ESE 2025

Main Exam Detailed Solutions

Mechanical Engineering

PAPER-II

EXAM DATE : 10-08-2025 | 02:00 PM to 05:00 PM

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ANALYSIS

Mechanical Engineering Paper-II
ESE 2025 Main Examination

Sl.	Subjects	Marks
1	Strength of Materials	96
2	Theory of Machines	72
3	Machine Design	72
4	Engineering Mechanics	20
5	Industrial Engineering	32
6	Manufacturing Engineering	72
7	Mechatronics and Robotics	76
8	Engineering Materials	40
		Total 480

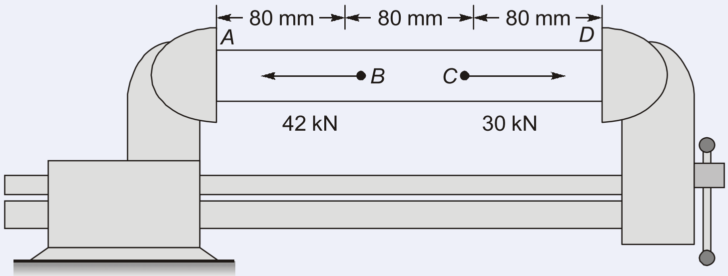
Scroll down for detailed solutions

SECTION : A

Q.1 (a) A steel tube (E = 200 GPa) with a 32 mm outer diameter and a 4 mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces as shown in figure are then applied to the tube. After these forces are applied the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine

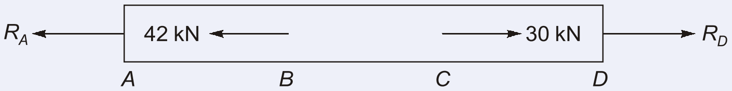
(i) the forces exerted by the vise on the tube at A and D

(ii) the change in length of the portion BC of the tube



[12 marks : 2025]

Solution:



Free body diagram of tube

Internal diameter of tube, $d_i = 32 - 2 \times 4 = 24$ mm

Outer diameter of tube, $d_o = 32$ mm

$$\text{Area of tube, } A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}\left(32^2 - 24^2\right) = 351.86 \text{ mm}^2$$

$$= 351.86 \times 10^{-6} \text{ m}^2$$

For A to B:

$$\delta_{AB} = \frac{R_A L}{EA} = \frac{R_A(0.08)}{200 \times 10^9 \times 351.86 \times 10^{-6}}$$

$$= 1.13682 \times 10^{-9} R_A$$

For B to C:

$$P = R_A + 42 \times 10^3$$

$$\delta_{BC} = \frac{\left(R_A + 42 \times 10^3\right)(0.08)}{\left(200 \times 10^9\right)\left(351.86 \times 10^{-6}\right)}$$

$$= 1.13682 \times 10^{-9} R_A + 47.746 \times 10^{-6}$$

For C to D:

$$P = R_A + 12 \times 10^3$$

$$\delta_{CD} = \frac{\left(R_A + 12 \times 10^3\right)(0.08)}{\left(200 \times 10^9\right)\left(351.86 \times 10^{-6}\right)}$$

$$= 1.13682 \times 10^{-9} R_A + 13.6418 \times 10^{-6}$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$= 3.4104 \times 10^{-9} R_A + 61.388 \times 10^{-6}$$

Due to Jaw movement, $\delta_{AD} = 2 \times 10^{-3}$ (Compression)

$$-0.2 \times 10^{-3} = 3.4104 \times 10^{-9} R_A + 61.388 \times 10^{-6}$$

$$-0.2 = 3.4104 \times 10^{-6} R_A + 61.388 \times 10^{-3}$$

$$R_A = -76.644 \text{ kN}$$ **Ans. (i)**

For overall, $\Sigma F_x = 0$

$$R_A + 42 = 30 + R_D$$

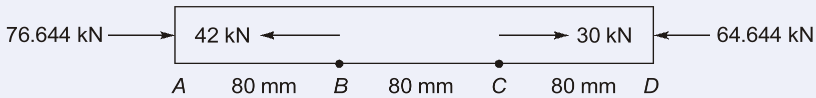
$$R_D - R_A = 12$$

$$R_D = 12 + R_A = 12 - 76.644 = -64.644 \text{ kN}$$ **Ans. (ii)**

$$\delta_{BC} = 1.13682 \times 10^{-9} (-76.644 \times 10^3) + 47.746 \times 10^{-6}$$

$$= -39.384 \times 10^{-6} \text{ m}$$

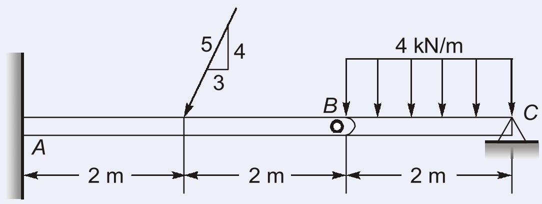
$$\delta_{BC} = -0.0394 \text{ mm}$$ **Ans. (iii)**



FBD with all the forces

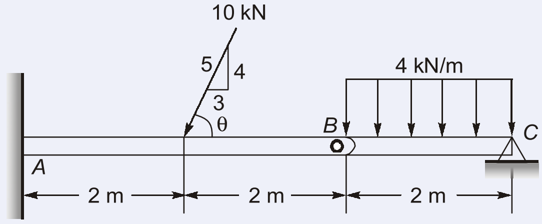
End of Solution

Q.1 (b) The compound beam shown in figure below is pin connected at *B*. Determine the components of reaction at its supports. Neglect its weight and thickness.

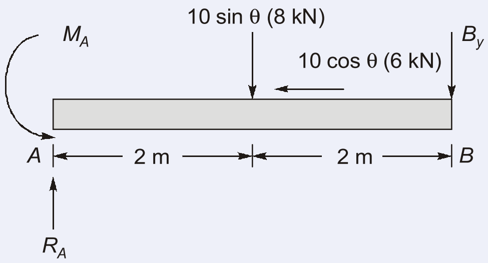


[12 marks : 2025]

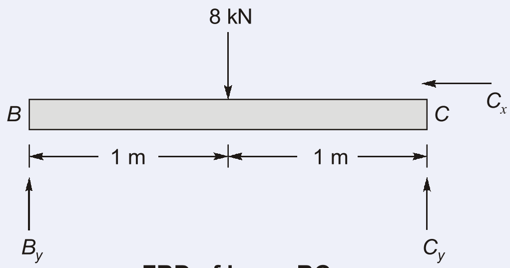
Solution:



$$\cos\theta = \frac{3}{5}, \quad \sin\theta = \frac{4}{5}$$



FBD of beam AB



FBD of beam BC

For BC: $B_y + C_y = 8$ kN [$\Sigma F_y = 0$]

$B_y \times 2 = 8 \times 1$ [$\Sigma M_C = 0$]

$B_y = 4$ kN

So, $C_y = 4$ kN

For whole beam equilibrium,

$C_x + 6 = 0$ [$\Sigma F_x = 0$]

$C_x = -6$ kN

For beam AB: $R_A = 8 + B_y$ [$\Sigma F_y = 0$]

$R_A = 8 + 4 = 12$ kN

Taking moment about A:

$M_A = 8 \times 2 + 4 \times 4$

$M_A = 16 + 16$

i.e., $M_A = 32$ kNm (CCW)

Summarizing reaction at all supports:

Moment at A, $M_A = 32$ kNm (CCW) **Ans.**

Reaction at A, $R_A = 12$ kN (Upward) **Ans.**

Horizontal reaction at C, $C_x = 6$ kN (Rightward x-direction) **Ans.**

Vertical reaction at C, $C_y = 4$ kN (Upward) **Ans.**

End of Solution

Q.1 (c) A compound cylinder is formed by shrinking one tube of 100 mm internal diameter and 25 mm wall thickness on to another tube of 100 mm external diameter and 25 mm wall thickness. Both the tubes are made of steel (with $E = 210$ GPa) and the shrinkage allowance based on radius is 0.02 mm. Determine the pressure set up at the junction due to shrinkage.

[12 marks : 2025]

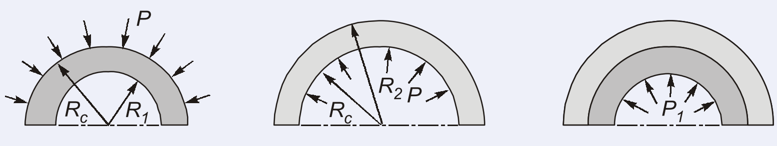
Solution:

The method of solution for compound cylinders constructed from similar materials is to break the problem down into three separate effects:

(a) shrinkage pressure only on the inside cylinder.

(b) Shrinkage pressure only on the outside cylinder.

(c) Internal pressure only on the complete cylinder, as shown in figure.



(a) Shrikage - internal Cylinder (b) Shrikage - external Cylinder (c) Internal pressure - compound cylinder

Method of solution for compound cylinders

For each of the resulting load conditions there are two known values of radial stress which enable the Lame constants to be determined in each case.

Condition (a) Shrinkage - Internal cylinder

At $r = R_1$, $\sigma_r = 0$

At $r = R_c$, $\sigma_r = -P$ (Compressive since it tends to reduce the wall thickness)

Condition (b) Shrinkage – External cylinder

At $r = R_2$, $\sigma_r = 0$

At $r = R_c$, $\sigma_r = -P$

Condition (c) Internal pressure – Compound cylinder

At $r = R_2$, $\sigma_r = 0$

At $r = R_1$, $\sigma_r = -P_1$

We know that Lame's equation or thick cylinders is given as

$$\sigma_r = A - \frac{B}{r^2} \quad \text{(Radial stress)}$$

$$\sigma_H = A + \frac{B}{r^2} \quad \text{(Hoop stress)}$$

The total interference or shrinkage allowance (on radius) for cylinders of the same material is given as

$$\Delta(\text{Shrinkage allowance}) = \frac{r}{E}\left[\sigma_{H_o} - \sigma_{H_i}\right]$$

For the given loading, let P be the required shrinkage pressure, then for the inner tube:

At $r = 0.025$ m, $\sigma_r = 0$ and

At $r = 0.05$ m, $\sigma_r = -P$

$$A - \frac{B}{(0.025)^2} = 0 = A - 1600B \quad \text{... (i)}$$

$$-P = A - \frac{B}{(0.05)^2} = A - 400B \quad \text{... (ii)}$$

Solving equation (i) and (ii),

$$-P = 1200B \text{ or } B = -\frac{P}{1200}$$

For equation (i), $A = 1600B = -\frac{P}{1200} \times 1600 = -\frac{4P}{3}$

Therefore, at the common radius the hoop stress is given by

$$\sigma_{H_i} = A + \frac{B}{(0.05)^2} = -\frac{4P}{3} + 400\left(-\frac{P}{1200}\right)$$

$$= -\frac{4P}{3} - \frac{P}{3} = -\frac{5P}{3} = -1.67P$$

Now, for outer tube:

At $r = 0.05$ m, $\sigma_r = -P$ and

At $r = 0.075$ m, $\sigma_r = 0$

$$-P = A - \frac{B}{(0.05)^2} = A - 400B \quad \text{... (iii)}$$

$$0 = A - \frac{B}{(0.075)^2} = A - 178B \quad \text{... (iv)}$$

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Subtracting (iv) – (iii), $P = 222B$ or $B = \dfrac{P}{222}$

From equation (iv), $A = 178B = \dfrac{178}{222}P$

Therefore, at the common radius the hoop stress is given by

$$\sigma_{H_o} = A + \frac{B}{0.05^2} = A + 400B$$

$$= \frac{178P}{222} + \frac{P \times 400}{222} = \frac{578P}{222} = 2.6P$$

$$\text{Shrinkage allowance} = 0.02 \times 10^{-3} = \frac{r}{E}\left(\sigma_{H_o} - \sigma_{H_i}\right)$$

$$0.02 \times 10^{-3} = \frac{50 \times 10^{-3}}{210 \times 10^9}\left[2.6P - (-1.67P)\right] \times 10^6$$

$$0.02 \times 10^{-3} = \frac{50 \times 10^{-3} \times 4.27 \times 10^6}{210 \times 10^9}P$$

$P = 19.67$ MPa **Ans.**

End of Solution

Q.1 (d) A pair of 20° full depth involute spur gears are in mesh. The larger gear has 48 teeth whereas the pinion has 12 teeth. The module is 10 mm. Determine

(i) the reduction in addendum of the gear to avoid interference, and

(ii) Contact ratio

[12 marks : 2025]

Solution:

Given data: $t = 12$, $T = 48$, $\phi = 20°$, $m = 10$ mm.

Addendum of the larger gear to avoid interference

$$= \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{10 \times 48}{2}\left[\sqrt{1 + \frac{1}{4}\left(\frac{1}{4} + 2\right)\sin^2 20°} - 1\right]$$

$$= 240\left[\sqrt{1 + 0.25 \times 2.25 \times 0.117} - 1\right]$$

$$= 240 \times (1.032382 - 1) = 7.77 \text{ mm}$$

Reduction in addendum $= 10 - 7.77 = 2.23$ mm **Ans. (i)**

Pitch circle radius of larger gear,

$$R = \frac{mT}{2} = 240 \text{ mm}$$

Pitch circle radius of pinion, $r = \dfrac{mt}{2} = \dfrac{10 \times 12}{2} = 60 \text{ mm}$

$$R_A = R + A_W = 240 + 10 = 250 \text{ mm}$$

$$r_A = r + A_p = 60 + 10 = 70 \text{ mm}$$

$$\text{Length of path of approach} = \sqrt{(R_A^2) - (R\cos\phi)^2} - R\sin\phi$$

$$= \sqrt{(250)^2 - (240\cos 20°)^2} - 240\sin 20°$$

$$= 107.88 - 82.08 = 25.79 \text{ mm}$$

$$\text{Length of path of recess} = \sqrt{(r_A^2) - (r\cos\phi)^2} - r\sin\phi$$

$$= \sqrt{(70)^2 - (60\cos 20°)^2} - 60\sin 20°$$

$$= 41.476 - 20.52 = 20.97 \text{ mm}$$

$$\text{Length of path of contact} = 25.79 + 20.97 = 46.76 \text{ mm}$$

$$\text{Length of arc of contact} = \frac{46.76}{\cos 20°} = 49.76 \text{ mm}$$

$$\text{Contact ratio} = \frac{\text{Length of arc of contact}}{\text{Circular pitch}} = \frac{49.76}{\pi \times 10}$$

$$= 1.584$$

Ans. (ii)

End of Solution

Q.1 (e) A rotating solid shaft of diameter d is under bending moment M and Torque T without any axial load. Determine the equivalent bending moment, M_e, using Maximum Normal Stress Theory (MNST) and Distortion Energy Theory (DE) and the equivalent twisting moment, T_e, using Maximum Shear Stress Theory (MSST). Effects due to fatigue and stress concentration are to be neglected.

[12 marks : 2025]

Solution:

Consider a point P on the periphery of the shaft. If d is the diameter, then owing to the bending moment M, the normal stress σ at P on a plane normal to the axis of the shaft is

$$\sigma = \frac{My}{I} = M \cdot \frac{d}{2} \times \frac{64}{\pi d^4} = \frac{32M}{\pi d^3}$$

The shearing stress on a transverse plane at P due to torsion T is

$$\tau = \frac{Td}{2I_p} = \frac{Td \times 32}{2\pi d^4} = \frac{16T}{\pi d^3}$$

Therefore, the principal stresses at P are

$$\sigma_{1,2} = \frac{1}{2}\sigma \pm \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}, \sigma_3 = 0$$

Maximum normal stress theory,

$$\sigma_{max} = \sigma_1 = \frac{\sigma}{2} + \frac{1}{2}\left(\sigma^2 + 4\tau^2\right)^{1/2} = \sigma_y$$

$$\sigma + (\sigma^2 + 4\tau^2)^{1/2} = 2\sigma_y$$

$$\frac{32M}{\pi d^3} + \frac{1}{\pi d^3} 32\left(M^2 + T^2\right)^{1/2} = 2\sigma_y$$

$$32M + 32\,(M^2 + T^2)^{1/2} = 2\pi d^3 \sigma_y$$

$$32[M + (M^2 + T^2)^{1/2}] = 2\pi d^3 \sigma_y$$

$$32\left[\frac{1}{2}\left\{M + \left(M^2 + T^2\right)^{1/2}\right\}\right] = \pi d^3 \sigma_y \quad \text{... (i)}$$

Also, $32M_e = \pi d^3 \sigma_y$ ... (ii)

From equation (i) and (ii)

Equivalent bending moment, $M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$ N.m **Ans.**

Distortion Energy Theory

$$U_d = \frac{(1+\nu)}{6E}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

$$= \frac{(1+\nu)}{3E}\sigma_y^2$$

With $\sigma_3 = 0$,

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2$$

Substituting for σ_1 and σ_2

$$\begin{bmatrix} \frac{1}{4}\sigma^2 + \frac{1}{4}\left(\sigma^2 + 4\tau^2\right) + \frac{1}{2}\sigma\left(\sigma^2 + 4\tau^2\right)^{1/2} + \frac{1}{4}\sigma^2 + \frac{1}{4}\left(\sigma^2 + 4\tau^2\right) \\ -\frac{1}{2}\sigma\left(\sigma^2 + 4\tau^2\right)^{1/2} - \frac{1}{4}\sigma^2 + \frac{1}{4}\left(\sigma^2 + 4\tau^2\right) \end{bmatrix}^{1/2} = \sigma_y$$

$$(\sigma^2 + 3\tau^2)^{1/2} = \sigma_y$$

Substituting for σ and T

$$\frac{16 \times 2}{2\pi d^3}\left(4M^2 + 3T^2\right)^{1/2} = \sigma_y$$

$$M_e = \frac{1}{2}\sqrt{4M^2 + 3T^2} \quad \textbf{Ans.}$$

Maximum Shear Stress Theory,

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}\left(\sigma^2 + 4\tau^2\right)^{1/2} = \frac{\sigma_y}{2}$$

$$(\sigma^2 + 4\tau^2)^{1/2} = \sigma_y = 2\tau_{max}$$

Substituting for σ and τ

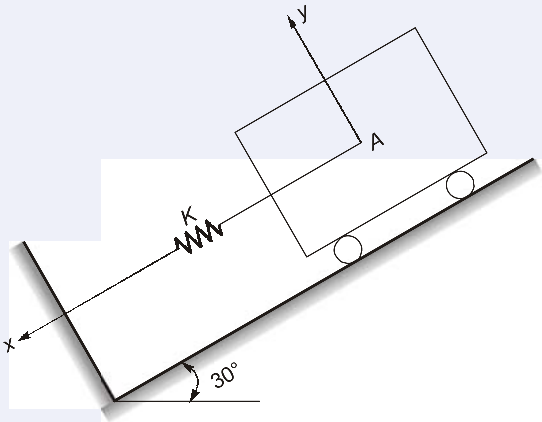
$$\frac{32}{\pi d^3}(M^2 + T^2)^{1/2} = 2\tau_{max}$$

$$\frac{16}{\pi d^3}(M^2 + T^2)^{1/2} = \tau_{max}$$

$$T_e = \sqrt{M^2 + T^2} \quad \textbf{Ans.}$$

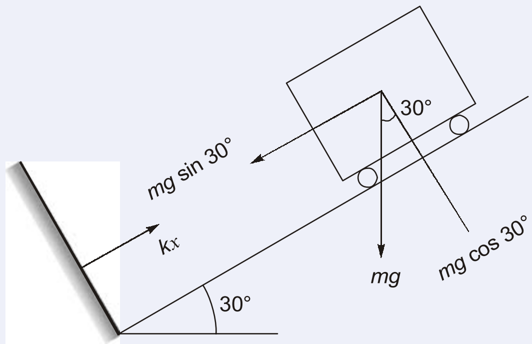
End of Solution

Q.2 (a) A cart *A* shown in figure having a mass of 200 kg is held on an incline so as to just touch the undeformed spring whose constant *K* is 50 N/mm. If body *A* is released very slowly, what distance down the incline must *A* move to reach an equilibrium configuration? If body *A* is released suddenly, what is its speed when it reaches the aforementioned equilibrium configuration for a slow release?



[20 marks : 2025]

Solution:



For slow release:

The gravitational force component down the incline is equal to spring force.

$$F_g = F_s$$

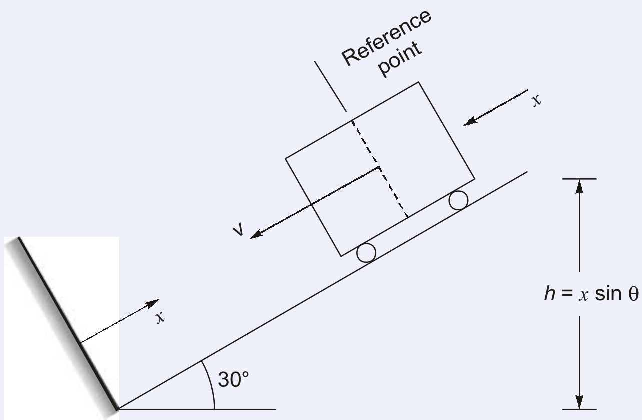
$$mg \sin 30° = kx$$

$$x = \frac{mg \sin 30°}{k} = \frac{200 \times 9.81 \times \sin 30°}{50 \times 1000}$$

$$= 0.01962 \text{ m or } 19.62 \text{ mm}$$ **Ans.**

For sudden release:

When the cat is released suddenly, we need to use the conservation of energy principal. The potential energy lost (Gravitational + Elastic) is converted into kinetic energy at the equilibrium position.



Let the initial position be the reference point for gravitational potential energy. The height change down the incline is $h = x \sin\theta$

Initial energy (Gravitational + Elastic + Kinetic) = 0

At equilibrium position (Distance x down the incline):

$$(PE)_{gravitational} = -mgh = -mg\,x\sin\theta$$

$$(PE)_{elastic} = \frac{1}{2}kx^2$$

$$(KE)_{final} = \frac{1}{2}mv^2$$

By the principal of energy conservation:

$$0 + 0 + 0 = \frac{1}{2}mv^2 - mgx\sin\theta + \frac{1}{2}kx^2$$

$$\frac{1}{2}mv^2 = mg\,x\sin\theta - \frac{1}{2}kx^2$$

$$mv^2 = 2mg\,x\sin\theta - kx^2$$

$$v = \sqrt{\frac{2mgx\sin\theta - kx^2}{m}} = \sqrt{\frac{2\times 200\times 9.81\times 0.01962\sin 30^\circ - 50000\times(0.01962)^2}{200}}$$

$$= \sqrt{\frac{38.4944 - 19.2472}{200}} = \sqrt{\frac{19.2472}{200}}$$

$$v = 0.31022 \text{ m/s}$$ Ans.

End of Solution

Q.2 (b) The torque produced by an engine is given by the expression

$T = (5000 + 1500 \sin 3\theta)$ N-m

where θ (theta) is the angle turned by the crank measured from some datum. The mean engine speed is 300 rpm and the flywheel and other rotating parts attached to the shaft have a mass of 450 kg with radius of gyration 500 mm. Determine:

(i) the power of the engine

(ii) percentage fluctuation of speed when the resisting torque is constant

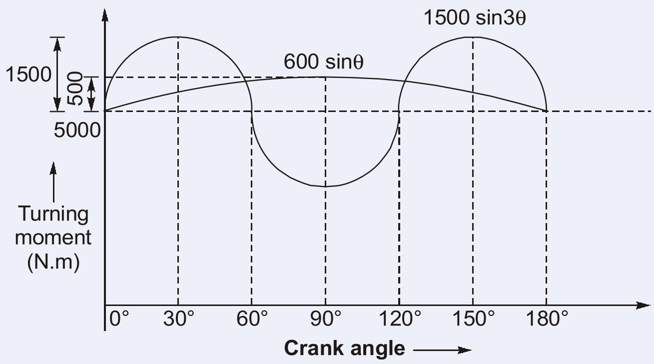
(iii) percentage fluctuation of speed when the resisting torque is $(5000 + 600 \sin\theta)$

[20 marks : 2025]

Solution:

Given: m = 450 kg, N = 300 rpm, k = 500 mm, $\omega = \dfrac{2\pi \times 300}{60} = 31.416$ rad/s

For the expression for torque being a function of 3θ, the cycle is repeated after every 120° of the crank rotation as shown in figure.



(i)
$$T_{mean} = \frac{1}{2\pi/3}\int_o^{2\pi/3} T d\theta = \frac{3}{2\pi}\int_o^{2\pi/3}(5000 + 1500\sin 3\theta)d\theta$$

$$= \frac{3}{2\pi}\left[5000\theta - \frac{1500}{3}\cos 3\theta\right]_o^{2\pi/3} = 5000 \text{ Nm}$$

$$P = T\omega = 5000 \times 31.416 = 157080 \text{ W}$$

(ii) At any instant, $\Delta T = T - T_{mean} = 5000 + 1500 \sin 3\theta - 5000 = 1500 \sin 3\theta$

ΔT is zero when 1500 sin3θ = 0

or when sin3θ = 0 or 3θ = 0° or 180° or θ = 0° or 60°

$$e_{max} = \int_{0°}^{60°} \Delta T dt = \int_{0°}^{60°}(1500 \sin 3\theta)d\theta$$

$$= \left[\frac{1500\cos 3\theta}{3}\right]_{0°}^{60°} = 1000 \text{ N.m}$$

$$K = \frac{e}{mk^2\omega^2} = \frac{1000}{450 \times (0.5)^2 \times (31.416)^2}$$

= 0.009006 or 0.9006%

(iii)
$$\Delta T = (5000 + 1500 \sin 3\theta) - (5000 + 600 \sin\theta)$$
$$= 1500 \sin 3\theta - 600 \sin\theta$$

ΔT is zero when 1500 sin3θ – 600 sinθ = 0

or, 1500 sin3θ = 600 sinθ

$$1500(3\sin\theta - 4\sin^3\theta) = 600 \sin\theta$$

$$3 - 4\sin^2\theta = \frac{600}{1500}$$

$$3 - 4\sin^2\theta = 0.4$$

$$4\sin^2\theta = 3 - 0.4$$

$$\sin^2\theta = \frac{3-0.4}{4} = 0.65$$

$$\sin\theta = \pm 0.806$$

or, $$\theta = \pm 53.73° \text{ and } \pm 126.27°$$

$$e_{max} = \int_{53.73}^{126.27} \Delta T d\theta = \int_{53.73}^{126.27} (1500\sin 3\theta - 600\sin\theta) d\theta$$

$$= \left[-\frac{1500\cos 3\theta}{3} + 600\cos\theta\right]_{53.73°}^{126.27°}$$

$$= -500 \cos 378.81° + 600 \cos 126.27°$$
$$+ 500 \cos 161.19 - 600 \cos 53.73°$$
$$= -1656.502 \text{ Nm}$$

$$K = \frac{e}{mk^2\omega^2} = \frac{1656.502}{450\times(0.5)^2\times(31.416)^2}$$

$$= 1.49\% \text{ or } 0.00149$$

End of Solution

Q.2 (c) The following data refers to a pair of spur gears with 20° full depth involute teeth:
Number of teeth on pinion = 24, Number of teeth on gear = 56,
Speed of pinion = 1200 rpm, Module = 3 mm,
Face width = 30 mm
Both gears are made of steel with an ultimate tensile strength of 600 N/mm². Using the velocity factor to account for the dynamic load and assuming service factor as 1.5, determine
(i) beam strength, and
(ii) rated power that the gears can transmit without bending failure, if the factor of safety is 1.5.

Take Lewis form factor for 24 teeth equal to 0.337, and velocity factor, $C_v = \frac{3}{3+v}$, where v is the pitch line velocity in m/s.

[20 marks : 2025]

Solution:

Given data: n = 1200 rpm, z_p = 24, z_g = 56, m = 3 mm, b = 30 mm, S_{ut} = 600 N/mm², C_s = 1.5, FOS = 1.5.

Since the same material is used for the pinion and the gear, the pinion is weaker than the gear.

Lewis form factor for pinion, Y = 0.337 (Given)

$$\sigma_b = \frac{1}{3}S_{ut} = \frac{1}{3}\times 600 = 200 \text{ N/mm}^2$$

$$S_b = mb\sigma_b Y = 3 \times 30 \times 200 \times 0.337$$
$$= 6066 \text{ N}$$ Ans. (i)

$$\text{Pitch line velocity, } V = \frac{\pi d_p n_p}{60000} = \frac{\pi \times (mz_p) n_p}{60000}$$

$$= \frac{\pi \times 3 \times 24 \times 1200}{60000} = 4.524 \text{ m/s}$$

$$\text{Velocity factor, } C_v = \frac{3}{3+v} = \frac{3}{3+4.524} = 0.3987$$

$$P_{\text{effective}} = \frac{C_s}{C_v} P_t = \frac{1.5 \times P_t}{0.3987} = (3.762 P_t)\ \text{N}$$

For bending failure criteria, $S_b = P_{\text{eff}}(\text{FOS})$

$$6066 = P_{\text{eff}} \times 1.5$$

$$P_{\text{eff}} = 4044 \text{ N}$$

$$P_t = \frac{4044}{3.762} = 1074.96 \text{ N}$$

$$\text{Torque, } T = P_t\left(\frac{mz_p}{2}\right) = 1074.96 \times \frac{3 \times 24}{2}$$

$$= 38698.56 \text{ N.mm}$$

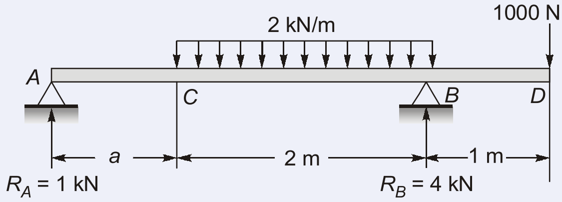
$$\text{Rated power (kW)} = \frac{2\pi n_p T}{60 \times 10^6} = \frac{2\pi \times 1200 \times 38698.56}{60 \times 10^6}$$

$$= 4.863 \text{ kW}$$

Ans. (ii)

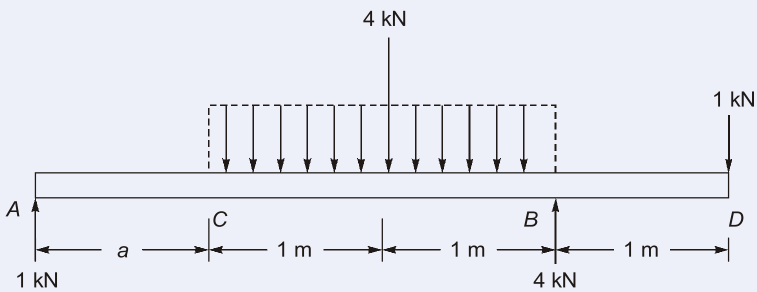
End of Solution

Q.3 (a) Find the value of a and draw the bending moment diagram for the beam shown in figure below.



[20 marks : 2025]

Solution:



Taking moment about C: $\Sigma M_C = 0$

$$1 \times a + 4 \times 1 - 4 \times 2 + 1 \times 3 = 0$$

$$a + 4 - 8 + 3 = 0$$

$$a - 1 = 0 \text{ or } a = 1 \text{ m} \qquad \textbf{Ans.}$$

Now, moment equation for each segment by considering a section x-x

$$M_{AC} = x$$

$$M_{BC} = x - \frac{2(x-1)^2}{2} = x - (x-1)^2 = 3x - x^2 - 1$$

$$M_{BD} = x - 4(x-2) + 4(x-3)$$
$$= x - 4x + 8 + 4x - 12 = x - 4$$

Moment of A = 0 kN.m

Moment at $C = (3x - x^2 - 1)_{x=1} = 3 - 1 - 1 = 1$ kN.m

Moment at $B = (3x - x^2 - 1)_{x=3} = 3 \times 3 - 9 - 1 = -1$ kN.m

Moment at $D = (x - 4)_{x=4} = 4 - 4 = 0$ kN.m

For maximum bending moment:

$$\frac{dM_{BC}}{dx} = 0$$

or, $3 - 2x = 0$ or $x = 1.5$ m

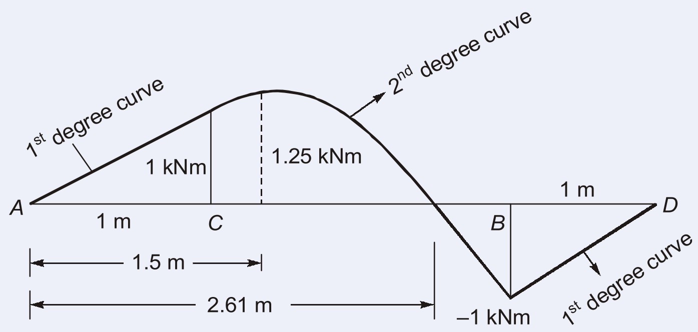
$$M_{max} = 3 \times 1.5 - (1.5)^2 - 1 = 4.5 - 2.25 - 1 = 1.25 \text{ kN.m}$$

For point of contraflexure:

$$M_{BC} = 0 \text{ or } 3x - x^2 - 1 = 0$$

Solving this, we get, $x = 0.382$ m or 2.61 m

By problem geometry, acceptable value of x is 2.61 m.



Bending Moment Diagram

End of Solution

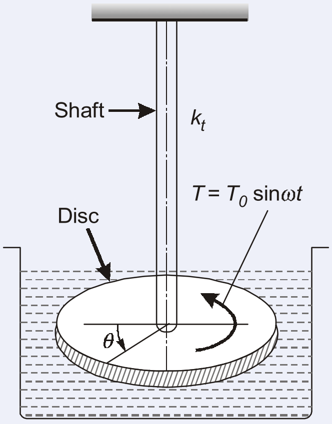
Q.3(b)(i) A torsional pendulum has a natural frequency of 200 cycles/min when vibrating in a vacuum. The mass moment of inertia of the disc is 0.2 kg-m². It is then immersed in oil and its natural frequency is found to be 180 cycles/min. Determine the damping constant.

If the disc, when placed in oil, is given an initial angular displacement of 2°, find its displacement at the end of first cycle.

[10 marks : 2025]

Solution:

Given: $f_n = 200$ cycles/min, $f_d = 180$ cycle/min.



Damped forced torsional vibration

$$f_d = f_n\sqrt{1-\xi^2}$$

$$180 = 200\sqrt{1-\xi^2}$$

$$\xi = 0.436$$

$$c_{tc} = 2I\omega_n = 4\pi I f_n \text{ [Critical damping coefficient]}$$

$$= 4\pi \times 0.2 \times \frac{200}{60} = 8.3776 \text{ N.m.s/rad}$$

$$c_t = c_{tc}\xi = 0.436 \times 8.3776 = 3.6526 \text{ N.m.s/rad} \quad \text{Ans.}$$

$$\theta(t) = e^{(-\xi\omega_n t)}\,(A \sin\omega_d t + B \cos\omega_d t)$$

$$\theta(0) = 2° = \frac{2\pi}{180} = \frac{\pi}{90}\text{radian} = B$$

$$\frac{d\theta(t)}{dt} = \omega_d\, e^{(-\xi\omega_n t)}\,[A \cos\omega_d t - B \sin\omega_d t]$$

$$- \xi\omega_n\, e^{(-\xi\omega_n t)}\,[A \sin\omega_d t + B \cos\omega_d t]$$

At $t = 0$,

$$\frac{d[\theta(t)]}{dt} = 0$$

$$A = \xi\frac{\omega_n}{\omega_d}B = \frac{0.436 \times 200}{180} \times \frac{\pi}{90} = 0.035$$

Time taken to complete one cycle,

$$t = T_d = \frac{2\pi}{\omega_d} = \frac{2\pi \times 60}{2\pi \times 180} = \frac{1}{3}s$$

$$\omega_d t = 2\pi$$

$$\theta = e^{\left[-\frac{0.436\times 2\pi\times 200}{60\times 3}\right]}\left(0.035\sin 2\pi + \frac{\pi}{90}\cos 2\pi\right)$$

$$= 0.04765 \times \frac{\pi}{90} \times \frac{180}{\pi} = 0.09512° \quad \text{Ans.}$$

End of Solution

Q.3(b)(ii) The natural frequency of vibration of a person standing on a horizontal floor is found to be 5.2 Hz while standing. Assuming damping to be negligible, determine:

(a) the equivalent stiffness of his body in the vertical direction if the mass of the person is 70 kg.

(b) the amplitude of vertical displacement of the person if the floor is subjected to a vertical harmonic vibration of frequency 5.3 Hz and amplitude 0.1 m due to an unbalanced rotating machine operating on the floor.

[10 marks : 2025]

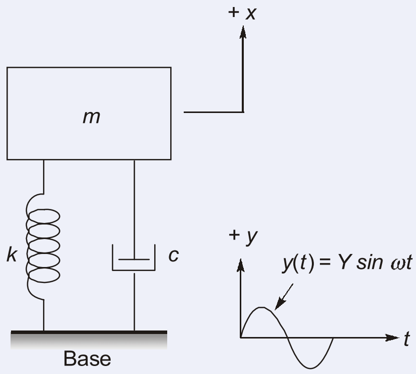
Solution:

$$f_n = 5.2 \text{ Hz}, \ \omega_n = 2\pi f_n = 2\pi \times 5.2$$
$$= 32.6726 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$k = m\omega_n^2 = (32.6726)^2 \times 70 = 74.725 \text{ kN/m}$$ **Ans.(a)**

$$\left.\begin{aligned} \omega &= 5.3 \text{ Hz} = 33.3009 \text{ rad/s} \\ \text{Amplitude}, \quad Y &= 0.1 \text{ m} \end{aligned}\right] \text{ Base excitation}$$



Vertical displacement (x) is given by

$$X = Y\left[\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}\right]^{\frac{1}{2}}$$

$$r = \frac{\omega}{\omega_n} = \frac{33.3009}{32.6726} = 1.0192; \ r^2 = 1.0388$$

$$\xi = 0 \text{ (no damping)}$$

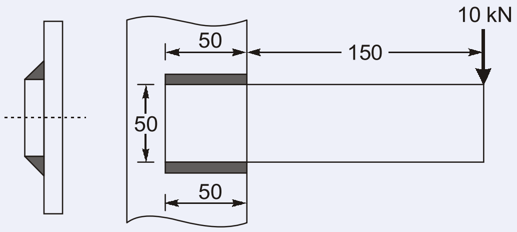
Above equation reduces to

$$X = Y\left[\frac{1}{(1-r^2)^2}\right]^{\frac{1}{2}} = \frac{Y}{1-r^2}$$

$$= \frac{0.1}{|1-1.0388|} = 2.5752 \text{ m}$$ **Ans.(b)**

End of Solution

Q.3 (c) A welded connection of steel plates as shown in figure is subjected to an eccentric force of 10 kN. Assuming static conditions, determine the throat dimension of the welds if the permissible shear stress is limited to 95 MPa.



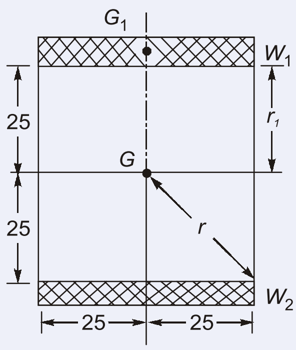
[20 marks : 2025]

Solution:

Let us assume t is the thread of each weld. There are two welds W_1 and W_2 and their threat area is given by,

$$A = 2(50\ t) = (100\ t)\ \text{mm}^2$$

Primary shear stress:
$$\tau_1 = \frac{10000}{100t} = \left(\frac{100}{t}\right)\ \text{N/mm}^2$$



The two welds are symmetrical and G is the centre of gravity of the two welds.

$$e = 150 + 25 = 175\ \text{mm}$$

$$M = P \times e = 10000 \times 175 = 1750000\ \text{N.mm}$$

From the above figure, the distance r of the farthest point in the weld from the centre of gravity is given by

$$r = \sqrt{(25)^2 + (25)^2} = 35.36\ \text{mm}$$

The polar moment of inertia J_1 of weld W_1 about G is given by

$$J_1 = A\left[\frac{l^2}{12} + r_1^2\right] = 50\ t\left[\frac{50^2}{12} + 25^2\right] = (41667\ t)\ \text{mm}^4$$

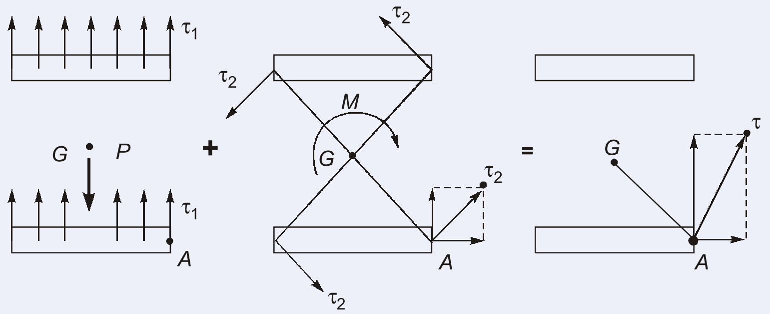
The welds are symmetrical. Therefore, the polar moment of inertia of two welds is given by

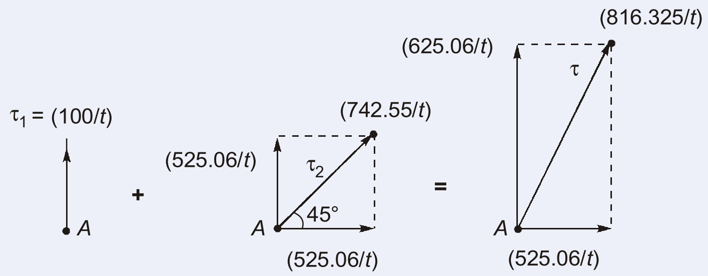
$J = J_1 + J_2 = 2J_1 = 2 \times 41667\ t = (83334\ t)\ \text{mm}^4$

The secondary shear stress is given by

$$\tau_2 = \frac{M_r}{J} = \frac{1750000 \times 35.36}{83334\ t} = \left(\frac{742.55}{t}\right)\ \text{N/mm}^2$$

The following figure shows the primary and secondary shear stresses. The vertical and horizontal components of these shear stresses are added and the resultant shear stress is determined.





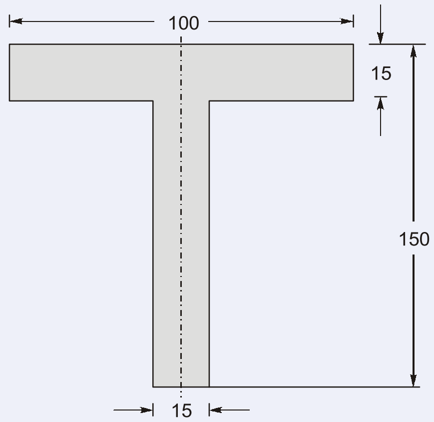
From the above figure,
$$\tau = \left(\frac{816.325}{t}\right) \text{ N/mm}^2$$

$$\frac{816.325}{t} = 95$$

$$t = \frac{816.325}{95} = 8.59 \text{ mm} \approx 8.6 \text{ mm}$$ **Ans.**

End of Solution

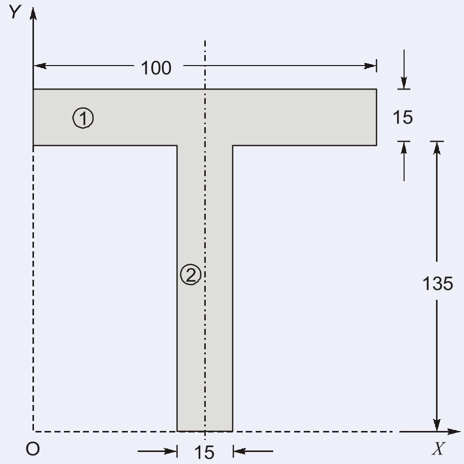
Q.4 (a) Find the position of the centroid of a *T*-section shown in figure below. The flange is 100 × 15 mm and the web is 135 × 15 mm. A cantilever of length 3000 mm and of section shown, with flange at the top, carries load *W* at the free end. What is the maximum value of *W* if the stress in the section is not to exceed 50 N/mm².



All dimensions in mm

[20 marks : 2025]

Solution:



By symmetry,

$$\bar{x} = 50 \text{ mm}$$

Distance of centroids of two rectangles (1) and (2) from OX axis:

$$y_1 = 135 + \frac{15}{2} = 142.5 \text{ mm}$$

$$y_2 = \frac{135}{2} = 67.5 \text{ mm}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{100 \times 15 \times 142.5 + 135 \times 15 \times 67.5}{100 \times 15 + 135 \times 15}$$

$$\bar{y} = \frac{100 \times 142.5 + 135 \times 67.5}{235} = 99.415 \text{ mm}$$

The centroid lies at 50 mm from *OY* axis and 99.415 mm from *OX* axis. Distance of CG from the upper extreme fibre,

$$y_1 = 150 - 99.415 = 50.585 \text{ mm}$$

Distance of CG from the bottom most fibre, $y_2 = 99.415$ mm

Since, $y_2 > y_1$, maximum stress will occur at bottom.

$$\sigma_{max} = \frac{My_2}{I_{xx}}$$

$$I_{xx} = I_{xx_1} + I_{xx_2} = \left[\frac{b_1d_1^3}{12} + a_1h_1^2\right] + \left[\frac{b_2d_2^3}{12} + a_2h_2^2\right]$$

$$\Rightarrow I_{xx} = \frac{100 \times 15^3}{12} + 100 \times 15 \times \left(50.585 - \frac{15}{2}\right)^2 + \frac{15 \times 135^3}{12} + 135 \times 15 \times \left(99.415 - \frac{135}{2}\right)^2$$

$$\Rightarrow \quad I_{xx} = 2812.6 \times 10^3 + 5138 \times 10^3 \text{ mm}^4$$

$$I_{xx} = 7950.6 \times 10^3 \text{ mm}^4$$

$$M = \frac{\sigma_{max} I_{xx}}{y_2} = \frac{50 \times 7950.6 \times 10^3}{99.415} = 3998692.35 \text{ N.mm}$$

$$W = \frac{3998692.35}{3000} = 1332.89 \text{ N} = 1.332 \text{ kN}$$ **Ans.**

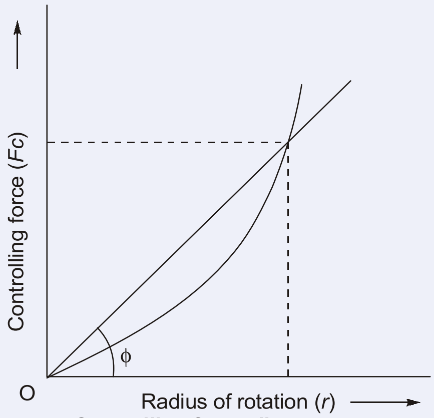
End of Solution

Q.4(b)(i) What is the controlling force of a governor? Draw a typical controlling force diagram for spring controlled governors and explain how it helps in establishing the stability or instability of a governor.

[8 marks : 2025]

Solution:

Controlling force: When a body rotates in a circular path, there is an inward radial force or centripetal force acting on it. In case of a governor running at a steady speed, the inward force acting on the rotating balls is known as controlling force. It is equal and opposite to the centrifugal reaction.



Controlling force diagram

∴ Controlling force, $F_C = m\omega^2 r$

The controlling force is provided by the weight of the sleeve and balls as in Porter governor and by the spring and weight as in Hartnell governor (or spring controlled governor).

When the graph between the controlling force (F_C) as ordinate and radius of rotation of the balls (r) as abscissa is drawn, then the graph obtained is known as controlling force diagram. This diagram enables the stability and sensitiveness of the governor to be examined and also shows clearly the effect of friction.

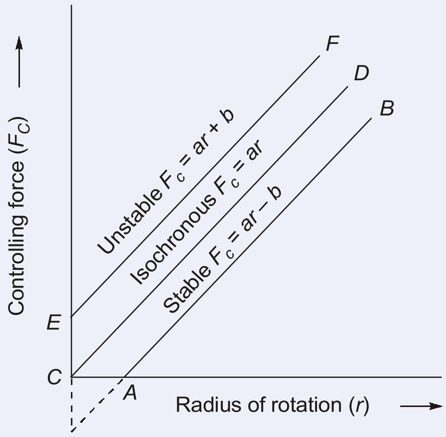
Controlling force diagram for spring-controlled governors:

The controlling force diagram for the spring controlled governors is a straight line, as shown in figure. We know that controlling force,

$$F_C = m\omega^2 r \text{ or } \frac{F_C}{r} = m\omega^2$$

The following points, for the stability of spring-controlled governors, may be noted:

1. For the governor to be stable, the controlling force (F_C) must increase as the radius of rotation (r) increases, i.e. $\frac{F_C}{r}$ must increase as r increases. Hence the controlling force line AB when produced must intersect the controlling force axis below the origin, as shown in figure.



The relation between the controlling force (F_C) and the radius of rotation (r) for the stability of spring controlled governors is given by the following equation

$$F_C = ar - b \quad \text{... (i)}$$

where a and b are constants.

2. The value of b in equation (i) may be made either zero or positive by increasing the initial tension of the spring. If b is zero, the controlling force line CD passes through the origin and the governor becomes isochronous because $\frac{F_C}{r}$ will remain constant for all radii of rotation.
   The relation between the controlling force and the radius of rotation, for an isochronous governor is, therefore,

$$F_C = ar \quad \text{... (ii)}$$

3. If b is greater than zero or positive, then $\frac{F_C}{r}$ decreases as r increases, so that the equilibrium speed of the governor decreases with an increase of the radius of rotation of balls, which is impracticable.
   Such a governor is said to be unstable and the relation between the controlling force and the radius of rotation is, therefore,

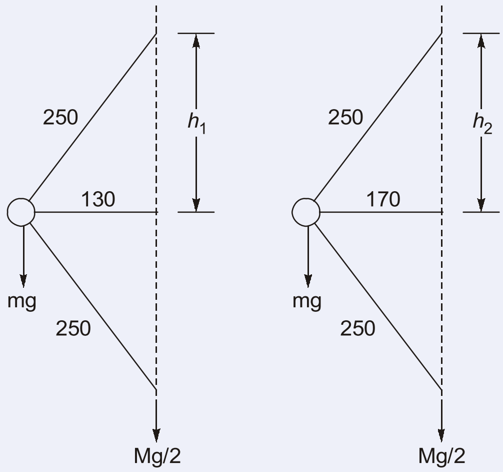
$$F_C = ar + b$$

End of Solution

Q.4(b)(ii) In case of a Porter governor each arm is 250 mm long and the arms are pivoted on the governor axis. The weight of the sleeve is 30 N and the weight of each ball is 50 N. The radii of rotation of the balls corresponding to minimum and maximum speeds are 130 mm and 170 mm respectively. Determine the range of speed. If the friction at the sleeve is taken equivalent to 25 N of load at the sleeve, determine how the speed range is modified.

[12 marks : 2025]

Solution:



Given: $mg = 50$ N, $Mg = 300$ N, $f = 25$ N

At minimum speed, $h = \sqrt{250^2 - 130^2} = 213.54$ mm

At maximum speed, $h = \sqrt{250^2 - 170^2} = 183.3$ mm

General formula for range of speed is given as

$$N^2 = \frac{895}{h}\left[\frac{2mg + (Mg \pm f)(1+k)}{2\,mg}\right]$$

Case 1 ($k = 1$, $f = 0$):

$$N^2 = \frac{895}{h}\left(\frac{M+m}{m}\right)$$

$$N_{min}^2 = \frac{895}{213.54}\left(\frac{50+300}{50}\right) \times 1000 = 29338.76$$

$$N_{min} = 171.286 \text{ rpm}$$

$$N_{max}^2 = \frac{895}{0.1833}\left(\frac{50+300}{50}\right) = 34178.94$$

$$N_{max} = 184.87 \text{ rpm}$$

Range of speed without friction = 184.87 – 171.286 = 13.584 rpm **Ans. (Part 1)**

Case 2 ($k = 1$, $f \neq 0$):

When friction at the sleeve is 25 N

At minimum speed, $$N^2 = \frac{895}{h}\left[\frac{mg + (Mg - f)}{mg}\right] = \frac{895}{0.21354}\left(\frac{50+300-25}{50}\right)$$

$$N_{min} = 165.05 \text{ rpm}$$

At maximum speed, $$N^2 = \frac{895}{0.1833}\left(\frac{50+300+25}{50}\right)$$

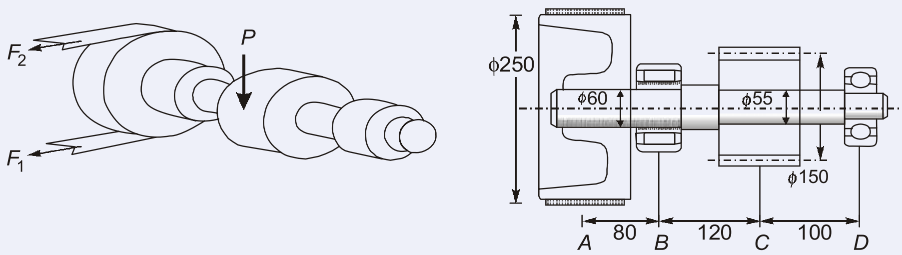
$$N_{max} = 191.36 \text{ rpm}$$

Range of speed with friction = 191.36 – 165.05

= 26.31 rpm **Ans. (Part 2)**

End of Solution

Q.4 (c) The horizontal shaft ABCD is mounted in bearings at *B* and *D* as shown in figure. A belt passes around the 250 mm diameter pulley fixed to the shaft at *A* and a gear pinion of 150 mm pitch diameter is mounted on the shaft at *C*. Shaft diameters and axial disposition of the components are as sketched. The belt strand tensions are horizontal and in the ratio $\frac{F_1}{F_2} = 4$, while the vertical reaction on the pinion, *P*, acts tangentially to the pinion's pitch circle. Ascertain the shaft's safety factor when transferring 20 kW from belt to pinion at a steady 450 rpm, taking the yield strength of the ductile shaft material to be 500 MPa. Fatigue and stress concentrations are to be neglected. All dimensions are in mm. Use Maximum Shear Stress Theory.



[20 marks : 2025]

Solution:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 450}{60} = 47.124 \text{ rad/s}$$

Torque, $$T = \frac{P}{\omega} = \frac{20 \times 10^3}{47.124} = 424.4 \text{ N.m}$$

Tight side tension on pulley = F_1

Slack side tension on pulley = F_2

$$\text{Torque on pulley} = (F_1 - F_2)\frac{d_p}{2}$$

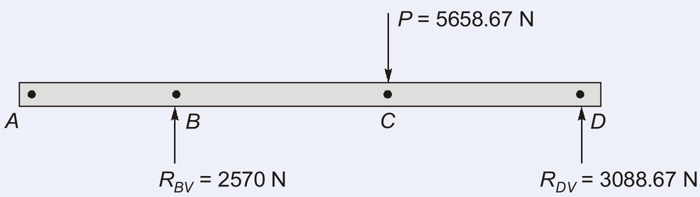
$$424.4 \times 10^3 = (4\ F_2 - F_2) \times \frac{250}{2}$$

$$F_2 = 1131.73 \text{ N}$$

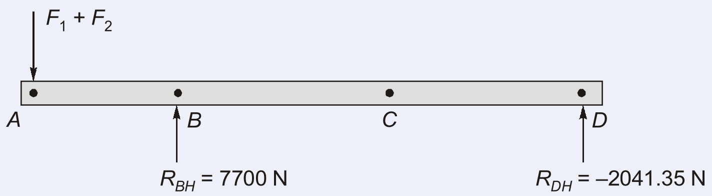
$$F_1 = 4 \times 1131.73 = 4526.92 \text{ N}$$

$$\text{Torque transmitted by gear} = P \times \frac{d_g}{2}$$

$$424.4 \times 10^3 = P \times \frac{150}{2}$$



FBD of shaft for vertical load



FBD of shaft for horizontal load

Bending moment at A, $M_A = 0$

Bending moment at B, $M_B = (F_1 + F_2)\ AB$

$= (1131.73 + 4526.92) \times 80 \times 10^{-3}$

$= 452.69$ N.m

Bending moment at C, $M_C = \sqrt{(R_{DV})^2 + (R_{DH})^2} \times CD$

$= \sqrt{(3088.67)^2 + (2041.35)^2} \times 100 \times 10^{-3}$

$= 370.23$ N.m

Bending moment at D, $M_D = 0$

Now, we have to find out critical section

Bending stress at B, $\sigma_B = \dfrac{32 \times 452.69 \times 10^3}{\pi \times 60^3} = 21.35$ MPa

Bending stress at C, $\sigma_C = \dfrac{32 \times 370.23 \times 10^3}{\pi \times 55^3} = 22.67$ MPa

As $\sigma_C > \sigma_B$, section C is critical

$$\tau_{max} = \frac{\sigma_{yield}}{2} = \frac{500}{2} = 250 \text{ N/mm}^2$$

Equivalent twisting moment, $T_e = \sqrt{M^2 + T^2}$

$T_e = \sqrt{(370.23)^2 + (424.4)^2}$

$T_e = 563.2$ N.m

$$\tau = \frac{16T_e}{\pi d^3} = \frac{16 \times 563.2 \times 10^3}{\pi \times 55^3} = 17.24 \text{ N/mm}^2$$

Factor of safety, $N = \dfrac{250}{17.24} = 14.5$ **Ans.**

End of Solution

SECTION : B

Q.5 (a) A manufacturing unit has to supply 4200 units of a product per year to the customer. The set-up cost per run is 75. Inventory carrying cost is 1.5 per unit per annum. Shortages are not permitted.

Determine the following:

(i) Economic order quantity

(ii) Optimum number of order per annum

(iii) Average annual inventory cost (minimum)

(iv) Optimum period of supply per optimum order

[12 marks : 2025]

Solution:

$$D = 4200 \text{ units/year}, \; A = \text{Rs.}75 \text{ per run}, \; H = \text{Rs.}1.5\text{/unit/year}$$

$$EOQ = \sqrt{\frac{2AD}{H}} = \sqrt{\frac{2 \times 75 \times 4200}{1.5}} = 648 \text{ units per procurement (rounded off)}$$

Ans.(i)

$$\text{Number of orders per annum} = \frac{4200}{648} = 6.48 \text{ orders per year} \approx 7 \text{ order per year}$$

Ans. (ii)

$$\text{Total minimum annual inventory cost} = \sqrt{.2\,ADH} = \sqrt{2 \times 4200 \times 75 \times 1.5}$$

$$= \text{Rs.}972.11$$

Ans. (iii)

Optimum period of supply per optimum order

$$\frac{1}{N} = \frac{1}{7} = 0.143 \text{ years}$$

Ans. (iv)

End of Solution

Q.5 (b) A round specimen of wrought iron of diameter 12.5 mm and gauge length of 100 mm was tested in tension up to fracture. Following observations were obtained:

Load up to yield point = 29.5 kN, Maximum load = 44 kN,

Load at time of fracture = 37 kN, Diameter at neck = 9.2 mm,

Total extension of specimen = 28.5 mm

Calculate:

(i) yield strength (ii) ultimate strength

(iii) actual breaking stress (iv) percentage elongation

(v) modulus of resilience at yield point stress

Take Young's modulus E=200 kN/mm^2.

[12 marks : 2025]

Solution:

$$\text{Original diameter, } d = 12.5 \text{ mm}$$

$$\text{Gauge length, } L = 100 \text{ mm}$$

$$\text{Diameter at neck, } d' = 9.2 \text{ mm}$$

Original cross-sectional area, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 12.5^2$

$= 122.72 \text{ mm}^2$

c/s area at neck, $A' = \frac{\pi}{4}d'^2 = \frac{\pi}{4} \times 9.2^2 = 66.476 \text{ mm}^2$

Yield strength, $\sigma_{yp} = \frac{\text{Load upto yield point}}{\text{Original c/s area}}$

$= \frac{29.5 \times 10^3}{122.72} = 240.385 \text{ N/mm}^2$ **Ans (i)**

UTS, $\sigma_{ut} = \frac{44 \times 10^3}{122.72} = 358.54 \text{ N/mm}^2$ **Ans (ii)**

Actual breaking strength, $\sigma_{bk} = \frac{\text{fracture load}}{\text{neck area}}$

$= \frac{37 \times 10^3}{66.476} = 556.6 \text{ N/mm}^2$ **Ans. (iii)**

Percentage elongation $= \frac{28.5}{100} \times 100$

$= 28.5\%$ **Ans. (iv)**

Modulus of resilience at yield point stress $= \frac{\sigma_{yp}^2}{2E}$

$$U_R = \frac{(240.385)^2 \times 10^6}{2 \times 200 \times 10^3} \frac{N}{m^2}$$

$= 144.46 \times 10^3 \text{ J/m}^3 = 0.1445 \text{ MJ/m}^3$ **Ans. (v)**

End of Solution

Q.5 (c) Two systems are represented by its system matrix as given below. Determine the characteristic equation, its roots and establish the stability of each of the system.

(i) $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ (i) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

[12 marks : 2025]

Solution:

(i)

We have to find out eigenvalues λ_1, λ_2 and λ_3 using characteristic equation of the matrix A,

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\det\left(\begin{bmatrix} -1-\lambda & 0 & 0 \\ 0 & -2-\lambda & 0 \\ 0 & 0 & -3-\lambda \end{bmatrix}\right) = 0$$

$$(\lambda + 1)(\lambda + 2)(\lambda + 3) = 0$$

Roots are $\lambda_1 = -1, \lambda_2 = -2, \lambda_3 = -3$

All given values are negative real parts, so the system is asymptotically stable.

(ii)

$$\det\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & -3 & 3-\lambda \end{bmatrix}\right) = 0$$

$$-\lambda\begin{vmatrix} -\lambda & 1 \\ -3 & 3-\lambda \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$-\lambda[-\lambda(3-\lambda) + 3] - 1 \times -1 = 0$$

$$-\lambda[-3\lambda + \lambda^2 + 3] + 1 = 0$$

$$+ 3\lambda^2 - \lambda^3 - 3\lambda + 1 = 0 = -(\lambda - 1)^3$$

$$\lambda_1, \lambda_2, \lambda_3 = 1$$

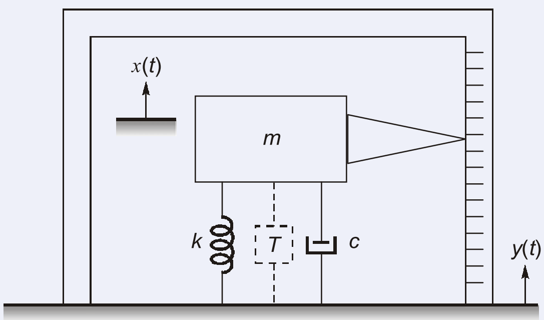
The roots are positive real parts so unstable system. Ans.

End of Solution

Q.5 (d) A machine component performing a harmonic motion and a vibrometer having a natural frequency of 5 rad/sec and damping ratio = 0.3 is attached to this machine component. If the difference between the maximum and the minimum recorded values is 6 mm, find the amplitude of motion of the vibrating component, when its frequency is 30 rad/sec.

[12 marks : 2025]

Solution:



The vibrating body is assumed to have a harmonic motion:

$$y(t) = Y \sin \omega t$$

The equation of motion of the mass m can be written as

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$m\ddot{z} + c\dot{z} + kz = m\ddot{y}\ [z = x - y]$$

$$m\ddot{z} + c\dot{z} + kz = m\omega^2\ Y \sin \omega_t$$

Steady state solution

$$Z = \frac{Y\omega^2}{\left[(k - m\omega^2)^2 + (c\omega)^2\right]^{\frac{1}{2}}} = \frac{r^2 Y}{\left[(1 - r^2)^2 + (2\xi r)^2\right]^{\frac{1}{2}}}$$

Where $r = \dfrac{\omega}{\omega_n}$ and $\xi = \dfrac{c}{2m\omega_n}$

Amplitude of recorded motion, $Z = \dfrac{\text{Peak to peak value}}{2} = 3\text{ mm}$

$$Z = \frac{Y\left(\frac{30}{5}\right)^2}{\left[(1 - 6^2)^2 + (2 \times 0.3 \times 6)^2\right]^{\frac{1}{2}}}$$

$$= \frac{36\ Y}{\sqrt{35^2 + 3.6^2}} = \frac{36\ Y}{35.185}$$

$$Y = 3 \times \frac{35.185}{36} = \frac{35.185}{12} = 2.932\text{ mm}$$ **Ans.**

End of Solution

Q.5 (e) Determine the inverse of the following transformation matrix:

$$T = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 2 \\ 0.369 & 0.819 & 0.439 & 5 \\ -0.766 & 0 & 0.643 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[12 marks : 2025]

Solution:

For 4 × 4 homogeneous matrix,

$$T = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 2 \\ 0.369 & 0.819 & 0.439 & 5 \\ -0.766 & 0 & 0.643 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse of T will be

$$T^{-1} = \begin{bmatrix} n_x & n_y & n_z & -P\cdot n \\ o_x & o_y & o_z & -P\cdot o \\ a_x & a_y & a_z & -P\cdot a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-(P\cdot n) = -(P_x n_x + P_y n_y + P_z n_z)$$
$$= -(2\times 0.527 + 5\times 0.369 + 3\times(-)\,0.766)$$
$$= -0.601$$

$$-(P\cdot o) = -(P_x o_x + P_y o_y + P_z o_z)$$
$$= -(2\times(-)\,0.574 + 5\times 0.819 + 3\times 0)$$
$$= -2.947$$

$$-(P\cdot a) = -(P_x a_x + P_y a_y + P_z a_z)$$
$$= -(2\times 0.628 + 5\times 0.439 + 3\times 0.643)$$
$$= -5.38$$

$$T^{-1} = \begin{bmatrix} 0.527 & 0.369 & -0.766 & -0.601 \\ -0.574 & 0.819 & 0 & -2.947 \\ 0.628 & 0.439 & 0.643 & -5.38 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End of Solution

Q.6 (a) In an organisation, manufacturing of a component is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers.

Activity	Estimated duration (weeks)		
($i - j$)	Optimistic	Most likely	Pessimistic
1 - 2	2	2	8
1 - 3	2	5	8
1 - 4	3	3	9
2 - 5	2	2	2
3 - 5	3	6	15
4 - 6	3	6	9
5 - 6	4	7	16

(i) Draw the network diagram of activities.

(ii) Find the expected duration and variance for each activity. What is the expected project length.

(iii) Determine the critical path.

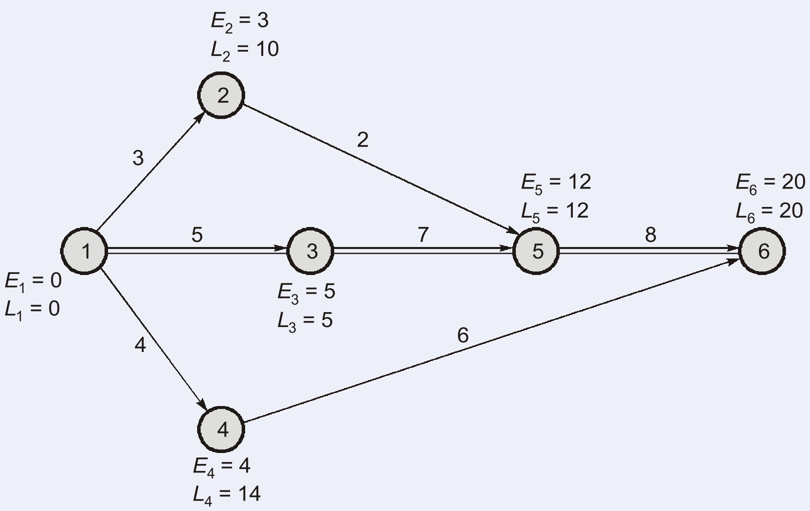
(iv) Calculate the variance and standard deviation of the project length.

(v) The earliest and latest expected completion time of each event.

[20 marks : 2025]

Solution:

Activity Sequence	Time duration (weeks)			$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
	t_0	t_m	t_p		
1 – 2	2	2	8	$\frac{2+4\times2+8}{6} = 3$	1
1 – 3	2	5	8	$\frac{2+5\times4+8}{6} = 5$	1
1 – 4	3	3	9	$\frac{3+4\times3+9}{6} = 4$	1
2 – 5	2	2	2	$\frac{2+4\times2+2}{6} = 2$	0
3 – 5	3	6	15	$\frac{3+4\times6+15}{6} = 7$	4
4 – 6	3	6	9	$\frac{3+4\times6+9}{6} = 6$	1
5 – 6	4	7	16	$\frac{4+4\times7+16}{6} = 8$	4



Network Diagram

Various path	Length of the path
1 - 2 - 5 - 6	3 + 2 + 8 = 13 Weeks
1 - 3 - 5 - 6	5 + 7 + 8 = 20 Weeks
1 - 4 - 6	4 + 6 = 10 Weeks

Expected project length = 20 weeks

Critical path = 1 - 3 - 5 - 6

Activity	Variance (σ^2)	Standard deviation
1 - 3	1	1
3 - 5	4	2
5 - 6	4	2
	$\Sigma\sigma^2 = 9$	

Variance of project length = 9

Standard deviation of project length = $\sqrt{9} = 3$ Weeks

End of Solution

Q.6 (b) In a machining operation, under orthogonal cutting condition with a cutting tool of rake angle 12°, the following data were observed:

Vertical component of cutting force = 1600 N

Horizontal component of cutting force = 1250 N

Chip thickness ratio = 0.25, Cutting speed = 200 m/min

Calculate the following:

(i) Friction force along the rake face, (ii) Normal force on the rake face

(iii) Resultant cutting force

(iv) Shear force along the shear plane

(v) Normal force on the shear plane

(vi) Work done in shear

(vii) Work done in friction

[20 marks : 2025]

Solution:

Given: F_C = 1600 N, F_T = 1250 N, r = 0.25, V = 200 m/min.

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} = \frac{0.25\cos 12°}{1 - 0.25\sin 12°} = 0.258$$

$$\phi = \tan^{-1}(0.258) = 14.46°$$

$$F = F_C \sin\alpha + F_T \cos\alpha$$
$$= 1600 \sin 12° + 1250 \cos 12°$$
$$= 1555.34 \text{ N}$$ Ans. (i)

$$N = F_C \cos\alpha - F_T \sin\alpha$$
$$= 1600 \cos 12° - 1250 \sin 12°$$
$$= 1305.15 \text{ N}$$ Ans. (ii)

$$R = \sqrt{F_C^2 + F_T^2} = \sqrt{1600^2 + 1250^2} = 2030.4 \text{ N}$$ Ans. (iii)

$$F_S = F_C \cos\phi - F_T \sin\phi$$
$$= 1600 \cos 14.46° - 1250 \sin 14.46°$$
$$= 1237.19 \text{ N}$$ Ans. (iv)

$$F_N = F_C \sin\phi + F_T \cos\phi$$
$$= 1600 \sin 14.46° + 1250 \cos 14.46°$$
$$= 1609.93 \text{ N}$$ Ans. (v)

$$W_{shear} = F_S V_S = \frac{F_S V \cos\alpha}{\cos(\phi - \alpha)}$$

$$= \frac{1237.19(\cos 12°) \times 200}{\cos(14.46° - 12°)} \times \frac{1}{60}$$

$$= 4037.57 \text{ Watt}$$ Ans. (vi)

$$W_{friction} = FV_C = FrV = \frac{1555.34 \times 0.25 \times 200}{60}$$

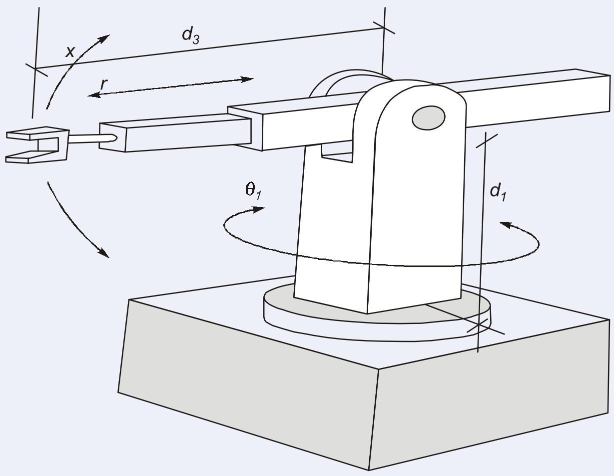
$$= 1296.12 \text{ Watt}$$ Ans. (vii)

End of Solution

Q.6 (c) Generate the forward kinematic model of the 3 DOF (RRP) Spherical Manipulator Arm, shown in the figure, by

(i) generating and drawing the frames using DH rules
(ii) generating the DH parameters table from the assigned frame
(iii) generating the individual transformation matrices, ${}^0T_1$, ${}^1T_2$ and ${}^2T_3$
(iv) generating the overall transformation matrix ${}^0T_3$

Note: {0}th frame will be at the base of the manipulator.



The homogeneous transformation matrix ${}^{i-1}T_i$ is given as

$${}^{i-1}T_i = \left[\begin{array}{ccc|c} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

[20 marks : 2025]

Solution:

Generating and drawing frames using D-H rules:

(i) We have 3DOF:

- **Joint 1:** Revolute (θ_1) - Base rotation about τ-axis.
- **Joint 2:** Revolute (θ_2) - Arm elevation about x-axis.
- **Joint 3:** Prismatic (d_3) - Linear extension (along z-axis of frame 2)

DH rules:

- Place z-axis along the joint axis.
- Place x-axis along the common normal between z_{i-1} and z_i
- Place origin at intersection or along common normal.

So frames:

- Frame {0} at base, z_0 is axis of joint 1.
- Frame {1} rotates with θ_1
- Frame {2} rotates with θ_2
- Frame {3} translates with d_3

(ii) DH parameter table

Standard DH parameters:

Link i	a_i	α_i	d_i	θ_i
1	0	$+\frac{\pi}{2}$	0	θ_1
2	0	$-\frac{\pi}{2}$	0	θ_2
3	0	0	d_3	0

Explanation:

- a_i = 0 (all axes intersect)
- α_i = +90° (between z_0 and z_1)
- α_2 = –90° (between z_1 and z_2)
- α_3 = 0 (Prismatic along z_2)

(iii) Individual transformation matrices:

$$^0T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Frame 0 → Frame 1 ($^0T_1$)

$$a_1 = 0,\ d_1 = 0,\ \alpha_1 = +90^\circ$$

$$^0T_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} C\theta_2 & 0 & -S\theta_2 & 0 \\ S\theta_2 & 0 & C\theta_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_2 = 0,\ d_2 = 0,\ \alpha_2 = -90^\circ$$

$$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

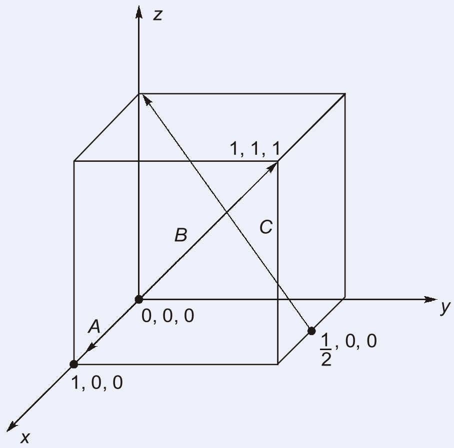
$$a_3 = 0,\ \theta_3 = 0,\ d_3 = d_3$$

Overall transformation, $\quad {}^0T_3 = {}^0T_1 \times {}^1T_2 \times {}^2T_3$

$$^0T_3 = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & C_1S_2d_3 \\ S_1C_2 & C_1 & S_1S_2 & S_1S_2d_3 \\ -S_2 & 0 & C_2 & C_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End of Solution

Q.7 (a) Explain the meaning of Miller indices in a unit crystal cell. Determine the Miller indices of directions *A*, *B* and *C* in figure below.



[20 marks : 2025]

[Starting point of direction C is given wrong, it is (1/2, 1, 0) instead of (1/2, 0, 0)]

Solution:

Miller's indices are a notation system in crystallography used to describe the orientation of crystal planes and directions in a crystal lattice. They are expressed as a set of three integers (*h*, *k*, *l*) that correspond to the intercepts of the crystal plane with the axes of the crystal lattice.

Definition and Calculation

1. Intercepts: Identify where the plane intersects the axes of the unit cell in terms of the unit cell dimensions (*a*, *b*, *c*).
2. Reciprocal: Take the reciprocal of the intercepts.
3. Clearing fractions: Multiply by the least common multiple (LCM) to convert the numbers to the smallest integers.

Direction A

1. Two points are 1, 0, 0 and 0, 0, 0
2. (1, 0, 0) - (0, 0, 0) = 1, 0, 0

3. No fractions to clear or integers to reduce
4. [1 0 0]

Direction B

1. Two points are 1, 1, 1 and 0, 0, 0
2. (1, 1, 1) - (0, 0, 0) = 1, 1, 1
3. No fractions to clear or integers to reduce
4. [1 1 1]

Direction C

1. Two points are 1/2, 1, 0 and 0, 0, 1
2. (0, 0, 1) - (1/2, 1, 0) = –1/2, –1, 1
3. 2 (–1/2, –1, 1) = –1, –2, 2
4. $[\bar{1}\,\bar{2}\,2]$ Ans.

End of Solution

Q.7(b)(i) A shaper machine is used to machine a medium carbon steel workpiece of 225 mm in length and 125 mm in width. The shaper machine is operated at 125 cutting strokes per minute, feed of 0.5 mm per stroke and a depth of cut of 5 mm. The forward stroke is completed in 220°.
Calculate the percentage of time when the tool is not contacting the workpiece also calculate the total machining time for machining the component. Assume that approach distance = 30 mm.

[10 marks : 2025]

Solution:

Number of strokes per min, S = 125 strokes/min
Length of workpiece, l = 225 mm
Width of workpiece, W = 125 mm
Feed, f = 0.5 mm per stroke
Depth of cut, d = 5 mm
Forward stroke angle, θ_f = 220°
Approach distance, A = 30 mm
Total stroke length, $L = l + 30 = 225 + 30 = 255$ mm

$$\text{Number of strokes for cutting} = \frac{\text{Width of workpiece}}{\text{Feed}} = \frac{125}{0.5} = 250$$

$$\text{Time for completing one stroke}, T = \frac{L}{S} = \frac{255}{125} = 2.04 \text{ min}$$

The percentage of time when the tool is not contacting the workpiece is given by

$$= \frac{360° - \theta_f}{360°} = \frac{360° - 220°}{360°} = 0.389 \text{ or } 38.9\%$$ Ans.

Total machining time = $T \times S_N = 2.04 \times 250 = 510$ minutes Ans.

End of Solution

Q.7(b)(ii) A milling operation is carried out on low carbon steel workpiece. A milling cutter of 125 mm diameter having 10 teeth is operated at 2.5 m/min to perform the milling operation. The table feed rate is 120 mm/min and depth of cut is 5 mm. Calculate the following:

(i) Length of the chip in up and down milling operation.

(ii) Change in path length from up to down milling.

[10 marks : 2025]

Solution:

$$\text{Spindle speed} = \frac{1000 \times 25}{\pi \times 125} = 63.662 \text{ rev/min}$$

$$\text{Feed rate of cutter} = \frac{120}{63.662} = 1.885 \text{ mm/rev.}$$

$$\text{Tooth contact angle, } \phi_c = \cos^{-1}\left[\frac{D-2d}{D}\right] = \cos^{-1}\left[\frac{125-10}{125}\right]$$

$$= 23.074° \text{ or } 0.4027 \text{ radian}$$

Up-milling path length is

$$U_L = \frac{D\phi_c}{2} + \frac{f\sqrt{d(D-d)}}{\pi D}$$

$$= \frac{125 \times 0.4027}{2} + \frac{1.885\sqrt{5(125-5)}}{\pi \times 125}$$

$$= 25.17 + 0.1176 = 25.2876 \text{ mm}$$ **Ans.**

Down-milling path length is $D_L = \frac{D\phi_c}{2} - \frac{f\sqrt{d(D-d)}}{\pi \times D}$

$$= 25.17 - 0.1176 = 25.0524 \text{ mm}$$ **Ans.**

Change in path length from up to down milling is

$$\Delta L = U_L - D_L = 2 \times 0.1176 = 0.2352 \text{ mm}$$ **Ans.**

End of Solution

Q.7(c)(i) For the given DH parameters table of a manipulation arm; generate the frames as per DH rules. Also give reasons for selection of origins of the frames and the corresponding coordinate axes (X, Y, Z), frame wise, i.e. from frame {0} to {3}.

	d_i	θ_i	a_i	α_i
${}^0T_1$	0	θ_1	0	90°
${}^1T_2$	0	θ_2	L_2	0
${}^2T_3$	0	θ_3	L_3	0

[10 marks : 2025]

Solution:

From D-H parameter Table:

Frame: 0-1

[From transformation frame (0) to frame (1) there is rotation of θ_1 about z-axis (CCW) & rotation of 90° about x-axis (CCW)]

$$ {}^{0}T_{1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Frame: 1-2

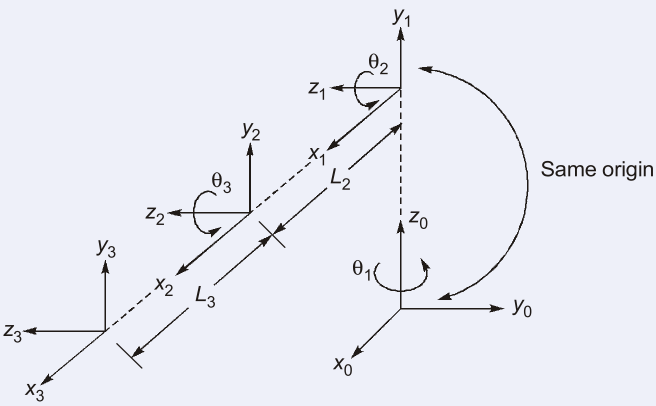
Rotation about z-axis by θ_2 [CCW] and translation (Link length) along x-axis

$$ {}^{1}T_{2} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Frame: 2-3

Rotation about z-axis by θ_3 (CCW) and link 3 translation along x-axis

$$ {}^{2}T_{3} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & L_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



End of Solution

Q.7(c)(ii) The overall transformation matrix of a 3 DOF manipulator arm is given below:

$$ {}^{0}T_{3} = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & -d_3\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & d_3\cos\theta_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(i) Use inverse kinematic modelling to generate the expressions for the joint parameters θ_1, d_2 and d_3.

(ii) Determine all possible values of θ_1, d_2 and d_3 for the above manipulator arm from the following overall transformation matrix.

$$ \begin{bmatrix} 0.866 & 0 & -0.5 & -5 \\ 0.5 & 0 & 0.866 & 8.66 \\ 0 & -1 & 0 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

[10 marks : 2025]

Solution:

(i) Given: Transformation matrix

$$^0T_3 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & -d_3\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & d_3\cos\theta_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let denote this

$$^0T_3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the rotation matrix, $r_{11} = \cos\theta_1$

$r_{21} = \sin\theta_1$

$$\theta_1 = \tan^{-1}\frac{r_{21}}{r_{11}}$$

From the position matrix, $P_x = -d_3\sin\theta_1 \Rightarrow d_3 = -\frac{P_x}{\sin\theta_1}$

$$P_y = d_3\cos\theta_1 \Rightarrow d_3 = \frac{P_y}{\cos\theta_1}$$

$$P_z = d_2$$

$\therefore$ $$d_3 = -\frac{P_x}{\sin\theta_1} \text{ or } \frac{P_y}{\cos\theta_1} \text{ and } d_2 = P_z$$

(ii) Given: Numerical matrix, $$^0T_3 = \begin{bmatrix} 0.866 & 0 & -0.5 & -5 \\ 0.5 & 0 & 0.866 & 8.66 \\ 0 & -1 & 0 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now comparing elements from the first column (rotation matrix)

$$\cos\theta_1 = 0.866$$

$$\sin\theta_1 = 0.5$$

$$\theta_1 = \tan^{-1}\frac{0.5}{0.866} = \tan^{-1}(0.577) \sim 30°$$

Now, $$d_3 = -\frac{P_x}{\sin\theta_1} = \frac{-(-5)}{0.5} = 10$$

and $d_2 = P_z = 15$

$\therefore$ $\theta_1 = 30°,\ d_2 = 15,\ d_3 = 10$

End of Solution

Q.8(a)(i) Explain the 'laws of corrosion'. Give examples of two metals following the law in each case. Oxidation loss on copper surface is 0.05 mm in 15 hours. How much will be the loss in 225 hours?

[10 marks : 2025]

Solution:

Faraday's Law: It states that the rate of corrosion (mass loss of metal) is directly proportional to the corrosion current the amount of electric charge flowing due to electrochemical reactions at the metal's surface. Faraday's law is fundamental to understanding and quantifying corrosion. It establishes a direct relationship between the amount of material corroded and the electrical charge passed during the electrochemical process of corrosion. Corrosion involves the flow of electric current between anodic and cathodic areas on a metal surface.

Faraday's First Law: The mass of a substance (metal in this case) transformed during electrolysis (corrosion) is directly proportional to the quantity of electricity passed through the system.

Faraday's Second Law: The masses of different substances transformed by the same amount of electricity are proportional to their equivalent weights.

Time-dependent power law: A time-dependent power law is a mathematical model used to describe how the severity or extent of corrosion changes over time, typically following an equation where the amount of material loss or gain is proportional to time raised to a specific exponent. This model is applied to various corrosion scenarios, such as the atmospheric corrosion of copper, high-temperature oxidation of alloys, and the pitting corrosion of aluminum in seawater, to analyze the processes involved, which often involve both diffusion and chemical reactions.

Example of Metals following Faraday's Law:

Iron (Fe): The rusting of iron (formation of iron oxide) follows Faraday's Law, where the amount of iron corroded is proportional to the measured corrosion current in the environment.

Zinc (Zn): In galvanic cells or as a sacrificial anode, the dissolution of zinc also strictly follows Faraday's Law, with mass loss accurately calculated from the anodic current.

Example of metals following time-dependent power laws:

- **Copper (Cu):** The atmospheric corrosion of copper, such as the development of a green patina, adheres to a time-dependent power law, where the weight loss over time is predictable by such mathematical forms.
- **Austenitic Stainless Steel:** Oxidation at elevated temperatures leads to weight gain due to oxide formation, and the rate follows a similar time-dependent power law involving both diffusion and surface reaction kinetics.

The rate of oxidation loss is calculated by dividing the given loss by the corresponding time.

The rate of oxidation loss, R, is calculated as:

$$R = \frac{\text{Oxidation loss}}{\text{Time}} = \frac{0.05}{15} = 0.00333 \text{ mm/hour}$$

The total oxidation loss for the new time period is calculated by multiplying the determined rate of oxidation loss by the new time.

The total oxidation loss, L is calculated as:

$$L = R \times \text{New time}$$
$$= 0.00333 \text{ mm/hour} \times 225 \text{ hours}$$
$$= 0.74925 \text{ mm}$$

Final answer:

The oxidation loss in 225 hours will be approximately 0.75 mm.

End of Solution

Q.8(a)(ii) What are the different 'top-down' and 'bottom-up' methods for the synthesis of nano structured materials. With a sketch briefly explain the mechanical high energy ball milling for the synthesis of nano materials.

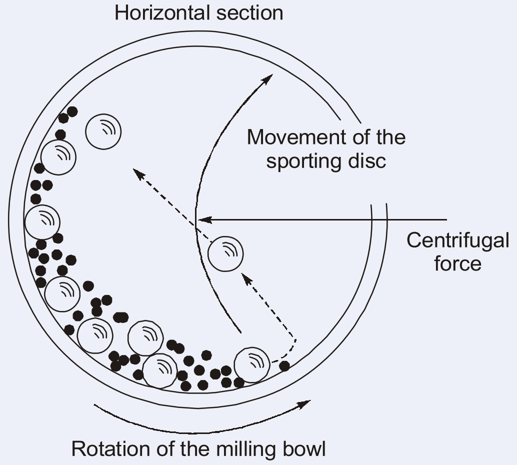
[10 marks : 2025]

Solution:

There are plenty of methods to synthesis nano materials. These methods are grouped into two categories namely Top-Down and Bottom-Up techniques. The techniques are classified based on the phase of the starting material. In the Top-down class of techniques, the starting material is in solid state, whereas in Bottom-Up techniques the starting material is either in gaseous state or in liquid state.

Top-down approach refers to a subtractive process in which a bulk starting material is divided into smaller ones of nanosize. Bottom-up approach refers to an additive process that starts with precursor atoms (or) molecules which combine to form nanosized structure. In bottom-up approach, nanostructures are built atom-by-atom or molecule-by-molecule. Depending on the requirement we have to select an appropriate method for preparation of nano materials.

S.No.	Top-down techniques	S.No.	Bottom-up techniques
1	Mechanical grinding (Ball milling)	1	Sol gel method
2	Lithography	2	Chemical Vapour Deposition
3	Etching	3	Physical Vapour Deposition
4	Erosion	4	Electrochemical Deposition
		5	Hydrothermal method
		6	Atomic layer deposition



High-energy ball milling (HEBM) synthesizes nanomaterials by subjecting a powder and milling balls to intense, high-energy mechanical collisions within a rotating container. This high-energy process repeatedly fractures, deforms, and welds the powder particles, breaking them down into nanometer-sized crystallites. The key factors are the high speed of the rotating mill and the balls, generating kinetic energy that induces mechanical and chemical changes to produce fine powders.

Ball milling is a mechanical process and thus all the structural and chemical changes are produced by mechanical energy.

1. Nano powders of 2 to 20 nm in size can be produced. The size of nano powder also depends upon the speed of the rotation of the balls.
2. It is an inexpensive and easy process.
3. This method produces crystal defects.

End of Solution

Q.8 (b) Calculate the power required to draw hot-drawn steel wire from 15 mm to 12.5 mm in diameter at 120 m/min. The coefficient of friction between the die and wire is 0.15 and die angle is 5°. Average flow stress for hot-drawn steel is 30 kgf/mm^2. Also calculate the maximum reduction possible. Assume that back pull = 0.

[20 marks : 2025]

Solution:

Given: d_o = 15 mm, μ = 0.15, d_f = 12.5 mm, α = 5°, v = 120 m/min, σ_{avg} = 30 kgf/mm^2.

$$\sigma_{avg} = 30 \times 9.807 \text{ N/mm}^2 \text{ or MPa}$$

$$= 294.21 \text{ MPa}$$

$$\sigma_d = \sigma_{avg}\left(\frac{1+B}{B}\right)\left[1-\left(\frac{d_f}{d_o}\right)^{2B}\right]$$

$$B = \mu\cot\alpha = 0.15 \times \cot 5° = 1.7145$$

$$\sigma_d = 294.21\left(\frac{1+1.7145}{1.7145}\right)\left[1-\left(\frac{12.5}{15}\right)^{2\times1.7145}\right]$$

$$= 216.51 \text{ MPa}$$

$$F_d = \sigma_d \times A_f = 216.5\times\frac{\pi}{4}(12.5)^2 = 26.5696 \text{ kN}$$

$$P = F_d u = 26.5696 \times \frac{120}{60}$$

$$P = 53.14 \text{ kW}$$

At maximum reduction, $\sigma_d = \sigma_f = \sigma_{avg}$

$$\sigma_d = \sigma_{avg}\left(\frac{1+B}{B}\right)\left[1-\left(\frac{A_{f\min}}{A_o}\right)^{B}\right]$$

$$1 = \left(\frac{1+1.7145}{1.7145}\right)\left[1-\left(\frac{A_{f\min}}{A_o}\right)^{1.7145}\right]$$

$$0.6316 = 1 - \left(\frac{A_{f\min}}{A_o}\right)^{1.7145}$$

$$\left(\frac{A_{f\min}}{A_o}\right)^{1.7145} = 0.3684$$

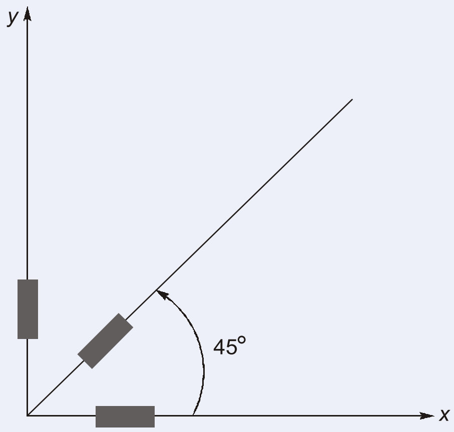
$$\left(\frac{A_{f\min}}{A_o}\right) = (0.3684)^{\frac{1}{1.7145}} = 0.5585$$

$$\%\text{Reduction} = \left(\frac{A_o - A_{f\min}}{A_o}\right) \times 100 = \left(1 - \frac{A_{f\min}}{A_o}\right) \times 100$$

$$= (1 - 0.5585) \times 100 = 44.143\%$$

End of Solution

Q.8 (c) A 3 element rectangular rosette is used at a certain point on a steel machine part as shown in the figure. Determine the principal strains and principal stresses using analytical expressions, if the measured strains are $\in_{0^\circ} = -220$ µm/m, $\in_{45^\circ} = 120$ µm/m and $\in_{90^\circ} = 220$ µm/m assuming $E = 200$ GPa and $\nu = 0.3$.



[20 marks : 2025]

Solution:

$$\in_x = \in_{0^\circ} = -220 \text{ µm/m}$$

$$\in_y = \in_{90^\circ} = 220 \text{ µm/m}$$

$$E = 200 \text{ GPa}, \ \nu = 0.3$$

$$\gamma_{xy} = 2\in_{45^\circ} - \in_x - \in_y$$

$$= 2 \times 120 + 220 - 220 = 240 \text{ µm/m}$$

$$\text{Principal strain, } \in_1, \in_2 = \frac{1}{2}(\in_x + \in_y) \pm \frac{1}{2}\sqrt{(\in_x - \in_y)^2 + \gamma_{xy}^2}$$

$$\Rightarrow \quad \in_{1,2} = \frac{10^{-6}}{2}\left[(-220+220) \pm \sqrt{(-440)^2 + (240)^2}\right]$$

$$\Rightarrow \quad \in_{1,2} = \frac{10^{-6}}{2}[0 \pm 501.198]$$

Major principal strain, $\in_1 = 250.59 \times 10^{-6}$

Minor principal strain, $\in_2 = -250.59 \times 10^{-6}$ **Ans.**

Major principal stress, $\sigma_1 = \frac{E}{1-\nu^2}[\nu \in_2 + \in_1]$

$$\sigma_1 = \frac{200 \times 10^3}{1-0.09}[0.3 \times -250.59 + 250.59] \times 10^{-6}$$

$\sigma_1 = 38.55$ MPa (Tensile) **Ans.**

$$\sigma_2 = \frac{E}{1-\nu^2}[\nu \in_1 + \in_2]$$

$$= \frac{200 \times 10^3}{1-0.09}[0.3 - 1] \times 250.59 \times 10^{-6}$$

$= -38.55$ MPa $= 38.55$ MPa (Compressive) **Ans.**

End of Solution

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