MADE EASY

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Detailed Solutions

ESE-2026 Mains Test Series

Mechanical Engineering Test No : 1

Section A : Thermodynamics [All Topics]

Section B : IC Engine + Refrigeration and Air-conditioning [All Topics]

Section A : Thermodynamics

1. (a) (i) Solution:

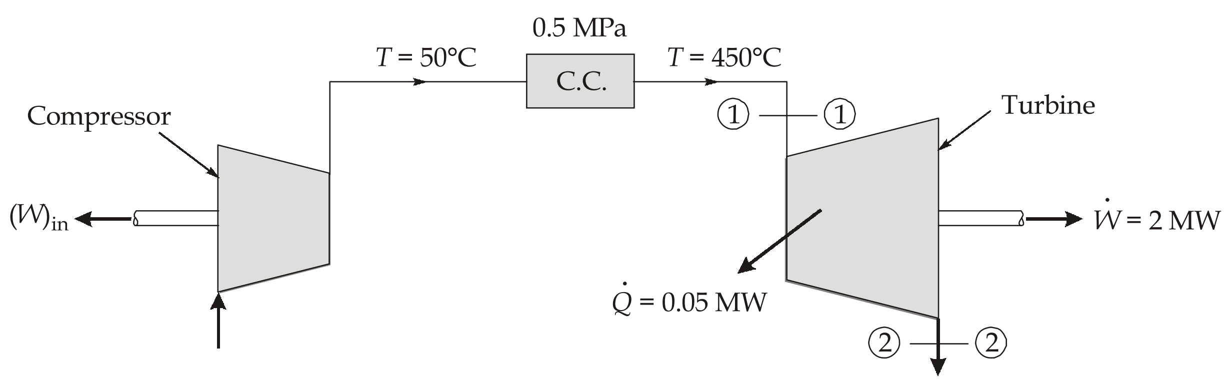
Assumptions:

(i) This is steady flow process.

(ii) Change in K.E. and P.E. are neglected.

(iii) Specific heats of air are constant.

(iv) Air is assumed to be ideal gas.



Using SFEE for turbine

$$\dot{m}\left[h_1 + \frac{V_1^2}{2000} + gz_1\right] + \dot{Q} = \dot{m}\left[h_2 + \frac{V_2^2}{2000} + gz_2\right] + \dot{W}$$

∵ Change in K.E. and P.E. are neglected.

$$\dot{m}h_1 + \dot{Q} = \dot{m}h_2 + \dot{W} \quad [\dot{m}, h, \dot{Q}, \dot{W} \text{ are used as standard notations}]$$

$$\dot{m}(h_1 - h_2) = \dot{W} - \dot{Q} \quad \text{...(i)}$$

∵ $\dot{W} = 2$ MW

$\dot{Q} = 0.05$ MW

Using, $P\dot{V} = \dot{m}RT$

$$\dot{m} = \frac{P\dot{V}}{RT}$$

$$= \frac{0.5 \times 10^6 \times 3.5}{0.287 \times 10^3 \times (273 + 50)}$$

$$\dot{m} = 18.877 \text{ kg/s}$$

From equation (i)

$$18.877 \times [c_pT_1 - c_pT_2] = [2 - 0.05] \times 10^3$$

$$[T_1 - T_2] = \frac{(2 - 0.05) \times 10^3}{18.877 \times c_p}$$

$$(450 + 273) - T_2 = \frac{1.95 \times 10^3}{18.877 \times 1.005} \quad \text{[Note for air } (c_p) = 1.005 \text{ kJ/kgK]}$$

$$T_2 = 723 - 102.786$$

$$T_2 = 620.214 \text{ K} = 347.21°\text{C}$$

1. (a) (ii) Solution:

1. Clausius inequality is stated as

$$\oint \frac{\delta Q}{T} \leq 0 \quad \text{[This is for a process]}$$

where δQ is the infinitesimal heat transfer to the system and T is absolute temperature at the boundary at which heat transfer occurs.

Physical significance: The Clausius inequality provides a quantitative statement of the second law of thermodynamics. It implies that complete conversion of heat into work in a cyclic process is impossible and establishes the direction of natural process. It also forms the mathematical basis for defining entropy as a property.

2. Consider a closed system undergoing a reversible process from state 1 to state 2. from Clausius inequality, for any cyclic process.

$$\oint \frac{\delta Q}{T} = 0 \qquad \text{(reversible)}$$

This implies that the integral of $\frac{\delta Q}{T}$ between two states is path independent. Hence, a property can be defined as:

$$ds = \frac{\delta Q_{rev}}{T}$$

on integrating between state 1 and 2

$$s_2 - s_1 = \int_1^2 \frac{\delta Q_{rev}}{T} \qquad \text{[For process]}$$

The property S is defined as entropy, which is a state function. It means for a cycle the change in this property is zero.

3. For a reversible process:

$$ds = \frac{\delta Q_{rev}}{T}$$

or $$\int \frac{\delta Q}{T} < 0 \qquad \text{[For is irreversible process]}$$

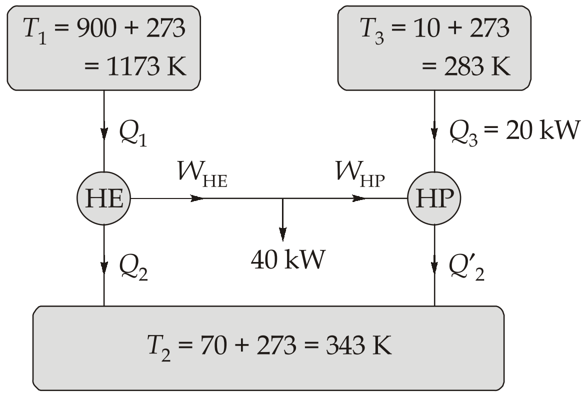
Entropy change of the system is equal to the entropy transfer due to heat interaction.

For an irreversible process:

$$ds > \frac{\delta Q}{T}$$

The entropy change of the system is greater than the entropy transfer due to heat interaction because of entropy generation due to irreversibilities such as friction, unrestrained expansion, and finite temperature gradients.

1. (b) Solution:



For heat pump, $(COP)_{HP} = \dfrac{T_2}{T_2 - T_3} = \dfrac{343}{343 - 283} = 5.716$

Also, $(COP)_{HP} = \dfrac{Q_2'}{W_{HP}} = \dfrac{W_{HP} + Q_3}{W_{HP}}$

$$5.716 = 1 + \frac{Q_3}{W_{HP}}$$

$$5.716 - 1 = \frac{20}{W_{HP}}$$

$$W_{HP} = 4.24 \text{ kW}$$

∴ Work output of heat engine $= W_{HP} + 40 = 4.24 + 40$

$$W_{HE} = 44.24 \text{ kW}$$

For heat engine, $\eta_{HE} = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{343}{1173} = 0.7075$

(i) $\eta_{HE} = \dfrac{W_{HE}}{Q_1}$

$$0.7075 = \frac{44.24}{Q_1}$$

$$Q_1 = 62.52 \text{ kW}$$

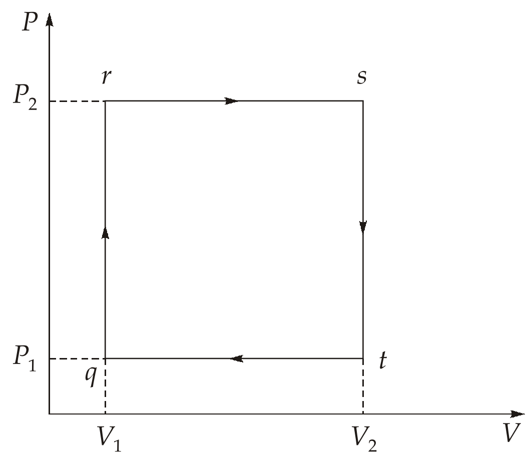
(ii) Rate of heat rejection to 70°C sink

- From HE, $Q_2 = Q_1 - W_{HE}$
  $= 62.52 - 44.24$
  $Q_2 = 18.280 \text{ kW}$
- From HP, $Q_2' = Q_3 + W_{HP}$

$$Q_2' = 20 + 4.24 = 24.24 \text{ kW}$$

$$\text{Total heat} = Q_2 + Q_2' = 18.280 + 24.24$$
$$= 42.52 \text{ kW}$$

1. (c) Solution:



(i) Net work done, per cycle, W_{net} = Area of cycle

$$W_{net} = (P_2 - P_1)(V_2 - V_1)$$

(ii) Heat absorbed by 1 mole of gas in one cycle,

$$Q_{in} = Q_{qr} + Q_{rs}$$

[As qr is constant volume and qs is constant pressure process]

$$Q_{in} = c_v\left(T_r - T_q\right) + c_p\left(T_s - T_r\right) \quad ...(i)$$

Now, for process qr, $\dfrac{T_q}{T_r} = \dfrac{P_1}{P_2}$

$\Rightarrow$ $T_q = T_r\left[\dfrac{P_1}{P_2}\right]$

Using ideal gas equation, for 1 mole of gas

$P_2V_1 = RT_r$ [This is at point r]

For process, rs, $\dfrac{T_r}{T_s} = \dfrac{V_1}{V_2}$ [$\because$ This is a constant pressure process]

$\Rightarrow$ $T_r = T_s\left[\dfrac{V_1}{V_2}\right]$

From equation (i),

$$Q_{in} = c_v T_r\left(1-\frac{P_1}{P_2}\right)+c_p T_r\left(\frac{V_2}{V_1}-1\right)$$

Thermal efficiency of the cycle,

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{(P_2-P_1)(V_2-V_1)}{\frac{P_2V_1}{R}\left[c_v\left(\frac{P_2-P_1}{P_2}\right)+c_p\left(\frac{V_2-V_1}{V_1}\right)\right]}$$

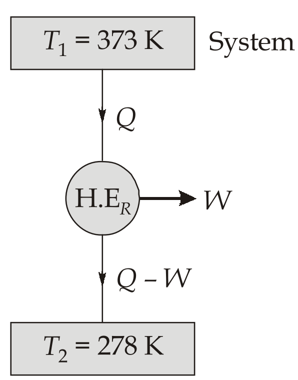
$$= \frac{R(P_2-P_1)(V_2-V_1)}{c_vV_1(P_2-P_1)+c_pP_2(V_2-V_1)}$$

$$\eta = \frac{c_p-c_v}{c_v\frac{V_1}{V_2-V_1}+c_p\frac{P_2}{P_2-P_1}}$$

Divide by c_v,
$$\eta = \frac{\frac{c_p}{c_v}-1}{\frac{V_1}{V_2-V_1}+\frac{c_p}{c_v}\frac{P_2}{P_2-P_1}} = \frac{\gamma-1}{\frac{V_1}{V_2-V_1}+\gamma\frac{P_2}{P_2-P_1}}$$

1. (d) Solution:

Given data: T_1 = 100 + 273 = 373 K; T_2 = 5 + 273 = 278 K



Final temperature of system is 278 K

Heat given by system,
$$Q = \int_{T_1}^{T_2} c_v dT$$

$\Rightarrow$
$$Q = \int_{T_1}^{T_2}(A+2BT)dT = \int_{373}^{278}(0.019+2\times5.8\times10^{-4}\times T)dT$$

$$= \left[0.019 \times T + 2 \times 5.8 \times 10^{-4} \times \frac{T^2}{2}\right]_{373}^{278}$$

$$= -37.675 \text{ J (flow from the system)}$$

Now, entropy change of system,

$$(\Delta s)_{system} = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{c_v dT}{T}$$

$$(\Delta s)_{system} = \int_{373}^{278} \left(\frac{A + 2BT}{T}\right) dT = [A \ln T + 2BT]_{373}^{278}$$

$$(\Delta s)_{system} = 0.019 \times \ln\left[\frac{278}{373}\right] + 2 \times 5.8 \times 10^{-4}[278 - 373]$$

$$(\Delta s)_{system} = -0.11578 \text{ J/K}$$

Now, entropy change of reservoir,

$$(\Delta s)_{res} = \frac{Q - W}{T_2}$$

$$(\Delta s)_{res} = \frac{37.675 - W}{278}$$

For maximum workput,

$$(\Delta s)_{universe} \geq 0$$

$$(\Delta s)_{res} + (\Delta s)_{system} \geq 0$$

$$\frac{37.675 - W}{278} + (-0.11578) \geq 0$$

On solving, $W \leq 5.486$ J

Maximum work, $W_{max} = 5.486$ J **Ans.**

1. (e) Solution:

Given : Inlet Condition of turbine, P_1 = 7 bar; T_1 = 850 K

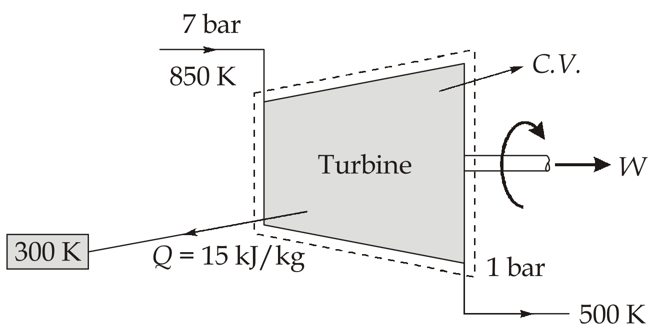
Outlet Condition of turbine, P_2 = 1 bar; T_2 = 500 K

Heat lost to surrounding, Q = 15 kJ/kg

Assumption:

1. This is steady flow process.

2. Heat transfer occurs with a thermal reservoir at 300 K



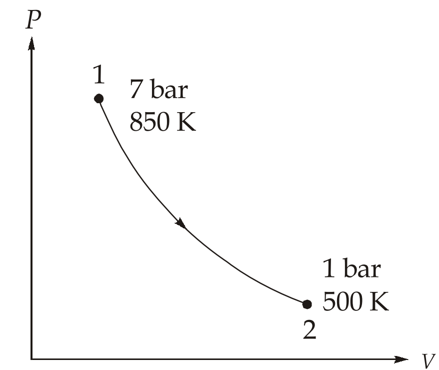


Fig. *P*-*V* diagram

(i) Entropy change in turbine (system)

$$\Delta s_{system} = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$\Delta s_{system} = 1.0\ln\left[\frac{500}{850}\right] - 0.287\ln\left[\frac{1}{7}\right] = 0.0278 \text{ kJ/kgK}$$

Entropy change of surrounding

$$\Delta s_{surr} = \frac{Q}{T_{surr}} = \frac{15}{300} = 0.05 \text{ kJ/kgK}$$

$$\text{Irreversibility, } I = 300[0.0278 + 0.05]$$

$$= 23.354 \text{ kJ/kg of air}$$

(ii) For actual work, using SFEE

$$h_1 + Q = W + h_2$$

$$\Rightarrow \quad W = (h_1 - h_2) + Q$$

$$W = c_p (T_1 - T_2) + Q$$

$$W = 1.0[850 - 500] + (-15)$$

$$W_{act} = 335 \text{ kJ/kg}$$ **Ans.**

(iii) Maximum work = Change in availability

$$W_{max} = \psi_1 - \psi_2$$

$$= (h_1 - T_0 s_1) - (h_2 - T_0 s_2)$$

$$= (h_1 - h_2) - T_0 (s_1 - s_2)$$

$$= c_p(T_1 - T_2) - T_0\left[c_p \ln\frac{T_1}{T_2} - R\ln\frac{P_1}{P_2}\right]$$

$$= 1.0 \times (850 - 500) - 300\left[1.0 \ln \frac{850}{500} - 0.287 \ln\left(\frac{7}{1}\right)\right]$$

$$W_{max} = \psi_1 - \psi_2 = 358.35 \text{ kJ/kg}$$ **Ans.**

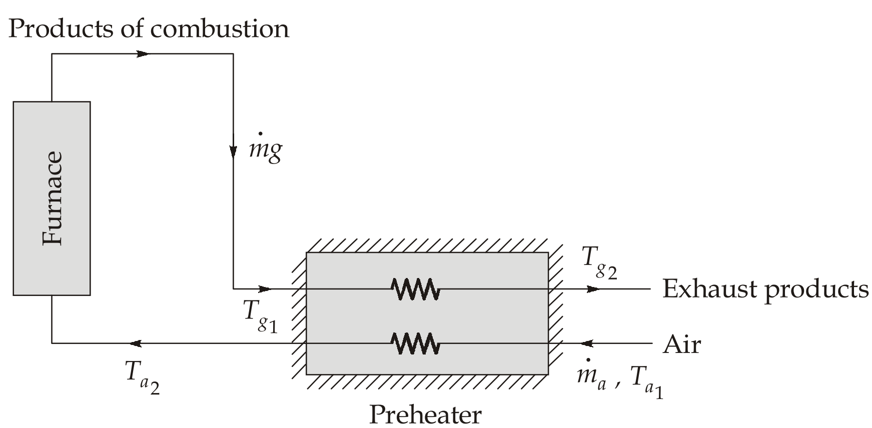
2. (a) Solution:

Given data : $\dot{m}_g$ = 20 kg/s; T_{g1} = 400°C = 673 K; T_{g2} = 200°C = 473 K;

$\dot{m}_a$ = 18 kg/s; T_{a1} = 30°C = 303 K, T0 = 300 K, C_{pg} = 1.03 kJ/kgK

Assumption:

1. This is steady state flow process.
2. Neglect the change in kinetic and potential energy.
3. Pressure losses in pipe are neglected.
4. No heat loss from the heat exchanger to surrounding.



(i)

$$\phi_1 = \text{Initial availability of products}$$

$$\phi_1 = (h_1 - h_0) - T_0(s_1 - s_0)$$

$$\phi_1 = c_{p_g}\left(T_{g1} - T_0\right) - T_0\left[c_{pg} \ln \frac{T_{g1}}{T_0}\right]$$

$$\phi_1 = 1.03(673 - 300) - 300\left[1.03 \ln \frac{673}{300}\right]$$

$$\phi_1 = 134.529 \text{ kJ/kg}$$

$$\phi_2 = \text{Final availability of products}$$

$$\phi_2 = (h_2 - h_0) - T_0(s_2 - s_0)$$

$$\phi_2 = 1.03 \times (473 - 300) - 300\left[1.03 \times \ln \frac{473}{300}\right]$$

$$\phi_2 = 37.498 \text{ kJ/kg}$$

(ii) Apply energy balance in heat exchanger

$$\text{Heat lost by products} = \text{Heat gained by air}$$

$$\dot{m}_g \times c_{p_g} \times (T_{g1} - T_{g2}) = \dot{m}_a \times c_{p_a} \times (T_{a_2} - T_{a_1})$$

$$20 \times 1.03 \times (673 - 473) = 18 \times 1.005 \times (T_{a_2} - 303)$$

$$T_{a_2} = 530.75 \text{ K}$$

Entropy change of the products,

$$(\Delta s)_{products} = \dot{m}_g c_{pg} \ln \frac{T_{g2}}{T_{g1}}$$

$$= 20 \times 1.03 \times \ln\left[\frac{473}{673}\right] = -7.264 \text{ kJ/K}$$

$$\text{Entropy change of air, } (\Delta s)_{air} = \dot{m}_a c_{p_a} \ln \frac{T_{a_2}}{T_{a_1}}$$

$$= 18 \times 1.005 \times \ln\left[\frac{530.75}{303}\right] = 10.140 \text{ kW/K}$$

$$(\Delta s)_{universe} = (\Delta s)_{system} + (\Delta s)_{surrounding}$$

$$= (\Delta s)_{product} + (\Delta s)_{air} + (\Delta s)_{surrounding}$$

$$= -7.264 + 10.140 + 0 \quad [\because \text{ No heat loss to surrounding}]$$

$$= 2.876 \text{ kW/K}$$

$$\text{Irreversibility} = T_0 \times (\Delta s)_{universe}$$

$$= 300 \times 2.876 = 862.949 \text{ kW}$$ **Ans.**

(iii) Let, T'_{a_2} be the final temperature of air

For reversible process,

$$(\Delta s)_{universe} = 0$$

or $$(\Delta s)_{product} + (\Delta s)_{air} = 0$$

$$-7.264 + \dot{m}_a c_{p_a} \ln\left[\frac{T'_{a_2}}{T_{a_1}}\right] = 0$$

$$-7.264 + 18 \times 1.005 \times \ln\left[\frac{T'_{a_2}}{303}\right] = 0$$

$$T'_{a_2} = 452.72 \text{ K}$$

So, heat transfer by products of combustion,

$$Q_1 = \dot{m}_g c_{p_g} \left[T_{g_2} - T_{g_1}\right]$$

$$= 20 \times 1.03 \times (673 - 473) = 4120 \text{ kW}$$

Heat given to the air preheater,

$$Q_2 = \dot{m}_a c_{p_a} \left[T'_{a_2} - T_{a_1}\right]$$

$$= 18 \times 1.005 \times [452.72 - 303]$$

$$= 2708.43 \text{ kW}$$

$\therefore$ From 1st law of thermodynamics

$$Q_1 = W + Q_2$$

$$W = Q_1 - Q_2$$

$$W = 4120 - 2708.43$$

$$W = 1411.56 \text{ kW}$$

Ans.

2. (b) Solution:

Given: $T_1 = 27 + 273 = 300$ K; $P_1 = 1$ bar; $T_2 = 164 + 273 = 437$ K; $P_2 = 4$ bar

Exit velocity, $C_2 = 110$ m/s

Assumption:

1. This is steady flow process.
2. Air is Assumed to be ideal gas.
3. Specific heats are constant.

(i) After isentropic compression

Using
$$\frac{T_{2s}}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{\gamma-1}{\gamma}}$$

$$T_{2s} = 300 \times \left[\frac{4}{1}\right]^{\frac{1.4-1}{1.4}} = 445.798 \text{ K}$$

Since the temperature is higher than the given temperature of 437 K, there is heat loss to the surroundings. The compression cannot be adiabatic. It must be polytropic.

(ii)
$$\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{n-1}{n}}$$
[n is the polytropic index of compression]

$$\frac{437}{300} = \left[\frac{4}{1}\right]^{\frac{n-1}{n}}$$

$$n = 1.3723$$ **Ans.**

(iii) Actual work of compression, using SFEE

$$W_a = h_1 - h_2 - \frac{c_2^2}{2000}$$

$$= c_p(T_1 - T_2) - \frac{c_2^2}{2000}$$

$$W_a = 1.003 \times (300 - 437) - \frac{110^2}{2000}$$

$$= -143.461 \text{ kJ/kg}$$

[Negative sign means work input to the system compression]

$$\text{Isothermal work, } W_T = -\int_1^2 v dp - \frac{c_2^2}{2000} = -RT_1 \ln\frac{P_2}{P_1} - \frac{c_2^2}{2000}$$

$$= -0.287 \times 300 \times \ln\frac{4}{1} - \frac{110^2}{2000}$$

$$= -125.409 \text{ kJ/kg}$$

$$\text{Isothermal efficiency, } \eta_T = \frac{W_T}{W_a} = \frac{125.409}{143.461}$$

$$\eta_T = 0.8742 \text{ or } 87.42\%$$

(iv) Minimum work input = Decrease in availability

$$W_{min} = \psi_1 - \psi_2$$

$$= h_1 - h_2 - T_0(s_1 - s_2) + \frac{c_1^2 - c_2^2}{2000}$$

$$= c_p(T_1 - T_2) - T_0\left[R \ln\frac{P_2}{P_1} - c_p \ln\frac{T_2}{T_1}\right] - \frac{c_2^2}{2000}$$

$$= 1.003 \times (300 - 437) - 300 \times \left[0.287 \times \ln\frac{4}{1} - 1.003 \times \ln\frac{473}{300}\right] - \left[-\frac{110^2}{2000}\right]$$

$$W_{min} = -125.817 \text{ kJ/kg}$$ **Ans.**

$$\text{Irreversibility, } I = W_{min} - W_{act}$$

$$I = -125.817 - (-143.461)$$

$$I = 17.643 \text{ kJ/kg}$$ **Ans.**

(v) Second law efficiency in case of work consuming device is given as

$$\eta_{II} = \frac{\text{Minimum work input}}{\text{Actual work input}}$$

$$\eta_{II} = \frac{125.817}{143.461} = 0.8770$$

$$\eta_{II} = 87.70\%$$ **Ans.**

2. (c) (i) Solution:

Given data: m = 20 kg; height, h = 20 m

First law of thermodynamics, $Q = \Delta E + W$

For adiabatic process, $0 = \Delta E + W_{ad}$

$$W_{on} = W_{ad} = E_f - E_i$$

Here, W_{on} is work done on the object by external drag force.

Considering an infinitesimal change with the object falling by a distance (dh), we can write

$$dE = \delta W = Fdx$$

$$dE = -Fdh \quad \text{...(i)}$$

$\because$ Negative sign is used because force and displacement are in opposite direction.

Since there is no thermal interaction and the only interaction is that involving mechanical energy transfer, so,

$$dE = d(KE + PE) = d\left(\frac{mV^2}{2}\right) + d(mgx)$$

$$= d\left(\frac{mV^2}{2}\right) - mgdh$$

From equation (i),

$$d\left(\frac{mV^2}{2}\right) - mgdh = -Fdh\,;$$

$\Rightarrow$
$$d\left(\frac{V^2}{2}\right) = \frac{(mg - F)dh}{m}$$

Substituting m = 20 kg; g = 9.8 m/s^2

$$d\left(\frac{V^2}{2}\right) = \frac{196 - \frac{V^2}{200} dh}{20}$$

$$dh = \frac{10}{196 - \frac{V^2}{200}} d(V^2)$$

On integrating, $$\int_0^{20} dh = \int_0^{V_f} \frac{10(dV^2)}{196 - \frac{V^2}{200}}$$

$\because$ at $h = 0$, $v = 0$; at $h = 20$, $v = v_f$

$$[h]_0^{20} = \left[-10 \times 200 \times \ln\left(196 - \frac{V^2}{200}\right)\right]_0^{V_f}$$

$$20 = -2000 \ln\left[\frac{196 - \frac{V_f^2}{200}}{196}\right]$$

On solving, $V_f = 19.749$ m/s

Considering the overall energy equation,

$$E_f - E_i = W_{on}$$

$$\left[\frac{1}{2} m V_f^2 + 0\right] - \left[\frac{1}{2} m V_i^2 + mgh\right] = W_{on}$$

$$\left[\frac{1}{2} \times 20 \times 19.749^2 + 0\right] - [0 + 20 \times 9.8 \times 20] = W_{on}$$

$$W_{on} = -19.77 \text{ J}$$ **Ans.**

i.e. 19.77 J of work is done by the object in overcoming air resistance.

2. (c) (ii) Solution:

The different types of work commonly encountered in thermodynamics are as follows:

1. Boundary work occurs when a system undergoes a change in volume against an external pressure, such as in piston cylinder arrangements.

$$\delta W = \int P dV$$

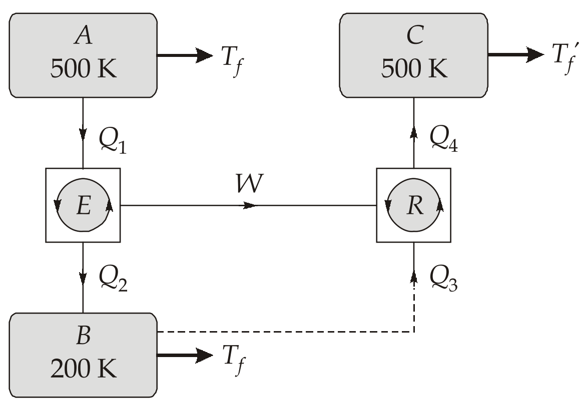
2. Flow work is required to push mass into or out of a control volume against pressure and is significant in open systems. It is given by Pv per unit mass.
3. Shaft work is associated with rotating devices such as turbine, compressors and pumps, where energy transfer occurs due to torque and angular displacement.
4. Electrical work is associated with electrical system due to flow of current under applied voltage.

$$\delta W = VIdt$$

Nature of work: Thermodynamic work is a path function, not a point state function. This is because the amount of work done during a process depends on the specific path followed by system plus initial and final state, not solely on the end states.

3. (a) Solution:

Let, the three identical bodies A, B and C having the same heat capacity 'c' be respectively at 500 K, 200 K and 500 K initially, and let us operate a heat engine and a refrigerator, as shown in Figure.



Let T_f be the final temperature of bodies A and B and T_f' be the final temperature of body C.

Change in entropy of bodies;

$$(\Delta s)_A = c\ln\left[\frac{T_f}{500}\right]$$

$$(\Delta s)_B = c\ln\left[\frac{T_f}{200}\right]$$

$$(\Delta s)_c = c\ln\left[\frac{T_f'}{500}\right]$$

$$(\Delta s)_{HE} = 0$$

$$(\Delta s)_{ref} = 0$$

Since, $(\Delta s)_{universe} \geq 0$

$(\Delta s)_A + (\Delta s)_B + (\Delta s)_C + (\Delta s)_{HE} + (\Delta s)_{ref} \geq 0$

$$c\ln\frac{T_f}{500} + c\ln\frac{T_f}{200} + c\ln\frac{T'_f}{500} \geq 0$$

$$c\ln\left[\frac{T_f^2 T'_f}{50000000}\right] \geq 0$$

For minimum value of T'_f

$$c\ln\left[\frac{T_f^2 T'_f}{50000000}\right] = 0$$

$$\Rightarrow \quad \frac{T_f^2 T'_f}{50000000} = 1 \quad \text{...(i)}$$

Now, heat interaction by each body

$$Q_1 = c(500 - T_f)$$

$$Q_2 = c(T_f - 200)$$

$$Q_4 = c(T'_f - 500)$$

Again, Q_1 = Heat removed from body A

= Heat discharged to bodies B and C

= $Q_2 + Q_4$

$$c(500 - T_f) = c(T_f - 200) + c(T'_f - 500)$$

$$0 = 2T_f - 1200 + T'_f$$

$$T'_f = 1200 - 2T_f \quad \text{...(ii)}$$

T'_f will be highest when T_f is minimum.

From equation (i) and (ii), we get

$$T_f^2(1200 - 2T_f) = 50000000$$

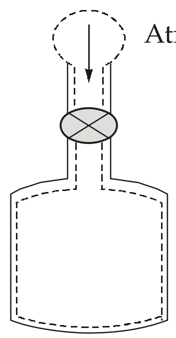
$$2T_f^3 - 1200T_f^2 + 50000000 = 0$$

On solving, $T_f = 279.128$ K

From equation (ii), $T_f' = 641.744$ K **Ans.**

3. (b) Solution:

Given data : $P_{atm} = 760$ mm Hg; $T_a = 27°C$



Assumption:

1. Specific heats are assumed to be constant
2. Change in KE and PE are neglected.

(i) Before filling, bottle is empty

m_1 = Mass of air = 0

u_1 = Internal energy = 0

After filling, m_2 = Mass of air

u_2 = Internal energy

Mass equation for control volume

$$(m_i - \cancel{m_e}^{\,0}) = m_2 - m_1$$

$$m_i = m_2 - m_1 \quad \text{...(i)}$$

Energy equation for the process

$$\frac{dE}{dt} = m_i\left[h_i + \frac{V_i^2}{2000} + gz_i\right] + Q - m_e\left[h_e + \frac{V_e^2}{2000} + gz_e\right] - \dot{W}$$

$$U_2 - U_1 = m_i h_i + \cancel{\dot{Q}}^{\,0} - \cancel{m_e}^{\,0} h_e - \cancel{\dot{W}}^{\,0} \quad [\dot{W} = 0 \text{ no work done by C.V}]$$

$$m_2 u_2 - \cancelto{0}{m_1} u_1 = (m_2 - \cancelto{0}{m_1})h_i$$

$\because$ Initial mass, $m_1 = 0$

$$m_2 u_2 = m_2 h_i$$

$$u_0 + 0.718 t_2 = u_i + (pv)_i \qquad \{\text{As } (pv)_i = 0.287\,[t_i + 273]\}$$

$$u_0 + 0.718 t_2 = u_o + 0.718 t_i + 0.287 \times (t_i + 273)$$

$$0.718 t_2 = 0.718 \times 27 + 0.287 \times (27 + 273)$$

$$t_2 = 146.916°\text{C}$$

(ii) if $m_1 \neq 0$

before filling, $V_1 = 0.04\ \text{m}^3$

$$P_1 = 500 \text{ mm Hg}$$

$$T_1 = 27°\text{C}$$

$$P_1 V_1 = m_1 \times 0.287 \times (t + 273)$$

$$m_1 = \frac{P_1 V_1}{0.287(t + 273)} = \frac{(0.5 \times 13.6 \times 9810) \times 0.04}{0.287 \times 300}$$

$$= 30.99 \text{ kg}$$

Similarly, after filling

$$m_2 = \frac{P_2 V_2}{0.287(t_2 + 273)} = \frac{(0.76 \times 13.6 \times 9810) \times 0.04}{0.287 \times (t_2 + 273)}$$

$$= \frac{14131.86}{t_2 + 273} \text{kg} \qquad [\text{Here } t_2 \text{ is in °C}]$$

Energy equation for this process,

$$m_2 u_2 - m_1 u_1 = m_i h_i + \cancelto{0}{\dot{Q}} - \cancelto{0}{m_e} h_e - \cancelto{0}{\dot{W}}$$

$$m_2 u_2 - m_1 u_1 = (m_2 - m_1) h_i$$

$$\left(\frac{14131.86}{t_2 + 273}\right) \times (u_0 + 0.718 t_2) - 30.99 \times (u_0 + 0.718 \times 27) = \left[\frac{14131.86}{t_2 + 273} - 30.99\right] \times \left[u_i + (pv)_i\right]$$

$$\left[\frac{14131.86}{t_2 + 273} \times 0.718 t_2 - 30.99 \times 19.386\right] = \left[\frac{14131.86}{t_2 + 273} - 30.99\right] \times [0.718 \times 27 + 0.287 \times 300]$$

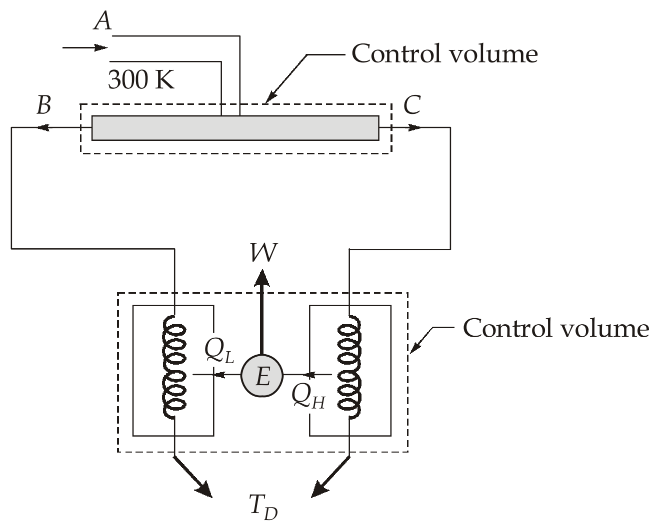
On solving, $t_2 = 59.484°\text{C}$ **Ans.**

3. (c) Solution:

Given : T_A = 300 K; P_A = 15 bar; T_B = 250 K; P_B = 1 bar; T_C = 315 K; P_C = 1 bar

Assumption :

1. Change in KE and PE are neglected.
2. No heat transfer from surroundings.



Considering the vortex tube as control volume with one stream entering and two streams leaving, we can write energy and mass conservation equation as :

$$\dot{m}_A h_A = \dot{m}_B h_B + \dot{m}_C h_c \quad ...(i)$$

$$\dot{m}_A = \dot{m}_B + \dot{m}_C \quad ...(ii)$$

Substituting values in (i) and (ii),

$$\dot{m}_A (c_p T)_A = \dot{m}_B (c_p T)_B + \dot{m}_C (c_p T)_C$$

$$1 \times 1 \times 300 = \dot{m}_B \times 1 \times 250 + \dot{m}_C \times 1 \times 315$$

$$300 = 250\dot{m}_B + 315\dot{m}_C \quad ...(iii)$$

$$1 = \dot{m}_B + \dot{m}_C \quad ...(iv)$$

On solving, (iii) and (iv) $\quad \dot{m}_B = 0.2307$ kg/s

$$\dot{m}_C = 0.7692 \text{ kg/s}$$

To find exit temperature of air T_D, consider control volume consisting of both heat exchangers and reversible engine, since all process are reversible, there is no net entropy generation.

$$(\Delta s)_{cv} + \cancelto{0}{(\Delta s)_{surr}} = 0$$

$$(\Delta s)_{BD} + (\Delta s)_{CD} = 0$$

$$\dot{m}_B c_p \ln\frac{T_D}{T_B} + \dot{m}_C c_p \ln\frac{T_D}{T_C} = 0$$

$$0.2307\ln\frac{T_D}{250} + 0.7692\ln\frac{T_D}{315} = 0$$

$$\ln\left(\frac{T_D}{250}\right) + \frac{0.7692}{0.2307}\ln\left(\frac{T_D}{315}\right) = 0$$

$$\ln\left(\frac{T_D}{250}\right) + 3.3342\ln\left(\frac{T_D}{315}\right) = 0$$

$$\ln\left(\frac{T_D}{250}\right) + \ln\left(\frac{T_D}{315}\right)^{3.3342} = 0$$

$$\ln\left[\left(\frac{T_D}{250}\right)\times\left(\frac{T_D}{315}\right)^{3.3342}\right] = 0$$

On solving, $T_D = 298.643$ K **Ans.**

Work output from reversible heat engine,

$$\dot{W} = \dot{Q}_H - \dot{Q}_L$$

$$= \dot{m}_c c_p (T_C - T_D) - \dot{m}_B c_p (T_D - T_B)$$

$$= 0.7692 \times 1 \times (315 - 298.64) - 0.2307 \times 1 \times (298.64 - 250)$$

$$\dot{W} = 1.3628 \text{ kW}$$ **Ans.**

$$\text{Exergy of stream } A = \phi_A = \dot{m}_A \times \left[(h_A - h_D) - T_0 (s_A - s_D)\right]$$

$$\phi_A = 1\times\left[c_p (T_A - T_D) - T_D\left\{c_p \ln\frac{T_A}{T_D} - R\ln\frac{P_A}{P_D}\right\}\right]$$

$$\Rightarrow \quad \phi_A = 1\times(300 - 298.64) - 298.64\left\{1\times\ln\frac{300}{298.64} - 0.287\ln\frac{15}{1}\right\}$$

$$\phi_A = 232.11 \text{ kW}$$ **Ans.**

The total exergy of the stream is clearly much greater than the power output obtained from the reversible engine *E* because of the irreversibilities within the vortex tube.

4. (a) Solution:

Consider entropy 'S' as a function of 'T' and 'P',

$$ds = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

Substituting this value of ds in TdS equation

$$TdS = dh - vdp$$

$$\Rightarrow \quad dh = TdS + vdp = \left[T\left(\frac{\partial S}{\partial T}\right)_P\right]dT + \left[T\left(\frac{\partial S}{\partial P}\right)_T + v\right]dP \quad \text{... (i)}$$

Consider enthalpy h as a function of T and P. Then

$$dh = \left[\left(\frac{\partial h}{\partial T}\right)_P\right]dT + \left[\left(\frac{\partial h}{\partial P}\right)_T\right]dP$$

$$= C_p dT + \left(\frac{\partial h}{\partial T}\right)_T dP \quad \text{[By definition of } C_p\text{]} \quad \text{... (ii)}$$

From Maxwell's equation

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \quad \Rightarrow \quad \left(\frac{\partial h}{\partial P}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_P$$

From equations (i) and (ii)

$$\therefore \quad dh = C_p dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]dP$$

In this case, enthalpy is constant.

$$\therefore \quad dh = 0$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{\partial}{\partial T}\left(\frac{RT}{P} + \frac{k}{RT}\right)_P = \frac{R}{P} - \frac{k}{RT^2}$$

$$\therefore \quad V - T\left(\frac{\partial V}{\partial T}\right)_P = V - \frac{RT}{P} + \frac{k}{RT} = \frac{k}{RT} + \frac{k}{RT} = \frac{2k}{RT}$$

$$\therefore \quad dh = C_P dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]dP = 0$$

$$\Rightarrow \quad -C_p\, dT = \frac{2k}{RT}dP$$

$$\Rightarrow \quad -T \cdot C_p dT = \frac{2k}{R}dP$$

Integrating both sides with in proper limits,

$$-C_p \int_{T_1}^{T_2} T \cdot dT = \frac{2K}{R} \int_{P_1}^{P_2} 1 \cdot dP$$

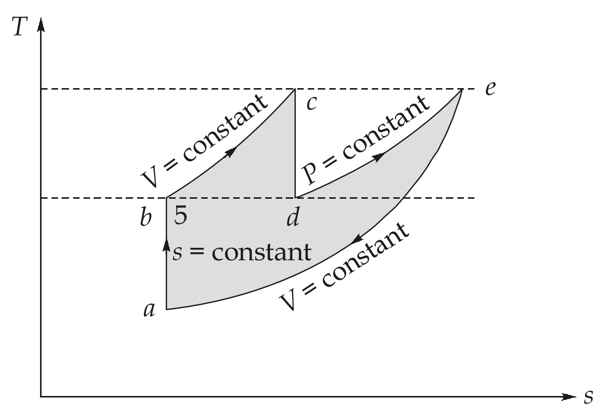
$$\frac{-C_p}{2}[T^2]_{T_1}^{T_2} = \frac{2k}{R}[P]_{P_1}^{P_2}$$

$$\Rightarrow \quad \frac{C_p(T_2^2 - T_1^2)}{4} = \frac{2k}{R}(P_1 - P_2)$$

$$\Rightarrow \quad \frac{T_2^2 - T_2^2}{4k} = \frac{P_1 - P_2}{C_p R}$$ Hence, proved.

4. (b) Solution:

Given : $T_b = T_d$; $T_c = T_e$



Process, $a - b$ is reversibly and adiabatically i.e. isentropic process.

$$(\Delta s)_I = 0$$

In process b - c; constant volume process from 1st law of thermodynamics.

$$\delta Q = du + PdV$$

at constant volume, $dV = 0$

$$\delta Q = du$$

$$\therefore \quad \Delta S_{II} = \int_b^c \frac{\delta Q}{T} = \int_b^c \frac{du}{T}$$

$$\Delta S_{II} = c_v \ln \frac{T_c}{T_b} \text{ kJ/kgK}$$

In process c - d : reversibly and adiabatically, i.e. isentropic process

$$\Rightarrow \quad (\Delta S)_{III} = 0$$

In process *d* - *e* : Constant pressure process, so we can write as

So,
$$TdS = dh - V\cancelto{0}{dP}$$

$$ds = \frac{dh}{T}$$

$$\Delta s_{IV} = \int_d^e c_p \frac{dT}{T}$$

$$\Delta s_{IV} = c_p \ln\frac{T_e}{T_d} \text{ kJ/kgK}$$

In process *e* – *a* : constant volume process

$$(\Delta s)_V = \int_e^a \frac{du}{T} = \int_e^a c_v \frac{dT}{T}$$

$$(\Delta s)_V = c_v \ln\frac{T_a}{T_e} \text{ kJ/kgK}$$

∴ For reversible process,

$$(\Delta s)_{total} = 0$$

$$\Rightarrow \cancelto{0}{(\Delta s)_I} + (\Delta s)_{II} + \cancelto{0}{(\Delta s)_{III}} + (\Delta s)_{IV} + (\Delta s)_V = 0$$

$$c_v \ln\frac{T_c}{T_b} + c_p \ln\frac{T_e}{T_d} + c_v \ln\frac{T_a}{T_e} = 0$$

$$c_v \ln\frac{T_c \times T_a}{T_b \times T_e} = c_p \ln\frac{T_d}{T_e}$$

$$\Rightarrow \ln\left(\frac{T_c \times T_a}{T_b \times T_e}\right) = \gamma \ln\left(\frac{T_d}{T_e}\right)$$

$$\frac{T_c \times T_a}{T_b \times T_e} = \left(\frac{T_d}{T_e}\right)^{\gamma}$$

$$\frac{\cancel{T_c} \times T_a}{T_d \times \cancel{T_c}} = \left(\frac{T_d}{T_e}\right)^{\gamma} \qquad [\text{As } T_b = T_d \text{ and } T_c = T_e]$$

$$\therefore \quad T_a = \frac{(T_d)^{\gamma+1}}{T_e^{\gamma}}$$

As, $T_b = T_d = 500$ K

$T_e = T_c = 850$ K, $\gamma = 1.4$

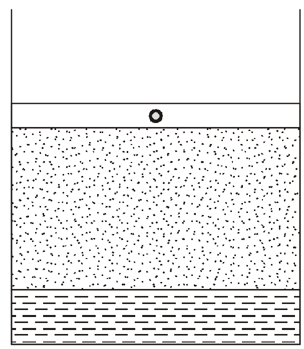
$$T_a = \frac{500^{1.4+1}}{850^{1.4}}$$

$$T_a = 237.87 \text{ K}$$ **Ans.**

4. (c) Solution:

Given data: $m = 1$ kg; $P_1 = 2$ bar $= 0.2$ MPa; $x_1 = 0.35$; $P_2 = 4$ bar $= 0.4$ MPa;

$P_3 = 2$ bar $= 0.2$ MPa



From steam table data:

At $P_1 = 2$ bar $= 0.2$ MPa $= P_3$

$v_f = 0.00106052\ \text{m}^3/\text{kg}$, $v_g = 0.88568\ \text{m}^3/\text{kg}$

$u_f = 504.49$ kJ/kg, $u_g = 2529.1$ kJ/kg

At $P_2 = 4$ bar $= 0.4$ MPa

$v_f = 0.00108355\ \text{m}^3/\text{kg}$, $v_g = 0.46238\ \text{m}^3/\text{kg}$

$u_f = 604.22$ kJ/kg , $u_g = 2553.1$ kJ/kg

Assumptions:

1. Reversible processes
2. Frictionless piston-cylinder arrangement
3. Massless piston
4. For wet steam liquid volume is neglected as compared to volume of vapour phase.

Process 1 → 2 : Isochoric (V = constant)

$$v_1 = v_{f1} + x_1 v_{fg1} \cong x_1 v_{g1} \quad \left(\because v_{f1} <<< v_{g1}\right)$$

$$v_1 = 0.35 \times 0.88568 \simeq 0.31\ \text{m}^3/\text{kg}$$

$$v_2 = v_1 \qquad \text{(Isochoric process)}$$

$$x_2 v_{g2} = 0.31 \text{ m}^3/\text{kg}$$

$$x_2 = \frac{0.31}{0.46238} \simeq 0.6704$$

Process $2 \rightarrow 3$: Pv = constant

$$P_2 v_2 = P_3 v_3$$

$$v_3 = \left(\frac{P_2}{P_3}\right) v_2 = \left(\frac{0.4}{0.2}\right) \times 0.31 = 0.62 \text{ m}^3/\text{kg}$$

$$v_3 \simeq x_3 v_{g3}$$

$$x_3 = \frac{v_3}{v_{g3}} = \frac{0.62}{0.88568} \simeq 0.7$$

(i) Heat addition during process $1 \rightarrow 2$

$$Q_{1 \rightarrow 2} = m(u_2 - u_1)$$

$$= 10\left[(u_{f2} + x_2 u_{fg2}) - (u_{f1} + x_1 u_{fg1})\right]$$

$$= 10[(604.22 + 0.6704 \times (2553.1 - 604.22) - (504.49 + 0.35 \times (2529.1 - 504.49)]$$

$$Q_{1 \rightarrow 2} = 6976.456 \text{ kJ}$$ **Ans.**

(ii) Heat and work transferred during Pv = constant process

$$W_{2 \rightarrow 3} = \int_2^3 P_{in} dv = m \int_{v_2}^{v_3} P dv \qquad \left[\text{As } Pv = c = P_2 v_2;\ P = \frac{P_2 v_2}{v}\right]$$

$$= m \int_{0.31}^{0.62} \frac{124}{v} dv \ ; \quad \left\{\begin{matrix} \because Pv = \text{constant} \\ P_2 v_2 = 4 \times 10^2 \times 0.31 = 124 \text{ kPa-m}^3 \end{matrix}\right\}$$

$$= 10 \times 124 \ln [v]_{0.31}^{0.62}$$

$$= 1240 \times \ln 2$$

$$W_{2 \rightarrow 3} \simeq 859.5 \text{ kJ}$$ **Ans.**

$$Q_{2 \rightarrow 3} = W_{2 \rightarrow 3} + \Delta U_{2 \rightarrow 3}$$

$$= 859.5 + m(u_3 - u_2)$$

$$= 859.5 + 10\left[\left(u_{f3} + x_3 u_{fg3}\right) - \left(u_{f2} + x_2 u_{fg2}\right)\right]$$

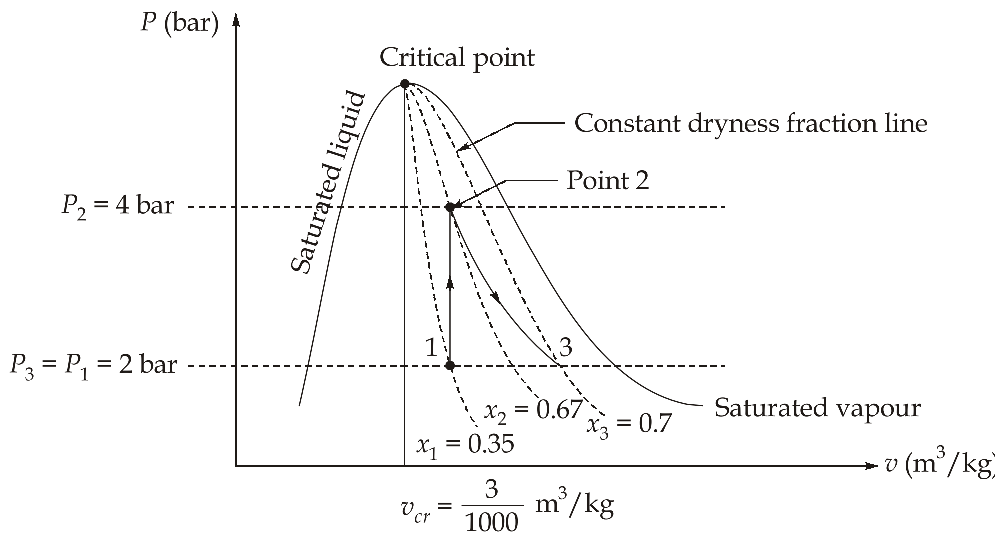
$$= 859.5 + 10\begin{bmatrix} 604.22 + 0.7(2553.1 - 604.22) \\ -(504.49 + 0.6704(2529.1 - 504.49)) \end{bmatrix}$$

$$Q_{2\to 3} = 1925.97 \text{ kJ}$$ **Ans.**

(iii) Since $x_3 \simeq 0.7$

Implies at the end of Pv = constant expansion-process the steam is wet hence temperature of steam will be saturation temperature corresponding to final pressure (2 bar).

$$T_{sat}\big|_{2bar} = 120.21°C \qquad \text{(from Table data)}$$



$$v_1, v_2, v_3 > v_{cr}$$

$$0 < x_1, x_2, x_3 < 1$$

Section B : IC Engine + Refrigeration and Air-conditioning

5. (a) Solution:

Given data : P_1 = 1 bar; T_1 = –6°C = 273 – 6 = 267 K; P_2 = 5.6 bar; T_3 = 273 + 19 = 292 K

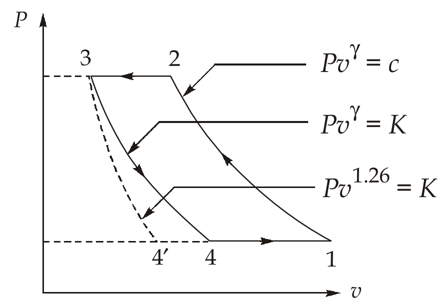
Assumption:

1. Air is considered as ideal gas.
2. Specific heats are constant with temperature and pressure.

(i) For process 1 - 2, $$T_2 = T_1\left[\frac{P_2}{P_1}\right]^{\frac{\gamma-1}{\gamma}} = 267\left(\frac{5.6}{1}\right)^{\frac{1.4-1}{1.4}} = 436.7 \text{ K}$$

For process 3 - 4, $$T_4 = \frac{T_3}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$= \frac{292}{\left(\frac{5.6}{1}\right)^{\frac{1.4-1}{1.4}}} = 178.49 \text{ K} \qquad [\because \; P_3 = P_2 \text{ and } P_4 = P_1]$$



Heat rejected in the cooler $= C_p(T_2 - T_3)$

$= 1(436.7 - 292)$

$= 144.7$ kJ/kg [Constant pressure process]

Heat absorbed in evaporator $= C_p(T_1 - T_4)$

$= 1(267 - 178.49)$

$= 88.51$ kJ/kg [Constant pressure process]

Work done = Heat rejected – Heat absorbed

$= 144.7 - 88.51 = 56.19$ kJ/kg

$$\text{COP} = \frac{\text{Heat absorbed (desired effect)[In evaporator]}}{\text{Work done}}$$

$$\text{COP} = \frac{88.51}{56.19} = 1.575$$

(ii) Since, expansion process follow $Pv^{1.26}$ = constant

$$T_4' = \frac{T_3}{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}} = \frac{292}{(5.6)^{(1.26-1)/1.26}} = 204.64 \text{ K}$$

Heat absorbed during expansion process $= \frac{\gamma - n}{n-1} C_v (T_3 - T_4')$

$$= \frac{1.4 - 1.26}{1.26 - 1} \times 0.7 \times (292 - 204.64) = 32.927 \text{ kJ/kg}$$

Heat absorbed in evaporator $= C_p(T_1 - T_4')$

$= 1 \times (267 - 204.64) = 62.36$ kJ/kg

$$\text{Total heat absorbed} = 32.927 + 62.36 = 95.287 \text{ kJ/kg}$$

$$\text{Work done} = \text{Total heat rejected} - \text{Total heat absorbed}$$

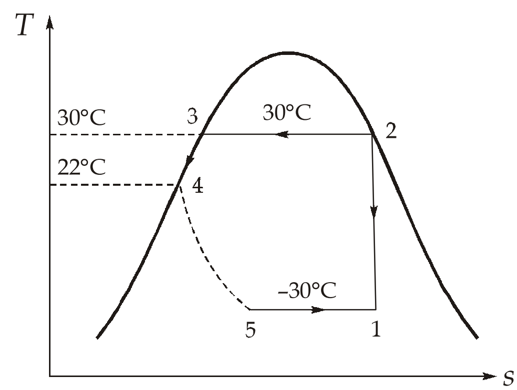
$$= 144.7 - 95.287$$

$$= 49.413 \text{ kJ/kg}$$

$$\text{COP} = \frac{\text{Desired effect}}{\text{Work done}} = \frac{62.36}{49.413} = 1.262$$

5. (b) Solution:

The processes of the system are represented on *T-s* chart as shown



(i)

$$h_3 = (h_f)_{\text{at 11.82 bar}} = 236.7 \text{ kJ/kg}$$

$$h_4 = h_3 - C_{p,l} \times (T_3 - T_4)$$

$$h_4 = h_3 - C_{p,l}(30 - 22)$$

$$= 236.7 - 1.19 \times 8 = 227.18 \text{ kJ/kg}$$

Considering the throttling process 4 - 5,

$$h_4 = h_5 = h_{f5} + x_5 h_{fg1} \qquad [\because \text{ Enthalpy remain constant}]$$

$$227.18 = 116.1 + x_5(h_{g1} - h_{f1})$$

$$227.18 = 116.1 + x_5(393.1 - 116.1)$$

$$x_5 = 0.401 \qquad \text{[This is the dryness fraction at state 5]}$$

Consider the isentropic compression 1 - 2

$$s_1 = s_2$$

$$s_{f1} + x_1 s_{fg1} = s_{g2}$$

$$0.8698 + x_1(1.803 - 0.8698) = 1.712$$

$$x_1 = \frac{1.712 - 0.8698}{1.803 - 0.8698} = 0.902$$

$$\text{Refrigeration effect, R.E.} = h_1 - h_5$$
$$\text{R.E.} = (x_1 - x_5) \times h_{fg1}$$
$$\text{R.E.} = (x_1 - x_5) \times (h_{g1} - h_{f1})$$
$$\text{R.E.} = (0.902 - 0.401) \times (393.1 - 116.1)$$
$$\text{R.E.} = 138.77 \text{ kJ/kg}$$
$$\text{Work done, } W = h_2 - h_1$$
$$W = h_{g2} - \left[h_{f1} + x_1\left(h_{g1} - h_{f1}\right)\right]$$
$$W = 414.5 - [116.1 + 0.902(393.1 - 116.1)]$$
$$= 48.546 \text{ kJ/kg}$$
$$\text{COP}_{\text{(theoretical)}} = \frac{R.E.}{W} = \frac{138.77}{48.546} = 2.858$$
$$\text{COP}_{\text{(act)}} = 0.70 \times (\text{COP})_{\text{th}}$$
$$= 0.70 \times 2.858 = 2$$

(ii) Required cooling effect = 110 × 3.5

$$\text{R.C.} = 385 \text{ kJ/sec}$$

∴ Rate of mass of refrigerant to be circulated,

$$\dot{m}_r = \frac{R.C.}{R.E.} = \frac{385}{138.77} = 2.774 \text{ kg/sec}$$

(iii) Compressor power $= \dot{m}_r(h_2 - h_1)$

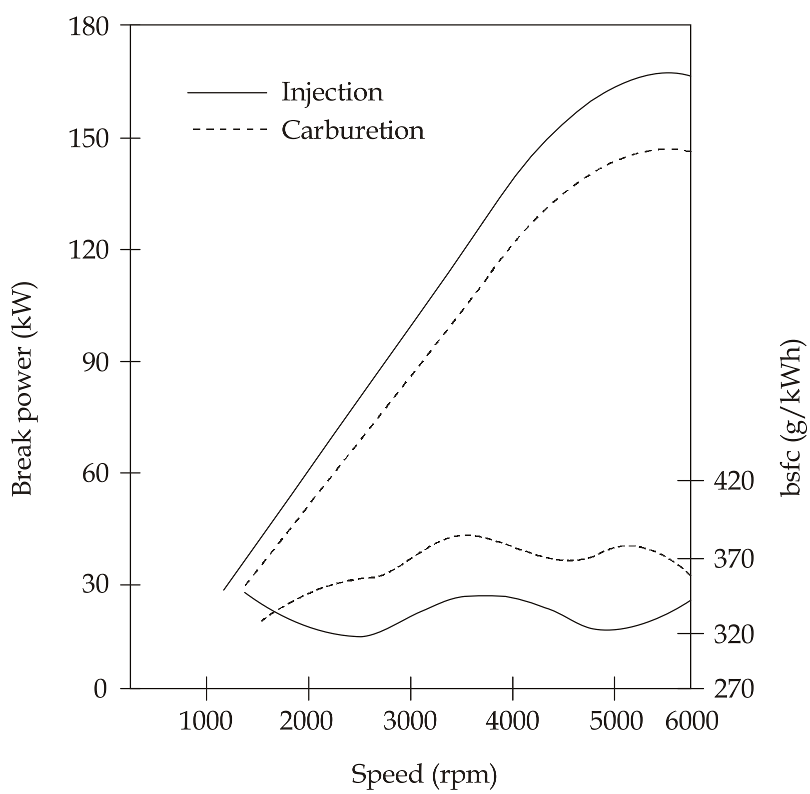
$$= 2.774 \times (414.5 - 365.954)$$
$$= 134.67 \text{ kW}$$

5. (c) Solution:

Advantages of multi-point fuel injection system over carburetted system:

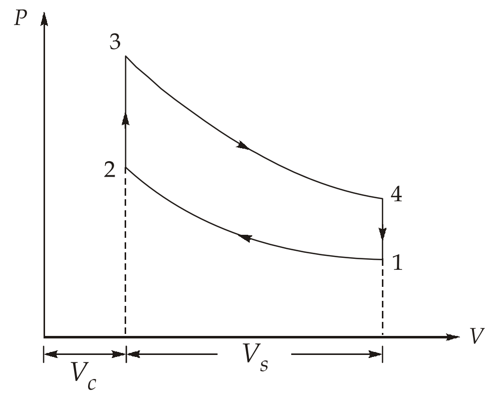
1. The elimination of intake manifold heating and carburettor pressure loss result in increasing volumetric efficiency and hence higher power output and torque.
2. Lower Specific fuel consumption since the amount of fuel injected is reduced at part throttle and during deceleration.
3. Less maldistribution to each cylinder.
4. Faster acceleration, since the fuel is injected into or close to the cylinder and need not flow through the manifold.
5. Easier starting, since atomisation of fuel does not depend on the cranking speed.

6. Less knocking tendency, since heat need not be supplied for better distribution. The temperature of mixture, therefore, is lowered. Thus, a fuel having a lower octane number or higher compression ratio can be used.
7. Since the combustible mixture is not in the intake manifold, the tendency of backfiring or popping in the carburettor is reduced.
8. For emission control, fuel injection exhibits the higher accuracy of fuel metering, and hence less exhaust emissions are produced.
9. Fuel response is practically instantaneous, so the flat spot is eliminated.
10. Engines fitted with injection systems can be used in high angles of tilt, the tilt may produce fuel surge problems in the float chamber of the carburettor.



Comparison of performance of the carburettor system with that of the fuel-injection system

5. (d) Solution:



Note : Heat is added in process 1 - 2 and heat is rejected in process 4 - 1.

Using first law of thermodynamics, for a closed system the workdone during a cycle,

$$W = \text{Heat supplied – heat rejected}$$

$$W = Q_S - Q_R = mc_v(T_3 - T_2) - mc_v(T_4 - T_1)$$

$$\Rightarrow \quad W = mc_v[(T_3 - T_2) - (T_4 - T_1)] \quad \text{...(i)}$$

For constant volume process 2 - 3,

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} = \alpha \quad \text{(Pressure ratio)} \quad \text{...(ii)}$$

For isentropic process 1 - 2,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1} \quad [r \rightarrow \text{Compression ratio}] \quad \text{...(iii)}$$

From (ii) and (iii), we get

$$T_3 = \alpha T_1 r^{\gamma-1} \quad \text{...(iv)}$$

and for process 3 - 4,

$$T_4 = \frac{T_3}{r^{\gamma-1}}$$

Substituting in equation (iv), we get

$$T_4 = \frac{\alpha T_1 r^{\gamma-1}}{r^{\gamma-1}} = \alpha T_1 \quad \text{...(v)}$$

Substituting the values of T_2, T_3 and T_4 in (i), we get

$$W = mc_v\left[(\alpha T_1 r^{\gamma-1} - T_1 r^{\gamma-1}) - (\alpha T_1 - T_1)\right]$$

or

$$W = mc_v T_1 (\alpha - 1)\left(r^{\gamma-1} - 1\right) \quad \text{...(vi)}$$

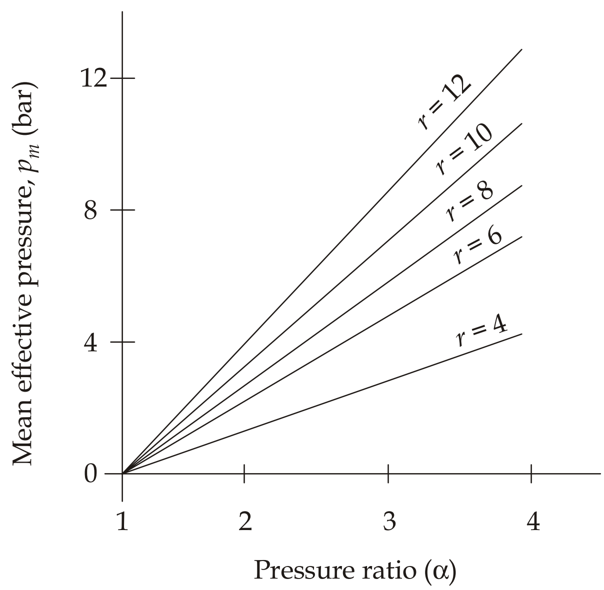
$$\text{Mean effective pressure, } P_m = \frac{\text{Workdone}}{\text{Swept volume}} = \frac{W}{V_s}$$

$$\text{Swept volume, } V_s = V_1 - V_2 = V_1\left(1 - \frac{1}{r}\right)$$

$$= \frac{mRT_1}{P_1}\left(1 - \frac{1}{r}\right) \quad \text{...(vii)}$$

$$\therefore \quad P_m = \frac{mc_vT_1(\alpha - 1)\left(r^{\gamma-1} - 1\right)P_1}{mRT_1\left(1 - \frac{1}{r}\right)}$$

$$\text{or} \quad P_m = \frac{P_1(\alpha - 1)\left(r^{\gamma-1} - 1\right)r}{(\gamma - 1)(r - 1)}$$



Mean effective pressure vs. pressure ratio for different values of compression ratio (*r*)

5. (e) Solution:

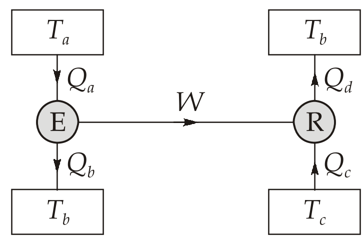
(i)

$$\eta_{\text{engine}} = \frac{W}{Q_a} = \frac{Q_a - Q_b}{Q_a} = \frac{T_a - T_b}{T_a}$$

$$W = Q_a\left(\frac{T_a - T_b}{T_a}\right) \quad \text{... (i)}$$

$$(\text{COP})_{\text{refrigerator}} = \frac{Q_c}{W} = \frac{Q_c}{Q_d - Q_c} = \frac{T_c}{T_b - T_c}$$

$$W = Q_c\left(\frac{T_b - T_c}{T_c}\right) \quad \text{... (ii)}$$



Now, from equations (i) and (ii), we get

$$\frac{Q_c}{Q_a} = \left(\frac{T_a - T_b}{T_a}\right)\left(\frac{T_c}{T_b - T_c}\right) = \frac{T_c}{T_a}\left(\frac{T_a - T_b}{T_b - T_c}\right)$$

(ii) If $Q_a = Q_c$, $T_a = 300°C = 573$ K, $T_c = 20°C = 253$ K

As $Q_c = Q_a$ so from above part we get,

$$1 = \left(\frac{T_a - T_b}{T_b - T_c}\right)\frac{T_c}{T_a}$$

$$\frac{T_a}{T_c} = \frac{T_a - T_b}{T_b - T_c}$$

$$\frac{573}{253} = \frac{573 - T_b}{T_b - 253}$$

$$2.265 = \frac{573 - T_b}{T_b - 253}$$

$\Rightarrow$ $$T_b = 351.01 \text{ K} \approx 78.01°C$$

(iii) $$\eta_{engine} = \frac{T_a - T_b}{T_a} = \frac{573 - 351.01}{573} = 0.3874 = 38.74\%$$

$$(COP)_{ref} = \frac{T_c}{T_b - T_c} = \frac{253}{351.01 - 253} = 2.581$$

6. (a) Solution:

Given data : $\eta_n = 0.9$, $\eta_e = 0.66$, $\eta_c = 0.83$, $x_e = 0.93$

$$P_a = 8 \text{ bar;}$$

Pressure of condenser, P_c = 5 cm of Hg (absolute)

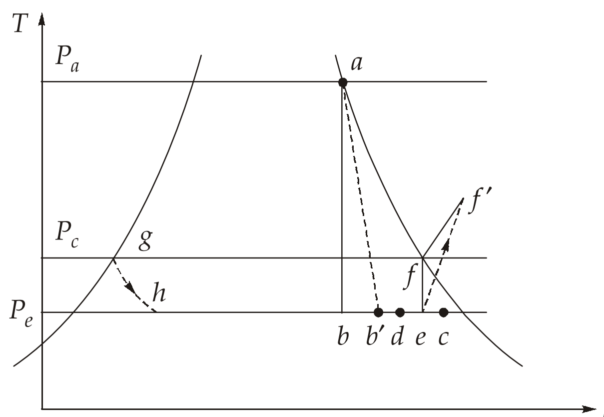
$$P_e = \text{Saturation pressure at } 6°C$$

From steam tables,

$$\text{At } P_a = 8 \text{ bar}; h_a = 2768.3 \text{ kJ/kg}$$
$$x_a = 1, T_a = 170.406°\text{C}$$
$$s_a = 6.6616 \text{ kJ/kgK}$$

At saturation pressure 6°C,

$$P_c = 0.0009354 \text{ MPa}$$
$$s_f = 0.09134 \text{ kJ/kgK}; h_f = 25.22 \text{ kJ/kg}$$
$$s_{fg} = 8.9080 \text{ kJ/kgK}; h_{fg} = 2486.7 \text{ kJ/kg}$$



As ab is isentropic expansion,

$$s_a = s_b$$
$$6.6616 = \left[s_f + x_b s_{fg}\right]_b$$
$$6.6616 = 0.09134 + x_b \times 8.9080$$
$$x_b = 0.738$$
$$h_b = 25.22 + 0.738 \times 2486.7$$
$$= 1835.185 \text{ kJ/kg}$$

Nozzle efficiency is given by

$$\eta_n = \frac{h_a - h_{b'}}{h_a - h_b}$$
$$0.9 = \frac{2768.3 - h_{b'}}{2768.3 - 1835.185}$$
$$h_{b'} = 1928.49 \text{ kJ/kg}$$

The point b and b' are on same pressure line

$$h_{b'} = (h_f)_{b'} + x_{b'}(h_{fg})_{b'}$$

$$x_{b'} = \frac{h_{b'} - (h_f)_{b'}}{(h_{fg})_{b'}} = \frac{1928.49 - 25.22}{2486.7} = 0.765$$

Entrainment efficiency is given by

$$\eta_e = \frac{h_a - h_d}{h_a - h_{b'}}$$

$$0.66 = \frac{2768.3 - h_d}{2768.3 - 1928.49}$$

$$h_d = 2214.03 \text{ kJ/kg}$$

Also, $$h_d = (h_f)_d + x_d(h_{fg})_d$$

$$2214.03 = 25.22 + x_d \times 2486.7$$

$$x_d = 0.880$$

At point *e*, $$x_e = 0.93 \quad \text{(given in problem)}$$

$$h_e = 25.22 + 0.93 \times 2486.7$$

$$h_e = 2337.85 \text{ kJ/kg}$$

Also, $$s_e = (s_f)_e + x_e(s_{fg})_e$$

$$= 0.09134 + 0.93 \times 8.9080$$

$$= 8.37578 \text{ kJ/kgK}$$

As, condenser pressure = 5 cm of Hg

$$P_c = \frac{(13.6 \times 10^3) \times 9.81 \times 0.05}{10^6} = 0.00667 \text{ MPa} \simeq 0.0067 \text{ MPa}$$

From steam table,

At 0.0067 MPa, $$h_f = 159.906 \text{ kJ/kg}, h_f = 2570.26 \text{ kJ/kg}$$

$$s_f = 0.54797 \text{ kJ/kgK}, s_g = 8.29022 \text{ kJ/kgK}, T_s = 38.176°\text{C}$$

Process *ef* is isentropic

i.e. $$s_e = s_f$$

But $s_e > (s_f)_{g \text{ at } 0.0067 \text{ MPa}}$, that means point *f* is in superheated region.

$\therefore$ $$s_e = s_f$$

$$= (s_f)_{g \text{ at } 0.0067 \text{ MPa}} + 2.1 \ln\left(\frac{T_{sup}}{T_s}\right)$$

$$8.37578 = 8.29022 + 2.1\ln\left(\frac{T_{sup}}{T_s}\right)$$

$$T_{sup} = 324.116 \text{ K or } 51.116°\text{C}$$

Enthalpy at point f, $h_f = (h_g)_{\text{at } 0.0067 \text{ MPa}} + 2.1(T_{sup} + T_s)$

$$h_f = 2570.26 + 2.1(51.116 - 38.176)$$

$$h_f = 2597.434 \text{ kJ/kg}$$

The compression efficiency is given by,

$$\eta_c = \frac{h_f - h_e}{h_{f'} - h_e}$$

$$0.83 = \frac{2597.434 - 2337.85}{h_{f'} - 2337.85}$$

$$h_{f'} = 2650.602 \text{ kJ/kg}$$

(i) Mass of motive steam required per kg of flashed vapour is given by

$$m = \frac{h_f - h_e}{\eta_n \cdot \eta_e \cdot \eta_c (h_a - h_b) - (h_f - h_e)}$$

$$= \frac{2597.434 - 2337.85}{0.9 \times 0.66 \times 0.83 \times (2768.3 - 1835.185) - (2597.434 - 2337.85)}$$

$$m = 1.295 \text{ kg of motive steam per kg of flashed vapour}$$

(ii) Refrigeration effect per kg of flash vapour = h_c – (Liquid heat per kg of make up water)

Also, at point c, $h_c + mh_d = (m+1)h_e$

$$h_c = (1.295 + 1) \times 2337.85 - 1.295 \times 2214.03$$

$$= 2498.197 \text{ kJ/kg}$$

∴ Refrigeration effect per kg of flash vapour = $h_c - 75.54 = 2498.197 - 75.54$

$$= 2422.657 \text{ kJ/kg}$$

6. (b) Solution:

Given : $r = 16$, $x = 0.05$, $c_p = 1.004$ kJ/kgK, $R = 0.287$ kJ/kgK, $\frac{dc_p}{c_p} = 0.03$

The efficiency of diesel engine is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}\left[\frac{\rho^\gamma - 1}{\gamma(\rho - 1)}\right]$$

or
$$1 - \eta = \left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$$

Here r is the compression ratio and ρ is cut-off ratio.
Taking log on both sides,

$$\ln(1-\eta) = -(\gamma-1)\ln r + \ln\left(\rho^{\gamma}-1\right) - \ln\gamma - \ln(\rho-1)$$

Differentiating the equation with respect to γ, assuming r and ρ as constant.

$$\Rightarrow \quad -\frac{1}{1-\eta}\frac{d\eta}{d\gamma} = -\ln r + \frac{\rho^{\gamma}\ln\rho}{\rho^{\gamma}-1} - \frac{1}{\gamma}$$

$$\Rightarrow \quad \frac{d\eta}{\eta} = \frac{1-\eta}{\eta}d\gamma\left[\ln r - \frac{\rho^{\gamma}\ln\rho}{\rho^{\gamma}-1} + \frac{1}{\gamma}\right] \quad \text{...(i)}$$

Now,

$$\because \quad \frac{R}{c_p} = \frac{\gamma-1}{\gamma} = 1 - \frac{1}{\gamma}$$

On differentiating the equation,

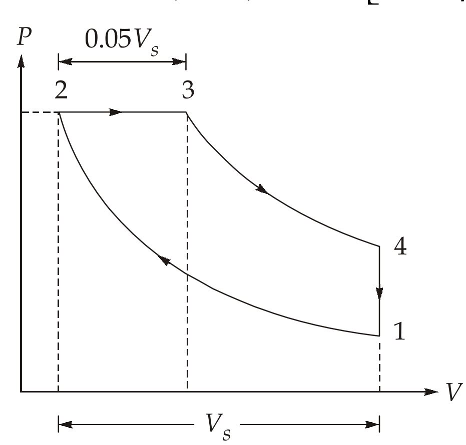
$$\Rightarrow \quad -\frac{R}{c_p^2}dc_p = \frac{1}{\gamma^2}d\gamma$$

$$\Rightarrow \quad d\gamma = \frac{-R}{c_p}\frac{dc_p}{c_p}\gamma^2 = -\frac{R}{c_p}\frac{dc_p}{c_p}\cdot\gamma\cdot\frac{(\gamma-1)c_p}{R}$$

$$\Rightarrow \quad d\gamma = -\gamma(\gamma-1)\frac{dc_p}{c_p} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{d\eta}{\eta} = -\left(\frac{1-\eta}{\eta}\right)\gamma(\gamma-1)\left[\ln r - \frac{\rho^{\gamma}\ln\rho}{\rho^{\gamma}-1} + \frac{1}{\gamma}\right]\frac{dc_p}{c_p} \quad \text{...(iii)}$$



Now, $r = 16$

$\Rightarrow$ $V_1 = 16V_2$

and $V_s = V_1 - V_2$

$= 15V_2$

$\Rightarrow$ $V_3 - V_2 = 0.05V_s$

$\Rightarrow$ $V_3 = 0.05 \times 15V_2 + V_2$

$\Rightarrow$ $V_3 = 1.75V_2$

$\Rightarrow$ Cut-off ratio, $\rho = \frac{V_3}{V_2} = 1.75$

and $\gamma = \frac{c_p}{c_p - R} = \frac{1.004}{1.004 - 0.287} = 1.4$

and $\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right]$

$= 1 - \left(\frac{1}{16}\right)^{0.4}\left[\frac{1.75^{1.4} - 1}{1.4 \times 0.75}\right]$

$\Rightarrow$ $\eta = 0.6264$

Substituting all the values in equation (iii), we get

$$\frac{d\eta}{\eta} = -\left(\frac{1 - 0.6264}{0.6264}\right)(1.4 \times 0.4)\left[\ln(16) - \frac{(1.75^{1.4})\ln(1.75)}{1.75^{1.4} - 1} + \frac{1}{1.4}\right]0.03$$

$\Rightarrow$ $\frac{d\eta}{\eta} = -(0.5964)(0.56)[2.4566]0.03$

$\Rightarrow$ $\frac{d\eta}{\eta} = -0.0246$

$\therefore$ $\frac{d\eta}{\eta} \times 100 = -2.46\%$

Therefore, the efficiency decreases by 2.46%.

6. (c) Solution:

Given : $K = 4$, 4 stroke, $D = 50$ mm = 0.05 m, $L = 75$ mm = 0.075, $N = 2500$ rpm, $a = 0.30$ m,

$W_{net} = 150$ N, $\dot{V}_f = 5.5$ litre/hour, $s = 0.75$, $CV = 42000$ kJ/kg

$W_{234} = 108$ N, $W_{134} = 101$ N, $W_{124} = 103$ N, $W_{123} = 110$ N

(i)

$$bp = T\omega = (W_{net} \times a)\left(\frac{2\pi N}{60}\right)$$

$$= (150 \times 0.3)\left(\frac{2\pi \times 2500}{60}\right) = 11.78 \text{ kW}$$

(ii)

$$P_{bmep} \times \dot{V}_s = bp$$

$$\Rightarrow \quad P_{bmep} \times \left[\frac{\pi}{4} D^2 L \frac{N}{2 \times 60} \times K\right] = bp$$

$$\Rightarrow \quad P_{bmep} \times \left[\frac{\pi}{4} \times (0.05)^2 (0.075) \frac{2500}{2 \times 60} \times 4\right] = 11.78$$

$$\Rightarrow \quad P_{bmep} = 960 \text{ kPa}$$

(iii)

$$\eta_{bth} = \frac{bp}{\dot{m}_f \times CV}$$

$$= \frac{11.78}{\left\{\frac{5.5 \times 10^{-3} \times 0.75 \times 1000}{3600}\right\} \times 42000}$$

$$= \frac{11.78}{(1.146 \times 10^{-3}) \times 42000} = 0.24478$$

$$= 24.47\%$$

(iv)

$$bsfc = \frac{\dot{m}_f}{bp} = \frac{1.146 \times 10^{-3} \times 3600}{11.78}$$

$$= 0.350 \text{ kg/kWh}$$

(v) Morse test

We know, $$ip_{1234} - FP = bp_{1234} \qquad \text{...(i)}$$

when cylinder (i) is not firing,

$$ip_{234} - FP = bp_{234} \qquad \text{...(ii)}$$

when cylinder (ii) is not firing,

$$ip_{134} - FP = bp_{134} \qquad \text{...(iii)}$$

when cylinder (iii) is not firing,

$$ip_{124} - FP = bp_{124} \quad ...(iv)$$

when cylinder (iv) is not firing,

$$ip_{123} - FP = bp_{123} \quad ...(v)$$

Now, from equation (i) – equation (ii), we get

$$ip_1 = bp_{1234} - bp_{234} \quad ...(vi)$$

Similarly, equation (i) – equation (iii), we get

$$ip_2 = bp_{1234} - bp_{134} \quad ...(vii)$$

equation (i) – equation (iv), we get

$$ip_3 = bp_{1234} - bp_{124} \quad ...(viii)$$

equation (i) – equation (v), we get

$$ip_4 = bp_{1234} - bp_{123} \quad ...(ix)$$

Now, sum of equation (vi), (vii), (viii) and (ix) gives

$$ip_{1234} = 4bp_{1234} - (bp_{234} + bp_{134} + bp_{124} + bp_{123}) \quad ...(x)$$

Now,
$$bp_{234} = (W_{234} \times a)\left(\frac{2\pi N}{60}\right) \times \frac{1}{1000}$$

$$= (108 \times 0.3)\left(\frac{2 \times \pi \times 2500}{60}\right) \times \frac{1}{1000} = 8.482 \text{ kW}$$

Similarly,
$$bp_{134} = (W_{134} \times a)\left(\frac{2\pi N}{60}\right) \times \frac{1}{1000}$$

$$= (101 \times 0.3)\left(\frac{2\pi \times 2500}{60}\right) \times \frac{1}{1000} = 7.9325 \text{ kW}$$

$$bp_{124} = (W_{124} \times a)\left(\frac{2\pi N}{60}\right) \times \frac{1}{1000}$$

$$= (103 \times 0.3)\left(\frac{2\pi \times 2500}{60}\right) \times \frac{1}{1000} = 8.089 \text{ kW}$$

and
$$bp_{123} = (W_{123} \times a)\left(\frac{2\pi N}{60}\right) \times \frac{1}{1000}$$

$$= (110 \times 0.3)\left(\frac{2\pi \times 2500}{60}\right) \times \frac{1}{1000} = 8.6393 \text{ kW}$$

Putting values in equation (x), we get

$$ip_{1234} = 4(11.78) - (8.482 + 7.9325 + 8.089 + 8.6393)$$

$$\Rightarrow \quad ip_{1234} = 13.9772 \text{ kW}$$

(vi)

$$\eta_m = \frac{bp}{ip} = \frac{11.78}{13.9772}$$

$$= 0.8420 = 84.20\%$$

(vii)

$$P_{imep} \times \dot{V}_S = ip$$

$$P_{imep} \times \left[\frac{\pi}{4} D^2 L \times \frac{N}{2 \times 60} \times K\right] = ip$$

$$P_{imep} \times \left[\frac{\pi}{4}(0.05)^2 (0.075) \times \left(\frac{2500}{2 \times 60}\right) \times 4\right] = 13.9772$$

$$P_{imep} = 1139.136 \text{ kPa}$$

7. (a) Solution:

The combustion reaction

$$C + O_2 \rightarrow CO_2$$

$$2H_2 + O_2 \rightarrow 2H_2O$$

$$S + O_2 \rightarrow SO_2$$

$\because$ 1 kg of fuel contains 0.85 kg C, 0.06 kg H_2, 0.018 kg O_2 and 0.006 kg S.

$\therefore$ N_2 in 1 kg fuel = 1 – 0.85 – 0.06 – 0.018 – 0.006 = 0.066 kg

Now, oxygen required for complete combustion.

- For 12 kg of C $\Rightarrow$ 32 kg O_2 required and produces 44 kg CO_2

$\therefore$ For 0.85 kg C $\Rightarrow \frac{32}{12} \times 0.85 = 2.267$ kg O_2 required

$\Rightarrow$ $\frac{44}{12} \times 0.85 = 3.117$ kg CO_2 produces

- 4 kg $H_2 \Rightarrow$ Needs 32 kg O_2 and produces 36 kg H_2O

$\therefore$ 0.06 kg $H_2 \Rightarrow$ Needs $\left(\frac{32}{4} \times 0.06 = 0.48 \text{ kg}\right) O_2$

$\Rightarrow$ Produces $\left(\frac{36}{4} \times 0.06 = 0.54 \text{ kg}\right) H_2O$

- $32 \text{ kg S} \Rightarrow \text{Require } 32 \text{ kg } O_2 \text{ and produces } 64 \text{ kg } SO_2$

$\therefore$ $0.006 \text{ kg S} \Rightarrow \text{Require } \frac{32}{32} \times 0.006 = 0.006 \text{ kg } O_2$

$\Rightarrow$ Produces 0.012 kg SO_2

Therefore, the amount of O_2 required per kg of complete combustion is (from air).

$$= 2.267 + 0.48 + 0.006 - 0.018$$

[As 1.8% O_2 already in fuel]

$$= 2.735 \text{ kg}$$

$\therefore$ Amount of theoretical air required per kg fuel

$$= \frac{2.735}{0.23} = 11.89 \text{ kg} \quad \{\therefore \text{ Air contains } 23\% \ O_2 \text{ by weight}\}$$

$\therefore$Stoichiometric air/fuel ratio = 11.89 **Ans.**

Now, since, 20% excess air is supplied.

$\therefore$ The actual quantity of air supplied per kg of fuel = 11.89 × 1.2 = 14.27 kg

The combustion products will be

$$CO_2 = 3.117 \text{ kg}$$
$$H_2O = 0.54 \text{ kg}$$
$$SO_2 = 0.012 \text{ kg}$$
$$O_2 \text{ in excess air} = 0.23 \times 0.2 \times 11.89 = 0.557 \text{ kg}$$
$$N_2 = N_2 \text{ in air} + N_2 \text{ in fuel}$$
$$= 0.77 \times 1.2 \times 11.89 + 0.066 = 11.052 \text{ kg}$$

Since, water vapour and SO_2 are condensed, the dry products of combustion include CO_2, N_2 and O_2.

$$CO_2 = 3.117 \text{ kg}$$
$$N_2 = 11.052 \text{ kg}$$
$$O_2 = 0.557 \text{ kg}$$
$$\text{Total weight} = 14.726 \text{ kg}$$

Percentage composition by weight

$$CO_2 = \frac{3.117}{14.726} \times 100 = 21.167\%$$ **Ans.**

$$N_2 = \frac{11.052}{14.726} \times 100 = 75.05\%$$ **Ans.**

$$O_2 = \frac{0.557}{14.726} \times 100 = 3.78\%$$ **Ans.**

Proportionate moles of

$$CO_2 = \frac{3.117}{44} = 0.0708$$

$$N_2 = \frac{11.052}{28} = 0.3947$$

$$O_2 = \frac{0.557}{32} = 0.0174$$

Total weight = 0.4829

Volumetric analysis of dry products:

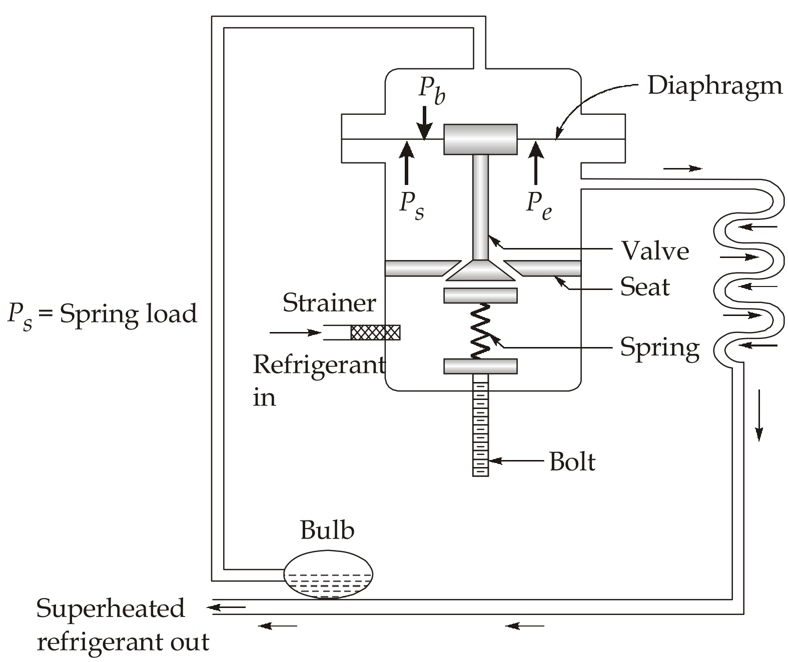
$$CO_2 = \frac{0.0708}{0.4829} \times 100 = 14.66\%$$ **Ans.**

$$N_2 = \frac{0.3947}{0.4829} \times 100 = 81.74\%$$ **Ans.**

$$O_2 = \frac{0.0174}{0.4829} \times 100 = 3.603\%$$ **Ans.**

7. (b) (i) Solution:

A thermostatic expansion valve is used to maintain a constant degree of superheat at exit of evaporator hence it is most effective for dry evaporators in preventing the slugging of the compressor since it does not allow the liquid refrigerant to enter the compressor.



It consists of a feeler bulb that is attached to the evaporator exit tube so that it senses the temperature of the exit of evaporator. The feeler bulb and the narrow tube contains some fluid that is called power fluid. The power fluid may be same as refrigerant or it may be different. In case if it is different from the refrigerant then the TEV is called TEV with cross charge.

Let, P_b is the pressure of power fluid, P_e is the saturation pressure corresponding to evaporator exit temperature and evaporator temperature is T_e, then the purpose of TEV is to maintain a temperature $T_e + \Delta T_s$) at evaporator exit where ΔT_s is the degree of superheat.

Force exerted on the top area A_b of bellows (diaphragm):

$$F_b = P_b A_b$$

Force exerted by evaporator pressure from bottom side of bellows:

$$F_e = P_e A_b$$

This is called external equalizer if the evaporator is large and has significant pressure drop; otherwise it is called TEV with internal equalizer.

The differences of force F_b and F_e is exerted on top of the middle which controls the opening of orifice and is equal to spring force f_s, i.e.

$$f_s = (P_b - P_e)A_b$$

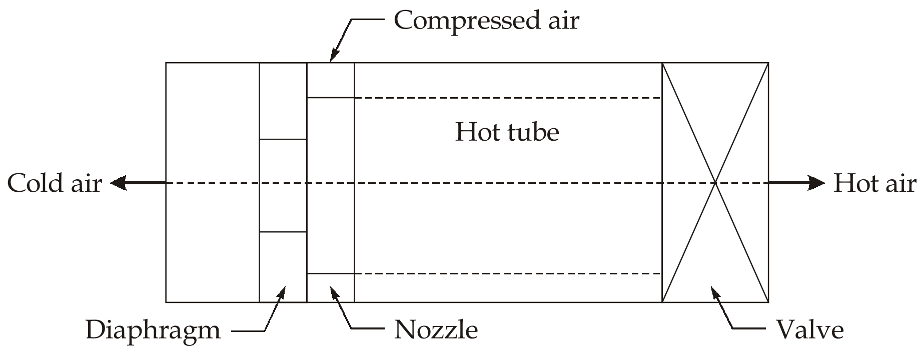
Also, $$\Delta T_s \propto (P_b - P_e)A_b$$

As the compressor starts, P_e decreases, so a positive spring force on the middle which opens the orifice and refrigerant flow starts.

When the cooling load increases, the refrigerant evaporates at a faster rate in evaporator than the compressor can suck. As a result, the saturation pressure (P_o) correspond to the temperature at the exit end of the evaporator and the degree of superheat increases. The increase in superheat causes the valve to open more and to allow more refrigerant to enter the evaporator. At the same time suction pressure (P) also enables the compressor to deliver increased refrigerating capacity.

When the cooling load decreases, the refrigerant evaporates at a slower rate than the compressor is able to suck. As a result, the evaporator pressure drops and the degree of superheat decreases. The valve tends to close and the compressor delivers less refrigerating capacity at a decreased suction pressure. Thus the thermostatic expansion valve is capable of meeting variable load requirements.

7. (b) (ii) Solution:



Vortex tube

The figure shows a Vortex tube refrigeration system. It consists of the following parts:

1. Nozzle
2. Diaphragm
3. Valve

Working: Compressed air is passed through the nozzle. Inside the nozzle, air expands and acquires high velocity. A vortex flow is created in the chamber and air travels in a spiral-like motion along the periphery of the hot side. The flow is restricted by the valve. When the pressure of the air near the valve is increased more than the outside by partially closing the valve, a reversed axial flow through the core of the hot side starts from the high-pressure region to the low-pressure region. During this period, energy transfer takes place between reversed stream and forward stream, and therefore the air stream through the core gets cooled below the inlet temperature of the air in the vortex tube, while air stream in the forward direction gets heated.

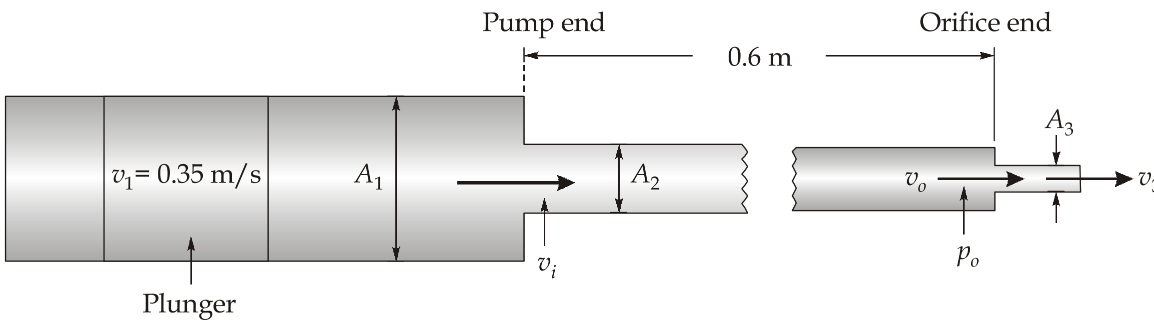
The cold stream escapes through the diaphragm hole into the cold side, while the hot stream is passed through the opening valve.

Advantages of vortex tube:

1. It only uses air as refrigerant, so there is no leakage problem.
2. Vortex tube is simple in design and it avoids control system.
3. There is a complete absence of moving parts in the vortex tube.
4. It is light in weight and requires less space.
5. Cheaper in initial cost.
6. Working expenses are less as compressed air is readily available.
7. Maintenance is simple and no expert attendant is required.

7. (c) Solution:

Injection system



Given : $A_2 = \frac{1}{20}A_1,\ A_3 = \frac{1}{50}A_2$

Initial pressure, $p_1 = 27$ bar

$\rho_f = 0.85 \times 1000 = 850\ \text{kg/m}^3$

$K = 1.8 \times 10^9\ \text{N/m}^2$

(i) The velocity of pressure disturbances

$$v_s = \sqrt{\frac{K}{\rho_f}} = \sqrt{\frac{1.80 \times 10^9}{850}} = 1455.21\ \text{m/s}$$

(ii) The time taken by the disturbance to travel through the pipeline,

$$\Delta t = \frac{\text{Length of the pipeline}}{v_s} = \frac{0.6}{1455.21} = 0.00041s$$

(iii) Let velocity of the fuel at inlet is v_i,

$\therefore \quad A_1 v_1 = A_2 v_i$

$\Rightarrow \quad v_i = \frac{A_1}{A_2}v_1 = 20 \times 0.35 = 7\ \text{m/s}$

Assuming initial velocity of this location is zero.

$\therefore \quad \Delta v = 7\ \text{m/s}$

$$\Delta p = \frac{K}{v_s}(\Delta v) = \frac{1.8 \times 10^9}{1455.21} \times 7 = 86.6 \times 10^5\ \text{N/m}^2$$

or $\quad \Delta p = 86.6$ bar

Pressure at the instant when velocity increased to 7 m/s,

$$= 86.6 + 27 = 113.6 \text{ bar}$$

(iv) The velocity of the fluid after passing the orifice,

$$v_3 = \sqrt{\frac{2\Delta p_1}{\rho_f}}$$

Velocity (v_0) just before the orifice can be obtained by

$$A_2 v_0 = A_3 v_3$$

$$\Rightarrow \quad v_0 = \frac{A_3}{A_2} v_3 = \frac{1}{50}\sqrt{\frac{2(p_0 - p_3)}{\rho_f}}$$

$$\because \quad p_0 = 27 + 86.6 + p_r \text{ (reflected pressure)}$$

$$p_3 = 30 \text{ bar}$$

Change in velocity from pump end to orifice end,

$$\Delta v = 7 - \frac{1}{50}\sqrt{\frac{2(27 + 86.6 + p_r - 30)\times 10^5}{850}}$$

$$= 7 - 0.3068\sqrt{83.6 + p_r}$$

The first reflected pressure wave resulting from Δv,

$$\Delta p_1 = p_r = \frac{K}{v_s}\cdot \Delta v$$

$$\Rightarrow \quad p_r = \frac{1.8\times 10^4}{1455.21}(7 - 0.3068\sqrt{83.6 + p_r})$$

$$p_r = 86.6 - 3.79\sqrt{83.6 + p_r}$$

We get, $p_r = 43.82$ bar

then, $\Delta v = 7 - 0.3068\sqrt{83.6 + 43.82}$

$$\Delta v = 3.54 \text{ m/s}$$

(iv) $p_0 = 27 + 86.6 + 43.82$

$$= 157.42 \text{ bar}$$

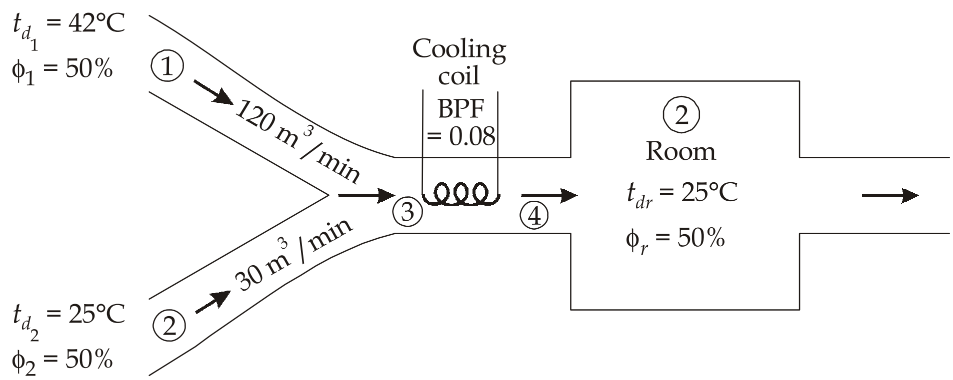
$$v_0 = \frac{1}{50}\sqrt{\frac{2(157.42 - 30)\times 10^5}{850}}$$

$$\Rightarrow \quad v_0 = 3.46 \text{ m/s}$$

8. (a) Solution:

Given : v_1 = 120 m^3/min; t_{d_1} = 42°C; ϕ_1 = 50%; v_2 = 30 m^3/min; t_{d_2} = 25°C; ϕ_2 = 50%; ADP = 11°C; BPF = 0.08

The flow diagram for the given condition is shown below



First of all, mark the initial condition of air at 42°C dry bulb temperature and 50% relative humidity on the psychrometric chart as point 1. Now mark the room condition at 25°C DBT and 50% RH as point 2.

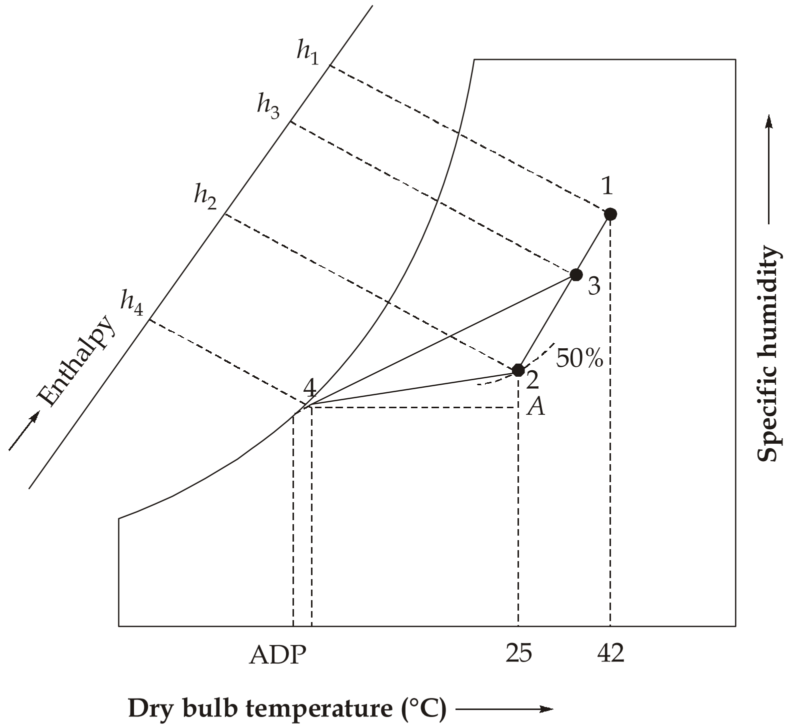
From the psychrometric chart,

Enthalpy: h_1 = 110.5kJ/kg dry air

h_2 = 50.5 kJ/kg dry air

Specific volume: v_{s_1} = 0.93 m^3 kg dry air

v_{s_2} = 0.86 m^3/kg dry air



$$m_{a1} = \frac{v_1}{v_{s_1}} = \frac{120}{0.93} = 129.03 \text{ kg/min}$$

$$m_{a2} = \frac{v_2}{v_{s_2}} = \frac{30}{0.86} = 34.88 \text{ kg/min}$$

∴ Mass flow rate of air flowing through the cooling coil at point 3,

$$m_{a3} = m_{a1} + m_{a2}$$
$$= 129.03 + 34.88$$
$$= 163.91 \text{ kg/min}$$

For energy balance,

$$m_{a_1}h_1 + m_{a_2}h_2 = m_{a_3}h_3$$

$$h_3 = \frac{129.03 \times 110.5 + 34.88 \times 50.5}{163.91}$$

$$= 97.73 \text{ kJ/kg of dry air}$$

Now, draw a constant enthalpy line corresponding to h_3 = 97.73 kJ/kg of dry air which intersects the line 1-2 at point 3.

From the Psychrometric chart, the dry bulb temperature of air entering the cooling coil at point 3 is

$$t_{d_3} = 38.4°C$$

Mark point S on the saturation curve such that ADP = 11°C and draw a line 3 - 5. The point 4 lies on this line.

Let, t_{d_4} = Dry bulb temperature of air leaving the coil

$$\text{BPF} = 0.08 = \frac{t_{d_4} - ADP}{t_{d_3} - ADP}$$

$$0.08 = \frac{t_{d_4} - 11}{38.4 - 11}$$

$$t_{d_4} = 13.19°C$$ **Ans.**

(i) Room sensible heat factor

From Psychrometric chart,

$$h_4 = 36.5 \text{ kJ/kg of dry air}$$

Point A, h_A = 49 kJ/kg of dry air

A is the intersection of horizontal line from point 4 and vertical line from point 2.

$$\text{RSHF} = \frac{h_A - h_4}{h_2 - h_4} = \frac{49 - 36.5}{50.5 - 36.5}$$

$$= 0.893$$ **Ans.**

(ii) Cooling capacity of the coil,

$$\text{C.C.} = m_{a_3}(h_3 - h_4)$$

$$= 163.91 \times (97.73 - 36.5)$$

$$= 10036.2098 \text{ kJ/min}$$

$$\text{In tonns, C.C.} = \frac{10036.2098}{210} = 47.79 \text{ TR}$$ **Ans.**

8. (b) Solution:

Given : d = 90 mm = 0.09 m; L = 120 mm = 0.12 m

$$\text{Compression ratio, } r = 10, \frac{\dot{m}_a}{\dot{m}_f} = 14.8$$

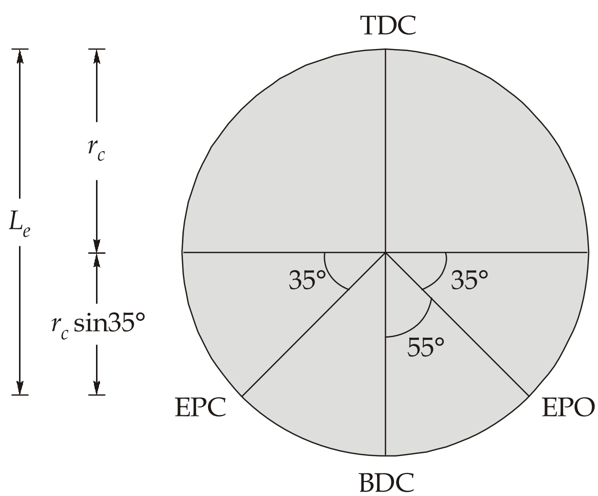
$$T_1 = 310 \text{ K}; p_e = 1.02 \text{ bar}, \dot{m}_a = 190 \text{ kg/h};$$

$$N = 4200 \text{ rpm}; R = 287 \text{ J/kgK}$$

$\therefore$ Scavenging ratio, $R_{sc} = \dfrac{\dot{m}_{\text{supp}}}{\dot{m}_{\text{ideal}}}$

$$\dot{m}_f = \frac{\dot{m}_a}{14.8} = \frac{190}{14.8} = 12.84 \text{ kg/h}$$

$\therefore$ $\dot{m}_{\text{supp}} = \dot{m}_a + \dot{m}_f = 202.84 \text{ kg/h}$



$$r_c = \frac{L}{2} = \frac{120}{2} = 60 \text{ mm}$$

Effective stroke, $L_e = r_c + r_c \sin 35° = 94.4 \text{ mm} = 0.0944 \text{ m}$

Corresponding swept volume,

$$V_{se} = \frac{\pi}{4} d^2 L_e$$

$\Rightarrow$ $V_{se} = \dfrac{\pi}{4}(0.09)^2(0.0944) = \left(6 \times 10^{-4}\right) m^3$

Total cylinder volume corresponding to L_e,

$$V = \left(\frac{r}{r-1}\right) V_{se} = \frac{10}{9} \times 6 \times 10^{-4} = 6.673 \times 10^{-4} m^3$$

and $\rho = \dfrac{p_e}{RT} = \dfrac{1.02 \times 10^5}{287 \times 310} = 1.146 \text{ kg/m}^3$

Mass of mixture per cycle, $m = \rho V = 1.146 \times 6.673 \times 10^{-4}$

$\Rightarrow$ $m = 7.65 \times 10^{-4}$ kg/cycle

Ideal rate of mass flow, $\dot{m}_{ideal} = m \times 60\, N$

$$= 7.65 \times 10^{-4} \times 60 \times 4200$$

$$= 192.79 \text{ kg/h}$$

(i) Scavenging ratio, $R_{sc} = \frac{202.84}{192.79} = 1.052$ **Ans.**

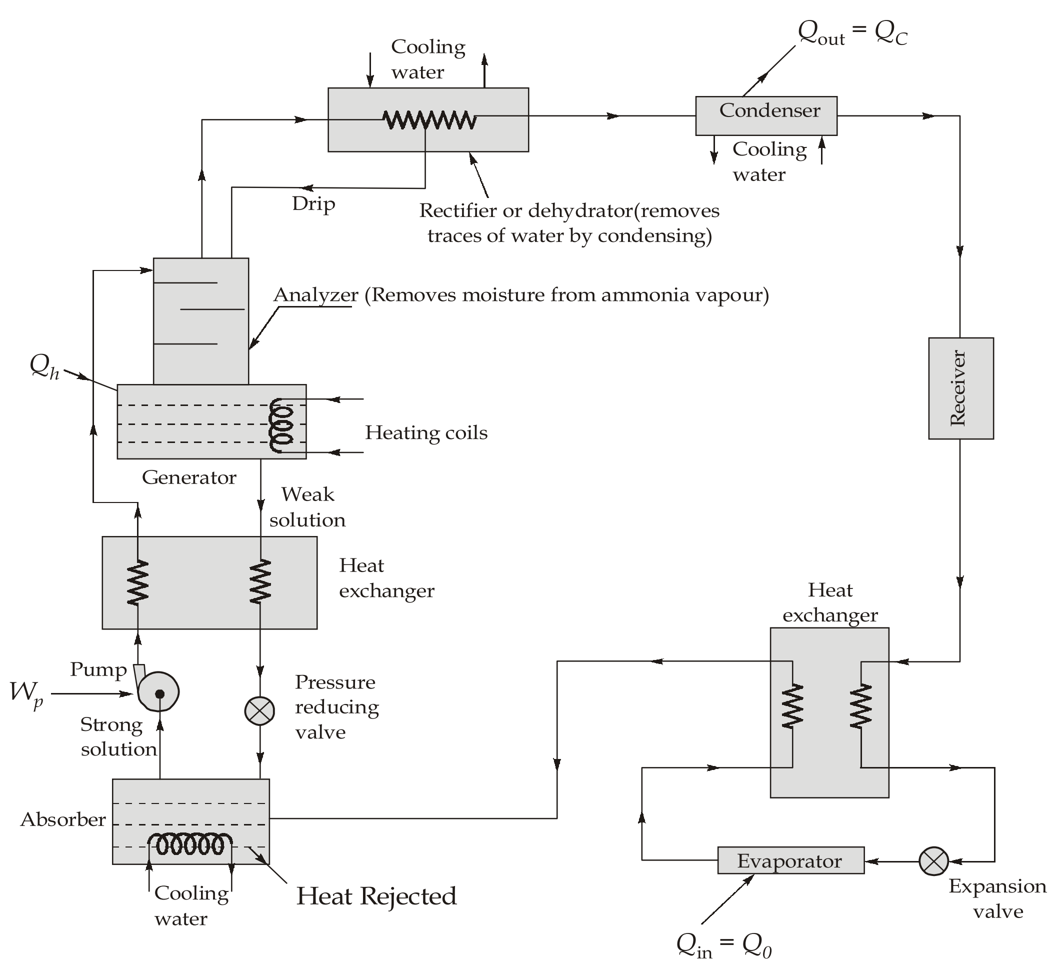
(ii) Scavenging efficiency, $\eta_{sc} = 1 - e^{-R_{sc}}$ **Ans.**

$$= 1 - e^{-1.052} = 0.6508$$

or $\eta_{sc} = 65.08\%$ **Ans.**

(iii) Trapping efficiency, $\eta_{tr} = \frac{\eta_{sc}}{R_{sc}} = \frac{65.08}{1.052} = 61.86\%$ **Ans.**

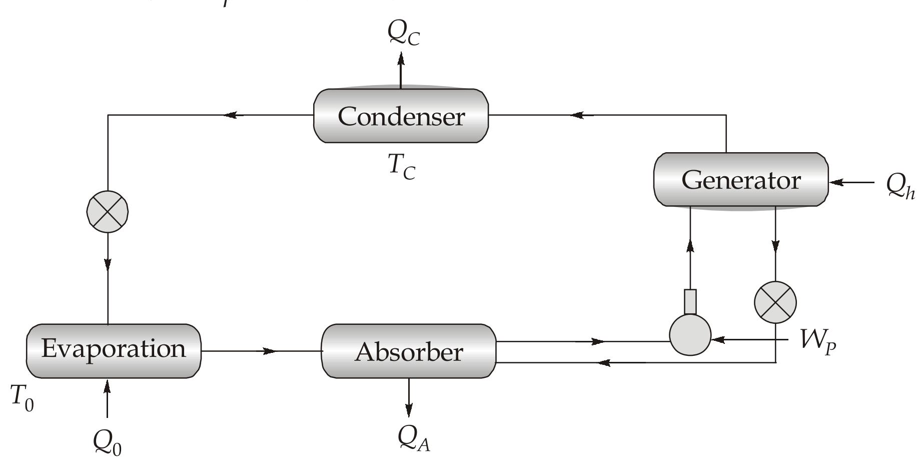
8. (c) Solution:



A simple vapour absorption system, consists of a condenser, an expansion device and an evaporator as in the vapour compression system and in addition, an absorber, a pump, a generator and a pressure reducing valve to replace the compressor. The schematic representation of the system is shown in figure in which various components of the system are arranged according to their pressures and temperatures.

The refrigerating effect is shown as Q_0 at temperature T_0 and the heat rejected in the condenser as Q_C at temperature $T_C = T_K$ of the environment. The compressor work is replaced by the heat supplied in the generator Q_h plus pump work W_P. Cooling must be done in the absorber to remove the latent heat of the refrigerant vapour as it changes into the liquid state by absorption by the weak solution. Heat rejected in the absorber be Q_A at absorber temperature $T_A = T_K$. Then the energy balance of the system is

$$Q_0 + W_p + Q_h = Q_h + Q_A$$



Schematic representation of simple vapour absorption system

The pump work $W_P = -\int vdP$ is very small compared to compressor work in the vapour compression system, as the specific volume v of the liquid is extremely small compared to that of the vapour ($v_f << v_g$). The energy consumption of the system is mainly in the generator in the form of heat supplied Q_h.

In the vapour-absorption system, the function of the compressor is accomplished in a three step process by the use of the absorber, pump and generator. Functions of these devices are given below:

(i) **Absorber :** Absorption of the refrigerant vapour by its weak or poor solution in a suitable absorbent or adsorbent, forming strong or rich solution of refrigerant in absorbent/adsorbent.

(ii) **Pump :** Pumping of the rich solution raising its pressure to the condenser pressure.

(iii) **Generator :** Distillation of the vapour from the rich solution leaving the poor solution for recycling.

Let, T_g be the temperature at which heat Q_g is supplied to the generator, T_C be the temperature at which heat Q_C is discharged to atmosphere or cooling water from the condenser and absorber and T_L be the temperature at which heat Q_L is absorbed in the evaporator.

For maximum possible COP, the vapour absorption system should be perfectly reversible. The entropy generation must be equal to zero.

$$\Delta s_{sys} + \Delta s_{surr} = 0$$

$$0 + \frac{Q_C}{T_C} - \frac{Q_g}{T_g} - \frac{Q_L}{T_L} = 0$$

$$\therefore \quad \frac{Q_g}{T_g} + \frac{Q_L}{T_L} = \frac{Q_C}{T_C} \quad \text{... (i)}$$

Also,

$$\frac{Q_C}{T_C} = \frac{Q_g + Q_L}{T_C} \quad \text{... (ii)}$$

From equation (i) and (ii),

$$\frac{Q_g}{T_g} - \frac{Q_g}{T_C} = \frac{Q_L}{T_C} - \frac{Q_L}{T_L}$$

or

$$Q_g\left(\frac{T_C - T_g}{T_g \times T_C}\right) = Q_L\left(\frac{T_L - T_C}{T_C \times T_L}\right)$$

or

$$Q_g = Q_L\left(\frac{T_L - T_C}{T_C \times T_L}\right)\left(\frac{T_g \times T_C}{T_C - T_g}\right) = Q_L\left(\frac{T_C - T_L}{T_C \times T_L}\right)\left(\frac{T_g \times T_C}{T_g - T_C}\right)$$

$$= Q_L\left(\frac{T_C - T_L}{T_L}\right)\left(\frac{T_g}{T_g - T_C}\right)$$

The maximum coefficient of performance (COP) of the system is given by

$$(\text{COP})_{max} = \frac{Q_L}{Q_g} = \left(\frac{T_L}{T_C - T_L}\right)\left(\frac{T_g - T_C}{T_g}\right)$$

The ideal refrigerant-absorbent combination should possess the following qualities:

1. The refrigerant should have high affinity for the absorber at low temperature and less affinity at high temperature.
2. The combination should have high degree of negative deviation from Raoult's law.
3. The mixture should have low specific heat and low viscosity.
4. The mixture (solution) should be non-corrosive.
5. The mixture should have a small heat.
6. The mixture should have low freezing point.

7. There should be a large difference in normal boiling points of the refrigerants and the absorbent. [Atleast 150 - 200°C]

$$\left[COP = \frac{Q_0}{Q_h}\right]$$

Commonly used refrigerant-absorbent combinations in VARS are.

(a) Ammonia-water combination; and (b) Lithium-bromide water combination.

The ammonia-water absorption system finds a significant place in large tonnage industrial applications.

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