MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

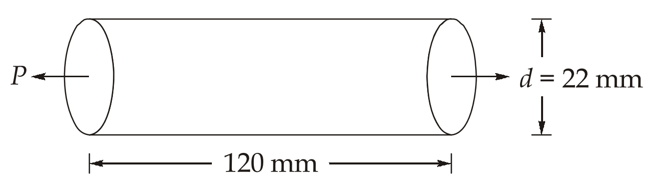
ESE-2026 Mains Test Series

Civil Engineering Test No : 3

Section A : Strength of Materials [All Topics]

1. (a) Solution:

Tension test:



$$P = 55725 \text{ N}$$

$$\Delta = 0.0765 \text{ mm}$$

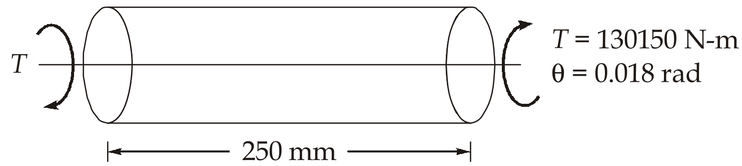
We know,

Elongation due to axial pull, $\Delta = \dfrac{PL}{AE}$

$$\Rightarrow \quad 0.0765 = \frac{55725 \times 120}{\frac{\pi}{4} \times 22^2 \times E}$$

$$\Rightarrow \quad E = 229950.65 \text{ N/mm}^2$$

Torsion test



$$T = 130150 \text{ N-m}$$
$$\theta = 0.018 \text{ rad}$$

As angular twist,

$$\theta = \frac{TL}{GJ}$$

$$\Rightarrow \quad 0.018 = \frac{130150 \times 10^3 \times 250}{G \times \frac{\pi}{32} \times 22^4}$$

$$\Rightarrow \quad G = 78599.73 \times 10^3 \text{ N/mm}^2$$

We know, the relationship, $E = 2G(1 + \mu)$

$$\Rightarrow \quad G = \frac{E}{2(1+\mu)}$$

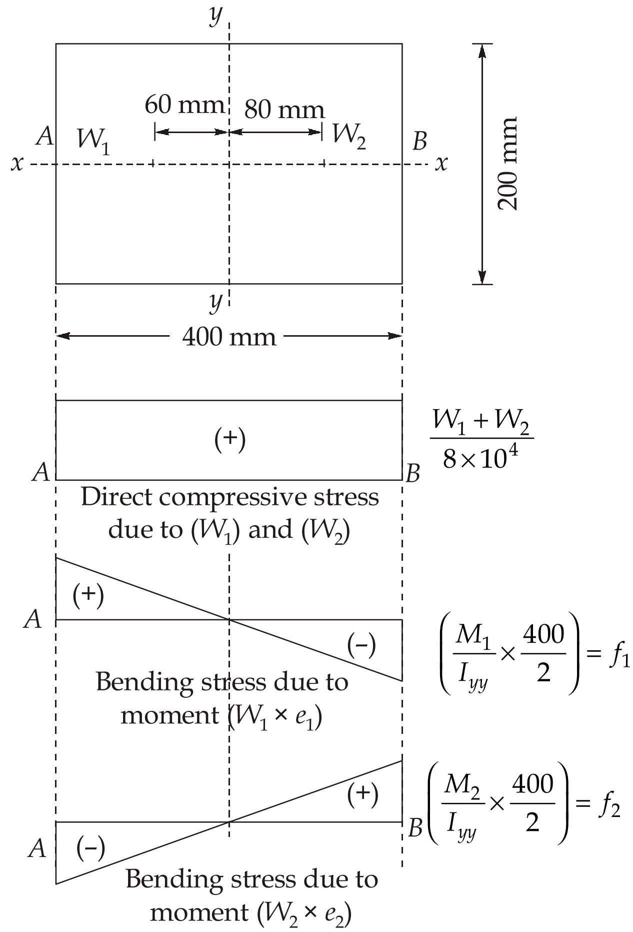
$$\Rightarrow \quad 78599.73 \times 10^3 = \frac{229950.65}{2(1+\mu)}$$

$$\Rightarrow \quad \mu = -0.998$$

Bulk modulus $K = \frac{E}{3(1-2\mu)} = \frac{229950.65}{3(1+2\times 0.998)}$

$$K = 25584.18 \text{ N/mm}^2$$

Q.1 (b) Solution:



Moment of inertia about y-axis, $I_y = \dfrac{200\times400^3}{12} = 10.67 \times 10^8\ \text{mm}^4$

Cross sectional area, $A = 200 \times 400 = 8 \times 10^4\ \text{mm}^2$

Direct compressive stress, $f_d = \dfrac{W_1+W_2}{A} = \dfrac{W_1+W_2}{8\times10^4}$

Now, bending moment due to W_1, $M_1 = W_1 \times 60 = 60W_1$

Similarly, bending moment due to W_2, $M_2 = W_2 \times 80 = 80W_2$

Now,bending stresses due to M_1, $f_1 = \dfrac{M_1}{I_y}\times\dfrac{400}{2} = \dfrac{60\,W_1}{10.67\times10^8}\times200 = \dfrac{W_1}{88916.67}$

(Compression at A and Tension at B)

Now, bending stress due to M_2, $f_2 = \dfrac{M_2}{I_y}\times\dfrac{400}{2} = \dfrac{80W_2}{10.67\times10^8}\times200 = \dfrac{W_2}{66687.5}$

Total stress at *A* and *B*,

$$\text{Stress at } A, f_A = f_d + f_1 - f_2$$

$$\Rightarrow \quad f_A = \frac{W_1 + W_2}{8 \times 10^4} + \frac{W_1}{88916.67} - \frac{W_2}{66687.5}$$

$$\text{Stress at } B, f_B = f_d - f_1 + f_2$$

$$\Rightarrow \quad f_B = \frac{W_1 + W_2}{8 \times 10^4} - \frac{W_1}{88916.67} + \frac{W_2}{66687.5}$$

Now, $f_A = 4f_B$

$$\Rightarrow \quad \frac{W_1 + W_2}{8 \times 10^4} + \frac{W_1}{88916.67} - \frac{W_2}{66687.5} = 4\left[\frac{W_1 + W_2}{8 \times 10^4} - \frac{W_1}{88916.67} + \frac{W_2}{66687.5}\right]$$

$$\frac{W_1}{88916.67} + \frac{4W_1}{88916.67} - \frac{W_2}{66687.5} - \frac{4W_2}{66687.5} = \frac{4(W_1 + W_2)}{8 \times 10^4} - \frac{(W_1 + W_2)}{8 \times 10^4}$$

$$\Rightarrow \quad \frac{5W_1}{88916.67} - \frac{5W_2}{66687.5} = \frac{3W_1}{8 \times 10^4} + \frac{3W_2}{8 \times 10^4}$$

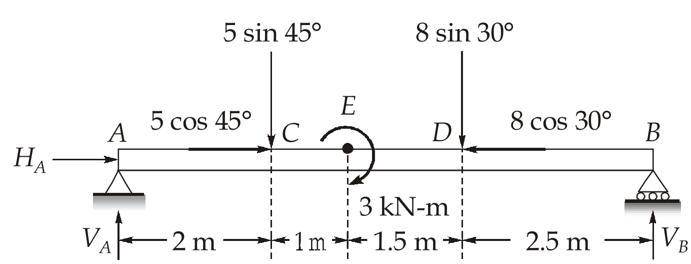
$$\Rightarrow \quad \frac{5W_1}{88916.67} - \frac{3W_1}{8 \times 10^4} = \frac{5W_2}{66687.5} + \frac{3W_2}{8 \times 10^4}$$

$$\Rightarrow \quad W_1 \times 1.87324 \times 10^{-5} = 1.12476 \times 10^{-4} \times 50$$

$$W_1 = 300.22 \text{ kN}$$

1. (c) Solution:

The given inclined forces may be resolved onto horizontal and vertical components.



Using equilibrium conditions

$\Sigma F_x = 0 \quad \Rightarrow H_A + 5\cos 45° - 8\cos 30° = 0 \quad \Rightarrow \quad H_A = 3.393$ kN ...(i)

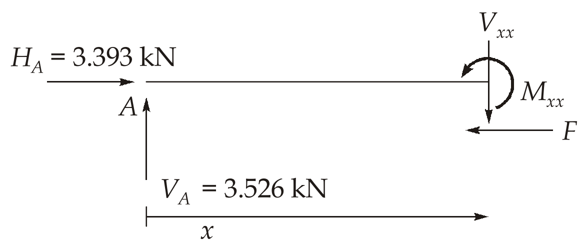
$\Sigma F_y = 0 \quad \Rightarrow V_A - 5\sin 45° - 8\sin 30° + V_B = 0 \quad \Rightarrow \quad V_A + V_B = 7.535$ kN ...(ii)

$\Sigma M_A = 0 \Rightarrow V_B \times 7 - (8\sin 30°) \times 4.5 - 3 - (5\sin 45°) \times 2 = 0$

$$V_B = 4.01 \text{ kN}$$

From (ii) $\quad V_A = 3.526$ kN

Portion AC

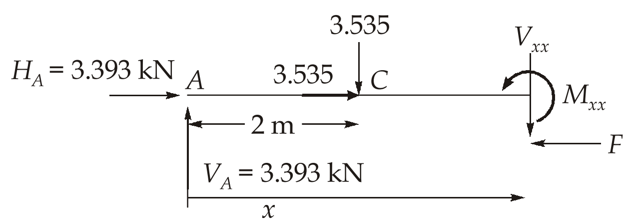


$$\text{Shear force, } V = 3.526 \text{ kN}$$

$$\text{Bending moment, } M = 3.526x \qquad (0 \le x \le 2\text{m})$$

At $\quad x = 2 \text{ m}, M(x = 2) = 7.052 \text{ kN-m}$

Portion CE



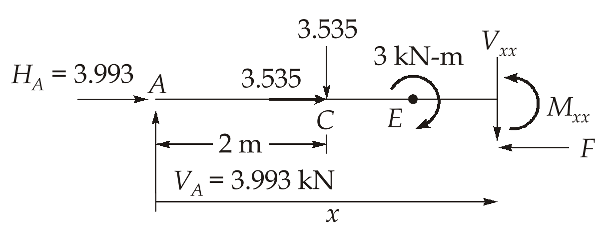
$$\text{Shear, } V_{xx} = 3.526 - 3.535 = -0.0095 \text{ kN}$$

$$M_{xx} = 3.526x - 3.535(x - 2) \qquad (2 \text{ m} \le x < 3 \text{ m})$$

At $\quad x = 2 \text{ m}, M(x = 2) = 7.052 \text{ kN-m}$

At $\quad x = 3 \text{ m}, M(x = 3) = 7.043 \text{ kN-m}$

Portion ED:



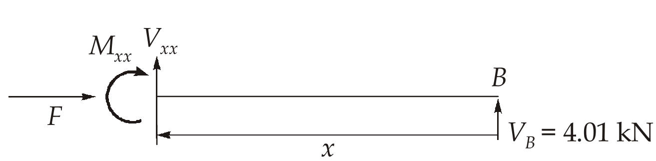
$$V_{xx} = -0.0095 \text{ kN}$$

$$M_{xx} = 3.526x - 3.535\,(x - 2) + 3 \qquad (3 \text{ m} \le x < 4.5 \text{ m})$$

At $\quad x = 3 \text{ m}, M(x = 3 \text{ m}) = 10.043 \text{ kN-m}$

At $\quad x = 4.5 \text{ m}, M(x = 4.5 \text{ m}) = 10.025 \text{ kN-m}$

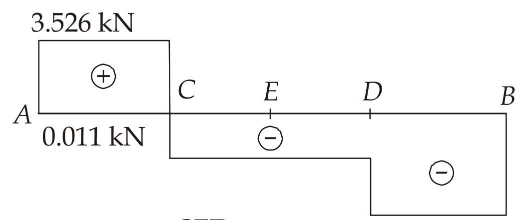
Portion DB:



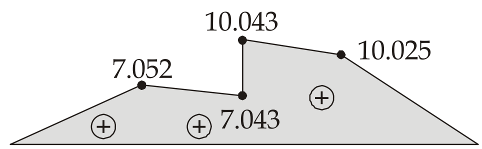
$$V_{xx} = -4.01 \text{ kN}$$

$$M_{xx} = 4.01x \qquad (0 \le x < 2.5 \text{ m})$$

At $\quad x = 2.5 \text{ m}, M(x = 2.5 \text{ m}) = 10.025 \text{ kN-m}$



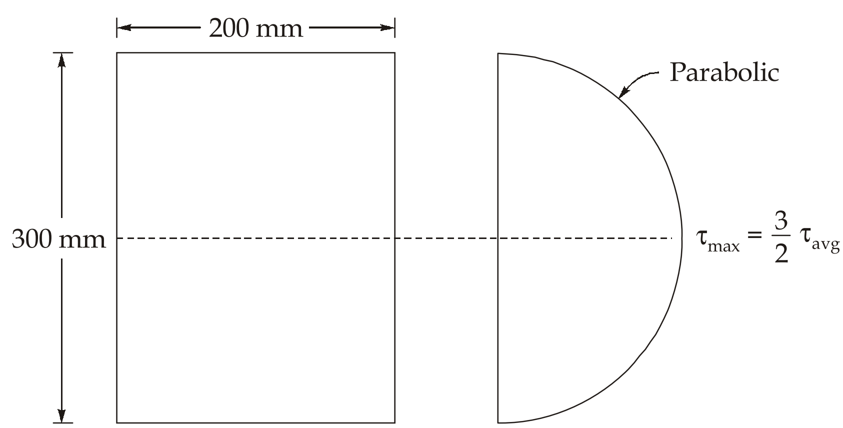
SFD



BMD

1. (d) Solution:

Given: Transverse shear force, $V = 100$ kN



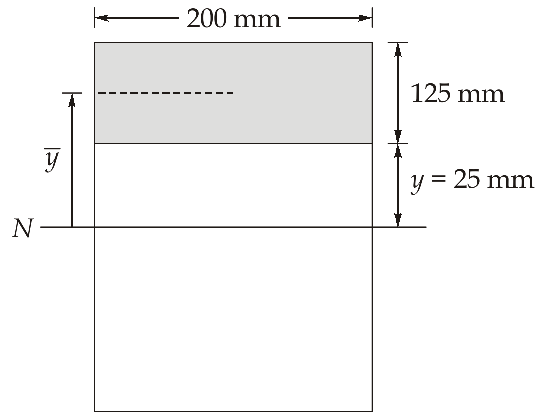
(i) Average shear stress in the section

$$\tau_{avg} = \frac{V}{A} = \frac{100 \times 10^3}{200 \times 300} = 1.67 \text{ N/mm}^2$$

(ii) Maximum shear stress will be developed at neutral axis

$$\tau_{max} = \frac{3}{2} \times 1.67 = 2.5 \text{ N/mm}^2$$

(iii)



Shear stress at average distance y from *NA*

$$\tau = \frac{V}{Ib}(A\bar{y})$$

where, A = area of shaded portion = 25000 mm²

$$\bar{y} = \frac{300}{2} - \frac{125}{2} = 87.5 \text{ mm}$$

$$I = \frac{bd^3}{12} = \frac{200 \times 300^3}{12} = 450 \times 10^6 \text{ mm}^4$$

$$\therefore \quad \tau = \frac{100 \times 10^3 \times 25000 \times 87.5}{450 \times 10^6 \times 200} = 2.430 \text{ N/mm}^2$$

1. (e) Solution:

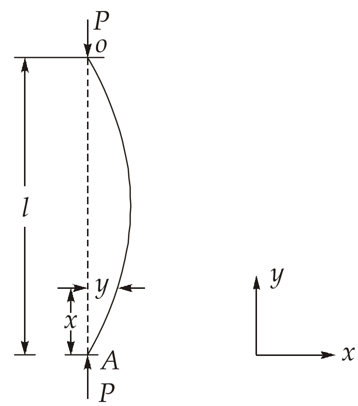
Euler's theory for long columns

Following assumptions are taken while developing theory for the buckling load of long columns:

1. The material of the column is homogenous and isotropic.
2. The compressive load on the column is fully axial.
3. The column fails only by buckling.
4. The weight of the column is neglected.
5. The column is initially straight and buckles suddenly at a particular load.
6. Pin joints are frictionless and fixed ends are rigid.

Derivation of buckline load:

Consider a strut initially straight acted upon by an axial load *P* through the centroid.



Considering a section at a distance of x from the end A, say the deflection is y.

B.M. at the section x-x, $M_{xx} = -Py$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} = -Py$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} + Py = 0 \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \alpha^2 y = 0 \qquad \text{where, } \alpha^2 = \frac{P}{EI}$$

The solution of this differential equation is

$$y = A\cos\alpha' x + B\sin\alpha' x \qquad \text{...(i)}$$

Where A and B are constants

At $x = 0, y = 0$ putting in equation (i)

$$A = 0$$

At $x = e, y = 0$ putting in equation (i)

$$B\sin(\alpha l) = 0$$

$$\Rightarrow \quad \sin(\alpha l) = 0$$

$$\Rightarrow \quad \alpha l = 0, \pi, 2\pi, 3\pi \ldots$$

$$\Rightarrow \quad \alpha = \frac{\pi}{l}$$

Euler's Crippling load, $P_e = \pi^2 EI$

$$P_e = \frac{\pi^2 EI}{l^2}$$

2. (a) Solution:

As per Castigliano's theorem

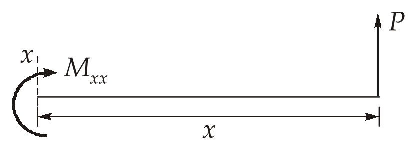
$$\Delta = \frac{\partial U}{\partial P}$$

where, U = total strain energy

$$= U_{AB} + U_{DE} + U_{BD}$$

due to symmetry $U_{AB} = U_{DE}$

For strain energy stored in 'straight limbs'

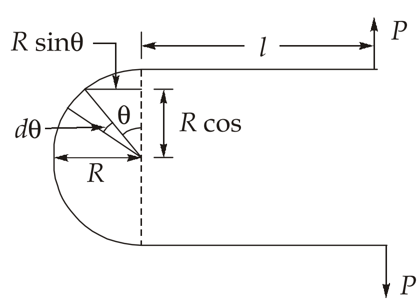


Bending moment at x-x section, $M_{xx} = Px$

$$U_{AB} = \int_0^l \frac{M^2 dx}{2EI} = \int_0^l \frac{(Px)^2\,dx}{2EI} = \frac{P^2 l^3}{6EI}$$

$$\therefore \quad U_{AB} = U_{DE} = \frac{P^2 l^3}{6EI}$$

For strain energy stored in circular portion



Bending moment at any angle q

$$M = P(l + R\sin\theta)$$

Stored energy in element of length $dl = Rdq$

$$du = \frac{M_{xx}^2 dx}{2EI} = \frac{[P(l + R\sin\theta)]^2 \times Rd\theta}{2EI}$$

$$U_{BD} = 2 \times \int_0^{\pi/2} \frac{[P(l + R\sin\theta)]^2 \times Rd\theta}{2EI}$$

$$U_{BD} = \int_0^{\pi/2} \frac{[P(l+R\sin\theta)]^2 R d\theta}{EI}$$

Total strain energy stored

$$U = U_{AB} + U_{BD} + U_{DE}$$

$\Rightarrow$ $$U = 2U_{AB} + U_{BD} \qquad (\because \ U_{AB} = U_{DE})$$

$\Rightarrow$ $$U = 2\times\frac{P^2l^2}{6EI} + \int_0^{\pi/2} \frac{[P(l+R\sin\theta)]^2 R d\theta}{EI}$$

Relative displacement between points A and E

$$\Delta_{AE} = \frac{\partial u}{\partial P}$$

$\Rightarrow$ $$\Delta_{AE} = \frac{\partial}{\partial P}\left[\frac{P^2l^2}{3EI} + \int_0^{\pi/2} \frac{[P(l+R\sin\theta)]^2 R d\theta}{EI}\right]$$

$\Rightarrow$ $$\Delta_{AE} = \frac{2Pl^3}{3EI} + \int_0^{\pi/2} \frac{2[P(l+R\sin\theta)](l+R\sin\theta) R d\theta}{EI}$$

$\Rightarrow$ $$\Delta_{AE} = \frac{2Pl^3}{3EI} + \frac{2PR}{EI}\int_0^{\pi/2} (l+R\sin\theta)^2 d\theta$$

$\Rightarrow$ $$\Delta_{AE} = \frac{2Pl^3}{3EI} + \frac{2PR}{EI}\left(\frac{\pi}{2}l^2 + \frac{\pi}{4}R^2 + 2Rl\right)$$

Here, $P = 80 \times 10^3$ N, $l = 125$ mm, $R = 700$ kN

$$EI = 2 \times 10^5 \times 45 \times 10^6 = 90 \times 10^{11} \text{ N.mm}^2$$

Now, $$\Delta_{AE} = \frac{2\times 80\times 10^3\times(125)^3}{3\times(90\times 10^{11})} + \frac{2\times 80\times 10^3\times 700}{90\times 10^{11}}\left(\frac{\pi}{2}\times 125^2 + \frac{\pi}{4}\times 700^2 + 2\times 700\times 125\right)$$

$$\Delta_{AE} = 7.284 \text{ mm}$$

2. (b) Solution:

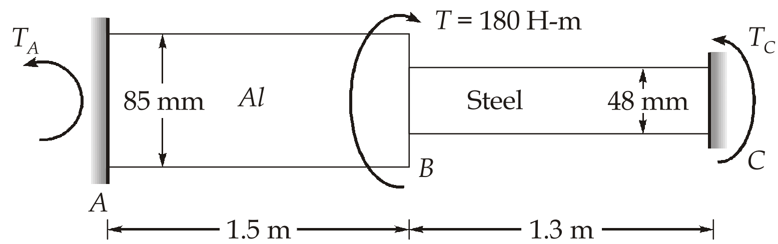
Given:

Shaft AB : Aluminium

$$d = 8.5 \text{ cm}$$
$$L_{AB} = 1.5 \text{ m}$$

Shaft BC: Steel

$$d = 4.8 \text{ cm}$$
$$L_{BC} = 1.3 \text{ m}$$



If T_A and T_B torsional resistance gets developed at end A and B

$\because$ $$\theta_{AB} = \theta_{BC}$$

$\Rightarrow$ $$\frac{T_A \times L_{AB}}{G_{al} \times J_{al}} = \frac{T_B \times L_{BC}}{G_s \times J_s}$$

$\Rightarrow$ $$\frac{T_A}{T_B} = \frac{0.3}{0.9} \times \left(\frac{85}{48}\right)^4 \times \frac{1.3}{1.5} = 2.841$$

Also, due to equilibrium condition

$$T_A + T_C = T = 180 \text{ Nm}$$

Now, $$2.841\, T_B + T_C = 180$$

$$T_C = 46.863 \text{ N-m}$$
$$T_A = 180 - T_C = 133.137 \text{ N-m}$$

Maximum shear stress developed in each shaft

In Aluminium bar: $$\tau_{al} = \frac{16T}{\pi d^3} = \frac{16 \times 133.137 \times 10^3}{\pi \times 85^3} = 1.104 \text{ N/mm}^2$$

In steel bar: $$\tau_{st} = \frac{16 \times 46.863 \times 10^3}{\pi \times 48^3} = 2.158 \text{ N/mm}^2$$

Angle of twist at junction B,

$$\theta_B = \frac{T_A \times L_{AB}}{G_{al} \times J_{al}} = \frac{133.137 \times 10^3 \times 1.5 \times 10^3}{0.3 \times 10^5 \times \frac{\pi}{32} \times (85)^4}$$

$$\theta_B = 1.2989 \times 10^{-3} \text{ rad}$$

OR

$$\theta_B = \frac{T_C L_{BC}}{G_{st} J_{st}} = \frac{46.863 \times 10^3 \times 1.3 \times 10^3}{0.9 \times 10^5 \times \frac{\pi}{32}(48)^4}$$

$$\theta_B = 1.298 \times 10^{-3} \text{ rad.}$$

2. (c) Solution:

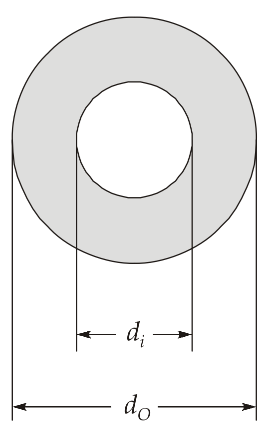
Given:

Internal diameter, d_i = 3.5 cm = 35 mm

External diameter, d_o = 5 cm = 50 mm

Compressive load, P = 230 kN

$$\text{Area, } A = \frac{\pi}{4} \times [50^2 - 35^2] = 1001.383 \text{ mm}^2$$



Short tubular column fails under crushing,

$$\therefore \quad \sigma_c = \frac{P}{A} = \frac{230 \times 10^3}{1001.383} = 229.682 \text{ N/mm}^2$$

Now, for strut with both ends fixed, $P_{failure}$ = 180,000 N

Unsupported length, L = 3 m

Effective length, l = 0.5 × 3 = 1.5 m

$$\text{Rankine formula, } P_r = \frac{\sigma_c A}{1+\alpha\lambda^2} \quad \text{...(i)}$$

Where,
$$\alpha = \text{Slenderness ratio} = \frac{kL}{r}$$
$$r = \text{radius of gyration} = \sqrt{\frac{I}{A}}$$
$$I = \frac{\pi}{64}\times\left[d_0^4 - d_l^4\right] = \frac{\pi}{64}\times\left[50^4 - 35^4\right]$$
$$I = 233134.400 \text{ mm}^4$$

So,
$$r = \sqrt{\frac{233134.400}{1001.383}} = 15.258\,\text{mm}$$

∴
$$\lambda = \frac{1.5\times1000}{15.258} = 98.309$$

From equation (i),

⇒
$$180000 = \frac{229.682\times1001.383}{1+\alpha\times98.309^2}$$

⇒
$$\alpha = \frac{1}{34793.017}$$

3. (a) Solution:

For the direction x and y

$$\in_x = \in_0 = -100\,\mu$$
$$\in_y = ?$$
$$\gamma_{xy} = ?$$

Using the relation for strain at any angle θ from x-axis,

$$\in_\theta = \left(\frac{\in_x + \in_y}{2}\right) + \left(\frac{\in_x - \in_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \quad \text{...(i)}$$

put θ = 60° in equation (i) we get

$$\in_{60^\circ} = \frac{-100+\in_y}{2} + \frac{-100-\in_y}{2}\cos 120^\circ + \frac{\gamma_{xy}}{2}\sin 120^\circ \quad \text{...(ii)}$$

put, θ = 120° in equation (ii) we get

$$\in_{120^\circ} = \frac{-100+\in_y}{2} + \frac{-100-\in_y}{2}\cos 240^\circ + \frac{\gamma_{xy}}{2}\sin 240^\circ \quad \text{...(iii)}$$

By adding equation (ii) and (iii)

$$\Rightarrow \quad \in_{60} + \in_{120} = \left(-100 + \in_y\right) + \left(\frac{-100 - \in_y}{2}\right) \times (-1)$$

$$\Rightarrow \quad 700 - 600 = -100 + \in_y + 50 + \frac{\in_y}{2}$$

$$\Rightarrow \quad \in_y = 100\ \mu$$

by substracting (ii) – (iii)

$$\Rightarrow \quad \in_{60} - \in_{120} = \frac{\gamma_{xy}}{2} \times (\sin 120 + \sin 240)$$

$$\Rightarrow \quad 700 - (-600) = \frac{\gamma_{xy}}{2} \times 1.732$$

$$\Rightarrow \quad \gamma_{xy} = 1501.155\ \mu$$

Principal strains

$$\Rightarrow \quad \in_{1,2} = \frac{\in_x + \in_y}{2} \pm \sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\Rightarrow \quad \in_{1,2} = \frac{-100 + 100}{2} \pm \sqrt{\left(\frac{-100 - 100}{2}\right)^2 + \left(\frac{1501.155}{2}\right)^2}$$

$$\Rightarrow \quad \in_{1,2} = \pm 757.209\ \mu$$

$$\Rightarrow \quad \in_1 = 757.209\ \mu, \in_2 = -757.209\ \mu$$

Principal stress

$$\because \quad \sigma_1 = \frac{E}{1-\mu^2}\left(\in_1 + \mu \in_2\right)$$

$$\Rightarrow \quad \sigma_1 = \frac{0.8 \times 10^5}{1 - 0.32^2} (757.209 - 0.32 \times 757.209) \times 10^{-6}$$

$$\Rightarrow \quad \sigma_1 = 45.89\ \text{N/mm}^2$$

Now,

$$\sigma_2 = \frac{E}{1-\mu^2}\left(\in_2 + \mu \in_1\right)$$

$$\Rightarrow \quad \sigma_2 = \frac{0.8 \times 10^5}{1 - 0.32^2} (-757.209 \times 0.32 \times 757.209) \times 10^{-6}$$

$$\Rightarrow \quad \sigma_2 = -45.89\ \text{N/mm}^2$$

3. (b) Solution:

At section 1-1, applied load is eccentric

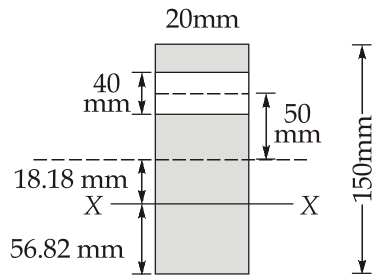
Let us analyse this section 1-1.

Net area of the plate at section 1-1,

$$A = (20 \times 150) - (20 \times 40) = 2200 \text{ mm}^2$$

Distance of the centroid from the bottom edge

$$= \frac{(20 \times 150 \times 75) - (20 \times 40 \times 125)}{2200} = 56.82 \text{ mm}$$



Section 1-1

Moment of inertia of the section about the centroidal axis *XX*,

$$\Rightarrow \quad I_{XX} = \left[\frac{20 \times 150^3}{12} + 20 \times 150 \times (75 - 56.82)^2\right] - \left[\frac{20 \times 40^3}{12} + 40 \times 20(125 - 56.82)^2\right]$$

$$\Rightarrow \quad I_{XX} = [5.625 \times 10^6 + 0.9915 \times 10^6] - [0.1067 \times 10^6 + 3.719 \times 10^6]$$

$$\Rightarrow \quad I_{XX} = 2.791 \times 10^6 \text{ mm}^4$$

Direct stress, $\sigma_d = -\frac{P}{A} = -\frac{120 \times 10^3}{2200} = -54.55 \text{ N/mm}^2$ (Tensile)

Eccentricity of the load, $e = 75 - 56.82 = 18.18$ mm

Stress at top due to eccentricity of the load,

$$(\sigma_b)_{Top} = \frac{Pe}{I_{XX}}(y_t) = -\frac{120 \times 10^3 \times 18.18}{2.791 \times 10^6} \times 93.18$$

$$= -72.83 \text{ N/mm}^2 \text{ (Tensile)}$$

Stress at bottom due eccentricity of the load,

$$(\sigma_b)_{bottom} = \frac{Pe}{I_{XX}}(y_b) = \frac{120 \times 10^3 \times 18.18}{2.791 \times 10^6} \times 56.82$$

$$= 44.41 \text{ N/mm}^2 \text{ (Comp.)}$$

$$\text{Resultant stress at top} = -54.55 - 72.83 = -127.38 \text{ N/mm}^2 \text{ (Tensile)}$$

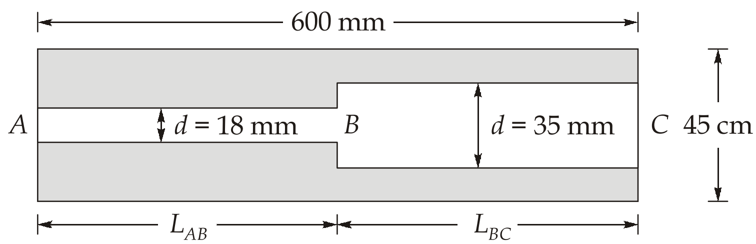
$$\text{Resultant stress at bottom} = -54.55 + 44.41 = -10.14 \text{ N/mm}^2 \text{ (Tensile)}$$

At section 2-2

Resultant stress at top and bottom

$$\sigma_b = \sigma_t = \frac{-P}{A} = -\frac{120 \times 10^3}{150 \times 20} = -40 \text{ N/mm}^2 \text{ (Tensile)}$$

3. (c) (i) Solution:



Given $\tau_{max} = 75 \text{ N/mm}^2, L = L_{AB} + L_{BC} = 600 \text{ mm}$

Max torque can be applied

For portion *AB*,

$$T_{AB} = \tau_{max} \times \frac{J_{AB}}{r_{max}}$$

$$\Rightarrow \quad T_{AB} = \frac{75 \times \frac{\pi}{32} \times [45^4 - 18^4]}{\left(\frac{45}{2}\right)}$$

$$\Rightarrow \quad T_{AB} = 1307573.078 \text{ N-mm}$$

$$\Rightarrow \quad T_{AB} = 1307.573 \text{ N-m}$$

For portion *BC*,

$$T_{BC} = \tau_{max} \times \frac{J_{BC}}{r_{max}}$$

$$\Rightarrow \quad T_{BC} = \frac{75 \times \frac{\pi}{32} \times [45^4 - 35^4]}{\left(\frac{45}{2}\right)} = 850848.01 \text{ N-mm}$$

$$\Rightarrow \quad T_{BC} = 850.848 \text{ N-m}$$

So maximum permissible applied torque on shaft will be minimum of above two,

$$T_{max} = 850.848 \text{ N-m}$$

Maximum power transmitted

$$P = T\omega$$

$$\Rightarrow \quad P = 850.848 \times \frac{2\pi \times 250}{60} \qquad \left(\because \ \omega = \frac{2\pi N}{60}\right)$$

$$\Rightarrow \quad P = 22275.148 \text{ watt} = 22.275 \text{ kW}$$

Further as given angle of twist of both portions are equal,

$$\theta_{AB} = \theta_{BC}$$

$$\frac{T_{AB} \times L_{AB}}{GJ_{AB}} = \frac{T_{BC} \times L_{BC}}{GJ_{BC}}$$

Also, $T'_{AB} = T'_{BC} = T_{max}$

$$\therefore \quad \frac{L_{AB}}{J_{AB}} = \frac{L_{BC}}{J_{BC}}$$

$$\Rightarrow \quad \frac{L_{AB}}{L_{BC}} = \frac{45^4 - 18^4}{45^4 - 35^4} = 1.537 \qquad \text{...(ii)}$$

From equation (i) and (ii),

$$\therefore \quad L_{AB} = \frac{1.537}{1+1.537} \times 600 = 363.5 \text{ mm}$$

$$\therefore \quad L_{BC} = 60 - L_{AB} = 236.5 \text{ mm}$$

3. (c) (ii) Solution:

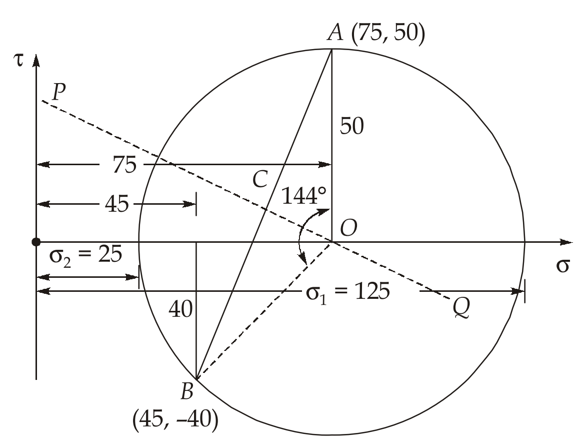
Normal and shear stress on planes A and B:

$$\sigma_A = 75 \text{ MPa}$$

$$\tau_A = 50 \text{ MPa}$$

$$\sigma_B = 45 \text{ MPa}$$

$$\tau_B = -40 \text{ MPa}$$



(Stress values in MPa)

$$\text{Slope of line } AB, m_{AB} = \frac{(50+40)}{(75-45)} = 3$$

$$\text{Slope of line } PQ, m_{PQ} = \frac{-1}{3} \qquad (\because AB \perp PQ)$$

$$\text{Mid point of } AB, C \equiv (60, 5)$$

Now line PQ passes through C (60, 5) and is having slope of $-\frac{1}{3}$ and thus equation of line PQ is,

$$y - 5 = \frac{-1}{3}(x-60)$$

For centre of Mohr's circle, $y = 0$

$$0 - 5 = \frac{-1}{3}(x-60)$$

$\Rightarrow$ $x = 75$

Centre of Mohr's circle = (75, 0)

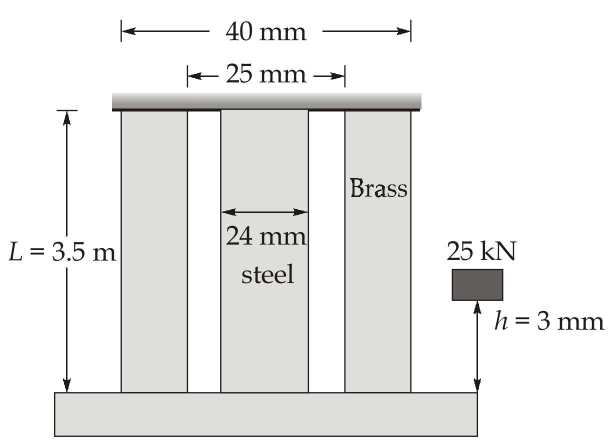
Radius of Mohr's circle, $r = \sqrt{(75-75)^2+(50-0)^2}$

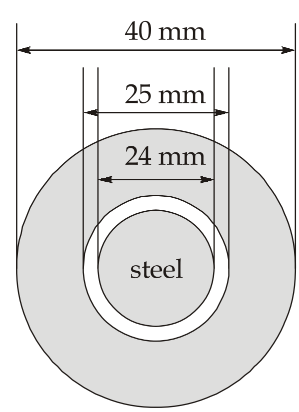
$\Rightarrow$ $r = 50$

Principal stress, $\sigma_{1,2} = 75 \pm r$

$\therefore$ $\sigma_1 = 125$ MPa and $\sigma_2 = 25$ MPa

4. (a) Solution:





C/S of compound bar

Assume after weight falls on the flange, compound bar elongates to max deflection of δ and max stress produced in the steel rod and brass tube are σ_s and σ_b respectively.

Using energy conservation–

Change in total (*KE* + *PE*) energy of weight

$$= \text{Strain energy stored in compound bar} \quad ...(i)$$

$\because$ Since weight is at rest before and after condition change in total energy of weight

$$= \text{change in } PE$$

$$= w \times (h + \delta)$$

$$= 25 \times 10^3 \times (3 + \delta) \text{ N-mm}$$

Strain energy stored in compound bar

$$U = \frac{{\sigma_s}^2}{2E_s} \times [A_b \times h] + \frac{{\sigma_b}^2}{2E_b} \times [A_b \times h]$$

Also at final condition

$$\Delta h_{\text{steel}} = \Delta h_{\text{brans}} = \delta$$

$$\frac{\sigma_s L_s}{Es} = \frac{\sigma_b L_b}{E_b}$$

$$\Rightarrow \quad \frac{\sigma_s}{\sigma_b} = \frac{E_s}{E_b} = 2$$

$$A_S = \frac{\pi}{4} \times 24^2 = 452.389 \text{ mm}^2$$

$$A_b = \frac{\pi}{4} \times (40^2 - 25^2) = 765.763 \text{ mm}^2$$

From equation (i)

$$\frac{\sigma_s^2}{2E_s} \times (A_s \times L) + \frac{\sigma_b^2}{2E_b}(A_b \times L) = 25 \times 10^3 \times (3 + \delta) \qquad (\because \ \sigma_s = 2\sigma_b)$$

and

$$\delta = \frac{\sigma_b \times L}{E_b}$$

$$\Rightarrow \quad \frac{(2\sigma_b)^2}{2E_s} \times A_s \times L + \frac{\sigma_b^2}{2E_b}(A_b \times L) = 25 \times 10^3 \left(3 + \frac{\sigma_b \times L}{E_b}\right)$$

$$\Rightarrow \quad \frac{(2\sigma_b)^2}{2 \times 2 \times 10^5} \times 452.389 \times 3500 + \frac{\sigma_b^2}{2 \times 1 \times 10^5} \times 765.763 \times 3500 = 25 \times 10^3 \left(3 + \frac{\sigma_b \times 3500}{1 \times 10^5}\right)$$

$$\Rightarrow \quad 29.234\ \sigma_b^2 - 875\ \sigma_b - 75 \times 10^3 = 0$$

$$\Rightarrow \quad \sigma_b = 67.78 \text{ N/mm}^2 \text{ (tensile)}$$

$$\sigma_s = 2 \times \sigma_b = 135.56 \text{ N/mm}^2 \text{ (Tensile)}$$

Maximum instantaneous elongation

$$\delta = \frac{\sigma_b L}{E_b} = \frac{67.78 \times 3500}{1 \times 10^5} = 2.372 \text{ mm}$$

4. (b) Solution:

Given: axial load $P = 750$ kg

Transverse shear load $V = 400$ kg

factor of safety $= 2.5$ Assume *cls* area of bar is A cm^2

Average axial stress

$$\sigma_x = \frac{750}{A} \text{ kg/cm}^2$$

Average shear stress

$$\tau_{xy} = \frac{400}{A} \text{ kg/cm}^2$$

(a) Maximum principal stress theory

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\Rightarrow \quad \sigma_{1/2} = \frac{750+0}{2A} \pm \sqrt{\left(\frac{750}{2A}\right)^2 + \left(\frac{400}{A}\right)^2}$$

$$\sigma_1 = \frac{923.293}{A} \text{ kg/cm}^2$$

$$\sigma_2 = \frac{-173.292}{A} \text{ kg/cm}^2$$

As per maximum principal stress theory,

$$\sigma_1 \le (f_y/\text{FOS})$$

$$\Rightarrow \quad \left(\frac{923.293}{A}\right) \le \frac{3010}{2.5}$$

$$\Rightarrow \quad A \ge 0.7668$$

$$\Rightarrow \quad \frac{\pi d^2}{4} \ge 0.7668$$

$$\Rightarrow \quad d \ge 0.988 \text{ cm}$$

$$\Rightarrow \quad d \ge 9.88 \text{ mm}$$

(b) **Maximum shear stress theory,**

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{923.293 + 173.292}{2A}$$

$$\Rightarrow \quad \tau_{max} = \frac{548.292}{A} \text{ kg/cm}^2$$

$$\because \quad \tau_{max} \leq \frac{f_y}{2 \times FOS}$$

$$\Rightarrow \quad \frac{548.292}{A} \leq \frac{3010}{2 \times 2.5}$$

$$\Rightarrow \quad \frac{\pi}{4} d^2 \geq 0.91078$$

$$\Rightarrow \quad d \geq 1.076 \text{ cm}$$

$$\Rightarrow \quad d \geq 10.76 \text{ mm}$$

(c) **Maximum shear strain energy theory,**

$$\frac{1}{12E}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \leq \frac{(f_y / FOS)^2}{6E}$$

$$\frac{1}{12E}\left[\left(\frac{923.293 + 173.292}{A}\right)^2 + \left(\frac{923.293 - 0}{A}\right)^2 + \left(\frac{0 - 173.292}{A}\right)^2\right] \leq \frac{\left(\frac{3010}{2.5}\right)^2}{6E}$$

$$\Rightarrow \quad \frac{2084998.743}{A^2} \leq 2\left(\frac{3010}{2.5}\right)^2$$

$$\Rightarrow \quad A \geq 0.848$$

$$\Rightarrow \quad d_s \geq 1.039 \text{ cm}$$

$$\Rightarrow \quad d_s \geq 10.39 \text{ mm}$$

(d) **Strain energy theory**

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right] \leq \frac{\left(\frac{f_y}{FOS}\right)^2}{2E}$$

$$\Rightarrow \quad \left(\frac{923.293}{A}\right)^2 + \left(\frac{-173.292}{A}\right)^2 - 2 \times 0.33 \times \frac{923.293 \times (-173.292)}{A^2} \leq \left(\frac{3010}{2.5}\right)^2$$

$$\Rightarrow \qquad \frac{988099.612}{A^2} \le \left(\frac{3010}{2.5}\right)^2$$

$$\Rightarrow \qquad A \ge 0.8256$$

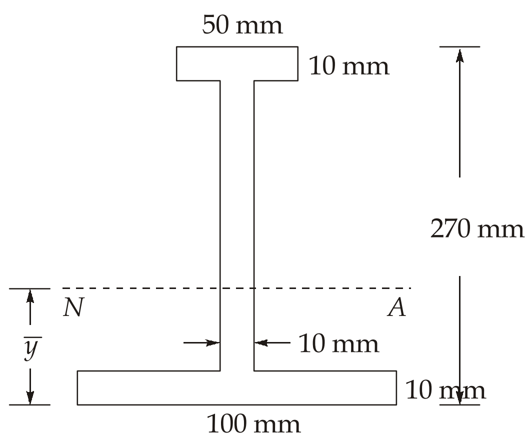
$$\Rightarrow \qquad d \ge 1.025 \text{ cm}$$

$$\Rightarrow \qquad d \ge 10.25 \text{ mm}$$

Q.4 (c) (i) Solution:

$$\text{Modulus ratio} = \frac{E_s}{E_w} = \frac{2\times10^5}{1\times10^4} = 20$$

The composite section may be replaced by an equivalent steel section. The 200 mm × 250 mm wooden component is replaced by a steel component 250 mm deep but having a width $\left(\frac{200}{20}\right) = 10$ mm.



Position of neutral axis from bottom most fibre, $\bar{y}$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$

$$\Rightarrow \qquad \bar{y} = \frac{(100\times10)\times5+(250\times10)\times135+(50\times10)\times265}{(100\times10)+(250\times10)+(50\times10)}$$

$$= \frac{475000}{4000} = 118.75 \text{ mm}$$

Moment of inertia about NA

$$I_{NA} = \left[\frac{100\times10^3}{12}+(100\times10)(118.75-5)^2\right]$$

$$+\left[\frac{10\times 250^3}{12}+(10\times 250)(118.75-135)^2\right]$$

$$+\left[\frac{50\times 10^3}{12}+(50\times 10)(265-118.75)^2\right]$$

$$= (0.833 + 1293.90 + 1302.08 + 66.02 + 0.416 + 1069.45) \times 10^4$$

$$= 3732.699 \times 10^4 \text{ mm}^4$$

Let the extreme stress in steel be 150 N/mm^2

$$\therefore \quad \text{Extreme stress in wood} = \frac{1}{20}\times\frac{141.25}{151.25}\times 150 = 7 \text{ N/mm}^2 < 7.5 \text{ N/mm}^2$$

$$\therefore \frac{M}{I}=\frac{\sigma}{y}$$

$$\Rightarrow \quad \frac{M}{3732.699\times 10^4} = \frac{150}{151.25}$$

$$\Rightarrow \quad M = 3.702 \times 10^7 \text{ Nmm}$$

$$\Rightarrow \quad M = 37.02 \text{ kNm}$$

Q.4 (c) (ii) Solution:

Moment of inertia of the beam section

$$I = \frac{150\times 200^3}{12}-\frac{100\times 150^3}{12} = 7.1875 \times 10^7 \text{ mm}^4$$

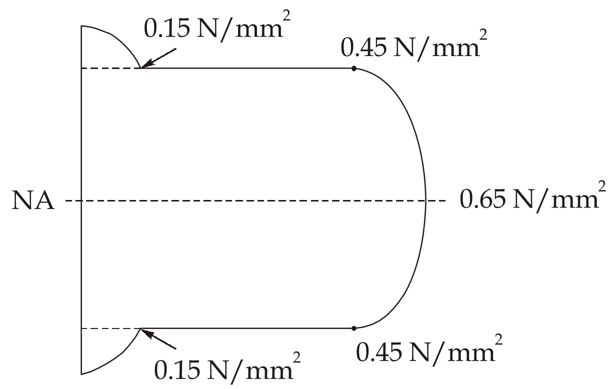
Shear stress distribution:

Shear stress at 100 mm from NA i.e., at extreme fibres = 0

Shear stress in flange at 75 mm from $NA = \frac{SA\bar{y}}{IB}$

$$= \frac{5000\times 150\times 25\times 87.5}{7.1875\times 10^7\times 150} = 0.15 \text{ N/mm}^2$$

Shear stress in the web at 75 mm from $NA = \frac{SA\bar{y}}{Ib} = \frac{150}{50}\times 0.15 = 0.45 \text{ N/mm}^2$



Shear stress distribution

$$\text{Shear stress at the } NA = \frac{SA\bar{y}}{Ib} = \frac{5000(150\times25\times87.5+2\times25\times75\times37.5)}{7.1875\times10^7\times(2\times25)}$$

$$= 0.65 \text{ N/mm}^2$$

Minimum pitch of screws connecting the flange and the web planks:

Horizontal shear stress in the web at the junction of flange and web

$$= 0.45 \text{ N/mm}^2$$

Let the pitch of screw be p mm

Consider one pitch length and thus horizontal shear force at this level for one pitch length

$$= 0.45 \times (2 \times 25)p = 22.5\,p \text{ N}$$

Equating the shear force per pitch length to the shearing strength of the two screws, we have,

$$22.5p = 1250 \times 2$$

$$p = 111.11 \text{ mm}$$

Section B : Highway Engineering-1 + Surveying and Geology-1 + Geo-technical & Foundation Engineering - 2 + Environmental Engineering - 2

5. (a) Solution:

Given data

Width of footing, $B = 3.0$ m

Length of footing, $L = 4.5$ m

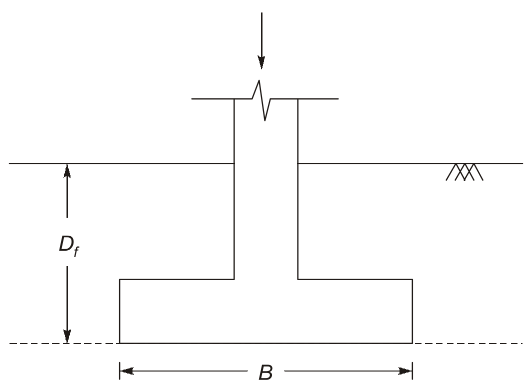
Depth of foundation, $D_f = 1.6$ m

Unit weight of soil, $\gamma = 1.65\ t/m^3$

Cohesion, $c = 1.2\ t/m^2$

Angle of internal friction, $\phi = 21.9°$

Factor of safety, $FOS = 2.5$



Since the angle of internal friction is less than 28°, local shear failure is assumed. The mobilized cohesion for local shear failure is taken as

$$c' = \frac{2}{3}c$$

$$c' = \frac{2}{3} \times 1.2 = 0.8\ t/m^2$$

and $$\tan\phi' = \frac{2}{3}\tan(21.9°)$$

$\Rightarrow$ $$\phi = 15°$$

Bearing capacity factors for local shear failure

$$N'_c = 9.7,\ N'_q = 2.7,\ N'_\gamma = 0.9$$

For a rectangular footing, Terzaghi's bearing capacity equation for local shear failure is

$$q_u = c'N'_c\left(1+0.3\frac{B}{L}\right)+\gamma D_f N'_q + 0.5\gamma B N'_\gamma\left(1-0.2\frac{B}{L}\right)$$

Substituting the given values,

$$\Rightarrow \quad q_u = 0.8 \times 9.7\left(1+0.3\times\frac{3.0}{4.5}\right) + 1.65 \times 1.6 \times 2.7 + 0.5 \times 1.65 \times 3.0 \times 0.9\left(1-0.2\times\frac{3.0}{4.5}\right)$$

$$\Rightarrow \quad q_u = 7.76 \times 1.2 + 7.128 + 2.2275 \times 0.867$$

$$\Rightarrow \quad q_u = 9.312 + 7.128 + 1.931$$

$$\Rightarrow \quad q_u = 18.371 \text{ t/m}^2$$

The net ultimate bearing capacity is obtained by subtracting the overburden pressure at the foundation level,

$$q_{nu} = q_u - \gamma D_f$$

$$q_{nu} = 18.371 - (1.65 \times 1.6)$$

$$q_{nu} = 18.371 - 2.64 = 15.731 \text{ t/m}^2$$

The net safe bearing capacity is calculated by applying the factor of safety,

$$q_{ns} = \frac{q_{nu}}{FOS}$$

$$q_{ns} = \frac{15.731}{2.5} = 6.292 \text{ t/m}^2$$

The safe bearing capacity is obtained by adding the overburden pressure,

$$q_s = q_{ns} + \gamma D_f$$

$$q_s = 6.292 + 2.64 = 8.932 \text{ t/m}^2$$

The maximum safe load on the footing is equal to the safe bearing capacity multiplied by the area of the footing,

$$Q_s = q_s(B \times L)$$

$$Q_s = 8.932 \times (3.0 \times 4.5)$$

$$Q_s = 8.932 \times 13.5 = 120.582\ t$$

5. (b) Solution:

Given data

Load on first plate, $Q_1 = 40$ kN

Diameter of first plate, $D_1 = 0.30$ m

Load on second plate, $Q_2 = 150$ kN

Diameter of second plate, $D_2 = 0.75$ m

Load on square foundation, $Q = 1200$ kN

Allowable settlement, $s = 25.4$ mm

According to Housel's method, the load corresponding to a given settlement is expressed as

$$Q = Am + Pn \quad \text{...(i)}$$

where A is the area, P is the perimeter, and m and n are soil constants.

For the circular plate of diameter 0.30 m,

$$A_1 = \frac{\pi}{4}(0.30)^2 = 0.070686 \text{ m}^2$$

$$P_1 = \pi \times 0.30 = 0.942478 \text{ m}$$

From equation (i),

$$40 = 0.070686\, m + 0.942478 \text{ n} \quad \text{...(ii)}$$

For the circular plate of diameter 0.75 m,

$$A_2 = \frac{\pi}{4}(0.75)^2 = 0.441786 \text{ m}^2$$

$$P_2 = \pi \times 0.75 = 2.356194 \text{ m}$$

From equation (i),

$$150 = 0.441786 \text{ m} + 2.356194 \text{ n} \quad \text{...(iii)}$$

Using equation (ii) and (iii)

$$m = 188.629 \text{ kN/m}^2,\ n = 28.294 \text{ kN/m}$$

Let the side of the square foundation be B.

For a square footing,

$$A = B^2,\ P = 4B$$

From equation (i)

$$\Rightarrow \quad 1200 = 188.629\, B^2 + 4\, B \times 28.294$$

$$\Rightarrow \quad 1200 = 188.629\, B^2 + 113.176\, B$$

$$\Rightarrow \quad 188.629\, B^2 + 113.176\, B - 1200 = 0$$

Solving the quadratic equation,

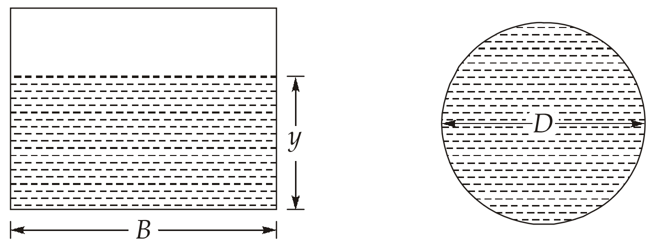
$$\Rightarrow \quad B = \frac{-113.176 + \sqrt{(113.176)^2 + 4 \times 188.629 \times 1200}}{2 \times 188.629}$$

$$\Rightarrow \quad B = \frac{-113.176 + 958.242}{377.258} = 2.24 \text{ m}$$

Hence, the required size of the square foundation is approximately 2.24 m × 2.24 m.

5. (c) (i) Solution:

Given: $B = 1.5\,y$



For hydraulically equivalent section, full flow discharge are equal provided same n, and gradient.

$$Q_{rec} = Q_{\text{circular}}$$

$$\Rightarrow \quad \left(\frac{A}{n}R^{2/3}S^{1/2}\right)_{rec} = \left(\frac{A}{n}R^{2/3}S^{1/2}\right)_{\text{circular}}$$

$$\Rightarrow \quad (By)\times\left[\frac{By}{2(B+y)}\right]^{2/3} = \left(\frac{\pi D^2}{4}\right)\times\left(\frac{D}{4}\right)^{2/3}$$

Putting $y = \dfrac{B}{1.5}$

$$\Rightarrow \quad B\times\frac{B}{1.5}\times\left[\frac{\left(\frac{B^2}{1.5}\right)}{2\times\left(B+\frac{B}{1.5}\right)}\right]^{2/3} = \frac{\pi D^2}{4}\times\left(\frac{D}{4}\right)^{2/3}$$

$$\Rightarrow \quad B^{8/3}\times 0.2279 = D^{8/3}\times 0.3168$$

$$\Rightarrow \quad \frac{B}{D} = 1.124$$

5. (c) (ii) Solution:

Given:

$$t_{20} = 11 \text{ days}$$

Substitute into the formula:

Relative stability $S = 100\,[1 - (0.794)^{11}] = 92.093\%$

Percent relative stability at 20°C = 92.093%

5. (d) Solution:

1. Spacing of Expansion Joints (L_e)

Given:

Gap: 2.0 cm $\Rightarrow \delta = 1.0$ cm = 0.01 m (because the joint can close by half its width)

Temperature change: $\Delta T = 24°C$

Coefficient of thermal expansion: $\alpha = 12 \times 10^{-6}/°C$

$$\delta = L_e \cdot \alpha \cdot \Delta T$$

$$\Rightarrow \quad 0.01 = L_e \times (12 \times 10^{-6}) \times 24$$

$$\Rightarrow \quad 0.01 = L_e \times 288 \times 10^{-6}$$

$$\Rightarrow \quad L_e = \frac{0.01}{288 \times 10^{-6}}$$

$$\Rightarrow \quad L_e \approx 34.73 \text{ m}$$

The slab can be up to 34.73 m long before needing an expansion joint.

2. Spacing of Contraction Joints (L_c)

These are used to prevent shrinkage cracks.

Given:

Allowable tensile stress of concrete: $\sigma_{at} = 0.9 \text{ kg/cm}^2 = 9000 \text{ kg/m}^2$

Unit weight of concrete: $w = 2400 \text{ kg/m}^3$

$$f = 1.5$$

$$\because \quad L_c = \frac{2 \cdot \sigma_{at}}{w \cdot f}$$

$$L_c = \frac{2 \times 9000}{2400 \times 1.5} = \frac{18000}{3600}$$

$$L_c = 5.0 \text{ m}$$

But in any unreinfreed case $L_c \not> 4.5$ m

$$\therefore \quad L_c = 4.5 \text{ m}$$

5. (e) (i) Solution:

Basis	Plane Surveying	Geodetic Surveying
Definition	Surveying in which the curvature of the earth is neglected.	Surveying in which the curvature of the earth is taken into account.
Area Covered	Suitable for small areas (generally less than about 250 km²).	Used for large areas such as states, countries, or continents. (> 250 km²)
Earth's Shape Assumption	Earth is assumed to be flat.	Earth is assumed to be spherical or spheroidal.
Geometry Used	Plane geometry is used.	Spherical trigonometry is used.
Level Surface	Level line is considered straight.	Level line is curved following the earth's curvature.
Instruments Used	Chain, compass, level, theodolite, total station.	Precise theodolites, EDM, GPS, Satellite-based instruments.
Application	Engineering projects, small land surveys, construction works.	National mapping, boundary surveys, control networks.
Control Points	Established by traversing or simple triangulation.	Established by triangulation, trilateration, and geodetic control networks.

5. (e) (ii) Solution:

Principles of Surveying

Surveying is based on two fundamental principles:

1. **Working from Whole to Part**

   This is the most important principle in surveying. In this method, the entire area to be surveyed is first covered by establishing a network of main control points with high accuracy. These control points form a framework (such as triangles or traverse stations). After fixing this primary control network, secondary control points and detailed features are located within this framework.

   Explanation:

   If small areas are surveyed independently without an overall control, errors may accumulate and distort the entire map. By first establishing accurate main control points and then filling in the details, errors are confined locally and do not propagate over the whole area. This ensures better accuracy and reliability of the survey.

   For example, in triangulation surveying, large well-conditioned triangles are first established. Smaller triangles are then formed inside them to locate details.

2. **Fixing the Position of a Point by at Least Two Independent Measurements**

   The position of any new point should be determined by at least two independent measurements from known reference points. These measurements may be in the form of:

- Two distances
- Two angles
- One distance and one angle

This provides a check on accuracy and reduces the possibility of error.

6. (a) Solution:

Different Types of Geosynthetics and Their Properties

Geosynthetics are polymeric products used in contact with soil, rock, water, or other civil engineering materials. They perform key functions such as reinforcement, separation, filtration, drainage, and containment.

1. Geotextiles

Primary Properties

- Made of woven or non-woven fabrics (polypropylene, polyester).
- Permeable to water and air.
- High tensile strength and flexibility.
- Good filtration and separation characteristics.
- Resistant to biological and chemical degradation.

Typical Applications

- Separation: Prevent mixing of subgrade soil with aggregates in roads.
- Filtration: Prevent soil particles from entering drainage layers.
- Drainage: Provide pathways for water flow.
- Protection: Protect geomembranes in landfill liners.

Key Design Considerations

- Permittivity & Water Flow Rate: For drainage and filtration.
- Aperture Size & Soil Gradation: To prevent clogging.
- Tensile Strength & Elongation: For reinforcement.
- Durability & UV Resistance: For exposure conditions.
- Compatibility with Soil Chemistry

2. Geomembranes

Primary Properties

- Impermeable sheets made of HDPE, LLDPE, PVC, or EPDM.
- Very low permeability (acts as a barrier).
- High tensile strength and puncture resistance.
- Chemical and UV resistant.

Typical Applications

- Landfill liners and caps
- Reservoir and canal lining
- Containment of hazardous liquids
- Mining tailings ponds

Key Design Considerations

- Permeability & Leak Tightness
- Puncture Resistance and Tensile Strength
- Seam Strength and Installation Quality
- Chemical Compatibility
- Temperature and UV Stability

3. Geogrids

Primary Properties

- High tensile strength in one or two directions.
- Made from polyester, polypropylene, or high-density polyethylene.
- Open grid structure for soil interlocking.
- High stiffness and low elongation.

Typical Applications

- Reinforced soil walls
- Mechanically stabilized earth (MSE) structures
- Embankments over weak soils
- Railway and road base reinforcement

Key Design Considerations

- Tensile Strength and Modulus at Small Strains
- Long-term Creep Resistance
- Soil-Geogrid Interaction (Pullout Resistance)
- Durability and UV Resistance
- Installation Damage and Overburden Protection

4. Geonets

Primary Properties

- Net-like structure (HDPE) with high transmissivity.
- High drainage capacity.

- Good flow in-plane but minimal strength.

Typical Applications

- Landfill leachate drainage layers
- Drainage of retaining walls
- Roof and green roof drainage
- Horizontal drainage layers

Key Design Considerations

- In-plane Flow Capacity
- Compression and Creep under Load
- Clogging Potential
- Compatibility with Adjacent Geotextiles

5. Geocomposites

Primary Properties

- Combination of two or more geosynthetic types.
- Can combine drainage, filtration, reinforcement, and separation.
- Optimized performance for specific functions.

Typical Applications

- Drainage composite layers in landfills
- Prefabricated vertical drains
- Drainage behind retaining walls
- Road drainage systems

Key Design Considerations

- Compatibility of Components
- Flow Capacity and Filtration Efficiency
- Strength and Deformation
- Long-term Performance

6. (b) (i) Solution:

Face Right: When the vertical circle of a theodolite is on the right hand side of the observer, the position is called face right and the observation made is called face right observation.

Face Left: When the vertical circle of a theodolite is on the left hand side of the observer, the position is called face left and the observation made is called face left observation.

The errors that are eliminated by changing face are below:

Error in fundamental lines are eliminate such as

1. Error due to line of collimation not being perpendicular to the horizontal axis.
2. Error due to horizontal axis not being perpendicular to the vertical axis.
3. Error due to line of collimation not being parallel to the axis of the altitude level.

6. (b) (ii) Solution:

Given:

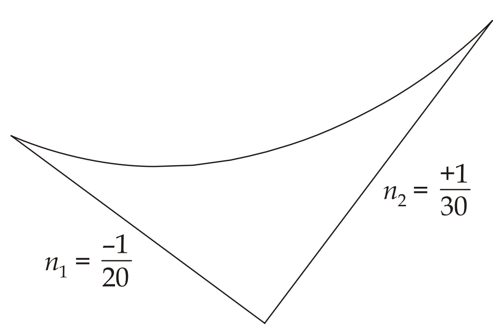
The gradients of the two intersecting roads are:

$$n_1 = -\frac{1}{20} = -0.05, n_2 = +\frac{1}{30} = +0.0333$$

Hence, the algebraic difference of gradients (total deviation angle) is:

$$N = n_2 - n_1 = 0.0333 - (-0.05) = 0.0833$$

Design speed:



Valley curve

$$v = 80 \text{ km/h} = \frac{80}{3.6} = 22.22 \text{ m/s}$$

Allowable rate of change of centrifugal acceleration:

$$C = 0.60 \text{ m/s}^3$$

Reaction time: $t = 2.5$ s

Coefficient of friction: $f = 0.35$

Length of Valley Curve Based on Comfort Condition

For valley curves, the comfort criterion limits the rate of change of centrifugal acceleration. The length of curve based on comfort is given by:

$$L = 2\left(\frac{Nv^3}{C}\right)^{1/2}$$

Substituting the given values:

$$L = 2\left(\frac{0.0833 \times (22.22)^3}{0.60}\right)^{1/2}$$

$$L = 2\sqrt{1523.53}$$

$$L = 78.06 \text{ m}$$

Length of Valley Curve Based on Headlight Sight Distance (HSD)

For valley curves, the headlight sight distance is taken equal to the stopping sight distance (SSD).

Stopping Sight Distance

$$SSD = vt + \frac{v^2}{2gf}$$

$$SSD = (22.22 \times 2.5) + \frac{(22.22)^2}{2 \times 9.81 \times 0.35}$$

$$\text{SSD} = 55.55 + 71.86$$

$$SSD = 127.41 \text{ m}$$

Length of Curve Based on HSD

Assuming $L > SSD$, the length of valley curve is:

$$L = \frac{NS^2}{1.5 + 0.035S}$$

Substituting values:

$$L = \frac{0.0833 \times (127.41)^2}{1.5 + 0.035 \times 127.41}$$

$$L = \frac{1352.12}{1.5 + 4.459} = \frac{1352.12}{5.959}$$

$$L = 226.90 \text{ m}$$

Since $L = 226.90$ m > $SSD = 127.41$ m, the assumption $L > SSD$ is valid.

Governing Length of Valley Curve

The required length of the valley curve is the maximum of the lengths obtained from comfort and headlight sight distance criteria:

- Length based on comfort: 78.06 m
- Length based on HSD: 226.90 m

$$L = 226.90 \text{ m}$$

6. (c) (i) Solution:

Determination of Maximum Visible Distance

To determine the total distance (D), we have to compute the distance from the observer to the horizon (d_1) and the distance from the lighthouse to the horizon (d_2), then add them. The formula for distance to horizon is:

$$d = 3.855\sqrt{h}$$

where,

$$d = \text{distance in km}$$

$$h = \text{height in metres}$$

Distance from Observer to Horizon (d_1):

Given height of observer's eye, $h_1 = 16$ m

$$d_1 = 3.855\sqrt{16}$$

$$\Rightarrow \quad d_1 = 3.855 \times 4$$

$$\Rightarrow \quad d_1 = 15.42 \text{ km}$$

Distance from Lighthouse to Horizon (d_2):

Given height of lighthouse, $h_2 = 64$ m

$$d_2 = 3.855\sqrt{64}$$

$$\Rightarrow \quad d_2 = 3.855 \times 8$$

$$\Rightarrow \quad d_2 = 30.84 \text{ km}$$

Total Distance (D)

$$D = d_1 + d_2$$

$$\Rightarrow \quad D = 15.42 + 30.84$$

$$\Rightarrow \quad D = 46.26 \text{ km}$$

Maximum distance at which the lighthouse is visible to the observer = 46.26 km

6. (c) (ii) Solution:

Design of oxidation pond may be carried out as follows:

1. Sewage Characteristics and Total Load:

Total Sewage Flow (Q),

$$Q = 1{,}200 \text{ persons} \times 150 \text{ litres/person/day}$$
$$\Rightarrow \quad Q = 180{,}000 \text{ litres/day}$$
$$\Rightarrow \quad Q = 180 \text{ m}^3\text{/day}$$

Total BOD Load per Day:

$$\text{BOD Load} = Q \times \text{BOD}_5$$
$$\Rightarrow \quad \text{BOD Load} = 180{,}000 \text{ L/day} \times 250 \text{ mg/L}$$
$$\Rightarrow \quad \text{BOD Load} = 45 \times 10^6 \text{ mg/day}$$
$$\Rightarrow \quad \text{BOD Load} = 45 \text{ kg/day}$$

2. Determination of Pond Area

Given Organic Loading Rate = 250 kg BOD/ha/day

Surface Area (A):

$$A = \text{(Total BOD produced per day)/(Organic loading rate)}$$
$$\Rightarrow \quad A = 45/250$$
$$\Rightarrow \quad A = 0.18\, ha = 1{,}800 \text{ m}^2$$

3. Dimensioning of the Pond

Assuming the length (L) is twice the width (B):

$$\because \quad L = 2B$$
$$\text{Area} = L \times B$$
$$\Rightarrow \quad (2B) \times B = 1{,}800$$
$$\Rightarrow \quad 2B^2 = 1{,}800$$
$$\Rightarrow \quad B^2 = 900$$
$$\Rightarrow \quad B = 30 \text{ m}$$

Length:

$$L = 2 \times 30$$
$$L = 60 \text{ m}$$

4. Tank Capacity and Detention Time

Liquid Capacity of Pond:

$$\text{Volume} = L \times B \times \text{depth}$$

$$\text{Assume water depth} = 1.5 \text{ m}$$

$$\text{Volume} = 60 \text{ m} \times 30 \text{ m} \times 1.5 \text{ m}$$

$$\text{Volume} = 2700 \text{ m}^3$$

Check for Detention Time (t_e):

$$t_e = \text{Volume/Daily flow}$$

$$t_e = 2700/180$$

$$t_e = 15 \text{ days}$$

Oxidation Pond detention time generally ranges from 10 to 30 days; hence, 15 days is acceptable.

6. (c) (iii) Solution:

Defects in Rigid Pavements

Rigid pavements are made of cement concrete and are designed to distribute wheel loads over a wide area. Despite their strength and durability, rigid pavement may develop various defects due to traffic loading, environmental effects, material deficiencies, or construction errors.

1. **Cracking:** Cracks are the most common defect in rigid pavements and may occur due to shrinkage, temperature stresses, or excessive loading.

   Longitudinal cracks: They run parallel to the centerline, often caused by warping stresses or poor joint spacing.

   Transverse cracks: They Run perpendicular to traffic direction, usually due to temperature variation or inadequate expansion joints.

   Corner cracks: They Occur at slab corners due to heavy wheel loads combined with poor load transfer.

2. **Faulting:** Faulting is the difference in elevation across joints or cracks.

   It is Caused by pumping of subgrade material, poor drainage, or inadequate load transfer.

   Results in uncomfortable riding quality and increased dynamic loads on slabs.

3. **Pumping:** Pumping refers to the ejection of water and fine soil particles through joints or cracks under wheel loads.

   It cccurs when water accumulates under the slab and traffic causes slab deflection.

   This leads to loss of support and eventual slab failure.

4. **Spalling:** Spalling is the breaking or chipping of concrete at joints, edges, or cracks.
   It is caused by freeze–thaw action, poor quality concrete, corrosion of dowel bars, or excessive stresses.
   Affects ride quality and joint performance.
5. **Scaling:** Scaling is the peeling or flaking of the concrete surface.
   It is generally caused by freeze–thaw cycles, use of de-icing salts, or improper finishing.
   Reduces surface durability and skid resistance.
6. **Blowups:** Blowups are sudden upward movements of slabs at joints or cracks.
   They occur due to thermal expansion when expansion joints are clogged.
   Can cause serious hazards to traffic.
7. **Settlement and Depression:** Settlement occurs when slabs sink due to weak or poorly compacted subgrade.
   This leads to uneven pavement surface and water accumulation.
   Often associated with poor drainage or soil erosion.
8. **Joint Seal Damage:** Failure of joint sealants allows water and debris to enter joints.
   It accelerates pumping, spalling, and corrosion of reinforcement.
   Regular maintenance is required to prevent further deterioration.

7. (a) Solution:

Given:

$$\text{Diameter } D = 0.5 \text{ m}$$

$$\text{Area of base } A_b = \frac{\pi}{4}D^2 = 0.196 \text{ m}^2$$

$$\text{Perimeter P} = \pi D = 1.571 \text{ m}$$

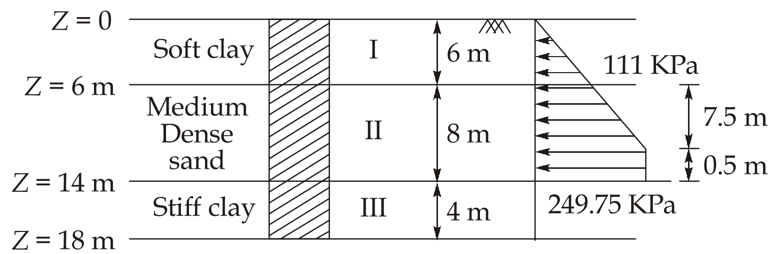
$$\text{Critical depth } L_c = 15D = 7.5 \text{ m}$$

$$\text{Unit weight } \gamma = 18.5 \text{ kN/m}^3$$

Layer 1 (0 – 6 m): $C_{u1} = 30 \text{ kN/m}^2, \alpha_1 = 0.8$

Layer 2 (6 – 14 m): $\phi = 32°, k = 1.2, \delta = 0.75\phi$

Layer 3 (14 – 18 m): $C_{u3} = 100 \text{ kN/m}^2, \alpha_3 = 0.45, N_c = 9$



Skin friction in Layer 1 (0 – 6 m):

$$Q_{s1} = \alpha_1 C_{u1} P L_1$$

$$Q_{s1} = 0.8 \times 30 \times 1.571 \times 6 = 226.195 \text{ kN}$$

Skin friction in Layer 2 (6 – 14 m) with arching effect:

At 6 m: $\sigma'_{v6} = \gamma \times 6 = 18.5 \times 6 = 111 \text{ kN/m}^2$

At 13.5 m: $\sigma'_{v13.5} = 18.5 \times 13.5 = 249.75 \text{ kN/m}^2$

From 13.5 m to 14 m, $\sigma'_v = 249.75 \text{ kN/m}^2$

$$\delta = 0.75\phi = 0.75 \times 32° = 24°$$

Friction from 6 m to 13.5 m:

$$\sigma'_{avg1} = \frac{111 + 249.75}{2} = 180.375 \text{ kN/m}^2$$

$$Q_{s2a} = \sigma'_{avg1}\, k(\tan\delta)P\,(13.5 - 6)$$

$$Q_{s2a} = 180.375 \times 1.2 \times \tan 24° \times 1.571 \times 7.5 = 1135.329 \text{ kN}$$

Friction from 13.5 m to 14 m:

$$Q_{s2b} = 100 \times 1.571 \times 0.5 = 78.55 \text{ kN}$$

$$(\because\ 249.75 \times 1.2 \times \tan 24° = 133.44 > 100)$$

$$Q_{s2} = Q_{s2a} + Q_{s2b} = 1213.88 \text{ kN}$$

Skin friction in Layer 3 (14 – 18 m):

$$Q_{s3} = \alpha_3 C_{u3} P L_3$$

$$Q_{s3} = 0.45 \times 100 \times 1.571 \times 4 = 282.78 \text{ kN}$$

End bearing in Layer 3:

$$Q_b = C_{u3} N_c A_b$$

$$Q_b = 100 \times 9 \times \frac{\pi}{4}(0.5)^2 = 176.715 \text{ kN}$$

Total ultimate load capacity:

$$Q_u = Q_{s1} + Q_{s2} + Q_{s3} + Q_b$$

$$Q_u = 226.195 + 1213.88 + 282.78 + 176.715 = 1899.57 \text{ kN}$$

Assume Factor of safety = 3

$$\text{Safe load capacity} = \frac{1899.57}{3}$$

$$= 633.19 \text{ kN}$$

Q.7 (b) Solution:

Input Data and Initial Conditions:

The characteristics of the sewage, receiving stream, and resulting mixture at the point of discharge are summarized below.

Flow Rates

Sewage flow rate, $Q_s = 1.5 \text{ m}^3/\text{s}$

Stream flow rate, $Q_r = 6.0 \text{ m}^3/\text{s}$

Total mixed flow,

$$Q_{mix} = Q_s + Q_r = 7.5 \text{ m}^3/\text{s}$$

BOD and Temperature

Sewage BOD_5 at 20°C = 200 mg/l (raw)

Stream BOD_5 = 1 mg/l

Temperature (sewage stream mixture) = 20°C

Dissolved Oxygen Conditions

Saturation DO at 20° C

$$DO_{sat} = 9.17 \text{ mg}/l$$

Stream Dissolved Oxygen

$$DO_r = 0.90 \times 9.17 = 8.253 \text{ mg}/l$$

Initial DO of Mixture

Assuming that the sewage has negligible dissolved oxygen:

$$DO_o = \frac{(1.5 \times 0) + (6.0 \times 8.253)}{7.5}$$

$$DO_o = 6.602 \text{ mg}/l$$

Initial Oxygen Deficit

$$D_o = DO_{sat} - DO_o = 9.17 - 6.602$$

$$D_o = 2.568 \text{ mg}/l$$

Allowable Critical Oxygen Deficit

$$D_c = DO_{sat} - DO_{min} = 9.17 - 4.5$$

$$D_c = 4.67 \text{ mg}/l$$

Determination of Maximum Allowable Ultimate BOD of Mixture

Deoxygenation constant:

$$k_d = 0.1 \text{ day}^{-1}$$

Reoxygenation constant:

$$k_r = 0.3 \text{ day}^{-1}$$

Self-Purification Factor

$$f = \frac{k_r}{k_d} = \frac{0.3}{0.1} = 3$$

Critical Deficit Relationship

Using the Streeter-Phelps critical deficit equation:

$$\Rightarrow \quad \frac{L_o}{D_c f} = \left[f\left\{1-(f-1)\frac{D_o}{L_o}\right\}\right]^{\frac{1}{f-1}}$$

$$\Rightarrow \quad \frac{L_o}{4.67 \times 3} = \left[3\left(1-2\frac{2.568}{L_o}\right)\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad L_o = 21.1 \text{ mg}/l \text{ and } 5.4 \text{ mg}/l$$

This represents the maximum allowable ultimate BOD of the mixed stream.

Conversion to Allowable 5-Day BOD of Mixture (for L_o = 21.1 mg/l).

The relationship between ultimate BOD and 5-day BOD (base-10):

$$BOD_5 = L_o(1 - 10^{-k_d t})$$

$$\Rightarrow \quad BOD_{mix,\,5} = 21.1(1 - 10^{-0.1 \times 5})$$

$$\Rightarrow \quad BOD_{mix,\,5} = 21.1(0.6838)$$

$$\Rightarrow \quad BOD_{mix,\,5} = 14.43 \text{ mg}/l$$

Allowable Sewage Effluent BOD

Applying a mass balance at the mixing point:

$$BOD_{mix,\,5} = \frac{Q_s BOD_{S,5} + Q_r BOD_{r,5}}{Q_{mix}}$$

$$\Rightarrow \quad 14.43 = \frac{1.5(BOD_{S,5}) + 6.0(1)}{7.5}$$

$$\Rightarrow \quad 1.5\,(BOD_{S,\,5}) = 108.225 - 6$$

$$\Rightarrow \quad BOD_{S,\,5} = 68.15 \text{ mg}/l$$

Required Treatment Efficiency

$$\Rightarrow \quad \eta = \left(\frac{BOD_{raw} - BOD_{S,5}}{BOD_{raw}}\right) \times 100$$

$$\Rightarrow \quad \eta = \left(\frac{200 - 68.15}{200}\right) \times 100$$

$$\Rightarrow \quad \eta = 65.93\%$$

Conversion to Allowable 5-Day BOD of Mixture (for L_o = 5.4 mg/l).

The relationship between ultimate BOD and 5-day BOD (base-10):

$$BOD_5 = L_o(1 - 10^{-k_d t})$$

$$\Rightarrow \quad BOD_{mix,\,5} = 5.4(1 - 10^{-0.1 \times 5})$$

$$\Rightarrow \quad BOD_{mix,\,5} = 5.4(0.6838)$$

$$\Rightarrow \quad BOD_{mix,\,5} = 3.69 \text{ mg}/l$$

Allowable Sewage Effluent BOD

Applying a mass balance at the mixing point:

$$BOD_{mix,\,5} = \frac{Q_s BOD_{S,5} + Q_r BOD_{r,5}}{Q_{mix}}$$

$$\Rightarrow \quad 3.69 = \frac{1.5(BOD_{S,5}) + 6.0(1)}{7.5}$$

$$\Rightarrow \quad 1.5\,(BOD_{S,\,5}) = 27.675 - 6$$

$$\Rightarrow \quad BOD_{S,\,5} = 14.45 \text{ mg}/l$$

Required Treatment Efficiency

$$\Rightarrow \quad \eta = \left(\frac{BOD_{raw} - BOD_{S,5}}{BOD_{raw}}\right) \times 100$$

$$\Rightarrow \quad \eta = \left(\frac{200 - 14.15}{200}\right) \times 100$$

$$\Rightarrow \quad \eta = 92.925\%$$

7. (c) (i) Solution:

1. Initial Latitudes and Departures

$$\text{Latitude} = l\cos\theta, \text{ Departure} = l\sin\theta$$

Your computed values are:

Line	Length (m)	WCB	Latitude (m)	Departure (m)
PQ	150.00	45°00′	+106.066	+106.066
QR	125.00	130°00′	–80.348	+95.756
RS	160.00	220°30′	–121.666	–103.910
SP	132.00	315.00′	+93.338	–93.338

Summations:

$$\Sigma L = -2.610 \text{ m}, \Sigma D = +4.574 \text{ m}$$

Error of Closure (Linear Error)

$$\Rightarrow \qquad e = \sqrt{(\Sigma L)^2 + (\Sigma D)^2}$$

$$\Rightarrow \qquad e = \sqrt{(-2.610)^2 + (4.574)^2} = \sqrt{27.75} = 5.267 \text{ m}$$

Direction of Error

$$\tan\beta = \left|\frac{\Sigma D}{\Sigma L}\right| = \frac{4.574}{2.610}$$

$$\Rightarrow \qquad \beta = 60°17'\,24.74''$$

Now the quadrant

- $\Sigma L = -2.610 \rightarrow$ South
- $\Sigma D = +4.574 \rightarrow$ East

This places the error in the South-East (SE) quadrant

For SE quadrant:

$$\text{WCB of error} = 180° - \beta$$

$$\text{WCB of error} = 180° - 60°17'\,24.74'' = 119°42'32.26''$$

Bowditch (Compass) Rule Adjustment

Total perimeter:

$$P = \text{sum of lengths of all sides} = 567.00 \text{ m}$$

Correction formulas:

$$C_L = -\left(\frac{l}{P}\right)\Sigma L, C_D = -\left(\frac{l}{P}\right)\Sigma D$$

Adjusted Latitudes and Departures

Line	Lat Corr	Adj. Lat	Dep Corr	Adj. Dep
PQ	+0.690	+106.756	–1.210	+104.856
QR	+0.575	–79.773	–1.008	+94.748
RS	+0.737	–120.929	–1.291	–105.201
SP	+0.608	+93.946	–1.065	–94.403

7. (c) (ii) Solution:

Bitumen used in pavement construction must satisfy requirements related to consistency, durability, temperature susceptibility, safety, and performance. To evaluate these properties, several laboratory tests are conducted. The important tests on bitumen are explained below.

1. **Penetration Test:** This test measures the hardness or consistency of bitumen. A standard needle is allowed to penetrate vertically into a bitumen sample under a load of 100 g for 5 seconds at 25°C. The depth of penetration, measured in tenths of a millimetre (0.1 mm), is reported as the penetration value. Higher penetration indicates softer bitumen, while lower penetration indicates harder grade bitumen.
2. **Ductility Test:** This test determines the tensile property and adhesive characteristics of bitumen. A standard briquette specimen is pulled apart at a specified rate (usually 50 mm/min) at 27°C. The distance in centimeters at which the bitumen thread breaks is recorded as ductility.

   Higher ductility indicates better flexibility and resistance to cracking.
3. **Softening Point Test (Ring and Ball Test):** This test determines the temperature at which bitumen softens to a specified degree. A steel ball is placed on a bitumen sample confined within a ring, and the temperature is gradually increased. The temperature at which the softened bitumen allows the ball to fall a specified distance is recorded as the softening point. It indicates temperature susceptibility and suitability in hot climates.
4. **Specific Gravity Test:** This test determines the density of bitumen relative to water. It is used in mix design calculations and quality control. The specific gravity typically ranges between 0.97 and 1.02 for paving bitumen.
5. **Flash and Fire Point Test:** This test determines the safety of heating bitumen.
   - **Flash Point:** Temperature at which vapors ignite momentarily.
   - **Fire Point:** Temperature at which vapors burn continuously.

   This test ensures safe handling and heating during mixing.

6. **Loss on Heating Test:** This test measures the loss in weight and change in properties after heating bitumen at a specified temperature for a fixed duration. It indicates volatility and short-term aging characteristics.
7. **Float Test:** It is used for very viscous bitumen or cutbacks where penetration test is not suitable. It measures the time required for water to break through a bitumen plug under specified conditions.

8. (a) (i) Solution:

Given data

Height of filling, $H = 12$ m

Factor of safety, $F_s = 1.25$

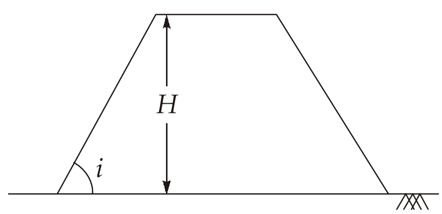
Cohesion, $c = 20$ kN/m^2

Angle of internal friction, $\phi = 15°$

Unit weight of soil, $\gamma = 17.0$ kN/m^3

Stability number at 30° slope, $S_n = 0.063$

Stability number at 45° slope, $S_n = 0.098$



The mobilized angle of internal friction is obtained by reducing the shear strength by the factor of safety. Thus,

$$\tan \phi_m = \frac{\tan \phi}{F_s}$$

$$\Rightarrow \quad \tan \phi_m = \frac{\tan 15°}{1.25} = \frac{0.2679}{1.25} = 0.214$$

$$\Rightarrow \quad \phi_m = \tan^{-1}(0.214) = 12.08° \approx 12°$$

The mobilized cohesion is given by

$$c_m = \frac{c}{F_s}$$

$$\Rightarrow \quad c_m = \frac{20}{1.25} = 16 \text{ kN/m}^2$$

The required stability number for the slope is

$$S_n = \frac{c_m}{\gamma H}$$

$$\Rightarrow \quad S_n = \frac{16}{17.0 \times 12} = \frac{16}{204} = 0.078$$

The required inclination is obtained by linear interpolation between the given stability numbers corresponding to slopes of 30° and 45°.

$$i = 30° + \frac{(0.078 - 0.063)}{(0.098 - 0.063)} \times (45° - 30°)$$

$$\Rightarrow \quad i = 30° + \frac{0.015}{0.035} \times 15°$$

$$\Rightarrow \quad i = 30° + 6.43° = 36.43°$$

8. (a) (ii) Solution:

Improving stability of Slopes

The slopes that are susceptible to failure by sliding can be improved and made safe and serviceable. Various methods are used to stabilise the slopes. The methods generally involve one or more of the following measures, which either reduce the mass which may cause sliding or improve the shear strength of the soil in the failure zone.

1. Slope flattening reduces the weight of the mass that tends to slide. It can be used wherever possible.
2. Providing a berm at the toe of the slope increases the resistance to movement. It is specially useful when there is a possibility of a base failure.
3. Drainage helps in reduces the seepage forces and hence increases the stability. The zone of subsurface water is lowered and infiltration of the surface water is prevented.
4. Densification by use of explosives, vibroflotation, or terra probe helps in increasing the shear strength of cohesionless soils and thus increasing the stability.
5. Consolidation by surcharging, electro-osmosis or other methods helps in increasing the stability of slopes in cohesive soils.
6. Grouting and injection of cement or other compounds into specific zones help in increasing the stability of slopes.
7. Sheet piles and retaining walls can be installed to provide lateral support and to increase the stability. However, the method is relatively expensive.
8. Stabilisation of the soil helps in increasing the stability of slopes.

In the interest of economy, relatively inexpensive methods, such as slope flattening and drainage control, are generally preferred.

8. (b) Solution:

True Difference in Elevation (ΔH)

In reciprocal levelling, the true difference in elevation between two points is obtained by taking the average of the apparent differences observed from both instrument stations.

Let:

a_1, b_1 = staff readings on A and B when the instrument is near A

a_2, b_2 = staff readings on A and B when the instrument is near B

$$\Delta H = \frac{(b_1 - a_1) + (b_2 - a_2)}{2}$$

Substituting the given values:

$$\Delta H = \frac{(2.685 - 1.455) + (2.045 - 0.925)}{2}$$

$$\Rightarrow \quad \Delta H = \frac{1.230 + 1.120}{2}$$

$$\Rightarrow \quad \Delta H = 1.175 \text{ m}$$

Since the staff reading at point B is greater than that at A, point B is lower than point A.

1. Reduced Level of Point B

Given: $RL_A = 120.500$ m

Since B is lower than A:

$$RL_B = RL_A - \Delta H$$

$$\Rightarrow \quad RL_B = 120.500 - 1.175$$

$$\Rightarrow \quad RL_B = 119.325 \text{ m}$$

2. Combined Error Due to Curvature and Refraction (e_{c+r})

The combined correction for curvature and refraction is given by:

$$e_{c+r} = 0.06735\, d^2$$

Where d is the distance in kilometers.

$$d = 800 \text{ m} = 0.8 \text{ km}$$

$$\Rightarrow \quad e_{c+r} = 0.06735 \times (0.8)^2$$

$$\Rightarrow \quad e_{c+r} = 0.043 \text{ m}$$

3. Collimation Error of the Instrument (e_{coll})

The apparent difference in elevation from the first instrument station is:

$$(b_1 - a_1) = 2.685 - 1.455 = 1.230 \text{ m}$$

The total observational error is therefore:

$$E = (b_1 - a_1) - H$$

$$\Rightarrow \quad E = 1.230 - 1.175$$

$$\Rightarrow \quad E = 0.055 \text{ m}$$

This total error consists of curvature, refraction, and collimation errors:

$$E = e_{c+r} + e_{coll}$$

$$\Rightarrow \quad 0.055 = 0.043 + e_{coll}$$

$$\Rightarrow \quad e_{coll} = 0.012 \text{ m}$$

8. (c) Solution:

1. Total Biomass Inventory ($V \times X_v$)

Given:

Reactor volume, $V = 3{,}000 \text{ m}^3$

Mixed liquor suspended solids (MLSS): $X = 3{,}500$ mg/L

Volatile fraction of MLSS: $f_v = 0.8$

Conversion to mixed liquor volatile suspended solids (MLVSS):

$$X_v = 0.8 \times 3{,}500 = 2{,}800 \text{ mg/L}$$

Biomass inventory: Total biomass to be maintained in tank in steady state

$$VX_v = 3{,}000 \text{ m}^3 \times 2{,}800 \text{ g/m}^3 \times 10^{-3}$$

$$VX_v = 8{,}400 \text{ kg}$$

2. Gross Biomass Growth Rate

Gross biomass growth due to substrate utilization is given by:

$$\text{Gross Growth} = Q \times Y \times (S_0 - S)$$

Where:

Flow rate, $Q = 12{,}000 \text{ m}^3/\text{day}$

Yield coefficient, $Y = 0.5$ kg VSS per kg of BOD removed

Influent substrate concentration, $S_o = 250$ mg/l

Effluent substrate concentration, $S = 25$ mg/l

$$\text{Gross Growth} = 12{,}000 \times 0.5 \times (250 - 25) \times 10^{-3}$$

$$\text{Gross Growth} = 1{,}350 \text{ kg/day}$$

3. Biomass Loss Due to Endogenous Decay

The biomass lost due to endogenous respiration is:

$$\text{loss} = VS_v \cdot k_e$$

Where:

$$k_e = 0.06 \text{ day}^{-1}$$

$$\text{loss} = 8{,}400 \times 0.06$$

$$\text{loss} = 504 \text{ kg/day}$$

4. The net biomass production rate is:

$$\text{Net Growth} = \text{Gross Growth} - \text{loss}$$

$$\text{Net Growth} = 1{,}350 - 504$$

$$\text{Net Growth} = 846 \text{ kg/day}$$

5. Determination of Sludge Age (θ_c)

At steady state, net biomass growth = net biomass removed

Sludge age (mean cell residence time) is defined as:

$$\theta_c = \frac{\text{Total Biomass Inventory}}{\text{Net Biomass Production Rate}}$$

$$\theta_c = \frac{8400}{846}$$

$$\theta_c = 9.93 \text{ days}$$

6. Biomass Loss in Effluent

$$\text{Effluent loss} = Q \cdot X_e$$

Where:

Effluent suspended solids concentration,

$$X_e = 15 \text{ mg}/l$$

$$\text{Effluent loss} = 12{,}000 \times 15 \times 10^{-3}$$

$$\text{Effluent loss} = 180 \text{ kg/day}$$

Biomass to Be Removed by Wasting

At steady state:

$$\text{Mass wasted} = \text{Net Growth} - \text{Effluent loss}$$

$$\text{Mass wasted} = 846 - 180$$

$$\text{Mass wasted} = 666 \text{ kg/day}$$

7. Wasting Flow Rate

Assuming wasting from the return sludge line:

Return sludge concentration,

$$X_u = 10{,}000 \text{ mg}/l = 10 \text{ kg/m}^3$$

$$Q_w = \frac{\text{Mass Wasted}}{X_u}$$

$$Q_w = \frac{666}{10} = 66.6 \text{ m}^3/\text{day}$$

8. Determination of Recycle Ratio (R)

A solids mass balance at the influent return sludge junction gives:

$$QX_0 + Q_r X_u = (Q + Q_r)X$$

Assuming influent suspended solids are negligible ($X_0 \approx 0$):

$$R = \frac{Q_r}{Q} = \frac{X}{X_u - X}$$

$$R = \frac{3500}{10000 - 3500}$$

$$R = 0.538 \text{ (or } 53.8\%)$$

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