Detailed Solutions

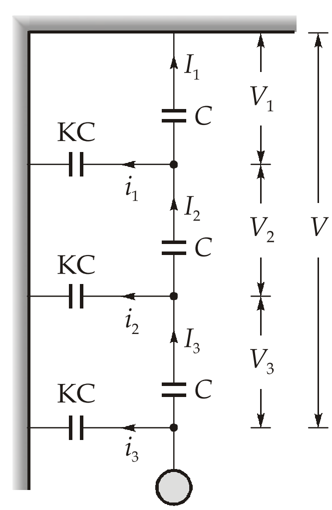
ESE-2026 Mains Test Series

Electrical Engineering Test No : 3

Section A : Power Systems

Q.1 (a) Solution:

The equivalent circuit,



It is given that $V_3 = 13.1$ kV and $V_2 = 11$ kV. Let K be the ratio of shunt capacitance to self capacitance of each unit. Applying Kirchoff's current law to junctions,

$$V_2 = V_1(1 + K)$$

$$V_1 = \frac{V_2}{1+K}$$

$$V_3 = V_2 + (V_1 + V_2)K$$

$$V_3 = V_2 + \left[\frac{V_2}{1+K} + V_2\right]K$$

$$V_3(1 + K) = V_2(1 + K) + [V_2 + V_2(1 + K)]K$$
$$= V_2[(1 + K) + K + (K + K^2)]$$
$$= V_2[1 + 3K + K^2]$$
$$13.3(1 + K) = 11[1 + 3K + K^2]$$
$$11K^2 + 19.9K - 2.1 = 0$$

Solving this equation,

We get $K = 0.1$

$$\therefore \quad V_1 = \frac{V_2}{1+K} = \frac{11}{1+0.1} = 10 \text{ kV}$$

Voltage between line and earth $= V_1 + V_2 + V_3 = 10 + 11 + 13.1$

$= 34.1$ kV

$\therefore$ Voltage of bus-bars $= 34.1 \times \sqrt{3} = 59$ kV

Q.1 (b) Solution:

The power loss due to corona for 3-phase is given by

$$P = \frac{3 \times 242.2(f+25)}{\delta}\sqrt{\frac{r}{d}}(V - V_c)^2 \times 10^{-5} \text{ kW/km}$$

As f, δ, r and d are the same for the two cases,

$$P \propto (V - V_C)^2$$

For first case, $P = 53$ kW

and $V = \frac{106}{\sqrt{3}} = 61.2$ kV

For second case, $P = 98$ kW

and $V = \frac{110.9}{\sqrt{3}} = 64$ kV

$\therefore$ $53 \propto (61.2 - V_C)^2$ ...(i)

and $98 \propto (64 - V_C)^2$ ...(ii)

$$\frac{98}{53} = \frac{(64 - V_C)^2}{(61.2 - V_C)^2}$$

Critical disruptive voltage, $V_C = 54$ kV

Let W kilowatt be the power loss at 113 kV

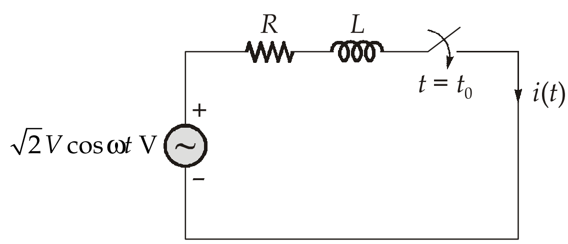
$$W = \left(\frac{113}{\sqrt{3}} - V_C\right)^2 = (65.2 - 54)^2 \qquad \text{...(iii)}$$

Dividing (iii)/(i) we get

$$\frac{W}{53} = \frac{(65.2-54)^2}{(61.2-54)^2}$$

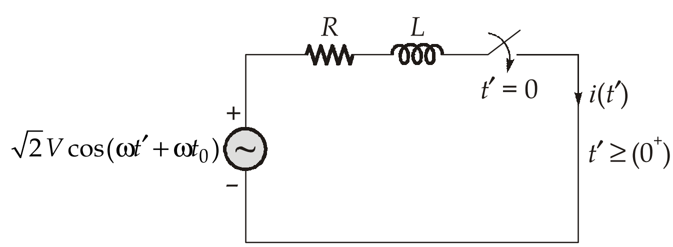
$$\therefore \qquad W = \left(\frac{11.2}{7.2}\right)^2 \times 53 = 128 \text{ kW}$$

Q.1 (c) Solution:



To find $i(t)$ for $t \geq 0^+$ we can convert the above circuit in following way

Let, $\qquad t' = t - t_0 \Rightarrow t = t' + t_0$



$$i(t') = \frac{\sqrt{2}V}{|z|}\cos(\omega t' + \omega t_0 - \alpha) + A\, e^{-\frac{t'}{\tau}} \qquad (t' \geq t_0^+)$$

Symmetrical short circuit current,

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$\alpha = \tan^{-1}\frac{\omega L}{R}$$

$$\tau = \frac{L}{R}$$

Now, $\qquad i(t' = 0^-) = i(t' = 0^+) = 0$

$$\Rightarrow \qquad \frac{\sqrt{2}V}{|Z|}\cos(\omega t_0 - \alpha) + A = 0$$

$$A = -\frac{\sqrt{2}V}{|Z|}\cos(\omega t_0 - \alpha)$$

Now change, $t' = t - t_0$

$$i(t) = \frac{\sqrt{2}V}{|Z|}\cos(\omega t - \alpha) - -\frac{\sqrt{2}V}{|Z|}\cos(\omega t_0 - \alpha)\, e^{-\frac{(t-t_0)R}{L}} \qquad t \geq t_0^+$$

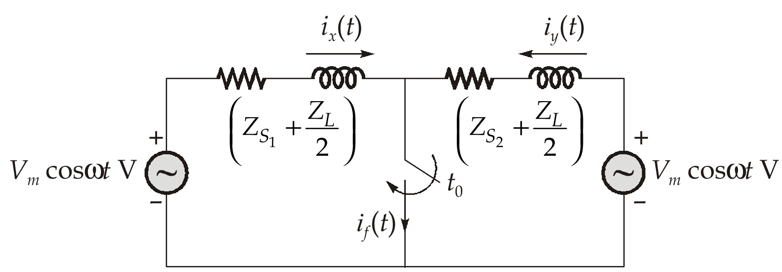
Now, if the initial value of dc off set (A) is $-\frac{\sqrt{2}V}{|Z|}$ it requires that $\cos(\omega t_0 - \alpha) = 1$

$$\Rightarrow \qquad \omega t_0 - \alpha = 0$$

$$\Rightarrow \qquad t_0 = \frac{\alpha}{\omega}$$

$$t_0 = \frac{\tan^{-1}\frac{\omega L}{R}}{\omega}$$

In problem given the circuit for 3-ϕ fault can be drawn as:



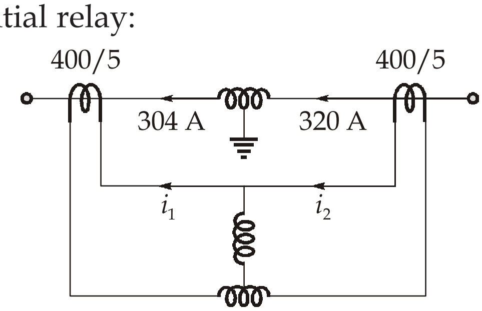
According to the derivation:

For initial value of dc off set to be $-\frac{\sqrt{2}\text{V}}{|Z|}$ we required

$$t_0 = \frac{\tan^{-1}\frac{\omega L}{R}}{\omega} = \frac{\tan^{-1}\frac{0.04}{0.004}}{2 \times 3.14 \times 50} = 4.68 \text{ msec.}$$

Q.1 (d) Solution:

Given percentage differential relay:



As we know,

For trip, $i_R = (i_2 - i_1) > I_0 + K\left(\frac{i_1 + i_2}{2}\right)$

Given, $K = 0.1;\ I_0 = 0.1$ A

Now, $i_1 = \frac{304}{(400/5)} = 3.8$ A

$i_2 = \frac{320}{(400/5)} = 4$ A

$\therefore \quad i_2 - i_1 = 0.2$ A

Also, $\frac{i_1 + i_2}{2} = 3.9$ A

Now, $I_0 + K\left(\frac{i_1 + i_2}{2}\right) = 0.1 + 0.1 \times 3.9 = 0.49$

$\because \quad i_R = 0.2 < I_0 + K\left(\frac{i_1 + i_2}{2}\right)$

$\therefore$ Relay will not operate to trip the breaker.

Q.1 (e) Solution:

Reactance of synchronous generator = 1.2 p.u.

Voltage of infinite bus bar = 1.0 p.u.

Reactance of line and transformer = 0.6 p.u.

No load voltage of generator = 1.2 p.u.

Maximum power, $P_{em} = \frac{V_t \times E_g}{X_s}$

where X_s is total reactance of the circuit.

$$P_{em} = \frac{1.2 \times 1}{1.2 + 0.6} = \frac{1.2}{1.8} = \frac{2}{3}$$

Given, $P = P_{em} \sin \delta_o = 0.8 P_{em}$

$$\delta_o = 53.13°$$

where $S_{Po} = \left.\frac{dP}{d\delta}\right|_{\delta_o} = P_{em} \cos \delta_o = 0.6 P_{em} = 0.6 \times \frac{2}{3} = 0.4$

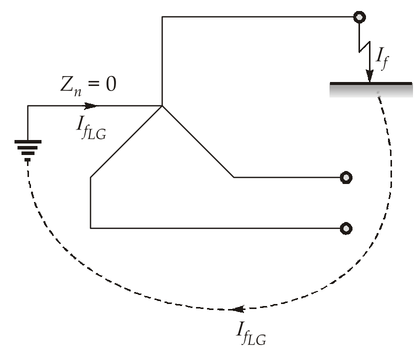
$$\omega_s = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad/s}$$

Natural frequency of oscillation

$$\omega_o = \sqrt{\frac{S_{po}\omega_s}{2H}}$$

$$f_o = \frac{1}{2\pi}\sqrt{\frac{S_{po}\omega_s}{2H}} = \frac{1}{2\pi}\sqrt{\frac{0.4\times 100\pi}{2\times 4}} = 0.63 \text{ Hz}$$

Q.2 (a) (i) Solution:



Given, $X_1 = 0.2$ pu, $X_2 = 0.2$ pu, $X_3 = 0.05$ pu

For a line-to-ground fault on generator, fault current is given by

$$I_{f_{LG}} = 3I_{R_0} = 3I_{R_1} = 3I_{R_2}$$

$$= \frac{3E}{z_1 + z_2 + z_0}$$

$$I_{f_{LG}} = \frac{3\times 1}{0.2+0.2+0.05} = \frac{3}{0.45} = \frac{20}{3} \text{ pu}$$

Also, base MVA = 100

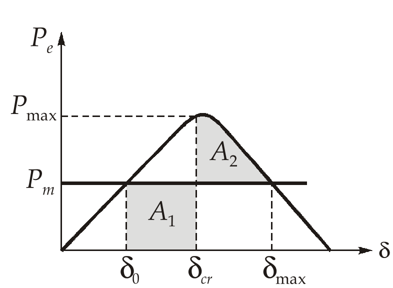
Base KVA = 25

$\therefore$ Base current, $I_b = \dfrac{100\times 10^6}{\sqrt{3}\times 25\times 10^3} = 2.309 \text{ kA}$

$\therefore$ Actual fault current in ampere $= (I_{f_{LG}})_{pu} \times I_b = \dfrac{20}{3}\times 2.309\times 10^3$

$\therefore$ $I_f = 15393$ Ampere

Q.2 (a) (ii) Solution:



Power generated by the generator,

$$P_e = P_{max} \sin\delta$$

At δ_0,

$$P_e = 1 \text{ p.u.}$$
$$P_{max} = 2 \text{ p.u.}$$
$$\delta_{cr} = \text{rotor angle at } t = t_c$$
$$P_e = P_{max} \sin\delta$$
$$1 = 2 \sin\delta_0$$
$$\delta_0 = 30°$$

- During fault, P_e become zero and the fault is cleared at δ_{cr}. Mechanical input to the generator remains constant, $P_m = 1$ p.u.
- Applying equal areal criterion,

$$A_1 = A_2$$

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta = P_m(\delta_{cr} - \delta_0) \quad \text{...(i)}$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin\delta - P_m)\, d\delta$$

$$= P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m(\delta_{max} - \delta_{cr}) \quad \text{...(ii)}$$

Equating equation (i) and (ii), we get

$$P_m(\delta_{cr} - \delta_0) = P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m(\delta_{max} - \delta_{cr})$$

$$\cos\delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max}$$

$$\cos\delta_{cr} = \frac{1}{2} \times (110 - 30) \times \frac{\pi}{180} + \cos 110$$

$$\delta_{cr} = 69.14°$$

Q.2 (b) Solution:

Given, Series impedance, $Z = (40 + j125)\ \Omega = 131.2\angle 72.3°\ \Omega$

Shunt admittance $Y = 10^{-3}\angle 90°\ ℧$

∵ the receiving end load is 50 MW at 220 kV, 0.8 pf lagging

∴

$$I_R = \frac{50}{\sqrt{3} \times 220 \times 0.8}\angle -36.9° = 0.164\angle -36.9° \text{ kA}$$

$$V_R = \frac{220}{\sqrt{3}}\angle 0° = 127\angle 0° \text{ kV}$$

(i) Short line approximation :

$$V_S = V_R + ZI_R$$
$$I_S = I_R$$

$\therefore \quad V_S = 127 + 0.164\angle{-36.9°} \times 131.2\angle 72.3°$

$= 145\angle 4.9°$ kV

$|V_s|_{\text{line}} = 251.2$ kV

$I_S = I_R = 0.164\angle{-36.9°}$ kA

Sending end power factor $= \cos(4.9 + 36.9) = \cos(41.8°)$

$= 0.745$ lagging

Sending end power $= \sqrt{3} \times 251.2 \times 0.164 \times 0.745$

$= 53.2$ MW

(ii) Nominal π method:

$$A = D = 1 + \frac{YZ}{2}$$

$$= 1 + \frac{1}{2} \times 10^{-3} \angle 90° \times 131.2\angle 72.3°$$

$$= 0.938\angle 1.2°$$

$$B = Z = 131.2\angle 72.3°\ \Omega$$

$$C = Y\left(1 + \frac{YZ}{4}\right) = Y + \frac{1}{4}Y^2 Z$$

$$C = 0.001\angle 90° + \frac{1}{4} \times 10^{-6} \angle 180° \times 131.2\angle 72.3°$$

$$C \approx 0.001\angle 90°$$

$$V_S = AV_R + BI_R$$

$$= 0.938\angle 1.2° \times 127 + 131.2\angle 72.3° \times 0.164\angle{-36.9°}$$

$$= 119.1\angle 1.2° + 21.5\angle 35.4° = 137.4\angle 6.2° \text{ kV}$$

$$V_{s\,\text{line}} = 238 \text{ kV}$$

$$I_S = CV_R + DI_R$$

$$= 0.001\angle 90° \times 127 + 0.938\angle 1.2° \times 0.164\angle{-36.9°}$$

$$= 0.127\angle 90° + 0.154\angle{-35.7°} = 0.13\angle 16.5° \text{ kA}$$

Sending end pf $= \cos(16.5° - 6.2°) = 0.984$ leading

Sending end power $= \sqrt{3} \times 238 \times 0.13 \times 0.984$

$= 52.7$ MW

(iii) Exact transmission line equations:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \frac{1}{Z_C} \sinh \gamma l & \cosh \gamma l \end{bmatrix}$$

$$\gamma l = (\alpha + j\beta) l = \sqrt{YZ}$$

$$= \sqrt{10^{-3} \angle 90° \times 131.2 \angle 72.3°} = 0.362 \angle 81.2°$$

$$\cosh \gamma l = \frac{1}{2}\left(e^{\alpha l} \angle \beta l + e^{-\alpha l} \angle -\beta l\right)$$

$$\gamma l = 0.0554 + j0.3577$$

$$\beta l = 0.3577 \text{ radian} = \angle 20.49°$$

$$e^{0.0554} \angle 20.49° = 1.057 \angle 20.49°$$

$$= 0.99 + j0.37$$

$$e^{-0.0554} \angle -20.49° = 0.946 \angle -20.49°$$

$$= 0.886 - j0.331$$

$\therefore$ $$\cosh \gamma l = 0.938 + j0.02$$

$$= 0.938 \angle 1.2°$$

$$\sinh \gamma l = 0.052 + j0.35$$

$$= 0.354 \angle 81.5°$$

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{131.2 \angle 72.3°}{10^{-3} \angle 90°}} = 362.21 \angle -8.85°$$

$$A = D = \cosh \gamma l = 0.938 \angle 1.2°$$

$$B = Z_C \sinh \gamma l = 362.21 \angle -8.85° \times 0.354 \angle 81.5°$$

$$= 128.2 \angle 72.65°$$

Now, $$V_S = 0.938 \angle 1.2° \times 127 \angle 0° + 128.2 \angle 72.65° \times 0.164 \angle -36.9°$$

$$= 119.13 \angle 1.2° + 21.03 \angle 35.75°$$

$$= 136.97 \angle 6.2° \text{ kV}$$

$$|V_S|_{\text{line}} = 237.23 \text{ kV}$$

$$C = \frac{1}{Z_C} \sinh \gamma l = \frac{1}{362.21 \angle -8.85°} \times 0.354 \angle 81.5°$$

$$= 9.77 \times 10^{-4} \angle 90.4°$$

$$I_S = 9.77 \times 10^{-4}\angle 90.4° \times 127 + 0.938\angle 1.2° \times 0.164\angle -36.9°$$

$$= 0.1286\angle 15.3° \text{ kA}$$

$$\text{Sending end pf} = \cos(15.3° - 6.2°)$$

$$= \cos(9.1°) = 0.987 \text{ leading}$$

$$\text{Sending end power} = \sqrt{3} \times 237.23 \times 0.1286 \times 0.987$$

$$= 52.15 \text{ MW}$$

(iv) Approximation,

$$A = D = 1 + \frac{YZ}{2} = 0.938\angle 1.2°$$

$$B = Z\left(1 + \frac{YZ}{6}\right) = Z + \frac{YZ^2}{6}$$

$$= 131.2\angle 72.3° + \frac{1}{6} \times 10^{-3}\angle 90° \times (131.2)^2\angle 144.6°$$

$$= 128.5\angle 72.7°$$

$$C = Y\left(1 + \frac{YZ}{6}\right)$$

$$= 0.001\angle 90° + \frac{1}{6} \times 10^{-6}\angle 180° \times 131.2\angle 72.3°$$

$$= 0.001\angle 90°$$

$$V_S = 0.938\angle 1.2° \times 127\angle 0° + 128.5\angle 72.7° \times 0.164\angle -36.9°$$

$$= 119.13\angle 1.2° + 21.07\angle 35.8°$$

$$= 136.2 + j14.82$$

$$= 137\angle 6.2° \text{ kV}$$

$$|V_s|_{\text{line}} = 237.3 \text{ kV}$$

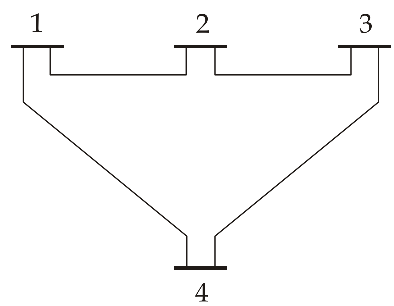
$$I_S = 0.13\angle 16.5°$$

$$\text{Sending end pf} = \cos(16.5° - 6.2°) = 0.984 \text{ leading}$$

$$\text{Sending end power} = \sqrt{3} \times 237.3 \times 0.13 \times 0.984 = 52.58 \text{ MW}$$

Q.2 (c) Solution:

(i)



Line (bus to bus)	R	X	$R + jX$
1-2	0.025	0.1	Z_{12}
2-3	0.02	0.08	Z_{23}
3-4	0.05	0.20	Z_{34}
4-1	0.04	0.16	Z_{41}

$$Y_{11} = y_{12} + y_{14} = \frac{1}{Z_{12}} + \frac{1}{Z_{41}}$$

$$y_{12} = \frac{1}{0.025 + j0.1} = 2.35 - j9.41$$

$$y_{14} = y_{41} = \frac{1}{0.04 + j0.16} = 1.47 - j5.88$$

$$Y_{11} = y_{12} + y_{41} = 2.35 - j\,9.41 + 1.47 - j\,5.88$$

$$Y_{11} = 3.82 - j\,15.29$$

$$Y_{12} = -y_{12} = -2.35 + j\,9.41$$

$$Y_{13} = 0$$

$$Y_{14} = -y_{14} = -1.47 + j\,5.88$$

$$Y_{22} = y_{12} + y_{23}$$

$$y_{23} = \frac{1}{Z_{23}} = \frac{1}{0.02 + j0.08} = 2.94 - j11.76$$

$$y_{12} = 2.35 - j\,9.41$$

$$Y_{22} = 5.29 - j\,21.17$$

$$Y_{21} = -y_{12} = -y_{21} = -2.35 + j\,9.41$$

$$Y_{23} = -y_{23} = -2.94 + j\,11.76$$

$$Y_{24} = 0$$

$$Y_{33} = y_{23} + y_{43}$$

$$y_{43} = \frac{1}{Z_{43}} = \frac{1}{0.05 + j0.2} = 1.18 - j4.71$$

$$Y_{33} = 4.12 - j\,16.46$$

$$Y_{31} = 0$$

$$Y_{32} = Y_{23} = -2.94 + j\,11.76$$

$$Y_{34} = Y_{43} = -y_{43} = -1.18 + j\,4.71$$

$$Y_{44} = y_{41} + y_{43} = 1.47 - j\,5.88 + 1.18 - j\,4.71$$

$$Y_{44} = 2.65 - j\,10.59$$

$$Y_{41} = Y_{14} = -1.47 + j\,5.88$$

$$Y_{42} = 0$$

$$Y_{43} = Y_{34} = -1.18 + j\,4.71$$

$$Y_{Bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 3.82 - j15.29 & -2.35 + j9.41 & 0 & -1.47 + j5.88 \\ -2.35 + j9.41 & 5.29 - j21.17 & -2.94 + j11.76 & 0 \\ 0 & -2.94 + j11.76 & 4.12 - j16.46 & -1.18 + j4.71 \\ -1.47 + j5.88 & 0 & -1.18 + j4.71 & 2.65 - j10.59 \end{bmatrix} \end{array}$$

(ii) When a new line is added between Bus-1 and Bus-3 following elements will be changing:

1. Y_{11}
2. Y_{33}
3. Y_{13}
4. Y_{31}

Q.3 (a) (i) Solution:

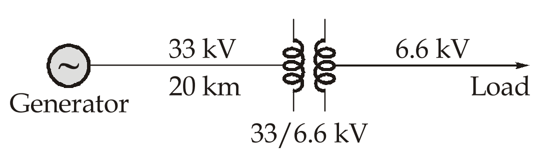


Figure shown the single diagram of the tranmission system. The voltage drop will be due to the impedance of tranmission line and also due to the impedance of transformer.

Resistance of each conductor = 20 × 0.4 = 8 Ω

Reactance of each conductor = 20 × 0.5 = 10 Ω

Let us transfer the impedance of transformer secondary to high tension side, i.e. 33 kV side

Equivalent resistance of transformer referred to 33 kV side

$$= \text{Primary resistance} + 0.35\left(\frac{33}{6.6}\right)^2$$

$$= 7.5 + 8.75 = 16.25\ \Omega$$

Equivalent reactance of transformer referred to 33 kV side

$$= \text{Primary reactance} + 0.65\left(\frac{33}{6.6}\right)^2$$

$$= 13.2 + 16.25 = 29.45\ \Omega$$

Total resistance of line and transformer

$$R = 8 + 16.25 = 24.25\ \Omega$$

Total reactance of line and transformer

$$X_L = 10 + 29.45 = 39.45\ \Omega$$

Receiving end voltage per phase

$$V_R = \frac{33000}{\sqrt{3}} = 19052\ \text{V}$$

Line current, $$I = \frac{2000\times10^3}{\sqrt{3}\times33000} = 35\ \text{A}$$

Using the approximate expression for sending end voltage V_S per phase

$$V_S = V_R + I\cos\phi_R + IX_L\sin\phi_R$$

$$= 19052 + 35 \times 24.25 \times 0.8 + 35 \times 39.45 \times 0.6$$

$$= 19052 + 679 + 828$$

$$= 20.599\ \text{kV}$$

Sending end line volgate $$= \sqrt{3}\times20.599 = 35.67\ \text{kV}$$

Sending end p.f., $$\cos\phi_s = \frac{V_R\cos\phi_R + IR}{V_S}$$

$$= \frac{19052\times0.8+35\times24.25}{20559} = 0.7826\ \text{lag}$$

Line losses $$= \frac{3I^2R}{1000}\text{kW} = \frac{3(35)^2\times24.25}{1000} = 89.12\ \text{kW}$$

Output power $$= 2000 \times 0.8 = 1600\ \text{kW}$$

∴ Transmission efficiency $$= \frac{1600}{1600+89.12}\times100 = 94.72\%$$

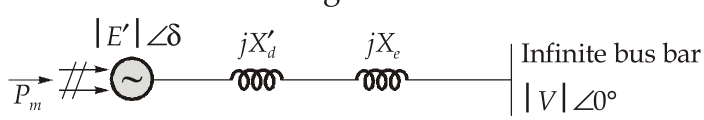
Q.3 (a) (ii) Solution:

From the given voltages,
$$I_R = \frac{V_{RN}}{R} = \frac{100\angle 0°}{R}$$
$$I_Y = \frac{V_{YN}}{jX_L} = \frac{100\angle -120°}{j10} = 10\angle{-210°}$$
$$I_B = \frac{V_{BN}}{-jX_C} = \frac{100\angle -240°}{-j10} = 10\angle{-150°}$$
For $I_N = 0$
$$I_R + I_Y + I_B = 0$$
$$\frac{100}{R} + 10\angle -210° + 10\angle -150° = 0$$
$$R = 5.77\ \Omega$$

Q.3 (b) (i) Solution:

For sudden change in mechanical input:

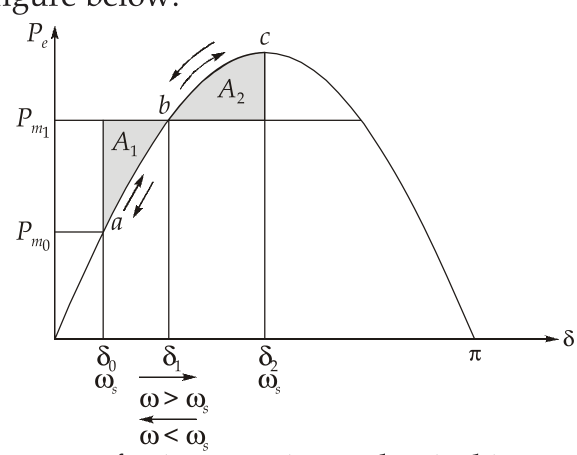
Figure shows the transient model of a single machine connected to infinite bus bar.



Single machine connected to an infinite bus-bar

The electrical power transmitted is given by
$$P_e = \frac{|E'||V|}{(X'_d + X_e)} \sin\delta = P_{max} \cdot \sin\delta$$
Under steady-state condition, $P_{m_0} = P_{e_0} = P_{max} \cdot \sin\delta$

Let the mechanical input to the rotor be suddenly increased to P_{m_1}. Due to the accelerating power $P_a = (P_{m_1} - P_e)$, rotor accelerates and its speed increases $(\omega > \omega_s)$ as a result δ also increases as shown in figure below.



Power angle curve for increase in mechanical input to generator

At angle δ_1, $P_a = (P_{m_1} - P_e) = P_{max} \sin\delta_1 = 0$ (at point *b*), but the rotor angle continues to increase as $\omega > \omega_s$. Now, P_a becomes negative (decelerating) hence, the rotor speed starts reducing but, the angle continues to increase till at angle δ_2, $\omega = \omega_s$ (at point *C*). At *C*, the decelerating area A_2 equals the accelerating area A_1 i.e. $\int_{\delta_0}^{\delta_2} P_a \cdot d\delta = 0$.

Since the rotor is deaccelerating, the speed reduces below ω_s and the rotor angle begins to reduce. The state point now traverses the P_e - δ curve in opposite direction as shown in figure above. The system oscillates about the new steady-state point *b* ($\delta = \delta_1$) with angle upto δ_0 and δ_2 on the two sides. When the oscillations decay out, the system settles to the new steady state where

$$P_{m_1} = P_e = P_{max} \sin\delta_1$$

Now, using figure, the two areas are

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m_1} - P_e) \cdot d\delta$$

and

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m_1}) \cdot d\delta$$

For the system to be stable, using equal area criterion, we have

$$A_1 = A_2$$

At point *b* and *c* of figure, powers are same, therefore,

$$P_{m_1} \sin\delta_2 = P_{m_1} \sin\delta_1 \text{ or } \sin\delta_2 = \sin\delta_1$$

or,

$$\delta_2 = (\pi - \delta_1) \qquad \text{...(i)}$$

Also, at point *b*, we have

$$P_{m_1} = P_m \sin\delta_1$$

or

$$\delta_1 = \sin^{-1}\left(\frac{P_{m_1}}{P_m}\right) \qquad \text{...(ii)}$$

From equation (i) and (ii), we have

$$\delta_2 = \pi - \sin^{-1}\left(\frac{P_{m_1}}{P_m}\right) \qquad \text{...(iii)}$$

$$\int_{\delta_0}^{\delta_1} (P_{m_1} - P_{max} \sin\delta)\, d\delta = \int_{\delta_1}^{\delta_2} (P_{max} \sin\delta - P_{m_1})\, d\delta$$

$$P_{m_1}[\delta_1-\delta_0]-P_{max}[\cos\delta_0-\cos\delta_1] = P_{max}[\cos\delta_1-\cos\delta_2-P_{m_1}[\delta_2-\delta_1]$$

$$P_{m_1}[\delta_2-\delta_0] = P_{max}[\cos\delta_0 - \cos\delta_2]$$

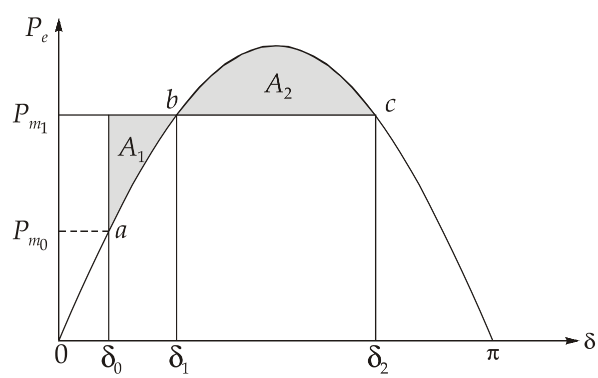
$$P_{m_1}[\pi-\delta_1-\delta_0] = P_{max}[\cos\delta_0 + \cos\delta_1]$$

$$\frac{P_{m_1}}{P_{max}}[\pi-\delta_1-\delta_0] = \cos\delta_0 + \cos\delta_1$$

$$[\pi - \delta_1 - \delta_0] \sin \delta_1 = \cos\delta_0 + \cos\delta_1$$

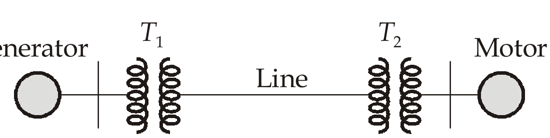
$$\cos \delta_1 = [\pi - \delta_1 - \delta_0] \sin\delta_1 - \cos\delta_0$$

Now, any further increase in P_{m1} means that the area available for A_2 is less than A_1, so that the excess kinetic energy causes δ to increase beyond point c and the decelerating power changes over to accelerating power and hence, the system becomes unstable. The system will remain stable even though the rotor oscillate beyond $\delta = 90°$, as long as the equal area criterion is met. The oscillations for $\delta = 90°$ is shown in figure below which is a stable system as A_2 (decelerating area) is greater than A_1 (accelerating area).



Transient stability with sudden increase in mechanical input for $\delta > 90°$

Q.3 (b) (ii) Solution:



Given :Gen, Motor : 25 MVA, 11 kV, $X'' = 15\%$

T/F : $X = 10\%$

Line : 25 MVA, 66 kV, $X = 10\%$

$$P_{3\phi} = 15 \text{ MW, pf} = 0.8 \text{ lead,}$$

$$P_{pv} = \frac{15 \text{ MW}}{25 \text{ MVA}} = 0.6 \text{ p.u.}$$

Terminal voltage of motor = 10.6 kV

Let base voltage at generator side = 11 kV

Base MVA = 25 MVA

Prefault Conditions :

$$\text{Terminal voltage of the motor in p.u.} = \frac{10.6}{11}$$

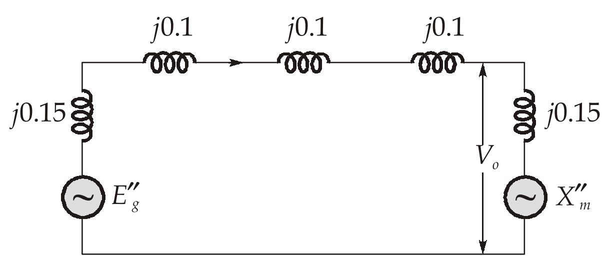
$$V_o = 0.963 \text{ p.u.}$$

$$(P_{\text{p.u.}}) = V_{\text{p.u.}} \times I_{\text{p.u.}} \times \cos\phi$$

$$\Rightarrow \quad \frac{15}{25} = 0.963 \times I_{pu} \times 0.8$$

$$I_{pu} = 0.778\angle 36.86$$

Equivalent circuit :



$$E''_g = V_o + I_{pu}(X_{T1} + V_{T2} + X_L + X''_g)$$

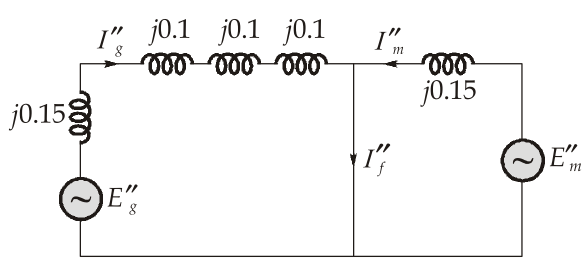
$$= 0.963\angle 0° + 0.778\angle 36.86\ (j0.1 + j0.1 + j0.1 + j0.15)$$

$$= 0.8034\angle 20.41°$$

$$E''_M = V_o - I_{pu}(X''_M)$$

$$= 0.963\angle 0° - 0.778\angle 36.86\ (j0.15) = 1.03\angle -5.16°$$

At fault :



$$I''_g = \frac{E''_g}{j0.15 + j0.1 + j0.1 + j0.1}$$

$$= \frac{0.8034\angle 20.41°}{0.45\angle 90°} = 1.785\angle -69.59° \text{ p.u.}$$

$$I''_{g\ (\text{actual})} = 1.785\angle -69.58 \times \frac{25}{\sqrt{3} \times 11} \text{ kA}$$

$$I''_{g\,(\text{actual})} = 2.34\angle{-69.58}\text{ kA}$$

$$I''_m = \frac{E''_m}{j0.15} = \frac{1.03\angle -5.16}{j0.15} = 6.87\angle{-85.6°}\text{ pu}$$

$$I''_{m\,(\text{actual})} = 6.87\angle -95.16 \times \frac{25}{\sqrt{3}\times 11}\text{ kA}$$

$$I''_{m\,(\text{actual})} = 9.01\angle{-95.16}\text{ kA}$$

$$I''_f = I''_g + I''_m$$

$$= 1.785\angle{-69.59°} + 6.87\angle{-95.16°}\text{ pu} = 8.51\angle{-89.96°}$$

$$I''_{f\,(\text{actual})} = 8.51\angle -89.96 \times \frac{25}{\sqrt{3}\times 11}\text{ kA}$$

$$I''_{f\,(\text{actual})} = 11.17\angle{-89.96}\text{ kA}$$

Subtransient current in generator = 2.34 kA

Subtransient current in motor = 9.01 kA

Subtransient fault current = 11.17 kA

Q.3 (c) Solution:

The Y_{bus} matrix of system can be assembled as

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$Y_{11} = \text{Sum of all the admittances connected to bus-1}$$

$$= y_{12} + y_{13} = -j5 - j2.5 = -j7.5\text{ pu}$$

$$Y_{12} = -y_{12} = j2.5 = Y_{21}$$

$$Y_{31} = Y_{13} = -y_{13} = j5$$

$$Y_{22} = y_{12} + y_{23} = -j4 - j2.5 = -j6.5$$

$$Y_{23} = -y_{23} = j4$$

$$Y_{33} = y_{31} + y_{32} = -j5 - j4 = -j9$$

Therefore,

$$Y_{bus} = j\begin{bmatrix} -7.5 & 2.5 & 5.0 \\ 2.5 & -6.5 & 4.0 \\ 5.0 & 4.0 & -9.0 \end{bmatrix}$$

Iteration 1 :

$$I_2^0 = \frac{S_2^*}{(V_2^0)^*} = \frac{P_2 - jQ_2}{(V_2^0)^*} = \frac{-1 + j0.8}{1.0 - j0} = -1 + j0.8$$

Now, $$V_2^{(1)} = \frac{I_2^0 - (Y_{21}V_1 + Y_{23}V_3^0)}{Y_{22}}$$

$$\left[\because V_i^{K+1} = \frac{1}{Y_{11}}\left\{\frac{P_i - jQ_i}{(V_i^K)^*} - \sum_{m=1}^{i-1} Y_{im}V_m^{K-1} - \sum_{m=i+1}^{n} Y_{im}V_m^K\right\}\right]$$

$$= \frac{-1 + j0.8 + (j2.5 \times 1.04 + j4 \times 1.005)}{-j6.5}$$

$$V_2^{(1)} = (0.8953 - j0.1538) \text{ pu} = 0.9084\angle{-9.75°} \text{ pu}$$

Since, bus-3 is a voltage-controlled bus, real power P and magnitude of bus voltage are specified. Now, reactive power Q_3 at bus-3 can be computed as

$$Q_3^{(1)} = Im[V_3^{(0)} \times I_3^*]$$

$$= Im[1.005 \times (Y_{31}V_1^{(0)} + Y_{32}V_2^{(1)} + Y_{33}V_3^{(0)})^*]$$

$$= Im[(1.005) * \{(j5 \times 1.04) + j4 \times (0.8954 - j0.1538) + (-j9.0) \times (1.005)\}^*]$$

$$Q_3^{(1)} = 0.2647$$

$$I_3^{(0)} = \frac{S_3^*}{(V_3^0)^*} = \frac{1 - j0.2647}{1.005} = 0.9954 - j0.2633$$

$$V_3^{(1)} = \frac{[I_3^* - (Y_{31}V_1^{(1)} + Y_{32}V_2^{(1)})]}{Y_{33}}$$

Since, bus-1 is slack bus, so its voltage remains unchanged.

$$= \frac{(0.9950 - j0.2635) - [j5 \times (1.04 + j0) + j4.0(0.8954 - j0.1538)]}{-j9.0}$$

$$V_3^{(1)} = 1.0050 + j0.0422$$

Since, the voltage magnitude at bus-3 is held constant, the real part of $V_3^{(1)}$ is modified as

$$e_3^{(1)} = \sqrt{(1.005)^2 - (0.0422)^2} = 1.0041$$

Hence, $$V_3^{(1)} = 1.0041 + j0.0422 = 1.005\angle 2.41° \text{ pu}$$

Q.4 (a) (i) Solution:

Length of cable, $l = 1 \text{ km} = 1000 \text{ m}$

Cable insulation resistance, $R = 495\ \text{M}\Omega = 495 \times 10^6\ \Omega$

Conductor radius, $r_1 = \frac{2.5}{2} = 1.25 \text{ cm}$

Resistivity of insulation, $\rho = 4.5 \times 10^{14}\ \Omega\text{-cm} = 4.5 \times 10^{12}\ \Omega\text{-m}$

Let r_2 cm be the internal sheath radius,

Now, $$R = \frac{\rho}{2\pi l}\log_{10}\frac{r_2}{r_1}$$

$$\ln\frac{r_2}{r_1} = \frac{2\pi lR}{\rho} = \frac{2\pi\times 1000\times 495\times 10^6}{4.5\times 10^{12}} = 0.69$$

$$2.3\log_{10}\frac{r_2}{r_1} = 0.69$$

$$\frac{r_2}{r_1} = e^{\frac{0.69}{2.3}} = 2$$

$$r_2 = 2r_1 = 2\times 1.25 = 2.5 \text{ cm}$$

$$\text{Insulation thickness} = r_2 - r_1 = 2.5 - 1.25$$
$$= 1.25 \text{ cm}$$

Q.4 (a) (ii) Solution:

$$S = 250 \text{ MVA},$$
$$\cos\phi = 0.8$$
$$\text{K.E.} = 1000 \text{ MJ}$$
$$P_e = 60 \text{ MW}$$
$$\delta_0 = 10°$$
$$t = 10 \text{ cycles} = \frac{10}{50} = 0.2 \text{ sec}$$
$$t = 5 \text{ cycles} \Rightarrow 0.1 \text{ sec}$$

Load is removed, $$P_e = 0$$
$$P_a = P_m - P_e$$

$\Rightarrow$ $$P_m = 60 \text{ MW},$$

$$M = \frac{HS}{180f} = \frac{\text{K.E.}}{180f} = \frac{1000}{180\times 50} = 0.111$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = 545.45 \text{ ele.degree/s}^2$$

Integrating the above equation, we get

$$\int\left(\frac{d^2\delta}{dt^2}\right)dt = \int\frac{P_a}{M}dt = 545.45\times t$$

Integrating it once again with dt

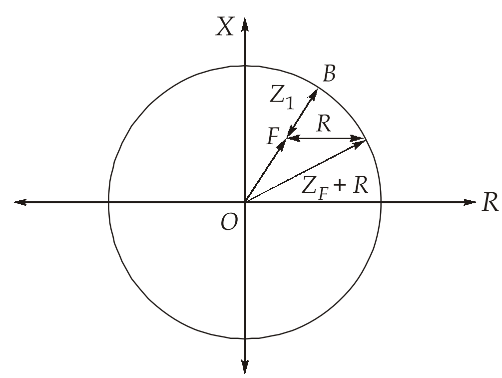
$$= \int 545.45 \times t\,(dt)$$

$$\delta = \frac{545.45 \times t^2}{2} = 2.7°$$

So, new value of power angle = 10° + 2.7° = 12.7°

Q.4 (b) (i) Solution:

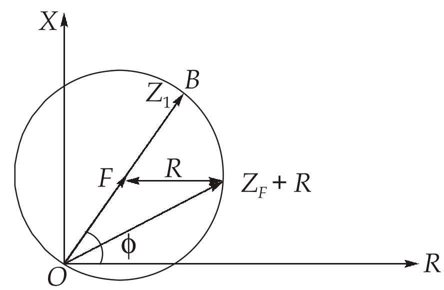
For multiphase fault or phase to ground fault an arc resistance is introduced in the fault path. In phase to ground path arc resistance is in series with tower footing resistance. The arc resistance is appreciable at higher voltages. Arc resistance is added in the impedance seen by the distance relay. Arc resistance and earth resistance collectively known as the fault resistance. In case of phase to phase fault, fault resistance consists of arc resistance only as there is no earth resistance in this case. Figure shows the effect of arc resistance on impedance relay.



The relay has been set to protect a line of impedance Z_{12}. If fault occurs at point F and an arc resistance R is introduced, the relay will measure $(Z_F + R)$. Z_F is the impedance of line upto point F. If value of arc resistance is greater than R, the impedance measured by relay will be greater than the radius of the circle and relay will fail to operate. Thus with arc resistance, the relay just operates. The maximum length of line which can be protected is OF with arc resistance 'R'. It may be seen that arc resistance cause the relay to under reach.

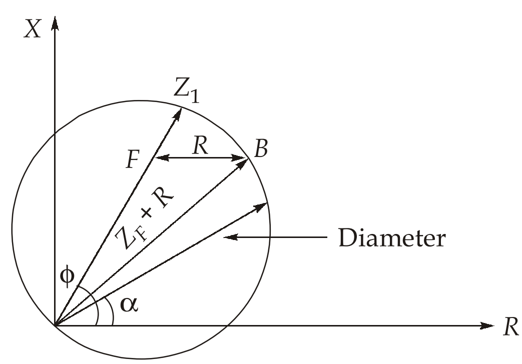
$$\text{Percentage under reach} = \frac{\text{BF}}{\text{OB}} \times 100\%$$

Similarly it can be seen that a line protected by MHO relay will have problem of under reach due to arc resistance. It can be seen from figure as under.



$$\text{Percentage under reach} = \frac{\text{BF}}{\text{OB}} \times 100\%$$

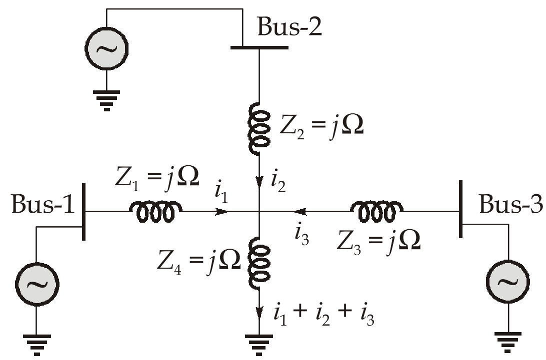
However the protection of under reach can be solved by shifting the MHO circle towards R-axis by making the characteristic angle of MHO circle α less than the characteristic angle of the line ϕ. The resulting characteristic will tolerate a greater value of arc resistance. Figure below shows the new characteristic:



MHO circle shift towards R-axis

In the case $Z_F + R$ may even be greater than Z_1, but less than the diameter of the circle. In such a case, the relay will operate so long as the point $Z_F + R$ remains within the characteristic circle. The maximum length of line that can be protected is Z_1 = OB cos (ϕ – α), where OB is diameter of the circle. The inclination of relay characteristic towards R-axis for lower voltage level is more and such characteristic have a greater tolerance for arc resistance. Arc resistance affects the performance of different types of relay to different extent. MHO relays is most affected, impedance relay is moderately affected and reactance relay is least affected by arc resistance.

Q.4 (b) (ii) Solution:



By KVL;
$$V_1 = j1i_1 + j1(i_1 + i_2 + i_3)$$
$$V_1 = 2ji_1 + ji_2 + ji_3$$
and
$$V_2 = ji_1 + 2ji_2 + ji_3$$
$$V_3 = ji_1 + ji_2 + 2ji_3$$

∴
$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2j & j & j \\ j & 2j & j \\ j & j & 2j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} 2j & j & j \\ j & 2j & j \\ j & j & 2j \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} \frac{-3j}{4} & \frac{j}{4} & \frac{j}{4} \\ \frac{j}{4} & \frac{-3j}{4} & \frac{j}{4} \\ \frac{j}{4} & \frac{j}{4} & \frac{-3j}{4} \end{bmatrix} \Omega$$

Q.4 (c) Solution:

Let current in section *AB* be $(x + jy)$

Current in section *BC*,

$$\bar{I}_{BC} = (x + jy) - 50\,(0.8 - j0.6)$$
$$= (x - 40) + j(y + 30)$$

Current in section *CD*,

$$\bar{I}_{CD} = [(x - 40) + j(y + 30)] - [120 + j0]$$
$$= (x - 160) + (y + 30)$$

Current in section DA,

$$\bar{I}_{DA} = [(x - 160) + j(y + 30)] - [70(0.866 - j0.5)]$$
$$= (x - 220.6) + j(y + 65)$$

Drop in section AB,

$$= \bar{I}_{AB}\bar{Z}_{AB} = (x + jy)(1 + j0.6)$$
$$= (x - 0.6y) + j(0.6x + y)$$

Drop in section BC,

$$= \bar{I}_{BC}\bar{Z}_{BC}$$
$$= [(x - 40) + j(y + 30)][(1.2 + j0.9)]$$
$$= (1.2x - 0.9y - 75) + j(0.9x + 1.2y)$$

Drop in section CD,

$$= \bar{I}_{CD}\bar{Z}_{CD}$$
$$= [(x - 160) + j(y + 30)][(0.8 + j0.5)]$$
$$= (0.8x - 0.5y - 143) + j(0.5x + 0.8y - 56)$$

Drop in section DA,

$$= \bar{I}_{DA}\bar{Z}_{DA}$$
$$= [(x - 220.6) + j(y + 65)][(3 + j2)]$$
$$= (3x - 2y - 791.8) + j(2x + 3y - 246.2)$$

Applying Kirchhoff's voltage low to mesh $ABCDA$,

We have

Drop in AB + Drop in BC + Drop in CD + Drop in DA = 0

$$= [(x + 0.6y) + j(0.6x + y) + [(1.2x - 0.9y - 75) + (0.9x + 1.2y)]$$
$$+ [(0.8x - 0.5y - 143) + j(0.5x + 0.8y - 56)]$$
$$+ [(3x - 2y - 791.8) + j(2x + 3y - 246.2)$$
$$= 0$$

$$(6x - 4y - 1009.8) + j(4x + 6y - 302.2) = 0$$

$\therefore$ $6x - 4y - 1009.8 = 0$

and $4x + 6y - 302.2 = 0$

Solving for x and y, we have

$$x = 139.7 \text{ A};$$
$$y = -42.8 \text{ A}$$

Current in section AB = $(139.7 - j42.8)$A

Current in section BC = $(x - 40) + j(y + 30) = (99.7 - j12.8)$A

$$\text{Current in section } CD = (-20.3 - j12.8)\text{A}$$

$$\text{Current in section } DA = (x - 220.6) + j(y + 65) = (-80.9 + j22.2)\text{A}$$

Voltage at supply end A,

$$V_A = \frac{11000}{\sqrt{3}} = 6351 \text{ V/phase}$$

Voltage at section B,

$$\vec{V}_B = \vec{V}_A - \vec{I}_{AB}\vec{Z}_{AB}$$

$$= (6351 + j0) - (139.7 - j42.8)(1 + j0.6)$$

$$= (6185.62 - j41.02) \text{ volts/phase}$$

Voltage at section C,

$$\vec{V}_C = \vec{V}_B - \vec{I}_{BC}\vec{Z}_{BC}$$

$$= (6185.62 - j0) - (99.7 - j12.8)(1.2 + j0.9)$$

$$= (6054.46 - j115.39) \text{ volts/phase}$$

Voltage at station D,

$$\vec{V}_D = \vec{V}_C - \vec{I}_{CD}\vec{Z}_{CD}$$

$$= (6054.46 - j115.39) - (-20.3 - j12.8) \times (0.8 + j0.5)$$

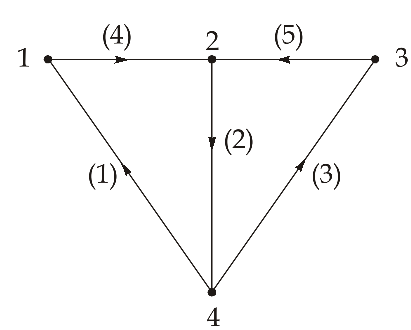
$$= (6064.3 - j95) \text{ volts/phase}$$

Section B : Digital Electronics-1 + Microprocessor-1 + Electrical Circuits-2 + Systems and Signal Processing -2

Q.5 (a) Solution:

(i) Graph of the network:

Considering the current sources as open circuit, the graph of the given circuit can be drawn as below:

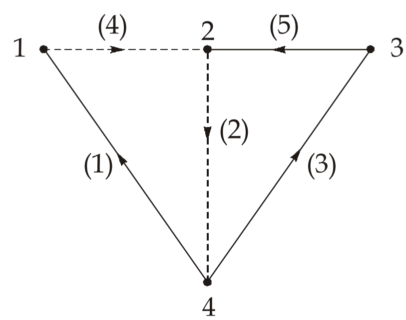


The graph has 4 nodes and 5 branches. Assume the orientations of the branches as shown in the graph.

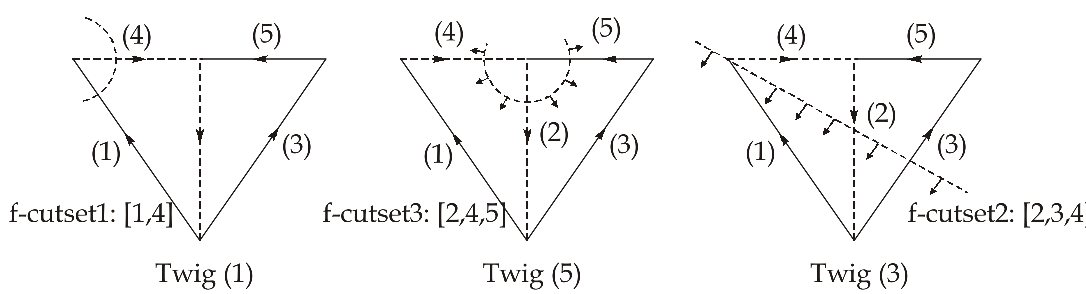
The incidence matrix is given by

$$A = \begin{array}{c} \text{Nodes} \\ \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \text{Branches} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Let us select a tree as shown below with branches 1, 3 and 5 as twigs.



For each twig, the cut--sets are obtained as below:



The cut-set matrix is obtained as follows:

$$Q_f = \begin{array}{c} \text{f-cutset} \\ \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \text{Branches} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

(ii) The reduced incidence matrix can be written as

$$[A_a] = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

The number of possible trees is given by

$$N = \det\{[A_a]\,[A_a]^T\}$$

$$N = \det\left\{\begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\right\}$$

$$N = \det\left\{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}\right\} = 8$$

$\therefore$ The number of possible trees is 8.

Q.5 (b) Solution:

(i)

$$\left(\frac{1}{2}\right)^n u[n] \xleftrightarrow{\text{Z.T.}} \frac{z}{z-\frac{1}{2}} \qquad |z| > \frac{1}{2} \qquad \text{...(i)}$$

$$\left(\frac{1}{3}\right)^n u[n] \xleftrightarrow{\text{Z.T.}} \frac{z}{z-\frac{1}{3}} \qquad |z| > \frac{1}{3} \qquad \text{...(ii)}$$

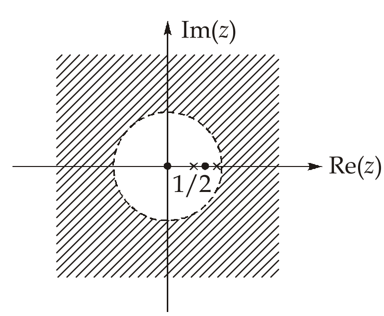
The ROCs in eqns. (i) and (ii) overlap, and resultant ROC:

$$|z| > \frac{1}{2}$$

Thus,

$$X(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-\frac{1}{3}} = \frac{2z\left(z-\frac{5}{12}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}; \qquad |z| > \frac{1}{2} \qquad \text{...(iii)}$$

From eqn. (iii) we see that $X(z)$ has two zeros at $z = 0$ and $z = \frac{5}{12}$ and two poles at $z = \frac{1}{2}$ and $z = \frac{1}{3}$ and that the ROC is $|z| > \frac{1}{2}$.



ROC (i)

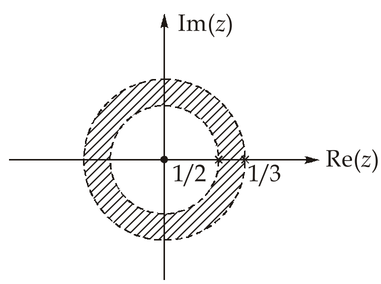
(ii)
$$\left(\frac{1}{3}\right)^n u[n] \leftrightarrow \frac{z}{z-\frac{1}{3}} \qquad |z| > \frac{1}{3} \qquad \text{...(iv)}$$

$$\left(\frac{1}{2}\right)^n u[-n-1] \leftrightarrow \frac{-z}{z-\frac{1}{2}} \qquad |z| < \frac{1}{2} \qquad \text{...(v)}$$

The ROCs in eqn. (iv) and (v) overlap, and thus

$$X(z) = \frac{z}{z-\frac{1}{3}} - \frac{z}{z-\frac{1}{2}} = -\frac{1}{6}\frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}; \quad \frac{1}{3} < |z| < \frac{1}{2} \qquad \text{...(vi)}$$

From eqn. (vi), we see that $X(z)$ has one zero at $z = 0$ and two poles at $z = \frac{1}{2}$ and $z = \frac{1}{3}$ and that the ROC is $\frac{1}{3} < |z| < \frac{1}{2}$.



ROC (ii)

(iii)
$$\left(\frac{1}{2}\right)^n u[n] \leftrightarrow \frac{z}{z-\frac{1}{2}} \qquad |z| > \frac{1}{2} \qquad \text{...(vii)}$$

$$\left(\frac{1}{3}\right)^n u[-n-1] \leftrightarrow -\frac{z}{z-\frac{1}{3}} \qquad |z| < \frac{1}{3} \qquad \text{...(viii)}$$

The ROCs in eqn. (vii) and (viii) do not overlap and hence, there is no common ROC, and thus $x[n]$ will not have $X(z)$.

Q.5 (c) Solution:

$$y(n+2) - \frac{3}{4}y(n+1) + \frac{1}{8}y(n) = x(n+2) + x(n+1)$$

Expressing the given equation in delay operator we have,

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + x(n-1)$$

Now $\qquad x(n) = u(n)$

$$\therefore \qquad X(z) = \frac{1}{1-z^{-1}}$$

(i) Calculation of ZIR:

$\therefore$ Consider $x(n) = 0$

$$\therefore \ Y(z) - \frac{3}{4}\left(z^{-1}Y(z) + y(-1)\right) + \frac{1}{8}\left(z^{-2}Y(z) + z^{-1}y(-1) + y(-2)\right) = 0$$

$$\therefore \qquad \left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right)Y(z) = \frac{1}{8}$$

$$\therefore \qquad Y(z) = \frac{\frac{1}{8}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{1}{8} \times \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$= \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 - \frac{1}{4}z^{-1}}$$

$$\therefore \qquad Y(z) = \frac{1}{4\left(1 - \frac{1}{2}z^{-1}\right)} - \frac{1}{8\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$\therefore \qquad y(n) = \frac{1}{4}\left(\frac{1}{2}\right)^n u(n) - \frac{1}{8}\left(\frac{1}{4}\right)^n u(n)$$

This is zero input response of system

(ii) Calculation of ZSR :

So consider initial conditions to be zero

$$\text{and} \qquad X(z) = \frac{1}{1-z^{-1}}$$

$$\therefore \ Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z) + z^{-1}X(z)$$

$$\therefore \qquad \left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right)Y(z) = (1 + z^{-1})\,X(z)$$

$$\therefore \qquad Y(z) = \frac{1 + z^{-1}}{\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-1}\right)} \cdot X(z)$$

$$= \frac{1 + z^{-1}}{(1 - z^{-1})\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)}$$

$$= \frac{A}{1-z^{-1}} + \frac{B}{1-\frac{1}{4}z^{-1}} + \frac{C}{1-\frac{1}{2}z^{-1}}$$

$$\therefore \quad Y(z) = \frac{\frac{16}{3}}{1-z^{-1}} + \frac{\frac{5}{3}}{1-\frac{1}{4}z^{-1}} + \frac{-6}{1-\frac{1}{2}z^{-1}}$$

$$\therefore \quad y(n) = \frac{16}{3}u(n) + \frac{5}{3}\left(\frac{1}{4}\right)^n u(n) - 6\left(\frac{1}{2}\right)^n u(n)$$

This is zero state response of the system.

Q.5 (d) Solution:

Assuming octal inputs as D_7 to D_0 with D_7 having highest priority and outputs as x, y, z and V where $(xyz)_2$ is a binary equivalent of highest priority input available and V is the valid bit indicator.

The truth table of this encoder can be written as follows :

D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	x	y	z	V
0	0	0	0	0	0	0	0	×	×	×	0
1	0	0	0	0	0	0	0	0	0	0	1
×	1	0	0	0	0	0	0	0	0	1	1
×	×	1	0	0	0	0	0	0	1	0	1
×	×	×	1	0	0	0	0	0	1	1	1
×	×	×	×	1	0	0	0	1	0	0	1
×	×	×	×	×	1	0	0	1	0	1	1
×	×	×	×	×	×	1	0	1	1	0	1
×	×	×	×	×	×	×	1	1	1	1	1

If $D_2 = D_6 = 1$, then Output =

x	y	z	V
1	1	0	1

Q.5 (e) Solution:

```
START:   LXI H, XX50H   ; Set up HL as a memory pointer for bytes
         MVI D, 00      ; Clear register D to set up a flag
         MVI C, 02      ; Set register C for comparison count
CHECK:   MOV A, M       ; Get data byte
         INX H          ; Point to next byte
         CMP M          ; Compare bytes
```

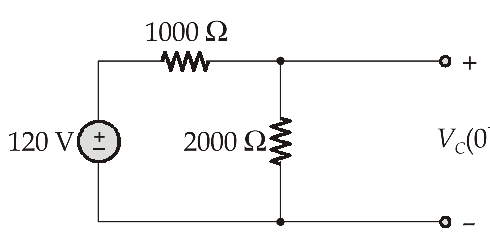
```
            JC NXTBYT     ; If (A) < second byte, do not exchange
            MOV B, M      ; Get second byte for exchange
            MOV M, A      ; Store first byte in second location
            DCX H         ; Point to first location
            MOV M, B      ; Store second byte in first location
            INX H         ; Get ready for next comparison
            MVI D, 01     ; Load 1 in D as a reminder for exchange
NXTBYT:     DCR C         ; Decrement comparison count
            JNZ CHECK     ; If comparison count ≠ 0, go back
            MOV A, D      ; Get flag bit in A
            RRC           ; Place flag bit D0 in Carry
            JC START      ; If flag is 1, exchange occurred
                          ; Start the next pass
            HLT           ; End of sorting
```

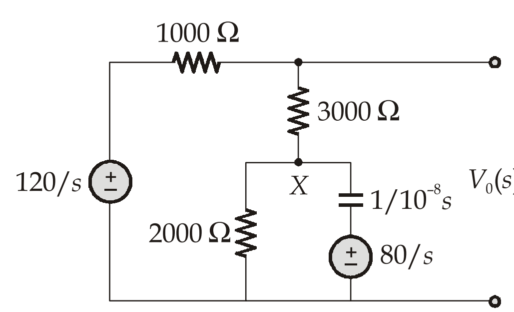
Q.6 (a) Solution:

With the switch closed, the initial voltage across the capacitor is

$$v_c(0^-) = \frac{120}{1000 + 2000} \times 2000 = 80 \text{ V}$$

After the switch is opened, the transformed circuit is shown in figure,





Applying KCL at node X, we get

$$\frac{V_X(s) - \frac{120}{s}}{4000} + \frac{V_X}{2000} + \frac{V_X - \frac{80}{s}}{\frac{1}{10^{-8}s}} = 0$$

$$\Rightarrow \quad V_X\left[\frac{1}{4000} + \frac{1}{2000} + 10^{-8}s\right] = \frac{120}{4000s} + 80 \times 100^{-8}$$

$$= \frac{0.03}{s} + 80 \times 100^{-8}$$

$$\Rightarrow \quad V_X = \frac{0.03}{s(0.00075 + 10^{-8}s)} + \frac{80 \times 100^{-8}}{0.00075 + 10^{-8}s}$$

$$= \frac{40}{s} - \frac{40}{s + 0.075 \times 10^6} + \frac{80}{s + 0.075 \times 10^{-6}}$$

$$= \frac{40}{s} + \frac{40}{s + 0.075 \times 10^6}$$

Taking inverse-Laplace transform,

$$V_X(t) = 40 + 40e^{-0.075 \times 10^6 t}$$

Therefore, the desired voltage is

$$V_0(t) = V_X(t) + \frac{120 - V_X(t)}{4000} \times 3000$$

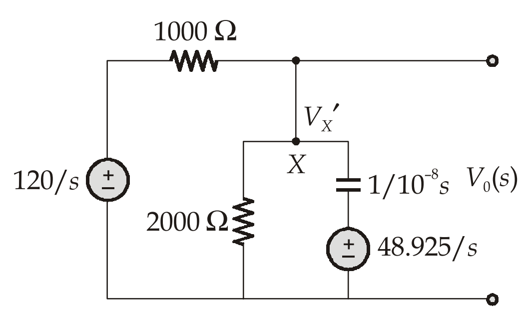
$$= 100 + 10e^{-0.075 \times 10^6 t} \qquad \text{for } 0 \le t \le 20 \ \mu s$$

At t = 20 µs, the voltage of node X is

$$V_X = 40 + 40e^{-0.075 \times 10^6 \times 20 \times 10^{-6}}$$

$$= 48.925 \text{ V}$$

When the switch is reclosed at t = 20 µs, the voltage across the capacitor will be 48.925 V. After reclosing the switch, the transformed circuit is shown in figure,



Now let the voltage of node X be V_X'

Applying KCL at node X, we get

$$\frac{V_X'(s) - \frac{120}{s}}{1000} + \frac{V_X'(s)}{2000} + \frac{V_X'(s) - \frac{48.925}{s}}{\frac{1}{10^{-8}s}} = 0$$

$$V_X'(s)\left[\frac{1}{1000}+\frac{1}{2000}+10^{-8}s\right] = \frac{120}{1000s}+48.925\times10^{-8}$$

$$\Rightarrow \quad V_X'(s)\left[0.0015+10^{-8}s\right] = \frac{0.12}{s}+48.928\times10^{-8}$$

$$\Rightarrow \quad V_X'(s) = \frac{0.12}{s(0.0015+10^{-8}s)}+\frac{48.925\times10^{-8}}{0.0015+10^{-8}s}$$

$$= \frac{0.12}{s(s+0.15\times10^{6})}+\frac{48.925}{s+0.15\times10^{6}}$$

$$\Rightarrow \quad V_X'(s) = \frac{80}{s}-\frac{31.075}{s+0.15\times10^{6}}$$

Taking inverse Laplace transform, we get $V_X'(t) = 80-31.075e^{-0.15\times10^{6}t}$

In this case, the output voltage V_0 is equal $V_X'(t)$. Since time t is to be counted from the instant the switch is reclosed, t is replaced by $(t - 20\times10^{-6})$

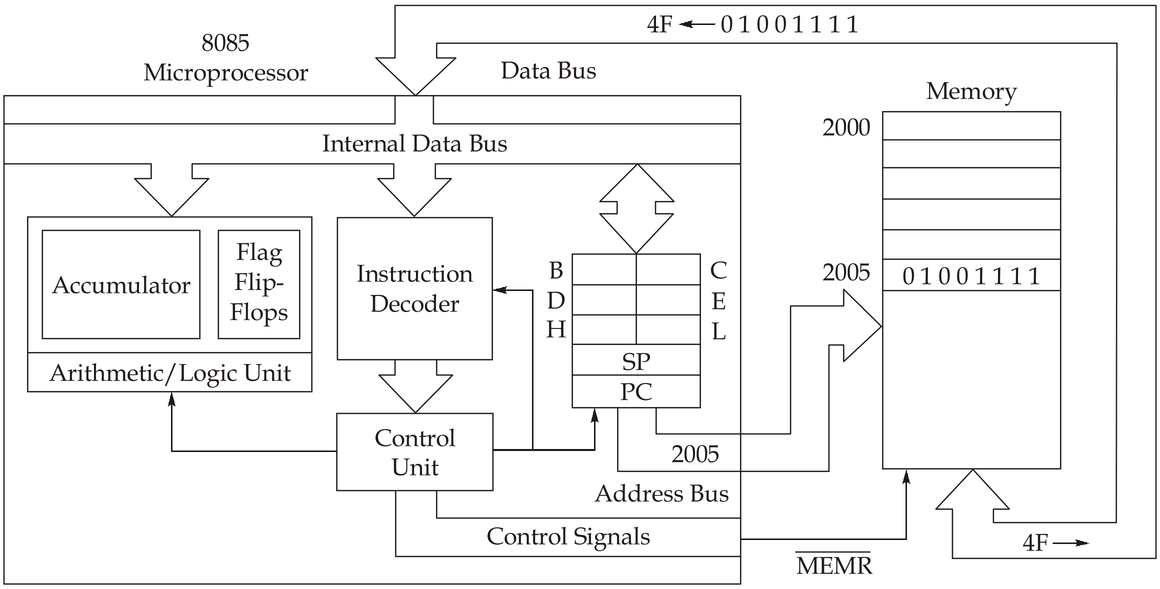
$$\therefore \quad V_0(t) = 80-31.075e^{-0.15\times10^{6}(t-20\times10^{-6})} \quad \text{for } t > 20\ \mu\text{s}$$

Q.6 (b) Solution:

To fetch the instruction located in memory location 2005H, the following steps are performed :

1. The program counter places the 16-bit address 2005H of the memory location on the address bus.
2. The control unit sends the Memory Read control signal ($\overline{\text{MEMR}}$, active low) to enable the output buffer of the memory chip.
3. The instruction (4FH) stored in the memory location is placed on the data bus and transferred (copied) to the instruction decoder of the microprocessor.
4. The instruction is decoded and executed according to the binary pattern of the instruction.

Figure shows how the 8085 microprocessor fetches the instruction using the address, the data, and the control buses.



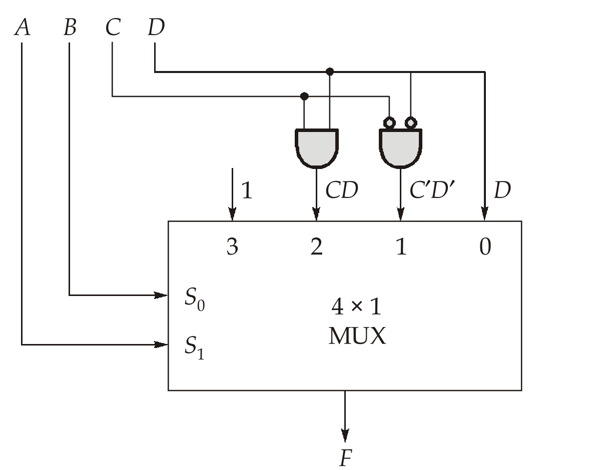
Q.6 (c) (i) Solution:

Truth table for the given expression :

$$F(A, B, C, D) = \Sigma(1, 3, 4, 11, 12, 13, 14, 15)$$

Dec	A (Select Lines)	B (Select Lines)	C (Inputs)	D (Inputs)	F (Output)
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

Dec	Selection Lines		Inputs		Output	
	A	*B*	*C*	*D*	*F*	
0	0	0	0	0	0	$F = D$
1	0	0	0	1	1	
2	0	0	1	0	0	
3	0	0	1	1	1	
4	0	1	0	0	1	$F = C'D'$
5	0	1	0	1	0	
6	0	1	1	0	0	
7	0	1	1	1	0	
8	1	0	0	0	0	$F = CD$
9	1	0	0	1	0	
10	1	0	1	0	0	
11	1	0	1	1	1	
12	1	1	0	0	1	$F = 1$
13	1	1	0	1	1	
14	1	1	1	0	1	
15	1	1	1	1	1	



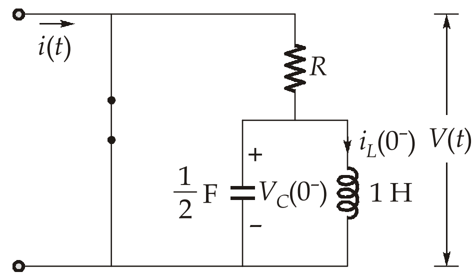
Q.6 (c) (ii) Solution:

```
        LXI  H, 2501 H
             MVI    B, 00
             MOV    A, M
             CMA
             ADI    01H
             STA    2503H
             JNC    GO
             INR    B
Go :         INX    H
```

```
MOV   A, M
CMA
ADD   B
STA   2504H
HLT
```

Q.7 (a) Solution:

Given, circuit has zero initial energy. At $t = 0^-$, switch is closed.

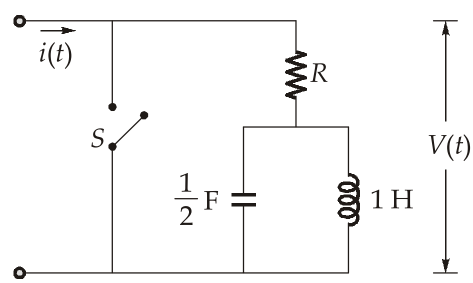


$$\therefore \quad i_L(0^-) = 0$$
$$V_C(0^-) = 0$$

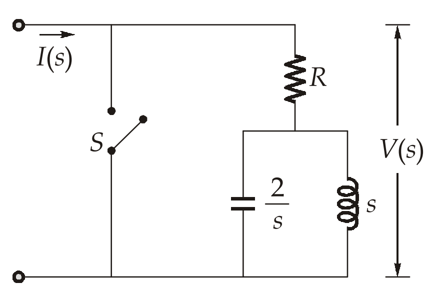
Since inductor current and capacitor voltage cannot change instantaneously.

$$i_L(0^-) = 0 = i_L(0^+)$$
$$V_C(0^-) = V_C(0^+) = 0$$

At $t > 0$, switch is opened.



by transforming above circuit into Laplace domain,



$$V(s) = I(s) \times Z(s)$$

where, $$Z(s) = R + s \parallel \frac{2}{s} = R + \frac{s \times \frac{2}{s}}{s + \frac{2}{s}} = R + \frac{2s}{s^2 + 2}$$

$$\frac{V(s)}{I(s)} = R + \frac{2s}{s^2 + 2}$$

where, $$V(s) = \frac{0.5 \times (\sqrt{2})}{s^2 + (\sqrt{2})^2} = \frac{0.5\sqrt{2}}{s^2 + 2} \quad \text{as } V(t) = 0.5 \sin \sqrt{2}t \, u(t)$$

$$I(s) = \frac{1}{(s + \sqrt{2})^2} = \frac{1}{s^2 + 2\sqrt{2}s + 2} \quad \text{as } i(t) = te^{-\sqrt{2}t} u(t)$$

∴ $$\frac{V(s)}{I(s)} = \frac{\frac{0.5\sqrt{2}}{s^2 + 2}}{\frac{1}{s^2 + 2\sqrt{2}s + 2}} = R + \frac{2s}{s^2 + 2}$$

$$0.5\sqrt{2}(s^2 + 2\sqrt{2}s + 2) = s^2R + 2s + 2R$$

$$0.5\sqrt{2}s^2 + 2s + \sqrt{2} = s^2R + 2s + 2R$$

On comparing, $R = 0.707\ \Omega$

Q.7 (b) Solution:

(i) Given : $K_1 = \frac{1}{2}, K_2 = \frac{1}{4}$

As, $$A_m(z) = A_{m-1}(z) + K_m z^{-m} A_{m-1}(z^{-1})$$

$$A_0(z) = 1 \text{ (initialize)}$$

$$A_1(z) = A_0(z) + K_1 z^{-1} A_0(z^{-1}) = 1 + \frac{1}{2}z^{-1} \times 1$$

$$= 1 + \frac{1}{2}z^{-1} \qquad (\because A_0(z) = 1; \therefore A_0(z^{-1}) = 1)$$

$$A_2(z) = A_1(z) + K_2 z^{-2} A_1(z^{-1})$$

$$= \left(1 + \frac{1}{2}z^{-1}\right) + \frac{1}{4}z^{-2}\left(1 + \frac{1}{2}z\right)$$

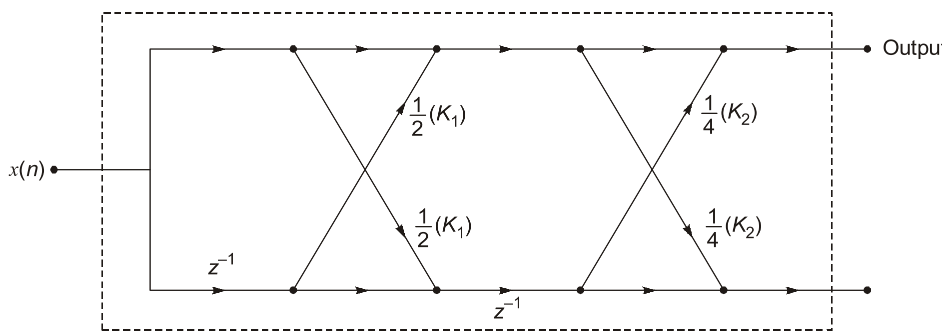
$$\left(\because A_1(z^{-1}) = A_1(z)\Big|_{z \text{ to } z^{-1}} = 1 + \frac{1}{2}z\right)$$

$$= 1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}+\frac{1}{8}z^{-1}$$

$$A_2(z) = 1+\frac{5}{8}z^{-1}+\frac{1}{4}z^{-2}$$

Thus, $$A_2(z) = \sum_{i=0}^{2} a_2(i)z^{-1}$$

$$a_2(0) = 1,\ a_2(1) = \frac{5}{8},\ a_2(2) = \frac{1}{4}$$



$$f_2(x) = x(n) \times A_2(n)$$
$$F_2(z) = X(z) \times A_2(z)$$
$$A_2(z) = 1+\frac{5}{8}z^{-1}+\frac{1}{4}z^{-2}$$

(ii) It is necessary to normalize H(z) so that the first coefficient is unity.

$$\tilde{H}(z) = 8\left[1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}+\frac{1}{8}z^{-3}\right]$$
$$= (8)H_3(z)$$
$$H_3(z) = 1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}+\frac{1}{8}z^{-3}$$
$$K_3 = a_3(z^{-3})$$
$$K_3 = \frac{1}{8} = 0.125$$

We use step down recursion to get lattice structure.

$$H_2(z) = \frac{1}{1-K_3^2}[H_3(z) - K_3 z^{-3} H_3(z^{-1})]$$

$$= \frac{1}{1-\left(\frac{1}{8}\right)^2}\left[\left(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}+\frac{1}{8}z^{-3}\right)-\frac{1}{8}\left(\frac{1}{8}+\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}+z^{-3}\right)\right]$$

$$H_2(z) = 1 + 0.4762z^{-1} + 0.1905z^{-2},\ K_2 = a_2(z^{-2})$$

$$K_2 = 0.1905$$

$$H_1(z) = \frac{1}{1-K_2^2}[H_2(z) - K_2 z^{-2} H_2(z^{-1})]$$

$$= \frac{1}{1-(0.1905)^2}[(1+0.4762z^{-1}+0.1905z^{-2})$$

$$-0.1905(0.1905+0.4762z^{-1}+z^{-2})]$$

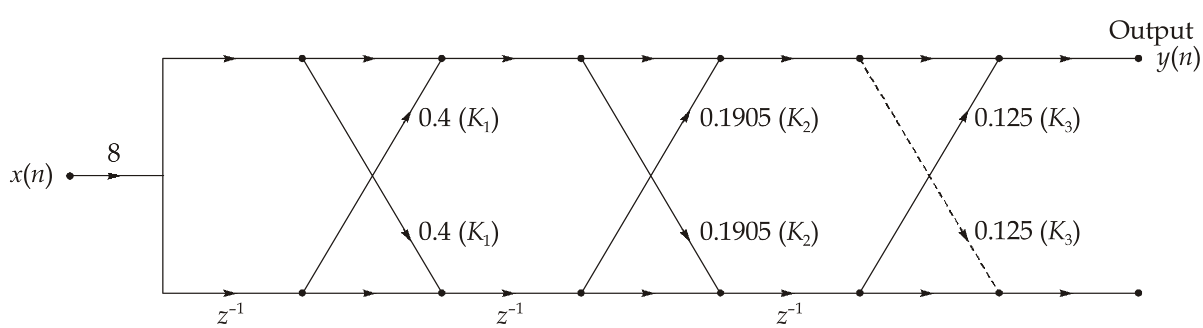
$$H_1(z) = 1 + 0.4z^{-1}$$

$$K_1 = a_1(z^{-1})$$

$$K_1 = 0.4$$

The lattice structure is as shown in figure

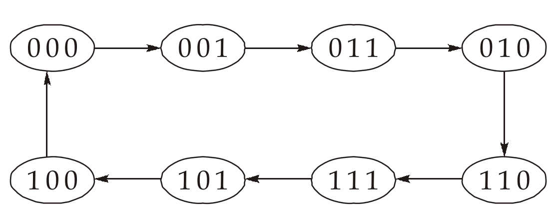
$$H(z) = \frac{Y(z)}{X(z)}$$



Q.7 (c) Solution:

Minimum no. of flip-flops required = 3

Sequence diagram:



Excitation table:

Present State			Next State			Required Excitations					
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	1	0	X	1	X	X	0
0	1	1	0	1	0	0	X	X	0	X	1
0	1	0	1	1	0	1	X	X	0	0	X
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	1	0	1	X	0	X	1	X	0
1	0	1	1	0	0	X	0	0	X	X	1
1	0	0	0	0	0	X	1	0	X	0	X

Minimization

K-Map for J_2

Q_2 \ Q_1Q_0	00	01	11	10
0				1
1	X	X	X	X

$$J_2 = Q_1\overline{Q}_0$$

K-Map for K_2

Q_2 \ Q_1Q_0	00	01	11	10
0	X	X	X	X
1	1			

$$K_2 = \overline{Q}_1\overline{Q}_0$$

K-Map for J_1

Q_2 \ Q_1Q_0	00	01	11	10
0		1	X	X
1			X	X

$$J_1 = \overline{Q}_2Q_0$$

K-Map for K_1

Q_2 \ Q_1Q_0	00	01	11	10
0	X	X		
1	X	X	1	

$$K_1 = Q_2Q_0$$

K-Map for J_0

Q_2 \ Q_1Q_0	00	01	11	10
0	1	X	X	
1		X	X	1

$$J_0 = \overline{Q}_2\overline{Q}_1 + Q_2Q_1$$

$$J_0 = Q_2 \odot Q_1$$

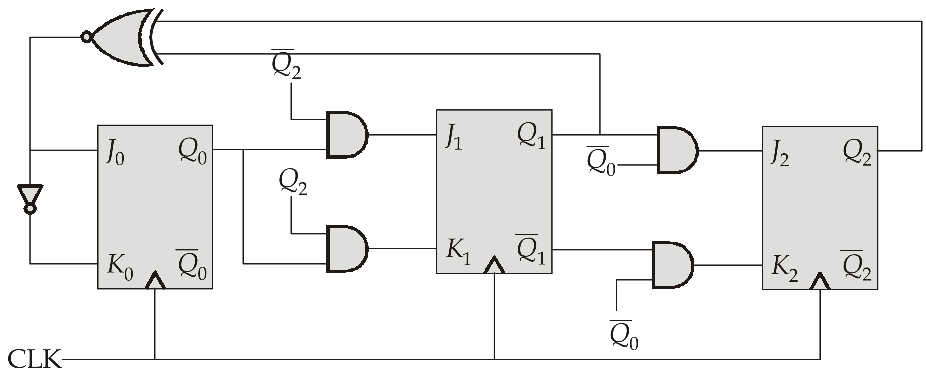
K-Map for K_0

Q_2 \ Q_1Q_0	00	01	11	10
0	X		1	X
1	X	1		X

$$K_0 = Q_1\overline{Q}_2 + Q_2\overline{Q}_1$$

$$K_0 = Q_2 \oplus Q_1$$

Logic Circuit:



Q.8 (a) Solution:

Truth Table :

Present State AB	Inputs EF	Next State AB	FF Outputs $J_AK_AJ_BJ_B$
0 0	0 0	0 0	0×0×
0 0	0 1	0 0	0×0×
0 0	1 0	1 1	1×1×
0 0	1 1	0 1	0×1×
0 1	0 0	0 1	0××0
0 1	0 1	0 1	0××0
0 1	1 0	0 0	0××1
0 1	1 1	1 0	1××1
1 0	0 0	1 0	×00×
1 0	0 1	1 0	×00×
1 0	1 0	0 1	×11×
1 0	1 1	1 1	×01×
1 1	0 0	1 1	×0×0
1 1	0 1	1 1	×0×0
1 1	1 0	1 0	×0×1
1 1	1 1	0 0	×1×1

J_A :

AB \ EF	00	01	11	10
00	0	0	0	1
01	0	0	1	×
11	×	×	×	×
10	×	×	×	×

$$J_A = BEF + B'EF'$$
$$J_A = (BF + B'F')E$$

K_A :

AB \ EF	00	01	11	10
00	×	×	×	×
01	×	×	×	0
11	0	0	1	0
10	0	0	0	1

$$K_A = BEF + \overline{B}E\overline{F}$$
$$K_A = J_A = (B'F' + BF)E$$

J_B :

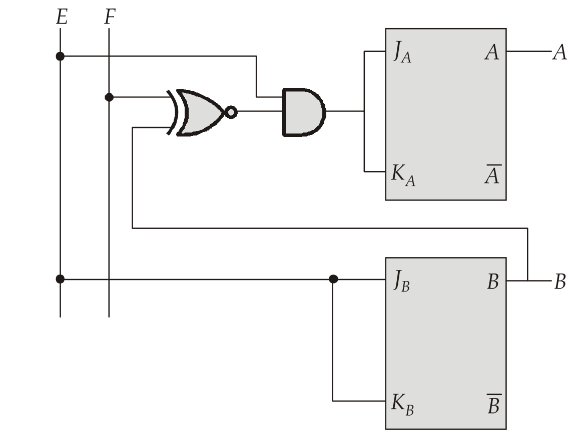
AB \ EF	00	01	11	10
00	0	0	1	1
01	×	×	×	×
11	×	×	×	×
10	0	0	1	1

$$J_B = E$$

K_B :

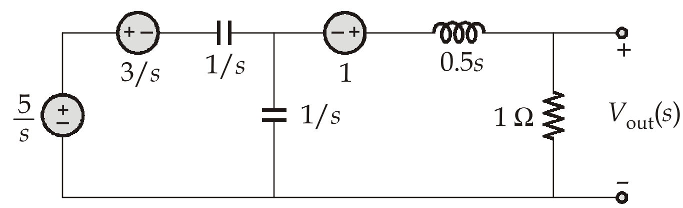
AB \ EF	00	01	11	10
00	×	×	×	×
01	0	0	1	1
11	0	0	1	1
10	×	×	×	×

$$K_B = J_B = E$$

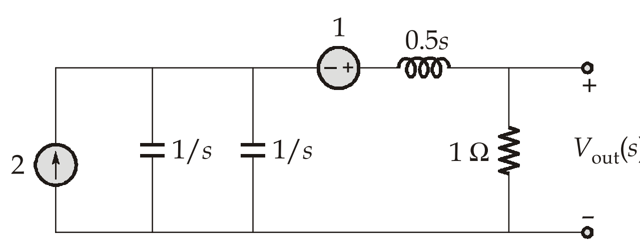


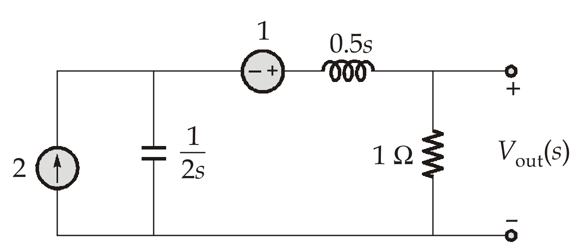
Q.8 (b) (i) Solution:

Apply Laplace transform to the circuit.

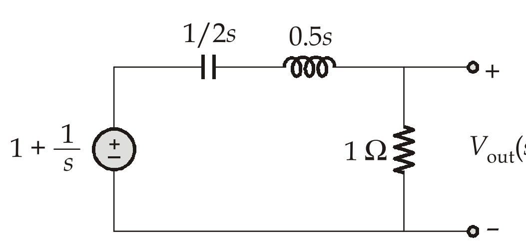


Perform a source transform and combine parallel capacitors.





Apply source transformation and combine series voltage source to produce a series circuit.



Use voltage division rule to find $V_{out}(s)$,

$$V_{out}(s) = \left(\frac{1}{\frac{1}{2s}+0.5s+1}\right)\left(1+\frac{1}{s}\right)$$

$$V_{out}(s) = \frac{2}{s+1}$$

Apply inverse Laplace transform

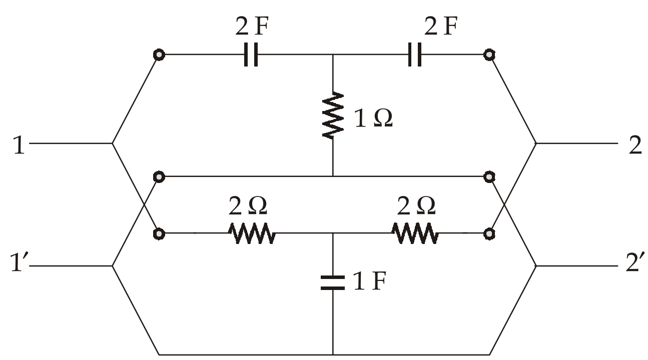
$$V_{out}(t) = L^{-1}\left[\frac{2}{s+1}\right] = 2e^{-t}u(t) \text{ Volts}$$

At $t = 1$ sec V_{out} is

$$V_{out}(1) = 2e^{-1} = 0.736 \text{ Volts}$$

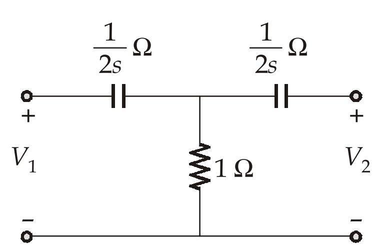
Q.8 (b) (ii) Solution:

Given network is divided into two sub-networks and these networks are connected in parallel.



The total Y-parameters of the network is obtained by addition of two individual sub-network Y-parameter.

i.e.,
$$[Y]_T = [Y]_A + [Y]_B$$

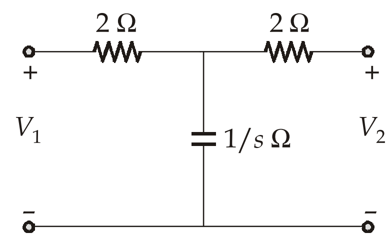


Z-parameters of the network is

$$[Z]_A = \begin{bmatrix} 1+\frac{1}{2s} & 1 \\ 1 & 1+\frac{1}{2s} \end{bmatrix}$$

$$[Z]_A = \begin{bmatrix} \frac{2s+1}{2s} & 1 \\ 1 & \frac{2s+1}{2s} \end{bmatrix}$$

$$[Y]_A = [Z]_A^{-1} = \begin{bmatrix} \frac{4s^2+2s}{4s+1} & \frac{-4s^2}{4s+1} \\ \frac{-4s^2}{4s+1} & \frac{4s^2+2s}{4s+1} \end{bmatrix}$$



Z-parameters of the network is

$$[Z]_B = \begin{bmatrix} 2+\frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & 2+\frac{1}{s} \end{bmatrix}$$

$$[Z]_B = \begin{bmatrix} \frac{2s+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{2s+1}{s} \end{bmatrix}$$

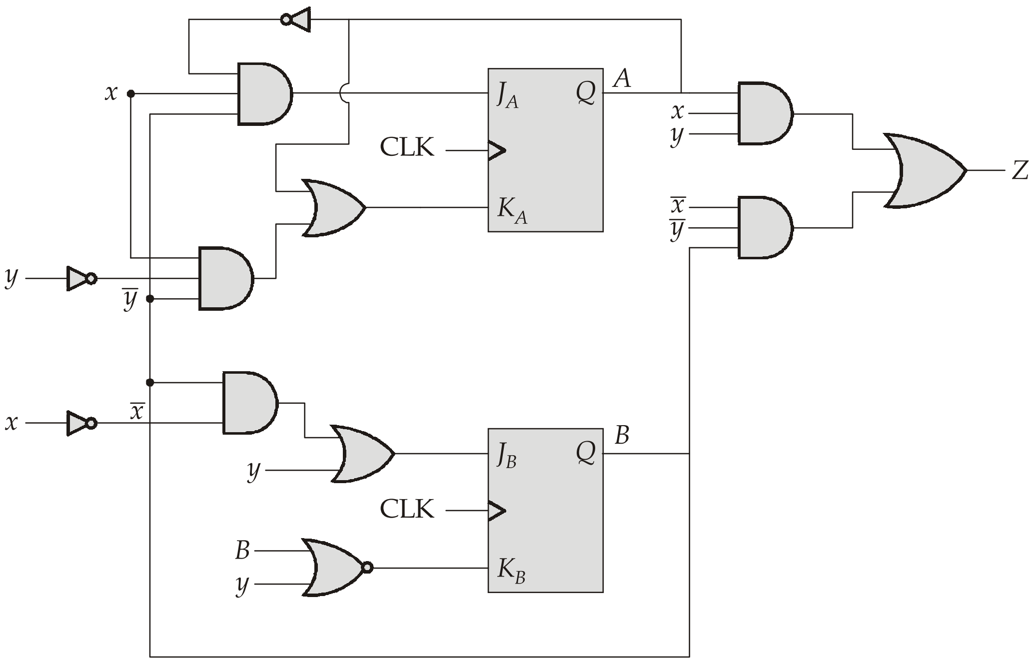
$$[Y]_B = [Z]_B^{-1} \Rightarrow \begin{bmatrix} \frac{2s+1}{4s+4} & \frac{-1}{4s+4} \\ \frac{-1}{4s+4} & \frac{2s+1}{4s+4} \end{bmatrix}$$

$$[Y]_T = [Y]_A + [Y]_B$$

$$[Y] = \begin{bmatrix} \frac{16s^3+32s^2+14s+1}{(4s+1)(4s+4)} & \frac{-(16s^3+16s^2+4s+1)}{(4s+1)(4s+4)} \\ \frac{-(16s^3+16s^2+4s+1)}{(4s+1)(4s+4)} & \frac{16s^3+32s^2+14s+1}{(4s+1)(4s+4)} \end{bmatrix}$$

Q.8 (c) Solution:

(i)



(ii)

Present State		Input		Output	Flip-flop inputs				Next State	
A	B	x	y	Z	J_A	K_A	J_B	K_B	$A(t+1)$	$B(t+1)$
0	0	0	0	0	0	0	0	1	0	0
0	0	0	1	0	0	0	1	0	0	1
0	0	1	0	0	0	0	0	1	0	0
0	0	1	1	0	0	0	1	0	0	1
0	1	0	0	1	0	0	1	0	0	1
0	1	0	1	0	0	0	1	0	0	1
0	1	1	0	0	1	1	0	0	1	1
0	1	1	1	0	1	0	1	0	1	1
1	0	0	0	0	0	1	0	1	0	0
1	0	0	1	0	0	1	1	0	0	1
1	0	1	0	0	0	1	0	1	0	0
1	0	1	1	1	0	1	1	0	0	1
1	1	0	0	1	0	1	1	0	0	1
1	1	0	1	0	0	1	1	0	0	1
1	1	1	0	0	0	1	0	0	0	1
1	1	1	1	1	0	1	1	0	0	1

(iii) For a J-K flip-flop, characteristic equation is given by

$$Q(t+1) = J\bar{Q}(t)+\bar{K}Q(t)$$

State equations for A and B,

$$
\begin{aligned}
A(t+1) &= J_A\overline{A} + \overline{K_A}A \\
&= (\overline{A}Bx)\overline{A} + (\overline{A + Bx\overline{y}})A \\
&= \overline{A}Bx + \left[\overline{A} \cdot \overline{Bx\overline{y}}\right]A \\
A(t+1) &= \overline{A}Bx \\
B(t+1) &= J_B\overline{B} + \overline{K}_B B \\
&= (B\overline{x} + y)\overline{B} + (\overline{\overline{B}\,\overline{y}})B \\
&= y\overline{B} + (B + y)B \\
&= y\overline{B} + B + yB \\
B(t+1) &= y + B
\end{aligned}
$$

❍❍❍❍