RPSC AEn-2024 Main Test Series

CIVIL ENGINEERING

Test 3

Test Mode : • Offline • Online

Subject : Structural Analysis

DETAILED EXPLANATIONS

1. Solution:

An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific section in a member as a unit load over that member.

2. Solution:

The equation of vertical rise in a parabolic arch is given by,

$$y = \frac{4h}{L^2}x(L-x)$$

where, L = Span of arch

h = crown height

3. Solution:

Castigliano's first theorem: "The partial derivative of the total strain energy in a structure with respect to the displacement at any one of the load points gives the value of corresponding load acting on the body in the direction of displacement".

Mathematically, $P_i = \dfrac{\partial U}{\partial \Delta_i}$

4. Solution:

Sway in portal frames may occur due to the following reasons:

(i) Unsymmetry in geometry of the frame.

(ii) Unsymmetry in loading.

(iii) Settlement of one end of a frame.

5. Solution:

Stiffness method (Displacement method) is suitable, when static indeterminacy is more than kinematic indeterminacy.

6. Solution:

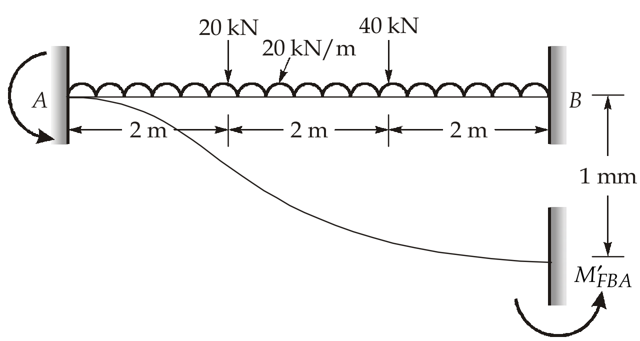
Slope deflection equation for moment at A in the member AB is given by,

$$M_{AB} = (M_F)_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

where, θ_A = slope at A, θ_B = slope at B.

Δ = Deflection of B w.r.t. tangent at A.

7. Solution:



$$M'_{FBA} = \frac{6EI\Delta}{L^2}$$

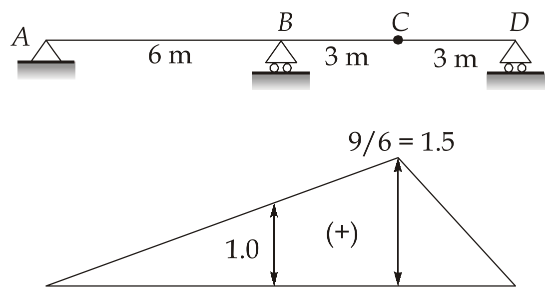
$$M_{FBA} = \underbrace{\frac{20 \times 2^2 \times 4}{6^2}}_{\text{due to point load 20 kN}} + \underbrace{\frac{40 \times 4^2 \times 2}{6^2}}_{\text{due to point load 40 kN}} + \overbrace{\frac{20 \times 6^2}{12}}^{\text{due to udl}} - \underbrace{\frac{6 \times 90 \times 10^3 \times 1 \times 10^{-3}}{6^2}}_{\text{due to support settlement}}$$

$\Rightarrow$ $M_{FBA} = 8.89 + 35.56 + 60 - 15$

$\Rightarrow$ $M_{FBA} \simeq 89.45$ kNm

8. Solution:

Applying Muller Breslau's principle to draw ILD for reaction at *B*. Give unit displacement at B in upward direction.

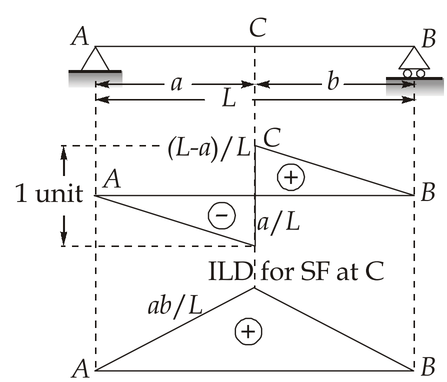


ILD for reaction at B

From above diagram,

Ordinate of ILD at *C* = 1.5

9. Solution:



ILD for BM at C

10. Solution:

A structure is externally unstable if all of its reactions are concurrent or parallel. As shown in figure (a), reactions V_1, H_1 and H_2 are concurrent and thus unstable. Hence, structure is externally unstable.

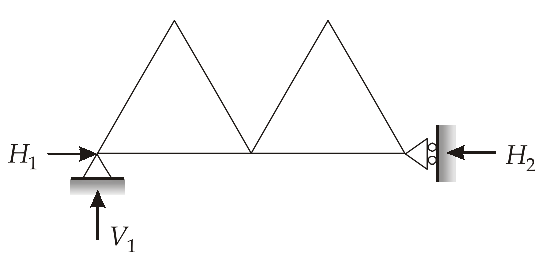


Fig. (a)

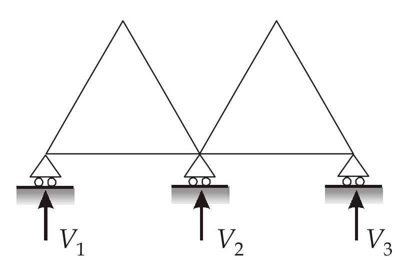


Fig. (b)

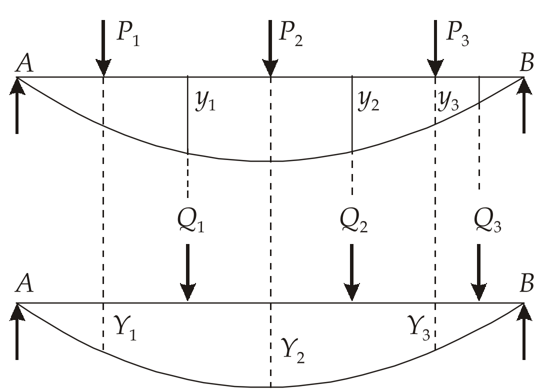
In figure (b), reactions V_1, V_2 and V_3 are parallel and unstable. Hence, this structure is also externally unstable.

11. Solution:

S. No.	Statically determinate structure	Statically indeterminate structure
1	Conditions of equilibrium are sufficient to analyze the structure completely.	Conditions of equilibrium are insufficient to analyze the structure completely.
2	The bending moment at a section or the force in any member is independent of the material of the components of the structure.	The bending moment at a section or the force in member is dependent upon the material of the components of the structure.
3	The bending moment at a section or the force in any member is independent of the cross-sectional areas of the components.	The bending moment at a section or the force in a member is dependent upon the cross-sectional areas of the components.
4	No stresses are caused due to temperature changes.	Stresses are generally caused due to temperature variations.
5	No stresses are caused due to lack of fit of members.	Stresses are caused due to lack of fit of members.

12. Solution:

Statement: For a structure whose material is elastic and follows Hooke's law and in which the supports are unyielding and the temperature is constant the work done by forces of system 1 acting through the displacements produced by forces of system 2 is equal to work done by forces of system 2 acting through the displacements produced by forces of system 1.



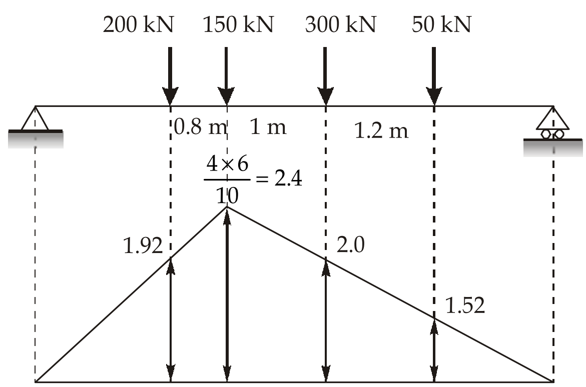
$$P_1Y_1 + P_2Y_2 + P_3Y_3 = Q_1y_1 + Q_2y_2 + Q_3y_3$$

13. Solution:

Allow the loads to cross the given section one after another and calculate average loads on AC and BC

Load crossing section C	Avg. load on AC	Avg. load on BC	Load remarks
50 kN	$\frac{650}{4}$	$\frac{50}{6}$	$AC > BC$
300 kN	$\frac{350}{4}$	$\frac{350}{6}$	$AC > BC$
150 kN	$\frac{200}{4}$	$\frac{500}{6}$	$BC > AC$

As 150 kN passes the section, average load on BC becomes higher than AC.



$$\text{Maximum BM at } D = 200 \times 1.92 + 150 \times 2.4 + 300 \times 2 + 50 \times 1.52$$
$$= 1420 \text{ kNm}$$

14. Solution:

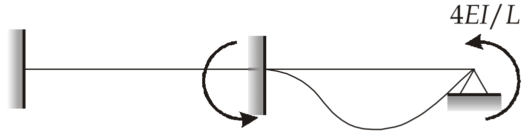
Joint	Member	Relative stiffness	Total stiffness	DF
B	BA	$I/4$	$\frac{5I}{6}$	$\frac{3}{10}$
	BC	$I/4$		$\frac{3}{10}$
	BD	$I/3$		$\frac{2}{5}$

$$M_{BD} = \frac{2}{5} \times 60 \text{ kNm}$$

$$\therefore \quad M_{DB} = \frac{2}{5} \times \frac{60}{2} = 12 \text{ kNm}$$

15. Solution:

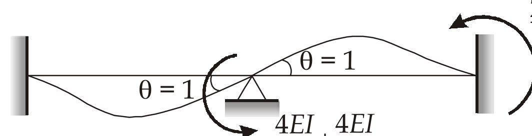
Give unit displacement in the direction (1)



$$k_{11} = \frac{4EI}{L}$$

$$k_{21} = \frac{2EI}{L}$$

Give unit displacement in the direction (2)



$$k_{12} = \frac{2EI}{L}; \qquad k_{22} = \frac{8EI}{L}$$

$$\therefore \quad \text{Stiffness matrix } [S] = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{8EI}{L} \end{bmatrix} = \frac{2EI}{L}\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$$

16. Solution:

Given:Supports are at same level (i.e., θ = 0°)

Span of cable, L = 130 m, Sag of cable, h = 15 m

Intensity of UDL, w = 1.5 kN/m, $\alpha = 5 \times 10^{-6}/°C$, t = 30°

$$\text{Horizontal reaction, } H = \frac{wL^2}{8H} = \frac{1.5 \times 130^2}{8 \times 15} = 211.25 \text{ kN}$$

$$\text{Change in horizontal reaction, } \frac{\delta H}{H} = -\frac{3}{16} \times \frac{L^2}{h^2} \times \alpha \times t$$

$$= -\frac{3}{16} \times \frac{130^2}{15^2} \times 5 \times 10^{-6} \times 30 = -2.1125 \times 10^{-3}$$

$$\therefore \quad \delta H = -2.1125 \times 10^{-3} \times 211.25 \text{ kN} = -0.446 \text{ kN}$$

There is a decrease in horizontal reaction.

17. Solution:

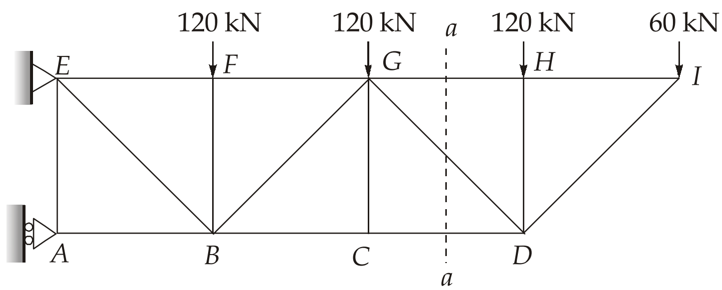
Following are the assumptions made in truss analysis:

(i) The centroidal axis of each member coincides with the line connecting the centers of the adjacent members and the members carry axial force only.

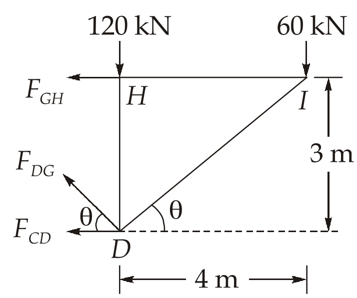
(ii) All members are connected only at their ends by frictionless hinges in plane trusses.

(iii) All loads and support reactions are applied only at the joints.

18. Solution:



Cut a section a-a, passing through the members CD, DG and GH.
Considering right part of the truss.



$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

Applying equilibrium equation,

$$\Sigma M_D = 0$$

$$-F_{GH} \times 3 + 60 \times 4 = 0$$

$$\Rightarrow \quad F_{GH} = 80 \text{ kN (T)}$$

$$\Sigma F_V = 0$$

$$F_{DG} \sin\theta - 120 - 60 = 0$$

$$\Rightarrow \quad F_{DG} = \frac{180}{\sin\theta} = \frac{180}{(3/5)} = 300 \text{ kN}$$

$$\therefore \quad F_{DG} = 300 \text{ kN(T)}$$

$$\Sigma F_H = 0$$

$$-F_{GH} - F_{CD} - F_{DG} \times \cos\theta = 0$$

$$\Rightarrow \quad -80 - F_{CD} - 300 \times \frac{4}{5} = 0$$

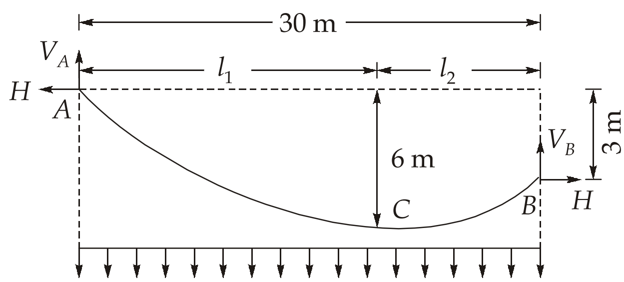
$$\Rightarrow \quad F_{CD} = -320 \text{ kN}$$

$$\therefore \quad F_{CD} = 320 \text{ kN(C)}$$

Hence, force in members CD, DG and GH are respectively, 320 kN (C), 300 kN(T) and 80 kN(T).

19. Solution:

Considering equilibrium of part *AC* of the cable,



$$l_1 = l\left(\frac{\sqrt{h_1}}{\sqrt{h_2}+\sqrt{h_2}}\right) = 30\left(\frac{\sqrt{6}}{\sqrt{6}+\sqrt{3}}\right) = 17.57 \text{ m}$$

$$l_2 = l\left(\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right) = 30\left(\frac{\sqrt{3}}{\sqrt{6}+\sqrt{3}}\right) = 12.43 \text{ m}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad V_A + V_B = 3 \times 30 = 90 \text{ kN} \qquad \text{...(i)}$$

Taking moments about *C*, (considering left portion)

$$V_A \times 17.57 - H \times 6 - 3 \times \frac{17.57^2}{2} = 0$$

$$\Rightarrow \quad V_A = \left(\frac{6H}{17.57} + \frac{3 \times 17.57}{2}\right) \qquad \text{...(ii)}$$

Taking moment aboutC, (Considering right portion)

$$V_B \times 12.43 - H \times 3 - 3 \times \frac{12.43^2}{2} = 0$$

$$\Rightarrow \quad V_B = \left(\frac{3H}{12.43} + \frac{3 \times 12.43}{2}\right) \qquad \text{...(iii)}$$

On putting the values of V_A and V_B in equation (i), we get

$$\left(\frac{6H}{17.57}+\frac{3\times17.57}{2}\right)+\left(\frac{3H}{12.43}+\frac{3\times12.43}{2}\right) = 90$$

$$\Rightarrow \quad 0.5828H = 45$$

$$\Rightarrow \quad H = 77.21 \text{ kN}$$

From equation (ii)

$$V_A = \frac{6\times77.21}{17.57}+\frac{3\times17.57}{2} = 52.72 \text{ kN}$$

From equation (iii)

$$V_B = \frac{3\times77.21}{12.43}+\frac{3\times12.43}{2} = 37.28 \text{ kN}$$

Maximum tension will occur at highest point i.e. A.

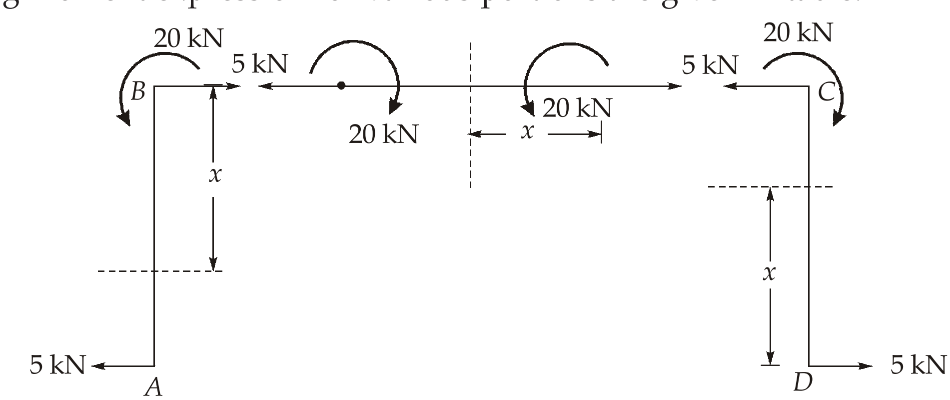
$$T_{max} = \sqrt{V_A^2+H^2} = \sqrt{(52.72)^2+(77.21)^2} = 93.50 \text{ kN}$$

Minimum tension will occur at lowest point i.e. C.

$$T_{min} = H = 77.21 \text{ kN}$$

20. Solution:

The bending moment expression for various portions are given in table.



Portion	CD	BC	AB
Origin	D	C	B
Limit	$0-4$	$0-3$	$0-4$
M_x (kNm)	$-5x$	-20	$-20+5x$

$$\text{Total strain energy, } U = \int_0^4 \frac{(-5x)^2}{2EI}dx+\int_0^3 \frac{(20)^2}{2EI}dx+\int_0^4 \frac{(-20+5x)^2}{2EI}dx$$

$\Rightarrow$ $$U = \frac{25}{2EI}\left[\frac{x^3}{3}\right]_0^4 + \frac{400}{2EI}[x]_0^3 + \frac{1}{2EI}\left[400x - 200\frac{x^2}{2} + \frac{25x^3}{3}\right]_0^4$$

$\Rightarrow$ $$U = \frac{266.67}{EI} + \frac{600}{EI} + \frac{1}{2EI}\left[1600 - 1600 + \frac{25\times 64}{3}\right]$$

$\Rightarrow$ $$U = \frac{1133.33}{EI} \quad \text{...(i)}$$

Now, $$\text{Work done} = \frac{1}{2}\times P\times\Delta = \frac{1}{2}\times 5\Delta = 2.5\Delta \quad \text{...(ii)}$$

From equation (i) and (ii)

$$2.5\Delta = \frac{1133.33}{EI}$$

$\Rightarrow$ $$2.5\Delta = \frac{1133.33}{8000} = 0.14167$$

$\Rightarrow$ $$\Delta = 0.05667 \text{ m} = 56.67 \text{ mm}$$

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