RPSC AEn-2024 Main Test Series

MECHANICAL ENGINEERING

Test 1

Test Mode : • Offline • Online

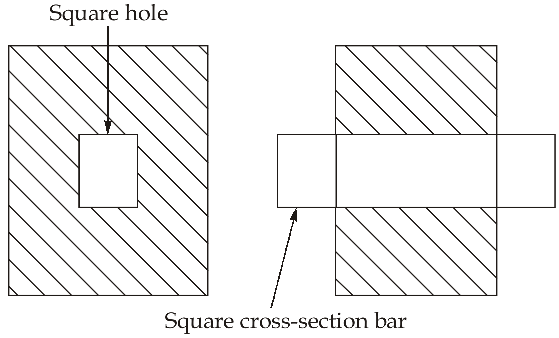
Subjects : Theory of Machines

DETAILED EXPLANATIONS

1. Solution:

Completely Constrained Motion: When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, the motion is known as completely constrained motion.

Examples: Motion of piston inside a cylinder in steam engine, the motion of square bar in a square hole is completely constrained motion as it follows only one direction of motion irrespective of the force applied.



2. Solution:

$$F = 3(n - 1) - 2j - h$$
$$= 3(7 - 1) - 2(12) - 1$$
$$= 18 - 24 - 1 = -7$$

3. Solution:

The inversions of a double slider crank mechanism are

(i) Elliptical Trammel

(ii) Scotch Yoke Mechanism

(iii) Oldham's coupling

4. Solution:

When a slider is free to move on a link which is free to rotate, slider has a linear velocity as well as rotational velocity. This types of motion provides an additional component of acceleration to the slider which is known as coriolis component of acceleration.

Coriolis component exists only if there are two coincident points which have two types of motion:

(i) Linear relative velocity of sliding.

(ii) Angular motion about fixed finite centres of rotation.

5. Solution:

The four methods to reduce gear-tooth interference are:

1. By increasing the pressure angle
2. By undercutting
3. By increasing the number of teeth
4. By stubbing.

6. Solution:

Grashof's law states that the sum of the shortest and longest links of a four -bar mechanism must be less than or equal to the sum of the remaining two links for continuous rotation of at least one link. It is the basic condition for crank existence.

7. Solution:

A steering mechanism ensures that the front wheels roll without lateral slip by making their axes meet at the instantaneous centre of turning. Proper steering geometry prevents tyre wear and maintains stable cornering.

8. Solution:

A Hooke's joint connects two shafts whose axes intersect at an angle and transmits torque between them. It accommodates mis-alignment but causes non-uniform angular velocity, especially at higher angles.

9. Solution:

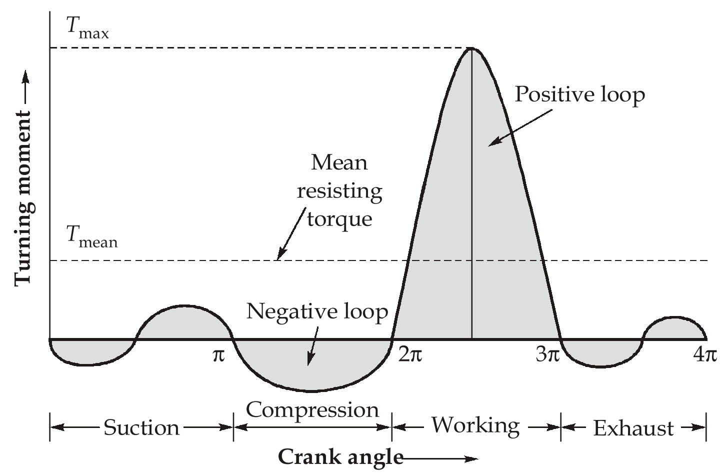
A governor automatically regulates engine speed by adjusting fuel or energy input in response to load variations. It maintains the mean speed constant despite fluctuations in load.

10. Solution:

Transmissibility is the ratio of the amplitude of transmitted force to the amplitude of exciting force. It indicates how effectively a system isolates vibrations, with values less than one showing successful isolation.

11. Solution:

A four stroke IC engine has a working stroke after the crank has turned through two revolutions, i.e. 720°(or 4π radians).



Turning moment diagram for a four stroke cycle internal combustion engine

During suction stroke, a negative loop is formed as the pressure inside the engine cylinder is less than the atmospheric pressure.

During compression stroke a high negative loop is formed because the work is done on the gases.

A large positive loop is formed during the working stroke and the work is done by the gases.

During exhaust stroke, the work is done on the gases therefore a negative loop is formed.

12. Solution:

(i) Base Circle: It is the smallest circle tangent to the cam profile (contour) drawn from the centre of rotation of a radial cam.

(ii) Trace point: It is a reference point on the follower to trace the cam profile such as the knife-edge of a knife-edged follower and centre of the roller of a roller follower.

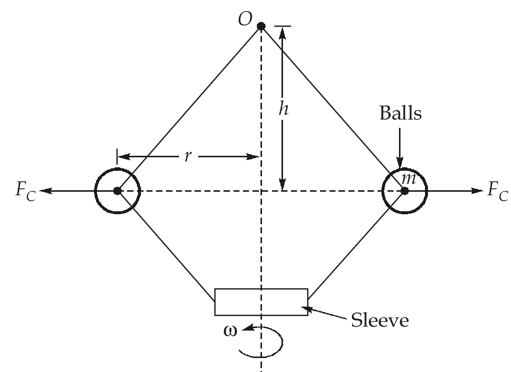
(iii) Pitch Curve: It is the curve drawn by the trace point assuming that the cam is fixed, and the trace point of the follower rotates around the cam.

(iv) Pressure Angle: The pressure angle, representing the steepness of the cam profile, is the angle between the normal to the pitch curve at a point and the direction of the follower motion. It varies in magnitude at all instants of the follower motion. A high value of the maximum pressure angle is not desired as it might jam the follower in the bearings.

(v) Prime Circle: The smallest circle drawn tangent to the pitch curve is known as the prime circle.

13. Solution:

Let, m = Mass of the balls (in kg);
r = Radius of the path of rotation of the ball (in m);
h = Height of the governor (in m)
ω = Angular velocity of the arm and ball about the spindle axis (rad/s);
T = Tension in the arm (in N);
F_c = Centrifugal force acting on the ball (in N)



(a) Simple Watt governor

Centrifugal force due to the balls, $F_c = m\omega^2 r$
Taking moment about the pivot O, we have

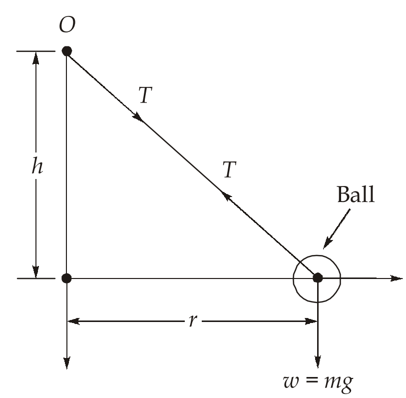
$$F_c \times h = mg \times r$$

or $$m\omega^2 r \times h = mg \times r$$

$$h = \frac{g}{\omega^2} = \frac{g}{\left(\frac{2\pi N}{60}\right)^2} = \frac{895}{N^2}$$

$$\therefore \quad N = \sqrt{\frac{895}{h}} \text{ rpm}$$

Hence, this is the required expression.



(b) Forces acting on ball

14. Solution:

The law of gearing states that for constant velocity ratio, the common normal at the point of tooth contact must always pass through the pitch point. Involute tooth profiles satisfy this law because the common normal remains aligned with the line of action at a constant pressure angle. This ensures smooth motion transfer even when centre distance varies slightly. Cycloidal gears also satisfy the law but are more sensitive to alignment errors. The involute profile's ease of manufacturing and robustness has made it the industry standard. Its uniform pressure angle provides predictable force transmission.

15. Solution:

A brake is an appliance used to apply frictional resistance to a moving body to stop or retard it by absorbing its kinetic energy. The absorbed kinetic energy is dissipated in form of heat to the surrounding atmosphere. Brake provides an artificial frictional resistance to the moving body.

The following are the different types of brakes:

1. Hydraulic brake
2. Electric brake
3. Mechanical brake: These can be further classified as follows:
   (i) Block or shoe brake,
   (ii) Band brake,
   (iii) Band and block brake,
   (iv) Internal expanding shoe brake.

16. Solution:

Laws of Static Friction :

1. The magnitude of the friction force is equal to the force and direction opposite to that force, which tends the body to move.
2. The magnitude of limiting friction (F) has a constant ratio with the normal reaction (N) between the two surfaces.

$$F/N = \text{Constant}$$

3. The friction force depends upon the roughness of the surfaces.
4. The friction force is independent of the area of contact between the two surfaces.

Laws of Kinetic Friction :

1. The friction force always acts in the direction opposite to that in which the body is moving.
2. The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant but it decreases slightly with increase of speed.

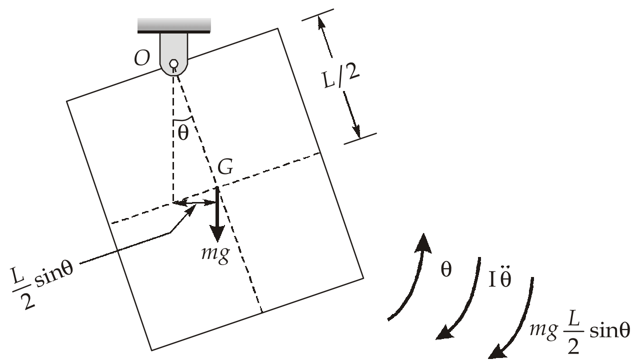
17. (a)

MOI of the body at suspended point,

$$I = I_G + m(OG)^2; \quad \text{(from parallel axis theorem)}$$

$$= \frac{mL^2}{6} + m\left(\frac{L}{2}\right)^2$$

$$I = \frac{5mL^2}{12}$$



Hence, $$\frac{5mL^2}{12}\ddot{\theta} + mg\frac{L}{2}\sin\theta = 0$$

As θ is very less,

∴ $$\sin\theta \simeq \theta$$

$$\frac{5mL^2}{12}\ddot{\theta} + mg\frac{L}{2}\theta = 0$$

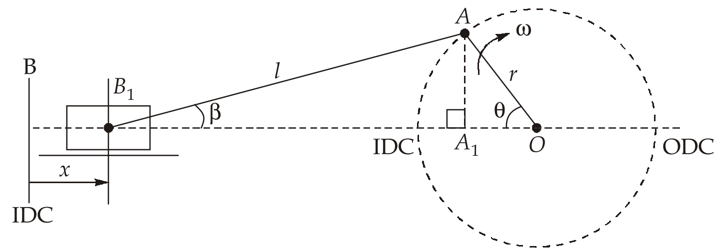
$$\ddot{\theta} + \frac{mg\frac{L}{2}}{\frac{5mL^2}{12}}\theta = 0$$

$$\omega_n = \sqrt{\frac{mg\frac{L}{2}}{\frac{5mL^2}{12}}} = \sqrt{\frac{12 \times mgL}{10mL^2}} = \sqrt{\frac{6g}{5L}} \text{ rad/s}$$

18. Solution:

Reciprocating engines generate unbalanced inertial forces due to the periodic acceleration and deceleration of pistons and connecting rods. These forces cause vibrations, shaking moments and heavy loads on bearings, especially at high speeds. To mitigate these effects, rotating counterweights and arrangement of multiple cylinders are used to balance primary and secondary forces. Proper balancing improves smoothness, reduces wear and increases engine life. It also enhances driver comfort and structural safety. Without balancing, engines would suffer severe vibration leading to mechanical failure.

19. Solution:



Single Slider Crank Mechanism: Figures shows a single slider crank mechanism in which the crank OA rotates with constant angular speed 'ω' and 'l' and 'r' are the lengths of the connecting rod and the crank respectively.

Let; x = Displacement of the piston

$$\text{Obliquity ratio, } n = \frac{l}{r}$$

At the moment when the crank has turned through angle 'θ' from IDC

$$x = BB_1 = BO - B_1O$$
$$= BO - (B_1A_1 + A_1O)$$
$$= (l + r) - (l\cos\beta + r\cos\theta)$$
$$= (nr + r) - (nr\cos\beta + r\cos\theta)$$
$$= r[(n + 1) - (n\cos\beta + \cos\theta)]$$

Where;
$$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{(AA_1)^2}{(B_1A)^2}}$$

$$= \sqrt{1-\frac{(r\sin\theta)^2}{l^2}} = \sqrt{1-\frac{\sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$x = r\left[(n+1)-\left(\sqrt{n^2-\sin^2\theta}+\cos\theta\right)\right]$$

Displacement of the piston,
$$x = r\left[(1-\cos\theta)+\left(n-\sqrt{n^2-\sin^2\theta}\right)\right]$$

Also, Velocity of the piston,
$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[r\left\{(1-\cos\theta)+n-\left(n^2-\sin^2\theta\right)^{1/2}\right\}\right]\frac{d\theta}{dt}$$

$$= r\left[(0+\sin\theta)+0-\frac{1}{2}\left(n^2-\sin^2\theta\right)^{-1/2}(-2\sin\theta\cos\theta)\right]\omega$$

Velocity of the piston, $$v = r\omega\left[\sin\theta+\frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$$

Acceleraton of the piston, $$a = \frac{dv}{dt}=\frac{dv}{d\theta}\frac{d\theta}{dt}=\frac{d}{d\theta}\left[r\omega\left(\sin\theta+\frac{\sin 2\theta}{2n}\right)\right]\omega$$

$$a = r\omega\left(\cos\theta+\frac{2\cos 2\theta}{2n}\right)\omega$$

$$a = r\omega^2\left(\cos\theta+\frac{\cos 2\theta}{n}\right)$$

Hence, these are the required expressions.

20. Solution:

Given: T_b = 27, T_c = 30, T_d = 24, T_e = 21.

Conditions	Revolutions of A (N_a)	Revolutions of E (N_e)	Revolutions of B-C $(N_b \text{ or } N_c)$	Revolutions of D (N_d)
Arm fixed: [Let gear E rotates by $+x$ rpm (clockwise)]	0	$+x$	$-x\times\frac{T_e}{T_c}=-x\times\frac{21}{30}$	$x\times\frac{21}{30}\times\frac{T_b}{T_d}=\frac{x\times21\times27}{30\times24}$
Arm free	y	$y+x$	$y-\frac{x\times21}{30}$	$y+\frac{x\times21\times27}{30\times24}$

As given is the question wheel E is fixed,

$\therefore$ $$y + x = 0$$

$$x = -y$$

Speed of driven shaft, (N_d) $$= y+x\times\frac{21\times27}{30\times24}=y+\frac{63x}{80}=y-\frac{63y}{80}$$

$$= \left(1-\frac{63}{80}\right)y=\frac{17y}{80}$$

Speed of driving shaft (N_a) $= y$

$$\frac{\text{Speed of driving shaft}(N_a)}{\text{Speed of driven shaft}(N_d)} = \frac{y}{\frac{17y}{80}}$$

$$\frac{N_a}{N_d} = \frac{80}{17}=4.706$$ **Ans.**

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