RPSC AEn-2024 Main Test Series

ELECTRICAL ENGINEERING | **Test 2**

Test Mode : • Offline • Online

Subjects : Field Theory + Electrical Materials

DETAILED EXPLANATIONS

PART-A

1. Solution:

Coulomb's law states that the electrostatic force between two point charges:

1. Acts along the straight that line joining the two charges.
2. Is directly proportional to the product of the two charges.
3. Is inversely proportional to the square of the distance between them.

2. Solution:

The total electric displacement or electric flux through any closed surface surrounding charges is equal to the net positive charge enclosed by that surface,

$$\psi = \oint_s \vec{D} \cdot d\vec{s} = Q = \int_v \rho\, dV$$

This is the integral form of Gauss's law.

3. Solution:

Magnetic field density due to infinite wire,

$$B = \frac{\mu_0 I}{2\pi a}$$

$$2 \times 10^{-6} = \frac{(4\pi)\times(10^{-7})}{2\pi}\times\frac{I}{1}$$

$$I = 10 \text{ A}$$

4. Solution:

Divergence of a vector $\bar{A}$ is given by

$$\nabla\cdot\bar{A} = \frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

$$\nabla\cdot\bar{A} = 2x + 12y + 3z^2$$

$$(\nabla\cdot\bar{A})_{(2,4,1)} = (2)\,(2) + 12(4) + 3(1)^2$$

$$= 55$$

5. Solution:

Electric field due to infinite line charge with charge density λ at a distance r

$$|\bar{E}| = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$|\bar{E}| = \frac{1}{2\pi\varepsilon_0(1)} = \frac{1}{2\pi\varepsilon_0}\text{V/m}$$

6. Solution:

$$P = N\,\alpha\,E = \varepsilon_0(\varepsilon_r - 1)E$$

$$= \epsilon_0\chi_e E$$

$$\alpha = \frac{\varepsilon_0(\varepsilon_r-1)}{N} = \frac{\epsilon_0\,\chi_e}{N} = \frac{8.854\times10^{-12}\times4.35\times10^{-4}}{2.7\times10^{25}}$$

$$= 1.43\times10^{-40}\ \text{F-m}^2$$

7. Solution:

A ferroelectric material is one that exhibits spontaneous polarization even when no external electric field is applied, and this polarization can be reversed by applying an external electric field.

8. Solution:

The applications of ferrites can be put broadly into four categories:

(i) Ferrites for permanent magnets - hard ferrites.

(ii) Ferrites for transformers and inductors - soft ferrites.

(iii) Data storage - rectangular loop ferrites.

(iv) Microwave applications = ferrites and garnets.

9. Solution:

The susceptibility decreases with rise in temperature because thermal agitation increases and disturbs the magnet's alignment.

10. Solution:

Dielectric losses increase with frequency, humidity, temperature, and applied electric field due to increased polarization lag higher leakage current and enhanced conduction paths within the dielectric.

PART-B

11. Solution:

$$Q = 20 \times 10^{-6} \text{ C},$$
$$V = 10 \times 10^{3} \text{ V},$$
$$\varepsilon_r = 10$$
$$\varepsilon_0 = 8.854 \times 10^{-12}$$
$$d = 5 \times 10^{-4} \text{ m}$$
$$C = \frac{Q}{V} = \frac{20\times10^{-6}}{10\times10^{3}} = 2\times10^{-9}\,\text{Farad}$$
$$\because \quad C = \frac{\varepsilon_r\varepsilon_0 A}{d}$$
$$A = \frac{Cd}{\varepsilon_0\varepsilon_r} = \frac{2\times10^{-9}\times5\times10^{-4}}{10\times8.854\times10^{-12}} = 10.294\times10^{-3}\,\text{m}^2$$

12. Solution:

Given, $\chi_m = -0.2 \times 10^{-5}$

and $H = 10^4 \text{ Am}^{-1}$

magnetization vector, $M = \chi_m \text{H}$

$$= -0.2 \times 10^{-5} \times 10^4$$
$$= -0.02 \text{ Am}^{-1}$$

Magnetic flux density, $B = \mu_0(H + M)$

$$= 4\pi \times 10^{-7} \times [10^4 - 0.02]$$
$$= 0.0116 \text{ T}$$

13. Solution:

Given, $\phi = 2.4 \times 10^{-5}$ Wb

and $A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$

The magnetic flux density is given by

$$B = \frac{\phi}{A} = \frac{2.4\times10^{-5}}{0.2\times10^{-4}} = 1.2\,\text{Wb/m}$$

The permeability is given by

$$\mu = \frac{B}{H} = \frac{1.2}{500} = 2.4\times10^{-3}\,\text{H/m}$$

The susceptibility is given by,

$$\chi_m = \frac{\mu}{\mu_0} - 1$$

$$= \frac{2.4\times10^{-3}}{4\times3.14\times10^{-7}} - 1 = 1909$$

14. Solution:

Soft ferromagnetic materials are easily magnetized and demagnetized, have low coercive field and show a narrow hysteresis loop, resulting in very low energy loss. They are used in transformer cores, motors, generators, and sensors.

Example: Silicon steel and iron-cobalt alloys.

Hard ferromagnetic materials are difficult to magnetize and demagnetize, possess high coercivity and exhibit a wide hysteresis loop. They retain strong magnetism and are used to make permanent magnets.

Example: Alnico and rare-earth metal alloys.

15. Solution:

Here, $\vec{R}_{21} = (-1, 1, -3) - (3, 1, 0)$

$$= (-4\hat{i} - 3\hat{k})$$

$\therefore$ $$\hat{a}_{21} = \frac{(-4\hat{i} - 3\hat{k})}{\sqrt{(-4)^2 + (-3)^2}} = \frac{-4\hat{i} - 3\hat{k}}{5}$$

Hence, the force on the charge Q_1 is

$$\vec{F}_{21} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2}\hat{a}_{21}$$

$$= \frac{50\times10^{-6}\times10\times10^{-6}}{4\pi\times\frac{10^{-9}}{36\pi}\times(5)^2}\times\frac{(-4\hat{i}-3\hat{k})}{5}$$

$$= 0.18(-0.8\hat{i}-0.6\hat{k})\text{N}$$

The force has a magnitude 0.18 N and a direction gives by the unit vector $(-0.8\hat{i}-0.6\hat{k})$

In component from

$$\vec{F}_{21} = (-0.144\hat{i}-0.108\hat{k})\text{N}$$

16. Solution:

Electric field lines start from positive charges and end on negative charges showing the direction of the electric field. They never intersect because a point cannot have two different field directions.

The density of lines indicates field strength. Closer the lines, stronger the field. Field lines are perpendicular to the surface of a charged conductor. These properties help visualize electric field distribution, understand force on charges and analyze conductor behaviour under electrostatic conditions.

17. Solution:

Gauss' law can be derived from Coulomb's law which states that the electric field due to a stationary point charge is:

$$\vec{E} = \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r$$

Using the expression from Coulomb's law, we get the total field at $\vec{r}$ by using an integral to sum the field at $\vec{r}$ due to the infinitesimal charge at each other point $\vec{r}$ in space, to give

$$\vec{E} = \frac{1}{4\pi\varepsilon}\int_v \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\text{dV}$$

where ρ is the charge density. If we take the divergence of both sides of this equation with respect to $\vec{r}$, and use the known theorem

$$\nabla\cdot\frac{\vec{r}'}{|\vec{r}'|^3} = 4\pi\delta(r')$$

where $\delta(r')$ is the Dirac delta function, the result is

$$\nabla\cdot\vec{E} = \frac{1}{\varepsilon}\int_v \rho(\vec{r}')\delta(r-\vec{r}')\delta(\vec{r}-\vec{r}')\text{dV}$$

using the shifting property of the Dirac delta function, we get

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

which is the differential form of Gauss' law.

18. Solution:

The properties of equipotential surface are:

- The electric field lines are perpendicular to the equipotential surfaces and are directed from higher to lower potentials.
- No work is required to move a particle along an equipotential surface.
- The equipotential surfaces for a flat surface with uniform charge distribution are planes parallel to the surface.

PART-C

19. Solution:

Non-uniform surface charge density of $\rho_s = \frac{5\rho}{\rho^2 + 1} \text{nC/m}^2$ at z = 2 plane when $\rho < 5$ and $\rho_s = 0$ when $\rho > 5$.

(i) The electric flux leaving the circular region $\rho < 5$, $z = 2$ is

$$Q = \int_0^{2\pi}\int_0^5 \rho_s \cdot \rho \, d\rho \, d\phi$$

$$= \int_0^{2\pi}\int_0^5 \frac{5\rho^2}{\rho^2 + 1} \cdot d\rho \, d\phi$$

$$= 2\pi\left[5\rho - 5\tan^{-1}\rho\right]_0^5 = 113.9 \text{ nC}$$

(ii) The flux crossing the z-plane in the direction $-\hat{a}_z$ is half the flux coming out of circular region $\rho < 5$.

$$\therefore \quad \psi = \frac{1}{2} \times 113.9 = 56.98 \simeq 57 \text{ nC}$$

(iii) The electric flux leaving the cylinder $\rho = 3$

$$\psi = \int_0^{2\pi}\int_0^3 \rho_s \rho \, d\rho \, d\phi$$

$$= \int_0^{2\pi}\int_0^{3} \frac{5\rho^2}{\rho^2+1}\cdot d\rho\, d\phi$$

$$= 2\pi\left[5\rho - 5\tan^{-1}\rho\right]_0^3 = 54.98 \text{ nC}$$

20. Solution:

Superconductors are classified into two groups: Type-I and Type-II.

Type-I:

Superconductors that show perfect diamagnetism up to the critical field H_c and go to the normal state as shown in figure (a) are called type-I superconductors.

The critical field of these superconductors are low and have small transistion temperature.

Type-I superconductors are not suitable for high field applications. They are also called soft superconductors (or ideal).

Soft metals such as lead or indium belong to this group.

Type-II:

Hard metals and alloys have different magnetization characteristics as shown schematically in figure (b), and are called type-II superconductors (or hard superconductors).

They are characterized by high transition temperature, high critical field and show incomplete Meissner effect, breakdown of Silsbee's rule, and broad transition region.

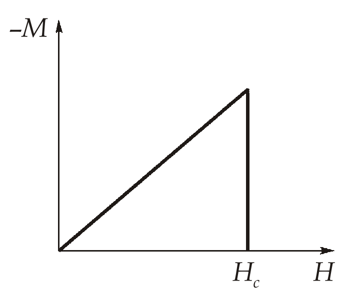

Fig. (a): Type-I Superconductor

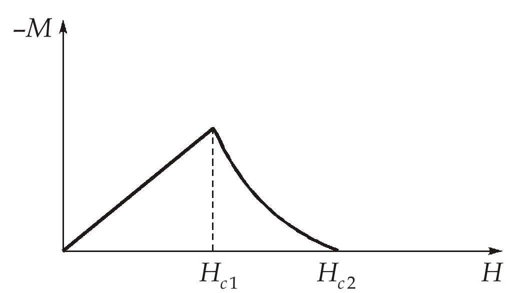

Fig. (b): Type-II Superconductor

Superconductivity is observed for dc and upto radio frequencies. It is not observed for higher frequencies. For a superconductor the resistance is zero only when the current is steady or varies slowly.

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