



**RPSC AEn-2024
Main Test Series**

**CIVIL
ENGINEERING**

Test 15

Test Mode : • Offline • Online

Subjects : Full Syllabus Test (Paper-I)

DETAILED EXPLANATIONS

1. Solution:

Over-consolidated soil is a soil that has experienced a past maximum effective stress greater than the present effective stress.

2. Solution:

Poisson's ratio is the ratio of lateral strain to longitudinal strain within the elastic limit.

$$\mu = \text{Lateral strain/Longitudinal strain}$$

For most engineering materials, its value ranges from 0 to 0.5.

3. Solution:

Maximum principal stress

$$\begin{aligned} &= \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} \\ &= \frac{1.5}{2} + \sqrt{\left(\frac{1.5}{2}\right)^2 + (1.20)^2} = 2.17 \text{ N/mm}^2 \end{aligned}$$

4. Solution:

Polar moment of inertia is the moment of inertia of a cross-section about an axis perpendicular to the plane of the section. It is used to describe resistance to torsional deformation.

5. Solution:

Since eccentricity effect is being neglected so column can be considered as concentrically loaded. Ultimate axial load carrying capacity of column.

$$P_u = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$$

$$= 0.45 \times 20 \times 250 \times 300 + (0.75 \times 415 - 0.45 \times 20) 4 \times \frac{\pi}{4} \times 16^2 = 918.1 \text{ kN}$$

6. Solution:

Void ratio is the ratio of volume of voids to the volume of solids while porosity is the ratio of volume of voids to the total volume of soil mass.

7. Solution:

Relaxation of steel is the gradual reduction in stress in prestressing steel when it is held at constant strain over a period of time.

8. Solution:

Angle of internal friction is the angle that represents the resistance offered by soil due to inter-particle friction. It is denoted by ϕ and is the slope of the failure envelope in shear strength analysis.

9. Solution:

$$\begin{aligned} \text{Gross diameter} &= \text{nominal diameter} + 1.5 && (\text{for } \phi \leq 25 \text{ mm}) \\ &= 20 + 1.5 = 21.5 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{If } \phi > 25 \text{ mm, gross diameter} \\ &= \text{nominal diameter} + 2.0 \end{aligned}$$

10. Solution:

Modular ratio is the ratio of modulus of elasticity of steel to modulus of elasticity of concrete.

$$m = E_s / E_c$$

11. Solution:

Carry-over factor is the ratio of moment induced at the far end of a member to the applied moment at the near end.

12. Solution:

A segment of block of material at and of member shears out due to the possible use of high bearing strength of steel and high strength bolts resulting in a smaller connection length.

13. Solution:

For normally consolidated soil,

$$K_o = \mu / (1 - \mu)$$

$$K_o = 0.30 / (1 - 0.30) = 0.30 / 0.70 = 0.43$$

Lateral earth pressure at rest,

$$P_o = K_o \sigma_v$$

$$P_o = 0.43 \times 200$$

$$P_o = 86 \text{ kN/m}^2$$

14. Solution:

$$\therefore \frac{G\theta}{L} = \frac{T}{J}$$

$$\therefore \theta = \frac{TL}{GJ} = \frac{100 \times 5}{50000}$$
$$= 0.01 \text{ rad}$$

$$\text{Torsional strain energy} = \frac{1}{2} T\theta$$
$$= \frac{1}{2} \times 100 \times 0.01$$
$$= 0.5 \text{ kN-m}$$

15. Solution:

The modes of shear failure under shallow foundations are:

1. General shear failure
2. Local shear failure
3. Punching shear failure

16. Solution:

A plastic hinge is a section in a structural member where the bending moment reaches the plastic moment capacity and the section undergoes large rotations without any increase in moment.

17. Solution:

Redundancy is the number of extra unknown forces or reactions in a structure beyond those required for static equilibrium.

18. Solution:

$$\frac{A_{st\min}}{bd} = \frac{0.85}{f_y}$$

or,

$$A_{st} = \frac{0.85}{500} \times 230 \times 500 \text{ mm}^2 = 195.5 \text{ mm}^2$$

$$n = \frac{A_{st}}{\frac{\pi d^2}{4}} = \frac{195.5}{\frac{\pi (12)^2}{4}} = 1.729 = 2 \text{ bars}$$

19. Solution:

Column buckling is the sudden lateral deflection of a slender column under axial compressive load when the load reaches a critical value.

Euler's critical load is given by

$$P_{cr} = \pi^2 EI / (L_e)^2$$

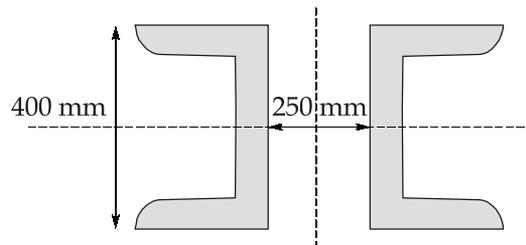
where

E = Modulus of elasticity

I = Moment of inertia

L_e = Effective length of column

20. Solution:



Lacing are designed to resist transverse shear equal to 2.5% of total axial force on the column taken into account.

$$\therefore V = \frac{2.5}{100} \times P$$

Here $P = 1600 \text{ kN}$;

$$V = \frac{2.5}{100} \times 1600$$

$$V = 40 \text{ kN}$$

21. Solution:

Serviceability limit states ensure satisfactory performance of structure under normal service loads without discomfort or damage.

They include:

1. Control of deflection to prevent excessive sagging.
2. Control of cracking to maintain durability and appearance.
3. Limitation of vibration where applicable.

22. Solution:

Stress in concrete at the level of tendon,

$$\begin{aligned} f_c &= \frac{P}{A} + \frac{Pe^2}{I} \\ &= \frac{150 \times 10^3}{120 \times 200} + \frac{150 \times 20^2 \times 12 \times 10^3}{120 \times 200^3} \\ &= 7 \text{ MPa} \end{aligned}$$

Loss of prestress due to elastic deformation,

$$\Delta f_s = \frac{E_s}{E_c} \times f_c = \frac{2.1 \times 10^5}{3.0 \times 10^4} \times 7 = 49 \text{ MPa}$$

Total stress in steel,

$$f_s = \frac{P}{A_s} = \frac{150 \times 10^3}{187.5} = 800 \text{ MPa}$$

∴ Percentage loss of stress

$$= \frac{\Delta f_s}{f_s} \times 100 = \frac{49}{800} \times 100 = 6.125\%$$

23. Solution:

Minimum pitch is the least centre-to-centre distance between adjacent bolts in a row. It should not be less than 2.5 times the nominal diameter of the bolt.

It is provided:

1. to prevent bearing and tearing failure of plate between bolts.
2. to provide sufficient space for proper tightening of bolts.
3. to avoid overlapping of washers and ensure proper installation.

24. Solution:

Nominal shear stress,

$$\tau_v = \frac{V_u}{bd} = \frac{200 \times 10^3}{250 \times 350} = 2.286 \text{ N/mm}^2 < \tau_{c,\max} \quad (\text{OK})$$

SF taken by stirrups

$$\begin{aligned} &= (\tau_v - \tau_c) bd \\ &= (2.286 - 0.62) \times 250 \times 350 = 145.75 \text{ kN} \end{aligned}$$

Now,

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$\begin{aligned} \Rightarrow S_v &= \frac{0.87 \times 250 \times 350 \times 2 \times \frac{\pi}{4} \times 10^2}{145.75 \times 10^3} \\ &= 82 \text{ mm} = 8.2 \text{ cm} \end{aligned}$$

25. Solution:

Mohr's circle is a graphical method used to determine principal stresses, maximum shear stress and their orientations in a plane stress condition. A circle is drawn using normal and shear stress components as coordinates. The centre lies at average normal stress and radius represents maximum shear stress.

Applications:

1. Determination of principal stresses
2. Determination of maximum shear stress
3. Finding orientation of principal planes

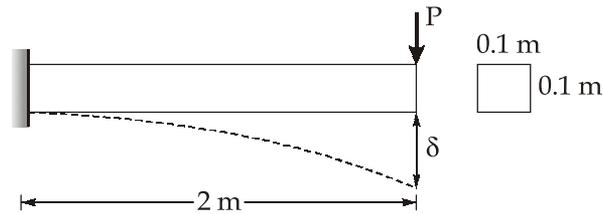
26. Solution:

Terzaghi's one-dimensional consolidation theory explains time-dependent settlement of saturated clay due to expulsion of pore water under applied load. Consolidation occurs because excess pore water pressure gradually dissipates with time.

Assumptions:

1. Soil is fully saturated.
2. Soil is homogeneous and isotropic.
3. Compression and drainage occur only in vertical direction.
4. Soil particles and pore water are incompressible.
5. Darcy's law is valid.

27. Solution:



$$I = \frac{(0.1)^4}{12}$$

Deflection,
$$\delta = \frac{Pl^3}{3EI}$$

$$\Rightarrow 5 \times 10^{-3} \text{ m} = \frac{P(2)^3}{3 \times 2 \times 10^{11} \times \frac{(0.1)^4}{12}}$$

$$\Rightarrow P = 3125 \text{ N}$$

Now,
$$M = Pl = 3125 \times 2 = 6250 \text{ Nm}$$

As,
$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \sigma_{\max} = \frac{M}{Z} = \frac{6250}{\frac{(0.1)^3}{6}}$$

$$= 37.5 \times 10^6 \text{ N/m}^2 = 37.5 \text{ MPa}$$

28. Solution:

Combined bending and direct stress occurs when a structural member is subjected to an axial load along with a bending moment. The direct stress is uniformly distributed over the cross-section, while bending stress varies linearly about the neutral axis.

Resultant stress at any fibre is:

$$\sigma = (P/A) \pm (My/I)$$

where P is axial load and M is bending moment.

29. Solution:

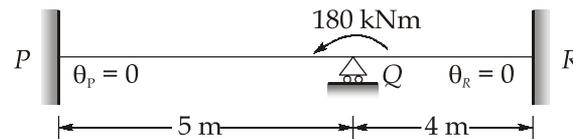
Fillet welds are preferred in comparison to butt welds due to the following reasons:

1. Fillet welds do not require edge preparation and finishing of members.
2. They are easier and more economical to fabricate in field conditions.
3. They generally develop lower residual stresses compared to butt welds.

30. Solution:

A doubly reinforced beam has reinforcement in both tension and compression zones. Under bending, tension steel resists tensile forces while compression steel assists concrete in resisting compressive stresses.

It is provided when depth of beam is restricted or when moment exceeds the capacity of a singly reinforced section. Compression steel also improves ductility and reduces long-term deflection.

31. Solution:

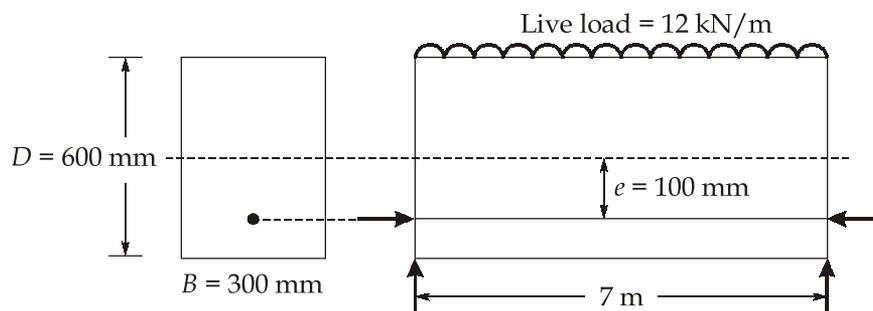
$$K_Q = K_{QP} + K_{QR}$$

$$= \frac{4EI}{5} + \frac{4EI}{4} = 1.8 EI$$

$$\theta_Q = \frac{M_Q}{K_Q} = \frac{180}{1.8 \times 10^4} = 0.01 \text{ rad}$$

32. Solution:

1. In constant head test, a saturated soil specimen is placed in a permeameter.
2. A constant hydraulic head is maintained across the specimen.
3. The discharge collected in a known time is measured.
4. Using Darcy's law, the coefficient of permeability is calculated from head difference, length, area and discharge.

33. Solution:**1. Calculation of load:**

$$\text{Dead load} = 0.30 \times 0.60 \times 1.0 \times 25$$

$$= 4.50 \text{ kN/m}$$

$$\text{Live load} = 12.00 \text{ kN/m}$$

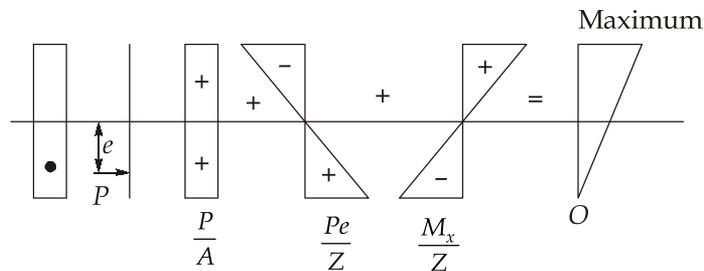
Total load = 16.50 kN/m

Bending moment at centre,

$$M_x = \frac{wL^2}{8} = \frac{16.50 \times 7^2}{8} = 101.0625 \text{ kN-m}$$

2. For no tension condition at mid span under live load.

Note: Live load will always act with dead load.



$$\text{Stress at bottom fibre, } (f_c)_{\text{bottom}} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_x}{Z}$$

where e is eccentricity and Z is section modulus.

For no tension at bottom, $(f_c)_{\text{bottom}} = 0$

$$\text{So, } \frac{P}{A} + \frac{Pe}{Z} = \frac{M_x}{Z}$$

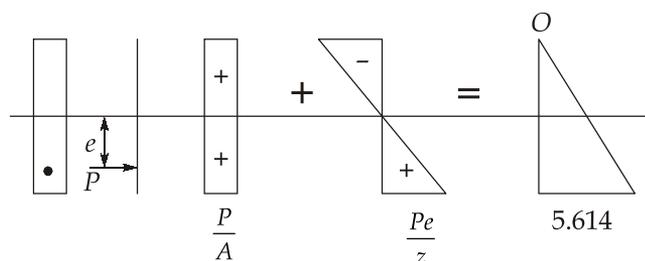
$$\Rightarrow P \left(\frac{Z}{A} + e \right) = M_x$$

$$\text{Now, } \frac{Z}{A} = \frac{BD^2}{6 \times BD} = \frac{D}{6}$$

$$\begin{aligned} \Rightarrow P &= \frac{M_x}{\left(\frac{D}{6} + e \right)} \\ &= \frac{101.0625 \times 10^6}{\left(\frac{600}{6} + 100 \right) \times 1000} \text{ kN} = 505.3125 \text{ kN} \end{aligned}$$

3. Stresses developed only due to DL.

(a) At end, moment = 0:



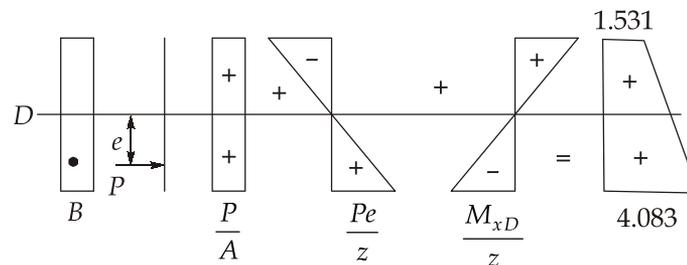
$$\frac{P}{A} = \frac{505.3125 \times 1000}{300 \times 600} = 2.807 \text{ N/mm}^2$$

$$\frac{Pe}{Z} = \frac{505.3125 \times 1000 \times 100}{\left(\frac{300 \times 600^2}{6}\right)} = 2.807 \text{ N/mm}^2$$

$$\text{At top} = \frac{P}{A} - \frac{Pe}{Z} = 2.807 - 2.807 = 0 \text{ N/mm}^2$$

$$\text{At bottom} = \frac{P}{A} + \frac{Pe}{Z} = 2.807 + 2.807 = 5.614 \text{ N/mm}^2$$

(b) At mid span:



$$\text{Due to DL} \quad M_{xD} = \frac{w_d L^2}{8} = \frac{4.50 \times 7^2}{8} = 27.5625 \text{ kN-m}$$

$$\frac{M_{xD}}{Z} = \frac{27.5625 \times 10^6}{\left(\frac{300 \times 600^2}{6}\right)} = 1.531 \text{ N/mm}^2$$

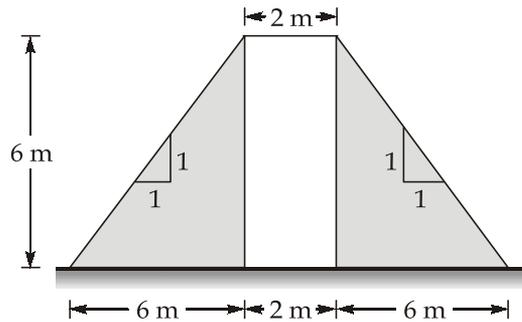
$$\begin{aligned} \text{Stress at Top} &= \frac{P}{A} - \frac{Pe}{Z} + \frac{M_{xD}}{Z} \\ &= 2.807 - 2.807 + 1.531 = 1.531 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Stress at bottom} &= \frac{P}{A} + \frac{Pe}{Z} - \frac{M_{xD}}{Z} \\ &= 2.807 + 2.807 - 1.531 = 4.083 \text{ N/mm}^2 \end{aligned}$$

34. Solution:

Given data: $\gamma_1 = 18 \text{ kN/m}^3$, $w_1 = 8\%$, $\gamma_{d2} = 20 \text{ kN/m}^3$, $w_2 = 10\%$, $G = 2.70$, $V_1 = ?$

$$\text{Area of embankment} = \left(\frac{2+14}{2} \right) \times 6 = 48 \text{ m}^2$$



Volume of embankment = Area \times Length = $48 \times 1 = 48 \text{ m}^3 = V_2$

Using subscript 1 and 2 for borrow pit and embankment respectively. All the symbols have their usual meanings

$$\begin{aligned} \therefore \gamma_{d2} &= \frac{G\gamma_w}{1 + e_2} \\ \Rightarrow 20 &= \frac{2.7 \times 9.81}{1 + e_2} \\ \Rightarrow e_2 &= \frac{2.7 \times 9.81}{20} - 1 = 0.32 \end{aligned}$$

Now, porosity of the embankment soil,

$$n_2 = \frac{e_2}{1 + e_2} = \frac{0.32}{1 + 0.32} = 0.2424$$

Degree of saturation of embankment soil can be given as

$$\begin{aligned} S_2 e_2 &= w_2 G \\ \Rightarrow S_2 &= \frac{w_2 G}{e_2} \\ \Rightarrow S_2 &= \frac{0.10 \times 2.70}{0.32} \times 100 \\ \Rightarrow S_2 &= 84.37\% \end{aligned}$$

Now, if V_1 is the volume of soil to be excavated from the borrow pit,

then

$$\frac{V_1}{1 + e_1} = \frac{V_2}{1 + e_2} = V_s$$

But to calculate V_1 , we require 'e₁' which can be calculated as

$$\gamma_{d_1} = \frac{G\gamma_w}{1 + e_1} \quad \dots (i)$$

$$\gamma_{d_1} = \frac{\gamma_1}{1 + w_1} \quad \dots (ii)$$

From (i) and (ii), we get $\frac{G\gamma_w}{1 + e_1} = \frac{\gamma_1}{1 + w_1}$

$$\Rightarrow \frac{2.70 \times 9.81}{1 + e_1} = \frac{18}{1 + 0.08}$$

$$\Rightarrow e_1 = \frac{2.70 \times 9.81 \times 1.08}{18} - 1 = 0.589$$

Now, we have

$$\frac{V_1}{1 + e_1} = \frac{V_2}{1 + e_2}$$

$$\Rightarrow V_1 = \left(\frac{1 + e_1}{1 + e_2} \right) \times V_2 = \left(\frac{1 + 0.589}{1 + 0.32} \right) \times 48 = 57.78 \text{ m}^3$$

Hence, 57.78 m³ of earth is to be excavated in the borrow area.

Total weight of Borrow pit soil = $V_1 \times \gamma_2 = 57.78 \times 18 = 1040.04 \text{ kN}$

Number of truck loads required = $\frac{\text{Total weight of embankment soil}}{\text{Capacity of truck per trip}}$

$$= \frac{1040.04}{80} = 13 \text{ number of truck loads, for construction of per meter length of embankment.}$$

35. Solution:

Reactions in real beam: By symmetry,

$$R_A = R_B = W$$

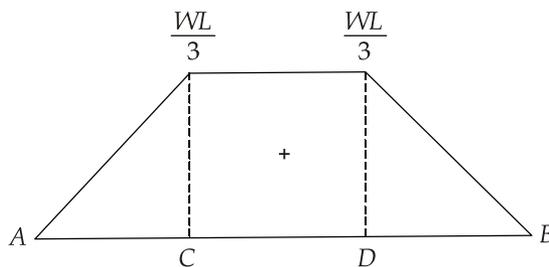


Fig. (i): BMD

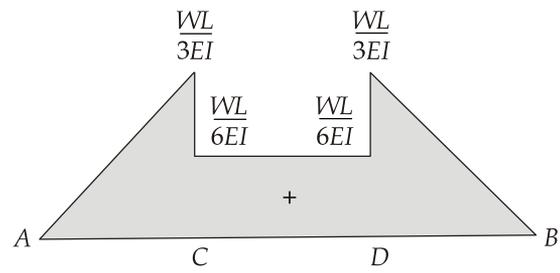


Fig. (ii): $\frac{M}{EI}$ diagram

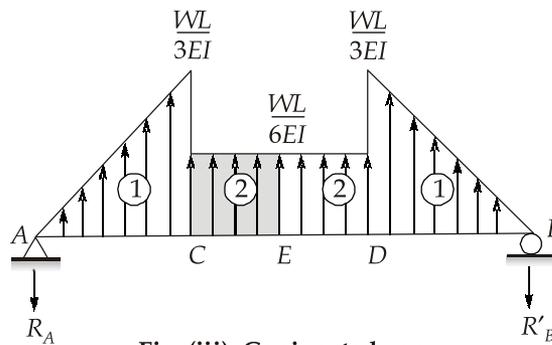


Fig. (iii): Conjugate beam

Reactions in conjugate beam:

By symmetry,

$$\begin{aligned}
 R_A' = R_B' &= \frac{1}{2} \times \text{Area of } \frac{M}{EI} \text{ diagram} \\
 &= \frac{1}{2} \left[\frac{1}{2} \times \frac{L}{3} \times \frac{WL}{3EI} + \frac{L}{3} \times \frac{WL}{6EI} + \frac{1}{2} \times \frac{L}{3} \times \frac{WL}{3EI} \right] \\
 &= \frac{1}{2} \left[\frac{WL^2}{18EI} + \frac{WL^2}{18EI} + \frac{WL^2}{18EI} \right] = \frac{WL^2}{12EI} (\downarrow)
 \end{aligned}$$

| S.N. | Part | Area | Centroid (from A) |
|------|------|--|---|
| 1. | 1 | $A_1 = \frac{1}{2} \times \frac{L}{3} \times \frac{WL}{3EI} = \frac{WL^2}{18EI}$ | $\bar{X}_1 = \frac{2}{3} \times \frac{L}{3} = \frac{2L}{9}$ |
| 2. | 2 | $A_2 = \frac{L}{6} \times \frac{WL}{6EI} = \frac{WL^2}{36EI}$ | $\bar{X}_2 = \frac{L}{3} + \frac{L}{12} = \frac{5L}{12}$ |

According to conjugate beam method “the deflection at any section in real beam is equals to BM at that section in conjugate beam.”

$$\begin{aligned}
 \therefore \text{ deflection at mid span, } y_E &= -R_A' \times \frac{L}{2} + A_1 \left(\frac{L}{2} - \bar{X}_1 \right) + A_2 \left(\frac{L}{2} - \bar{X}_2 \right) \\
 &= -\frac{WL^2}{12EI} \times \frac{L}{2} + \frac{WL^2}{18EI} \left(\frac{L}{2} - \frac{2L}{9} \right) + \frac{WL^2}{36EI} \left(\frac{L}{2} - \frac{5L}{12} \right) \\
 &= -\frac{WL^3}{24EI} + \frac{5WL^3}{324EI} + \frac{WL^3}{432EI} = -\frac{31WL^3}{1296EI} \\
 \therefore y_E &= \frac{31}{1296} \frac{WL^3}{EI} (\downarrow)
 \end{aligned}$$

According to conjugate beam method "the slope at any section in real beam is equals to shear force at that section in conjugate beam."

$$\begin{aligned}\therefore \theta_A &= \text{SF at A in conjugate beam} \\ &= -R_A' = -\frac{WL^2}{12EI}\end{aligned}$$

$$\therefore \theta_A = \frac{WL^2}{12EI} \text{ (clockwise)}$$

and by symmetry, $\theta_B = \theta_A = \frac{WL^2}{12EI}$ (anticlockwise)

36. Solution:

Given: $b = 230 \text{ mm}$, $D = 500 \text{ mm}$, $d' = 40 \text{ mm}$, $f_{ck} = 20 \text{ MPa}$, $f_y = 415 \text{ MPa}$

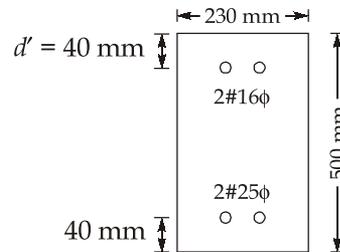
Effective depth, $d = D - d' = 500 - 40 = 460 \text{ mm}$

$$A_{st} = 2 \times \frac{\pi}{4} \times (25)^2 = 981.75 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times (16)^2 = 402.12 \text{ mm}^2$$

For, Fe 415, $x_{u, \max} = 0.48 d = 0.48 \times 460 = 220.8 \text{ mm}$

Here $\frac{d'}{d} = \frac{40}{460} = 0.087$



From the above table, the stress in compression reinforcement can be interpolated.

$$\begin{aligned}\therefore f_{sc} &= 355 - \left(\frac{355 - 353}{0.10 - 0.05} \right) \times (0.087 - 0.05) \\ &= 355 - 1.48 = 353.52 \text{ MPa.}\end{aligned}$$

Now, $C = T$

$$0.36 f_{ck} b x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$0.87 \times 415 \times 981.75 = 0.36 \times 230 \times 20 \times x_u + (353.52 - 0.45 \times 20) \times 402.12$$

$$x_u = 130.388 \text{ mm}$$

Limiting depth of NA,

$$\begin{aligned} x_{u \text{ lim}} &= 0.48 d && \text{(For Fe 415)} \\ &= 0.48 \times 460 = 220.8 \text{ mm} \end{aligned}$$

$\therefore x_u < x_{u \text{ lim}}$ (Section is under reinforced)

The **moment of resistance** is given by

$$\begin{aligned} M_u &= 0.36 f_{ck} b x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d') \\ &= 0.36 \times 20 \times 230 \times 130.388 \times (460 - 0.42 \times 130.388) \\ &\quad + (353.52 - 0.45 \times 20) \times 402.12 \times (460 - 40) \\ &= 145.7 \text{ kN-m} \end{aligned}$$

37. Solution:

Analysing the given rigid frame by slope deflection method which is a stiffness method.

Fixed end moments

$$\bar{M}_{AB} = 0; \bar{M}_{BA} = 0$$

$$\bar{M}_{BC} = - \left(\frac{10 \times 5^2}{12} + \frac{20 \times 5}{8} \right) = - 33.33 \text{ kN-m}$$

$$\bar{M}_{CB} = + 33.33 \text{ kN-m}$$

Slope deflection equations

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L} \left(2\theta_A + \theta_B - \frac{3\delta}{L} \right)$$

$$\Rightarrow M_{AB} = 0 + \frac{2EI}{4} (0 + \theta_B - 0)$$

$$\Rightarrow M_{AB} = 0.5EI\theta_B \quad \dots \text{(i)}$$

and

$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{L} \left(2\theta_B + \theta_A - \frac{3\delta}{L} \right)$$

$$\Rightarrow M_{BA} = 0 + \frac{2EI}{4} (2\theta_B + 0 - 0)$$

$$\Rightarrow M_{BA} = EI\theta_B \quad \dots \text{(ii)}$$

and

$$M_{BC} = \bar{M}_{BC} + \frac{2E(2I)}{L} \left(2\theta_B + \theta_C - \frac{3\delta}{L} \right)$$

$$\Rightarrow M_{BC} = - 33.33 + \frac{4EI}{5} (2\theta_B + \theta_C)$$

$$\Rightarrow M_{BC} = 0.8 EI (2\theta_B + \theta_C) - 33.33 \quad \dots \text{(iii)}$$

and
$$M_{CB} = \bar{M}_{CB} + \frac{2E(2I)}{L} \left(2\theta_C + \theta_B - \frac{3\delta}{L} \right)$$

$$\Rightarrow M_{CB} = +33.33 + \frac{4EI}{5} (2\theta_C + \theta_B)$$

$$M_{CB} = 0.8EI (\theta_B + 2\theta_C) + 33.33 \quad \dots \text{(iv)}$$

Equilibrium equations

(a)
$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow EI\theta_B + 0.8EI(2\theta_B + \theta_C) - 33.33 = 0$$

$$\Rightarrow 2.6\theta_B + 0.8\theta_C = \frac{33.33}{EI} \quad \dots \text{(v)}$$

(b)
$$M_{CB} = 0$$

$$\Rightarrow 0.8EI(\theta_B + 2\theta_C) + 33.33 = 0$$

$$\Rightarrow 0.8\theta_B + 1.6\theta_C = \frac{-33.33}{EI} \quad \dots \text{(vi)}$$

Solving (v) and (vi), we get

$$\theta_B = \frac{22.725}{EI}; \theta_C = \frac{-32.194}{EI}$$

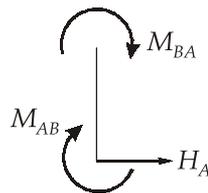
\therefore From (i),
$$M_{AB} = 0.5EI \times \frac{22.725}{EI} = 11.36 \text{ kN-m}$$

From (ii),
$$M_{BA} = EI \times \frac{22.725}{EI} = 22.725 \text{ kN-m}$$

From (iii),
$$M_{BC} = 0.8EI \left(2 \times \frac{22.725}{EI} - \frac{32.194}{EI} \right) - 33.33 = -22.725 \text{ kN-m}$$

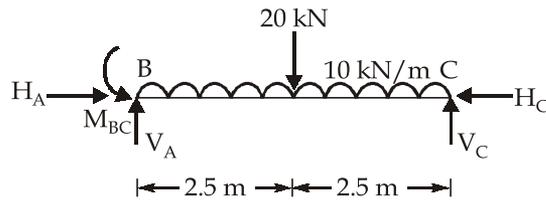
From (iv),
$$M_{CB} = 0$$

Reactions



$$H_A = \frac{M_{AB} + M_{BA}}{L_{AB}} = \frac{11.36 + 22.725}{4} = 8.52 \text{ kN}$$

$$\Sigma M_C = 0$$



$$\Rightarrow V_A \times 5 - 22.725 - 20 \times 2.5 - 10 \times 5 \times 2.5 = 0$$

$$\Rightarrow 5V_A = 197.725$$

$$\Rightarrow V_A = 39.545 \text{ kN}$$

and $V_C = 20 + (10 \times 5) - V_A$

$$\Rightarrow V_C = 70 - 39.545$$

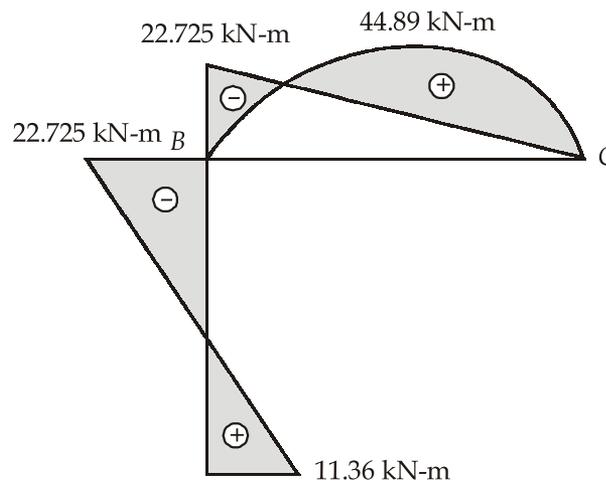
$$\Rightarrow V_C = 30.455 \text{ kN}$$

Also $H_A - H_C = 0$

$$\Rightarrow H_A = H_C = 8.52 \text{ kN}$$

Free BMD of portion BC

$$M_{\text{centre of BC}} = \frac{20 \times 5}{4} + \frac{10 \times 5^2}{8} = 56.25 \text{ kN-m}$$



Maximum sagging moment at the centre of beam BC

$$= 56.25 - \frac{22.75}{2} = 44.89 \text{ kN-m}$$

