MADE EASY

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Detailed Solutions

ESE-2026 Mains Test Series

Mechanical Engineering Test No : 3

Section A : Fluid Mechanics and Turbo Machinery **[All Topics]**

Section B : Strength of Materials & Mechanics-1 + Thermodynamics-2 + IC Engine-2 + Refrigeration and Air-Conditioning-2 **[Part Syllabus]**

Section A : Fluid Mechanics and Turbo Machinery

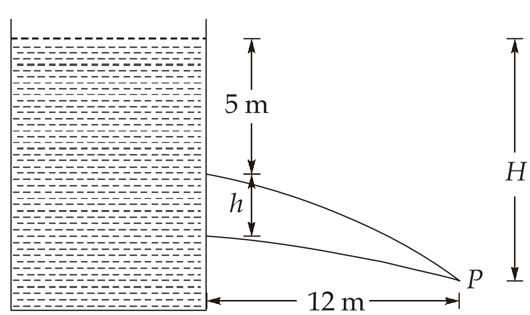
1. (a) Solution:

Let, P be the point of intersection of two jets. If t is the time taken for any liquid particle flowing in the jet from the upper orifice to reach the point P from the plane of the orifice, then

$$12 = u_1 t \quad \text{...(i)}$$

$$H - 5 = \frac{1}{2} g t^2 \quad \text{...(ii)}$$

where, u_1 is the velocity of discharge at the plane of upper orifice and H is the vertical distance of P from the water level in the tank.



Eliminating t from equations,

$$H - 5 = \frac{1}{2}g\left(\frac{144}{u_1^2}\right)$$

$$\frac{u_1^2}{g}(H-5) = 72 \quad \text{...(iii)}$$

Again, applying the Bernoulli's equation between the top water level and the discharge plane of the upper orifice

$$u_1^2 = 2g \times 5$$

$$u_1 = \sqrt{2g \times 5} = \sqrt{10g}$$

Substituting the value of u_1^2 in equation (iii),

$$\frac{10g}{g}(H-5) = 72$$

$$H = 12.2 \text{ m}$$

Similarly for the jet from lower orifice,

$$12 = u_2 t$$

$$(H - 5 - h) = \frac{1}{2}gt^2$$

Eliminating t

$$\frac{u_2^2}{g}(H-5-h) = 72 \quad \text{...(iv)}$$

Also, $u_2 = \sqrt{2g(5+h)}$

Substituting the value of u_2 in equation (iv)

$$\frac{2g(5+h)(12.2-5-h)}{g} = 72$$

$$2(5 + h)(7.2 - h) = 72$$

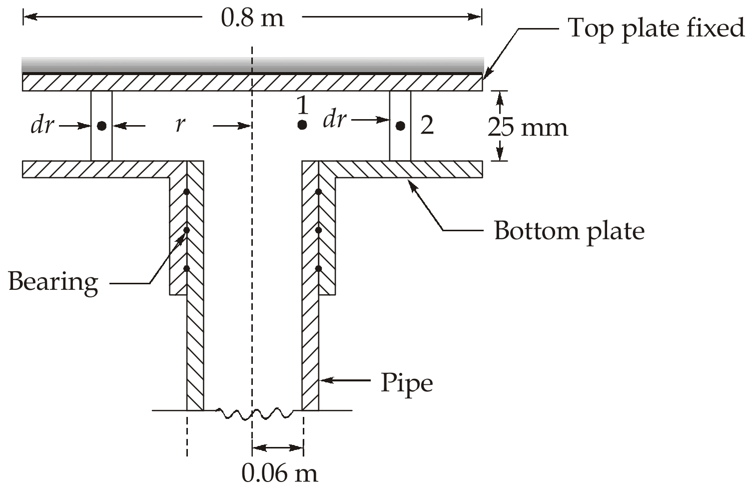
$$36 - 5h + 7.2h - h^2 = 36$$

$$h^2 - 2.2h = 0$$

which gives, $h = 0$ or $h = 2.2$ m

Therefore, the distance between the orifices is 2.2 m.

1. (b) Solution:



Consider an element area of width *dr* (annular) in the flow region at a distance r as shown in the figure. The pressure at this location as compared to point 1 can be determined by Bernoulli's equation.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2, \quad P_1 \text{ is atmospheric}$$

As $z_1 = z_2$

$$\frac{P_2 - P_1}{\rho g} = \frac{V_1^2 - V_2^2}{2g} \quad \text{...(i)}$$

Using continuity equation,

At annular section 1,

$$Q = (2\pi r_1 h)V_1$$

$$0.100 = 2\pi \times 0.06 \times 0.025 \times V_1$$

$$V_1 = 10.61 \text{ m/s}$$

$$\frac{0.100}{2\pi \times 0.025 \times r} = V_2$$

$$V_2 = \frac{0.6366}{r} \text{ m/s}$$

Substituting values in equation (i),

$$\frac{P_2 - P_1}{\rho g} = \frac{10.610^2 - \left(\frac{0.6366}{r}\right)^2}{2g}$$

$$P_2 - P_1 = \frac{\rho}{2}\left[10.610^2 - \frac{0.6366^2}{r^2}\right]$$

Force on the element area of bottom plate = $(2\pi r dr)(P_2 - P_1)$

$$dF = \rho \times \pi r dr\left[112.572 - \frac{0.4053}{r^2}\right]$$

$$F = \rho \times \pi \int_{0.06}^{0.4}\left(112.572 r dr - \frac{0.4053}{r}dr\right)$$

$$F = 1000 \times \pi\left[\frac{112.572 r^2}{2} - 0.4053 \ln r\right]_{0.06}^{0.4}$$

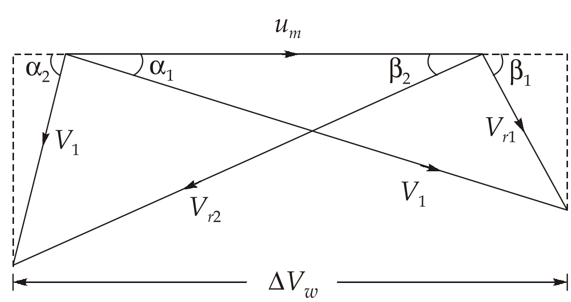
$$F = 1000 \times \pi\left(\frac{112.572}{2}(0.4^2 - 0.06^2) - 0.4053 \ln\frac{0.4}{0.06}\right)$$

$$F = 1000 \times \pi\,[8.803 - 0.7689]$$

$$= 25239.86 \text{ N}$$

1. (c) Solution:

Given: Mean blade diameter, D_m = 0.8 m, Speed of rotation of blade, N = 3000 rpm, Steam velocity at nozzle inlet, V_1 = 300 m/s, Nozzle angle, α_1 = 20°, $\beta_1 = \beta_2$, k = 0.86, Axial thrust, F_a = 140 N



Now, $$u_m = \frac{\pi D_m N}{60} = \frac{\pi \times 0.8 \times 3000}{60} = 125.6 \text{ m/s}$$

From velocity triangle, $$\tan\beta_1 = \frac{V_1 \sin\alpha_1}{V_1\cos\alpha_1 - u_m} = \frac{300\sin 20°}{300\cos 20° - 125.6}$$

$\therefore$ $$\beta_1 = 33.28° = \beta_2$$

Also, $$V_1 \sin\alpha_1 = V_{r1} \sin\beta_1$$

$$\therefore \quad V_{r1} = \frac{300\sin 20°}{\sin 33.28} = 186.98 \text{ m/s}$$

$$\therefore \quad V_{r2} = 0.86 \times V_{r1} = 0.86 \times 186.98$$

or, $$V_{r2} = 160.8 \text{ m/s}$$

$$\text{Axial thrust, } F_a = \dot{m}[V_{r1}\sin\beta_1 - V_{r2}\sin\beta_2]$$

$$F_a = \dot{m} \times V_{r1}\sin\beta_1[1-k]$$

or, $$140 = \dot{m} \times 186.98\sin 33.28°[1-0.86]$$

$$\therefore \quad \dot{m} = 9.74 \text{ kg/s}$$

$$\Delta V_w = V_{r2}\cos\beta_2 + V_{r1}\cos\beta_1$$

$$= V_{r1}\cos\beta_1 (1+k)$$

$$\Delta V_w = 186.98\cos 33.28° (1.86)$$

$$= 290.74 \text{ m/s}$$

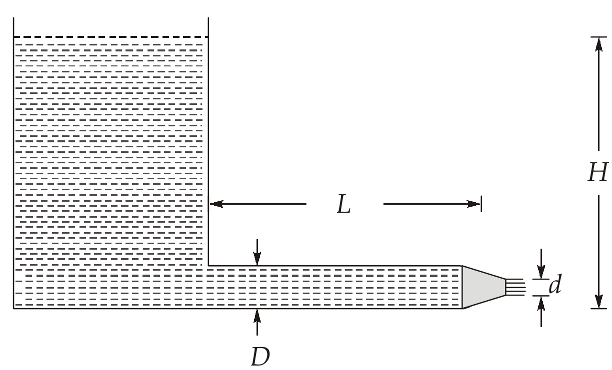
$$\therefore \quad \text{Power, } P = \dot{m} \times \Delta V_w \times u_m$$

$$= 9.74 \times 290.74 \times 125.6$$

$$= 355.67 \text{ kW}$$

Ans.

1. (d) Solution:



Let, v = velocity in the nozzle and V = velocity in the pipe; H = total head

Total head, H = Velocity head at the nozzle + Head lost in friction

$$H = \frac{v^2}{2g} + \frac{fLV^2}{2gD} \quad \text{...(i)}$$

by continuity equation,

$$\frac{\pi}{4}d^2 v = \frac{\pi}{4}D^2 V$$

$$V = v\left(\frac{d^2}{D^2}\right)$$

$$\frac{v^2}{2g}\left[1+\frac{fL}{D}\left(\frac{d}{D}\right)^4\right] = H$$

$$\frac{v^2}{2g} = \frac{H}{\left[1+\frac{fL}{D}\left(\frac{d}{D}\right)^4\right]} \quad \text{...(ii)}$$

$$\text{Power of jet, } P = \rho Q g H_{net} = \rho Q g\left(\frac{v^2}{2g}\right)$$

$$P = \rho Q g\left[H - \frac{fLV^2}{2gD}\right]$$

$$P = \rho g\frac{\pi}{4}d^2 v\left[H - \frac{fL}{D}\left(\frac{d}{D}\right)^4\frac{v^2}{2g}\right]$$

For maximum power $\frac{dP}{dv} = 0$

$$H - 3\frac{fL}{D}\left(\frac{d}{D}\right)^4\cdot\frac{v^2}{2g} = 0$$

$$\frac{v^2}{2g} = \frac{H}{3\left(\frac{fL}{D}\right)\left(\frac{d}{D}\right)^4} \quad \text{...(iii)}$$

From equation (ii) and (iii),

$$\frac{H}{\left[1+\frac{fL}{D}\left(\frac{d}{D}\right)^4\right]} = \frac{H}{3\left(\frac{fL}{D}\right)\left(\frac{d}{D}\right)^4}$$

$$1+\frac{fL}{D}\left(\frac{d}{D}\right)^4 = 3\left(\frac{fL}{D}\right)\left(\frac{d}{D}\right)^4$$

$$d^4 = \frac{D^5}{2fL}$$

$$d = \left(\frac{D^5}{2fL}\right)^{1/4}$$

Ans.

1. (e) (i) Solution:

Parameters Affecting Flight Performance : Two parameters which affect the performance of the jet propulsion cycle are – the forward speed of the aircraft and the altitude at which the aircraft flies. These two do not enter into the analysis of shaft power cycle. The effect of the two parameters are discussed in the following sections :

Effect of Forward Speed : The forward speed affects the inlet pressure and temperature of the compressor. The inlet duct to the compressor acts as a diffuser. The air which enters the diffuser at flight speed is slowed down to the speed acceptable to compressor, and at the same time raising its pressure and temperature. This increase of pressure and temperature due to aircraft speed is known as **ram effect**, or simply ram. It becomes more and more prominent as the flight speed increases. For a given aircraft speed and ram efficiency, the ram pressure ratio increases as the ambient temperature decreases at high altitudes.

The second effect of aircraft forward speed is in relation to propulsive efficiency. As flight velocity increases, the inlet drag also increases. If there were no ram effect, the net specific thrust, I_{sp}, would decrease. This is because the jet velocity remains the same. With ram, the increase of inlet temperature reduces the gross-thrust to some extent. However, the increase of inlet pressure more than compensates for this. Because of this, the cycle pressure ratio increases without shaft work being necessary. The overall effect of forward speed on inlet drag and ram is to reduce somewhat the net specific thrust.

Effect of Altitude : The effect of altitude on a turbojet is by virtue of reduction of ambient pressure and temperature. The temperature of atmosphere varies considerably and continuously with location and time, so that the standard atmosphere is used for calculating the performance at various altitudes. The most general data used are those for International Standard Atmosphere or ICON atmosphere (International Commission On Navigation). It corresponds approximately to average values found in the middle latitudes. It is normalized by using a linear decrease of temperature or lapse rate of 1.98°C per 300 m of altitude, starting with a ground level temperature of 15°C. With the temperature fixed, the pressure can be calculated according to the principles of hydrostatics using 1.03 bar as the ground level pressure.

1. (e) (ii) Solution:

Given : $C.V.$ = 43 MJ/kg, $\dot{m}_F$ = 0.25 kg/hr/N, F_t = 10 kN; V_a = 500 m/s, $\dot{m}_a$ = 27 kg/s

Now, $$\dot{m}_F = \frac{0.25}{3600} \times 10000$$

$$\dot{m}_F = 0.69 \text{ kg/s}$$

∴ $$A/F = \frac{27}{0.69} = 39.13 : 1$$

Thrust Power, $$P_T = F_t . V_a$$
$$= 10 \times 500$$
$$= 5000 \text{ kW}$$

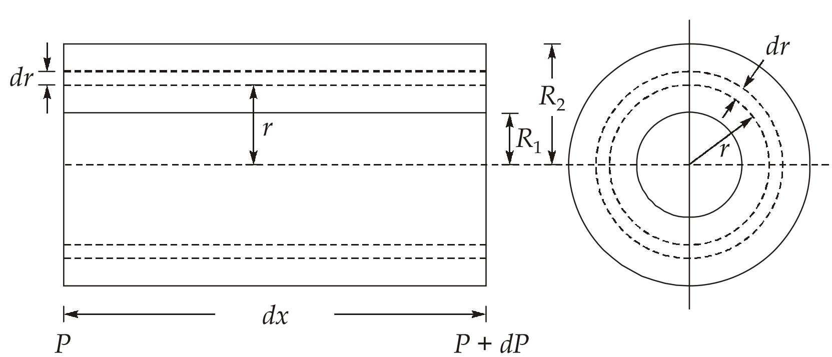
Heat input, $$Q_S = \dot{m}_F \times C.V.$$
$$= 0.69 \times 43000$$
$$= 29670 \text{ kW}$$

∴ Efficiency, $$\eta = \frac{P_T}{Q_S} = \frac{5000}{29670} \times 100$$

∴ $$\eta = 16.85\%$$

2. (a) Solution:

Consider an annular element of radius r, thickness dr and length dx. Further let the velocity at radius r be u and pressure at two sections dx apart be P and $\left[P + \left(\frac{\partial P}{dx}\right)dx\right]$ respectively.



Force on the element due to pressure difference is,

$$= P \times 2\pi r dr - \left[P + \left(\frac{\partial P}{dx}\right)dx\right](2\pi r dr)$$

$$= -\frac{\partial P}{\partial x}(2\pi r dr) \qquad \text{Acting from right to left}$$

Let, shear stress on the inner and outer surfaces of the element be τ and $\left(\tau + \frac{\partial \tau}{\partial r} dr\right)$ respectively. The net viscous force acting on the element from right to left is

$$= \left[\tau + \left(\frac{\partial \tau}{\partial r}\right) dr\right] \times \left(2\pi(r + dr)dx\right) - \tau(2\pi r)dx$$

$$= 2\pi\left[\tau + r\left(\frac{\partial \tau}{\partial r}\right)\right] drdx$$

Since the fluid is not accelerating (i.e. is moving with constant velocity), sum of the pressure and viscous forces acting on the element must be zero. Thus,

$$-\frac{\partial P}{\partial x}(2\pi r dr) + 2\pi\left[\tau + r\left(\frac{\partial \tau}{\partial r}\right)\right] drdx = 0$$

$$\left(\tau + r\frac{\partial \tau}{\partial r}\right) = r\frac{\partial P}{\partial x}$$

$$\frac{d}{dr}(\tau r) = r\frac{dP}{dx}$$

From Newton's law of viscosity,

$$\tau = \mu\frac{\partial u}{\partial r}$$

$$\frac{d}{dr}\left(r\mu\frac{du}{dr}\right) = r\frac{dP}{dx}$$

$$\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{r}{\mu}\frac{dP}{dx}$$

Integrating with respect to r

$$r\frac{du}{dr} = \frac{r^2}{2\mu}\frac{dP}{dx} + A$$

$$\frac{du}{dr} = \frac{r}{2\mu}\frac{dP}{dx} + \frac{A}{r} \qquad \text{...(i)}$$

Another integration w.r.t. r gives

$$u = \frac{r^2}{4\mu}\frac{dP}{dx} + A\ln r + B$$

Using boundary conditions

At $r = R_1$, $u = 0$

At $r = R_2$, $u = 0$

$$\frac{R_1^2}{4\mu}\frac{dP}{dx} + A\ln R_1 + B = 0 \quad \text{...(ii)}$$

$$\frac{R_2^2}{4\mu}\frac{dP}{dx} + A\ln R_2 + B = 0 \quad \text{...(iii)}$$

On solving equation (ii) and (iii),

$$A = \frac{\frac{\left(R_1^2 - R_2^2\right)}{4\mu}\left(\frac{dP}{dx}\right)}{\ln\left(\frac{R_2}{R_1}\right)}$$

Local velocity u would be maximum when

$$\frac{du}{dr} = 0$$

From equation (i),

$$0 = \frac{r}{2\mu}\left(\frac{dP}{dx}\right) + \frac{A}{r}$$

$$\frac{r^2}{2\mu}\left(\frac{dP}{dx}\right) = -A$$

On substituting the value of A, we get

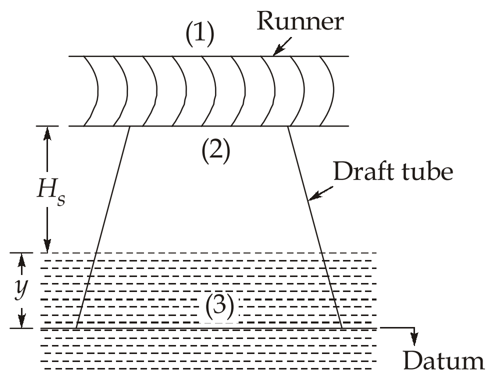
$$r^2 = \frac{2\mu\left(R_2^2 - R_1^2\right)\left(\frac{dP}{dx}\right)}{\left(\frac{dP}{dx}\right)4\mu\ln\left(\frac{R_2}{R_1}\right)}$$

$$r = \sqrt{\frac{R_2^2 - R_1^2}{2\ln\left(\frac{R_2}{R_1}\right)}}$$

2. (b) Solution:

Given : $H_s = 5$ m; $V_2 = 10$ m/s; $A_3 = 25$ m^2; $d_2 = 2$ m

Let points, 1, 2 and 3 denotes the runner entrance, runner exit, and at the outlet end of draft tube, respectively.



Applying Bernoulli's equation at (2) and (3) we get, consider datum at (3)

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + H_s + y = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + 0 + h_f$$

$$\therefore \quad \frac{P_3}{\rho g} = \frac{P_{atm}}{\rho g} + y$$

Substituting and solving we get,

(i) $$\frac{P_2}{\rho g} = \frac{P_{atm}}{\rho g} - \left[H_s + \frac{V_2^2 - V_3^2}{2g}\right] + h_f$$

or, $$\frac{P_2}{\rho g} = -\left[H_s + \frac{V_2^2 - V_3^2}{2g}\right] + h_f$$

$$\because \quad h_f = \frac{1}{2} \times \frac{V_3^2}{2g}$$ [Head loss in friction]

Thus, $$\frac{P_2}{\rho g} = -\left[H_s + \frac{V_2^2 - V_3^2}{2g}\right] + \frac{V_3^2}{4g}$$

Now, by continuity at (2) and (3) we have,

$$\frac{\pi}{4}(2)^2 \times V_2 = 25 \times V_3$$

or, $$\frac{\pi}{4}(4) \times 10 = 25 \times V_3 \Rightarrow V_3 = 1.256 \text{ m/s}$$

$$\therefore \quad \frac{P_2}{\rho g} = -\left[5 + \frac{10^2 - 1.256^2}{2 \times 9.81}\right] + \frac{1.256^2}{4 \times 9.81}$$

$$= -[10.016] + 0.04 = -9.975 \text{ m}$$ **Ans.**

(ii) Total head at the top of the draft tube,

$$H_{DTi} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$= -9.975 + \frac{10^2}{2 \times 9.81} + 5 = 0.1218 \text{ m}$$ **Ans.**

(iii) Power at the outlet of runner,

$$P = \rho g Q\, H_{DTi}$$

$$= 9.81 \times 1000 \times \frac{\pi}{4} \times (2)^2 \times 10 \times 0.1218$$

$$= 37.54 \text{ kW}$$ **Ans.**

(iv) Total head at the end of draft tube,

$$H_{DTe} = \frac{V_3^2}{2g} = \frac{1.256^2}{2 \times 9.81} = 0.08 \text{ m}$$

Power at the end of the draft tube,

$$P' = \rho g Q\, H_{DTe}$$

$$= 10^3 \times 9.81 \times \frac{\pi}{4} \times (2)^2 \times 10 \times 0.08$$

$$= 24.65 \text{ kW}$$ **Ans.**

(v) Power lost in the draft tube,

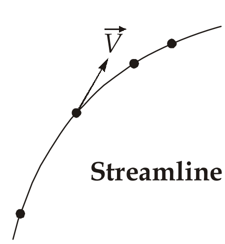
$$P_{\text{lost}} = \rho g Q\, h_f$$

$$= 10^3 \times 9.81 \times \frac{\pi}{4} \times (2)^2 \times 10 \times \frac{1.256^2}{4 \times 9.81}$$

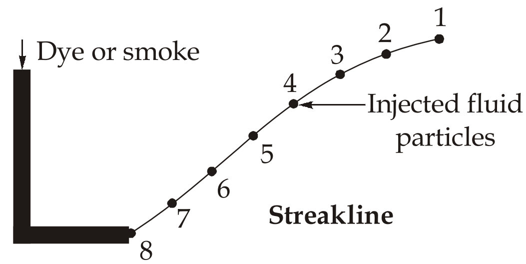
$$P_{\text{lost}} = 12.38 \text{ kW}$$ **Ans.**

2. (c) (i) Solution:

Stream line/ Flow line: It is an imaginary line drawn in the flow field in such a manner such that tangent to the stream line at any point gives direction of velocity to the flow at that point. Since a stream line is always tangent to the velocity vector at any point, therefore there can be no component of velocity at right angles to a stream lines.

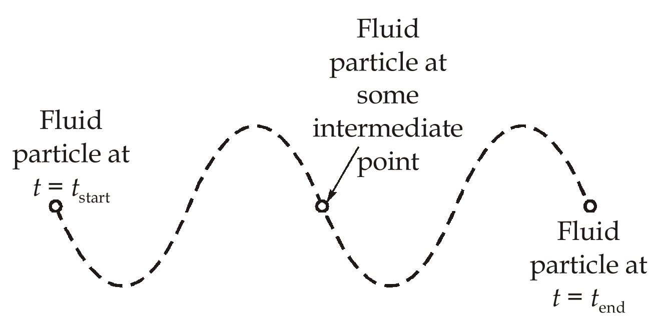


Streak line/ Filament line : It is a line formed by joining all the fluid particles that have crossed a given point in the flow field over a period of time. It is used for tracing the movement of gases.



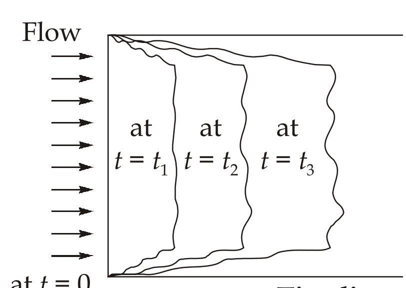
Streakline

Pathline: A pathline is the line traced by a single fluid particle as it moves through space over a period of time. It shows direction of single fluid particle at different instants of time.



Pathline

Timeline: It is a line formed by joining all the fluid particles which are adjacent to each other at any given instant of time.



Timeline

2. (c) (ii) Solution:

Given : $\phi = y^2 - x^2 + Axy$

The velocity components in x and y directions, respectively is given by

$$u = \frac{-\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \text{ and } v = \frac{-\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

The x-direction velocity component is

$$u = \frac{\partial \phi}{\partial x} \Rightarrow u = -2x + Ay$$

The y-direction velocity component is

$$v = \frac{\partial \phi}{\partial y} \Rightarrow v = 2y + Ax$$

The stream function can be obtained by integrating either u with respect to y or v with respect to x:

$$\psi = A\frac{y^2}{2} + f(x) + C_1$$

Differentiating the stream function with respect to x, we get

$$\frac{\partial \psi}{\partial x} = f'(x) = -v$$

or, $$f'(x) = -2y - Ax$$

Integrating with respect to x,

$$f(x) = -2xy - \frac{Ax^2}{2} + C_2$$

The stream function is, thus, given by

$$\psi = A\frac{(y^2 - x^2)}{2} - 2xy + C_1 + C_2 = A\frac{(y^2 - x^2)}{2} - 2xy + C$$

The streamlines passing through the points (1, 3) and (1, 6) are obtained as

$$\psi|_{(1,3)} = A\frac{(3^2 - 1^2)}{2} - 2\times(1)\times(3) + C = 4A - 6 + C$$

$$\psi|_{(1,6)} = A\frac{(6^2 - 1^2)}{2} - 2\times(1)\times(6) + C = 17.5A - 12 + C$$

The discharge between the streamlines is equal to the difference of stream functions,

$$\psi|_{(1,6)} - \psi|_{(1,3)} = q$$

$$13.5A - 6 = 12$$

$$\Rightarrow \quad A = \frac{18}{13.5} = \frac{4}{3}$$

3. (a) Solution:

Given : $C_V = 0.97$; $\eta_g = 0.95$; $\beta_2 = 15°$; $k = 0.85$

(i)

$$\text{Mechanical power output of the turbine} = \frac{\text{Electrical power output}}{\text{Generator efficiency}} = \frac{10}{0.95} = 10.53 \text{ MW}$$

$$\text{Pelton wheel efficiency, } \eta = \frac{P}{\rho g Q H}$$

where, Q is the flow rate through the turbine.

Then, $$Q = \frac{P}{\eta \times \rho g H} = \frac{10.53 \times 10^6}{0.87 \times 10^3 \times 9.81 \times 762} = 1.62 \text{ m}^3/\text{s}$$

(ii)

If d_j is the diameter of the jet, we can write,

$$Q_j = \left(\frac{\pi}{4}\right) \times d_j^2 C_V (2gH)^{1/2}$$

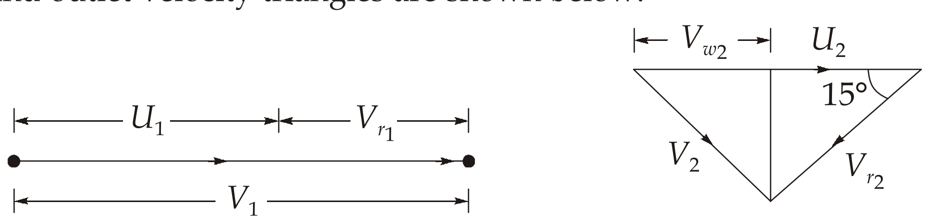
where, C_V is the coefficient of velocity.

then $$1.62 = \left(\frac{\pi}{4}\right) \times d_j^2 \times 0.97 \times (2 \times 9.81 \times 762)^{1/2}$$

which gives, $$d_j = 0.132 \text{ m} = 132 \text{ mm}$$

(iii)

The inlet and outlet velocity triangles are shown below:



Inlet **Outlet**

$$\text{Jet velocity, } V_1 = C_V (2gH)^{1/2} = 0.97 \times [2 \times 9.81 \times 762]^{1/2}$$
$$= 118.6 \text{ m/s}$$

$$\text{Mean bucket velocity, } U_1 = U_2 = 0.46 \times 118.6 = 54.56 \text{ m/s}$$

From the inlet velocity triangle,

$$V_{w1} = V_1 = 118.6 \text{ m/s}$$
$$V_{r1} = V_1 - U_1 = 118.50 - 54.56 = 63.94 \text{ m/s}$$
$$V_{r2} = 0.85 \times 63.94 = 54.35 \text{ m/s}$$

From the outlet velocity triangle,

$$V_{w2} = U_2 - V_{r2}\cos\beta_2 = 54.56 - 54.35 \times \cos 15°$$
$$= 2.06 \text{ m/s}$$

Therefore, the force exerted by the jet on the bucket is given by

$$F = \rho Q\,(V_{w1} - V_{w2}) = 10^3 \times 1.62\,[118.5 - 2.06] \text{ N}$$
$$= 188.63 \text{ kN}$$

Considering the ratio of mean bucket circle diameter D to the jet diameter d as 10,

$$D = 10 \times 0.132 = 1.32 \text{ m}$$

Again, $$U_1 = \frac{\pi D N}{60}$$

Hence, $$N = \frac{54.56 \times 60}{\pi \times 1.32} = 789.51 \text{ rpm}$$

Frequency of generator, $$f = p \times \frac{N_{syn}}{60}$$

Where, p = Number of pair of poles

$$p = 4 \text{ gives } N_{\text{syn}} = \frac{60 \times 50}{4}$$
$$= 750 \text{ rpm which is nearest to 789 rpm}$$

Therefore, we choose, N_{syn} = 750 rpm

Now, $$D(\text{revised}) = \frac{1.32 \times 789.5}{750} = 1.39 \text{ m}$$

3. (b) Solution:

Let, l be the length of wooden piece. For floating equilibrium of composite cylinder.

Weight of cylinder ≤ Weight of liquid of same volume as that of cylinder

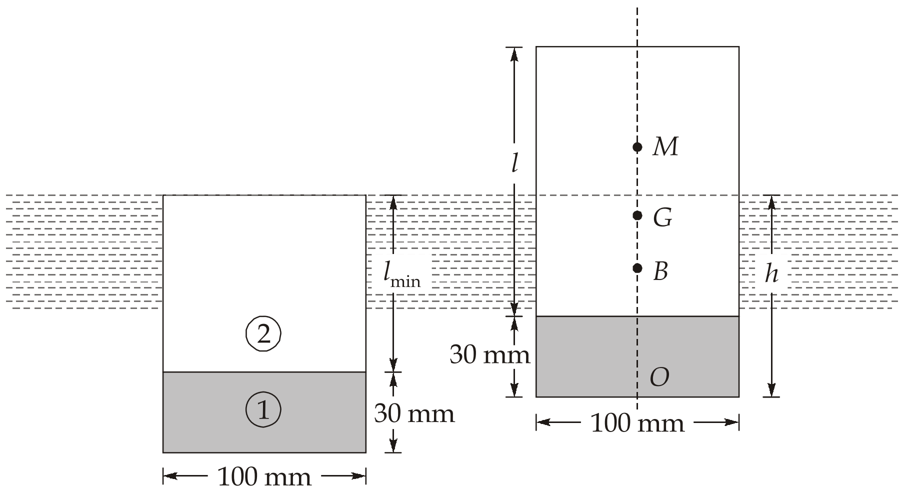
$$\frac{\pi}{4} \times (0.1)^2 \times [0.03 \times 5 + 0.85l] \le \frac{\pi}{4} \times (0.1)^2 \times (0.03 + l)$$

$$l \ge 0.8 \text{ m}$$

Hence, minimum length of the wooden portion;

$$l_{\text{Minimum}} = 0.8 \text{ m} = 800 \text{ mm}$$

The minimum length corresponds to the situation when the cylinder will just float with its top edge at the free surface. For any length l greater than 800 mm, the cylinder will always float in equilibrium with a part of its length submerged. The upper limit of l would be decided from the consideration of stable equilibrium of the cylinder.



(i) (ii)

For stable equilibrium,

$$\text{Metacentric height} > 0$$

The location of centre of gravity G of the composite cylinder can be found as

$$OG = \frac{W_1X_1 + W_2X_1}{W_1 + W_2}$$

$$OG = \frac{5\times\frac{\pi}{4}(100)^2\times 30\times\frac{30}{2} + 0.85\times\frac{\pi}{4}\times(100)^2\times l\times\left(30+\frac{l}{2}\right)}{5\times\frac{\pi}{4}\times(100)^2\times 30 + 0.85\times\frac{\pi}{4}\times(100)^2\times l}$$

$$OG = \frac{5\times 30\times 15 + 0.85\times l\times\left(30+\frac{l}{2}\right)}{5\times 30 + 0.85\times l}\ \text{mm}$$

$$OG = \frac{0.425l^2 + 25.5l + 2250}{0.85l + 150}\ \text{mm}$$

The submerged length h of the wooden cylinder is found from the consideration of floating equilibrium as

$$\text{Weight of cylinder} = \text{Buoyancy force}$$

$$\frac{\pi}{4}\times(100)^2[30\times 5 + l\times 0.85] = \frac{\pi}{4}\times 100^2\times h$$

$$150 + 0.85l = h \quad \text{...(i)}$$

The location of centre of buoyancy B can therefore be expressed as

$$OB = \frac{h}{2} = \frac{150 + 0.85l}{2}$$

Now, $BG = OG - OB$

$$BG = \frac{0.425l^2 + 25.5l + 2250}{0.85l + 150} - \frac{150 + 0.85l}{2}$$

$$BG = \frac{0.0638l^2 - 102l - 9000}{0.85l + 150}$$

The location of metacentre M above buoyancy B can be found out as

$$BM = \frac{I}{V} = \frac{\frac{\pi}{64} \times (100)^4}{\frac{\pi}{4} \times (100)^2 \times h} = \frac{625}{h}$$

Substituting h from equation (i)

$$BM = \frac{625}{150 + 0.85l}$$

Therefore, $GM = BM - BG$

$$GM = \frac{625}{150 + 0.85l} - \frac{0.0638l^2 - 102l - 9000}{0.85l + 150}$$

$$GM = \frac{102l - 0.0638l^2 + 9625}{0.85l + 150}$$

For stable equilibrium, $GM > 0$

$$\frac{102l - 0.0638l^2 + 9625}{0.85l + 150} > 0$$

$$0.0638l^2 - 102l - 9625 < 0$$

So, $(l - 1688.113)(l + 89.367) < 0$

The length can never be negative.

Hence, physical possible condition is

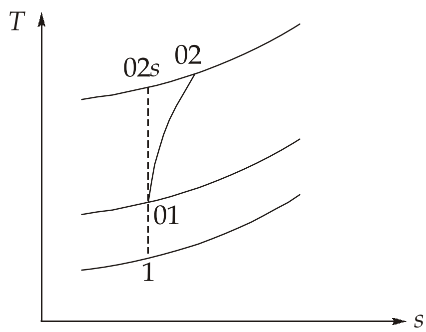
$$l - 1688.113 < 0$$

or $\quad l < 1688.113$ mm

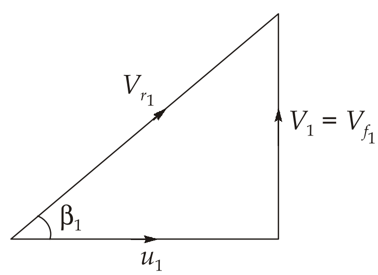
The limits of the length of the wooden portion are : 800 mm < l < 1688.11 mm.

3. (c) Solution:

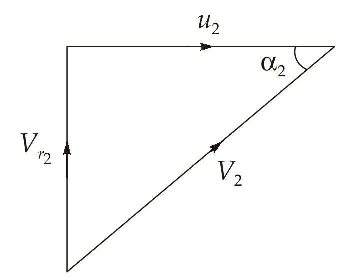
Given: N = 12000 rpm, Q = 700 m^3/min, r_p = 5 : 1, $\eta_{isen,c}$ = 0.82, P_{01} = 1 bar, T_{01} = 293 K



T-s plot



Inlet velocity triangle



Outlet velocity triangle

Now, $$T_{02s} = T_{01}\left(r_p\right)^{\gamma-1/\gamma} = 293\times(5)^{(1.4-1)/1.4}$$

$$= 464.06 \text{ K}$$

and $$T_{02} = T_{01} + \frac{T_{02s}-T_{01}}{\eta_{isen,c}} = 293 + \frac{464.05-293}{0.82} = 501.61 \text{ K}$$

$\therefore$ $$\text{Power input, } P = \dot{m}c_p\left(T_{02}-T_{01}\right) = \dot{m}\left(V_{w_2}u_2\right)$$

or, $$c_p(T_{02} - T_{01}) = u_2{}^2 \qquad (\because V_{w_2} = u_2)$$

$\therefore$ $$u_2 = \sqrt{1005\times(501.59-293)} = 457.87 \text{ m/s}$$

Now, $$T_1 = T_{01} - \frac{V_{f1}^2}{2C_P} \qquad \left(\because V_1 = V_{f1}\right)$$

$$= 293 - \frac{60^2}{2\times 1005} = 291.2 \text{ K}$$

and $$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\gamma/\gamma-1} = \left(\frac{291.2}{293}\right)^{1.4/0.4} = 0.978 \text{ bar}$$

$\therefore$ $$\rho_1 = \frac{P_1}{RT_1} = \frac{0.978\times 10^5}{287\times 291.2} = 1.171 \text{ kg/m}^3$$

$\therefore$ $$\dot{m} = \rho\dot{Q} = 1.171\times\frac{700}{60} = 13.66 \text{ kg/s}$$

$\therefore$ Power, $P = \dot{m} \times u_2^2 = 13.66 \times 457.85^2$

$= 2863.84$ kW **Ans (i)**

Now, $u_2 = \dfrac{\pi D_2 N}{60}$

$\therefore$ $457.85 = \dfrac{\pi \times D_2 \times 12000}{60}$

$D_2 = 0.729$ m **Ans (ii)**

Also, $D_1 = \dfrac{D_2}{2} = \dfrac{0.729}{2} = 0.36$ m **Ans (ii)**

Now, Discharge, $\dot{Q} = \pi D_1 B_1 V_{f1}$

$\therefore$ $B_1 = \dfrac{700}{60 \times 3.14 \times 0.36 \times 60} = 0.172$ m **Ans (ii)**

Further, $u_1 = \dfrac{\pi D_1 N}{60} = \dfrac{3.14 \times 0.36 \times 12000}{60} = 226.08$ m/s

$\therefore$ Blade angle at inlet, $\beta_1 = \tan^{-1}\left(\dfrac{V_{f1}}{u_1}\right) = \tan^{-1}\left(\dfrac{60}{226.08}\right) = 14.86°$ **Ans (iii)**

Diffuser inlet angle, $\alpha_2 = \tan^{-1}\left(\dfrac{V_{f2}}{u_2}\right)$ $[\because V_{f2} = V_{f1}]$

$= \tan^{-1}\left(\dfrac{60}{457.85}\right) = 7.46°$ **Ans (iii)**

4. (a) Solution:

Given: $D = 0.2$ m, $L = 0.2$ m, $H_s = 5$ m, $H_d = 35$ m, $l_s = 6.5$ m, $l_d = 40$ m, $d_s = d_d = 0.075$ m, $N = 30$ rpm, $H_{atm} = 10.3$ m of water, $f = 0.05$

For suction stroke, acceleration head,

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 30}{60} = 3.14 \text{ rad/s}$$

$$A = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$a_s = \frac{\pi}{4} \times 0.075^2 = 4.41 \times 10^{-3} \text{ m}^2$$

Now, $h_{as} = \frac{6.5}{9.81} \times \frac{0.0314}{4.41 \times 10^{-3}} \times 3.14^2 \times 0.1 = 4.65 \text{ m}$

Frictional head loss, $h_{fs} = \frac{f \times l_s}{2gd_s} \times \left(\frac{A}{a_s} \times \omega r\right)^2$

$$= \frac{0.05 \times 6.5}{2 \times 9.81 \times 0.075} \times \left(\frac{0.0314 \times 3.14 \times 0.1}{4.41 \times 10^{-3}}\right)^2 = 1.10 \text{ m}$$

∴ Pressure heads in metres of water during suction stroke will be as follows:

At Beginning $= H_{atm} - H_s - h_{as}$

$= 10.3 - 5 - 4.65$

$= 0.65$ m of water (Abs) **Ans.**

At Middle $= H_{atm} - H_s - h_{fs}$

$= 10.3 - 5 - 1.10$

$= 4.2$ m of water (Abs) **Ans.**

At End $= H_{atm} - H_s + h_{as}$

$= 10.3 - 5 + 4.65$

$= 9.95$ m of water (Abs) **Ans.**

For delivery stroke, $h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$

$$= \frac{40}{9.81} \times \frac{0.0314}{4.41 \times 10^{-3}} \times 3.14^2 \times 0.1 \qquad [\because a_s = a_d]$$

$h_{ad} = 28.62$ m

Frictional head, $h_{fd} = \frac{f \times l_d}{2gd_d}\left(\frac{A}{a_d}\omega r\right)^2$

$$= \frac{0.05 \times 40}{2 \times 9.81 \times 0.075}\left(\frac{0.0314}{4.41 \times 10^{-3}} \times 3.14 \times 0.1\right)^2$$

$= 6.769$ m

∴ Pressure heads in metres of water during delivery strokes will be as follows.

At Beginning $= H_{atm} + H_d + h_{ad}$

$= 10.3 + 35 + 28.62$

$= 73.92$ m of water (Abs) **Ans.**

At Middle $= H_{atm} + H_d + h_{fd}$

$= 10.3 + 35 + 6.769$

$= 52.07$ m of water (Abs) **Ans.**

At End $= H_{atm} + H_d - h_{ad}$

$= 10.3 + 35 - 28.62$

$= 16.68$ m of water (Abs) **Ans.**

Now, Theoretical discharge, $Q_{th} = \frac{ALN}{60} = \frac{0.0314 \times 0.2 \times 30}{60}$

$Q_{th} = 3.14 \times 10^{-3}\ m^3/s$ **Ans.**

Power required to drive the pump,

$$P = \rho g Q\left[H_s + H_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right]$$

$$= 10^3 \times 9.81 \times 3.14 \times 10^{-3}\left[5 + 35 + \frac{2}{3}(1.10 + 6.769)\right]$$

$= 1.39$ kW **Ans.**

4. (b) Solution:

Cubical tank of sides 2 m means dimensions of tank are 2 m × 2 m × 2 m

Depth of water, $h_2 = 0.8$ m

Depth of oil, $h_1 = 2 - 0.8 = 1.2$ m

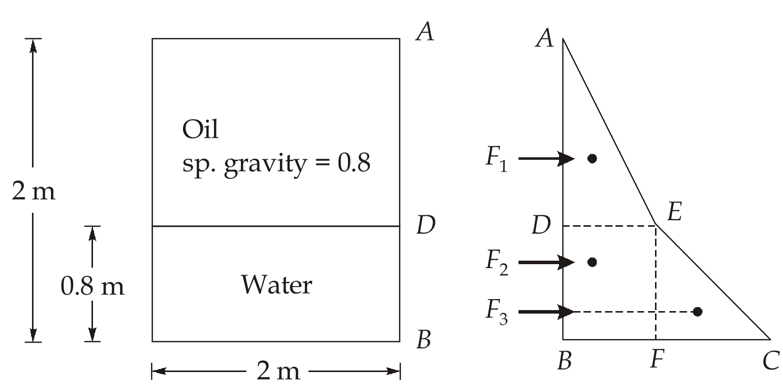
Specific gravity of oil = 0.8

Density of oil, $\rho_1 = 0.8 \times 1000 = 800\ kg/m^3$

Density of water, $\rho_2 = 1000\ kg/m^3$

(i)

Total pressure on one vertical side is calculated by drawing pressure diagram.



Intensity of pressure at A, $P_A = 0$

Intensity of pressure at D, $P_D = \rho_1 g h_1$

$= 800 \times 9.81 \times 1.2$

$= 9417.6 \text{ N/m}^2$

Intensity of pressure at B, $P_B = \rho_1 g h_1 + \rho_2 g h_2$

$= 800 \times 9.81 \times 1.2 + 1000 \times 9.81 \times 0.8$

$= 9417.6 + 7848 = 17265.6 \text{ N/m}^2$

Hence in pressure diagram,

$$DE = 9417.6 \text{ N/m}^2$$

$$BC = 17265.6 \text{ N/m}^2$$

$$FC = 7848 \text{ N/m}^2$$

The pressure diagram is split into triangle ADE, rectangle BDEF and triangle EFC. The total pressure force consists of the following components.

1. Force, F_1 = Area of triangle of ADE × Width of tank

$$= \left(\frac{1}{2} \times AD \times DE\right) \times 2$$

$$= \frac{1}{2} \times 1.2 \times 9417.6 \times 2 = 11301.12 \text{ N}$$

This force will be acting at C.G. of the triangle ADE i.e. at a distance of $\frac{2}{3} \times 1.2 = 0.8$ m below A.

2. Force, F_2 = Area of rectangle BDEF × Width of tank

$= (BD \times DE) \times 2$

$= 0.8 \times 9417.6 \times 2$

$= 15068.16$ N

This force will be acting at the C.G. of the rectangle BDEF i.e. at a distance $1.2 + \frac{0.8}{2} = 1.6$ m below A.

3. Force, F_3 = Area of triangle of EFC × Width of tank

$$= \frac{1}{2} \times EF \times FC \times 2$$

$$= \frac{1}{2} \times 0.8 \times 7848 \times 2$$

$$= 6278.4 \text{ N}$$

This force will be acting at the C.G. of the triangle EFC i.e. at a distance $1.2 + \frac{2}{3} \times 0.8 = 1.7333$ m below A.

∴ Total pressure force on one vertical side of the tank,

$$F = F_1 + F_2 + F_3$$

$$= (11301.12 + 15068.16 + 6278.4) \text{ N}$$

$$= 32647.68 \text{ N}$$

(ii) Position of centre of pressure,

Let, total force F is acting at a depth of h^* from the free surface of liquid, i.e. from A.

Taking the moments of all forces about A, we get

$$F \times h^* = F_1 \times 0.8 + F_2 \times 1.6 + F_3 \times 1.7333$$

$$h^* = \frac{11301.12 \times 0.8 + 15068.16 \times 1.6 + 6278.4 \times 1.7333}{32647.68}$$

$$h^* = 1.3487 \text{ m from A}$$ **Ans.**

4. (c) Solution:

By Karman momentum integral equation,

$$\tau_o = \rho U^2 \frac{\partial}{\partial x}\left[\int_o^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right]$$

Put, $\frac{y}{\delta} = \eta$, $dy = \delta \cdot d\eta$ and the limits of η are 0 and 1.

$$\frac{u}{U} = f(\eta) = 2\eta - 2\eta^3 + \eta^4$$

Substituting:

$$\tau_o = \rho U^2 \frac{d\delta}{dx}\left[\int_o^1 f(\eta)(1 - f(\eta)) d\eta\right]$$

But, $$\int_o^1 f(\eta)(1 - f(\eta)) d\eta = \int_o^1 \left(2\eta - 2\eta^3 + \eta^4\right)\left(1 - 2\eta + 2\eta^3 - \eta^4\right) d\eta$$

$$= \int_o^1 \left(2\eta - 4\eta^2 - 2\eta^3 + 9\eta^4 - 4\eta^5 - 4\eta^6 + 4\eta^7 - \eta^8\right) d\eta$$

$$= \left[\eta^2 - \frac{4\eta^3}{3} - \frac{2\eta^4}{4} + \frac{9\eta^5}{5} - \frac{4\eta^6}{6} - \frac{4\eta^7}{7} + \frac{4\eta^8}{8} - \frac{\eta^9}{9}\right]_o^1$$

$$= \frac{37}{315}$$

Therefore, $$\tau_o = \frac{37}{315}\rho U^2 \frac{d\delta}{dx} \quad \text{...(i)}$$

From the boundary conditions for a laminar boundary layer,

$$\tau_o = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu U}{\delta}\left[\frac{df(\eta)}{d\eta}\right]_{\eta=0}$$

Since, $$f(\eta) = 2\eta - 2\eta^3 + \eta^4$$

$$\left[\frac{df(\eta)}{d\eta}\right]_{\eta=0} = [2 - 6\eta + 4\eta^3]_{\eta=0} = 2$$

$\Rightarrow$ $$\tau_o = \frac{2\mu U}{\delta} \quad \text{...(ii)}$$

Equating the two expression for τ_o

$$\frac{2\mu U}{\delta} = \frac{37}{315}\rho U^2 \frac{d\delta}{dx}$$

$$\delta d\delta = \frac{630}{37}\frac{\mu}{\rho U}dx$$

On integrating, $$\frac{\delta^2}{2} = \frac{630}{37}\frac{\mu}{\rho U}x + C$$

Where, C = Constant of integration

B.C. : At $x = 0$, $\delta = 0$

$\Rightarrow$ $C = 0$

$$\delta^2 = \frac{1260}{37}\frac{\mu x}{\rho U} = \frac{34.054 x^2}{\left(\frac{\rho U x}{\mu}\right)}$$

$$\frac{\delta}{x} = \frac{5.835}{\sqrt{Re_x}} \quad \text{where } Re_x = \frac{\rho U x}{\mu}$$

$$\delta = \frac{(5.835)x}{\sqrt{Re_x}}$$

Ans.

Shear stress: Substituting the value of δ in the equation (ii)

$$\tau_o = \frac{2\mu U}{\delta} = \frac{2\mu U}{5.835x}\sqrt{\left(\frac{\rho U x}{\mu}\right)}$$

$$= 0.3427\frac{\mu^{1/2}U^{3/2}\rho^{1/2}}{x^{1/2}}$$

$$\tau_o = 0.6855 \times \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U x}\right)^{1/2}$$

$$\Rightarrow \quad \tau_o = \left(\frac{1}{2}\rho U^2\right)\frac{0.6855}{\sqrt{\text{Re}_x}} \quad \textbf{Ans.}$$

i.e.

$$\frac{\tau_o}{\rho U^2/2} = C_f = \frac{0.6855}{\sqrt{\text{Re}_x}}$$

Force on one side of plate:

Let F_D be force on one side of plate of unit width and length L,

$$F_D = \int_o^L \tau_o dx = \frac{\rho U^2}{2}\int_o^L \frac{0.6855}{(\rho U x/\mu)^{1/2}}dx$$

$$= \frac{\rho U^2}{2} \times \left(\frac{\mu}{\rho U}\right)^{1/2} \times \int_o^L \frac{0.6855}{(x)^{1/2}}dx$$

$$= \frac{\rho U^2}{2} \times \left(\frac{\mu}{\rho U}\right)^{1/2} \times 0.6855\left[2\sqrt{x}\right]_o^L$$

$$F_D = \frac{\rho U^2}{2} \times \left(\frac{\mu}{\rho U}\right)^{1/2} \times 1.371 \times L^{1/2}$$

$$\frac{F_D}{\frac{1}{2}\rho U^2 L} = C_{Df} = \frac{1.371}{(\rho U L/\mu)^{1/2}} = \frac{1.371}{\sqrt{\text{Re}_L}}$$

$$\Rightarrow \quad F_D = \left(\frac{1}{2}\rho U^2\right)(L)\left(\frac{1.371}{\sqrt{\text{Re}_L}}\right) \quad \textbf{Ans.}$$

Section B : Strength of Materials & Mechanics-1 + Thermodynamics-2 + IC Engine-2 + Refrigeration and Air-Conditioning-2

5. (a) Solution:

The proper combination of temperature, humidity and air movement, which induce a feeling such that any human being feels comfortable is known as human comfort condition.

Factors governing optimum effective temperature are:

1. Temperature of air
2. Relative humidity of air
3. Velocity of air
4. Purity of air
5. Standing wall temperature

Cooling load estimate:

Room load

(A) Room sensible heat (RSH)

(i) Solar and transmission heat gain through walls, roofs, etc.

(ii) Solar and transmission heat gain through glass.

(iii) Transmission gain through partition walls, ceiling, floor, etc.

(iv) Infiltration

(v) Internal heat gain not accounted above.

(vi) By passed outside air load.

The sum of items (i) to (vi) gives the effective room sensible heat (ERSH).

(B) Room latent heat (RLH)

(i) Infiltration

(ii) Internal heat gain from people, steam, appliances, etc.

(iii) Vapour transmission

(iv) Additional heat gain

(v) Supply duct leakage loss

The sum of (i) to (v) gives the effective room latent heat ERLH and ERLH + ERSH = ERTH (Effective room total heat).

Heating load estimate:

The following points are considered:

(i) Transmission Heat Loss: The transmission heat loss from walls, roofs, etc. is calculated on the basis of outside and inside air temperature difference $Q = UA(t_1 - t_o)$.

(ii) Solar Radiation: There is generally no solar radiation present and hence, there is no solar heat gain at the time of the peak load which normally occurs in the early hours of the morning.

(iii) Internal Heat Gains: Internal heat gains from occupants, lights motors and machinery, etc. diminish the heating requirement. These negative loads should be accounted for in application such as theatres, assembly halls, stores, office, building etc.

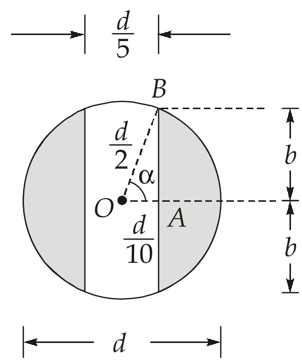
5. (b) Solution:

Given : Diameter of solid bar = d mm

Allowable tensile stress = σ_{allow} MPa

(i)

For the cross-section of the bar shown below



Let, α = Angle in radian

From ΔOAB,

$$\cos\alpha = \frac{OA}{OB}$$

$$\alpha = \cos^{-1}\left[\frac{d/10}{d/2}\right]$$

$$\alpha = 78.463° = 1.3694 \text{ rad}$$

$$b = \sqrt{\left(\frac{d}{2}\right)^2 - \left(\frac{d}{10}\right)^2} = \frac{\sqrt{6}}{5}d$$

$$\text{Area of dashed part, } A = 2\times\left[\frac{(2\alpha)}{2}\times\left(\frac{d}{2}\right)^2 - \frac{1}{2}\times\frac{d}{10}\times\frac{2\times\sqrt{6}d}{5}\right]$$

$$A = \left[\frac{\alpha d^2}{2} - \frac{\sqrt{6}d^2}{25}\right]$$

Now, $P_{allow} = \sigma_{allow} \times A$

$$P_{allow} = \sigma_{allow} \times \left[\frac{\alpha d^2}{2} - \frac{\sqrt{6}d^2}{25}\right]$$ **Ans.**

(ii) As, $d = 50$ mm and $\sigma_{allow} = 140$ MPa

$$P_{allow} = 140 \times \left[\frac{1.3694 \times (50)^2}{2} - \frac{\sqrt{6} \times (50)^2}{25}\right]$$

$$P_{allow} = 205.35 \text{ kN}$$ **Ans.**

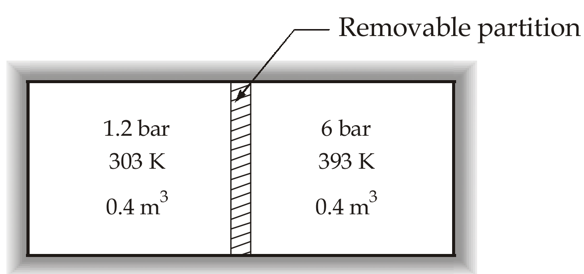
5. (c) Solution:

Given data : $P_1 = 1.2$ bar; $P_2 = 6$ bar; $T_1 = 30 + 273 = 303$ K; $T_2 = 120 + 273 = 393$ K

Assumptions :

(i) Air is considered as ideal gas

(ii) Specific heats are constant with temperature and pressure.



Let us indicate by subscript 1 and 2 the state of air in two compartments, and by subscript 3 the state of air in the vessel after the removal of partition and mixing of air.

After mixing, $V_3 = V_1 + V_2 = 0.8 \text{ m}^3$

and, $m_3 = m_1 + m_2$

Since, no heat or work interaction taking place

$$U_3 = U_1 + U_2$$

$$m_3 u_3 = m_1 u_1 + m_2 u_2$$

$$u_3 = \frac{m_1 u_1 + m_2 u_2}{m_3} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} \quad \text{...(i)}$$

here, $$m_1 = \frac{P_1 V_1}{RT_1} = \frac{1.2 \times 10^5 \times 0.4}{0.287 \times 10^3 \times 303} = 0.5519 \text{ kg}$$

$$m_2 = \frac{P_2 V_2}{RT_2} = \frac{6 \times 10^5 \times 0.4}{0.287 \times 10^3 \times 393} = 2.127 \text{ kg}$$

Since, for an ideal gas, $du = c_v dT$, taking datum as 0°C, we write $u = c_v T$,

From equation (i),
$$c_v T_3 = \frac{m_1 c_v T_1 + m_2 c_v T_2}{m_1 + m_2}$$

$$T_3 = \frac{0.5519 \times 303 + 2.127 \times 393}{0.5519 + 2.127} = 374.45 \text{ K} \text{ or } 101.46°C$$

Final pressure,
$$P_3 = \frac{m_3 R T_3}{V_3}$$
$$= \frac{(0.5519 + 2.127) \times 0.287 \times 374.45}{0.8} \text{ kPa}$$
$$= 3.598 \text{ bar}$$

$$\Delta S_1 = S_3 - S_1$$
$$= m_1\left[c_v \ln\frac{T_3}{T_1} + R\ln\frac{V_3}{V_1}\right]$$
$$= 0.5519 \times \left[0.718 \ln\frac{374.45}{303} + 0.287 \ln\frac{0.8}{0.4}\right]$$
$$= 0.19337 \text{ kJ/K}$$

$$\Delta S_2 = S_3 - S_2$$
$$= 2.127 \times \left[0.718 \ln\frac{374.45}{393} + 0.287 \ln\frac{0.8}{0.4}\right]$$
$$= 0.3493 \text{ kJ/K}$$

$$(\Delta S)_{system} = \Delta S_1 + \Delta S_2$$
$$= 0.19337 + 0.3493 = 0.54267 \text{ kJ/K}$$

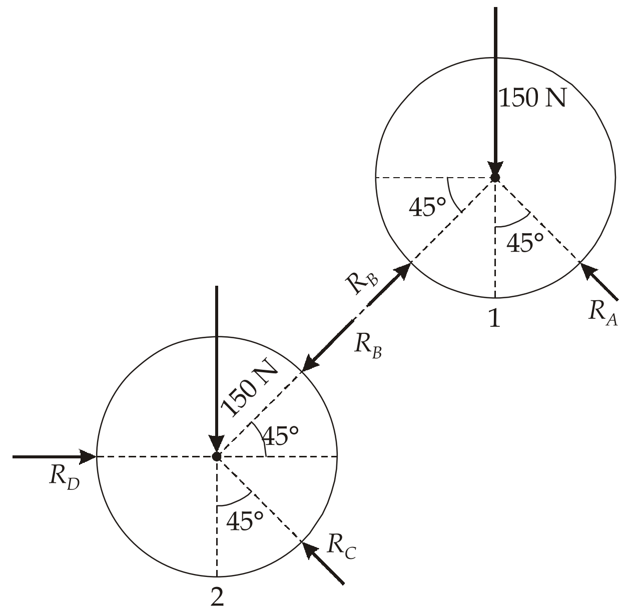
Since, there is no heat interaction with surrounding

$$(\Delta S)_{surrounding} = 0$$
$$(\Delta S)_{universe} = (\Delta S)_{system} + (\Delta S)_{surrounding}$$
$$= 0.54267 + 0 = 0.54267 \text{ kJ/K}$$

Irreversibility,
$$I = T_0\left(\Delta S_{system} + \Delta S_{surrounding}\right)$$
$$= 300 \times 0.54267 = 162.80 \text{ kJ}$$

5. (d) Solution:

Given : $W_1 = W_2 = 150$ N



FBD of two rollers

From the FBD of roller 1;

$\Sigma F_H = 0$

$$\Rightarrow \quad R_A \sin 45° = R_B \cos 45°$$

$$\therefore \quad R_A = R_B \quad \text{...(i)}$$

Also, $\Sigma F_V = 0$

$$R_A \cos 45° + R_B \sin 45° = W$$

$$\Rightarrow \quad \frac{2 \times R_A}{\sqrt{2}} = 150$$

$$\Rightarrow \quad R_A = \frac{150}{\sqrt{2}} = 106.07 \text{ N} \quad \text{or } 75\sqrt{2} \text{ N} \quad \text{Ans.}$$

From equation (i), $R_A = R_B = 106.07$ N **Ans.**

From the FBD of roller 2:

$\Sigma F_H = 0$

$$\Rightarrow \quad R_B \cos 45° + R_C \sin 45° = R_D \quad \text{...(ii)}$$

$$\Rightarrow \quad 75\sqrt{2} \times \frac{1}{\sqrt{2}} + \frac{R_C}{\sqrt{2}} = R_D$$

$$75 + \frac{R_C}{\sqrt{2}} = R_D \quad \text{...(iii)}$$

$\Sigma F_V = 0$

$$\Rightarrow \quad R_B \sin 45° + W = R_C \cos 45°$$

$$75\sqrt{2} \times \frac{1}{\sqrt{2}} + 150 = R_C \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad R_C = 225\sqrt{2} \text{ N or } 318.19 \text{ N}$$ **Ans.**

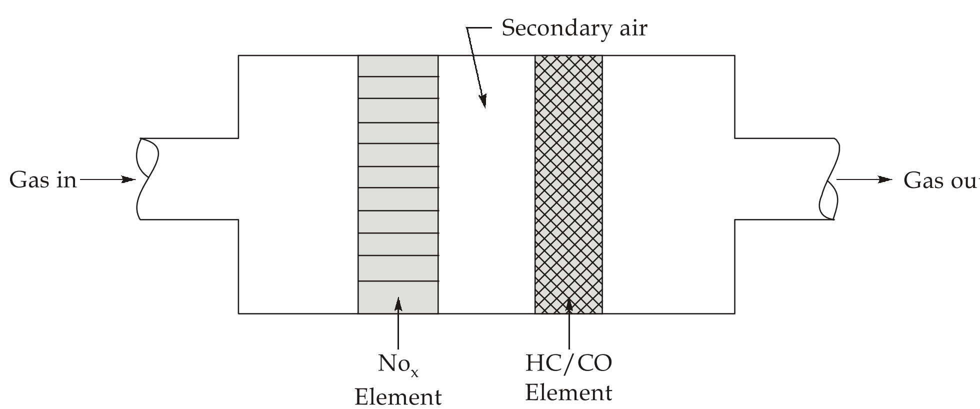
From equation (iii),

$$75 + \frac{225\sqrt{2}}{\sqrt{2}} = R_D$$

$$\Rightarrow \quad R_D = 300 \text{ N}$$ **Ans.**

5. (e) Solution:

Construction: Catalytic converter is usually a stainless steel container mounted somewhere along the exhaust pipe of the engine. Inside this container there is a porous ceramic structure through which the exhaust gas flow. In most converters, the ceramic is a single honeycomb structure with many flow passages as shown in figure. Some converters use loose granular ceramic with the gas passing between the packed spheres. Volume of the ceramic structure of a catalytic converter is generally about half the displacement volume of the engine. Catalytic converters for CI engines need larger flow passages because of the solid soot in the exhaust gases.



Catalytic converter

Working: The surface of the ceramic passages contains small embedded particles of catalytic material that promote the oxidation reactions in the exhaust gas as it passes. Aluminum oxide (alumina) is the base ceramic material used for most catalytic converters. Alumina can withstand the high temperatures, it remains chemically neutral, it has very low thermal expansion, and it does not thermally degrade with age. The catalyst materials most commonly used are platinum, palladium, and rhodium. Palladium and platinum promote the oxidation of CO and HC.

$$CO + \frac{1}{2}O_2 \rightarrow CO_2$$

$$C_xH_y + 2O_2 \rightarrow xCO_2 + yH_2O$$

Rhodium promotes the reaction of NO_x in one or more of the following reactions:

$$NO + CO \rightarrow \frac{1}{2}N_2 + CO_2$$

$$2NO + 5\,CO + 3\,H_2O \rightarrow 2NH_3 + 5CO_2$$

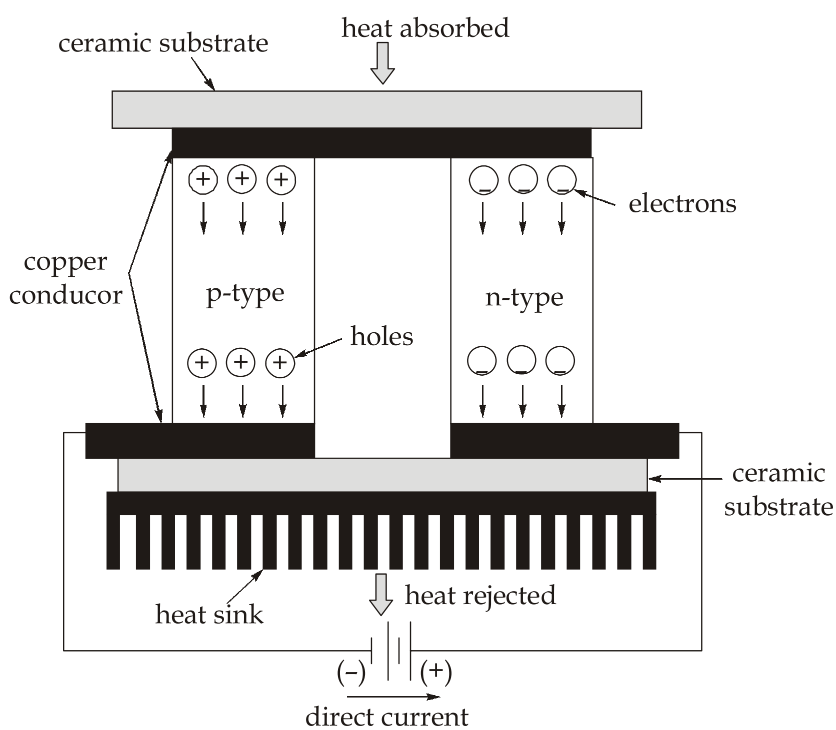
$$2NO + CO \rightarrow N_2O + CO_2$$

$$NO + H_2 \rightarrow \frac{1}{2}N_2 + H_2O$$

$$2NO + 5H_2 \rightarrow 2NH_3 + 2H_2O$$

$$2NO + H_2 \rightarrow N_2O + H_2O$$

6. (a) Solution:



Schematic of Thermoelectric cooler

The thermoelectric effect is the direct conversion of temperature differences to electric voltage and vice verse via a thermocouple. A thermoelectric device creates voltage when there is a different temperature on each side. Conversely, when a voltage is applied to it, it creates a temperature difference. At the atomic scale, an applied temperature gradient causes charge carriers in the material to diffuse from the hot side to the cold side. This effect can be used to generate electricity, measure temperature change of objects. Because the direction of heating and cooling is determined by the polarity of the applied voltage, thermoelectric devices can be used as temperature controllers.

Thermoelectric refrigeration modules are solid-state heat pumps that operate on the Peltier effect. A thermoelectric modules consists of an array of p-type semiconductor elements that are heavily doped with electrical carriers. The elements are arranged into array that is electrically connected in series but thermally connected in parallel. This array is then affixed to two ceramic substrates, one on each side of the elements.

The p-type semiconductor is doped with certain atoms that have fewer electrons than necessary to complete the atomic bonds within the crystal lattice. When a voltage is applied, there is a tendency for conduction electrons to complete the atomic bonds. When conduction electrons do this, they leave "holes" which essentially dropping in and being bumped out of the holes and moving on to the next available hole. In effect, it is the holes that are acting as the electrical carries. Now, electrons move much more easily in the copper conductors but not so easily in the semiconductors. When electrons leave the p-type and enter into the copper on the cold-side, holes are created in the p-type as the electrons jump out to a higher energy level to match the energy level of the electrons already moving in the copper. The extra energy to create these holes comes by absorbing heat. Meanwhile, the newly created holes travel downwards to the copper on the hot side. Electrons from the hot-side copper move into the p-type and drop into the holes, releasing the excess energy in the form of heat. The n-type semiconductor is doped with atoms that provide more electrons than necessary to complete the atomic bonds within the crystal lattice. When a voltage is applied, these extra electrons are easily moved into the conduction band. However, additional energy is required to get the n-type electrons to match the energy level of the incoming electrons from the cold-side copper. The extra energy comes by absorbing heat. Finally, when the electrons leave the hot-side of the n-type, they once again can move freely in the copper. They drop down to a lower energy level, and release heat in the process. The above explanation points out that heat is always absorbed at the cold side of the n and p type elements, and heat is always released at the hot side of thermoelectric element. The heat pumping capacity of a thermoelectric refrigeration module is proportional to the current and is dependent on the element geometry, number of couples, and material properties.

Application areas of thermo electric cooling system are:

(i) Electronic enclosures.

(ii) Laser diodes

(iii) Laboratory instruments

(iv) Temperature baths

(v) Telecommunications equipment

(vi) Temperature control in missiles and space systems.

6. (b) Solution:

Given : P_1 = 20.98 MPa; T_c = 647.3 K; T_1 = 633.22°C = 906.22 K; P_c = 220.9 bar;

T_c = 439.03°C = 712.03; m = 18.02 kg/Kmol

Water vapour is a closed system because it is contained in a rigid closed container.

(i)

$$\text{Reduced temperature, } T_{R_1} = \frac{906.22}{647.3} = 1.4$$

$$\text{and} \quad \text{Reduced pressure, } P_{R_1} = \frac{20.98}{22.09} = 0.949 \simeq 0.95$$

For these values of reduced temperature and reduced pressure, the value of z obtained from the compressibility chart is approximately 0.9.

We know that
$$z = \frac{Pv}{RT}$$

So, the specific volume at state 1 can be determined as follows:

$$v_1 = z_1 \frac{RT_1}{P_1}$$

$$= 0.9\left[\frac{8314}{18.02}\right] \times \left[\frac{906.22}{20.98 \times 10^6}\right] = 0.01794 \text{ m}^3/\text{kg}$$

(ii)

Since both mass and volume remain constant, the water vapour cools at constant specific volume and thus, at constant V_R', using the value for specific volume determine in part (i), the constant V_R' value is

$$V_R' = \frac{vP_c}{RT_c}$$

$$= \frac{0.01794 \times \left(22.09 \times 10^6\right)}{\left(\frac{8314}{18.02}\right) \times 647.3} = 1.33$$

and
$$T_{R_2} = \frac{712.03}{647.3} = 1.10$$

Locating the point on the compressibility chart where V'_R = 1.33 and T_R = 1.10, the corresponding value for P_R is approximately 0.68.

So, $$P_2 = P_c (P_{R2})$$
$$= 22.09 \times 0.68 = 15.02 \text{ MPa}$$ **Ans.**

6. (c) Solution:

Given data : d_1 = 140 mm; d_2 = 80 mm; L = 1600 mm; E = 6 GPa; P = 130 kN; δ_a = 9.0 mm

(i)

$$\delta_a = 9.0 \text{ m}$$

According to question,

$$\delta_{total} = \delta_a$$

$$\frac{\frac{PL}{4}}{\frac{\pi}{4}\left(d_1^2 - d_{max}^2\right)\times E} + \frac{\frac{PL}{4}}{\frac{\pi}{4}\left(d_1^2\right)\times E} + \frac{\frac{PL}{2}}{\frac{\pi}{4}\left(d_2^2\right)\times E} = \delta_a$$

On putting the values,

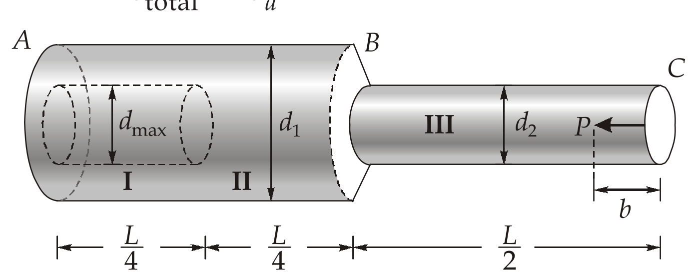
$$\frac{10^3}{6\times10^3}\times\left[\frac{130\times\frac{1600}{4}}{\frac{\pi}{4}\times\left(140^2 - d_{max}^2\right)} + \frac{130\times\frac{1600}{4}}{\frac{\pi}{4}\times140^2} + \frac{130\times\frac{1600}{2}}{\frac{\pi}{4}\times80^2}\right] = 9$$

$$\frac{130\times\frac{1600}{4}\times10^3}{\frac{\pi}{4}\times6\times10^3}\left[\frac{1}{140^2 - d_{max}^2} + \frac{1}{140^2} + \frac{2}{80^2}\right] = 9$$

$$d_{max} = 131.86 \text{ mm}$$

(ii) Now, if d_{max} is instead set at $\frac{d_2}{2}$ i.e. d_{max} = 40 mm and δ_a = 3.0 mm

$$\delta_{total} = \delta_a$$



$$\delta_I + \delta_{II} + \delta_{III} = \delta_a$$

$$\frac{\frac{PL}{4}}{\frac{\pi}{4}\times\left(d_1^2-d_{\max}^2\right)E}+\frac{\frac{PL}{4}}{\frac{\pi}{4}\times d_1^2E}+\frac{P\left(\frac{L}{2}-b\right)}{\frac{\pi}{4}\times d_2^2E} = \delta_a$$

On putting the values,

$$\frac{10^3}{6\times10^3}\times\left[\frac{130\times\frac{1600}{4}}{\frac{\pi}{4}\times\left(140^2-40^2\right)}+\frac{130\times\frac{1600}{4}}{\frac{\pi}{4}\times140^2}+\frac{130\times\left(\frac{1600}{2}-b\right)}{\frac{\pi}{4}\times80^2}\right] = 3$$

$$b = 376.85 \text{ mm}$$

(iii) $d_{\max}$ = 120 mm, δ_a = 5.0 mm

$$\delta_{total} = \delta_a$$

$$\frac{P\times x}{\frac{\pi}{4}\left(d_1^2-d_{\max}^2\right)\times E}+\frac{P\left(\frac{L}{2}-x\right)}{\frac{\pi}{4}\times d_1^2E}+\frac{\frac{PL}{2}}{\frac{\pi}{4}d_2^2\times E} = \delta_a$$

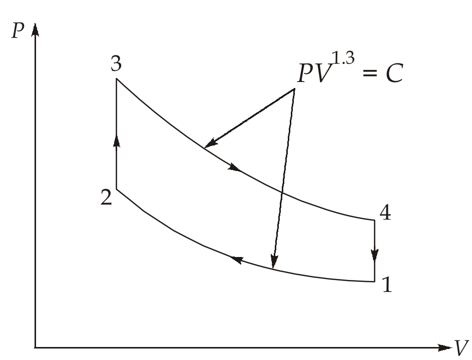
On putting the values

$$\frac{10^3}{6\times10^3}\left[\frac{130\times x}{\frac{\pi}{4}\times\left(140^2-120^2\right)}+\frac{130\times\left(\frac{1600}{2}-x\right)}{\frac{\pi}{4}\times140^2}+\frac{130\times\left(\frac{1600}{2}\right)}{\frac{\pi}{4}\times80^2}\right] = 5$$

$$x = 109.205 \text{ mm}$$

7. **(a) Solution:**

Given : Compression ratio, r = 9; Air fuel ratio = 14 : 1



The Stoichiometric equation can be written as

$$C_8H_{18} + \frac{25}{2}\left[O_2 + \frac{79}{21}N_2\right] \rightarrow 8CO_2 + 9H_2O + \frac{25}{2} \times \frac{79}{21}N_2$$

$$\text{Stoichiometric air/fuel ratio} = \frac{12.5 \times \left(32 + \frac{79}{21} \times 28\right)}{(12 \times 8) + (1 \times 18)} = 15.06$$

With the given air/fuel ratio as 14, the mixture is rich in fuel. Therefore the combustion will be incomplete.

The chemical reaction becomes:

$$C_8H_{18} + y\left[O_2 + \frac{79}{21}N_2\right] \rightarrow xCO + (8-x)CO_2 + 9H_2O + \frac{79}{21}yN_2$$

$$\frac{\text{Air}}{\text{Fuel}} = 14 = \frac{y\left(32 + \frac{79}{21} \times 28\right)}{(12 \times 8) + (1 \times 18)}$$

$$y = 11.62$$

By oxygen balance:

$$y = \frac{x}{2} + (8-x) + 4.5$$

$$11.62 = 0.5x + 8 - x + 4.5$$

$$0.5x = 8 + 4.5 - 11.62$$

$$x = 1.76$$

The chemical equation now becomes:

$$C_8H_{18} + 11.62\left(O_2 + \frac{79}{21}N_2\right) \rightarrow 1.76CO + 6.24CO_2 + 9H_2O + \frac{79}{21} \times 11.62N_2$$

$$\text{Number of moles before combustion} = 1 + 11.62 + \left(\frac{79}{21} \times 11.62\right) = 56.33$$

Number of moles after combustion = 1.76 + 6.24 + 9 + 43.71 = 60.71

$$\text{Molecular expansion} = \frac{60.71 - 56.33}{56.33} \times 100 = 7.78\%$$ **Ans.**

(i) P_1 = 1.2 bar; T_1 = 65 + 273 = 338 K; n = 1.3; r = 9

$$\frac{T_2}{T_1} = (r)^{n-1}$$

$$T_2 = 338 \times (9)^{0.3} = 653.42 \text{ K}$$

$$q_{2\text{-}3} = c_v(T_3 - T_2)$$

$$\frac{44000}{15} = 0.71(T_3 - 653.42)$$

$$T_3 = 4784.87 \text{ K}$$ **Ans.**

For pressure,

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3}$$

$$P_3 = \frac{V_1}{V_3} \times \frac{T_3}{T_1} \times P_1$$

$$= 9 \times \frac{4784.87}{338} \times (1.2) = 152.89 \text{ bar}$$

(ii)

Since the mass of the reactants and products is the same and specific heats are assumed same, the temperature of the products with molecular expansion will remain the same as without molecular expansion, only pressure will change.

$\therefore$

$$T_3 = 4784.87 \text{ K}$$ **Ans.**

$$PV = n\bar{R}T$$

$$P \propto n$$

$$\frac{P_3'}{P_3} = \frac{n'}{n}$$

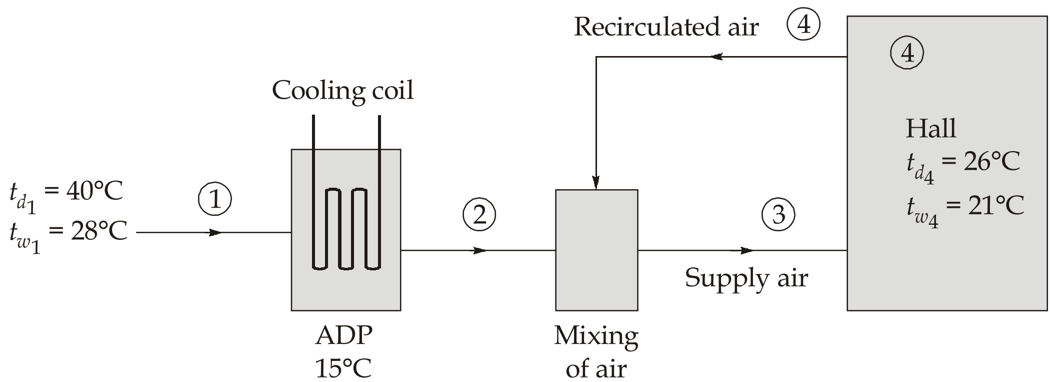
where n is the number of moles of products without molecular expansion and n' is the number of moles of products with molecular expansion.

$$P_3' = 152.89 \times \frac{60.71}{56.33} = 164.78 \text{ bar}$$ **Ans.**

7. (b) Solution:

Given data : ADP = 15°C; $t_{d_4} = 26°C$; $t_{w_1} = 28°C$; $t_{w_4} = 21°C$; $t_{d_1} = 40°C$;

$Q_{S_4} = 48$ kW; $Q_{L_4} = 18$ kW, $v_1 = 28$ m^3/min



The line diagram can be drawn as discussed below:

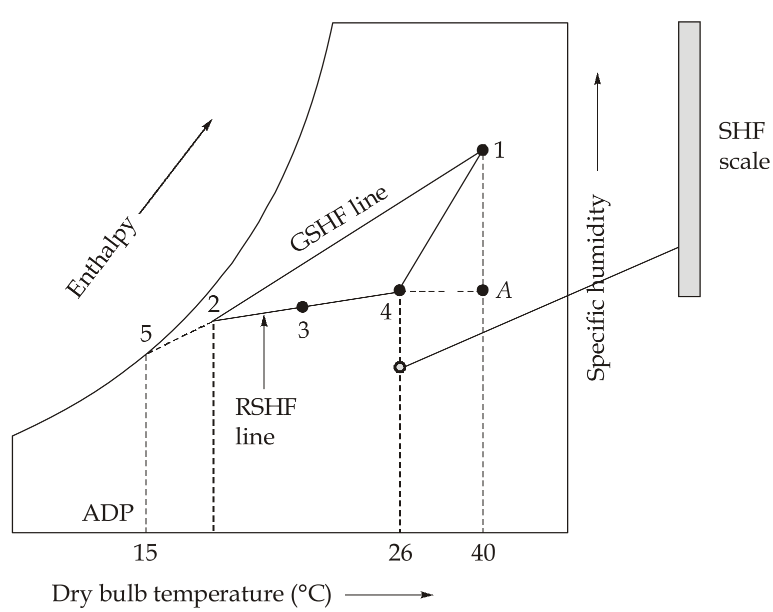
First of all, locate the condition of outside air i.e. at 40°C dry bulb temperature and 28°C wet bulb temperature on the psychrometric chart as point 1, now mark the condition of air in the hall, i.e. at 26 dry bulb temperature and 21°C wet bulb temperature at point 4. Mark point A by drawing vertical and horizontal lines from points 1 and 4 respectively. Since 28 m^3/min of outside air at 40°C and 28°C is supplied directly into the room through ventilation and infiltration, therefore the sensible heat and latent heat of 28 m^3/min infiltrated air are added to hall in addition to Sensible heat load of 48 kW and latent heat load of 18 kW.

From Psychrometric chart,

Enthalpy, h_1 = 90 kJ/kg of dry air

h_4 = 61 kJ/kg of dry air

h_A = 76 kJ/kg of dry air



Also, specific volume of air at point 1,

$$v_{s_1} = 0.915 \text{ m}^3/\text{kg of dry air}$$

∴ Mass of infiltrated air into the hall,

$$\dot{m}_a = \frac{V_1}{v_{s_1}} = \frac{28}{0.915} = 30.601 \text{ kg/min}$$

Latent heat load due to infiltrated air,

$$Q_{L_1} = \dot{m}_a(h_1 - h_A)$$

$$= 30.601 \times (90 - 76) = 428.41 \text{ kJ/min}$$

$$= 7.14 \text{ kW}$$

Sensible heat load due to infiltrated air,

$$Q_{s_1} = \dot{m}_a(h_A - h_4)$$

$$= 30.601 \times (76 - 61) = 459.015 \text{ kJ/min}$$

$$= 7.65 \text{ kW}$$

∴ Total room sensible heat load,

$$\text{RSH} = Q_{S4} + Q_{S1}$$

$$= 48 + 7.65 = 55.65 \text{ kW}$$

and total room latent heat load,

$$\text{RLH} = Q_{L4} + Q_{L1}$$

$$\text{RLH} = 18 + 7.14$$

$$= 25.14 \text{ kW}$$

Now, room sensible heat factor

$$\text{RSHF} = \frac{RSH}{RSH + RLH}$$

$$\text{RSHF} = \frac{55.65}{55.65 + 25.14} = 0.689$$

Now, mark this calculated value of RSHF on the sensible heat factor scale and join with the alignment circle (i.e. 26°C DBT and 50% RH). From point 4, draw a line 4 - 5 (known as RSHF line) parallel to this line. Since the outside air marked at point 1 is passed through the cooling coil with ADP = 15°C, therefore join point 1 with ADP = 15°C on the saturation curve. This line is the GSHF line and intersects the RSHF line at point 2, which represents the condition of air leaving the cooling coil. Also, 60% of air from the hall is recirculated and mixed with the conditioned air after the cooling coil. The mixing condition of air is shown at point 3 such that

$$\frac{\text{Length 2 - 3}}{\text{Length 2 - 4}} = 0.6$$

This gives the condition of air entering the hall

(i) Condition of air after the coil and before, the recirculated air mixes with it. i.e. point 2

From Psychrometric chart

$$t_{d_2} = 22°C$$

$$t_{w_2} = 19.3°C, \phi_2 = 79\%$$

(ii) Condition of air entering the hall, i.e. after mixing with recirculated air. i.e. point 3

From Psychrometric chart

$$t_{d_3} = 24.4°C$$

$$t_{w_3} = 20.3°C, \phi_3 = 70\%$$

(ii) Mass of fresh air entering the cooler.

$$\dot{m}_f = \frac{\text{Total heat removed}}{h_4 - h_2} = \frac{RSH + RLH}{h_4 - h_2}$$

$$m_f = \frac{55.65 + 25.14}{61 - 55.5} = 14.69 \text{ kg/s}$$

(iv) Bypass factor of cooling coil

$$\text{BPF} = \frac{t_{d_2} - ADP}{t_{d_1} - ADP} = \frac{22 - 15}{40 - 15} = 0.28$$

7. (c) Solution :

Force equilibrium in vertical direction,

$\Sigma F_v = 0$

$$\Rightarrow \quad R_A + R_B = w(a + L) \quad \text{...(i)}$$

Taking moment about A, $\Sigma M_A = 0$

$$\Rightarrow \quad \frac{w(L+a)^2}{2} = R_B \times L$$

$$\Rightarrow \quad R_B = \frac{w(L+a)^2}{2L}$$

From equation (i), we get

$$R_A = w(a+L) - R_B = w(L+a) - \frac{w(L+a)^2}{2L} = \frac{w(a+L)(L-a)}{2L}$$

$$\Rightarrow \quad R_A = \frac{w(a+L)(L-a)}{2L}$$

From the above reaction force at B, R_B will be always acting upward (i.e. for $L < a$, $L = a$ and $L > a$)

But, if $L > a$, R_A will act upwards

If $L = a$, $R_A = 0$

If $L < a$, R_A will act downwards

Now, for $w = 4$ kN/m, $L = 8$ m and $a = 4$ m

$$R_A = \frac{w(a+L)(L-a)}{2L} = \frac{4(4+8)(8-4)}{2\times 8} = 12 \text{ kN } (\uparrow)$$

$$R_B = \frac{w(L+a)^2}{2L} = \frac{4(8+4)^2}{2\times 8} = 36 \text{ kN} (\uparrow)$$

S.F.D.: for AB, $\quad F_x = R_A - wx = 12 - 4 \times x$

At $x = 0$, $\quad F_A = 12$ kN

At $x = L = 8$ m, $\quad F_B\text{(left)} = 12 - 4 \times 8 = -20$ kN

$$\text{S.F. is zero at } x = \frac{12}{4} = 3 \text{ m}$$

For BC, $\quad F_x = 12 - 4x + 36 = 48 - 4x$

At $x = 8$ m, $\quad F_B\text{(right)} = 48 - 4 \times 8 = 16$ kN

At $x = 12$ m, $\quad F_C = 48 - 4 \times 12 = 0$

B.M.D.: for AB, $\quad M_x = R_A x - \frac{w \times x^2}{2} = 12x - \frac{4x^2}{2} = 12x - 2x^2$

At $x = 0$, $M_A = 0$

At $x = L = 8$ m, $M_B = 12 \times 8 - 2 \times 8^2 = -32$ kNm

For maximum B.M., $\quad \frac{dM_x}{dx} = 0$

$$\Rightarrow \quad 12 - 2 \times 2 \times x = 0$$

$$\Rightarrow \quad x = 3 \text{ m}$$

Hence, the B.M. is maximum where S.F. is zero.

$$M_{max} = 12 \times 3 - 2 \times 3^2 = 18 \text{ kNm}$$

For B.M. to be zero in AB, we have $12x - 2x^2 = 0$

which gives, $x = 6$ m

For BC,
$$M_x = R_A \cdot x - \frac{w \times x^2}{2} + R_B(x-8)$$

$$M_x = 12x - \frac{4x^2}{2} + 36(x-8)$$

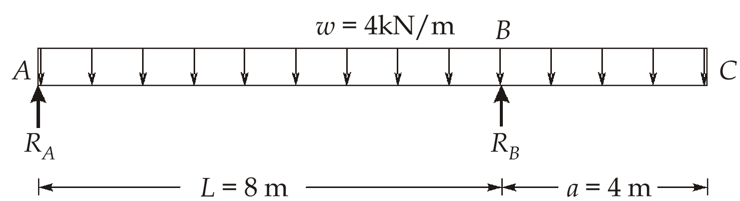
$$= 12x - 2x^2 + 36x - 288$$

$$= 48x - 2x^2 - 288$$

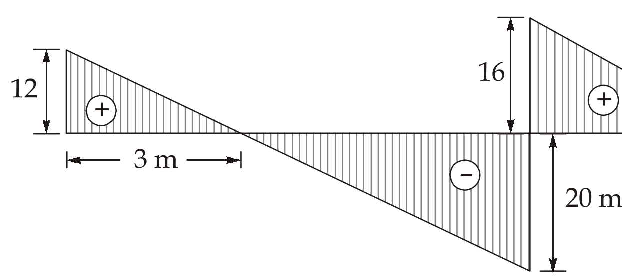
At $x = L = 8$ m, $M_B = 48 \times 8 - 2 \times 8^2 - 288 = -32$ kNm

At $x = L + a = 12$ m, $M_C = 48 \times 12 - 2 \times 12^2 - 288 = 0$

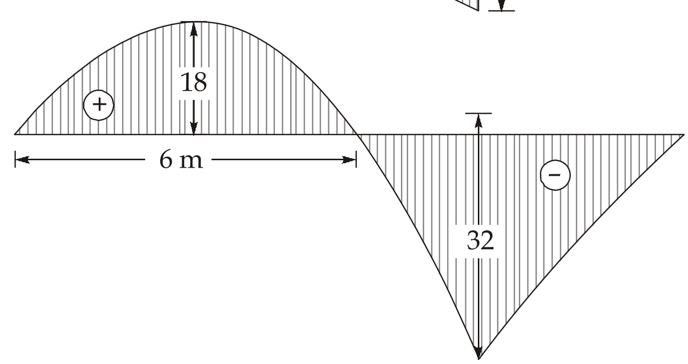
Loading diagram :



S.F.D.

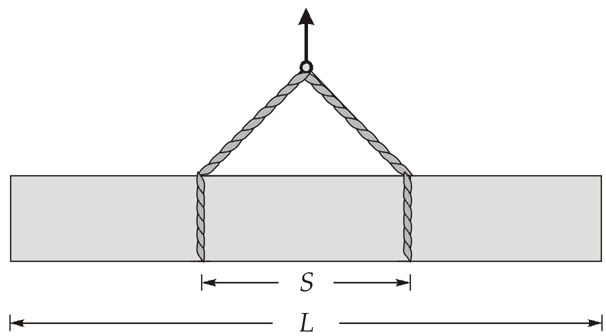


B.M.D.

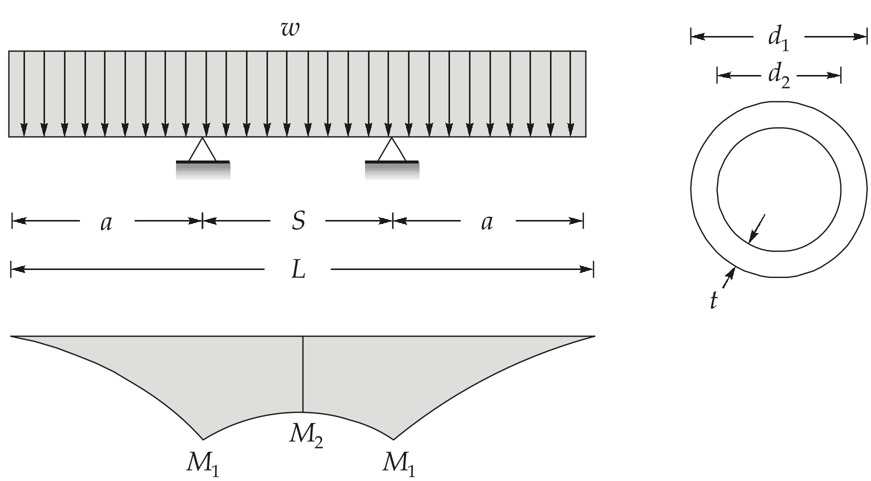


8. (a) Solution:

Given data : d_o = 180 mm; t = 6 mm; γ = 20 kN/m^3; L = 18 m; S = 5 m



$$d_i = d_o - 2t$$
$$= 180 - 2 \times 6 = 168 \text{ mm}$$
$$a = \frac{L-S}{2} = \frac{18-5}{2} = 6.5 \text{ m}$$
$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4} \times \left(180^2 - 168^2\right)$$
$$= 3279.8 \text{ mm}^2$$
$$w = \gamma \times A = \frac{20 \times 10^3 \times 3279.8}{10^6} = 65.596 \text{ N/m}$$
$$I = \frac{\pi}{64} \times \left(d_o^4 - d_i^4\right) = \frac{\pi}{64} \times \left(180^4 - 168^4\right)$$
$$I = 12.427 \times 10^6 \text{ mm}^4$$



$$(M_1)_{\text{at } s = 10.544 \text{ m}} = -\frac{wa^2}{2} = \frac{-65.596 \times 6.5^2}{2} = -1385.72 \text{ kNm}$$

$$M_2 = -\frac{wL}{4}\left(\frac{L}{2} - 5\right) = \frac{-65.596 \times 18}{4} \times \left[\frac{18}{2} - 5\right] = -1180.728 \text{ Nm}$$

(i) Maximum bending stress

$$\sigma_{max} = \frac{M_{max}}{I} y = \frac{M_1}{I}\left(\frac{d_o}{2}\right)$$

$$\sigma_{max} = \frac{1385.72 \times 10^3}{12.427 \times 10^6} \times \left(\frac{180}{2}\right) = 10.0358 \text{ MPa}$$

(ii) For bending stress to be minimized, beam must have a point of contraflexture and $M_1 + M_2 = 0$

$$M_1(s) = -w\frac{\left(\frac{L-S}{2}\right)^2}{2}$$

$$M_2(s) = -\frac{wL}{4}\left(\frac{L}{2} - S\right)$$

$\because$ $$M_1 + M_2 = 0$$

$$-w\frac{\left(\frac{L-S}{2}\right)^2}{2} - \frac{wL}{4}\left(\frac{L}{2} - S\right) = 0$$

On solving, $$S = 0.58579L = 10.544 \text{ m}$$

$$S = 10.544 \text{ m}$$

So, $$[M_1]_{\text{at } S = 10.544 \text{ m}} = -\frac{w}{2}\left(\frac{18 - 10.544}{2}\right)^2 = -455.826 \text{ Nm}$$

$$\sigma_{min} = \frac{[M_1]_{\text{at } 10.544 \text{ m}}}{I}\left(\frac{d_o}{2}\right)$$

$$\sigma_{min} = \frac{455.826 \times 10^3}{12.427 \times 10^6} \times \left(\frac{180}{2}\right)$$

$$= 3.301 \text{ MPa}$$ **Ans.**

(iii) Either $M_{1,\,max}$ $(S = 0)$ or $M_{2,\,max}$ $(S = L)$ will lead to maximum bending stress

1. Support at $\frac{L}{2}$ or $\frac{1}{2}$ beam is a cantilever with maximum moment $\frac{wL^2}{8}$

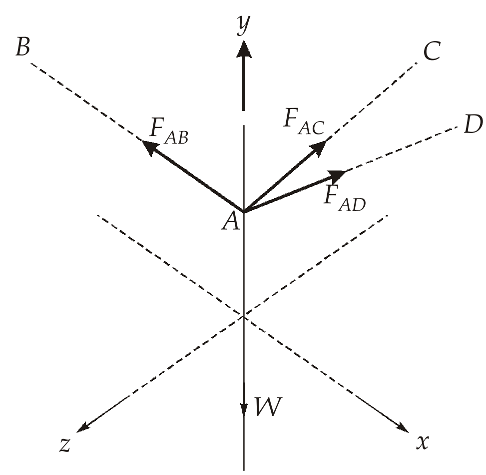
$$\sigma_{max_1} = \left(\frac{wL^2}{8}\right) \times \frac{(d_o/2)}{I} = \frac{65.596 \times 18^2 \times 10^3 \times 180}{8 \times 12.427 \times 10^6 \times 2}$$

$$= 19.24 \text{ MPa}$$

2. Or simply supported beam $S = L$ under uniform load, so max moment, $\frac{wL^2}{8}$

$$\sigma_{max} = \frac{wL^2}{8} \times \frac{d_o/2}{I} = 19.24 \text{ MPa}$$

8. (b) Solution:



$$\text{Weight, } W = 15 \times 9.81 = 147.15 \text{ N}$$

Coordinate of A, B, C, D are

$$A \equiv (0, 1.4, 0)$$
$$B \equiv (-0.3, 2, 1)$$
$$C \equiv (0, 2, -1)$$
$$D \equiv (2, 2, 0)$$

Unit vectors, $e_{AB} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{[-0.3, (2-1.4), (1-0)]}{\sqrt{0.3^2 + 0.6^2 + 1^2}}$

$$= \frac{-0.3\vec{i} + 0.6\vec{j} + 1\vec{k}}{1.204} = -0.249\vec{i} + 0.498\vec{j} + 0.831\vec{k}$$

Similarly,

$$e_{AC} = 0.515\vec{j} - 0.858\vec{k}$$
$$e_{AD} = 0.958\vec{i} + 0.287\vec{j}$$

Forces are:

$$\overrightarrow{F_{AB}} = -0.249 F_{AB}\vec{i} + 0.498 F_{AB}\vec{j} + 0.831 F_{AB}\vec{k}$$
$$\overrightarrow{F_{AC}} = 0.515 F_{AC}\vec{j} - 0.858 F_{AC}\vec{k}$$
$$\overrightarrow{F_{AD}} = 0.958 F_{AD}\vec{i} + 0.287 F_{AD}\vec{j}$$
$$\vec{W} = -147.15\vec{j}$$

Equations of equilibrium are

$$\Sigma F_x = 0 \Rightarrow \quad -0.249F_{AB} + 0.958F_{AD} = 0 \quad \text{...(i)}$$

$$\Sigma F_y = 0 \Rightarrow \quad 0.498F_{AB} + 0.515F_{AC} + 0.287F_{AD} = 147.15 \quad \text{...(ii)}$$

$$\Sigma F_z = 0 \Rightarrow 0.831F_{AB} - 0.858F_{AC} = 0 \quad \text{...(iii)}$$

Solving equations (i), (ii) and (iii), we get

$$F_{AB} = 137.35 \text{ N}$$
$$F_{AC} = 133.02 \text{ N}$$
$$F_{AD} = 35.69 \text{ N}$$

8. (c) (i) Solution:

Equivalence ratio (ϕ) is defined as the ratio of the actual fuel/air ratio $(F/A)_a$ to the stoichiometric fuel/air ratio $(F/A)_s$. It may also be defined as stoichiometric air/fuel ratio $(A/F)_s$ to actual air/fuel ratio $(A/F)_a$. Thus,

$$\phi = \frac{(F/A)_a}{(F/A)_s} = \frac{(A/F)_s}{(A/F)_a}$$

If the equivalence ratio (ϕ) is greater than unity, the mixture is said to be rich and if ϕ is less than unity the mixture is said to be weak or lean. The spark-ignition engines may normally run with both rich and weak mixtures but the compression-ignition engines normally run with weak mixtures only. The inverse of equivalence ratio ϕ is called the relative air/fuel ratio (λ). Therefore,

$$\lambda = \frac{1}{\phi} = \frac{(A/F)_a}{(A/F)_s}$$

For lean mixtures, $\phi < 1, \lambda > 1$

For stoichiometric mixtures, $\phi = \lambda = 1$

For rich mixtures, $\phi > 1, \lambda < 1$

8. (c) (ii) Solution:

(a) The combustion equation of an unknown hydrocarbon fuel can be written as

$$C_mH_n + xO_2 + yN_2 \rightarrow 14CO_2 + 0.6CO + 5O_2 + 80.4N_2 + zH_2O$$

On balancing the elements on both sides of equation,

$$\text{C balance : } m = 14 + 0.6 = 14.6$$

$$\text{N}_2 \text{ balance : } y = 80.4;$$

Also, we know $$\frac{y}{x} = \frac{79}{21}$$

$$\Rightarrow \quad x = \frac{80.4 \times 21}{79} = 21.372$$

$$\text{O}_2 \text{ balance : } 21.372 = 14 + \frac{0.6}{2} + 5 + \frac{z}{2}$$

$$\Rightarrow \quad z = 4.144$$

$$\text{H}_2 \text{ balance : } \frac{n}{2} = z = 4.144$$

$$\Rightarrow \quad n = 8.288$$

The combustion equation becomes:

$$C_{14.6}H_{8.288} + 21.372O_2 + 80.4N_2 \rightarrow 14CO_2 + 0.6CO + 5O_2 + 80.4N_2 + 4.144H_2O$$

$$\text{Air/fuel ratio} = \frac{21.372 \times 32 + 80.4 \times 28}{14.6 \times 12 + 8.288 \times 1}$$

$$= 15.996 \simeq 16$$ **Ans.**

(b) The stoichiometric combustion equation is

$$C_{14.6}H_{8.288} + x\left[O_2 + \frac{79}{21}N_2\right] \rightarrow yCO_2 + zH_2O + x\frac{79}{21}N_2$$

Balancing the equation on both sides

$$\text{C Balance, } 14.6 = y$$

$$H \text{ Balance, } 8.288 = 2z$$

$$\Rightarrow \quad z = 4.144$$

$$O \text{ Balance, } 2x = 2y + z$$

$$2x = 2 \times 14.6 + 4.144$$

$$\Rightarrow \quad x = 16.672$$

$$C_{14.6}H_{8.288} + 16.672O_2 + 16.672 \times \frac{79}{21}N_2 \rightarrow 14.6CO_2 + 4.144H_2O + 16.672 \times \frac{79}{21}N_2$$

$$\text{Air/fuel ratio (stoichiometric)} = \frac{16.672 \times 32 + 16.672 \times \frac{79}{21} \times 28}{14.6 \times 12 + 8.288 \times 1} = 12.478 \simeq 12.5$$

$$\therefore \quad \text{Percentage theoretical air} = \frac{16}{12.5} \times 100 = 128\%$$ **Ans.**

(c) Fuel composition:

$$C = \frac{14.6 \times 12}{[14.6 \times 12 + 8.288 \times 1]} \times 100 = 95.48\%$$ Ans.

$$H = \frac{8.288 \times 1}{14.6 \times 12 + 8.288 \times 1} \times 100 = 4.51\%$$ **Ans.**