RPSC AEn-2024 Main Test Series

ELECTRICAL ENGINEERING

Test 7

Test Mode : • Offline • Online

Subjects : Control Systems

DETAILED EXPLANATIONS

PART-A

1. Solution:

- Except oscillators, in positive feedback, we always have unstable systems.
- Closed loop system is complex and costly.

2. Solution:

Control systems are used in various engineering applications: including robotics, aerospace systems, automatic cruise control, industrial automation, and HVAC (Heating, ventilation and air conditioning) systems.

3. Solution:

They monitor and regulate voltage, frequency and other parameters to ensure a stable and reliable supply of electricity.

4. Solution:

Number of forward paths, P_K = one $(G_1G_2G_3)$

Number of loops = two $(-G_1G_2H_1, -G_2G_3H_2)$

Number of non-touching loops = 0, $\Delta_K = 1$

$$\Delta = 1 + G_1G_2H_1 + G_2G_3H_2$$

$$\therefore \quad \text{T.F.} = \frac{C(s)}{R(s)} = \frac{P_K\Delta_K}{\Delta} = \frac{G_1G_2G_3}{1+G_1G_2H_1+G_2G_3H_2}$$

5. Solution:

Close loop T.F. = $\mathcal{L}$(impulse response)

$$= \mathcal{L}(-te^{-t} + 2e^{-t}) = \frac{-1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$\text{C.L.T.F.} = \frac{2s+1}{(s+1)^2}$$

$\because$ $$\text{C.L.T.F.} = \frac{\text{O.L.T.F.}}{1+\text{O.L.T.F.}} \text{ for unity feedback}$$

i.e. $$\text{C.L.T.F.} = \left.\frac{G(s)}{1+G(s)}\right|_{H(s)=1}$$

$\therefore$ $$G(s) = \left.\frac{\text{C.L.T.F.}}{1-\text{C.L.T.F.}}\right|_{H(s)=1}$$

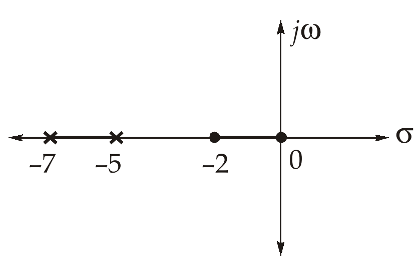
On putting the value of C.L.T.F.

We get, $$G(s) = \frac{2s+1}{s^2}$$

6. Solution:

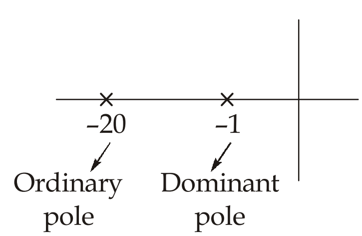
A signal flow graph is a graphical representation of the relationships between the variables of a set of linear algebraic equations. It consists of a network in which nodes representing each of the system variables are connected by directed branches.

7. Solution:



Root locus exist between –7 to –5 and –2 to 0.

8. Solution:



$$\text{Transfer function} \rightarrow \frac{40\left(1+\frac{s}{40}\right)}{20\left(1+\frac{s}{20}\right)(1+s)}$$

$$\text{Answer} = \frac{2}{s+1}$$

9. Solution:

Type: 2

Poles: 0, 0, 2, 25

Zeros: 5

$$\text{Gain} = m \log \omega + 20\log K$$

$$54 = -40 \log(0.1) + 20\log K$$

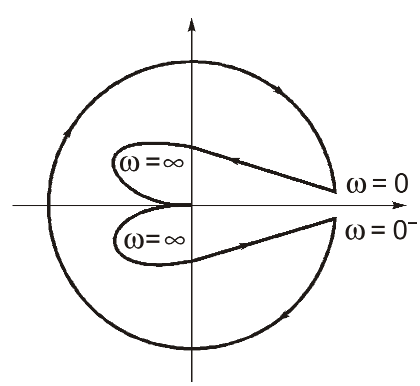
$$\Rightarrow \quad K = 5$$

$$\therefore \quad \text{T.F.} = \frac{5(1+0.2s)}{s^2(1+0.5s)(1+0.04s)}$$

$$= \frac{50(s+5)}{s^2(s+2)(s+25)}$$

10. Solution:

Nyquist plot of $G(s)\ H(s) = \dfrac{s+3}{s^2(s-3)}$ is as shown below:



From the Nyquist plot $G(s)H(s)$ encircle $-1 + j0$ once in clockwise direction.

Alternate solution:

Characteristic equation,

$$1 + G(s)\ H(s) = 0$$

$$s^2(s - 3) + (s + 3) = 0$$

$$s^3 - 3s^2 + s + 3 = 0$$

Using Routh's array,

s^3	1	1
s^2	–3	3
s^1	2	0
s^0	3	

There are two sign changes, hence two poles in right half of s-plane exist.

$Z = 2, \quad P = 1 \quad \Rightarrow \quad N = P - Z = -1$

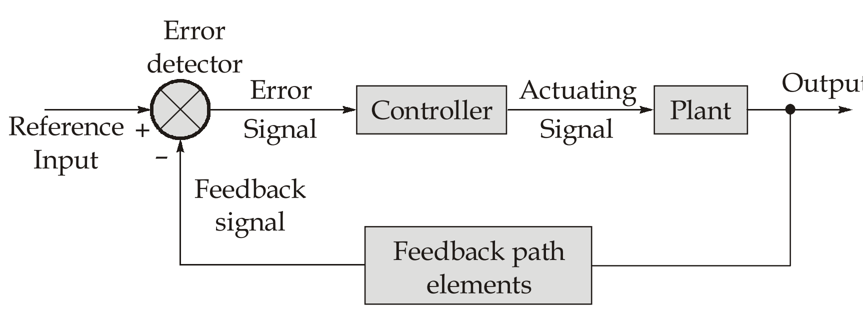
One encirclement in clockwise direction.

PART-B

11. Solution:

Closed-Loop Control System:

- Feedback control systems are often referred to as closed-loop control systems.
- In a closed-loop control system, the actuating error signal which is the difference between the input signal and the feedback signal (which may be the output signal itself or a function of the output signal and its derivative and/or integrals) is fed to the controller so as to reduce the error to bring the output of the system to a desired value.



12. Solution:

Advantages:

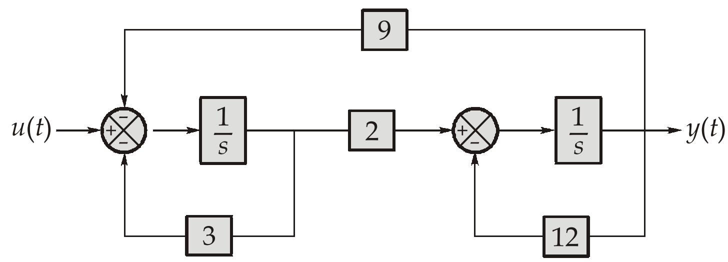
- Effect of parameter variation reduces.
- The gain of system reduces by a factor $(1 + GH)$.
- The bandwidth of the system increases.
- Effect of internal disturbance reduces.

Disadvantages:

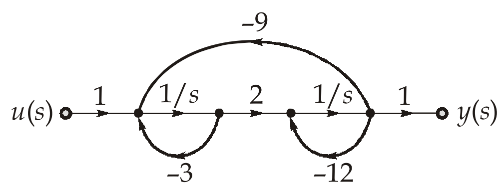
- The system is complex and costly.
- System may become unstable.

13. Solution:

Integrator are represented as 1/s in s-domain



As per the block diagram, the corresponding signal flow graph is drawn



One forward path $P_1 = 2/s^2$

The individual loops are,

$$L_1 = -\frac{3}{s}, \, L_2 = -\frac{12}{s} \text{ and } L_3 = -\frac{18}{s^2}$$

L_1 and L_2 are non-touching loops

$$L_1L_2 = \frac{36}{s^2}$$

The loops touches the forward path $\Delta_1 = 1$

The graph determinant is

$$\Delta = 1 - (L_1 + L_2 + L_3) + L_1L_2$$

$$= 1 + \frac{3}{s} + \frac{12}{s} + \frac{18}{s^2} + \frac{36}{s^2}$$

Applying mason's gain formula,

$$G(s) = \frac{Y(s)}{U(s)} = \frac{P_1\Delta_1}{\Delta}$$

$$= \frac{2/s^2}{1 + \frac{3}{s} + \frac{12}{s} + \frac{18}{s^2} + \frac{36}{s^2}}$$

$$= \frac{2}{s^2 + 15s + 54} = \frac{2}{(s+9)(s+6)}$$

$$= \frac{1}{27\left(1 + \frac{s}{9}\right)\left(1 + \frac{s}{6}\right)}$$

14. Solution:

Steady state value of response = 0.75

Input is unit-step. So steady state error

$$e_{ss} = 1 - 0.75 = 0.25$$

$$\text{Error} = E(s) = \frac{R(s)}{1+G(s)H(s)}$$

where,
$$G(s) = \frac{k}{(s+1)(s+2)}$$

$$H(s) = 1 \text{ and } R(s) = \frac{1}{s}$$

Steady state error using find value theorem,

$$e_{ss} = \underset{s\to 0}{\text{Lt}}\, sE(s) = \underset{s\to 0}{\text{Lt}} \frac{sR(s)}{1+G(s)H(s)}$$

$$= \underset{s\to 0}{\text{Lt}} \frac{s\cdot\frac{1}{s}}{1+\frac{k}{(s+1)(s+2)}} = \frac{1}{1+k/2}$$

$$\Rightarrow \quad 0.25 = \frac{1}{1+k/2}$$

$$\Rightarrow \quad 1+\frac{k}{2} = 4$$

$$\Rightarrow \quad k = 6$$

15. Solution:

$$G_1(s) = \frac{k}{s(s+1)(s+2)}$$

$$\Rightarrow \quad 1+\frac{k}{s(s+1)(s+2)} = 0$$

$$\Rightarrow \quad s(s^2+3s+2)+k = 0$$

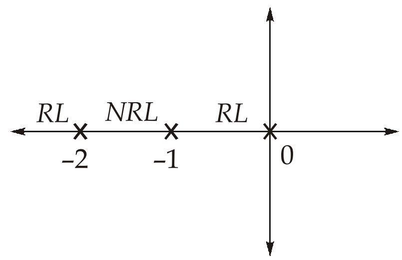
$$\Rightarrow \quad s^3+3s^2+2s+k = 0$$

$$\Rightarrow \quad k = -s^3-3s^2-2s \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{dk}{ds} = 0$$

$$\frac{dk}{ds} = -3s^2-6s-2 = 0$$

$$\Rightarrow \quad s = -0.422, -1.577$$



Only s = –0.422 lie on Root locus. Therefore breakaway point is s = –0.42.

16. Solution:

CE is
$$1 + G(s)H(s) = 0$$

$$\Rightarrow \quad 1 + \frac{s^2 + s + 1}{s^3 + 2s^2 + 2s + K} = 0$$

$$\Rightarrow \quad s^3 + 3s^2 + 3s + (1 + K) = 0$$

R.H. criteria:

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 3 & (1+K) \\ s^1 & 9-(1+K) & 0 \end{array}$$

For marginal stability,
$$9 - (1 + K) = 0 \quad \Rightarrow \quad K = 8$$

17. Solution:

$$G(s) = \frac{k}{s(s+3)(s+10)} \text{ and } H(s) = 1$$

Characteristic equation,
$$\Rightarrow \quad 1 + G(s)\,H(s) = 0$$

$$1 + \frac{k}{s(s+3)(s+10)} = 0$$

$$\Rightarrow \quad s(s + 3)(s + 10) + k = 0$$

$$\Rightarrow \quad s^3 + 13s^2 + 30s + k = 0$$

Routh-Array

$$\begin{array}{c|cc} s^3 & 1 & 30 \\ s^2 & 13 & k \\ s^1 & \dfrac{13\times 30 - k}{13} & \\ s^0 & k & \end{array}$$

According to Routh-Hurwitz criterion.
For a stable system, signs of first column do not change

$$k > 0 \quad \text{and} \quad \frac{13 \times 30 - k}{13} > 0$$

Therefore system to be stable $0 < k < 390$.

18. Solution:

$$T(s) = \left(\frac{1+3Ts}{1+Ts}\right)$$

Frequency at which $\angle T(j\omega)$ is maximum,

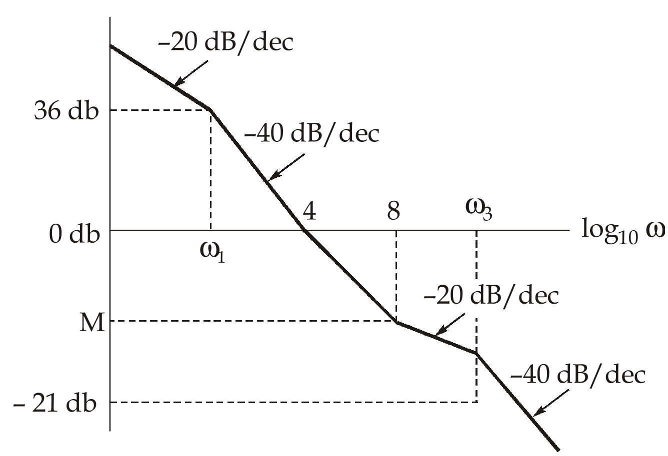
$$(\omega_m) = \frac{1}{T\sqrt{\alpha}}$$

$$\alpha = \frac{1}{1/3} = 3$$

$$\omega_m = \frac{1}{T\sqrt{3}} = \sqrt{\frac{1}{3T^2}}$$

PART-C

19. Solution:



From above the bode plot,
The slope in between (ω_1 and 4) is –40 dB/dec.

$$\therefore \qquad -40 = \frac{0-36}{\log 4 - \log \omega_1}$$

$$\therefore \qquad \omega_1 = 0.5 \text{ rad/sec}$$

The slope between (4 and 8) is (–40 dB/dec)

$$-40 = \frac{0 - M}{\log 4 - \log 8}$$

$$\therefore \quad M = -12 \text{ dB}$$

The slope in between (8 and ω_3) is –20dB/dec

$$-20 = \frac{-12 - (-21)}{\log 8 - \log \omega_3}$$

$$\therefore \quad \omega_3 = 22.547$$

For finding the value of (k)

$$20 \log k - 20 \log 0.5 = 36 \text{ dB}$$

$$\therefore \quad k = 31.54$$

From all the result, $$\left(\frac{T}{F}\right) = \frac{(31.54)\left(1+\frac{s}{8}\right)}{s\left(1+\frac{s}{0.5}\right)\left(1+\frac{5}{22.55}\right)} = \frac{45\,(s+8)}{s(s+0.5)(s+22.55)}$$

20. Solution:

(i) Selecting $x_1(t)$ and $x_2(t)$ as state variables.

$$\dot{x}_1(t) = \frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\dot{x}_2(t) = \frac{dx_2(t)}{dt} = -2x_2(t) + u(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t)$$

$$\dot{x} = AX + BU$$

So, $$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$y(t) = x_1(t) = [1 \quad 0]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$y = CX + DU$$

So, $$C = [1 \quad 0] \text{ and } D = 0$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}$$

$$\text{Transfer function} = C[sI - A]^{-1} B + D$$

$$= [1 \quad 0]\frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0$$

$$= \frac{[1 \quad 0]\begin{bmatrix} 2(s+2)+1 \\ s+3 \end{bmatrix}}{(s+2)(s+3)}$$

$$= \frac{2s+5}{(s+2)(s+3)} = \frac{2s+5}{s^2+5s+6}$$

(ii)

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+2} - \frac{1}{s+3} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$\text{State transition matrix} = \mathcal{L}^{-1}[(sI - A)^{-1}]$$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+2} - \frac{1}{s+3} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

❍❍❍❍