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ESE 2025

Main Exam Detailed Solutions

Electronics & Telecom. Engineering

PAPER-II

EXAM DATE : 10-08-2025 | 2:00 PM to 5:00 PM

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ANALYSIS

Electronics and Telecom. Engineering Paper-II
ESE 2025 Main Examination

Sl.	Subjects	Marks
1.	Analog and Digital Communication Systems	90
2.	Control Systems	80
3.	Microprocessors and Microcontrollers	40
4.	Electromagnetics	80
5.	Signals and Systems	30
6.	Computer Organization and Architecture	80
7.	Advanced Communication	40
8.	Advanced Electronics	40
	Total	**480**

Scroll down for detailed solutions

Section-A

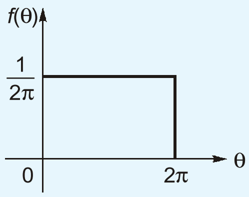
Q.1 (a) A random process $Y(t)$ is obtained by multiplication of a stationary process $X(t)$ with a sinusoidal wave $\cos(2\pi f_c t + \theta)$ where the phase θ is a random variable that is uniformly distributed over the interval $[0, 2\pi]$.

Express the power spectral density of random process $Y(t)$ in terms of power spectral density of $X(t)$. Assume that random variable θ is independent of $X(t)$.

[10 marks : 2025]

Solution:

$$Y(t) = X(t)\cos(2\pi f_c t + \theta)$$



Given that $X(t)$ and θ are independent.

Thus, auto-correlation function of $Y(t)$ is given by

$$ACF[Y(t)] = ACF[X(t)] \cdot ACF[\cos(2\pi f_c t + \theta)]$$

We have,

$$ACF[\cos(2\pi f_c t + \theta)] = E[\cos(2\pi f_c t + \theta) \cdot \cos(2\pi f_c (t - \tau) + \theta)]$$

$$= E\left[\frac{1}{2}\{\cos(4\pi f_c t - 4\pi f_c \tau + 2\theta) + \cos(2\pi f_c \tau)\}\right]$$

$$= \frac{1}{2}E[\cos(4\pi f_c t - 4\pi f_c \tau + 2\theta)] + \frac{1}{2}E[\cos(2\pi f_c \tau)]$$

Since θ is uniformly distributed over $[0, 2\pi]$. Thus,

$$ACF[\cos(2\pi f_c t + \theta)] = \frac{1}{2}\int_0^{2\pi} \cos(4\pi f_c t - 4\pi f_c \tau + 2\theta)\frac{1}{2\pi}d\theta + \frac{1}{2}\int_0^{2\pi}\cos(2\pi f_c \tau)\frac{1}{2\pi}d\theta$$

$$= 0 + \frac{1}{4\pi}\cos(2\pi f_c \tau) \times 2\pi$$

$$= \frac{1}{2}\cos(2\pi f_c \tau)$$

$$\therefore \quad R_Y(\tau) = R_X(\tau) \cdot \frac{1}{2}\cos 2\pi f_c \tau = \frac{1}{2}R_X(\tau)\left(\frac{e^{j2\pi f_c \tau} + e^{-j2\pi f_c \tau}}{2}\right)$$

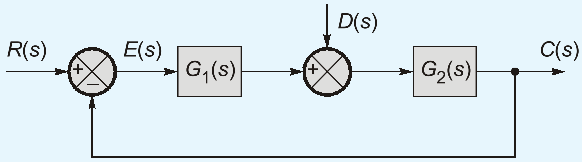
We know that the Fourier Transform of the auto-correlation function gives the power spectral density i.e. $R_X(\tau) \leftrightarrow S_X(f)$. Thus, using the time-shifting property of Fourier Transform, we obtain the power spectral density of $Y(t)$ as

$$S_Y(f) = \frac{1}{2}\left[\frac{S_X(f - f_c) + S_X(f + f_c)}{2}\right]$$

$$S_Y(f) = \frac{S_X(f - f_c) + S_X(f + f_c)}{4}$$

End of Solution

Q.1 (b) Consider the system shown below:

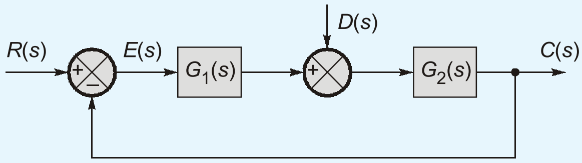


Here, $$G_1(s) = \frac{100(s+5)}{s+2} \;;\quad G_2(s) = \frac{5}{s(s+4)}$$

Determine the steady state error due to unit step input and a step disturbance of 10 unit.

[10 marks : 2025]

Solution:



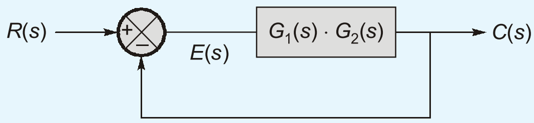
Given, $$G_1(s) = \frac{100(s+5)}{s+2}\;;\quad G_2(s) = \frac{5}{s(s+4)}$$

The steady-state error $E(s)$ is the difference between the desired value and the actual system output, given as $E(s) = R(s) - C(s)$.

(i) **Steady state error due to unit step input:** $[D(s) = 0]$

For a unit-step input, $$R(s) = \frac{1}{s}$$

Considering $D(s) = 0$, the block diagram can be drawn as below,



$$\frac{E(s)}{R(s)} = \frac{1}{1+G_1(s)G_2(s)}$$

$$\Rightarrow \quad E(s) = \left[\frac{1}{1+\dfrac{100(s+5)}{s+2}\times\dfrac{5}{s(s+4)}}\right]\times\frac{1}{s}$$

The steady-state error is given by

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{1}{1+\dfrac{100(s+5)}{s+2}\times\dfrac{5}{s(s+4)}}$$

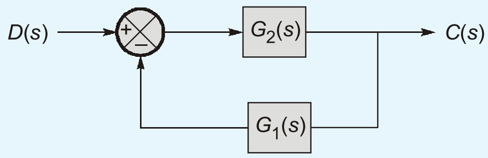
$$e_{ss} = \frac{1}{\infty} = 0$$

(ii) **Steady state error due to step disturbance of 10 unit:** $[R(s) = 0]$

For a step disturbance of 10 unit,

$$D(s) = \frac{10}{s}$$

Considering $R(s) = 0$, the block diagram can be drawn as below:



Here,
$$\frac{C(s)}{D(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)}$$

We have, $E(s) = -C(s)$. Thus,

$$E(s) = \left[\frac{-G_2(s)}{1+G_1(s)G_2(s)}\right]D(s)$$

$$E(s) = \left[\frac{\frac{-5}{s(s+4)}}{1+\frac{100(s+5)}{s+2}\times\frac{5}{s(s+4)}}\right]\times\frac{10}{s}$$

$$E(s) = \left(\frac{-5(s+2)}{s(s+2)(s+4)+500(s+5)}\right)\times\left(\frac{10}{s}\right)$$

The steady-state error is given by

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{-50(s+2)}{s(s+2)(s+4)+500(s+5)}$$

$$e_{ss} = \frac{-50\times 2}{500\times 5} = \frac{-1}{25}$$

End of Solution

Q.1 (c) **Find out the time complexity of the following code segment:**

for ($i = n/2$; $i < n$; i ++)

for ($k = 1$; $k < i$; $k * = 2$)

count += $n * n$;

[10 marks : 2025]

Solution:

The provided code segment is a nested loop. To determine its time complexity, we need to analyze how the number of operations scales as the input size n increases. Considering the code segment,

for ($i = n/2$; $i < n$; i ++) ; outer loop

for ($k = 1$; $k < i$; $k* = 2$) ; inner loop

count += $n * n$; 0(1) operation

Outer loop: Starts at $i = n/2$ and runs while $i < n$ incrementing i by 1 in each iteration.

Number of iterations = $n - \frac{n}{2} = \frac{n}{2}$

So, outer loop has time complexity $O(n)$.

Inner loop: Starts at $k = 1$ and iterates as long as k is less than i, with k doubling in each step.

This is log arithmetic loop. The number of times the loop runs for a given value of i is approximately $\log_2(i)$. The value of $\log_2(i)$ varies as i goes from $n/2$ to $n - 1$. The term $\log_2(i)$ has:

- minimum value of $\log_2 n/2 = \log_2 n - 1$
- maximum value of $\log_2 n - 1 \approx \log_2 n$

So, the inner loop has time complexity $O(\log_2 n)$

Overall Time Complexity: Since the number of outer loop iterations is $O(n)$ and the number of inner loop iterations is of the order of $O(\log_2 n)$, the total number of operations is approximately the product of these two complexities given by

$$O(n \log_2 n)$$

End of Solution

Q.1 (d) Determine the divergence of the vector field $\vec{A}$ and evaluate them at the specified point:

$$\vec{A} = \rho z \sin\phi \hat{a}_\rho + 3\rho z^2 \cos\phi \hat{a}_\phi \quad \text{at } (5, \pi/2, 1).$$

[5 + 5 = 10 marks : 2025]

Solution:

Given: $\vec{A} = \rho z \sin\phi \hat{a}_\rho + 3\rho z^2 \cos\phi \hat{a}_\phi$ at $(5, \pi/2, 1)$.

The expression for the divergence in a general curvilinear system is given by

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}(h_2 h_3 A_u) + \frac{\partial}{\partial v}(h_3 h_1 A_v) + \frac{\partial}{\partial w}(h_1 h_2 A_w)\right]$$

In cylindrical co-ordinate system,

$$u, v, w = \rho, \phi, z$$
$$h_1, h_2, h_3 = 1, \rho, 1$$

Comparing the given vector with $\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$, we have, $A_\rho = \rho z \sin\phi$, $A_\phi = 3z^2 \cos\phi$ and $A_z = 0$

$$\therefore \quad \vec{\nabla} \cdot \vec{A} = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho \cdot \rho z \sin\phi) + \frac{\partial}{\partial \phi}(3\rho z^2 \cos\phi)\right]$$

$$\Rightarrow \quad \vec{\nabla} \cdot \vec{A} = \frac{1}{\rho}\left[2\rho z \sin\phi - 3\rho z^2 \sin\phi\right]$$

$$\Rightarrow \quad \vec{\nabla} \cdot \vec{A} = (2z - 3z^2)\sin\phi$$

Now, $$\vec{\nabla} \cdot \vec{A}\Big|_{5, \pi/2, 1} = [2(1) - 3(1)^2]\sin\pi/2 = -1$$

End of Solution

Q.1 (e) Use Euler path method to find out optimal gate ordering for the stick diagram and layout of CMOS implementation of the Boolean expression $\overline{A(B+C)+DE}$.

[10 marks : 2025]

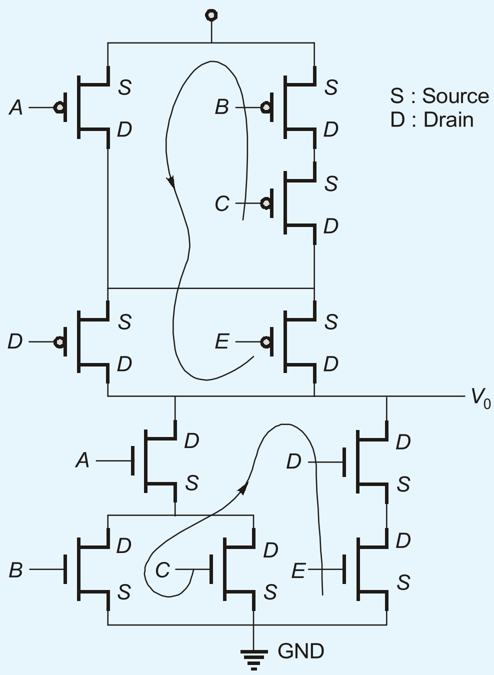
Solution:

Given, Boolean expression, $\overline{A(B+C)+DE}$

Euler path method:

1. **Boolean expression to CMOS implementation:**

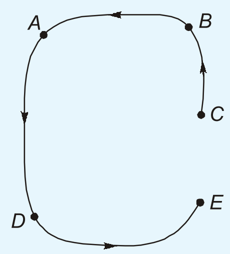
   The first step is to convert the Boolean expression into its CMOS transistor-level representation. This involves creating both the NMOS (pull-down) and PMOS (pull-up) networks as shown below.



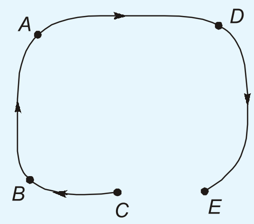
2. **Euler's path**

   Euler's path is obtained by tracing the entire MOS transistor in the circuit such that each transistor can be traced only once by considering any location as a starting point. The Euler path method aims to find a common Euler path in both the NMOS and PMOS networks. This common path dictates the gate ordering in the stick diagram.

   Euler's path for PMOS (pull-up) network

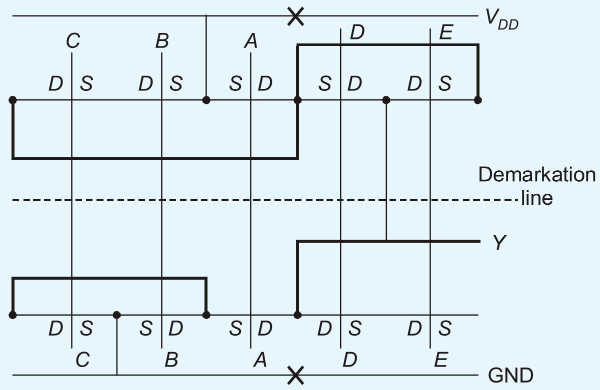


Euler's path for NMOS (pull-down) network

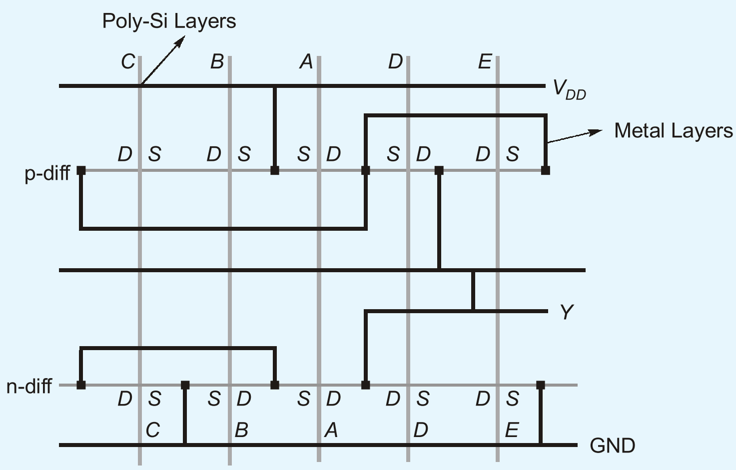


3. **Stick diagram:**

The stick diagram visually represents the Layout, showing the placement of transistors, polysilicon, diffusion (both n-type and p-type), and metal layers. The Euler path helps in arranging the transistors such that connections between them are minimized and the diffusion paths are continuous.



4. **Layout**



End of Solution

Q.1 (f) A bit stream 10011101 is received (LSB is received first). The transmitter is using standard CRC method with the generator polynomial $x^3 + 1$ Show the actual bit stream transmitted. Show that the error is detectable at the receiver's side.

[5 × 2 = 10 marks : 2025]

Solution:

Given: Received code : 10011101

Generator polynomial : $x^3 + 1 = x^3 + 0.x^2 + 0.x + 1$

Generator code = 1001

Receiver side: The received code is reversed because the LSB is received first.

Received code = 10111001

For detection of error, divide the received code with generator code

```
1001)10111001(10101
     1001↓↓
       1010
       1001↓↓
         1101
         1001
          100
```

Reminder is not zero ($R(x) \neq 0$). Thus, the received code word contains error.

The error is detectable at the receiver side due to non-zero syndrome.

The transmitted code word is

```
  1 0 1 1 1 0 0 1
⊕           1 0 0
  ---------------
  1 0 1 1 1 1 0 1
```

Sender side/transmitter side:

With generator polynomial of degree 3, the codeword length is $k + 3$, where k represents the bit stream length, Here, $k + 3 = 8$ i.e. $k = 5$. Thus, the bit stream transmitted is 10111.

To transmit the bit stream, we append three 0s at the end of message and the transmitted code word is obtained as below,

```
1001)10111000(10101
     1001↓↓
       1010
       1001↓↓
        01100
         1001
          101
```

Transmitted code word = 10111101

which is the same as obtained above. Thus, the actual bit stream transmitted is "10111".

End of Solution

Q.2 (a) Consider that, two input signals of a Binary Phase Shift Keying (BPSK) receiver are $\pm \sin 2\pi f_c t$ (where 'f_c' is carrier frequency). Draw the functional block diagram of a BPSK receiver to recover the bit stream of '0' and '1'. Give the necessary mathematical interpretation of the signals.

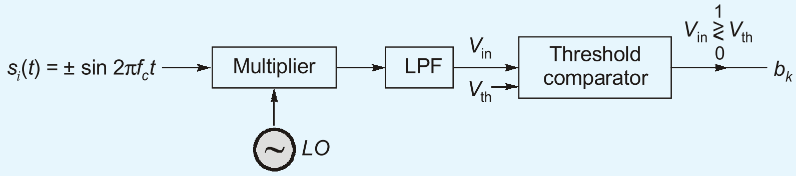
[20 marks : 2025]

Solution:

For the Binary Phase Shift Keying, assume the received signal

$$s_1(t) = \sin 2\pi f_c t; \text{ when bit '1' is transmitted}$$
$$s_2(t) = -\sin 2\pi f_c t; \text{ when bit '0' is transmitted}$$

The block diagram of the BPSK receiver consists of a multiplier with local oscillator circuit, a low pass filter and a comparator circuit as shown below:



BPSK Receiver

The received BPSK signal is multiplied with a synchronized, locally generated carrier and then passed through the low pass filter to recover the original baseband signal.

Let $$(LO)_{o/p} = \sin 2\pi f_c t$$

Output of multiplier,

$s_1(t)$ is received: $$\sin^2 2\pi f_c t = \frac{1-\cos 4\pi f_c t}{2}$$

$s_2(t)$ is received: $$-\sin^2 2\pi f_c t = \frac{-1+\cos 4\pi f_c t}{2}$$

Output of Low Pass Filter,

$s_1(t)$ is received: $$V_{in(1)} = \frac{1}{2}$$

$s_2(t)$ is received: $$V_{in(2)} = \frac{-1}{2}$$

The Low-Pass Filter (LPF) output provides a signal corresponding to the received bit. The comparator checks this signal against a threshold voltage (decision level). If the amplitude is greater than the threshold, it decides the bit as "1" and if it is less than the threshold, it decides the bit as "0" i.e.

$$b_k = \begin{cases} 1; \text{ if } V_{in} > V_{th} \\ 0; \text{ if } V_{in} < V_{th} \end{cases}$$

where, Threshold voltage, $$V_{th} = \frac{V_{in(1)} + V_{in(2)}}{2} = \frac{\frac{1}{2} + \left(-\frac{1}{2}\right)}{2} = 0$$

Output of Threshold Comparator,

$$b_k = \begin{cases} 1; \text{ if } V_{in} > 0 \\ 0; \text{ if } V_{in} < 0 \end{cases}$$

End of Solution

Q.2 (b) Show that the maximum phase lead of a lead compensator occurs at frequency $\sqrt{ab}$, where $(-a)$ and $(-b)$ are the locations of zero and pole respectively of the lead compensator.

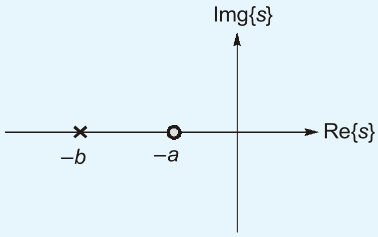
[20 marks : 2025]

Solution:

Given: Location of zero $= -a$

Location of pole $= -b$

Lead compensator: In a lead compensator, the zero is located closer to the origin than the pole. The placement of the zero closer to the origin results in a positive, or leading phase shift being added to the system. Here, $b > a$.



Transfer function of lead compensator is,

$$T(s) = \frac{(s+a)}{(s+b)}$$

Substituting $s = j\omega$, we get

$$T(j\omega) = \frac{(j\omega + a)}{(j\omega + b)}$$

The phase angle,

$$\angle T(j\omega) = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

To get maximum phase lead, differentiate $\angle T(j\omega)$ with respect to 'ω' and make equal to zero.

$$\frac{d\angle T(j\omega)}{d\omega} = \left(\frac{a}{a^2+\omega^2}\right) - \left(\frac{b}{b^2+\omega^2}\right) = 0$$

$$\Rightarrow \quad \frac{a}{a^2+\omega^2} = \frac{b}{b^2+\omega^2}$$

$$a(b^2+\omega^2) = b(a^2+\omega^2)$$

$$ab^2 + a\omega^2 = ba^2 + b\omega^2$$

$$(a-b)(\omega^2) = ba^2 - ab^2$$

$$(a-b)\omega^2 = ba(a-b)$$

$$\omega^2 = ba$$

$$\omega = \sqrt{ab}$$

$\therefore$ The maximum phase lead of a lead compensator occurs at frequency $\sqrt{ab}$.

End of Solution

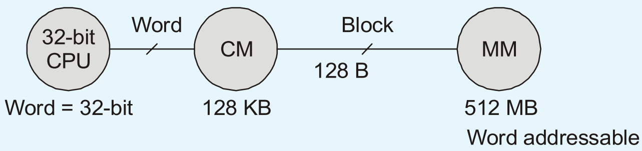
Q.2 (c) A system has 512 mega bytes of main memory and 128 kilo bytes of cache memory. Memory is word addressable with 32 bit word size. Cache memory is 8-way associative with 128 byte block size. Calculate the number of tag bits required for this set associative cache mapping scheme.

[20 marks : 2025]

Solution:

Given, Main Memory (MM) size = 512 MB
Block size = 128 B
Cache Memory (CM) size = 128 kB
Word size = 32 bit = 4 B

Memory is Word addressable



- Number of words in Cache Memory = $\frac{128K}{4}$ = 32 k words
- Number of words in Main Memory = $\frac{512\text{ M}}{4}$ = 128 M words
- Number of lines in Cache Memory (CM) = $\frac{32K}{32} = 1K = 2^{10}$
- Block size = $\frac{128\text{ B}}{4} = 32 = 2^5$ words $\Rightarrow$ Block offset = 5 bits
- MM address has $\log_2 128$ M = 27 bits
- Given the cache memory is 8-way associative,

$$\text{Number of sets } (S) = \frac{\text{Number of Blocks}}{\text{Associativity}} = \frac{1K}{8} = \frac{2^{10}}{2^3} = 2^7$$

$\Rightarrow$ Set offset = 7 bits

Address format:

Tag	Set Offset	Word Offset
15 bits	$\log_2 27$ = 7 bits	$\log_2 32$ = 5 bits

(27-bits total)

Thus, Tag bits = 27 – (7 + 5) = 27 – 12 = 15 bits

End of Solution

Q.3 (a) (i) Amplitude Modulation (AM) transmitter with a carrier power of 900 W, transmits a power of 1.1 kW, when modulated with a single sine wave. Calculate the percentage of modulation.

If the same carrier is simultaneously modulated with one more sine wave of 50% modulation, calculate the total transmitted power.

[4 + 6 = 10 marks : 2025]

(ii) Frequency modulation (FM) broadcast system is having a maximum frequency deviation of 75 kHz and modulating frequency of 15 kHz. Calculate the modulation Index and the required bandwidth using Carson's rule. Discuss whether this FM broadcast system is narrowband or wideband. Justify your answer.

[10 marks : 2025]

Solution:

(i) Given carrier power, $P_c = 900$ W

AM Transmitter Power, $P_t = 1.1$ kW $= 1100$ W

The total power of AM transmitter is given by,

$$P_t = P_c\left[1+\frac{\mu^2}{2}\right], \text{ where } \mu \text{ is the modulation index.}$$

$$1100 = 900\left[1+\frac{\mu^2}{2}\right]$$

On solving, we get $\mu = 0.666$

When a carrier is simultaneously modulated by two sine waves with modulation index $\mu_1 = 0.666$ and $\mu_2 = 0.5$, the overall modulation index (μ_t) is given by

$$\mu_t^2 = \mu_1^2 + \mu_2^2 = 0.44 + 0.25 = 0.69$$

The total transmitted power,

$$P_t = P_c\left[1+\frac{\mu_t^2}{2}\right] = 900\left[1+\frac{0.69}{2}\right] = 1210.5 \text{ W}$$

(ii) For a FM signal, it is given that

Frequency deviation, $\Delta f = 75$ kHz

Modulating frequency, $f_m = 15$ kHz

The modulation index of FM signal is given by

$$\beta = \frac{\Delta f}{f_m} = \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5$$

Since $\beta > 1$, it is a Wide Band Frequency Modulated (WBFM) signal. Using Carson's rule,

$$\text{B.W} = 2[\Delta f + f_m]$$

$$= 2[75 \text{ kHz} + 15 \text{ kHz}] = 180 \text{ kHz}$$

End of Solution

Q.3 (b) (i) The forward path transfer function of a negative unity feedback system is given by $G(s) = \dfrac{K}{s(s+T)}$. Determine the values of K and T such that all the roots of characteristic equation are in the left-half plane of the vertical line passing through $s = -a$.

[10 marks : 2025]

(ii) A negative feedback control system has the forward path transfer function $G(s) = \dfrac{K(s+0.5)}{s^2(s+12)}$ and feedback transfer function $H(s) = 1$. Find the value of K at the breakaway points.

[10 marks : 2025]

Solution:

(i) Characteristic equation of the system is given by $1 + G(s)H(s) = 0$ i.e.

For unity feedback, $H(s) = 1$. Thus, the characteristic equation is obtained as

$$1+\frac{K}{s(s+T)} = 0$$

$$s^2 + Ts + K = 0 \qquad \text{...(i)}$$

Routh Hurwitz Criteria is used to determine the poles in the left-half or right-half of s-plane. For getting poles in left half of $s = -a$ line, replace s by $(s - a)$ in equation (i),

$$(s-a)^2 + T(s-a) + K = 0$$

$$s^2 + a^2 - 2as + Ts - Ta + K = 0$$

$$s^2 + s(T-2a) + (K - Ta + a^2) = 0$$

By using Routh Hurwitz Criteria,

s^2	1	$(K - Ta + a^2)$
s^1	$T - 2a$	
s^0	$(K - Ta + a^2)$	

For all the poles to lie to the left half of the line $s = -a$, there should be no sign change in the first column of the Routh array i.e.

$$T - 2a > 0$$

$$T > 2a \qquad \text{...(i)}$$

and

$$K - Ta + a^2 > 0$$

$$K > Ta - a^2 \qquad \text{...(ii)}$$

For getting poles in left half of $s = -a$ line,

$$T > 2a \quad \text{and} \quad K > Ta - a^2$$

(ii) Characteristic equation of the system is given by, $1 + G(s)H(s) = 0$

$$1+\frac{K(s+0.5)}{s^2(s+12)} = 0$$

$$s^3 + 12s^2 + K(s + 0.5) = 0$$

$$K(s + 0.5) = -(s^3 + 12s^2)$$

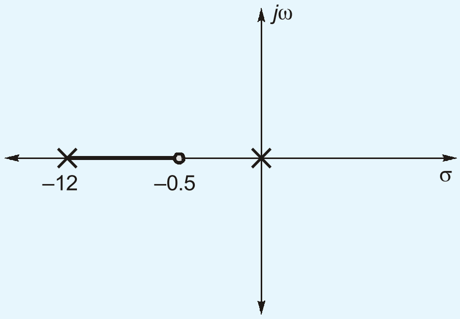
$$-K = \frac{(s^3+12s^2)}{(s+0.5)}$$

For break points,

$$\frac{dK}{ds} = \frac{(s+0.5)(3s^2+24s)-(s^3+12s^2)}{(s+0.5)^2} = 0$$

$$3s^3 + 24s^2 + 1.5s^2 + 12s - s^3 - 12s^2 = 0$$
$$2s^3 + 13.5s^2 + 12s = 0$$
$$s(2s^2 + 13.5s + 12) = 0$$
$$s = 0\ ;\ -1.05\ ;\ -5.69$$

The root locus exists on real axis to left of an odd number of poles and zeros of open loop transfer function, $G(s)H(s)$, that are on the real axis as shown below:



Thus, $s = 0$, $s = -1.05$ and $s = -5.69$ are valid breakpoints. Here, $s = 0$ and $s = -5.69$ are breakaway points and $s = -1.05$ is a break-in point.

For all points located on the root locus, the magnitude condition is always satisfied. Thus, $|G(s)H(s)| = 1$ i.e. $|G(s)| = 1$. The values of the gain (K) at the breakaway points are determined by applying the magnitude condition as below:

$$|G(s)| = 1 \Rightarrow \left|\frac{K(s+0.5)}{s^2(s+12)}\right| = 1$$

$$K = \left|\frac{s^2(s+12)}{s+0.5}\right|$$

For break-away point $s = 0$,

$$K = \left|\frac{s^2(s+12)}{s+0.5}\right|_{s=0} = 0$$

For break-away point $s = -5.69$,

$$K = \left|\frac{s^2(s+12)}{s+0.5}\right|_{s=-5.69} = \left|\frac{32.38\times 6.32}{-5.19}\right| = 39.36$$

End of Solution

Q.3 (c) An embedded system for a plant control uses two processes $P1$ and $P2$. High priority process $P1$ reads temperatures from two sensors at regular interval t and updates the latest temperature values in two fixed memory locations $T1$ and $T2$ sequentially. Low priority process $P2$ uses the values stored in locations $T1$ and $T2$ to calculate the average of this set of values. If the average of any set of values happens to be more than 50, then $P2$ calls a function to sound an alarm. The loop time of $P2$ is variable, but is ensured to be always less than t.

(i) Write an indicative pseudocode describing the above situation and mention what can go wrong in this case.

[10 marks : 2025]

(ii) Suggest appropriate operating system mechanism for solving the possible code problem with appropriate modifications in the pseudocode for part (i) above.

[10 marks : 2025]

Solution:

(i) Pseudo code

Process P1 : // High priority

Loop every t seconds:

Temp1 = Read – Sensor1 ()

T1 = Temp1

Temp2 = Read – Sensor2 ()

T2 = Temp2

End loop

Process P2 : // Low priority

Loop:

Value1 = T1 (Read memory location T1)

Value2 = T2 (Read memory location T2)

$$\text{Avg} = \frac{(\text{Value1} + \text{Value2})}{2}$$ (Calculate the average)

If Avg > 50 ;

Send alarm ()

End loop

The above code leads to a race condition problem. If P1 updates T1 but hasn't yet updated T2 and P2 runs in between P2 will read old value of T2 and new value of T1 leading to:

- Incorrect average calculation.
- False alarm (or) missed alarm.

This happens because P1 and P2 access shared memory without synchronization.

(ii) The most appropriate operating system mechanism to solve this problem is a mutex (mutual exclusion) lock. A mutex is a simple synchronization primitive that provides exclusive access to a shared resource. By using a mutex, we can ensure that P1 and P2 do not access the shared memory locations (T1 and T2) simultaneously. The process that acquires the mutex first has exclusive access, while the other process must wait until the mutex is released. This guarantees that any read or write operation on T1 and T2 happens as an atomic, indivisible block of code, preventing data inconsistency.

Modified pseudocode with mutex:

Mutex temp_lock

Process P1:

Loop every *t* seconds:

Lock (temp_lock)

```
        temp1 = read_sensor1 ()
        T1 = temp1
        temp2 = read_sensor2 ()
        T2 = temp2
        unlock (temp_lock)
End loop
```

Process P2:

```
Loop:
        Lock (temp_lock)
        Value1 = T1
        Value2 = T2
        Unlock (temp_lock)
        Avg = (Value1 + Value2)/2
        if (Avg > 50) ;
        Sound_alarm ()
End loop
```

End of Solution

Q.4 (a) (i) Plot the entropy function of a Binary Memoryless Source (BMS). List the important observations from the drawn plot.

[10 marks : 2025]

(ii) Plot the curve of transition probability versus Channel capacity for a Binary Symmetric Channel (BSC). List the significant observation from the drawn plot.

[10 marks : 2025]

Solution:

(i) A Binary Memoryless Source (BMS) has two possible symbols '0' and '1', where the symbols are independent of each other.

Let $P(0) = P(x_1) = P$

$P(1) = P(x_2) = 1 - P$

Entropy of the source, $$H(s) = -\sum_{i=1}^{2} P(x_i)\log_2 P(x_i)$$

$$H(s) = -\{P\log_2 P + (1-P)\log(1-P)\}$$

To determine the value of P for which the entropy of the source is maximized,

$$\frac{d}{ds}H(s) = \frac{-1}{\log 2}\{1 - \log P - 1 - \log(1-P)\} = 0$$

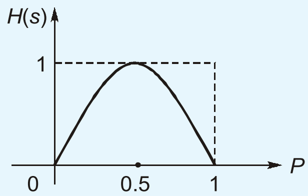
$\Rightarrow$ $\log P = \log(1-P) \Rightarrow P = 1 - P$

$\Rightarrow$ $P = \frac{1}{2} = 0.5$

We have,

$$H(s)_{max} = -\left\{\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}\right\} = 1\,\text{bit/symbol}$$

For $P = 0$ and $P = 1$, $H(s) = 0$. Thus, the entropy function can be plotted as below,



Thus, H(s) is maximum when $P = 0.5$ i.e. uncertainty about the source is maximum when $P = 0.5$.

No uncertainty about the source when $P = 0$ and 1.

(ii) Let transition probability of binary symmetric channel = P. Thus, we have

$$P(1/0) = P(0/1) = P$$

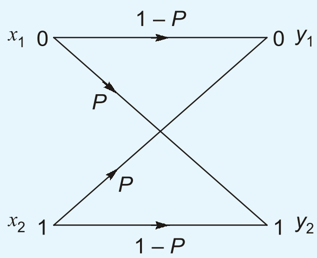
$$\text{Channel capacity, } C = \underset{\{P(x_i)\}}{\text{Max}}\; I(X;Y)$$

where

$$I(X;Y) = H(Y) - H\left(\frac{Y}{X}\right)$$

We have,

$$H\left(\frac{Y}{X}\right) = -\sum_{i=1}^{2}\sum_{k=1}^{2} P(x_i, y_k)\log_2 P\left(\frac{y_k}{x_i}\right) \quad \text{...(i)}$$



Let $P(0) = \alpha$ and $P(1) = 1 - \alpha$.

Thus,

$$[P(x)] = [\alpha \quad 1-\alpha]$$

$$\left[P\left(\frac{Y}{X}\right)\right] = \begin{matrix} & y_1 & y_2 \\ x_1 & 1-P & P \\ x_2 & P & 1-P \end{matrix}$$

$$[P(X, Y)] = [P(X)]_d\left[P\left(\frac{Y}{X}\right)\right]$$

$$[P(X, Y)] = \begin{bmatrix} \alpha & 0 \\ 0 & 1-\alpha \end{bmatrix}\begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix}$$

$$[P(X, Y)] = \begin{bmatrix} \alpha(1-P) & \alpha P \\ P(1-\alpha) & (1-P)(1-\alpha) \end{bmatrix}$$

Using equation (i),

$$H\left(\frac{Y}{X}\right) = -\{\alpha(1-P)\log_2(1-P) + \alpha P\log_2 P + P(1-\alpha)\log_2 P + (1-P)(1-\alpha)\log_2(1-P)\}$$

We get,

$$H\left(\frac{Y}{X}\right) = -\{(1-P)\log_2(1-P) + P\log_2 P\}$$

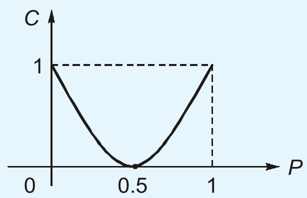
$$I(X;Y) = H(Y) + P\log_2 P + (1-P)\log_2(1-P)$$

$$C = \underset{\{P(x_i)\}}{\text{Max}}\; I(X;Y)$$

Max $\{H(Y)\}$ = 1 when output symbols 0 and 1 are equiprobable. Thus, channel capacity is given by

$$C = 1 + P\log_2 P + (1-P)\log_2(1-P) = 1 - H(P)$$

Using the result obtained in part (i), $H(P)$ is maximum when $P = 0.5$



Thus, C is maximum when $P = 0$ and 1 with C_{max} = 1 bit/sec
and C is minimum when $P = 0.5$ with $C_{min} = 0$

End of Solution

Q.4 (b) (i) Sketch the polar plot for $G(s)H(s) = \dfrac{1}{s^4(s+2)}$.

[12 marks : 2025]

(ii) How do you count the number of encirclements of the

(A) Origin

(B) $(-1 + j0)$

points? Use examples to justify the answer.

[4 × 2 = 8 marks : 2025]

Solution:

(i) We have,

$$G(s)H(s) = \frac{1}{s^4(s+2)}$$

Put $s \to j\omega$,

$$G(j\omega)H(j\omega) = \frac{1}{(j\omega)^4(2+j\omega)} = \frac{1}{\omega^4(2+j\omega)}$$

$$|G(j\omega)H(j\omega)| = \frac{1}{\omega^4\sqrt{\omega^2+4}}$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\left(\frac{\omega}{2}\right)$$

The magnitude and phase for different values of ω is given below,

ω	$\lvert G(j\omega)H(j\omega)\rvert$	$\angle G(j\omega)H(j\omega)$
0	∞	0°
∞	0	–90°

Intersection of polar plot with imaginary/real axis:

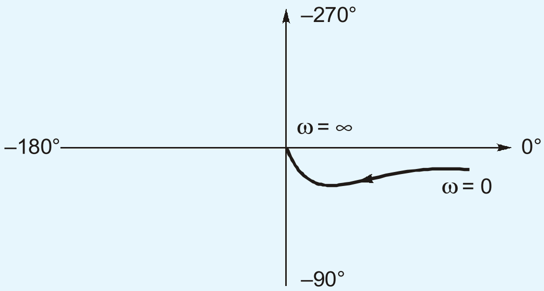
We have,

$$G(j\omega)H(j\omega) = \frac{1}{\omega^4(2+j\omega)} = \frac{1}{\omega^4}\left[\frac{2}{4+\omega^2} - j\frac{\omega}{4+\omega^2}\right] \quad \text{...(i)}$$

For intersection of polar plot with real axis, $\text{Im}\{G(j\omega)H(j\omega)\} = 0$. For intersection of polar plot with imaginary axis, $\text{Re}\{G(j\omega)H(j\omega)\} = 0$.

From equation (i), we see that the polar plot intersects the real axis and imaginary axis for $\omega = \infty$ at origin.

Using above, the polar plot can be drawn as below:



(ii) In the nyquist plot obtained for open loop transfer function, the number of encirclements with respect to origin N_{origin} is used to determine open loop system stability, and the number of encirclements with respect to $(-1 + j0)$, N_{-1+j0} is used to determine closed loop system stability with negative feedback.

We have, N_{origin} = Number of open loop poles in RHS of s-plane – Number of open loop zeros in RHS of s-plane

and N_{-1+j0} = Number of open loop poles in RHS of s-plane – Number of closed loop poles in RHS of s-plane

Example: Let, $G(s)H(s) = \dfrac{s+2}{s-1}$

⇒ Number of open loop poles on RHS of s-plane, $P = 1$

Number of open loop zeros on RHS of s-plane, $Z = 0$

Hence, Number of anti-clockwise encirclements about origin,

$$N_{origin} = P - Z = 1 - 0 = 1$$

The closed-loop poles are given by,

$$1 + G(s)H(s) = 0 \Rightarrow 1 + \frac{s+2}{s-1} = 0$$

$$2s + 1 = 0 \Rightarrow s = -\frac{1}{2}$$

Hence, no closed loop poles lie in the RHS of s-plane i.e. $P' = 0$. Hence, Number of anti-clockwise encirclements about $(-1 + j0)$,

$$N_{-1+j0} = P - P' - 1 - 0 = 1$$

To confirm the same, we draw the Nyquist plot. Substituting $s = j\omega$, we get

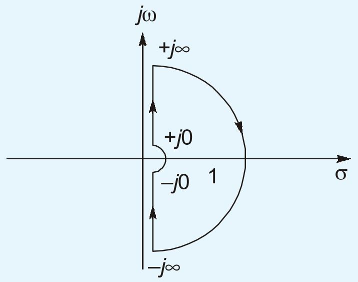
$$G(j\omega)\,H(j\omega) = \frac{j\omega + 2}{j\omega - 1}$$

$$\text{Magnitude, } |G(j\omega)H(j\omega)| = \frac{\sqrt{\omega^2+4}}{\sqrt{\omega^2+1}}\;; \quad \begin{matrix}\omega = 0 \rightarrow |GH| = 2 \\ \omega = \infty \rightarrow |GH| = 1\end{matrix}$$

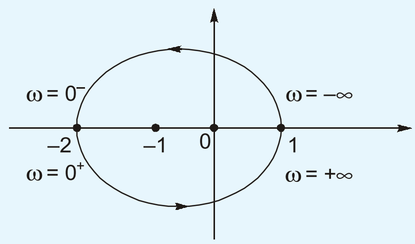
$$\text{Phase, } \angle G(j\omega)H(j\omega) = \tan^{-1}\left(\frac{\omega}{2}\right) - 180^\circ + \tan^{-1}(\omega);$$

$$\omega = 0 \rightarrow \angle GH = -180^\circ$$
$$\omega = \infty \rightarrow \angle GH = 0^\circ$$

Nyquist plot:



Nyquist Contour



1. **Origin:**

   Number of encirclements about origin, $N_{origin} = 1$

2. $(-1 + j0)$: Number of encirclements about $(-1 + j0)$, $N_{-1+j0} = 1$

Thus, the number of encirclements about origin and $(-1+j0)$ can be counted with the information of open-loop and closed-loop poles and zeros.

End of Solution

Q.4 (c) A demand paged virtual memory system uses 4 page frames. Consider following string of memory references:

1 2 4 0 5 7 4 3 4 0

Indicate the page frames for the reference string and determine the number of page faults for

(i) FIFO (First In First Out) page replacement algorithm

[10 marks : 2025]

(ii) LRU (Least Recently Used) page replacement algorithm

[10 marks : 2025]

Solution:

Given, Memory References: 1, 2, 4, 0, 5, 7, 4, 3, 4, 0

(i) FIFO page replacement algorithm: It replaces the oldest page in the main memory when a page fault occurs.

1	1	1	1	5	5	5	5	0
–	2	2	2	2	7	7	7	7
–	–	4	4	4	4	3	3	3
–	–	–	0	0	0	0	4	4
1-Miss	2-Miss	4-Miss	0-Miss	5-Miss	7-Miss	3-Miss	4-Miss	0-Miss

∴ Number of page faults = 9

(ii) LRU Page Replacement algorithm: It replaces the least recently used page when a page fault occurs.

1	1	1	1	5	5	5	5	5	0
–	2	2	2	2	7	7	7	7	7
–	–	4	4	4	4	4	4	3	3
–	–	–	0	0	0	0	4	4	4
1-Miss	2-Miss	4-Miss	0-Miss	5-Miss	7-Miss	4-Hit	3-Miss	4-Hit	0-Miss

Hence, Number of page faults = 8

End of Solution

Section-B

Q.5 (a) **Consider a telephone channel having a bandwidth of 3 kHz and the channel capacity of 30 kbps. Calculate the Signal-to-Noise ratio of this digital telephone communication system.**

[10 marks : 2025]

Solution:

Given, channel bandwidth, B = 3 kHz ; Channel capacity, C = 30 kbps

As per the Shannon-Hartley Theorem, the channel capacity is related to the bandwidth and signal to noise ratio as,

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

$$30\text{ K} = 3K\log_2\left(1+\frac{S}{N}\right)$$

$$10 = \log_2\left(1+\frac{S}{N}\right)$$

$$1+\frac{S}{N} = 2^{10} = 1024$$

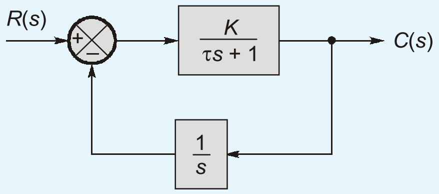
$$\frac{S}{N} = 1023$$

End of Solution

Q.5 (b) Figure approximately represents a differentiator. Its transfer function

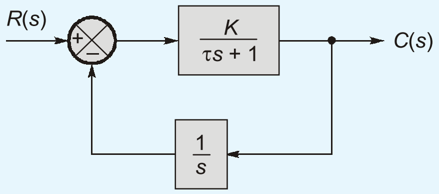
$$\frac{C(s)}{R(s)} = \frac{Ks}{s(\tau s+1)+K}. \text{ Note that } \lim_{\tau \to 0, K \to \infty} \frac{C(s)}{R(s)} = s.$$

Find the step, ramp and parabolic error constants for this system, where the ideal system is assumed to be a differentiator.

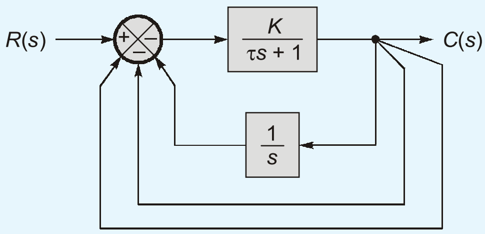


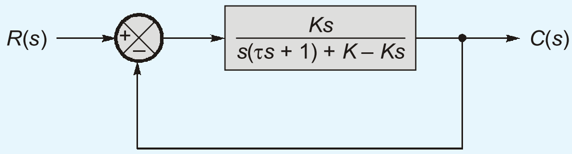
[10 marks : 2025]

Solution:



To find the error constants, convert non-unity feedback system to unity feedback system.





$$\therefore \qquad G(s) = \frac{\dfrac{K}{\tau s+1}}{1+\left(\dfrac{K}{\tau s+1}\right)\left(1-\dfrac{1}{s}\right)}$$

$$= \frac{Ks}{s(\tau s+1)+K-Ks}, \quad H(s) = 1$$

(i) Step error constant/position error constant:

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_p = \lim_{s \to 0}\left[\frac{Ks}{s(\tau s+1)+K-Ks}\right] = \frac{0}{0+K-0} = 0$$

(ii) Ramp error constant/velocity error constant:

$$K_v = \lim_{s \to 0} sG(s)H(s) = \lim_{s \to 0} s\left[\frac{Ks}{s(\tau s + 1) + K - Ks}\right]$$

$$\Rightarrow \qquad K_v = 0$$

(iii) Parabolic error constant/acceleration error constant:

$$K_a = \lim_{s \to 0} s^2G(s)H(s) = \lim_{s \to 0} s^2\left[\frac{Ks}{s(\tau s + 1) + K - Ks}\right]$$

$$\Rightarrow \qquad K_a = 0$$

End of Solution

Q.5 (c) Draw precedence graph for the schedule of transactions *X*, *Y*, and *Z* shown below and find if the schedule is conflict serializable?

X	Y	Z
Read (A)		
Write (B)		
	Read (C)	
	Write (A)	
	Write (C)	
	Commit	
Read (B)		
Write (A)		
Commit		
		Write (A)
		Commit

↓ time

[10 marks : 2025]

Solution:

To create the precedence graph,

(a) Create a node *T* in the graph for each participating transaction in the schedule.

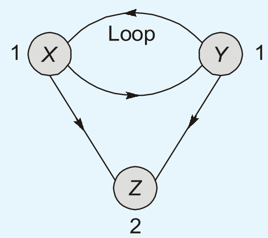
(b) For the conflicting operation read(*X*) and write(*X*): If a transaction T_j executes a read(*X*) after T_i executes a write(*X*), draw an edge from T_i to T_j in the graph.

(c) For the conflicting operation write(*X*) and read(*X*): If a transaction T_j executes a write(*X*) after T_i executes a read(*X*), draw an edge from T_i to T_j in the graph.

(d) For the conflicting operation write(*X*) and write(*X*): If a transaction T_j executes a write(*X*) after T_i executes a write(*X*), draw an edge from T_i to T_j in the graph.

The schedule *S* is serializable if there is no cycle in the precedence graph.

Precedence graph:



- $X \rightarrow Y$ Read (A) - Write (A) conflict.
- $X \rightarrow Z$ Read (A) - Write (A) conflict.
- $Y \rightarrow X$ Write (A) - Write (A) conflict.
- $Y \rightarrow Z$ Write (A) - Write (A) conflict.
- $X \rightarrow Z$ Write (A) - Write (A) conflict.

In the graph, loop (cycle) is formed. So, the given schedule is not conflict serializable.

End of Solution

Q.5 (d) Find the maximum effective area of a $\lambda/2$ wire dipole operating at 300 MHz. How much power is received with an incident plane wave of strength 2 mV/m?

[5 + 5 = 10 marks : 2025]

Solution:

The effective area of an antenna is related to the directive gain (G_d) as,

$$A_{eff} = \frac{\lambda^2}{4\pi} G_d(\theta, \phi)$$

$$\Rightarrow \quad A_{eff}\big|_{max} = \frac{\lambda^2}{4\pi} G_d\big|_{max}$$

$$\Rightarrow \quad A_{eff}\big|_{max} = \frac{\lambda^2}{4\pi} D$$

where D is the directivity of half-wave dipole = 1.64

$$\therefore \quad A_{eff}\big|_{max} = \frac{(c/f)^2}{4\pi} \times D = \frac{\left(\frac{3 \times 10^8}{3 \times 10^8}\right)^2}{4\pi} \times 1.64$$

$$\Rightarrow \quad A_{eff}\big|_{max} = 0.130 \text{ m}^2$$

Further,

$$A_{eff} = \frac{P_r}{P_{avg}} = \frac{\text{Power Density}}{\text{Radiated Power}}$$

$$\Rightarrow \quad P_r = A_{eff} * P_{avg} = A_{eff} * \frac{E^2}{2\eta},$$

where E is the field strength and η is the impedance of free space.

$$\Rightarrow \quad P_r = 0.130 \times \frac{(2 \times 10^{-3})^2}{2 \times 120\pi} = 0.69 \text{ nW}$$

$\therefore$ Power received, $P_r = 0.69$ nW

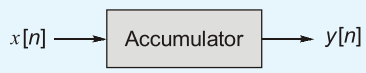
End of Solution

Q.5 (e) Show that the accumulator is the inverse system of a backward difference system.

[10 marks : 2025]

Solution:

The accumulator system's output at time 'n' is the sum of all input values up to that time.



$$y[n] = \sum_{K=-\infty}^{n} x[K] \quad \text{...(i)}$$

The backward difference system's output at time 'n' is the difference between the present input and previous input as given below:

$$y[n] = x[n] - x[n-1] \quad \text{...(ii)}$$

When the output of the backward difference system is fed as input to the accumulator, the output is,

$$z[n] = \sum_{K=-\infty}^{n} y[K] \quad \text{...(iii)}$$

$$= \sum_{K=-\infty}^{n} (x[k] - x[k-1]) \quad \text{(Using equation (ii))}$$

$$\Rightarrow \; z[n] = (x[n] - x[n-1]) + (x[n-1] - x[n-2]) + (x[n-2] - x[n-3]) + \ldots + (x[-\infty] - x[-\infty - 1])$$

$$\Rightarrow \; z[n] = x[n] \quad [\because x[-\infty] = 0]$$

Since applying the accumulator to the output of the backward difference system results in the original input signal, thus, accumulator is the inverse system of a backward difference system.

End of Solution

Q.5 (f) **A double-heterojunction InGaAsP light emitting diode (LED) used in a Fiber optic communication system emitting a peak wavelength of 1310 nm has radiative and non radiative recombination times of 30 ns and 100 ns respectively. The drive current is 40 mA.**

Calculate:

(i) Bulk recombination life time

(ii) Internal Quantum efficiency

(iii) Internal power of the LED

[3 + 3 + 4 = 10 marks : 2025]

Solution:

Given: Wavelength, λ = 1310 nm ; Radiative time, τ_r = 30 ns

Non-radiative time, τ_{Nr} = 100 ns ; Drive current, I = 40 mA

(i) Bulk recombination lifetime

$$\frac{1}{\tau_b} = \frac{1}{\tau_r} + \frac{1}{\tau_{Nr}}$$

$$\frac{1}{\tau_b} = \frac{1}{30 \times 10^{-9}} + \frac{1}{100 \times 10^{-9}} = \frac{13}{300} \times 10^{9}$$

$$\tau_b = \frac{300}{13} \times 10^{-9} = 23.08 \text{ ns}$$

(ii) **Internal quantum efficiency:**

The internal quantum efficiency is the ratio of the radiative recombination rate to the bulk recombination rate i.e.

$$\eta_{int} = \frac{\tau_b}{\tau_r} = \frac{23.08}{30} \simeq 0.769$$

$$\%\eta_{int} = 76.9\%$$

(iii) Internal power of the LED:

The number of photons emitted per second is given by

$$N_{photon} = \eta_{int}\frac{I}{e}$$

Since, energy of a photon is given by $E = \frac{hc}{\lambda}$. Thus, internal power of LED is given by

$$P = \eta_{int} \times \frac{h \times c}{\lambda}\cdot\frac{I}{e} = 0.769 \times \frac{6.626\times10^{-34}\times3\times10^{8}\times40\times10^{-3}}{1310\times10^{-9}\times1.602\times10^{-19}}$$

$$P = 0.02913\ \text{W}$$

End of Solution

Q.6 (a) **The z-component of magnetic field for the dominant mode propagating in air-filled waveguide in the z-direction at 10 GHz is given by the following expression:**

$$H_z(x,z) = 10\cos(43.74\pi x)e^{-j\beta_z z},\ \text{A/m}$$

Find:

(i) the cutoff wave number

(ii) the broader dimension of the guide

(iii) the phase velocity

(iv) the wave impedance

[5 + 5 + 5 + 5 = 20 marks : 2025]

Solution:

Given, dominant mode i.e. TE_{10}

$$f = 10\ \text{GHz}$$

$$H_z = 10\cos(43.74\pi x)e^{-j\beta_z z}\ \text{A/m}$$

(i) Cut-off wave number ; K_C:

$$K_C = \omega_c\sqrt{\mu\epsilon} = \frac{\omega_C}{c}$$

[$\because$ Waveguide is airfilled and ω_c is the cut-off frequency]

For TE_{mn} mode, $$H_z = H_o\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z}$$

$\therefore$ For TE_{10} mode, $$H_z = H_0\cos\left(\frac{\pi}{a}x\right)e^{-j\beta z}$$

So, on comparing the given expression with above, we get,

$$\frac{\pi}{a} = 43.74\pi$$

$\therefore$ $$a = 0.023\ \text{m}$$

Hence, $f_c|_{TE_{10}} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.023} = 6.52 \text{ GHz}$

$\therefore$ $\omega_c = 2\pi f_c = 40.96 \text{ G rad/s}$

$\Rightarrow$ $K_c = \frac{40.96 \times 10^9}{3 \times 10^8} = 136.54$

$\therefore$ Cut-off wave number, $K_c = 136.54 \text{ rad/m}$

(ii) **Broader dimension of the guide, *a*:**

$$a = 0.023 \text{ m (or) } 2.3 \text{ cm}$$

(iii) Phase velocity, v_p:

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1-\left(\frac{6.52}{10}\right)^2}}$$

$\Rightarrow$ $v_p = 3.96 \times 10^8 \text{ m/s}$

(iv) Wave impedance, $\eta_{TE_{10}}$

$$\eta_{TE_{10}} = \frac{\eta_{TEM}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{120\pi}{\sqrt{1-\left(\frac{6.52}{10}\right)^2}}$$

$\Rightarrow$ $\eta_{TE_{10}} = 497.21\ \Omega$

End of Solution

Q.6 (b) Using overlap add method of block filtering find the output of a filter with impulse response $h[n] = \{\underset{\uparrow}{2}, 2, 0, 0 \ldots\}$ and input $x[n] = \{\underset{\uparrow}{1}, -2, 3, 0, -1, 2\}$.

Take L (value of nonoverlapping blocks) = 3.

[20 marks : 2025]

Solution:

Given that,

$$x(n) = \{\underset{\uparrow}{1}, -2, 3, 0, -1, 2\}$$

$$h(n) = \{\underset{\uparrow}{2}, 2\} \Rightarrow \text{Length of impulse response, } M = 2$$

Value of non-overlapping blocks, $L = 3$

Now, we will form '2' sequences from $x(n)$ of length

$$[L + (M-1)] = 3 + (M-1) = 3 + (2-1) = 4$$

i.e., $x_1(n) = \{\underset{\uparrow}{1}, -2, 3, 0\}$

[$(M-1)$ no. of zeros will be padded at the end to bring the length to $L + (M-1)$.]

$$x_2(n) = \{\underset{\uparrow}{0} -1, 2, 0\}$$

$$h(n) = \{\underset{\uparrow}{2}, 2, 0, 0\}$$

[**Note:** Length of $h(n)$ is to be $[L + (M-1)] = 4$. Thus, impulse response $h[n]$ is padded with zeros at the end of sequence.]

Let us calculate '2' output signals $y_1(n)$ and $y_2(n)$ using the method overlap and add.

where $\quad y_1(n) = x_1(n) * h(n)$

and $\quad y_2(n) = x_2(n) * h(n)$

1st convolution:

$$y_1(n) = x_1(x) * h(n)$$
$$= \{1, -2, 3, 0\} * \{2, 2, 0, 0\}$$

$$\Rightarrow \quad \begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & -2 \\ -2 & 1 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 0 & 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \\ 6 \end{bmatrix}$$

2nd convolution:

$$y_2(n) = x_2(n) * h(n)$$
$$= \{0, -1, 2, 0\} * \{2, 2, 0, 0\}$$

$$\Rightarrow \quad \begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 & -1 \\ -1 & 0 & 0 & 2 \\ 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 2 \\ 4 \end{bmatrix}$$

Now, add the signals $y_1(n)$ and $y_2(n)$ by using the method overlap and add.

	2	–2	2	6			
				0	–2	2	4
$y(n) =$	$\{\underset{\uparrow}{2}$	–2	2	6	–2	2	4}
				←(n – 1)→			

Thus, $\quad y(n) = \{2, -2, 2, 6, -2, 2, 4\}$

End of Solution

Q.6 (c) (i) The loss computed for a single-mode optical fiber cable is given as 0.25 dB/km. Determine the optical power at a distance 100 km from a light source of 0.1 mW.

[10 marks : 2025]

(ii) Consider that 6 GHz is the receiving frequency at satellite transponder from the earth station and 4 GHz is the output frequency of the satellite transponder. Draw the block diagram of the satellite transponder and explain the functionality of each block.

[10 marks : 2025]

Solution:

(i) Given: $\alpha = 0.25$ dB/km ; $L = 100$ km ; $P_{in} = 0.1$ mW

The loss due to attenuation is given by

$$\text{Loss (in dB)} = \alpha L = 10\log_{10}\frac{P_{in}}{P_{out}}$$

where P_{in} is the input power and P_{out} is the output power at a distance L. Thus,

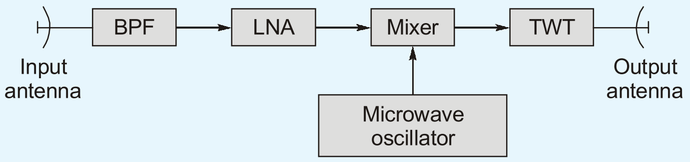
$$P_{out} = P_{in}\, 10^{-\alpha L/10}$$

$$P_{out} = 0.1\times 10^{-3}\times 10^{-\frac{0.25\times 100}{10}}$$

$$P_{out} = 0.316\times 10^{-6}\text{ W}$$

$$P_{out} = 0.316\ \mu\text{W}$$

(ii) Block diagram of satellite transponder:



Input Antenna: It receives the uplink signal from the earth station at 6 GHz.

Band Pass Filter (BPF): It filters unwanted frequencies and noise allowing only the desired frequencies.

Low Noise Amplifier (LNA): Low noise amplifier amplifies the weak incoming signal with minimal added noise.

Mixer: The mixer converts the 6 GHz uplink frequency into 4 GHz downlink frequency using microwave oscillator.

TWT: Travelling wave tube boosts the converted signal to a level suitable for transmission. TWT acts as power amplifier.

Output Antenna: It transmits the amplified 4 GHz signal back to earth station.

End of Solution

Q.7 (a) Given a uniform plane wave in air as

$$\vec{E}_i = 40\cos(\omega t - \beta z)\,\hat{a}_x + 30\sin(\omega t - \beta z)\hat{a}_y,\ \text{V/m.}$$

Find $\vec{H}_i$. If the wave encounters a perfectly conducting plate normal to the z-axis at $z = 0$, find the reflected wave $\vec{E}_r$ and $\vec{H}_r$. Assume $\epsilon_r = 1$.

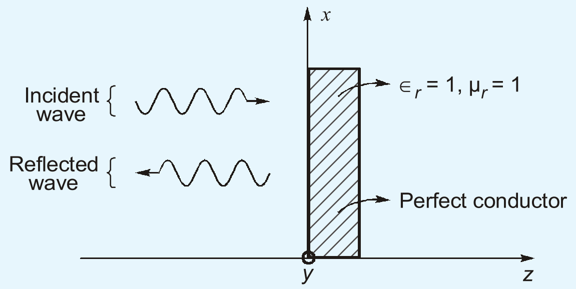
[20 marks : 2025]

Solution:

The given incident wave is travelling along z-axis, thus $\hat{a}_k = \hat{a}_z$. We have,

$$\vec{H}_i = \frac{\hat{a}_k\times\vec{E}_i}{\eta} = \frac{\hat{a}_z\times\left[40\cos(\omega t-\beta z)\hat{a}_x + 30\sin(\omega t-\beta z)\hat{a}_y\right]}{120\pi}$$

$$\vec{H}_i = \frac{1}{3\pi}\cos(\omega t-\beta z)\hat{a}_y - \frac{1}{4\pi}\sin(\omega t-\beta z)\hat{a}_x\ \text{A/m}$$



We can write, $$\vec{E}_i = E'_{i0}\cos(\omega t - \beta z)\,\hat{a}_x + E''_{i0}\sin(\omega t - \beta z)\,\hat{a}_y$$

The reflected wave travels in ($-z$) direction. Thus, we can write

$$\vec{E}_r = E'_{ro}\cos(\omega t + \beta z)\,\hat{a}_x + E''_{ro}\sin(\omega t + \beta z)\hat{a}_y$$

Now, at conductor surface, $\Gamma = -1$

$\therefore$ $$\Gamma = \frac{E'_{r_o}}{E_{i_o}} = -1 \quad \Rightarrow E'_{r_o} = -E'_{i_o} = -40$$

Similarly, $$\Gamma = \frac{E''_{r_o}}{E''_{i_o}} = -1 \quad \Rightarrow E''_{r_o} = -E''_{i_o} = -30$$

$$\vec{E}_r = -40\cos(\omega t - \beta z)\hat{a}_x - 30\sin(\omega t + \beta z)\hat{a}_y \text{ V/m}$$

We have, $$\vec{H}_r = \frac{\hat{a}_k \times \vec{E}_r}{\eta} = \frac{-\hat{a}_z \times \left[-40\cos(\omega t + \beta z)\hat{a}_x - 30\sin(\omega t + \beta z)\hat{a}_y\right]}{120\pi}$$

$\Rightarrow$ $$\vec{H}_r = \frac{1}{3\pi}\cos(\omega t + \beta z)\hat{a}_y - \frac{1}{4\pi}\sin(\omega t + \beta z)\hat{a}_x \text{ A/m}$$

End of Solution

Q.7 (b) A light bulb L turns ON and OFF depending on the positions of two switches P and Q. P and Q never change positions simultaneously. L is OFF when P is in OFF position irrespective of the position of switch Q. L turns ON when Q toggles its position while P is in ON position, and then remains ON until P goes to OFF position. Design an asynchronous circuit to implement the above logic. Derive the minimal-sum Boolean expressions for the output and next state variables in terms of the inputs and present state variables.

[20 marks : 2025]

Solution:

Let's define the states as:

Inputs: P, Q

P = 0: Switch P is OFF and P = 1: Switch P is ON

Q = 0: Switch Q is OFF and Q = 1: Switch Q is ON

Output: L

L = 0; Light bulb is OFF and L = 1; Light bulb is ON

Given,

- L is OFF when P is in OFF position, regardless of Q.
- L turns ON when Q toggles its position, while P is ON, and remains ON until P goes to OFF.

Assuming a state variable S. From the given conditions,

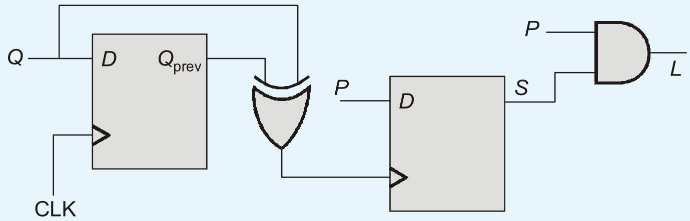
- If Q toggles (i.e., $Q_{prev} \oplus Q$ changes from 0 to 1), then $S = P$
- If $P = 0$, $S = 0$

Thus, we can use the clock of the flip-flop corresponding to state S as $Q^+ \oplus Q$. Thus, we can write the state equation as

$$Q_{prev} = Q \text{ [Triggered by External clock]}$$
$$S = P \text{ [Triggered by } Q_{prev} \oplus Q\text{]}$$

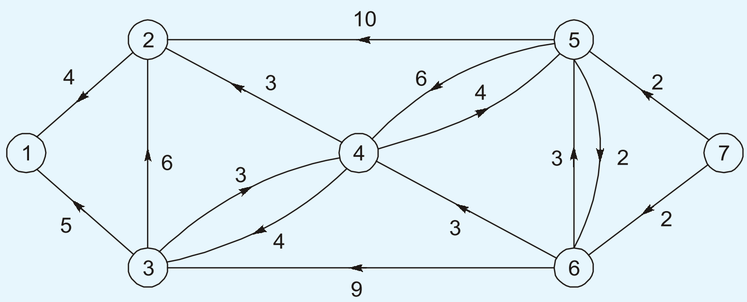
Since the output goes off as P becomes zero, thus we have

$$L = S.P$$



End of Solution

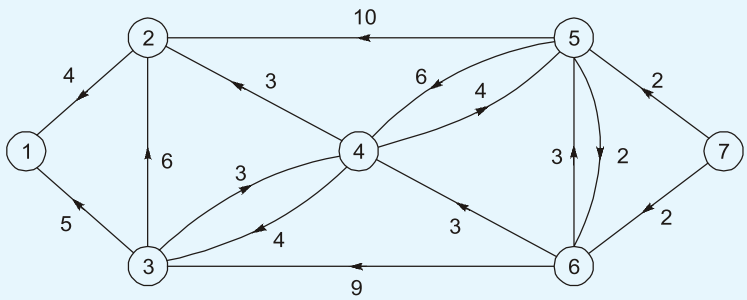
Q.7 (c) Find the shortest path tree from every node to node 1 for the graph of figure shown using the Bellman-Ford and Dijkstra algorithms.



[10 + 10 = 20 marks : 2025]

Solution:

Given that



(1) The directed edges and the costs/weights of the tree are

The reversed graph for shortest path to node 1

$1 \to 2 = 4$	$2 \to 1 = 4$
$1 \to 3 = 5$	$3 \to 1 = 5$
$2 \to 3 = 6$	$3 \to 2 = 6$
$2 \to 5 = 10$	$5 \to 2 = 10$
$2 \to 4 = 3$	$4 \to 2 = 3$
$3 \to 4 = 3$	$4 \to 3 = 3$
$3 \to 6 = 9$	$6 \to 3 = 9$
$4 \to 3 = 4$	$3 \to 4 = 4$
$4 \to 5 = 6$	$5 \to 4 = 6$
$4 \to 6 = 3$	$6 \to 4 = 3$
$5 \to 4 = 4$	$4 \to 5 = 4$
$5 \to 6 = 3$	$6 \to 5 = 3$
$5 \to 7 = 2$	$7 \to 5 = 2$
$6 \to 5 = 2$	$5 \to 6 = 2$
$6 \to 7 = 2$	$7 \to 6 = 2$
$7 \to 6 = 3$	$6 \to 7 = 3$

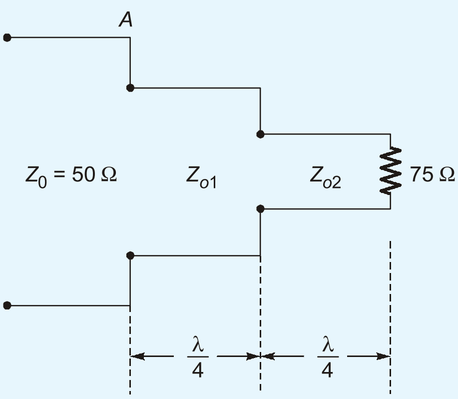
(2) Bellman ford from node 1 on reversed graph *n* distance to 1 is zero.

Node	Distance to 1	Predecessor
1	0	–
2	4	1
3	5	1
4	8	2
5	12	4
6	11	4
7	14	5

(3) The Dijkstra will yield the same distances here because there are no negative weights.

End of Solution

Q.8 (a) (i) Two $\lambda/4$ transformers in tandem are to connect a 50 Ω line to a 75 Ω load as given below:



Determine the characteristic impedance Z_{o1} if Z_{o2} = 30 Ω and there is no reflected wave to the left of *A*.

[10 marks : 2025]

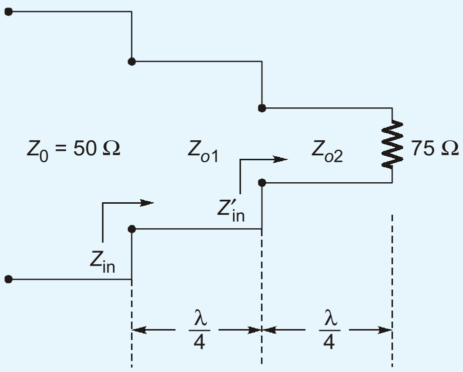
(ii) A distortionless transmission line operating at 250 MHz has R = 30 Ω/m, L = 200 nH / m and C = 80pF / m After how many meters travelling along the line, will the voltage wave get reduced to 30% of its initial value.

[10 marks : 2025]

Solution:

(i) It is given that there is no reflections to the left of A. Thus, the line with characteristic impedance Z_0 = 50 Ω is matched, i.e.

$$Z_0 = Z_{in}$$



For a $\frac{\lambda}{4}$ line, the input impedance is given by

$$Z_{in} = \frac{Z_0^2}{Z_L},$$

where Z_0 is the characteristic impedance of the line and Z_L is the load impedance.

From the above circuit,

$$Z'_{in} = \frac{Z_{02}^2}{75} = \frac{30 \times 30}{75} = 12\ \Omega$$

and

$$Z_{in} = \frac{Z_{01}^2}{Z'_{in}} = \frac{Z_{01}^2}{12}$$

Using equation (i),

$$Z_0 = \frac{Z_{01}^2}{12} \Rightarrow Z_{01} = \sqrt{12 \times 50}$$

$$Z_{01} = 24.5\ \Omega$$

(ii) Given: Transmission line is distortionless

Operating frequency, f = 250 MHz

Primary constants of the line, R = 30 Ω/m, L = 200 nH/m, C = 80 pF/m

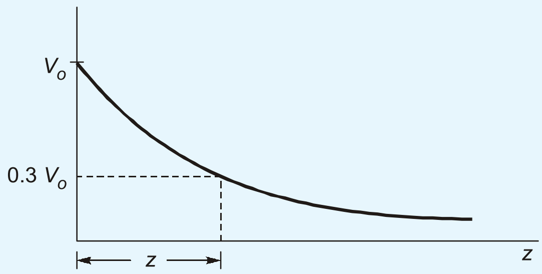
For a distortionless line, $LG = RC$

and

$$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}}$$

⇒

$$\alpha = 30\sqrt{\frac{80 \times 10^{-12}}{200 \times 10^{-9}}} = 0.6\ \text{Np/m}$$



The amplitude of the wave decays exponentially with distance and the attenuation constant (α) defines the rate of this decay as given below,

$$V = V_o\, e^{-\alpha z}$$

Assume at distance 'z', the voltage wave get reduced to 30% of its initial value. We have,

$$\Rightarrow \quad 0.3V_o = V_o\, e^{-0.6z}$$

$$\Rightarrow \quad z = -\frac{\ln(0.3)}{0.6}$$

$$\therefore \quad z = 2\text{ m}$$

End of Solution

Q.8 (b) A data byte read from port 0 of 8051 microcontroller is to be sent out serially from bit 0 of port 1. Each bit duration for serial transmission is 2 milliseconds and least significant bit goes out first. Write assembly language program using timer 0 for the delay. Assume crystal clock frequency to be 12 MHz.

[20 marks : 2025]

Solution:

Program to be implemented for 8051 microcontroller:

- To read 'P0' for 8-bit data.
- Transfer the data bitwise serially from LSB through bit 0 of port 1 i.e. P1.0.
- Delay for each bit is 2 million seconds, using Timer-0 during serial transmission.

Delay calculation: using Timer-0

- For 8051, clock frequency $f_{clk} = \frac{f_{crystal}}{12} = \frac{12\text{ MHz}}{12} = 1\text{MHz}.$

  $\therefore \quad T = 1$ Machine cycle $= \frac{1}{f_{clk}} = \frac{1}{1\times 10^6} = 1\mu s$

  The TMOD register for 8051 microcontroller is given as below:

MSB ... LSB

GATE	C/T	M1	M0	GATE	C/T	M1	M0
TIMER 1				TIMER 0			

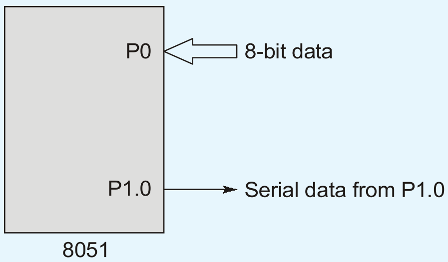
- When $C/\overline{T}$ bit is set to one, the timer will function as event counter and it is programmed to zero for timer operation.
- **GATE:** When set to 1, the timer/counter is enabled only while the INT pin is high and TR control bit is set. When cleared to 0, the timer is enabled whenever the TR control bit is set.

- M1 and M0 selects the mode of the timer:
  - 00: 13-bit Timer mode (8 Bit of THx and 5 Bit of TLx).
  - 01: 16-bit Timer mode
  - 10: 8-bit Auto-reload mode (TLx reload with the THx value each time when TLx overflows).
  - 11: Split Timer mode - Split 16 bit timer into two 8 bit timers(THx and TLx).
- Using Timer-0 in Mode-1 i.e. 16-bit Timer mode, the control word is '01 H'.

  For 2 ms delay

$$\text{Delay} = (65{,}536 - \text{Count value}) \times \text{Clock period}$$
$$2\text{ ms} = (65{,}536 - \text{C.V}) \times 1\ \mu\text{s}$$
$$\text{Count value} = 65{,}536 - \frac{2\times 10^{-3}}{1\times 10^{-6}}$$
$$= 65{,}536 - 2000 = (63{,}536)_{10} = \text{F830H}$$

The value that must be loaded into THO and TLO register is F8H and 30H to generate a 2 ms delay.



Algorithm:

1. Declare port-0 (P0) as input port by sending 1's to P0.
2. Declare port 1, bit 0 as O/P pin by sending 1 to P1.0.
3. Initialize R0 with 08 H to count transfer of 8-bits of P0.
4. Read 'P0' into accumulator.
5. Rotate accumulator content 1-bit right with carry. It shifts the LSB to carry.
6. If CY → 1 ; send '1' to P1.0, else '0' to P1.0.
7. Call delay subroutine for 2 ms.
8. Decrement R0. If R0 is not zero, output the next bit serially and jump to Step 5, otherwise if R0 is zero, end the program.
9. Write a delay subroutine separately.

Program Required:

```
         MOV TMOD, # 01H  ;  Timer in mode –1(16-bit timer)
         MOV P0, # FFH    ;  P0 as i/p port
         SETB P1.0        ;  Port-1, bit 0 i.e. P1.0 as o/p pin
Repeat : MOV R0, #08H     ;  Count of 8 in R0 for 8-bits
         MOV A, P0        ;  Read data from P0 to ACC
         CLR C            ;  Clear carry flag
Again :  RRC A            ;  Rotate 'ACC' content 1-bit right
         JC One           ;  Check if Cy = 1. If yes, jump to One.
         CLR P1.0         ;  Clear P1.0 if Cy = 0
```

	A CALL Delay	;	Call delay of 2 ms
	SJMP Count	;	Jump to decrement register R0
One :	SET P1.0	;	Set P1.0 i.e. '1' is sent to bit 0 of Port 1
	A CALL Delay	;	Call delay of 2 ms
Count :	DJNZ R0 Again	;	Jump to Again for shifting next bit serially if R0 is not zero
Repeat :	SJMP Repeat	;	End the Program
Delay :	MOV TL0, # 30H	;	Lower byte of count in TL0 register
	MOV TH0, # 0F8H	;	Higher byte of count in TH0 register
	SETB TR0	;	Start Timer-0
Check :	JNB TF0, Check	;	Check for timer over flow
	CLR TR0	;	Stop Timer-0
	CLR TF0	;	Clear timer flag
	RET	;	Return to main program

End of Solution

Q.8 (c) (i) The downlink of a satellite communication operated at 4 GHz, the receiving antenna is a parabolic reflector with a diameter of 3.6 m and efficiency is 0.7. Calculate the gain of the receiving antenna in dB.

[10 marks : 2025]

(ii) The numerical aperture of an optical fiber is 0.3. Calculate the acceptance angle for the meridional rays. Further calculate the acceptance angle for the skew rays which change direction by 90° at each reflection.

(Assume that refractive index, n_a of air is 1)

[10 marks : 2025]

Solution:

(i) **Given:** Parabolic reflector, $f = 4$ GHz ; $D = 3.6$ m ; $\eta = 0.7$; $G_r = ?$

The gain of a parabolic reflector is given by,

$$G_r = \eta\pi^2\left(\frac{D}{\lambda}\right)^2 = \eta\pi^2\left(\frac{D\times f}{c}\right)^2 = 0.7\times\pi^2\times\left(\frac{3.6\times4\times10^9}{3\times10^8}\right)^2$$

$$G_r = 15917.697 \simeq 15918$$

In decibels, $$G_r = 10\log 15918 \simeq 42 \text{ dB}$$

(ii) Given: NA = 0.3 ; $2\gamma = 90° \Rightarrow \gamma = 45°$ (for skew rays)

For meridional rays, $$\theta_a = \sin^{-1}(\text{NA})$$

$$\theta_a = \sin^{-1}(0.3) = 17.45°$$

For skew rays, $$\theta_{as} = \sin^{-1}\left[\frac{NA}{\cos\gamma}\right]$$

$$\theta_{as} = \sin^{-1}\left(\frac{0.3}{\cos 45°}\right) = 25.10°$$

End of Solution

■■■■