|_{\text{max}}$$

$$(\Delta f_{\text{max}})_{\text{PM}} = \frac{k_p}{2\pi}\left(\frac{A_m}{2}\right) = \frac{k_p A_m}{4\pi}$$

For $(\Delta f_{\text{max}})_{\text{PM}} = (\Delta f_{\text{max}})_{\text{FM}}$

$$\frac{k_p A_m}{4\pi} = k_f A_m$$

$$\frac{k_p}{k_f} = 4\pi \text{ rad/Hz}$$

(ii) **1.** The problem specifies high-side injection ($f_{LO} > f_c$), thus

$$f_{LO} = f_c + f_{IF}$$

Minimum carrier frequency:

$$f_{c,\text{min}} = 550 \text{ kHz}$$

Maximum carrier frequency:

$$f_{c,\max} = 1650 \text{ kHz}$$
$$\text{Fixed } f_{IF} = 450 \text{ kHz}$$

Thus, minimum LO frequency:

$$\begin{aligned} f_{LO,\min} &= f_{c,\min} + f_{IF} \\ &= 550 \text{ kHz} + 450 \text{ kHz} \\ &= 1000 \text{ kHz} \end{aligned}$$

Maximum LO frequency:

$$\begin{aligned} f_{LO,\max} &= f_{c,\max} + f_{IF} \\ &= 1650 \text{ kHz} + 450 \text{ kHz} \\ &= 2100 \text{ kHz} \end{aligned}$$

So, the local oscillator must tune from 1000 kHz to 2100 kHz.

2. For an LC oscillator, the frequency is given by $f = \dfrac{1}{2\pi\sqrt{LC}}$. Since $f \propto \dfrac{1}{\sqrt{C}}$, we have:

$$\frac{f_{\max}}{f_{\min}} = \sqrt{\frac{C_{\max}}{C_{\min}}}$$

$$\frac{C_{\max}}{C_{\min}} = \left(\frac{f_{LO,\max}}{f_{LO,\min}}\right)^2$$

Substituting the calculated oscillator frequencies:

$$\frac{C_{\max}}{C_{\min}} = \left(\frac{2100}{1000}\right)^2 = (2.1)^2$$

$$\frac{C_{\max}}{C_{\min}} = 4.41$$

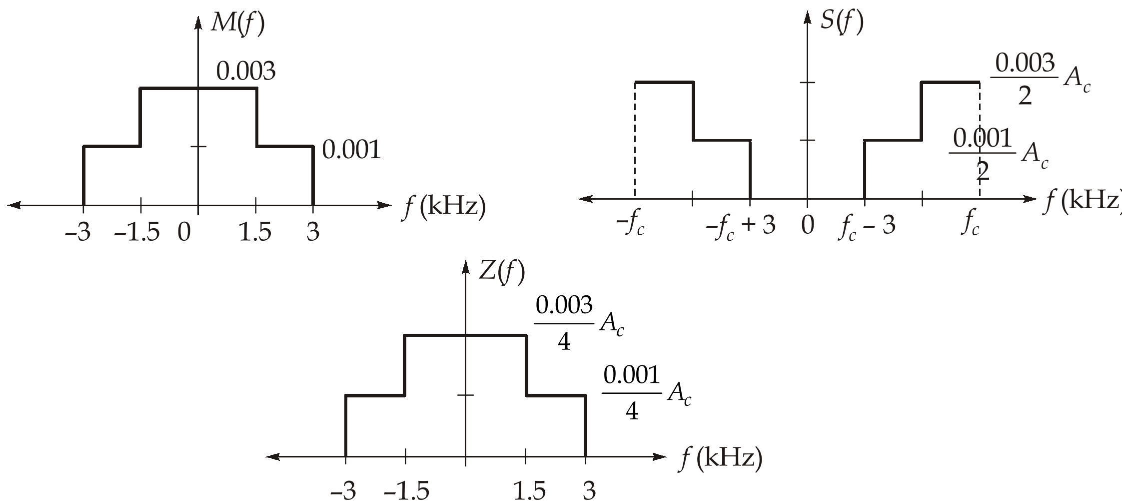
3. In a mixer, the local oscillator frequency f_{LO} is mixed with the signal frequency f_c to produce f_{IF}.

- High-side injection: $f_{LO} = f_c + f_{IF}$
- Low-side injection: $f_{LO} = f_c - f_{IF}$

High-side injection is preferred because it requires a smaller capacitance ratio in the tuning circuit. If low-side injection were used for a 550-1650 kHz range with f_{IF} = 450 kHz, the f_{LO} would need to range from 100 kHz to 1200 kHz, requiring a larger and impractical capacitive range.

Q.2 (a) Solution:

(i)



$$\text{Power in } s(t) = \int_{-\infty}^{\infty} |s(t)|^2\, dt = \int_{-\infty}^{\infty} |S(f)|^2\, df$$

$$= 2\left[1500\left(\frac{0.001\,A_c}{2}\right)^2 + 1500\left(\frac{0.003A_c}{2}\right)^2\right] = 100 \text{ mW}$$

$$= \frac{1}{2}\left[1500A_c^2 \times 10^{-6} + 1500A_c^2 \times 9 \times 10^{-6}\right] = 100 \times 10^{-3}$$

$$\Rightarrow \quad A_c^2 = 13.33 \Rightarrow A_c = 3.65$$

(ii) From the spectrum of $z(t)$, we can write

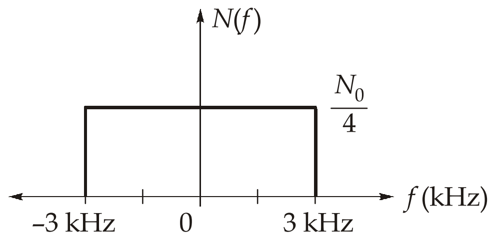
$$\text{Power of } z(t) = 2\left[1500\left(\frac{0.003A_c}{4}\right)^2 + 1500\left(\frac{0.001A_c}{4}\right)^2\right] = 25 \text{ mW}$$

(iii) $n(t)$ is AWGN with, $S_n(f) = \dfrac{N_0}{2}$

$n(t)\cos 2\pi f_c t$ will have PSD of $\dfrac{1}{4}S_n(f - f_c) + \dfrac{1}{4}S_n(f + f_c)$

$$= \frac{1}{4}\left(\frac{N_0}{2}\right) + \frac{1}{4}\left(\frac{N_0}{2}\right) = \frac{N_0}{4}$$

The noise affecting the demodulator output is as shown below,



$$\text{Noise power} = \frac{6000}{4}(0.0001\times10^{-3}) = 0.15\text{ mW}$$

(iv)

$$\text{SNR at output} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{25\text{ mW}}{0.15\text{ mW}} = 166.67 \approx 22.2\text{ dB}$$

Q.2 (b) Solution:

(i) For sinusoidal signal $m(t) = A\cos(2\pi ft)$. To avoid slope overload distortion,

$$\frac{\sigma}{T_s} \geq \left|\frac{d}{dt}m(t)\right|_{max} = 2\pi fA_{max} = \omega A_{max}$$

$$A_{max} = \frac{\sigma f_s}{\omega}$$

or,

$$\sigma = \frac{\omega A_{max}}{f_s} = \frac{2\pi\times3.4\times10^3}{64000},$$

$$\sigma = 0.3337$$

(ii)

$$N_0 = \frac{\sigma^2 B}{3f_s} = \frac{(0.3337)^2(3.4\times10^3)}{3(64000)} = 0.00197$$

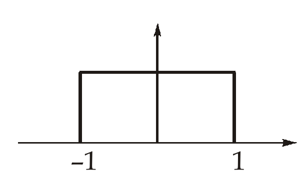
(iii) For sinusoidal signal, $S_0 = \frac{A^2}{2} = 0.5$

$$\text{SNR} = \frac{S_0}{N_0} = \frac{0.5}{0.00197} = 253.80 = 24.04\text{ dB}$$

(iv) For uniform distribution, $S_0 = \frac{(1-(-1))^2}{12} = \frac{1}{3}$

So,

$$\frac{S_0}{N_0} = \frac{0.333}{0.00197} = 22.28\text{ dB}$$



(v) The minimum transmission bandwidth BT for a baseband signal (using Nyquist criterion) is:

$$B_T = \frac{R_b}{2} = \frac{64\text{ kHz}}{2} = 32\text{ kHz}$$

Q.2 (c) Solution:

(i) Output bandpass noise will have non zero power within frequencies in the band 49 to 51 MHz.

So, Power, $P = \int_{-\infty}^{\infty} S_n(f)|H(f)|^2\, df$

$$P = 2\int_{49\times10^6}^{51\times10^6} 10^{-8}\left(1-\frac{f}{10^8}\right)df$$

$$P = 2\times10^{-8}\left[f-\frac{f^2}{2\times10^8}\right]_{49\times10^6}^{51\times10^6}$$

$$P = 2\times10^{-8}\left\{\left[51\times10^6-\frac{(51\times10^6)^2}{2\times10^8}\right]-\left[49\times10^6-\frac{(49\times10^6)^2}{2\times10^8}\right]\right\}$$

$P = 1.99 \times 10^{-2}$ Watt

(ii) The narrowband noise $y(t)$ can be represented as:

$$n(t) = n_c(t)\cos(2\pi(50\times10^6)t) - n_q(t)\sin(2\pi(50\times10^6)t,$$

where $n_c(t)$ is the in-phase component and $n_q(t)$ is the quadrature component. Power content in both $n_c(t)$ and $n_q(t)$ will be same as $n(t)$.

$$\therefore \quad E\left[\left(n_c^2(t)\right)\right] = E\left[\left(n_q^2(t)\right)\right] = E\left[\left(n^2(t)\right)\right] = 1.99\times10^{-2}\text{ Watt}$$

(iii) PSD of $n_c(t)$ and $n_q(t)$ is given by

$$S_{nc}(f) = S_{nq}(f)$$

$$= \begin{cases} S_n(f-f_c)+S_n(f+f_c)\,; & |f|\le B/2\text{ Hz} \\ 0\,; & \text{otherwise} \end{cases}$$

For the range $|f| \le 1$ MHz:

$$S_{nc} = S_{nq}(f)$$

$$= S_n(f-50\times10^6) + S_n(f+50\times10^6);\ |f| \le 1\times10^6\text{ Hz}$$

We have, $S_n(f + 50\times10^6) = 10^{-8}\left(1-\frac{50\times10^6+f}{10^8}\right) = 10^{-8}\left(0.5-\frac{f}{10^8}\right)$ ...(i)

$$S_n(f - 50\times10^6) = 10^{-8}\left(1-\frac{50\times10^6-f}{10^8}\right) = 10^{-8}\left(0.5+\frac{f}{10^8}\right) \quad \text{...(ii)}$$

On adding (i) and (ii)

$$S_{nc}(f) = S_{nq}(f) = 10^{-8}\ \text{W/Hz}; \quad |f| \le 10^6\,\text{Hz}$$
$$0 \quad ; \quad \text{otherwise}$$

(iv) The auto-correlation function $R_{nc}(\tau)$ is the inverse Fourier Transform of $S_{nc}(f)$. We have,

$$R_{nc}(\tau) = \int_{-10^6}^{10^6} 10^{-8} e^{j2\pi f\tau} df$$

$$= 10^{-8}\times\left[\frac{e^{j2\pi f\tau}}{j2\pi\tau}\right]_{-10^6}^{10^6} = 10^{-8}\times\frac{\sin(2\pi\times10^6\tau)}{\pi\tau}$$

Equivalently, $R_{nc}(\tau) = 2\times10^{-2}\,\text{sinc}(2\times10^6\tau)$ where $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

Q.3 (a) Solution:

(i) Given angle-modulated signal,

$$u(t) = 5\cos[2\pi f_c t + 40\sin(500\pi t) + 20\sin(1000\pi t) + 10\sin(2000\pi t)]$$

The instantaneous phase,

$\theta(t) = 2\pi f_c t + 40\sin(500\pi t) + 20\sin(1000\pi t) + 10\sin(2000\pi t)$ ...(i)

1. Let $u(t)$ be the PM signal.

The standard form of the PM signal is given by

$$u(t) = A_c\cos[2\pi f_c t + k_p m(t)]$$

where instantaneous phase,

$$\theta(t) = 2\pi f_c t + k_p m(t) \quad \text{...(ii)}$$

On comparing equation (i) with equation (ii), we get

$$k_p m(t) = 40\sin(500\pi t) + 20\sin(1000\pi t) + 10\sin(2000\pi t) \quad \text{...(iii)}$$

(a) Maximum phase deviation occurs when all sine terms reach their peak values.

$$\Delta\phi_{max} = 40 + 20 + 10 = 70$$

$\therefore \quad \Delta\phi_{max} = 70$ radians

(b) Given, $k_p = 5$ rad/volt

From equation (iii), message signal $m(t)$ can be written as,

$$m(t) = \frac{1}{k_p}[40\sin(500\pi t) + 20\sin(1000\pi t) + 10\sin(2000\pi t)]$$

$$m(t) = \frac{1}{5}[40\sin(500\pi t) + 20\sin(1000\pi t) + 10\sin(2000\pi t)]$$

$$\therefore \quad m(t) = 8\sin(500\pi t) + 4\sin(1000\pi t) + 2\sin(2000\pi t)$$

2. Let $u(t)$ be the FM signal,

The standard form of the FM signal is given by

$$u(t) = A_c \cos\left[2\pi f_c t + k_f \int m(t)dt\right]$$

where, instantaneous phase,

$$\theta(t) = 2\pi f_c t + k_f \int m(t)dt \quad \text{...(iv)}$$

On comparing equation (i) with equation (iv), we get

$$k_f \int m(t)dt = 40\sin(500\pi t) + 20\sin(1000\pi t) + 10\sin(2000\pi t)$$

(a) Maximum frequency deviation: $\Delta f_{i\,max}$

We know that instantaneous frequency,

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

$$\therefore \quad f_i(t) = \frac{1}{2\pi}\frac{d}{dt}[40\sin(500\pi t) + 20\sin(1000\pi t) + 10\sin(2000\pi t)]$$

$$= \frac{1}{2\pi}[20000\pi\cos(500\pi t)] + 20000\pi\cos(1000\pi t) + 20000\pi\cos(2000\pi t)]$$

Maximum frequency deviation occurs at $t = 0$.

$$\therefore \Delta f_{i\,max} = \frac{1}{2\pi}[20000\pi + 20000\pi + 20000\pi]$$

$$\therefore \Delta f_{i\,max} = 30000 \text{ Hz}$$

(b) For an FM signal,

$$\frac{d\theta}{dt} = 2\pi f_c + k_f m(t)$$

but from equation (i),

$$\frac{d\theta}{dt} = 2\pi f_c + 20000\pi[\cos(500\pi t) + \cos(1000\pi t) + \cos(2000\pi t)]$$

Thus,

$$k_f m(t) = 20000\pi[\cos(500\pi t) + \cos(1000\pi t) + \cos(2000\pi t)]$$

given, $k_f = 10000\pi$ rad/sec-V

$$\therefore \quad m(t) = \frac{20000\pi}{10000\pi}[\cos(500\pi t) + \cos(1000\pi t) + \cos(2000\pi t)]$$

$$\therefore \quad m(t) = 2\cos(500\pi t) + 2\cos(1000\pi t) + 2\cos(2000\pi t)$$

(ii) Let us assume for BPSK signal, the transmitted signal

$$S_i(t) = A\cos\omega_c t \text{ for binary 1}$$
$$= -A\cos\omega_c t \text{ for binary 0}$$

E_b is the average bit energy

$$E_b = \frac{A^2 T}{2}$$

$$A = \sqrt{\frac{2E_b}{T}}$$

We can write,

$$S_1(t) = \sqrt{\frac{2E_b}{T}}\cos\omega_c t \text{ and } S_2(t) = -\sqrt{\frac{2E_b}{T}}\cos\omega_c t$$

Consider the basis function with unit energy as

$$\phi(t) = \sqrt{\frac{2}{T}}\cos\omega_c t$$

The output of the correlator at $t = T$

$$z(T) = a_1(T) + n_0(T) \qquad i = 1, 2$$

where

$$a_1(T) = \sqrt{\frac{2}{T}} \cdot \sqrt{\frac{2E_b}{T}} \int_0^T \cos\omega_c t(\cos\omega_c t + \theta)\, dt$$

$$a_1(T) = \sqrt{E_b}\cos\theta$$

Similarly, $a_2(T) = -\sqrt{E_b}\cos\theta$

The probability of error for a binary signalling system with additive white Gaussian noise (AWGN) is given by

$$P_e = Q\left(\frac{a_1 - a_2}{2\sigma}\right), \qquad \text{where } \sigma^2 = \frac{N_0}{2}$$

With a phase error of 'θ', the signal responses of correlator becomes $\sqrt{E_b}\cos\theta$ or $-\sqrt{E_b}\cos\theta$. Thus,

$$P_e = Q\left(\frac{2\sqrt{E_b}\cos\theta}{2\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{2E_b\cos^2\theta}{N_0}}\right)$$

Q.3 (b) Solution:

(i) **1.** We have, $[P(X, Y)] = [P(X)]_d\, P[Y|X]$

$$[P(X, Y)] = \begin{bmatrix}\alpha & 0\\ 0 & 1-\alpha\end{bmatrix}\begin{bmatrix}1-p & p\\ p & 1-p\end{bmatrix}$$

or
$$[P(X, Y)] = \begin{bmatrix}\alpha(1-p) & \alpha p\\ (1-\alpha)p & (1-\alpha)(1-p)\end{bmatrix} = \begin{bmatrix}P(x_1, y_1) & P(x_1, y_2)\\ P(x_2, y_1) & P(x_2, y_2)\end{bmatrix}$$

We know that $H(Y \mid X) = -\sum_{j=1}^{n}\sum_{i=1}^{m} P(x_i, y_j)\log_2 P(y_j | x_i)$

By above expression, we have

$$H(Y \mid X) = -P(x_1, y_1)\log_2 P(y_1 | x_1) - P(x_1, y_2)\log_2 P(y_2 \mid x_1) - P(x_2, y_1)\log_2 P(y_1 | x_2) - P(x_2, y_2)\log_2 P(y_2 | x_2)$$

or
$$H(Y \mid X) = -\alpha(1-p)\log_2(1-p) - \alpha p\log_2 p - (1-\alpha)\, p\log_2 p - (1-\alpha)(1-p)\log_2(1-p)$$

or
$$H(Y \mid X) = -p\log_2 p - (1-p)\log_2(1-p)$$

We know that $I(X; Y) = H(Y) - (Y | X) = H(Y) + p\log_2 p + (1-p)\log_2(1-p)$

2. We know that

$$P[(Y)] = [P(X)]\,[P(Y|X)]$$

For $\alpha = 0.5$ and $p = 0.1$, we have

$$P[(Y)] = [0.5 \quad 0.5]\begin{bmatrix}0.9 & 0.1\\ 0.1 & 0.9\end{bmatrix} = [0.5 \quad 0.5]$$

Thus, $P(y_1) = P(y_2) = 0.5$

Now, using the expression

$$H(Y) = -\sum_{j=1}^{n} P(y_j)\log_2 P(y_j), \text{ we have}$$

$$H(Y) = -P(y_1)\log_2 P(y_1) - P(y_2)\log_2 P(y_2)$$

or $H(Y) = -0.5\log_2 0.5 - 0.5\log_2 0.5 = 1$ bits/symbol

We have, $p\log_2 p + (1-p)\log_2(1-p) = 0.1\log_2 0.1 + 0.9\log_2 0.9 = -0.469$

Thus, $I(X; Y) = 1 - 0.469 = 0.531$ bits

3. When $\alpha = 0.5$ and $p = 0.5$, we have

$$[P(Y)] = [0.5 \quad 0.5]\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} = [0.5 \quad 0.5]$$

$$H(Y) = 1 \text{ bits}$$

and, $p \log_2 p + (1-p)\log_2(1-p) = 0.5 \log_2 0.5 + 0.5 \log_2 0.5 = -1$

Thus, $\quad I(X;Y) = 1 - 1 = 0$

(ii) Given 'X' is a Gaussian random variable with $m = 0$, $\sigma = 1$.

As we know PDF for Gaussian random variable 'X' with mean 'm' and variance 'σ' is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

Substituting $m = 0$, $\sigma = 1$, we get

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Now, for random variable Y

$$Y = aX + b \quad \text{...(i)}$$

As we know PDF of Y

$$f_Y(y) = f_X(x)\left|\frac{dX}{dY}\right| \quad \text{...(ii)}$$

Differentiate (i) w.r.t 'x'

$$\frac{dY}{dX} = a \Rightarrow \frac{dX}{dY} = \frac{1}{a} \quad \text{...(iii)}$$

$\therefore$ using (iii) in (ii), we get

$$f_Y(y) = f_X\left(\frac{y-b}{a}\right) \cdot \frac{1}{|a|}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi a^2}} e^{-\frac{(y-b)^2}{2a^2}}$$

Q.3 (c) Solution:

(i) Given: The normalized autocorrelation function of the input signal for a lag of one sampling interval is

$$\rho_x(1) = \frac{R_x(1)}{R_x(0)} = 0.75$$

$$\text{Error variance} = R_x(0) - R_x(1)R_x^{-1}(0)R_x(1)$$

$$= R_x(0) - R_x(0)\rho_x^2(1)$$

$$= R_x(0)\left[1-\rho_x^2(1)\right]$$

where $R_x(0)$ = Input signal variance

Processing gain, $$G_p = \frac{\text{Input signal variance}}{\text{Error variance}}$$

$$G_p = \frac{R_x(0)}{R_x(0)[1-\rho_x^2(1)]}$$

$$= \frac{1}{\left[1-\rho_x^2(1)\right]} = \frac{1}{\left[1-(0.75)^2\right]}$$

$$= \frac{16}{7} = 2.286$$

Processing gain in dB,

$$G_p(\text{dB}) = 10\log_{10}(2.286) = 3.59 \text{ dB}$$

(ii) $$V(t) = \sum_{i=1}^{N}[\cos\omega_c t\cdot\cos(\omega_i t+\theta_i) - \sin\omega_c t\cdot\sin(\omega_i t+\theta_i)]$$

Now, consider the signal,

$$M(t) = \cos(\omega_i t + \theta_i)$$

Its Hilbert transform will be

$$M_h(t) = \cos\left(\omega_i t + \theta_i - \frac{\pi}{2}\right)$$

$$= \sin(\omega_i t + \theta_i)$$

The expression of upper sideband in SSB-SC modulated signal is given by

$$\phi_{\text{USB}} = M(t)\times\cos\omega_c t - M_h(t)\sin\omega_c t$$

$$= \cos(\omega_i t+\theta_i)\cos\omega_c t - \sin(\omega_i t+\theta_i)\sin\omega_c t$$

- Thus, $V(t)$ is the USB SSB modulated signal of

$$= \sum_{i=1}^{N} \cos(\omega_i t + \theta_i)$$

- The expression for lower sideband is

$$= \sum_{i=1}^{N} [\cos\omega_c t \cos(\omega_i t + \theta_i) + \sin\omega_c t \sin(\omega_i t + \theta_i)]$$

- The expression for total DSB-SC signal is given by the addition of USB and LSB SSB-SC modulated signals as,

$$s(t) = \sum_{i=1}^{N} \cos\omega_c t \cdot \cos(\omega_i t + \theta_i)$$

Q.4 (a) Solution:

(i) The generator matrix is given by $G = [P \mid I_k]$. For a (5, 1) linear block code, number of message bits $k = 1$ and the codeword length, $n = 5$. Given,

$$G = [1 \quad 1 \quad 1 \quad 1 \quad 1 \;\vdots\; 1]$$

The parity check matrix is therefore obtained as $H = [I_{n-k} \mid P^T]$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & 0 & \vdots & 1 \\ 0 & 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Syndrome, $S = eH^T$

1. Here e is error pattern, for single error we have

Error pattern	Syndrome
0 0 0 0 1	1 1 1 1
0 0 0 1 0	0 0 0 1
0 0 1 0 0	0 0 1 0
0 1 0 0 0	0 1 0 0
1 0 0 0 0	1 0 0 0

2. For two errors in the received code word, we have

Error pattern	Syndrome
0 0 0 1 1	1 1 1 0
0 0 1 0 1	1 1 0 1
0 1 0 0 1	1 0 1 1
1 0 0 0 1	0 1 1 1
0 0 1 1 0	0 0 1 1
0 1 0 1 0	0 1 0 1
1 0 0 1 0	1 0 0 1
0 1 1 0 0	0 1 1 0
1 0 1 0 0	1 0 1 0
1 1 0 0 0	1 1 0 0

(ii) 1. For any stationary or ergodic process, the mean square value is equal to the value of auto-correlation function at $\tau = 0$.

$$E[X^2(t)] = R_X(0)$$

Substituting $\tau = 0$ in the given equation

$$R_X(0) = \frac{25(0)^2 + 36}{6.25(0)^2 + 4}$$

$$R_X(0) = \frac{36}{4} = 9$$

Therefore, the mean-square value $E[X^2(t)] = 9$.

For an ergodic process with no periodic components, the square of the mean value is equal to the limit of the auto-correlation function as $\tau \to \infty$.

$$\mu_X^2 = \lim_{\tau \to \infty} R_X(\tau)$$

To evaluate this limit, divide the numerator and denominator by τ^2:

$$\mu_X^2 = \lim_{\tau \to \infty} \frac{25 + \frac{36}{\tau^2}}{6.25 + \frac{4}{\tau^2}}$$

$$\mu_X^2 = \frac{25 + 0}{6.25 + 0} = \frac{25}{6.25} = 4$$

Taking the square root, the mean value $|\mu_X| = 2$

2. The variance (σ_X^2) represents the AC Power of the process and is defined as the difference between the mean-square value and the square of the mean.

$$\text{Variance } (\sigma_X^2) = E[X^2(t)] - [E[X(t)]]^2$$
$$\sigma_X^2 = R_X(0) - \mu_X^2$$
$$\sigma_X^2 = 9 - 4 = 5$$

Hence, the variance of the process $X(t)$ is 5.

3. **Physical significance of $R_X(0)$**

The value $R_X(0)$ represents the Total Average Power of the random process.

For any stationary process, the auto-correlation is always maximum at the origin, meaning $|R_X(\tau)| \le R_X(0)$. This physically implies that a signal is most strongly correlated with itself at zero time-shift.

Physical significance of $R_X(\infty)$

The value $R_X(\infty)$ represents the DC Power (or the square of the mean) of the process. If $R_X(\infty) = 0$, it physically signifies that the random process has zero mean (no DC component).

Q.4 (b) Solution:

(i) To find PSD of $n_2(t)$, we are finding its ACF first. The ACF of $n_2(t)$ is given by

$$R_{N_2}(t_1, t_2) = E\,[n_2(t_1)\, n_2(t_2)]$$
$$= E\Big[\{n_1(t_1)\cos(2\pi f_c t_1 + \theta) - n_1(t_1)\sin(2\pi f_c t_1 + \theta)\}\cdot\big[\{n_1(t_2)\cos(2\pi f_c t_2 + \theta) - n_1(t_2)\sin(2\pi f_c t_2 + \theta)\}\big]\Big]$$
$$= E\,[n_1(t_1)n_1(t_2)\cos(2\pi f_c t_1 + \theta)\cos(2\pi f_c t_2 + \theta) - n_1(t_1)n_1(t_2)\cos(2\pi f_c t_1 + \theta)\sin(2\pi f_c t_2 + \theta) - n_1(t_1)n_1(t_2)\sin(2\pi f_c t_1 + \theta)\cos(2\pi f_c t_2 + \theta) + n_1(t_1)n_1(t_2)\sin(2\pi f_c t_1 + \theta)\sin(2\pi f_c t_2 + \theta)]$$

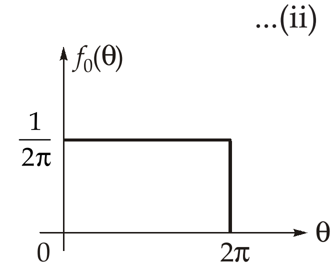
As we know,

$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$
$$\sin(A + B) = \sin A\cos B + \cos A\sin B \qquad \text{...(i)}$$

By using equation (i), we can write as:

$$R_{N_2}(t_1, t_2) = E\,[n_1(t_1)n_1(t_2)\cos 2\pi f_c(t_1 - t_2) - n_1(t_1)n_1(t_2)\sin[2\pi f_c(t_1 + t_2) + 2\theta])]$$
$$= E\,[n_1(t_1)n_1(t_2)]\,E\,[\cos[2\pi f_c(t_1 - t_2)] - E[n_1(t_1)n_1(t_2)] \times E\{\sin(2\pi f_c(t_1 + t_2) + 2\theta)\} \qquad \text{...(ii)}$$

Given that θ is uniformly distributed R.V.



Because of this we know,

$E\,\{\sin(2\pi f_c(t_1 + t_2) + 2\theta\} = 0$

By equation (ii),

$$R_{N_2}(t_1, t_2) = E[n_1(t_1)n_1(t_2) \cos 2\pi f_c(t_1 - t_2)]$$

or $$R_{N_2}(t_1, t_2) = R_{N_1}(t_1, t_2) \cos 2\pi f_c \tau$$

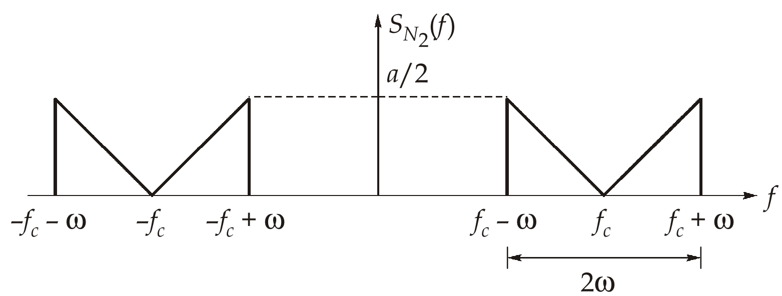
($\because n_1(t)$ is stationary, so we can write $t_1 - t_2 = \tau$)

so, $$R_{N_2}(\tau) = R_{N_1}(\tau) \cos 2\pi f_c\tau \quad \text{...(iii)}$$

Taking Fourier transform of equation (iii),

$$S_{N_2}(f) = \frac{1}{2}\left[S_{N_1}(f+f_c)+S_{N_1}(f-f_c)\right]$$

and $S_{N_1}(t)$ is given in question. So, $S_{N_2}(f)$ can be drawn as



(ii) 1. Given,

$$\phi_{AM}(t) = A_c[1+\cos(2\pi f_m t)]\cos[2\pi f_c t]$$

$$\text{noise, } n(t) = n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

$$\overline{n_c^2(t)} = \overline{n_s^2(t)} = 2N_0 f_m$$

Let $r(t)$ be the output of adder,

$$\therefore \quad r(t) = \phi_{AM}(t) + n(t)$$

$$= A_c[1 + \cos(2\pi f_m t)] \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

Output of multiplier,

Let $$m(t) = r(t) \cdot 2 \cos(2\pi f_c t)$$

$$\therefore \quad m(t) = [A_c[1 + \cos(2\pi f_m t)]\cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)]2\cos(2\pi f_c t)$$

Signal part, Let $$s_1(t) = A_c[1 + \cos(2\pi f_m t)][1 + \cos(4\pi f_c t)]$$

Noise part, Let $$n'(t) = n_c(t)[1 + \cos(4\pi f_c t)] - n_s(t) \sin(4\pi f_c t)$$

After passing through lowpass filter (with cutoff frequency of f_m)

we get, $$s_L(t) = A_c[1 + \cos 2\pi f_m t] + n_c(t)$$

After DC block (which removes constant A_c),

$$y(t) = A_c \cos(2\pi f_m t) + n_c(t)$$

2. Signal to noise ratio = $\frac{S_0}{N_0}$

From the output of system,

$$y(t) = A_c \cos[2\pi f_m t] + n_c(t)$$

$$\therefore \quad \text{Signal power, } S_0 = \frac{A_c^2}{2}$$

$$\text{Noise power} = \overline{n_c^2(t)} = 2N_0 f_m \quad \text{(given)}$$

$$\therefore \quad \left(\frac{S}{N}\right)_0 = \frac{A_c^2}{2 \times 2N_0 f_m} = \frac{A_c^2}{4N_0 f_m}$$

3. Given,

$$A_c = 10 \text{ V}$$

$$f_m = 10 \text{ kHz} = 10^4 \text{ Hz}$$

$$N_0 = 10\ \mu\text{W/Hz} = 10 \times 10^{-6} \text{ W/Hz}$$

$$\therefore \quad \left(\frac{S}{N}\right)_0 = \frac{A_c^2}{4N_0 f_m}$$

$$= \frac{10^2}{4 \times 10 \times 10^{-6} \times 10 \times 10^3}$$

$$\therefore \quad \left(\frac{S}{N}\right)_0 = 250 \quad \text{(or)} \quad 23.98 \text{ dB}$$

Q.4 (c) Solution:

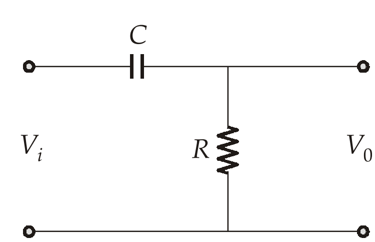
(i)

$$x_c(t) = A_c \cos\left[\omega_c t + \int_{-\infty}^{t} m(t)dt\right]$$

For a RC high pass filter,

$$\frac{V_0}{V_i} = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{sRC + 1}$$

$$\frac{V_0}{V_i} = \frac{j\omega RC}{1 + j\omega RC}$$



High pass RC filter

Given $\omega RC << 1$

$$\Rightarrow \quad \frac{V_0}{V_i} = j\omega RC \qquad \text{...(i)}$$

Using the property of Fourier transform,

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$$

Rewriting equation (i) in time domain

$$V_0(t) = RC\frac{d}{dt}V_i(t)$$

If
$$V_i(t) = x_c(t) = A_c \cos\left[\omega_c t + \int_{-\infty}^{t} m(t)dt\right]$$

then,
$$V_0(t) = RC\frac{d}{dt}\left[A_c \cos\left[\omega_c t + \int_{-\infty}^{t} m(t)dt\right]\right]$$

$$V_0(t) = -RCA_c(\omega_c + m(t))\sin\left[\omega_c t + \int_{-\infty}^{t} m(t)dt\right]$$

The envelope of $V_0(t)$ is given by $RC\ A_c\ (\omega_c + m(t)$, which contains the message signal. Thus, an envelope detector can demodulate the signal.

(ii) **1.** Let P_n denote the probability that a binary symbol is in error on transmission through the complete system. Then P_n is also the probability of odd number of errors, since an even no. of errors restores the original value. So probability of an even no. of errors is $1 - P_n$. Hence,

$$P_n = P_{n-1}(1 - P_1) + (1 - P_{n-1})P_1$$
$$= (1 - 2P_1)\, P_{n-1} + P_1$$
$$P_n = P_{n-1}(1 - 2P_1) + P_1$$
$$P_n = (1 - 2P_1)^2 P_{n-2} + (1 - 2P_1)P_1 + P_1$$
$$P_n = (1 - 2P_1)^3 P_{n-3} + (1 - 2P_1)^2 P_1 + (1 - 2P_1)P_1 + P_1$$
$$P_n = (1 - 2P_1)^{n-1} P_1 + (1 - 2P_1)^{n-1} P_1 \ldots\ldots + (1 - 2P_1)^2 P_1 + (1 - 2P_1)P_1 + P_1$$

$$P_n = P_1\sum_{i=0}^{n-1}(1-2P_1)^i$$

$$P_n = P_1\left[\frac{(1-2P_1)^n - 1}{(1-2P_1) - 1}\right]$$

$$P_n = \frac{1}{2}\left[1-(1-2P_1)^n\right]$$

2. If P_1 is very small and n is not too large, then

$$(1 - 2P_1)^n \approx 1 - 2P_1 n$$

Thus,
$$P_n \approx \frac{1}{2}[1-(1-2P_1 n)]$$
$$P_n \approx P_1 n$$

Section B : Digital Circuit-1 + Microprocessors and Microcontroller Network Theory-2 + Signals and Systems-2

Q.5 (a) Solution:

(i) Given, $(70)_8 + (122)_6 = (211)_X$

Converting the given system into its decimal equivalent,

$$2(X^2) + 1(X^1) + 1(X^0) = 8^1 \times 7 + 8^0 \times 0 + 6^2 \times 1 + 6^1 \times 2 + 6^0 \times 2$$
$$2X^2 + X + 1 = 56 + 36 + 12 + 2$$
$$2X^2 + X + 1 = 106$$
$$2X^2 + X = 105$$
$$X(2X + 1) = 105$$
$$X(2X + 1) = 7 \times 15$$
$$X(2X + 1) = 7(2 \times 7 + 1)$$

On comparing, we get $X = 7$

(ii) Given base system,

$$(131)_{12} = (X)_8 + (78)_9$$

Converting into decimal equivalent,

$$12^2 \times 1 + 12^1 \times 3 + 12^0 \times 1 = (X)_8 + (7 \times 9^1 + 8 \times 9^0)$$
$$144 + 36 + 1 = (X)_8 + (63 + 8)$$
$$(181)_{10} = (71)_{10} + (X)_8$$
$$\therefore \quad (X)_8 = (181)_{10} - (71)_{10} = (110)_{10}$$

Converting above result into binary equivalent,

2	110
2	55 – 0
2	27 – 1
2	13 – 1
2	6 – 1
2	3 – 0
2	1 – 1
	0 – 1

$$\therefore \quad (110)_{10} = (1101110)_2 = (156)_8$$

$$\therefore \quad (X)_8 = (156)_8 \quad \Rightarrow \quad X = 156$$

Q.5 (b) Solution:

Here, $R = 8\ \text{k}\Omega$, $L = 0.2$ mH, $C = 8\ \mu$F, $V = 10 \sin \omega t$ (V)

(i)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2\times10^{-3}\times8\times10^{-6}}} = \frac{10^5}{4} = 25\ \text{krad/s}$$

(ii)

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$= -\frac{1}{2\times8\times10^3\times8\times10^{-6}} + \sqrt{\left(\frac{1}{2\times8\times10^3\times8\times10^{-6}}\right)^2 + \frac{1}{0.2\times10^{-3}\times8\times10^{-6}}}$$

$$\Rightarrow \quad \omega_1 = 24.992\ \text{krad/s}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$= \frac{1}{2\times8\times10^3\times8\times10^{-6}} + \sqrt{\left(\frac{1}{2\times8\times10^3\times8\times10^{-6}}\right)^2 + \frac{1}{0.2\times10^{-3}\times8\times10^{-6}}}$$

$$\Rightarrow \quad \omega_2 = 25.008\ \text{krad/s}$$

(iii) Quality Factor, $Q = \omega_0 RC = 25 \times 10^3 \times 8 \times 10^3 \times 8 \times 10^{-6} = 1600$

$$\therefore \quad \text{BW} = (\omega_2 - \omega_1) = \frac{\omega_0}{Q} = \frac{25\times10^3}{1600} = 15.625\ \text{rad/s}$$

(iv) At $\omega = \omega_0$, $Y = \dfrac{I}{R} \quad \Rightarrow \quad Z = R = 8\ \text{k}\Omega$

$$\therefore \quad I = \frac{V}{Z} = \frac{10\angle-90^\circ}{8000} = 1.25\angle-90^\circ\ (\text{mA})$$

As the entire current flows through R at resonance, the average power dissipated at $\omega = \omega_0$ is,

$$P = \frac{1}{2}|I|^2 R = \frac{1}{2}(1.25\times10^{-3})^2(8\times10^3) = 6.25\ \text{mW}$$

or

$$P = \frac{V_m^2}{2R} = \frac{10^2}{2\times8\times10^3} = 6.25\ \text{mW}$$

As ω_1 and ω_2 are half power frequencies. Thus, at $\omega = \omega_1 = \omega_2$, the power dissipated is,

$$P = \frac{V_m^2}{4R} = \frac{10^2}{4 \times 8 \times 10^3} = 3.125 \text{ mW}$$

Q.5 (c) Solution:

Given, length of the filter M = 7

Hence, the order of the filter is M-1

$\therefore$ order = 7 – 1 = 6

From the given coefficients, it is clear that the filter follows:

$$h(n) = h(M - 1 - n)$$

Hence, it is linear phase filter and odd length symmetric.

Now, $h(n) = \{h(0),\ h(1),\ h(2),\ h(3),\ h(4),\ h(5),\ h(6)\}$

Virtual Y-axis

Due to even symmetry about virtual Y-axis

$$h(0) = h(6) = -0.3$$
$$h(1) = h(5) = 0.4$$
$$h(2) = h(4) = 0.2$$
$$h(3) = 0.5$$

The output of FIR filter is

$$y(n) = \sum_{m=0}^{M-1} h(m)x(n-m)$$

$$y(n) = h(0)\,[x(n) + x(n-6)] + h(1)\,[x(n-1) + x(n-5)] + h(2)\,[x(n-2) + x(n-4)] + h(3) \cdot x(n-3)$$

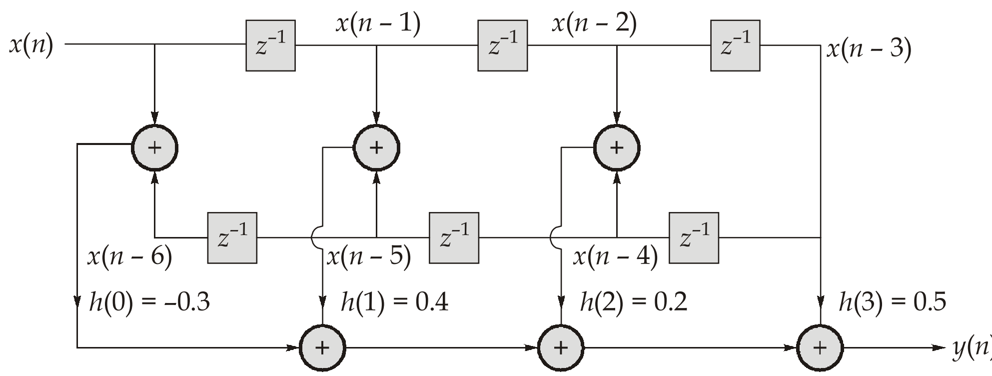
Here, number of delay elements = $M - 1 = 6$

No. of multipliers = $\dfrac{M+1}{2} = \dfrac{7+1}{2} = 4$

(for Linear phase FIR filter due to symmetry)

Multipliers are reduced in Direct Form-II of FIR filters.

Filter Design:



Q.5 (d) Solution:

```
         LDA   1100H
         MOV   C, A       ;  Initialize counter
         SUB   A          ;  Clear Accumulator
         LXI   H, 2100H   ;  Initialize pointer
LOOP:    ADD   M          ;  SUM = SUM + DATA
         INX   H          ;  Increment pointer
         DCR   C          ;  Decrement counter
         JNZ   LOOP       ;  if count ≠ 0 repeat
         STA   3200H      ;  Store sum
         HLT              ;  Terminate program execution
```

Q.5 (e) Solution:

(i) 1. **Setup time:** The setup time, denoted as t_{su}, is defined as the period of time immediately preceding the active clock edge during which the excitation input must be stable i.e., the excitation input must be "setup" atleast t_{su} prior to the active clock edge so that it can be reliably latched.

2. **Hold time:** Hold time denoted as t_h is defined as period of the time immediately following the active clock edge during which input should not change.

(ii)
$$N = (18.6)_9$$

Converting to base 10 via series substitution yields

$$N_{10} = 1 \times 9^1 + 8 \times 9^0 + 6 \times 9^{-1} + ...$$
$$= 9 + 8 + 0.666 ....$$
$$= (17.666...)_{10}$$

Converting from base 10 to base 11 via radix divide produces

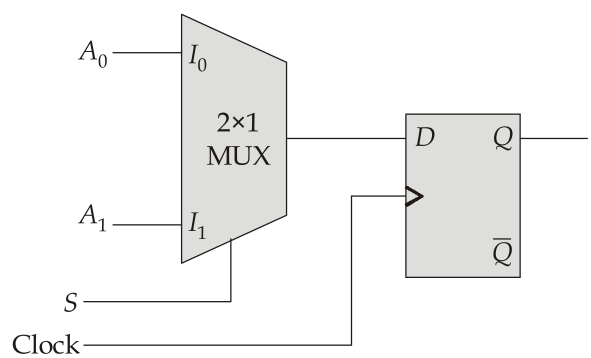
$$\begin{array}{r|ll} 11 & 17 & 6 \\ \hline & 1 & 1 \\ \cline{2-2} & 0 & \end{array} \quad \text{and} \quad \begin{array}{l} 0.666 \times 11 = 7.326 \\ 0.326 \times 11 = 3.586 \\ 0.586 \times 11 = 6.446 \end{array}$$

Putting integer and fraction parts together,

$$N_{11} = (16.736)_{11}$$

Q.6 (a) Solution:

Given sequential circuit,



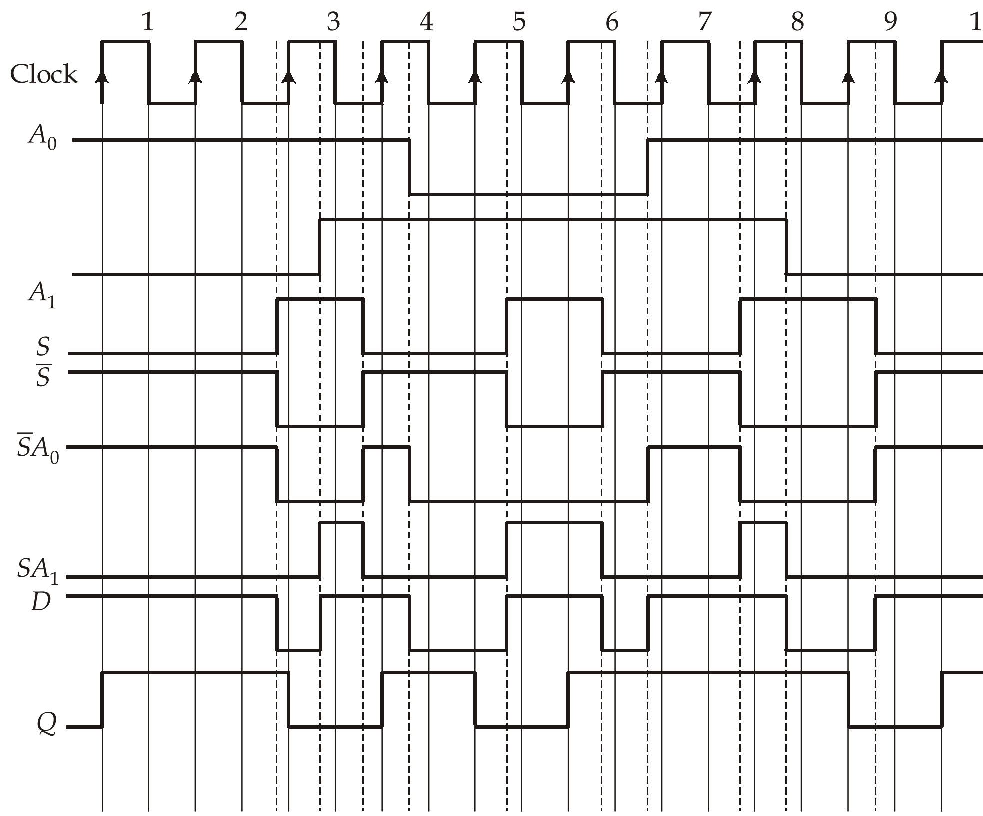
Let input to D-flip-flop, $D = Y$

where Y is the output of 2×1 MUX

We have,
$$Y = \bar{S}I_0 + SI_1$$
$$Y = \bar{S}A_0 + SA_1$$
$$\therefore \quad D = \bar{S}A_0 + SA_1$$

Output of D-flip-flop; $Q = D$ at each positive edge of the clock.

∴ The timing diagram for input D and output Q of the circuit is as follows:



Q.6 (b) Solution:

We know that,

z-parameter are given as

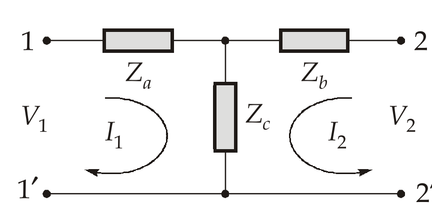
$$\left.\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned}\right\} \qquad ...(1)$$

and y-parameter are given as

$$\left.\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}\right\} \qquad ...(2)$$

and also $$[z] = [y]^{-1} \text{ and } [y] = [z]^{-1} \qquad ...(3)$$

(i) By KVL,



$$(Z_a + Z_c)I_1 + Z_cI_2 = V_1$$

and $$Z_cI_1 + (Z_b + Z_c)I_2 = V_2$$

On comparing with eq. (1), we get,

z-parameter matrix as

$$[z] = \begin{bmatrix} [Z_a + Z_c] & Z_c \\ Z_c & [Z_b + Z_c] \end{bmatrix}$$

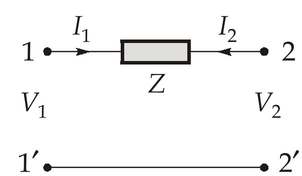
Now, y-parameter of the circuit can be obtained as,

$$[y] = [z]^{-1}$$

$$[y] = \frac{1}{[Z_a + Z_c][Z_b + Z_c] - Z_c^2} \begin{bmatrix} [Z_b + Z_c] & -Z_c \\ -Z_c & [Z_a + Z_c] \end{bmatrix}$$

$$[y] = \begin{bmatrix} \dfrac{Z_b + Z_c}{Z_a Z_b + Z_a Z_c + Z_b Z_c} & \dfrac{-Z_c}{Z_a Z_b + Z_a Z_c + Z_b Z_c} \\ \dfrac{-Z_c}{Z_a Z_b + Z_a Z_c + Z_b Z_c} & \dfrac{Z_a + Z_c}{Z_a Z_b + Z_a Z_c + Z_b Z_c} \end{bmatrix}$$

(ii) By KCL,



$$I_1 = \frac{V_1 - V_2}{Z} = \frac{1}{Z}V_1 - \frac{1}{Z}V_2$$

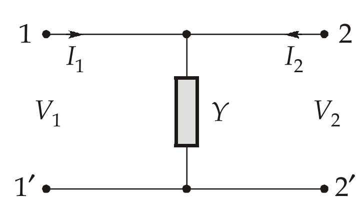
and $$I_2 = \frac{V_2 - V_1}{Z} = -\frac{1}{Z}V_1 + \frac{1}{Z}V_2$$

On comparing with eq. (2), we get, y-parameter matrix as

$$[y] = \begin{bmatrix} \dfrac{1}{Z} & -\dfrac{1}{Z} \\ -\dfrac{1}{Z} & \dfrac{1}{Z} \end{bmatrix}$$

Since, $\Delta y = y_{11}y_{22} - y_{12}\, y_{21} = 0$, the z-parameters do not exist for this network.

(iii) By KVL,



$$V_1 = \frac{I_1 + I_2}{Y} = V_2$$

or, $$V_1 = \left(\frac{1}{Y}\right)I_1 + \left(\frac{1}{Y}\right)I_2$$

and $$V_2 = \left(\frac{1}{Y}\right)I_1 + \left(\frac{1}{Y}\right)I_2$$

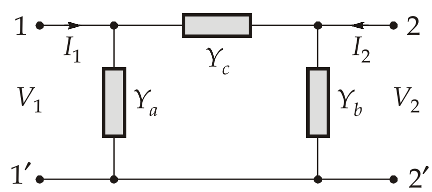
On comparing with eq. (1), we get, z-parameter matrix as

$$[z] = \begin{bmatrix} \dfrac{1}{Y} & \dfrac{1}{Y} \\ \dfrac{1}{Y} & \dfrac{1}{Y} \end{bmatrix}$$

Since, $\Delta z = z_{11}z_{22} - z_{12}z_{21} = 0$, the y-parameters do not exist for this network.

(iv) By KCL,

$$I_1 = Y_aV_1 + (V_1 - V_2)Y_c = V_1(Y_a + Y_c) - V_2Y_c$$
$$I_2 = Y_bV_2 + (V_2 - V_1)Y_c = -V_1Y_c + V_2(Y_b + Y_c)$$



On comparing with eq. (2), we get, y-parameter matrix as

$$[y] = \begin{bmatrix} Y_a + Y_c & -Y_c \\ -Y_c & Y_b + Y_c \end{bmatrix}$$

Now, z-parameter of the circuit are

$$[z] = [y]^{-1}$$

$$[z] = \frac{1}{[Y_a + Y_c][Y_b + Y_c] - Y_c^2}\begin{bmatrix} [Y_b + Y_c] & Y_c \\ Y_c & [Y_a + Y_c] \end{bmatrix}$$

$$[z] = \begin{bmatrix} \dfrac{Y_b + Y_c}{Y_aY_b + Y_aY_c + Y_bY_c} & \dfrac{Y_c}{Y_aY_b + Y_aY_c + Y_bY_c} \\ \dfrac{Y_c}{Y_aY_b + Y_aY_c + Y_bY_c} & \dfrac{Y_a + Y_c}{Y_aY_b + Y_aY_c + Y_bY_c} \end{bmatrix}$$

Q.6 (c) Solution:

(i) **Direct addressing mode :** The operand's offset is given in the instruction as a 16 bit displacement. In this addressing mode, a 16-bit memory address (offset) is directly specified in the instruction as a part of it.

Example : MOV AX, [0020H]. Here, the 16-bit data to be moved to the register AX is present in a memory location in the data segment. The offset addresses of the memory location in the data segment is 0020H. The effective address of memory is calculated as "10H × [DS] + 0020H".

Register addressing mode : In this addressing mode, the data is stored in a register and it is referred using the particular register. All the registers, except IP, may be used in this mode.

Example : MOV AX, BX ; here, the 16-bit data to be moved to the register AX is present in the register BX.

Register indirect addressing mode : In this addressing mode, the offset address of the memory location is specified indirectly, using the offset registers. In this addressing mode, the offset address is in either BX or SI or DI register and the default segment register is either DS or ES.

Example : MOV AX, [BX]; here the 16-bit data to be moved to the register AX is present in a memory location in the data segment. The offset address of the memory location in the data segment is in the register BX. The effective address of the memory location is "10H × [DS] + [BX]".

Indexed addressing mode : In this addressing mode, the offset address of the memory location is specified indirectly using one of the index register. DS is the default segment register for index registers SI and DI in case of string instructions. This addressing mode is a special case of the "register indirect addressing mode".

Example : MOV AX, [SI]; here the 16-bit data to be moved to the register AX is present in a memory location in the data segment. The offset address of the memory location in the data segment is in the register SI. The effective address of the memory location is "10H × [DS] + [SI]".

Register relative addressing mode : In this addressing mode, the offset address of the memory location is obtained by adding an 8-bit or 16-bit displacement to the content of any one of the registers BX, BP, SI and DI. The default segment register for this addressing mode is either DS or ES.

Example : MOV AX, 50H[BX]; here the offset address of the 16-bit data to be moved to the register AX is present in a memory location which is obtained by adding 50H to the content of the register BX. The effective address of the memory location is "10H × [DS] + 50H + [BX]".

Based indexed addressing mode : In this addressing mode, the offset address of the memory location is obtained by adding the content of a base register [BX or BP] to the content of an indexed register (SI and DI). The default segment register for this addressing is either DS or ES.

Example : MOV AX, [BX] [SI] ; here the 16-bit data to be moved to the register AX is present in a memory location in the data segment. The offset address of the memory location in the data segment is obtained by adding the content of the register BX to the content of the register SI. The effective address of the memory location is "10H × [DS] + [BX] + [SI]".

Relative based indexed addressing mode : In this addressing mode, the offset address of the memory location is obtained by adding an 8-bit or 16-bit displacement to the sum of the content of a base register (BX or BP) and the content of an indexed register (SI and DI). The default segment register for this addressing mode is either DS or ES.

Example : MOV, AX, 50H [BX] [SI]; here the 16-bit data to be moved to the register AX is present in a memory location in the data segment. The offset address of the memory location in the data segment is obtained by adding 50H to the sum of the content of the register BX and the content of the register SI. The effective address of the memory location is "10H × [DS] + 50H + [BX] + [SI]".

Immediate addressing mode : In this addressing mode, immediate data is a part of instruction and appears in the form of successive byte or bytes.

Example: MOV AX, 0020H; here the 16-bit data to be moved to the register AX is 0020H.

Implied Addressing Mode: In this addressing mode, the operand is implicitly defined within the opcode itself, requiring no explicit operand field.

Example: CLC instruction is used to reset the carry flag to zero.

(ii) The overall delay produced by the given subroutine program can be calculated by determining the total number of T-states required to execute the program.

The total number of T-states required to execute the program can be determined by analysing the given program as shown in the following table:

	Instruction	Number of times executed	Number of T-states for one time execution
	MVI B, Data_8 bit	1	7
LOOP:	DCR B	N	4
	JNZ LOOP	$(N-1) \Rightarrow$ true $1 \Rightarrow$ false	$10 \Rightarrow$ true $7 \Rightarrow$ false
	RET	1	10

The total delay produced by the subroutine in terms of T-states can be given by,

$$\text{Delay} = (1 \times 7T) + (N \times 4T) + [(N-1) \times 10T + 1 \times 7T] + (1 \times 10T)$$

$$= (14N + 14)\,T = 14(N+1)\,T$$

The time delay corresponding to one T-state is,

$$T = \frac{1}{f_{CLK}} = \frac{1}{2\,\text{MHz}} = 0.5\,\mu\text{s} \qquad \because \text{Given that, } f_{CLK} = 2\,\text{MHz}$$

So, the total delay produced by the program is,

$$\text{Delay} = 14(N+1)\frac{1}{2}\mu s = 7(N+1)\ \mu s \quad \text{...(i)}$$

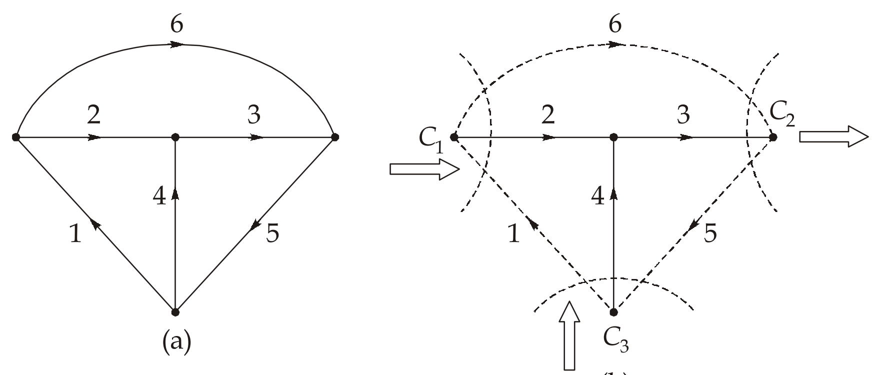
Required value of N, to produce a delay of 70 μs;

From the result in equation (i),

$$\text{Delay} = 7(N+1)\ \mu s$$
$$7(N+1)\ \mu s = 70\ \mu s$$
$$N+1 = 10$$
$$N = 9$$

Q.7 (a) Solution:

The graph of the network is shown in figure below. Further, a suitable tree is selected as shown below:



The fundamental cut-sets are identified as

f-cut-set-1 : [1, 2, 6]

f-cut-set-2 : [3, 5, 6]

f-cut-set-3 : [1, 4, 5]

The fundamental cutset matrix is given as

$$Q = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ C_1 & -1 & 1 & 0 & 0 & 0 & 1 \\ C_2 & 0 & 0 & 1 & 0 & -1 & 1 \\ C_3 & 1 & 0 & 0 & 1 & -1 & 0 \end{array}$$

The node equations are given as

$$[Q][Y_b][Q^T][V_t] = [Q] \times \{[I_s] - [Y_b][V_s]\} = -[Q][Y_b][V_s] \quad \{\text{since } I_s = 0 \text{ here}\}$$

Here,

$$[Q][Y_b][Q^T] = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9 & 0.5 & -0.2 \\ 0.5 & 0.8 & 0.2 \\ -0.2 & 0.2 & 0.5 \end{bmatrix}$$

$$[Q][Y_b][V_s] = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 910 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -182 \\ 0 \\ 182 \end{bmatrix}$$

Thus, the KCL equations are

$$= \begin{bmatrix} 0.9 & 0.5 & -0.2 \\ 0.5 & 0.8 & 0.2 \\ -0.2 & 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} V_{t2} \\ V_{t3} \\ V_{t4} \end{bmatrix} = \begin{bmatrix} 182 \\ 0 \\ -182 \end{bmatrix}$$

Solving by Cramer's rule, we get the tree-branch voltages as

$$V_{t2} = \frac{\begin{vmatrix} 182 & 0.5 & -0.2 \\ 0 & 0.8 & 0.2 \\ -182 & 0.2 & 0.5 \end{vmatrix}}{\begin{vmatrix} 0.9 & 0.5 & -0.2 \\ 0.5 & 0.8 & 0.2 \\ -0.2 & 0.2 & 0.5 \end{vmatrix}} = \frac{18.2}{0.127} = 143.3 \text{ V}$$

$$V_{t3} = \frac{\begin{vmatrix} 0.9 & 0.5 & 182 \\ 0.5 & 0.8 & 0 \\ -0.2 & 0.2 & -182 \end{vmatrix}}{0.127} = \frac{-38.22}{0.127} = -300.94 \text{ V}$$

The branch voltages are thus given by $V_b = Q^T V_t$. Thus,

$$\begin{bmatrix} V_{b1} \\ V_{b2} \\ V_{b3} \\ V_{b4} \\ V_{b5} \\ V_{b6} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 143.3 \\ -14.33 \\ -300.94 \end{bmatrix} = \begin{bmatrix} -444.24 \\ 143.3 \\ -14.33 \\ -300.94 \\ 315.27 \\ 128.97 \end{bmatrix} \text{V}$$

The branch currents are given by $I_b = Y_b (V_b + V_s)$. Thus,

$$\begin{bmatrix} I_{b1} \\ I_{b2} \\ I_{b3} \\ I_{b4} \\ I_{b5} \\ I_{b6} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -444.24 + 910 \\ 143.3 \\ -14.33 \\ -300.94 \\ 315.27 \\ 128.97 \end{bmatrix} = \begin{bmatrix} 93.152 \\ 28.662 \\ -1.433 \\ -30.094 \\ 63.054 \\ 64.485 \end{bmatrix} \text{A}$$

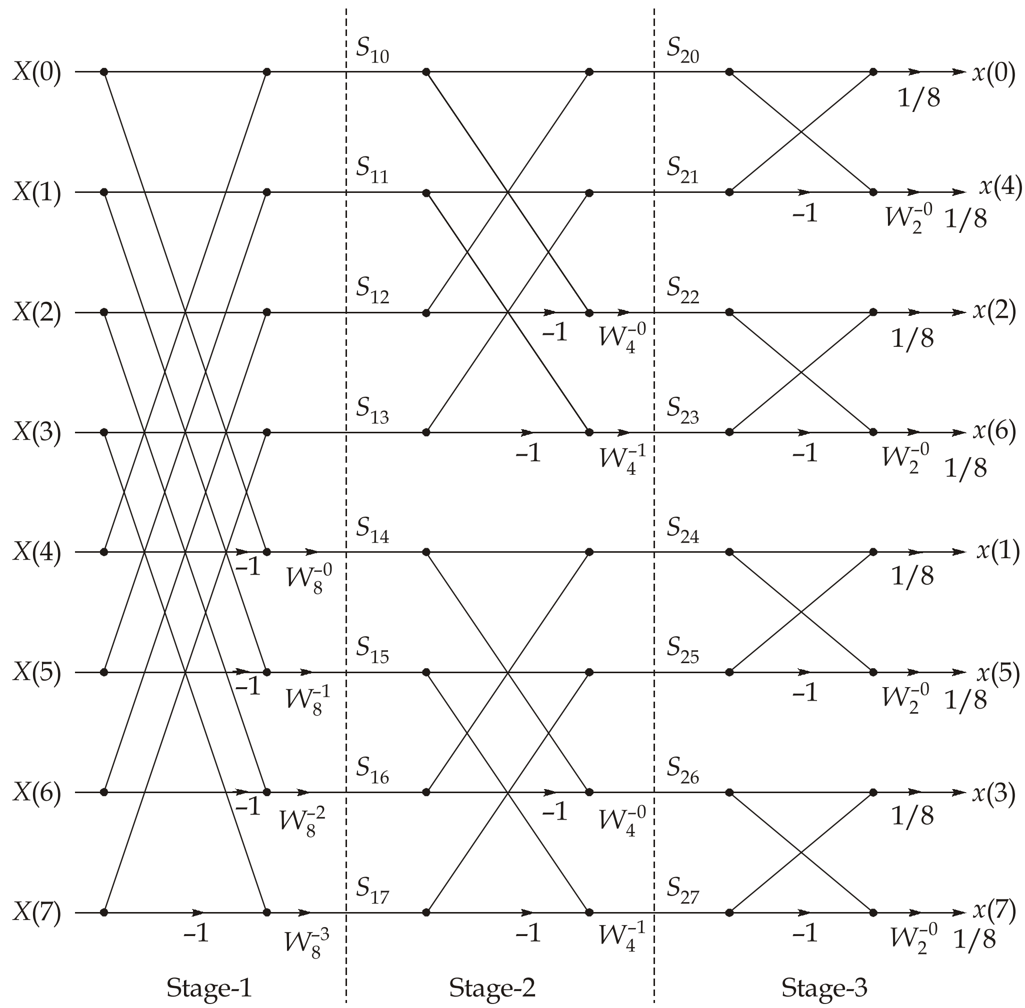
Q.7 (b) Solution:

Given, $N = 8$

$$\therefore \quad W_N^K = e^{-j\left(\frac{2\pi}{N}\right)K}$$

$$\left.\begin{aligned} W_8^{-0} &= 1 \\ W_8^{-1} &= 0.707 + j0.707 \\ W_8^{-2} &= j \\ W_8^{-3} &= -0.707 + j0.707 \end{aligned}\right| , \quad \left.\begin{aligned} W_4^{-0} &= 1 \\ W_4^{-1} &= j \end{aligned}\right| , \quad W_2^0 = 1$$

The DIT – Inverse FFT butterfly diagram for N =8 would be as follows:



Output of Stage-1

$$S_{10} = X(0) + X(4) = 255 - 85 = 170$$

$$S_{11} = X(1) + X(5) = 48.63 + j166.05 + (-78.63 - j46.05)$$

$$= -30 + j120$$

$$S_{12} = X(2) + X(6) = -51 + j102 - 51 - j102 = -102$$
$$S_{13} = X(3) + X(7) = -78.63 + j46.05 + 48.63 - j166.05$$
$$= -30 - j120$$
$$S_{14} = [X(0) - X(4)]\,W_8^{-0} = 255 + 85 = 340$$
$$S_{15} = [X(1) - X(5)]\,W_8^{-1}$$
$$= (0.707 + j0.707)[48.63 + j166.05 + 78.63 + j46.05]$$
$$S_{15} = -60 + j240$$
$$S_{16} = [X(2) - X(6)]\,W_8^{-2} = j[-51 + j102 + 51 + j102] = -204$$
$$S_{17} = [X(3) - X(7)]\,W_8^{-3}$$
$$= (-0.707 + j0.707)[-78.63 + j46.05 - 48.63 + j166.05]$$
$$S_{17} = -60 - j240$$

Output of Stage-2

$$S_{20} = (S_{10} + S_{12}) = 170 - 102 = 68$$
$$S_{21} = (S_{11} + S_{13}) = -30 + j120 - 30 - j120 = -60$$
$$S_{22} = (S_{10} - S_{12})\,W_4^{-0} = 1(170 + 102) = 272$$
$$S_{23} = (S_{11} - S_{13})\,W_4^{-1} = j[-30 + j120 + 30 + j120] = -240$$
$$S_{24} = (S_{14} + S_{16}) = 340 - 204 = 136$$
$$S_{25} = (S_{15} + S_{17}) = -60 + j240 - 60 - j240 = -120$$
$$S_{26} = (S_{14} - S_{16})\,W_4^{-0} = 1[340 + 204] = 544$$
$$S_{27} = (S_{15} - S_{17})\,W_4^{-1} = j[-60 + j240 + 60 + j240] = -480$$

Output of Stage-3

$$x(0) = \frac{1}{8}(S_{20} + S_{21}) = \frac{(68-60)}{8} = 1$$
$$x(4) = \frac{1}{8}(S_{20} - S_{21})W_2^{-0} = \frac{(68+60)}{8} = 16$$
$$x(2) = \frac{1}{8}(S_{22} + S_{23}) = \frac{(272-240)}{8} = 4$$
$$x(6) = \frac{1}{8}(S_{22} - S_{23})W_2^{-0} = \frac{(272+240)}{8} = 64$$

$$x(1) = \frac{1}{8}(S_{24} + S_{25}) = \frac{(136 - 120)}{8} = 2$$

$$x(5) = \frac{1}{8}(S_{24} - S_{25})W_2^{-0} = \frac{(136 + 120)}{8} = 32$$

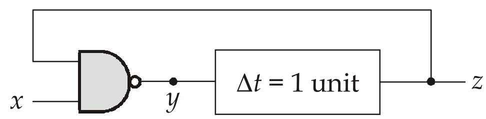
$$x(3) = \frac{1}{8}(S_{26} + S_{27}) = \frac{(544 - 480)}{8} = 8$$

$$x(7) = \frac{1}{8}(S_{26} - S_{27})W_2^{-0} = \frac{(544 + 480)}{8} = 128$$

Hence, $x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$

Q.7 (c) Solution:

(i)

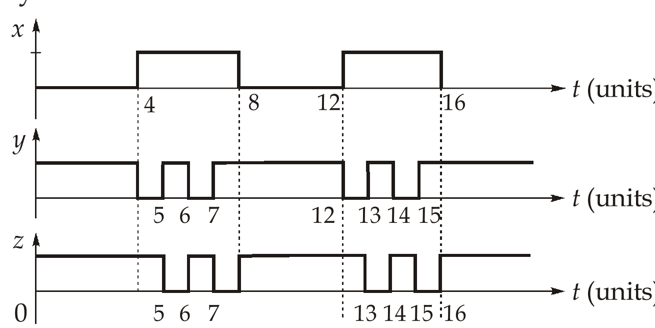


The output z is given by

$$z(t + \Delta t) = \overline{x(t)\,z(t)} = \overline{x}(t) + \overline{z}(t)$$

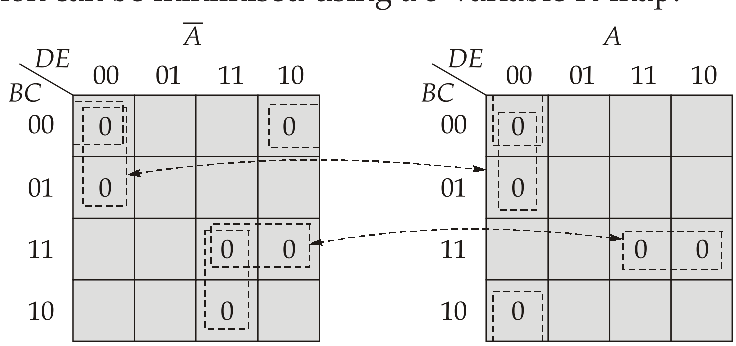
Consequently the timing diagram is obtained as:

Whenever x changes from 0 to 1, the output z starts oscillating with a period of $2\Delta t$ (2 unit in this case). However, when x changes from 1 to 0, the output z becomes 1 after a time delay of Δt.



(ii) $f(A, B, C, D, E) = \prod M(0, 2, 4, 11, 14, 15, 16, 20, 24, 30, 31)$

Above function can be minimised using a 5-variable K-map.

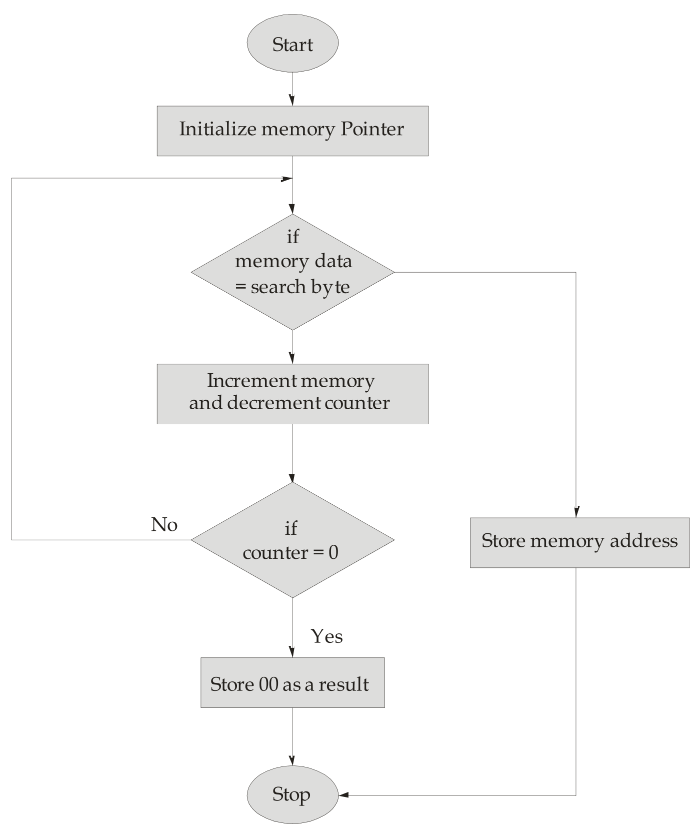


$$f = (B + D + E)(\overline{B} + \overline{C} + \overline{D})(A + B + C + E)\,(A + \overline{B} + \overline{D} + \overline{E})(\overline{A} + C + D + E)$$

Q.8 (a) Solution:

	LXI	H, 1000H	;	Initialize memory pointer
	MVI	B, 32H	;	Initialize counter with 50_{10} = 32H
LOOP1:	MOV	A, M	;	Get the number
	CMP	C	;	Compare with the given byte
	JZ	L1	;	Go to L1 if match occurs
	DCR	B	;	Decrement counter
	JNZ	LOOP1	;	If not zero, repeat
	LXI	H, 0000H		
	SHLD	1100H	;	Store 00 at 1100H and 1101H
	JMP	L2		
L1:	SHLD	1100H	;	Store memory address
L2:	HLT		;	Stop

Flow chart:



Q.8 (b) Solution:

(i) Here, $R = 9\ \Omega$ and $L = 8$ mH, $i = 5 + 100\sin(1000t + 45°) + 100\sin(3000t + 60°)$ A

For dc component

Current, $I_0 = 5$ A, $Z_0 = R = 9\ \Omega$

$$\therefore \quad V_0 = I_0 \times R = 5 \times 9 = 45 \text{ V}$$

For first-harmonic component

Current, $I_1 = 100\angle 45°$ A

Impedance, $Z_1 = R + j\omega L = 9 + j1000 \times 8 \times 10^{-3}$

$$= (9 + j8) = 12.04\angle 41.63°\ (\Omega)$$

$$\therefore \quad V_1 = I_1 Z_1 = 100\angle 45° \times 12.04\angle 41.63°$$

$$= 1204\angle 86.63°\ (\text{V})$$

For third-harmonic component

Current, $I_3 = 100\angle 60°$ A

Impedance, $Z_{31} = R + j3\omega L$

$$= 9 + j3 \times 1000 \times 8 \times 10^{-3}$$

$$= (9 + j24) = 25.63\angle 69.44°(\Omega)$$

$$\therefore \quad V_3 = I_3 Z_3 = 100\angle 60° \times 25.63\angle 69.44°$$

$$= 2563\angle 129.44°\ (\text{V})$$

$\therefore$ The applied voltage is given as

$$v = 45 + 1204\sin(1000t + 86.63°) + 2563\sin(3000t + 129.44°)\ (\text{V})$$

(ii) The fundamental frequency current,

$$I_1 = \frac{250\sin 314t}{R + j\omega L} = 17.7\sin(314t - 45°) \qquad \text{...(i)}$$

The third harmonic current,

$$I_3 = \frac{50\sin(942t + 30°)}{R + j3\omega L} = 1.583\sin(942t - 41.6°) \qquad \text{...(ii)}$$

Equating the magnitude in equation (i),

$$\frac{250}{\sqrt{R^2 + \omega^2 L^2}} = 17.7 \quad \Rightarrow \quad R^2 + \omega^2 L^2 = 199.495 \qquad \text{...(iii)}$$

Equating the angles in equation (i)

$$314t - \tan^{-1}\frac{\omega L}{R} = 314t - 45°$$

$$\Rightarrow \qquad \tan^{-1}\frac{\omega L}{R} = 45° \Rightarrow \frac{\omega L}{R} = 1 \Rightarrow \omega L = R$$

Putting in equation (iii),

$$\Rightarrow \qquad (\omega L)^2 = 99.747 \Rightarrow \omega L = 9.987 = R$$

$$\therefore \qquad L = \frac{9.987}{314} = 0.0318$$

$$\therefore \qquad R = 9.987\ \Omega \quad L = 0.0318\ \text{H}$$

$$\text{Power factor} = \frac{\text{Average power}}{\text{Apparent power}} = \frac{\frac{V_1 I_1}{2}\cos\phi_1 + \frac{V_3 I_3}{2}\cos\phi_3}{\sqrt{\frac{V_1^2}{2} + \frac{V_3^2}{2}} \times \sqrt{\frac{I_1^2}{2} + \frac{I_3^2}{2}}}$$

$$= \frac{\frac{250 \times 17.7}{2}\cos 45° + \frac{50 \times 1.583}{2}\cos 41.6°}{\sqrt{\frac{250^2 + 50^2}{2}} \times \sqrt{\frac{17.7^2 + 1.583^2}{2}}} = 0.69$$

Q.8 (c) Solution:

(i) Given,

inputs: x, y, z (3-bit binary input)

outputs: A, B, C (3-bit binary output)

given condition:

For inputs 0 – 3 $\Rightarrow$ output = input + 1

For inputs 4 – 7 $\Rightarrow$ output = input – 2

Truth table:

x	y	z	Decimal input	Output (in decimal)	A	B	C
0	0	0	0	0 + 1 = 1	0	0	1
0	0	1	1	1 + 1 = 2	0	1	0
0	1	0	2	2 + 1 = 3	0	1	1
0	1	1	3	3 + 1 = 4	1	0	0
1	0	0	4	4 – 2 = 2	0	1	0
1	0	1	5	5 – 2 = 3	0	1	1
1	1	0	6	6 – 2 = 4	1	0	0
1	1	1	7	7 – 2 = 5	1	0	1

For output *A*:

$$A = \Sigma m(3, 6, 7)$$

Using 3-variable K-map:

x \ yz	00	01	11	10
0			1	
1			1	1

$$\therefore \quad A = yz + xy$$

For the output *B*:

$$B = \Sigma m(1, 2, 4, 5)$$

x \ yz	00	01	11	10
0		1		1
1	1	1		

$$\therefore \quad B = \overline{y}z + x\overline{y} + \overline{x}y\overline{z}$$

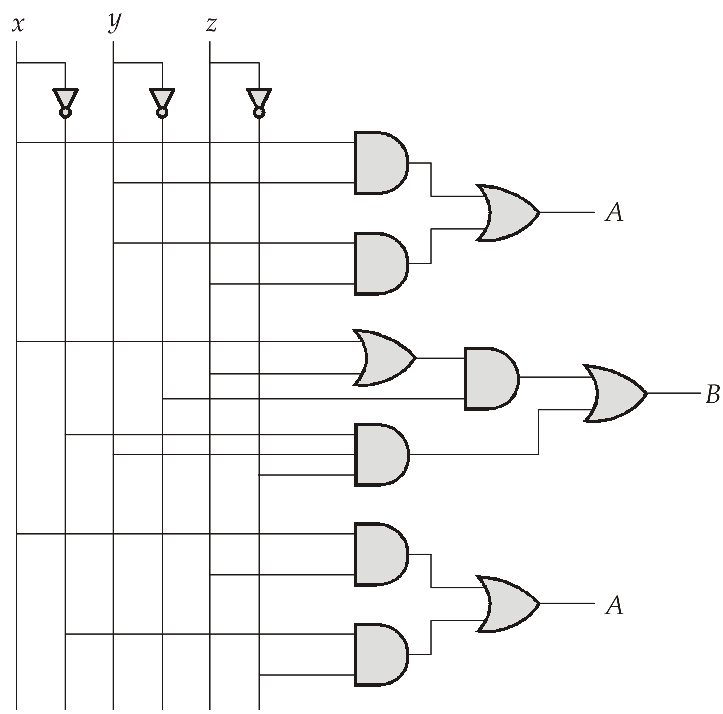
$$B = \overline{y}(x+z) + \overline{x}y\overline{z}$$

For the output *C*:

$$C = \Sigma m(0, 2, 5, 7)$$

x \ yz	00	01	11	10
0	1			1
1		1	1	

$$C = xz + \overline{x}\,\overline{z}$$



(ii) Given crystal frequency,

$$f = 5 \text{ MHz}$$

The microprocessor internally divides the crystal frequency by 2. Thus,

$$\text{clock frequency, } f_{clk} = \frac{5 \text{ MHz}}{2} = 2.5 \text{ MHz}$$

$$\text{clock period, } T = \frac{1}{2.5 \times 10^6} = 0.4 \, \mu\text{sec}$$

Program			T-states
DELAY :	MVI	D, 10 H	7T
Loop 2:	LXI	B, 2030 H	10T
Loop 1:	DCX	B	6T
	MOV	A, C	4T
	ORA	B	4T
	JNZ	Loop 1	10T/7T
	DCR	D	4T
	JNZ	Loop 2	10T/7T
	RET		10T

The Loop 1 is executed (2030H) = 8240 times. When the condition is true (i.e. 8239 times), the jump condition takes 10T-states and 7T-States when the condition is false i.e. 8240^{th} time.

Each iteration of Loop 1 takes

$$(6 + 4 + 4 + 10) = 24\text{T-states}$$

$\therefore$ For 8239 iterations,

$$\text{T-states} = 8239 \times 24 = 197736 \text{ T-states}$$

$$\text{Last iteration, } 6 + 4 + 4 + 7 = 21\text{T-states}$$

$\therefore$ Total jump Loop 1 executes,

$$197736 + 21 = 197757 \text{ T-states}$$

Similarly, JNZ Loop 2, is executed 15 times when the condition is true and Last iteration (No jump) = 7T-states

Total T-states in outer loop:

Inner loop per outer iteration:

$$10 + 197781 + 4 + 10 = 197781\text{T-states}$$

For 15 iterations,

$$197781 \times 15 = 2966715 \text{ T-states}$$

Final iteration,

$$10 + 197757 + 4 + 7 = 197778 \text{ T-states}$$

Add initialization and RET:

$$7 + 2966715 + 197778 + 10 = 3164510 \text{ T-states}$$

$\therefore$ Total Execution time, $T = 3164510 \times 0.4\ \mu\text{sec}$

$$T = 1.265 \text{ sec}$$

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