RPSC AEn-2024 Main Test Series

CIVIL ENGINEERING | **Test 2**

Test Mode : • Offline • Online

Subject : Strength of Materials

DETAILED EXPLANATIONS

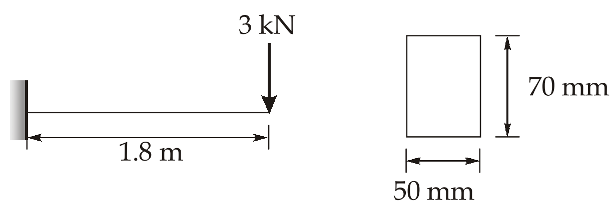
1. Solution:

Tenacity is the ability of a material to withstand tensile force without breaking. It is commonly expressed as the maximum tensile stress a material can bear before failure.

2. Solution:

Fretting is the surface damage that occurs when two surfaces in contact undergo slight periodic relative motion under load. It leads to the formation of oxidised debris, pitting, and surface cracks.

3. Solution:



$$\text{Maximum bending stress, } \sigma = \frac{\text{Maximum bending moment}}{\text{Section modulus}}$$

$$\Rightarrow \quad \sigma = \frac{3\times1000\times1.8\times1000}{\frac{50\times70^2}{6}} = 132.245 \text{ N/mm}^2$$

4. Solution:

True stress is related to engineering stress and engineering strain by the following expression:

$$\text{True stress} = \text{Engineering stress} \times (1 - \text{Engineering strain})$$

$$\Rightarrow \quad \sigma_t = \sigma_{Engg}(1 - \in_{Engg})$$

5. Solution:

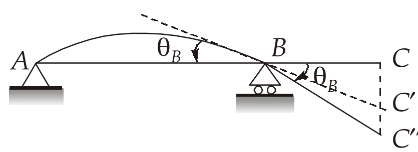
$$\text{Modulus of resilience} = \frac{\sigma_y^2}{2E}$$

$$\sigma_y = \frac{\text{Load}}{\text{Area}} = \frac{28 \times 1000}{120} = 233.33 \text{ N/mm}^2$$

$$\text{Modulus of resilience} = \frac{233.33^2}{2 \times 200 \times 1000} = 0.1361 \text{ N-mm/mm}^3$$

6. Solution:

Deflection profile is as shown below.



7. Solution:

As per the maximum principal strain theory (Saint Venant's theory), failure occurs when the principal tensile strain in the material reaches the elastic limit strain in simple tension, or when the minimum principal strain reaches the elastic limit strain in simple compression.

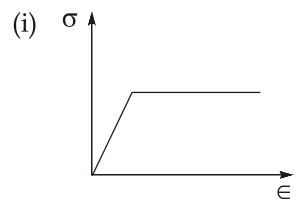
8. Solution:

A shear center is the point in a cross-section where a transverse load must act to produce bending without any twisting.

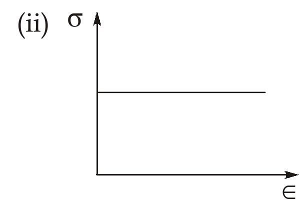
9. Solution:

Proof stress is the stress necessary to cause a permanent extension equal to a definite percentage (generally 0.2%) of the original gauge length.

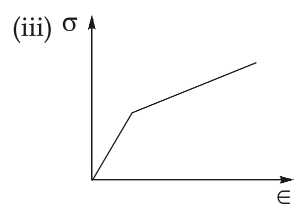
10. Solution:



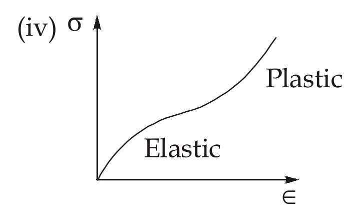
Elastoplastic material



Ideal plastic material



Elastoplastic material
with strain hardening



Viscoelastic material

11. Solution:

Principal stresses are given by,

$$\sigma_{P1} = \frac{E}{1-\mu^2}(\in_{P1} + \mu \in_{P2})$$

and
$$\sigma_{P2} = \frac{E}{1-\mu^2}(\in_{P2} + \mu \in_{P1})$$

$\therefore$
$$\sigma_{P1} = \frac{200\times 10^3}{1-0.3^2}(280\times 10^{-6} + 0.3\times 95\times 10^{-6})$$

$\Rightarrow$
$$\sigma_{P1} = 67.8 \text{ N/mm}^2$$

and,
$$\sigma_{P2} = \frac{200\times 10^3}{1-0.3^2}(95\times 10^{-6} + 0.3\times 280\times 10^{-6})$$

$$\sigma_{P2} = 39.34 \text{ N/mm}^2$$

12. Solution:

In pure torsion, the maximum tensile stresses act on planes inclined at 45° to the shaft axis. Brittle materials are weak in tension; hence the shaft cracks along the 45° plane where the tensile stress reaches its maximum value. These 45° planes wrapped around the circular shaft form a helical shaft.

13. Solution:

$$\text{Torque required, } T = \frac{60P}{2\pi N} = \frac{60 \times 120 \times 10^6}{2 \times \pi \times 250} = 4.584 \times 10^6 \text{ N-mm}$$

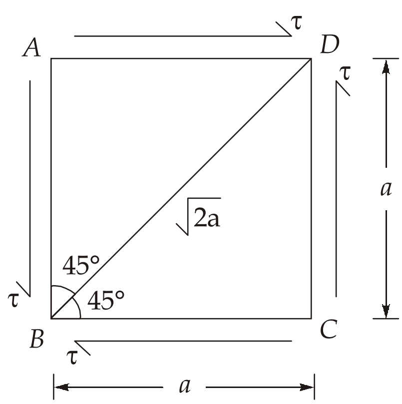
$$\text{We know, } \frac{\tau}{R} = \frac{T}{J} \Rightarrow \quad T = \frac{\tau J}{R}$$

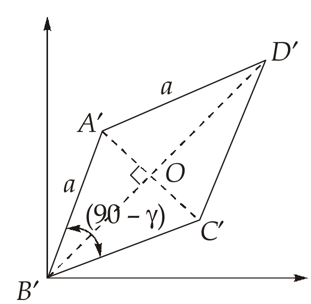
$$\Rightarrow \quad \tau = \frac{TR}{J} = \frac{4.584 \times 10^6 \times \frac{70}{2}}{\frac{\pi}{32} \times 70^4} = 68.06 \text{ N/mm}^2$$

$$\text{Angle of twist, } \theta = \frac{\tau L}{GR} \qquad \left(\because \frac{\tau}{R} = \frac{G\theta}{L}\right)$$

$$\Rightarrow \quad \theta = \frac{68.06 \times 5000}{90 \times 10^3 \times \frac{70}{2}} = 0.108 \text{ rad}$$

14. Solution:





Let normal strain along diagonal BD be $\in$, elongation due to pure shear be δ and shear strain be γ.

$$\therefore \quad \delta = \in \times \sqrt{2}a \qquad \text{...(i)}$$

$$\text{In } \Delta A'OB', \quad \cos\left(45° - \frac{\gamma}{2}\right) = \frac{(\sqrt{2}a + \delta)/2}{a}$$

$$\Rightarrow \quad \cos 45° \cos\frac{\gamma}{2} + \sin 45° \sin\frac{\gamma}{2} \simeq \frac{1}{\sqrt{2}} + \frac{\delta}{2a}$$

$$\cos\frac{\gamma}{2} = 1 \text{ and } \sin\frac{\gamma}{2} \simeq \frac{\gamma}{2} \qquad [\because \gamma \text{ is very small}]$$

$$\therefore \quad \frac{1}{\sqrt{2}}(1) + \frac{1}{\sqrt{2}}\left(\frac{\gamma}{2}\right) = \frac{1}{\sqrt{2}} + \frac{\delta}{2a}$$

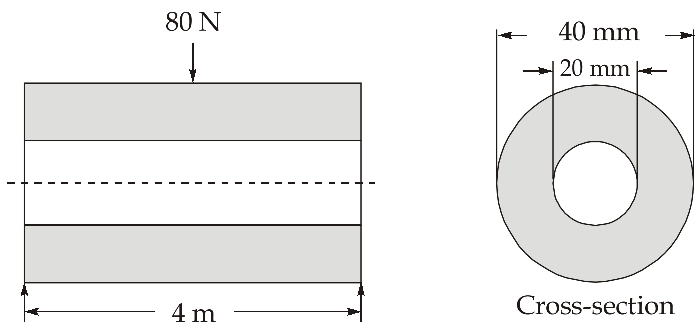
$$\Rightarrow \quad \frac{1}{\sqrt{2}}\left(1+\frac{\gamma}{2}\right) = \frac{1}{\sqrt{2}}+\frac{\delta}{2a}$$

$$\Rightarrow \quad \frac{\gamma}{2} = \frac{\delta}{\sqrt{2}a}$$

From (*i*),
$$\frac{\delta}{\sqrt{2}a} = \in$$

$$\therefore \quad \frac{\gamma}{2} = \in$$

15. Solution:



Maximum bending moment,

$$M_{max} = \frac{WL}{4} = \frac{80\times 4}{4} = 80\,\text{Nm}$$

Moment of inertia of pipe,

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$\Rightarrow \quad I = \frac{\pi}{64}(40^4 - 20^4) = 11.781 \times 10^4 \text{ mm}^4$$

Maximum bending stress is given by

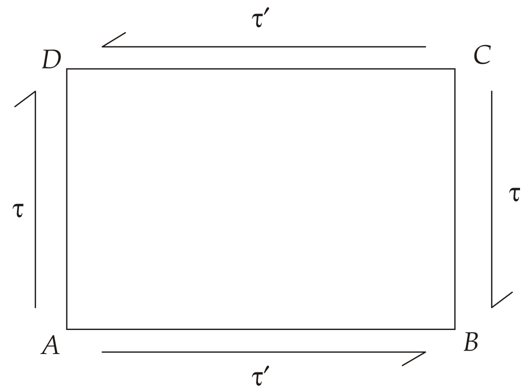
$$\sigma_{max} = \frac{M}{I} y_{max}$$

$$\Rightarrow \quad \sigma_{max} = \frac{80\times 10^3}{11.781\times 10^4}\times\left(\frac{40}{2}\right)$$

$$\Rightarrow \quad \sigma_{max} = 13.58 \text{ N/mm}^2$$

16. Solution:

Whenever a shear stress τ is applied on parallel surfaces of a body then to keep the body in equilibrium a shear stress τ' is induced on remaining parallel surfaces of body. These stresses form a couple. As shown in figure, the couple formed due to shear stress τ produces clockwise moment. For equilibrium this couple is balanced by couple developed by τ'. This resisting shear stress τ' is known as complementary shear stress.



$$\text{Couple produced by } \tau = (\tau BC) \times AB$$

$$\text{Couple produced by } \tau' = (\tau' CD) \times BC$$

For equilibrium,

$$(\tau BC) \times AB = (\tau' CD) \times BC$$

$$\Rightarrow \quad \tau = \tau' \qquad (\because AB = CD)$$

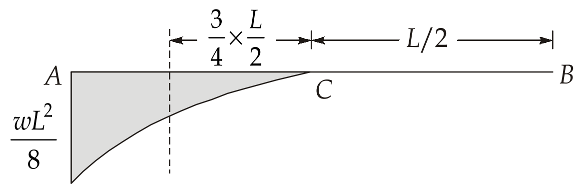
17. Solution:

If a length of a beam is subjected to a constant bending moment and no shear force (i.e., zero shear force), then the stresses will be set up in that length of the beam due to B.M. only and that length of the beam is said to be in pure bending or simple bending. The stresses set up in that length of beam are known as bending stresses.

The following are the important assumptions:

1. The material of the beam is homogeneous (the material is same throughout) and isotropic (elastic properties in all directions are same).
2. The value of Young's modulus of elasticity is the same in tension and compression.
3. The transverse sections which were plane before bending remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into simple circular arcs with a common center of curvature.
5. The radius of curvature is large compared with the dimensions of the cross-section of beam.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

18. Solution:



From the moment area theorem,

$$\theta_B = \text{Area of } \left(\frac{M}{EI}\right) \text{ diagram between A and B}$$

$\Rightarrow$
$$\theta_B = -\left(\frac{1}{3}\frac{wL^2}{8}\times\frac{L}{2}\right)\times\frac{1}{EI} = -\frac{wL^3}{48EI} = \frac{wL^3}{48EI} \text{ (clockwise)}$$

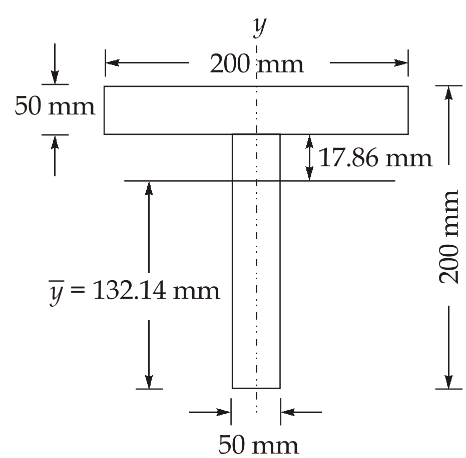
Vertical deflection at B is given by,

$$\Delta_B = \text{Moment of } \left(\frac{M}{EI}\right) \text{ diagram about } B$$

$\Rightarrow$
$$\Delta_B = \left[-\frac{wL^3}{48EI}\right]\times\left[\frac{3}{4}\left(\frac{L}{2}\right)+\frac{L}{2}\right]$$

$\Rightarrow$
$$\Delta_B = -\frac{7wL^4}{384EI} = \frac{7wL^4}{384EI}(\downarrow)$$

19. Solution:



Distance of neutral axis from the bottom edge of T-section is,

$$\bar{y} = \frac{200\times50+(150\times50)\times75}{200\times50+150\times50}$$

$\Rightarrow$
$$\bar{y} = 132.14 \text{ mm}$$

Area moment of inertia about NA is,

$$I_{NA} = \frac{200\times 50^3}{12} + 200\times 50(175-132.14)^2 + \frac{50\times 150^3}{12} + 50\times 150\left(132.14-\frac{150}{2}\right)^2$$

$\Rightarrow$ $$I_{NA} = 5.9\times 10^7 \text{ mm}^4$$

Shear stress at the neutral axis is,

$$\tau_{NA} = \frac{V}{I_{NA}b}(A\bar{y})$$

$\Rightarrow$ $$\tau_{NA} = \frac{180\times 10^3}{5.9\times 10^7\times 50}\left[50\times 132.14\left(\frac{132.14}{2}\right)\right]$$

$\Rightarrow$ $$\tau_{NA} = 26.64 \text{ N/mm}^2$$

Now, shear stress in the web just at the junction of web and flange is,

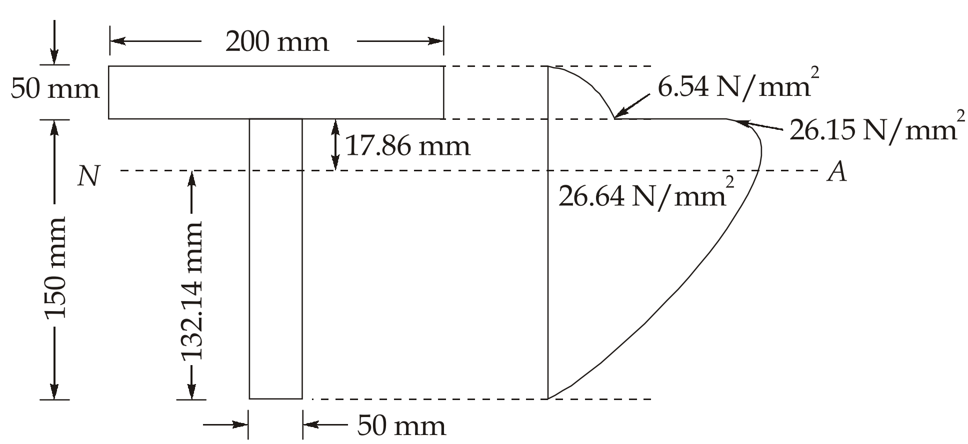
$$\tau_{web} = \frac{180\times 10^3}{5.9\times 10^7\times 50}\left[(200\times 50)\left(\frac{50}{2}+17.86\right)\right]$$

$\Rightarrow$ $$\tau_{web} = 26.15 \text{ N/mm}^2$$

Shear stress in the flange just at the junction of the flange and web is,

$$\tau_{flange} = \frac{180\times 10^3}{5.9\times 10^7\times 200}\left[(200\times 50)\left(\frac{50}{2}+17.86\right)\right]$$

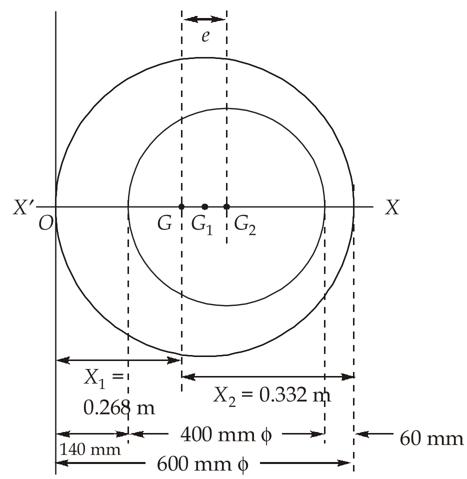
$$\tau_{flange} = 6.54 \text{ N/mm}^2$$



Beam cross-section   Shear stress distribution

20. Solution:

Given: $D_1 = 600$ mm, $D_2 = 400$ mm, $P = 2000$ kN



From figure, $GG_1 = OG_1 - X_1$

⇒ $GG_1 = 300 - 268 = 32$ mm

Now, $GG_2 = X_2 - 200 - 60$

⇒ $GG_2 = 332 - 260 = 72 \text{ mm} = e$

$$\text{Area, } A = \frac{\pi}{4}\left(600^2 - 400^2\right) = 157079.6327 \text{ mm}^2$$

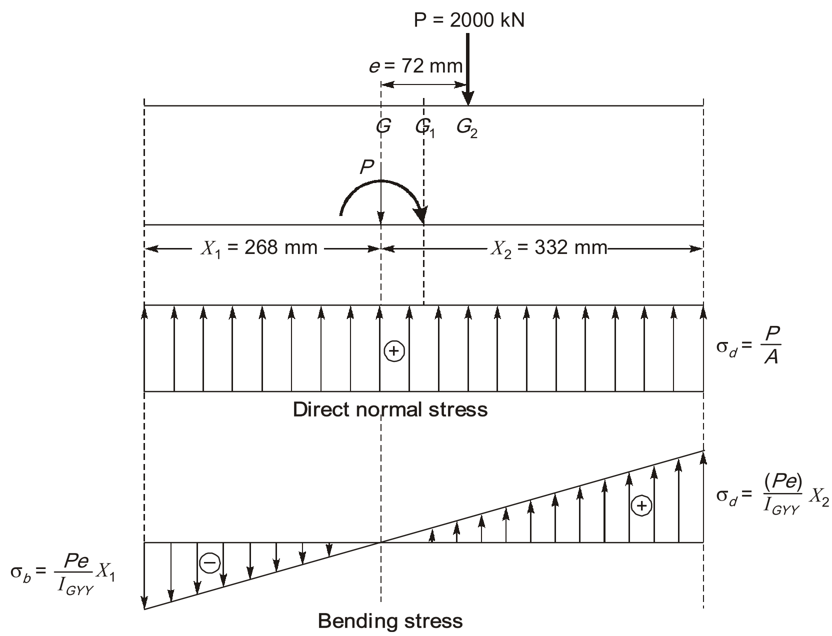
Area moment of inertia about centroidal y-axis is,

$$I_{Gyy} = \left[\frac{\pi}{64}D_1^4 + \frac{\pi}{4}D_1^2\left(GG_1\right)^2\right] - \left[\frac{\pi}{64}D_2^4 + \frac{\pi}{4}D_2^2\left(GG_2\right)^2\right]$$

⇒ $$I_{Gyy} = \left(\frac{\pi}{64}(600)^4 + \frac{\pi}{4}(600)^2\,32^2 - \frac{\pi}{64}\times 400^4 - \frac{\pi}{4}\times 400^2 \times 72^2\right)$$

⇒ $$I_{Gyy} = 474.318 \times 10^7 \text{ mm}^4$$

Now,



Maximum stress:

$$\sigma_{max} = \sigma_d + \sigma_b = \frac{P}{A} + \frac{Pe}{I_{Gyy}} X_2$$

$$\Rightarrow \quad \sigma_{max} = \frac{2000 \times 10^3}{157079.6327} + \frac{2000 \times 10^3 \times 72}{474.318 \times 10^7} \times 332$$

$$\Rightarrow \quad \sigma_{max} = 22.81 \text{ MPa (Compressive)}$$

Minimum stress:

$$\sigma_{min} = \sigma_d - \sigma_b = \frac{P}{A} - \frac{Pe}{I_{Gyy}} X_1$$

$$\Rightarrow \quad \sigma_{min} = \frac{2000 \times 10^3}{157079.6327} - \frac{2000 \times 10^3 \times 72}{474.318 \times 10^7} \times 268$$

$$\Rightarrow \quad \sigma_{min} = 4.596 \text{ MPa (Compressive)}$$

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