ESE 2025

Main Exam Detailed Solutions

Electronics & Telecom. Engineering

PAPER-I

EXAM DATE : 10-08-2025 | 09:00 AM to 12:00 PM

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ANALYSIS

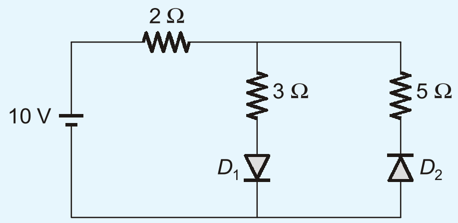
Electronics and Telecom. Engineering Paper-I
ESE 2023 Main Examination

Sl.	Subjects	Marks
1.	Basic Electronics Engineering (EDC)	32
2.	Materials Science	42
3.	Electronic Measurements and Instrumentation	72
4.	Network Theory	144
5.	Analog Circuits	106
6.	Digital Circuits	52
7.	Basic Electrical Engineering	32
	Total	**480**

Scroll down for detailed solutions

Section-A

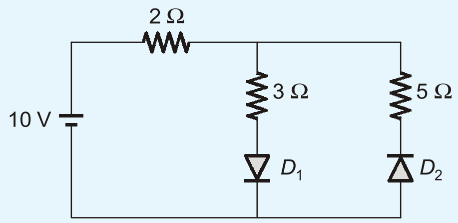
Q.1 (a) Find the current flowing in the circuit as given in the figure, where two ideal diodes are connected in parallel:



[12 marks : 2025]

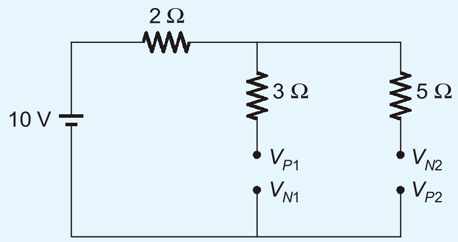
Solution:

To determine the current flowing in the circuit, we have to determine whether the diodes D_1 and D_2 are in ON/OFF state.



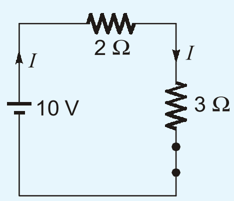
Assume both the diodes are in non-conducting state:

In the non-conducting state, the diodes can be considered as open circuit. Hence, the circuit can be drawn as below:



As, $V_{P1} > V_{N1}$; Diode D_1 is in conducting state. As, $V_{P2} < V_{N2}$; Diode D_2 is in non-conducting state.

Assuming the diode D_1 as conducting (short-circuit) and diode D_2 as non-conducting (open-circuit), the circuit can be drawn as below,



$$I = \frac{10}{5} = 2 \text{ A}$$

End of Solution

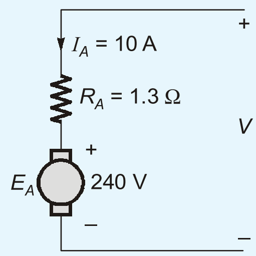
Q.1 (b) A certain d.c. motor has R_A = 1.3 Ω, I_A = 10 A and produces a back e.m.f. E_A = 240 V, while operating at a speed of 1200 r.p.m. Determine the voltage applied to the armature, the developed torque and the developed power.

[12 marks : 2025]

Solution:

We have,

Armature resistance, R_A = 1.3 Ω
Armature current, I_A = 10 A
Back emf, E_A = 240 V
Operating speed, N_r = 1200 rpm



1. Applied voltage, V:
The applied voltage (V) to the armature of a d.c. motor is the sum of the back e.m.f. (E_A) and the voltage drop across the armature resistance (R_A) due to the armature current (I_A).

$$V = I_A R_A + E_A$$
$$V = I_A R_A + 240$$
$$V = 13 + 240 = 253 \text{ volt}$$

2. Developed torque, T_d:

$$T_d = \frac{P_{del}}{\omega}, \text{ where } P_{del} = \text{Developed power} = E_A I_A$$
$$T_d = \frac{E_A I_A}{\frac{2\pi N}{60}} = \frac{240 \times 10}{\frac{2\pi \times 1200}{60}}$$
$$T_d = 19.09 \text{ Nm}$$

3. Developed power: P_d

$$P_d = E_A \cdot I_A = 240 \times 10 = 2.4 \text{ kWatt}$$

End of Solution

Q.1 (c) Consider a unit cell of simple cubic structure. Find the angle between the normals to pair of planes whose Miller indices are (i) [1 0 1] and [0 1 0], and (ii) [2 1 1] and [1 0 1].

[12 marks : 2025]

Solution:

In case of a cubic crystal, the direction [*hkl*] is perpendicular to the plane with miller indices (*hkl*). Therefore, the angle between the normals to the planes is the same as the angle between two planes represented by the Miller indices $(h_1k_1l_1)$ and $(h_2k_2l_2)$ given as

$$\cos\theta = \frac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2}\sqrt{h_2^2 + k_2^2 + l_2^2}}$$

(i) For the pair of planes with Miller indices [1 0 1] and [0 1 0],

$$\cos\theta = \frac{0+0+0}{\sqrt{2}\sqrt{1}} = 0 \Rightarrow \theta = 90°$$

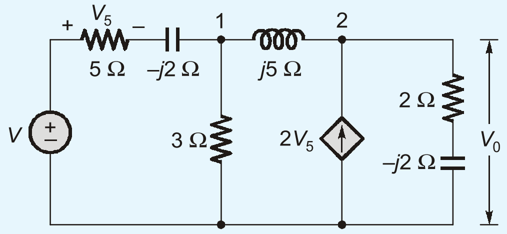
(ii) For the pair of planes with Miller indices [2 1 1] and [1 0 1],

$$\cos\theta = \frac{2+0+1}{\sqrt{4+1+1}\sqrt{1+0+1}} = \frac{3}{\sqrt{6}\sqrt{2}}$$

$$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$$

End of Solution

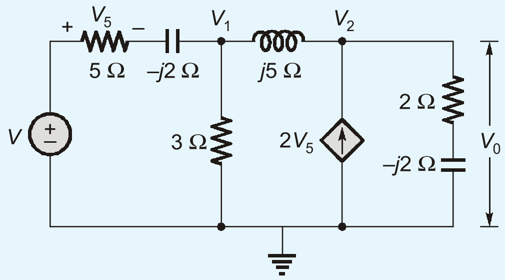
Q.1 (d) For the circuit shown in the figure, calculate the value of the voltage V which gives $V_0 = 5\angle 0°$ volts :



[12 marks : 2025]

Solution:

We have,



On applying KCL at node 1, we get,

$$\frac{V_1 - V}{5 - j2} + \frac{V_1}{3} + \frac{V_1 - V_2}{j5} = 0 \qquad [\because V_2 = V_0 = 5\angle 0°]$$

$$V_1\left[\frac{1}{5 - j2} + \frac{1}{3} + \frac{1}{j5}\right] - \frac{V}{(5 - j2)} = \frac{5}{j5}$$

$$0.52\angle{-14.52°}\, V_1 - 0.186\angle 21.80° = 1\angle{-90°} \qquad \text{...(1)}$$

On applying KCL at node 2, we get

$$\frac{V_2 - V_1}{j5} + \frac{V_2}{(2 - j2)} = 2V_5$$

where, $$V_5 = \left[\frac{(V - V_1)}{5 - 2j} \times 5\right]$$

Thus, $$\frac{V_2 - V_1}{j5} + \frac{V_2}{2 - j2} = 2\left[\frac{5(V - V_1)}{(5 - 2j)}\right]$$

$$\frac{V_2 - V_1}{j5} + \frac{V_2}{2 - j2} = \frac{10V}{5 - 2j} - \frac{10V_1}{(5 - 2j)}$$

Substituting $V_2 = V_0 = 5\angle 0°$, we have

$$V_1\left[\frac{-1}{j5} + \frac{10}{5 - 2j}\right] - \frac{10V}{(5 - 2j)} = \left[\frac{-1}{2 - j2} - \frac{1}{j5}\right]5\angle 0°$$

$$(1.94\angle 27.29°)V_1 - 1.86\angle 21.80° = 1.27\angle{-168.69°} \qquad \text{...(2)}$$

Using Carson's rule, we get

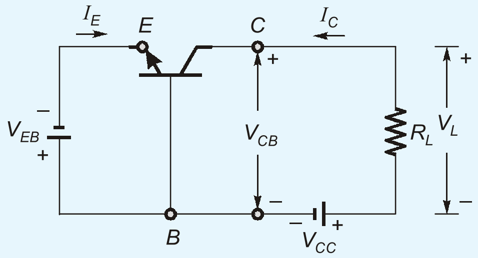
$$V = \frac{\begin{vmatrix} 0.52\angle -14.52° & 1\angle -90° \\ 1.94\angle 27.29° & 1.27\angle -168.69° \end{vmatrix}}{\begin{vmatrix} 0.52\angle -14.52° & -0.186\angle 21.80° \\ 1.94\angle 27.29° & -1.86\angle 21.80° \end{vmatrix}}$$

$$V = \frac{0.66\angle -183.21° - 1.94\angle -62.71°}{-0.9672\angle 7.28° + 0.36\angle 49.09°}$$

$$= \frac{-1.55 + j1.76}{-0.72 + j0.15} = 3.19\angle -36.86° \text{V}$$

End of Solution

Q.1 (e) Shown below is an *n*-*p*-*n* transistor biased in the active region:



Assume that the emitter is much more heavily doped than the base.

(i) Plot the potential variation across the emitter and collector junction.

(ii) Plot the minority carrier concentration in each section of the transistor.

(iii) Show how this transistor configuration works as an amplifier.

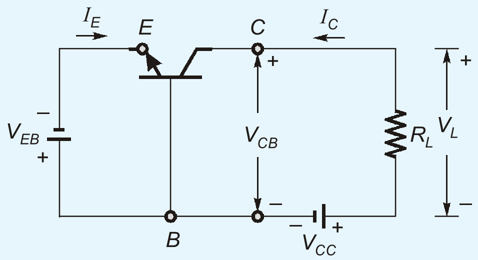
(iv) Plot the collector current against base to emitter voltage for a silicon transistor when it is varied from –0.4 V to +0.8 V. Indicate the cutoff, active and saturation regions.

(v) From the transistor characteristics, write the analytical expressions for the collector current and the emitter current.

(vi) Show how these equations are used to replace the *n*-*p*-*n* transistor with two back diodes in shunt with two dependent current sources.

[12 marks : 2025]

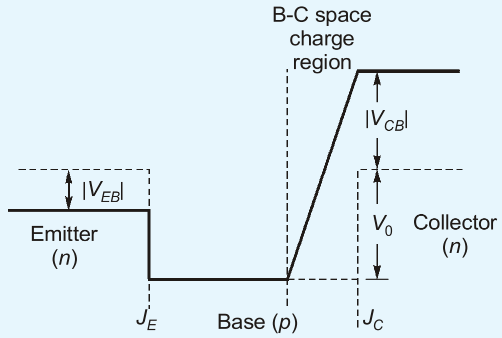
Solution:



Given, emitter is much more heavily doped than the base.

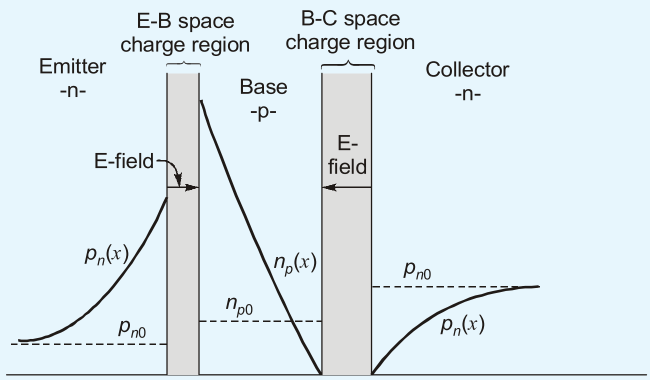
(i) It is given that the transistor is biased in active region i.e. emitter – base junction is forward biased and collector – base junction is reverse biased. The reverse biasing of the collector junction increases the collector – base barrier by $|V_{CB}|$, and forward biasing of the emitter junction lowers the emitter base barrier by $|V_{EB}|$.

Since E-B junction is forward biased, the E-B space charge region width is negligible and due to reverse-biased C-B junction, the E-B space charge region width is increased. Thus, the potential variation across the transistor is as shown below,



(ii) **Minority carrier concentration in each section of the transistor:**

- The quantities p_{n0} and n_{p0} gives the thermal equilibrium concentration of the holes and electrons respectively. The emitter doping being much higher, the value of p_{n0} in the emitter is less than n_{p0} in the base.
- Since the E-B junction is forward biased, the holes are injected into the emitter and electrons are injected into the base. This injection causes the minority carrier concentration at the junction - holes in the emitter and electrons in the base to increase slightly above its thermal equilibrium value. The concentration of these minority carriers decreases away from the junction , towards their respective thermal equilibrium values.
- Since the C-B junction is reverse biased, the minority carriers i.e. holes from collector and electrons from base drift across the junction. Therefore, the concentration of minority carriers—electrons in the base and holes in the collector is approximately zero at the C-B junction and increases away from it, towards their respective thermal equilibrium values.



(iii) Forward biasing E-B junction injects electrons into the base. Because the base is thin and lightly doped, most injected electrons diffuse across the junction and are collected by the reverse-biased C-B depletion region contributing to collector current. Thus, a small change in V_{BE} causes an exponential change in collector current:

$$I_C \approx I_S e^{V_{BE}/V_T} \quad \text{(Forward active).}$$

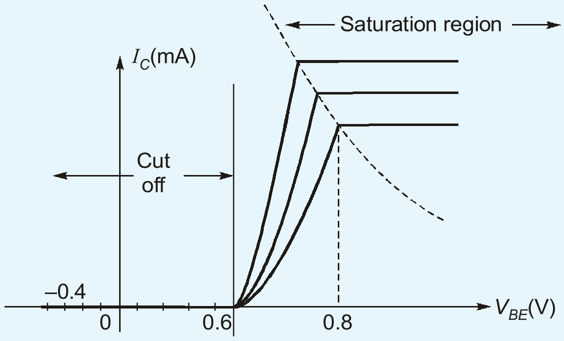
This change in current can be converted into a large voltage change across a load resistor, resulting in voltage amplification.

(iv) The I_C vs V_{CE} characteristics of the transistor is as shown below,

(a) **Cut-off region:** When the base-emitter voltage (V_{BE}) is less than the "cut-in voltage" (approximately 0.6 V), the transistor is in the cut-off region and the collector current is zero.

(b) **Active region:** As V_{BE} increases beyond the cut-in voltage, a small increase in V_{BE} leads to a exponential increase in the base current (I_B), which in turn causes an increase in the collector current (I_C). In this region, the collector current is proportional to the base current i.e. $I_C = \beta I_B$.

(c) **Saturation region:** When V_{BE} is increased further (around 0.8 V), the transistor enters the saturation region. In this state, both the emitter-base and collector-base junctions become forward-biased leading to a saturation in the collector current.



(v) Analytical expressions for collector current and emitter current:
Let I_{ES} and I_{CS} be the E-B and C-B diode saturation currents, and α_F and α_R are the forward and reverse common-base transport factors ($\alpha_F \le 1$, $\alpha_R << \alpha_F$ because the emitter is much more heavily doped). Using the Ebers-Moll model, the collector and emitter current is given by

$$I_C = -I_{CS}\left(e^{V_{BC}/V_T} - 1\right) + \alpha_F\, I_{ES}\left(e^{V_{BE}/V_T} - 1\right),$$

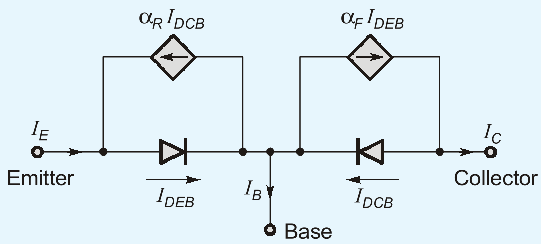
$$I_E = I_{ES}\left(e^{V_{BE}/V_T} - 1\right) - \alpha_R\, I_{CS}\left(e^{V_{BC}/V_T} - 1\right),$$

$$I_B = I_E - I_C$$

(vi) Using the analytical equations obtained in part (v), the transistor can be analyzed as a two back-to-back diodes connected in series along with two dependent current sources in shunt with the diode as shown below:

$$I_{DEB} = I_{ES}\left(e^{V_{BE}/V_T} - 1\right)$$

$$I_{DCB} = I_{CS}\left(e^{V_{BC}/V_T} - 1\right)$$



- Two current sources in shunt model transistor action (carrier transport across the base):
  (i) A source from emitter to collector of value $\alpha_F I_{DEB}$ (controlled by E-B diode current).
  (ii) A source from collector to emitter of value $\alpha_R I_{DCB}$ (controlled by the C-B diode current).

$\therefore$ The two diodes account for the junction currents themselves; and the two controlled sources "transfer" a fraction (α_F, α_R) of one junction's injected carriers to the other terminal.

End of Solution

Q.2 (a) (i) **The magnetic field of the earth is approximately 3 x 10^{-5} T (tesla). At what distance from a long-distance wire carrying a steady current of 10 A is the field equal to 10 percent of the earth's field? Suggest at least two ways to help reduce the effect of electric circuits on the navigation compass in a boat or an airplane.**

[12 marks : 2025]

(ii) **A typical deep cycle battery (used for electric trolling motors for fishing boats) is capable of delivering 12.6 V and 10 A for a period of 10 hours. How much charge flows through the battery in this interval? How much energy does the battery deliver?**

[8 marks : 2025]

Solution:

(i) Given, Magnetic field, $B_{earth} = 3 \times 10^{-5}$ T

Current, $I = 10$ A

The magnetic field around an infinitely long, straight wire carrying a current (I) at a distance 'r' is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

where, $\mu_0 = 4\pi \times 10^{-7}$ H/m

Let 'r' be the distance from the wire at which the field 'B' is equal to 10 percent of the earth's field i.e. $B = 0.1B_{earth}$

$$\therefore \quad r = \frac{\mu_0 I}{2\pi \times B}$$

$$= \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 3 \times 10^{-6}}$$

$$r = 0.67 \text{ m}$$

The following ways can be used to reduce the effect of electric circuits on the navigation compass in a boat or an airplane:

1. Increase the distance between the compass and electric circuits/wiring.
2. Use magnetic shielding (soft iron or mu-metal) around sensitive instruments.
3. Twist current-carrying wires together to reduce net magnetic field.
4. Route cables symmetrically to cancel fields.

(ii) Given, Battery voltage, $V = 12.6$ V

Current, $I = 10$ A

Time, $t = 10$ hours

Charge flow through battery,

$$Q = I \times t$$
$$= 10 \times 10 \times 3600$$
$$= 360000 \text{ C}$$
$$\therefore \quad Q = 3.6 \times 10^5 \text{ C}$$

Energy delivered by battery,

$$E = V \times I \times t$$
$$= 12.6 \times 10 \times 36000s$$
$$E = 4.54 \text{ MJ}$$

(or)
$$E = \frac{4.54 \times 10^6}{3.6 \times 10^6} \simeq 1.26 \text{ kWh}$$

End of Solution

Q.2 (b) During a laboratory experiment, a student tried to build an inverting amplifier as shown in Fig. (i). The student accidentally reversed the connection of the two input terminals and obtained the circuit of Fig. (ii). The student was greatly surprised that the circuit no longer behaved as expected. Calculate the gain in both the cases and explain the stability of both the circuits. Assume open-loop gain of op-amp as 2×10^5, $R_i = \infty$, $R_0 = 0$ and stray capacitance of 1 pF across the input terminals :

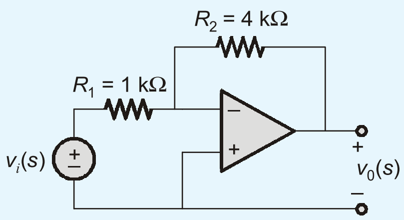


Fig. (i)

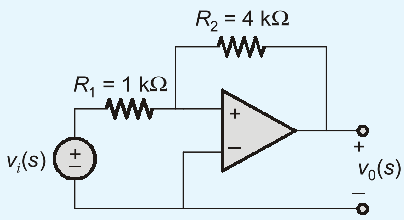
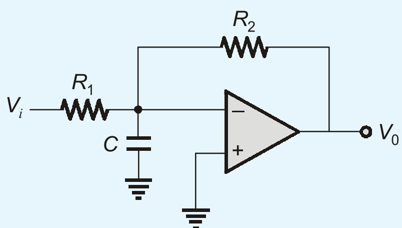


Fig. (ii)

[20 marks : 2025]

Solution:

Considering the circuit given in Fig. (i),



For an op-amp,
$$V_0 = A_{OL}(V^+ - V^-)$$
$$V_0 = A_{OL}(0 - V^-)$$
$$V_0 = -A_{OL} \times V^-$$
$$V^- = \frac{-V_0}{A_{OL}} \quad \text{...(1)}$$

Applying KCL at the inverting terminal,

$$(V_i - V^-)G_1 + (V_0 - V^-)G_2 + (0 - V^-)sC = 0, \text{ where } G_1 = \frac{1}{R_1} \text{ and } G_2 = \frac{1}{R_2}$$

$$V_iG_1 + V_0G_2 - V^-(G_1 + G_2 + sC) = 0$$

$$V_iG_1 + V_0G_2 + \frac{V_0}{A_{OL}}(G_1 + G_2 + sC) = 0 \quad \text{...using equation (1)}$$

$$V_0\left[G_2 + \frac{G_1 + G_2 + sC}{A_{OL}}\right] = -V_iG_1$$

$$\frac{V_0}{V_i} = \frac{-G_1}{G_2 + \frac{G_1 + G_2 + sC}{A_{OL}}}$$

$$A_{CL} = \frac{-G_1 \times A_{OL}}{G_1 + G_2(1 + A_{OL}) + sC}$$

The transfer function has a pole at,

$$s = \frac{-[G_1 + G_2(1 + A_{OL})]}{C}$$

$$s = \frac{-[1 + 0.25(1 + 2 \times 10^5)] \times 10^{-3}}{1 \times 10^{-12}}$$

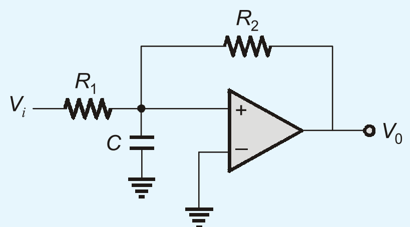
$$s = -5 \times 10^{13} \text{ rad/sec}$$

Thus, the pole lie in the left half of s-plane. So, the above circuit is stable.

We can write,

$$A_{CL} = \frac{-4}{1 + \frac{s}{5 \times 10^{13}}}$$

Considering the circuit given in Fig. (ii),



$$V_0 = A_{OL}(V^+ - V^-)$$

$$V_0 = A_{OL}V^+$$

$$V^+ = \frac{V_0}{A_{OL}} \quad \text{...(1)}$$

Applying KCL at the non-inverting terminal,

$$(V_i - V^+)G_1 + (0 - V^+)sC + (V_0 - V^+)G_2 = 0$$

$$V_iG_1 - V^+(G_1 + G_2 + sC) + V_0G_2 = 0$$

$$V_iG_1 - \frac{V_0}{A_{OL}}(G_1 + G_2 + sC) + V_0G_2 = 0$$

$$V_0\left[\frac{G_1+G_2+sC}{A_{OL}}-G_2\right] = V_iG_1$$

$$\frac{V_0}{V_i} = \frac{G_1}{\dfrac{G_1+G_2+sC}{A_{OL}}-G_2}$$

$$A_{CL} = \frac{G_1\times A_{OL}}{G_1+G_2(1-A_{OL})+sC}$$

Above circuit has pole at,

$$s = \frac{-[G_1+G_2(1-A_{OL})]}{C}$$

$$s = \frac{-[1+0.25(1-2\times10^5)]\times10^{-3}}{1\times10^{-12}}$$

$$s = 5\times10^{13}\text{ rad/sec}$$

Thus, the pole lie in the right half of s-plane. So, the above circuit is not stable.

We can write, $$A_{CL} = \frac{-4}{1-\dfrac{s}{5\times10^{13}}}$$

End of Solution

Q.2 (c) The state equation of a linear time-invariant system is expressed by

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}[r(t)]$$

(i) Calculate the state transition matrix.

(ii) Find the state vector $x(t)$ for $t \geq 0$, when $r(t) = u(t)$.

Assume the initial state to be zero.

[10 + 10 = 20 marks : 2025]

Solution:

Given, $$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}r(t) = Ax + Bu$$

(i) The state transition matrix:

$$\phi(t) = L^{-1}[sI-A]^{-1} \quad ...(1)$$

Here, $$[sI-A] = s\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix} = \begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}$$

$$[sI-A] = \begin{bmatrix}s & -1\\ 1 & s+2\end{bmatrix}$$

$$[sI-A]^{-1} = \frac{\text{Adj}[sI-A]}{|sI-A|} = \frac{\begin{bmatrix}s+2 & 1\\ -1 & s\end{bmatrix}}{(s(s+2)+1)}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}}{(s^2 + 2s + 1)} = \frac{\begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}}{(s+1)^2}$$

$$\therefore \qquad L^{-1}[sI - A]^{-1} = L^{-1}\begin{bmatrix} \frac{s+1}{(s+1)^2} + \frac{1}{(s+1)^2} & \frac{1}{(s+1)^2} \\ \frac{-1}{(s+1)^2} & \frac{s+1-1}{(s+1)^2} \end{bmatrix}$$

$$\Rightarrow \qquad \phi(t) = L^{-1}\begin{bmatrix} \frac{1}{(s+1)} + \frac{1}{(s+1)^2} & \frac{1}{(s+1)^2} \\ \frac{-1}{(s+1)^2} & \frac{1}{(s+1)} - \frac{1}{(s+1)^2} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} (e^{-t} + te^{-t})u(t) & te^{-t}u(t) \\ -te^{-t}u(t) & (e^{-t} - te^{-t})u(t) \end{bmatrix}$$

(ii) **State vector $x(t)$ for $t \geq 0$, when $r(t) = u(t)$:**

We know, $X(s) = [sI - A]^{-1}[x(0) + BU(s)]$

Given, $x(0) = 0$

$\therefore$ $X(s) = [sI - A]^{-1}\, BU(s)$

$$= \begin{bmatrix} \frac{s+2}{(s+1)^2} & \frac{1}{(s+1)^2} \\ \frac{-1}{(s+1)^2} & \frac{s}{(s+1)^2} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{(s+1)^2} \\ \frac{s}{(s+1)^2} \end{bmatrix}\begin{bmatrix} \frac{1}{s} \end{bmatrix} = \begin{bmatrix} \frac{1}{s(s+1)^2} \\ \frac{1}{(s+1)^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s} - \frac{1}{(s+1)} - \frac{1}{(s+1)^2} \\ \frac{1}{(s+1)^2} \end{bmatrix}$$

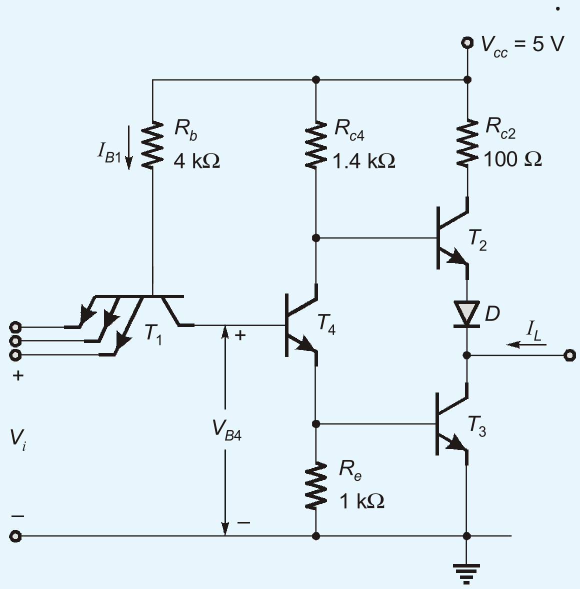
Take inverse Laplace transform on both sides,

$$x(t) = \begin{bmatrix} (1 - e^{-t} - te^{-t})u(t) \\ te^{-t}u(t) \end{bmatrix}$$

End of Solution

Q.3 (a) Explain the operation of the TTL gate circuit shown below, clearly mentioning the roles of the transistors T_2 and T_4, and the diode D.

Assume that when all the inputs are at logic 1, the transistors T_3 and T_4 are both in saturation :



[20 marks : 2025]

Solution:

The given TTL gate represents a NAND Gate with totem-pole output.

The totem pole configuration have three stages:

1. Multi-emitter input stage
2. Phase shift stage
3. Totem pole/Active pull-up stage

If any one of the input is LOW or all the inputs are LOW, then the corresponding emitter base junction of transistor T_1 becomes forward biased causing $V_{B_1} = 0.2 + 0.8 = 1\text{ V}$ and collector base junction becomes reverse biased (i.e. T_1 is in active region), and as a result T_4 and T_3 are turned off. The voltage V_{C_4} rises and causes T_2 and diode D to be turned ON and hence Y = HIGH.

Now, if all the inputs are HIGH, the emitter base junction of transistor T_1 is reverse biased and V_{B_1} approaches to V_{CC}, which causes T_4 and T_3 to go into deep saturation. As T_3 is in saturation, the transistor T_2 and diode D goes to cutoff state. Therefore output Y = LOW.

Operation:

TTL NAND gate:

When all inputs (A, B, C) are high, the BE junction of T_1 will be reverse biased but BC junction of T_1 will be forward biased. Current will flow from to C_1 to B_4. This will turn on T_4 and thus T_3. Due to T_3 being on, output Y will be zero.

When one or more of the inputs is low, BE will be forward biased and T_1 will turn on. This will lead to T_4 and T_3 being turned off and Y will be logic 1.

A	B	C	T_1	T_4	T_3	T_2	Y
0	0	0	Active	Cut-off	Cut-off	Saturation	High
0	0	1	Active	Cut-off	Cut-off	Saturation	High
0	1	0	Active	Cut-off	Cut-off	Saturation	High
0	1	1	Active	Cut-off	Cut-off	Saturation	High
1	0	0	Active	Cut-off	Cut-off	Saturation	High
1	0	1	Active	Cut-off	Cut-off	Saturation	High
1	1	0	Active	Cut-off	Cut-off	Saturation	High
1	1	1	Inverse Active	Saturation	Saturation	Cut-off	Low

Here, transistor T_4 is the pull-down transistor, sinking current to create a logic LOW output when conducting. T_3 is the pull-up transistor, sourcing current to create a logic HIGH output when T_4 is off. The purpose of using diode D is to ensure that the transistor T_2 is off, when T_3 is ON. In its absence, T_2 would go into saturation causing power loss.

End of Solution

Q.3 (b) (i) A thermistor, having β = 3100K, has a resistance of 1050 Ω at 20°C. The thermistor is used for the measurement of temperature and the resistance measured is 2300 Ω. Find the measured temperature if the thermistor is described by the relation $R = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$ where the symbols have their standard meanings.

[10 marks : 2025]

(ii) The following are the data for a Hay's bridge :

R_1 = 1 kΩ ± 1 part in 10 K, R_2 = 16.8 kΩ ± 1 part in 10 K,

R_3 = 833 ± 0.25 Ω, C = 1.43 ± 0.001 μF

The supply frequency is 50 ±0.1 Hz and the bridge's balanced conditions are

$$L = \frac{CR_1R_2}{1+\omega^2C^2R_3^2} \quad \text{and} \quad R = \frac{R_1R_2R_3C^2\omega^2}{1+\omega^2C^2R_3^2}$$

Calculate the values of L and R of the coil, and their limits of error.

[10 marks : 2025]

Solution:

(i) Given, β = 3100 K

`

R_0 = 1050 Ω at T_0 = 20°C = 273 + 20 = 293 K

Measured R = 2300 Ω

It is given that the resistance of thermistor varies with temperature as,

$$R = R_0\left(\exp\left(\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right)\right)$$

$$\frac{R}{R_0} = \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

$$\ln\left[\frac{R}{R_0}\right] = \beta\left[\frac{1}{T} - \frac{1}{T_0}\right]$$

Substituting the given values,

$$\ln\left[\frac{2300}{1050}\right] = 3100\left[\frac{1}{T} - \frac{1}{293}\right]$$

$$T \simeq 272.8 \text{ K}$$

Convert to °C

$$T = 272.8 - 273 \quad \Rightarrow \quad T = -0.2°\text{C}$$

(ii) Given,

$$R_1 = 1 \text{ k}\Omega \pm (1/10000) \times 100\%$$
$$= 1 \text{ k}\Omega \pm 0.01\%$$
$$= 1 \text{ k}\Omega \pm 0.1\ \Omega$$
$$R_2 = 1.68 \text{ k}\Omega \pm (1/10000) \times 100\%$$
$$= 1.68 \text{ k}\Omega \pm 0.01\%$$
$$= 16800 \pm 1.68\ \Omega$$
$$R_3 = 833 \pm 0.25\ \Omega$$
$$C = 1.43 \pm 0.001\ \mu\text{F}$$
$$f = 50 \pm 0.1 \text{ Hz}$$

From the given expressions, the values of R and L of the coil can be calculated as below,

$$L = \frac{CR_1R_2}{1+\omega^2C^2R_3^2}; \qquad R = \frac{R_1R_2R_3C^2\omega^2}{1+\omega^2C^2R_3^2}$$

$$L = \frac{(1.43\times10^{-6})\times1000\times16800}{1+(2\pi(50))^2\cdot(1.43\times10^{-6})^2(833)^2} = 21.07 \text{ H}$$

$$R = \frac{(1000)(16800)(833)(1.43\times10^{-6})^2(2\pi\times50)^2}{1+(2\pi(50)^2\cdot(1.43\times10^{-6})^2\cdot(833)^2}$$

$$R = 2477.54\ \Omega$$

The relative error for various parameters can be determined as below,

Parameter	Value	Tolarance given	Relative error (fraction)
R_1	1000 Ω	±1 Part in 10,000	0.0001 (0.01%)
R_2	16800 Ω	±1 Part in 10,000	0.0001 (0.01%)
R_3	833 Ω	±0.25 Ω	0.0003 (0.03%)
C	1.43×10^{-6}	±0.001μF	0.0007 (0.07%)
f	50	±0.1 Hz	0.002 (0.2%)

We have,

$$L = \frac{CR_1R_2}{1+\omega^2C^2R_3^2}, \qquad R = \frac{R_1R_2R_3C^2\omega^2}{1+\omega^2C^2R_3^2}$$

Assume,

$$D = 1+\omega^2C^2R_3^2$$

We have,

$$\omega^2C^2R_3^2 = 0.13984$$

$$D = 1+\omega^2C^2R_3^2 = 1.13984$$

Relative Error in D:

$$D = 1 + X; \quad \text{where } X = \omega^2 C^2 R_3^2$$

$$\frac{\Delta X}{X} = \frac{2\Delta\omega}{\omega} + \frac{2\Delta C}{C} + \frac{2\Delta R_3}{R_3}$$

$$= \pm[2(0.002) + 2(0.0007) + 2(0.0003)]$$

$$= \pm 0.0060 \text{ (or) } \pm 0.6\%$$

Absolute change in D:

$$\Delta D = \Delta X \text{(since 1 is constant)}$$

$$\Delta D = X * 0.0060 = \pm 0.13984 * 0.0060 = \pm 0.000839$$

Relative error in D:

$$\frac{\Delta D}{D} = \pm\frac{0.000839}{1.13984} \simeq \pm 0.000737$$

$$= \pm 0.0737\%$$

Relative error in L:

$$L = \frac{CR_1R_2}{D}$$

$$\frac{\Delta L}{L} = \frac{\Delta C}{C} + \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta D}{D}$$

$$= \pm(0.0007 + 0.0001 + 0.0001 + 0.000737)$$

$$= \pm 0.001637 \quad \text{(or)} \quad \pm 0.1637\%$$

$$\Delta L = \pm 21.07 \times 0.001637 = \pm 0.0345 \text{H}$$

Relative error in R:

$$R = \frac{R_1R_2R_3C^2\omega^2}{D}$$

$$\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} + \frac{2\Delta C}{C} + \frac{2\Delta\omega}{\omega} + \frac{\Delta D}{D}$$

$$\frac{\Delta R}{R} = \pm(0.0001 + 0.0001 + 0.0003 + 0.0014 + 0.0040 + 0.000737)$$

$$\frac{\Delta R}{R} = \pm 0.006637$$

$$\Delta R = \pm 0.006637 \times 2477.54$$

$$\Delta R = \pm 16.44\ \Omega$$

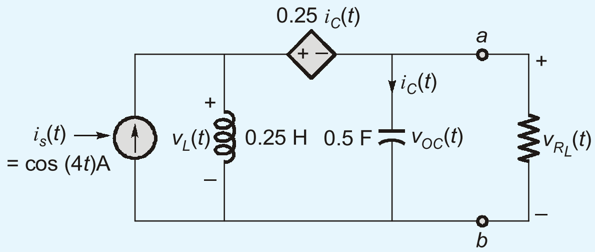
Thus, we get

$$L = 21.07 \pm 0.0345 \text{ H}$$

$$R = 2477.54 \pm 16.44\ \Omega$$

End of Solution

Q.3 (c) Find the Thevenin's equivalent of the circuit shown in the figure if ω = 4 rad/s. Also determine the voltage $v_{R_L}(t)$, when a 1.2 Ω load is connected to terminals a-b:

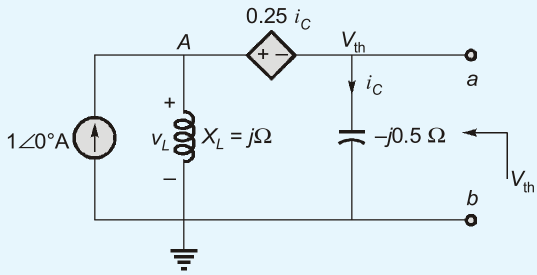


[20 marks : 2025]

Solution:

Step I: Thevenin Voltage, V_{th}: To calculate Thevenin voltage across *a-b*, we remove the load resistance. The circuit can thus be drawn as below,

For ω = 4 rad/sec
$$X_C = \frac{1}{j\omega C} = \frac{1}{j4\times 0.5} = -j0.5\,\Omega$$
$$X_L = j\omega L = j4 \times 0.25 = j1\ \Omega$$

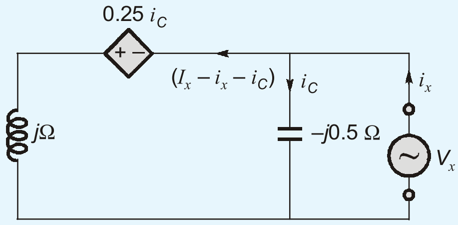


$$V_A - V_{th} = 0.25\, i_C$$
$$V_A - V_{th} = 0.25\left(\frac{V_{th}}{-j0.5}\right)$$
$$V_A = V_{th}\left[1 - \frac{0.25}{j0.5}\right]$$
$$V_A = [1.11\angle 26.56°]V_{th} \qquad ...(1)$$

Applying KCL at node *A*,
$$\frac{V_A}{j} + \frac{V_{th}}{-j0.5} = 1 \qquad ...(2)$$

Using (1) in (2), we get
$$\frac{1.11\angle 26.56° V_{th}}{j} + \frac{V_{th}}{-j0.5} = 1$$
$$V_{th} = 0.89\angle -63.43°\ \text{Volt}$$

Step II: Z_{th}: To calculate Z_{th}, the independent current source is open-circuited and we connect a voltage source V_x as shown below,



Using KCL, we can write

$$\frac{V_x}{-j0.5} + \frac{V_x + 0.25 i_C}{j} = i_x$$

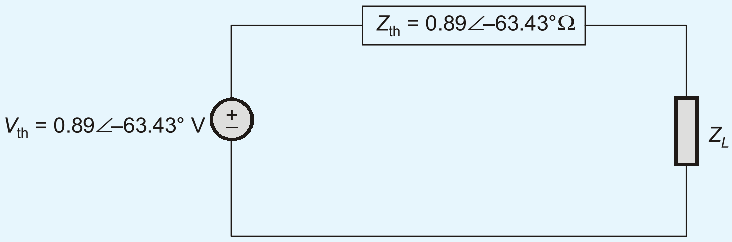
$$\frac{V_x}{-j0.5} + \frac{V_x + 0.25\left(\frac{V_x}{-j0.5}\right)}{j} = i_x$$

$$V_x\left[\frac{1}{-j0.5} + \frac{1}{j} + \frac{0.25}{j}\left(\frac{V_x}{-j0.5}\right)\right] = i_x$$

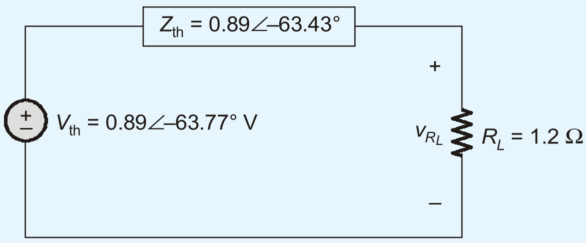
$$\frac{V_x}{i_x} = \frac{1}{\left[\frac{1}{j} + \frac{1}{-j0.5} + 0.5\right]}$$

$$Z_{th} = 0.89\angle{-63.43°}\ \Omega$$

Step III: Thevenin Equivalent Circuit



Step IV: When $Z_L = R_L = 1.2\ \Omega$



$$v_{R_L} = \frac{[0.89\angle -63.43°](1.2)}{1.2 + 0.89\angle -63.43°}$$

$$v_{R_L} = 0.6\ \angle{-37°}\ \text{volt}$$

Thus, $$v_{R_L}(t) = 0.6\cos(4t - 37°)\ \text{volt}$$

End of Solution

Q.4 (a) Consider that a double-heterojunction LED emitting at a peak wavelength 1400 nm has radiative and non-radiative recombination times of 20 ns and 80 ns respectively. The drive current is 30 mA and the refractive index of the light source material is 3.0. Calculate the power emitted from the device.

[20 marks : 2025]

Solution:

Given,

$$\text{wavelength, } \lambda = 1400 \text{ nm} = 1.4 \times 10^{-6} \text{ m}$$

$$\text{Radiative lifetime, } \tau_r = 20 \text{ nsec}$$

$$\text{Non radiative lifetime, } \tau_{nr} = 80 \text{ nsec}$$

$$\text{Drive current, } I = 30 \text{ mA}$$

$$\text{Refractive index, } n = 3$$

Power emitted by double-heterojunction LED is calculated by multiplying the external photon flux (number of photons per second) and the energy of the emitted photons.

$$P_{out} = N_{photons} \times E_{photon}$$

Quantum efficiency for light-emitting diodes (LEDs) is defined as the ratio of number of photons emitted to the number of electrons injected. Thus, we have

$$N_{photons} = \eta_{total} \times \frac{I}{q}$$

where the number of electrons per second

$$= \frac{I}{q} = \frac{0.03}{1.6 \times 10^{-19}} \simeq 1.875 \times 10^{17} \text{ electrons/sec}$$

and quantum efficiency, $\eta_{total} = \eta_{int} \times \eta_{ext}$

$$\text{Internal quantum efficiency, } \eta_{int} = \frac{\frac{1}{\tau_r}}{\left(\frac{1}{\tau_r}\right) + \left(\frac{1}{\tau_{nr}}\right)}$$

$$\eta_{int} = \frac{\frac{1}{20 \times 10^{-9}}}{\left(\frac{1}{20 \times 10^{-9}}\right) + \left(\frac{1}{80 \times 10^{-9}}\right)}$$

$$\eta_{int} = 0.8$$

For an LED with refractive index n, the external quantum efficiency is given by

$$\eta_{ext} \simeq \frac{1}{n(n+1)^2}$$

$$\simeq \frac{1}{3 \times 4^2} = 0.0208$$

$$\therefore \quad \eta_{total} = \eta_{inter} \times \eta_{ext}$$

$$= 0.8 \times 0.0208$$

$$= 0.01664$$

$$\therefore \quad N_{photons} = \eta_{total} \times \frac{I}{q}$$

$$= 0.01664 \times 1.875 \times 10^{17} = 3.12 \times 10^{15} \text{ photons/sec}$$

Energy of a photon is given by

$$E_{photon} = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34})(3\times10^{8})}{1.4\times10^{-6}}$$

$$= 1.419 \times 10^{-19}\ \text{J}$$

$\therefore$ Power emitted from LED, $P_{out} = 3.12 \times 10^{15} \times 1.419 \times 10^{-19}$

$$P_{out} = 4.427 \times 10^{-4}\ \text{W} \approx 0.44\ \text{mW}$$

End of Solution

Q.4 (b) A spherical nanoparticle has diameter of 10 nm. Determine the surface area to volume ratio and explain how this property affects the behaviour of nanomaterials compared to bulk materials.

[20 marks : 2025]

Solution:

Given, diameter of a spherical nanoparticle

$$d = 10\ \text{nm}$$

$\Rightarrow$ Radius, $r = \frac{d}{2} = 5\ \text{nm}$

Surface area of nanoparticle, $S.A = 4\pi r^2$

Volume (V) of nanoparticle, $V = \frac{4}{3}\pi r^3$

Surface area to volume ratio (SA/V)

$$= \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r} = \frac{3}{5\times10^{-9}} = 0.6\ \text{nm}^{-1}$$

Effect of SA/V on Nanomaterial behaviour:

The high surface area to volume ratio in nanoparticles significantly alters their physical and chemical properties compared to bulk materials.

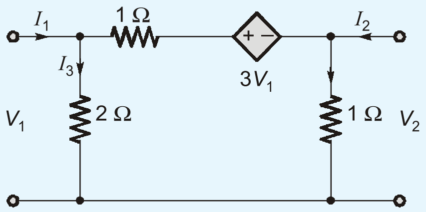
1. Increased Reactivity
   - More atoms are exposed on the surface and surface atoms are less tightly bound.
   - This leads to greater chemical reactivity useful in catalysis and sensors.
2. Enhanced Mechanical Properties
   - Nanoparticles can be stronger or harder due to their surface effects.
   - For example, nanomaterials can have higher tensile strength than bulk counter parts.
3. Optical properties
   - Nanoparticles exhibit size-dependent optical behavior (quantum effects)
   - Gold nanoparticles, for instance, change color based on size.
4. Thermal and Electrical properties:
   - Conductivity and heat resistance can be different due to limited mean free paths for electrons and phonons.
5. Biological Interactions:
   - Nanoparticles can easily penetrate cells and interact with biomolecules, important in drug delivery systems.

End of Solution

Q.4 (c) (i) A CR tube has an anode-screen distance of 30 cm. The accelerating potential is 1 kV. The tube is placed with its axis vertical. Find the maximum deflection of the spot due to the earth's magnetic field having $B = 0.018 \times 10^{-3}$ Wb/m^2.

[10 marks : 2025]

(ii) Calculate the Y-parameters for the network shown in the figure :



[10 marks : 2025]

Solution:

(i) Given for a CR tube,

$L = 30$ cm, $V_a = 1$ kV, $B = 0.018 \times 10^{-3}$ Wb/m^2

Let l_d = distance between the deflecting plates

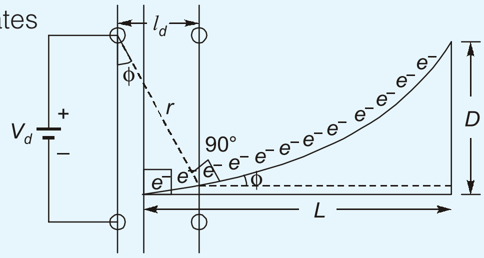
V_d = Applied potential difference across deflecting plates

D = Deflection of the beam

V_a = Accelerating potential

B = Magnetic flux density

L = distance between the centre of plates to the centre of screen



Due to the magnetic field, the electron beam moves along a circular arc. The force on the electrons due to magnetic field, Bev is equal to the centrifugal force i.e.

$$Bev = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Be} \qquad ...(i)$$

For an accelerating potential V, we have Gain in K.E. = Loss in P.E. i.e.

$$\frac{1}{2}mv^2 = qV_a \Rightarrow v = \sqrt{\frac{2qV_a}{m}} \qquad ...(ii)$$

From the given figure,

$$\tan\phi = \frac{D}{L}$$

We have, $\phi = \frac{l_d}{r}$. Since ϕ is very small, thus $\tan\phi \approx \phi$. Thus, we get

$$\frac{D}{L} = \frac{l_d}{r}$$

Using equation (i), the magnetic deflection is given by

$$D = \frac{Ll_dBq}{mv}$$

Using equation (ii), we get

$$D = L \cdot l_d \cdot B\sqrt{\frac{q}{2mV_a}}$$

where q = charge of electron = 1.6×10^{-19} Coulomb

m = mass of electron = 9.11×10^{-31} kg

Substituting the given values, we get

$$\frac{D}{l_d} = 30 \times 10^{-2} \times 0.018 \times 10^{-3} \times \sqrt{\frac{1.6 \times 10^{-19}}{2 \times 9.11 \times 10^{-31} \times 1 \times 10^3}}$$

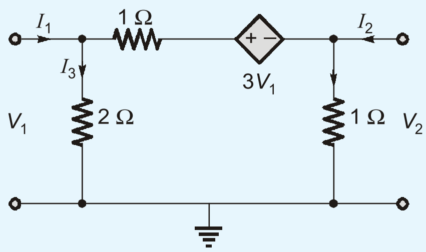
$$\frac{D}{l_d} = 0.54 \times 10^{-5} \times 0.29 \times \sqrt{10^{-19} \times 10^{28}}$$

$$= 0.1566 \times 10^{-5} \times \sqrt{10^9} = 0.1566 \times 10^{-5} \times 10^4 \times \sqrt{10}$$

$$\frac{D}{l_d} = 0.4952 \times 10^{-1} \Rightarrow \frac{D}{l_d} = 0.04952$$

For l_d = 1 cm $\Rightarrow$ Max. deflection = D_{max} = 0.04952 cm

(ii) We have,



Applying KCL at node 1,

$$\frac{V_1}{2} + \frac{V_1 - 3V_1 - V_2}{1} = I_1$$

$$I_1 = -2V_1 + \frac{V_1}{2} - V_2$$

$$I_1 = \frac{-3V_1}{2} - V_2 \quad \text{...(i)}$$

Applying KCL at node 2,

$$\frac{V_2}{1} + V_2 + 3V_1 - V_1 = I_2$$

$$I_2 = +2V_1 + 2V_2 \quad \text{...(ii)}$$

Comparing equations (i) and (ii) with Y-parameter equations,

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

we get the y-parameters matrix as,

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{-3}{2}\mho & -1\mho \\ 2\mho & 2\mho \end{bmatrix}$$

End of Solution

Section-B

Q.5 (a) Derive a relation for the value of the capacitor for frequency error compensation of a moving-iron voltmeter in terms of its parameters and the series resistance.

[12 marks : 2025]

Solution:

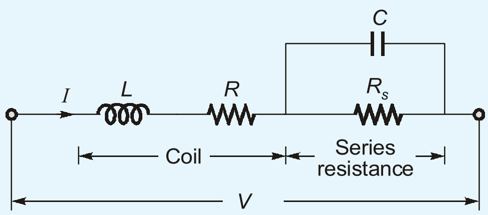
Frequency errors: Changes in frequency may cause errors due to changes of reactance of the working coil and also due to changes of magnitude of eddy currents set up in the metal parts of instrument.

Reactance of instrument coil: The change of reactance of the instrument coil is important in case of voltmeters where an additional resistance R_s is used in series with the instrument coil. Let the resistance and inductance of the instrument coil be R and L. Then the current I in the instrument coil for given applied voltage V is given by:

$$I = \frac{V}{\sqrt{(R+R_s)^2 + \omega^2 L^2}}$$

The deflection of the moving-iron voltmeter depends upon the current through the coil. Therefore, the deflection for a given voltage will be less at high frequencies than at low frequencies. To some extent, compensation to this type of error is possible by connecting a capacitor C across the series resistance R_s as shown in figure.

The idea of shunting the series resistor is to make the circuit behave like a pure resistance so that the frequency changes have no effect on the readings of the instrument. Thus, the compensated instrument will have a power factor nearly equal to unity. As the resistance of the meter, R, is considerably smaller than the series multiplier resistance R_s, it can be assumed that it is sufficient to ensure that the magnitude Z of the impedance of the circuit formed by L, R_s and C has a value R_s when used on a.c.



Now,

$$Z = j\omega L + \frac{R_s}{1 + j\omega CR_s}$$

$$= j\omega L + \frac{R_s - j\omega CR_s^2}{1 + \omega^2 C^2 R_s^2}$$

Since $\omega CR_s << 1$, we can write

$$Z = j\omega L + (R_s - j\omega CR_s^2)(1 - \omega^2 C^2 R_s^2)$$

$$= j\omega L + R_s - j\omega CR_s^2 - \omega^2 C^2 R_s^3 + j\omega^3 C^3 R_s^4$$

$$= R_s - \omega^2 C^2 R_s^3 + j[\omega L - \omega CR_s^2(1 - \omega^2 C^2 R_s^2)]$$

$$= R_s - \omega^2 C^2 R_s^3 + j[\omega L - \omega CR_s^2] \quad \text{...as } \omega^2 C^2 R_s^2 \ll 1$$

$$= R_s(1-\omega^2C^2R_s^2) + j\omega(L - CR_s^2)$$

or
$$|Z|^2 = R_s^2(1-\omega^2C^2R_s^2)^2 + \omega^2(L - CR_s^2)^2$$

This must equal R_s^2 in order that the a.c. calibration at all frequencies and d.c. calibration is the same.

∴
$$R_s^2 = R_s^2(1-\omega^2C^2R_s^2)^2 + \omega^2(L - CR_s^2)^2$$
$$= R_s^2(1-2\omega^2C^2R_s^2 + \omega^4C^4R_s^4) + \omega^2(L - CR_s^2)^2$$
$$= R_s^2(1-2\omega^2C^2R_s^2) + \omega^2(L - CR_s^2)^2$$

as $\omega^4C^4R_s^4 << 1$

$$= R_s^2 - 2\omega^2C^2R_s^4 + \omega^2L^2 + \omega^2C^2R_s^4 - 2\omega^2LCR_s^2$$

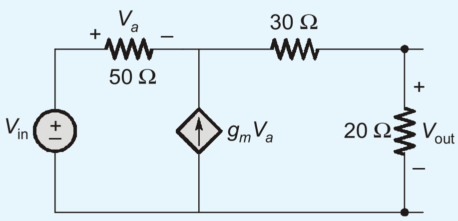
or
$$L^2 - 2LCR_s^2 - C^2R_s^4 = 0$$

or
$$L = \frac{2CR_s^2 + \sqrt{4C^2R_s^4 + 4C^2R_s^4}}{2} = 2.41CR_s^2$$

∴
$$C = \frac{1}{2.41}\frac{L}{R_s^2} = 0.41\frac{L}{R_s^2}$$

End of Solution

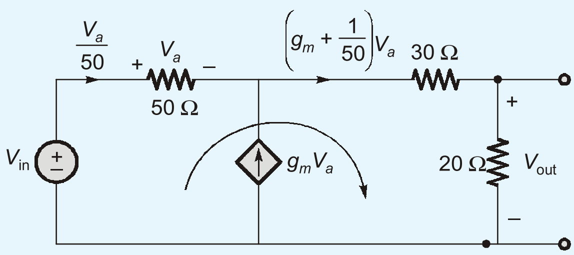
Q.5 (b) For the circuit shown in the figure, find $\frac{V_{out}}{V_{in}}$ in terms of the parameter g_m. Then find $\frac{V_{out}}{V_{in}}$, when $g_m = 2$:



[12 marks : 2025]

Solution:

Considering the given circuit,



We have,

$$V_{out} = 20\left[g_m + \frac{1}{50}\right]V_a$$

$$V_{out} = \left[20g_m + \frac{2}{5}\right]V_a \quad \text{...(i)}$$

On applying KVL around the outer loop,

$$-V_{in} + V_a + 30\left[g_m + \frac{1}{50}\right]V_a + V_{out} = 0$$

$$-V_{in} + \left[1 + 30g_m + \frac{3}{5}\right]V_a + V_{out} = 0 \quad \text{...(ii)}$$

Substitute equation (i) in (ii),

$$-V_{in} + \left[\frac{8}{5} + 30g_m\right]\left[\frac{V_{out}}{20g_m + \frac{2}{5}}\right] + V_{out} = 0$$

$$-V_{in} + \left[1 + \frac{30g_m + \frac{8}{5}}{20g_m + \frac{2}{5}}\right]V_{out} = 0$$

∴

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \left(\frac{30g_m + \frac{8}{5}}{20g_m + \frac{2}{5}}\right)} = \frac{1}{1 + \frac{150g_m + 8}{100g_m + 2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{100g_m + 2}{250g_m + 10}$$

For $g_m = s$,

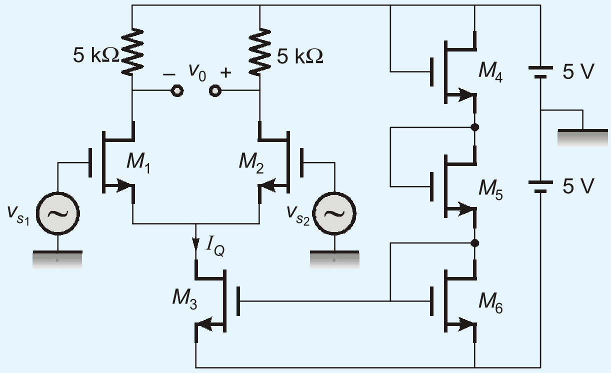
$$\frac{V_{out}}{V_{in}} = \frac{200 + 2}{500 + 10} = \frac{202}{510}$$

⇒

$$\frac{V_{out}}{V_{in}} \simeq 0.3961$$

End of Solution

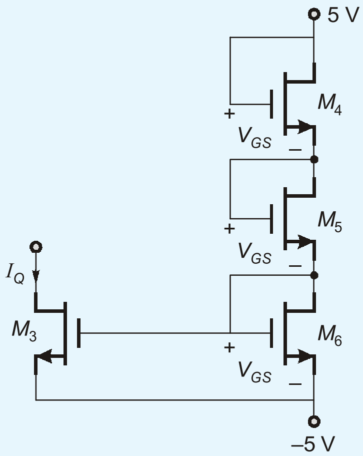
Q.5 (c) For the circuit shown in the figure, all the MOSFETs are identical. Assume $\mu_n C_{ox} = 0.1$ mA/V^2, $V_{tn} = 1$ V, $\lambda = 0$ and $I_Q = 1$ mA. Calculate $\frac{W}{L}$ ratio and voltage gain $A_d = \frac{v_0}{v_{s_1} - v_{s_2}}$:



[12 marks : 2025]

Solution:

Consider current Mirror part of the circuit.



Since all the MOSFETs are identical, we assume $V_{GS4} = V_{GS5} = V_{GS6} = V_{GS}$.

Using KVL : $5 = V_{GS} + V_{GS} + V_{GS} - 5$

$$\Rightarrow \quad 3V_{GS} = 10 \quad \Rightarrow \quad V_{GS} = \frac{10}{3}\text{V}$$

For MOSFETs M_4, M_5 and M_6, Drain is shorted to Gate, thus $V_{DS} > V_{GS} - V_T$ i.e. MOSFETs are in saturation. The drain current,

$$I_{DS} = \frac{\mu_n C_{ox}}{2} \times \frac{W}{L}(V_{GS} - V_{TN})^2$$

$$1 = \frac{0.1}{2} \times \frac{W}{L}\left(\frac{10}{3} - 1\right)^2$$

$$\Rightarrow \quad \frac{W}{L} = \frac{180}{49} = 3.6735$$

For M_1 and M_2 : $\quad I'_{DS} = \frac{I_Q}{2} = 0.5 \text{ mA}$

Thus, $\quad g_m = \sqrt{2\mu_n C_{ox} \times \frac{W}{L} \times I'_{DS}}$

$$g_m = \sqrt{2 \times 0.1 \times 10^{-3} \times \frac{180}{49} \times 0.5 \times 10^{-3}}$$

$$g_m = 0.6061 \text{ m℧}$$

The differential amplifier has a dual input balanced output. Thus, differential voltage gain,

$$A_d = \frac{V_0}{V_{S_1} - V_{S_2}} = g_m \times R_D = 0.6061 \times 5$$

$$= 3.03$$

End of Solution

Q.5 (d) Design a J-K counter for states 1, 2, 4, 5, 7, 8, 10, 11, ... What would happen if the circuit were turned ON and the first state it entered was a don't care state? [12 marks : 2025]

Solution:

Since the highest state of the sequence is 11 = $(1011)_2$, hence the counter requires four J-K flip flops.

Given states: 1, 2, 4, 5, 7, 8, 10, 11

Unused states: 0, 3, 6, 9, 12, 13, 14, 15

Q	Q^+	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Excitation Table for JK Flip-Flop

State Table:

Present State				Next State				Flip-flop Inputs							
Q_3	Q_2	Q_1	Q_0	Q_3^+	Q_2^+	Q_1^+	Q_0^+	J_3	K_3	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	X	X	X	X	X	X	X	X	X	X	X	X
0	0	0	1	0	0	1	0	0	X	0	X	1	X	X	1
0	0	1	0	0	1	0	0	0	X	1	X	X	1	0	X
0	0	1	1	X	X	X	X	X	X	X	X	X	X	X	X
0	1	0	0	0	1	0	1	0	X	X	0	0	X	1	X
0	1	0	1	0	1	1	1	0	X	X	0	1	X	X	0
0	1	1	0	X	X	X	X	X	X	X	X	X	X	X	X
0	1	1	1	1	0	0	0	1	X	X	1	X	1	X	1
1	0	0	0	1	0	1	0	X	0	0	X	1	X	0	X
1	0	0	1	X	X	X	X	X	X	X	X	X	X	X	X
1	0	1	0	1	0	1	1	X	0	0	X	X	0	1	X
1	0	1	1	0	0	0	1	X	1	0	X	X	1	X	0
1	1	0	0	X	X	X	X	X	X	X	X	X	X	X	X
1	1	0	1	X	X	X	X	X	X	X	X	X	X	X	X
1	1	1	0	X	X	X	X	X	X	X	X	X	X	X	X
1	1	1	1	X	X	X	X	X	X	X	X	X	X	X	X

Minimization of expression for flip-flops inputs using K-maps:

J_3: Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	0	X	0
01	0	0	1	X
11	X	X	X	X
10	X	X	X	X

$J_3 = Q_1\,Q_0$

K_3: Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	X	X	X	X
10	0	X	1	0

$K_3 = Q_0$

J_2: Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	0	X	1
01	X	X	X	X
11	X	X	X	X
10	0	X	0	0

$J_2 = \overline{Q_3} \cdot Q_1$ or $J_2 = \overline{Q_3} \cdot \overline{Q_0}$

K_2: Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	X	X	X
01	0	0	1	X
11	X	X	X	X
10	X	X	X	X

$K_2 = Q_1$

J_1: Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	1	X	X
01	0	1	X	X
11	X	X	X	X
10	1	X	X	X

$J_1 = Q_3 + Q_0$

K_1: Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	X	X	1
01	X	X	1	X
11	X	X	X	X
10	X	X	1	0

$K_1 = Q_0 + \overline{Q_3}$

J_0: Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	X	X	0
01	1	X	X	X
11	X	X	X	X
10	0	X	X	1

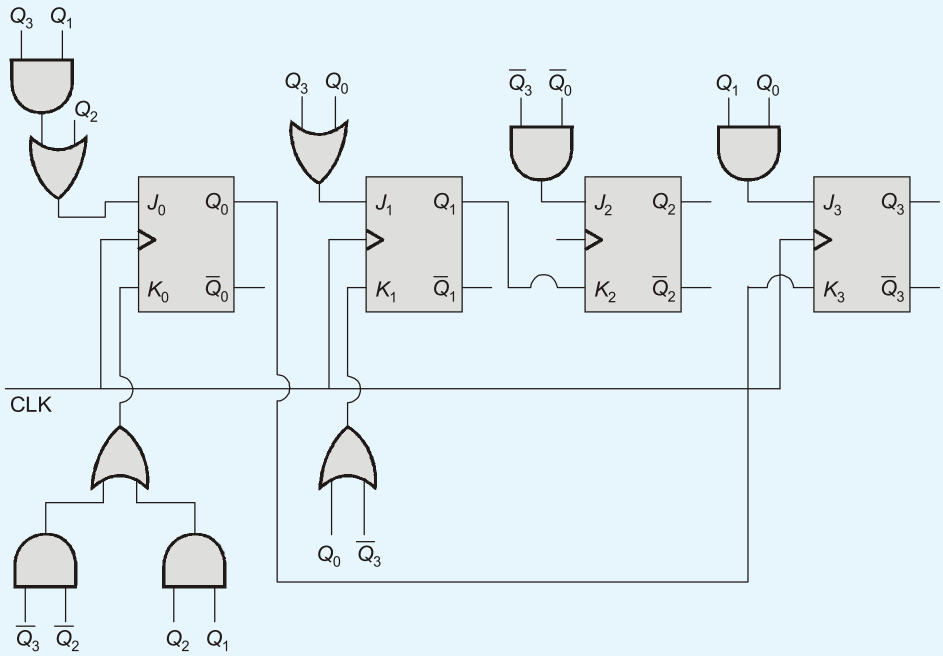
$J_0 = Q_2 + Q_3Q_1$

K_0: Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	1	X	X
01	X	0	1	X
11	X	X	X	X
10	X	X	0	X

$K_0 = \overline{Q_3} \cdot \overline{Q_2} + Q_2Q_1$

Logic Circuit:

The J-K counter for the given sequence is as shown below:

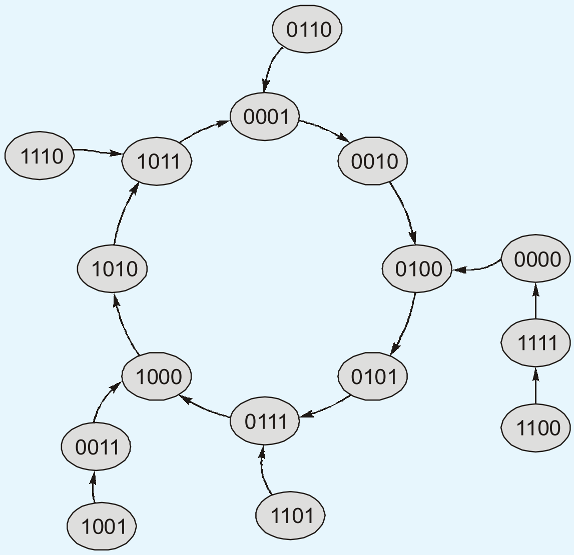


Checking for unused state:

For the above logic circuit, the next state when the circuit enters into the unused states can be determined as below,

Unused State				Flip-Flop Inputs								Next State			
Q_3	Q_2	Q_1	Q_0	J_3	K_3	J_2	K_2	J_1	K_1	J_0	K_0	Q_3^+	Q_2^+	Q_1^+	Q_0^+
0	0	0	0	0	0	1	0	0	1	0	1	0	1	0	0
0	0	1	1	1	1	0	1	1	1	0	1	1	0	0	0
0	1	1	0	0	0	1	1	0	1	1	1	0	0	0	1
1	0	0	1	0	1	0	0	1	1	0	0	0	0	1	1
1	1	0	0	0	0	0	0	1	0	1	0	1	1	1	1
1	1	0	1	0	1	0	0	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	0	1	1	1	0	1	1
1	1	1	1	1	1	0	1	1	1	1	1	0	0	0	0

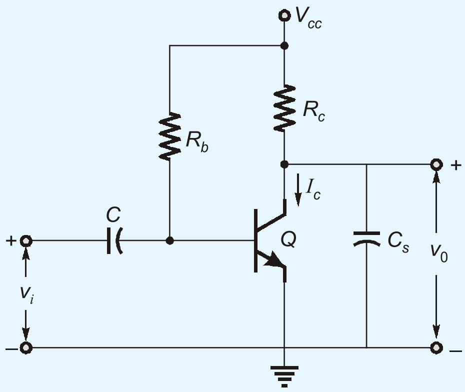
The complete state diagram of the counter can be drawn as below,

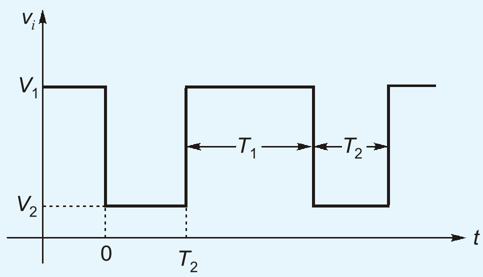


From the above state diagram, we can see that if the circuit entered into the unused state when turned ON, it will enter into the used state in finite number of clock cycles. Thus, the lockout condition does not occur.

End of Solution

Q.5 (e) The transistor Q acts as a switch in the given circuit for the applied input v_i that varies with time as shown below :





Plot the variation of the collector current I_c and the output voltage v_0, assuming that the time constants are small compared to T_1 or T_2.

[12 marks : 2025]

Solution:

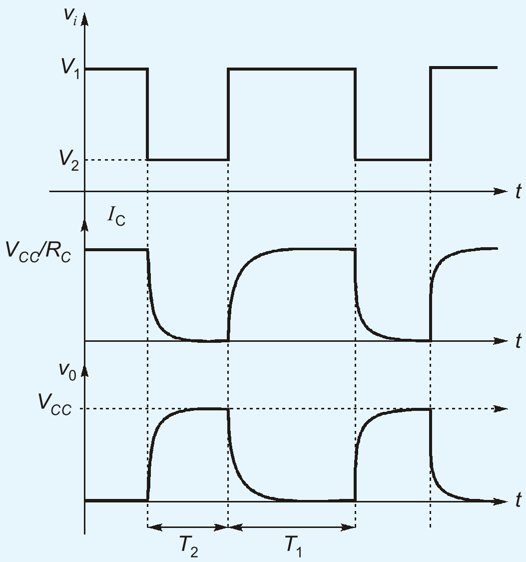
Transistor is acting as switch:

If $V_i = V_1$: Transistor becomes ON and the capacitor discharges to zero . Thus, v_0 becomes zero and the collector current,

$$I_C = \frac{V_{CC} - 0}{R_C} = \frac{V_{CC}}{R_C}$$

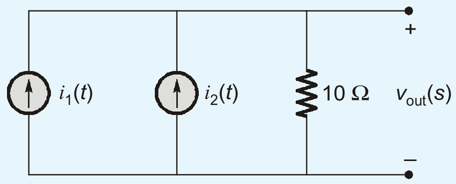
If $V_1 = V_2$: Transistor becomes OFF and the capacitor charges to V_{CC}. Thus, v_0 becomes V_{CC} and I_C becomes 0.

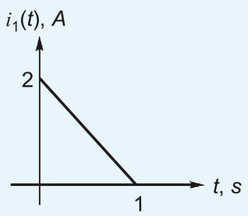
Since the time constant are small compared to T_1 or T_2, the capacitor charges/discharges completely during the time interval. Considering gradual charging and discharging of capacitor, we can draw waveforms as shown below.

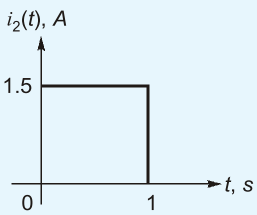


End of Solution

Q.6 (a) The circuit of the figure has two sources of excitation $i_1(t)$ and $i_2(t)$. Compute $v_{out}(s)$:



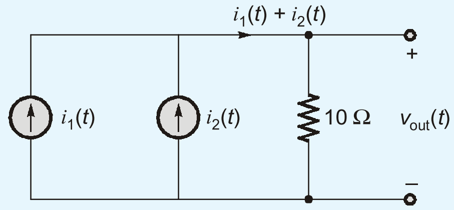


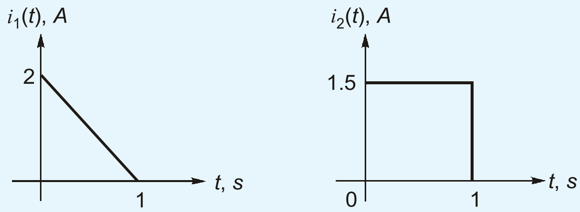


[20 marks : 2025]

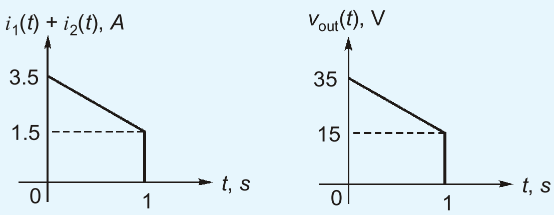
Solution:

Given,





We have, $$v_{out}(t) = 10 \times [i_1(t) + i_2(t)]$$



$\therefore$ $$v_{out}(t) = -20t + 35; \quad 0 < t < 1$$

To obtain the Laplace Transform, we can write $v_{out}(t)$ as,

$$v_{out}(t) = (35 - 20t)[u(t) - u(t-1)]$$

$$v_{out}(t) = 35u(t) - 35u(t-1) - 20tu(t) + 20(t-1)u(t-1) + 20u(t-1)$$

$$v_{out}(t) = 35u(t) - 15u(t-1) - 20tu(t) + 20(t-1)u(t-1)$$

We know that,

$$u(t) \longleftrightarrow \frac{1}{s}$$

$$tu(t) \longleftrightarrow -\frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2}$$ (Multiplication by t property)

$$x(t - t_0) \longleftrightarrow e^{-st_0} X(s)$$ (Time shifting property)

Using the above properties, we can write the Laplace Transform of $v_{out}(t)$ as,

$$V_{out}(s) = \frac{35}{s} - \frac{15e^{-s}}{s} - \frac{20}{s^2}(1 - e^{-s})$$

End of Solution

Q.6 (b) Consider the circuit given below with the following parameters :

R_{c1} = 3 K, R_{c2} = 500 Ω, R_e = 50 Ω, R' = R_s = 1.2 K, h_{fe} = 50, h_{ie} = 1.1 K and $h_{re} = h_{oe} = 0$

Analyze the circuit for—

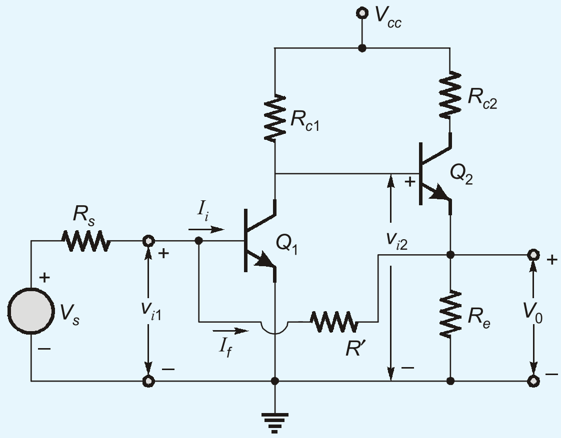
(i) reverse transmission factor, β;

(ii) transfer gain;

(iii) voltage gain with feedback;

(iv) input resistance with feedback;

(v) output resistance with feedback.

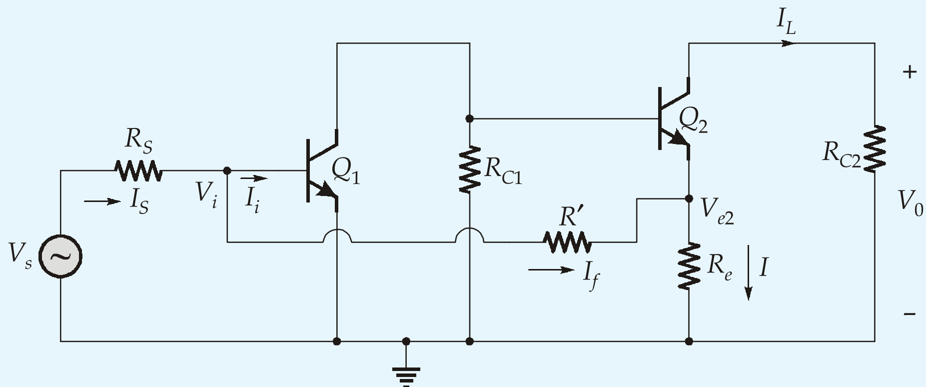


[20 marks : 2025]

Solution:

Given, R_{C1} = 3 kΩ, R_{C2} = 500 Ω, R_e = 50 Ω, R' = R_S = 1.2 kΩ, h_{fe} = 50, h_{ie} = 1.1 kΩ, and $h_{re} = h_{ce} = 0$.

Small signal equivalent circuit:



Step 1: Identify the type of feedback:

R' is not connected directly to output node ⇒ Current sampling

R' is connected directly to input node ⇒ shunt mixing

∴ The given feedback amplifier has current shunt feedback (current amplifier)

Step 2: Calculate the feedback factor (β):

KCL : $I_f = I_L + I$

$\Rightarrow \quad I = I_f - I_L$

We have, feedback current,

$$I_f = \frac{V_i - V_{e2}}{R'}$$

Put $V_i = 0 \Rightarrow$

$$I_f = \frac{-V_{e2}}{R'} = \frac{-IR_e}{R'} = \frac{-(I_f - I_L)R_e}{R'}$$

$$\Rightarrow \quad I_f\left[1 + \frac{R_e}{R'}\right] = \frac{I_L R_e}{R'}$$

$$\Rightarrow \quad I_f(R' + R_e) = I_L R_e$$

$$\Rightarrow \quad \frac{I_f}{I_L} = \beta = \frac{R_e}{R' + R_e} = \frac{50}{1200 + 50} = \frac{5}{125} = \frac{1}{25}$$

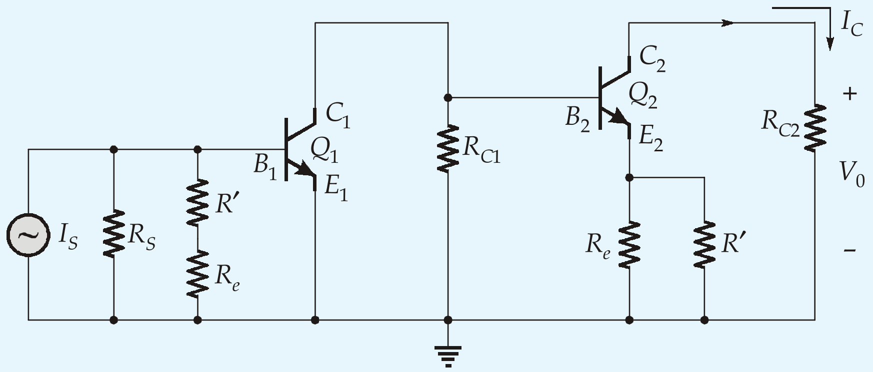
$$\Rightarrow \quad \beta = \frac{1}{25}$$

Step 3: Draw circuit without feedback:

Break output loop $\Rightarrow$ R' and R_E appear in series between B_1 and ground.

Ground input node $\Rightarrow$ R' and R_E appear in parallel between E_2 and ground.

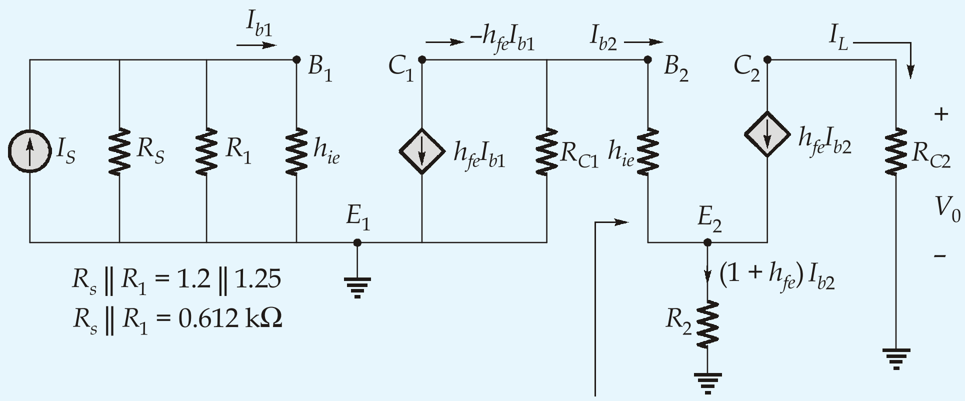
The small signal equivalent circuit without feedback can be drawn as below,



$$R_1 = R' + R_e = 1200 + 50 = 1250\ \Omega = 1.25\ \text{k}\Omega$$

$$R_2 = R' \parallel R_e = 1200 \parallel 50 = \frac{1200 \times 50}{(1200 + 50)} = 48\ \Omega$$

Step 4: Replace Q_1 and Q_2 with h-parameter model:



$$R_{i2} = h_{ie} + (1 + h_{fe}) R_2 = 1100 + (51)(48) = 3548\ \Omega = 3.548\ \text{k}\Omega$$

From the circuit,
$$I_{b1} = \frac{I_s(R_S \parallel R_1)}{(R_S \parallel R_1) + h_{ie}}$$

$$I_{b2} = \frac{-h_{fe}I_{b1}R_{C1}}{R_{C1} + R_{i2}}$$

$\therefore$ Current gain,
$$A_I = \frac{I_L}{I_S} = \left(\frac{I_L}{I_{b2}}\right)\cdot\left(\frac{I_{b2}}{I_{b1}}\right)\cdot\left(\frac{I_{b1}}{I_S}\right)$$

$$= (-h_{fe})\cdot\left(\frac{-h_{fe}R_{C1}}{R_{C1}+R_{i2}}\right)\cdot\left(\frac{R_S \parallel R_1}{R_S \parallel R_1 + h_{ie}}\right)$$

$$= (-50)\cdot\left(\frac{-50\times 3}{3+3.548}\right)\cdot\left(\frac{0.612}{0.612+1.1}\right) = 409.45$$

Resistance seen by current source 'I_s' is

$$R_{in} = R_S \parallel R_1 \parallel h_{ie} = 1.2 \parallel 1.25 \parallel 1.1 = 0.612 \parallel 1.1 = 0.3933\ \text{k}\Omega$$

Output resistance, $R_{C2} = 500\ \Omega$

Desensitivity, $D = 1 + \beta A_I$

$$= 1+\left(\frac{1}{25}\times 409.45\right) = 17.378$$

With feedback:

Current gain with feedback,
$$A_{If} = \frac{A_I}{1+\beta A_I} = \frac{409.45}{17.378} = 23.56$$

For a current shunt amplifier, the input resistance decreases by a factor of $(1 + \beta A_I)$ and output impedance increases by a factor of $(1 + \beta A_I)$. Thus, Input resistance with feedback, $R_{in\,f} = \dfrac{R_{in}}{1+\beta A_I} = \dfrac{393.3}{17.378} = 22.63\ \Omega$

Output resistance with feedback, $R_{of} = R_0(1+\beta A_I)$

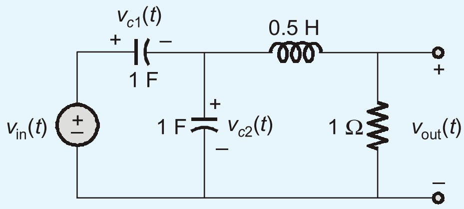
$= 500 \times 17.378 = 8.689\ \text{k}\Omega$

Voltage gain with feedback, $A_{vf} = \dfrac{V_0}{V_s} = \dfrac{I_L R_{c2}}{I_s R_s} = \dfrac{A_{If} R_{c2}}{R_s} = \dfrac{23.56 \times 0.5}{1.2}$

$\Rightarrow \quad A_{vf} = 9.8167$

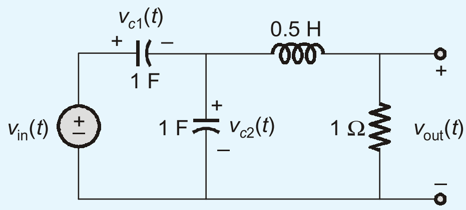
End of Solution

Q.6 (c) Consider the circuit in which $v_{in}(t) = 5u(t)$ V, $v_{c1}(0^-) = 3$ V, $v_{c2}(0^-) = 0$ V and $i_L(0^-) = 2$ A. Find $v_{out}(t)$:



[20 marks : 2025]

Solution:



Given,

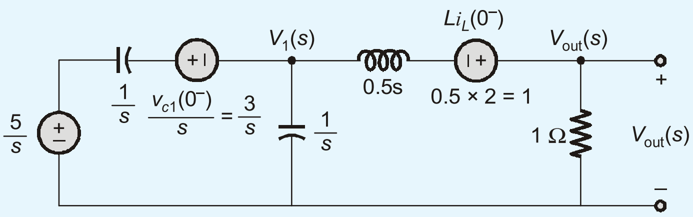
$$v_{in}(t) = 5u(t)\ \text{V}$$
$$v_{c1}(0^-) = 3\ \text{V}$$
$$v_{c2}(0^-) = 0\ \text{V}$$
$$i_L(0^-) = 2\ \text{A}$$

We can write,

$$V_L(s) = LsI(s) - Li(0^-)$$

$$V_C(s) = \frac{I(s)}{Cs} + \frac{v_c(0^-)}{s}$$

Thus, the s-domain equivalent of the given circuit can be drawn as,



Apply KCL at node $V_1(s)$:

$$\frac{V_1(s) + \frac{3}{s} - \frac{5}{s}}{\left(\frac{1}{s}\right)} + \frac{V_1(s)}{\left(\frac{1}{s}\right)} + \frac{V_1(s) + 1 - V_{out}(s)}{0.5s} = 0$$

$$V_1(s)\left(s + s + \frac{1}{0.5s}\right) - \frac{V_{out}(s)}{0.5s} = 2 - \frac{1}{0.5s}$$

$$V_1(s)\left[\frac{2(s^2+1)}{s}\right]-V_{out}(s)\left[\frac{2}{s}\right] = 2\left(\frac{s-1}{s}\right)$$

$$V_1(s)[2(s^2+1)] - 2V_{out}(s) = 2(s-1) \quad ...(1)$$

Apply KCL at node $V_{out}(s)$:

$$\frac{V_{out}(s)-1-V_1(s)}{0.5s}+\frac{V_{out}(s)}{1} = 0$$

$$V_{out}(s)\left[1+\frac{2}{s}\right]-\frac{2}{s}V_1(s) = \frac{2}{s}$$

$$V_{out}(s)\left[\frac{s}{2}+1\right]-V_1(s) = 1$$

$\Rightarrow$

$$V_1(s) = -1+V_{out}(s)\left[\frac{s+2}{2}\right] \quad ...(2)$$

Substitute equation (2) in (1),

$$\left\{-1+V_{out}(s)\left[\frac{s+2}{2}\right]\right\}\left[2(s^2+1)\right]-2V_{out}(s) = 2(s-1)$$

$$-2(s^2+1)+(s^2+1)(s+2)V_{out}(s)-2V_{out}(s) = 2(s-1)$$

$$V_{out}(s)[(s^2+1)(s+2)-2] = 2(s-1)+2(s^2+1)$$

$$V_{out}(s) = \frac{2s-2+2s^2+2}{s^3+2s^2+s+2-2} = \frac{2s^2+2s}{s(s^2+2s+1)}$$

$$V_{out}(s) = \frac{2s(s+1)}{s(s^2+2s+1)} = \frac{2(s+1)}{(s^2+2s+1)} = \frac{2(s+1)}{(s+1)^2}$$

$\therefore$

$$V_{out}(s) = \frac{2}{(s+1)}$$

Taking inverse Laplace transform on both sides,

$$v_{out}(t) = 2e^{-t}u(t)$$

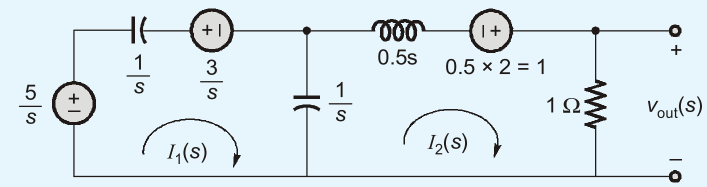
Alternate Method:

Given,

$$v_{in}(t) = 5u(t)\text{ V},$$

$$v_c(0^-) = 3\text{ V},\ v_{c2}(0^-) = 0\text{ V},\ i_L(0^-) = 2\text{ A}$$

The s-domain equivalent of the given circuit is,



Apply KVL around loop 1:

$$-\frac{5}{s}+\left(\frac{1}{s}+\frac{1}{s}\right)I_1(s)-\frac{1}{s}I_2(s)+\frac{3}{s} = 0$$

$$\frac{2}{s}I_1(s)-\frac{1}{s}I_2(s) = \frac{2}{s}$$

$$2I_1(s)-I_2(s) = 2 \quad ...(1)$$

Apply KVL around loop 2:

$$\left(0.5s + 1 + \frac{1}{s}\right) I_2(s) - \frac{1}{s} I_1(s) - 1 = 0$$

$$(0.5s^2 + s + 1)I_2(s) - I_1(s) - s = 0$$

$$I_1(s) = (0.5s^2 + s + 1)I_2(s) - s \quad \text{...(2)}$$

Substitute equation (2) in(1),

$$2[(0.5s^2 + s + 1)I_2(s) - s] - I_2(s) = 2$$

$$(s^2 + 2s + 2 - 1)I_2(s) = 2 + 2s$$

$$I_2(s) = \frac{2(s+1)}{(s^2 + 2s + 1)} = \frac{2(s+1)}{(s+1)^2} = \frac{2}{(s+1)}$$

We have, $V_{out}(s) = 1 \times I_2(s) = \dfrac{2}{(s+1)}$

Taking inverse Laplace transform on both sides,

$$v_{out}(t) = 2e^{-t} u(t)$$

End of Solution

Q.7 (a) (i) A first-order thermometer is used for the measurement of temperature of air cycling at a rate of 1 cycle every 5 minutes. The time constant of the thermometer is 20 seconds. Calculate the attenuation of the indicated temperature in percent. If the temperature undergoes a sinusoidal variation of 20°C, calculate the indicated variation in temperature.

[10 marks : 2025]

(ii) Compare and contrast Type-I and Type-II superconductors based on the following parameters:

(1) Magnetic field behaviour
(2) Critical magnetic field
(3) Material examples
(4) Meissner effect
(5) Applications

[10 marks : 2025]

Solution:

(i) Cycling rate : 1 cycle every 5 minutes ⇒ Period of the temperature cycle = 300 seconds

Time constant (τ) = 20 seconds

Input sinusoidal temperature variation = 20°C

The frequency (f) of the temperature cycle is

$$f = \frac{1}{\text{cycle time}} = \frac{1}{300} \text{ Hz}$$

$$\text{Angular frequency } (\omega) = 2\pi f = \frac{2\pi}{300} \simeq 0.02094 \text{ rad/sec}$$

The transfer function of a first-order system is given by

$$T(s) = \frac{1}{1 + \tau s}$$

For a first order system subjected to sinusoidal input, the amplitude attenuation is given by

$$\text{Attenuation factor} = \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

Substitute $\omega = 0.02094$ rad/sec, $\tau = 20$ sec

$$\omega\tau = 0.4188$$

$$\text{Attenuation factor} = \frac{1}{\sqrt{1+(0.4188)^2}} = 0.922$$

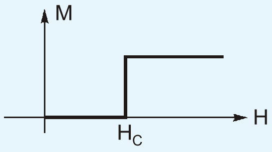
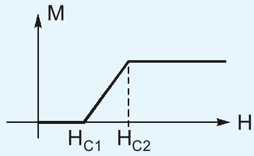
The attenuation in percent is

$$\text{Attenuation} = \{1 - 0.922\} \times 100 = 7.8\%$$

The indicated variation is the input-variation multiplied by attenuation factor.

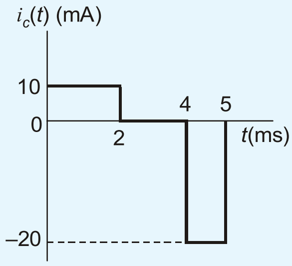
$$\text{Indicated variation} = 20 * 0.922 = 18.44°\text{C}$$

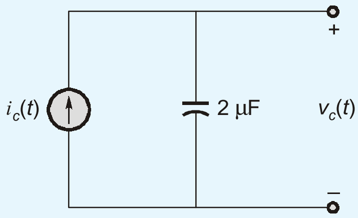
(ii) Comparison of Type-I and Type-II superconductors

Parameters	**Type-I Superconductor**	**Type-II Superconductor**
1. Magnetic field behaviour	 Exhibits an abrupt transition from superconducting to normal state at a critical field, H_C. It exhibit perfect diamagnetism until H_C.	 It loses superconductivity gradually and has two critical fields: H_{C1} (below which full diamagnetism) and H_{C2} (above which normal state). Between H_{C1} and H_{C2}: Mixed/Vortex state.
2. Critical magnetic field	Only one (H_C), The critical fields of Type I superconductors is generally low.	Two critical fields (H_{C1} and H_{C2}). H_{C2} can be very high.
3. Material examples	Pure elemental metals: Pd (lead), Hg (mercury), Al (Aluminium), Sn (tin).	Typically alloys and compounds: YBCO, NbTi, BSCCO.
4. Meissner effect	Complete Meissner effect below H_C (perfect flux expulsion)	It shows partial Meissner effect. It exhibit complete Meissner effect only below H_{C1}. In the mixed state, some flux penetrates in quantized vortices; thus full flux expulsion is lost.
5. Applications	Mostly academic and laboratory demonstrations	Widely used in practical high-field applications: Superconducting magnets (MRI, NMR, particle accelerators), superconducting wires/tapes, power devices, fault-current limiters, etc.

End of Solution

Q.7 (b) The current through a 2μF capacitor is shown in the figure. At $t = 0$, the voltage is zero. Sketch the voltage and power waveform with respect to the time (scaled voltage and power):

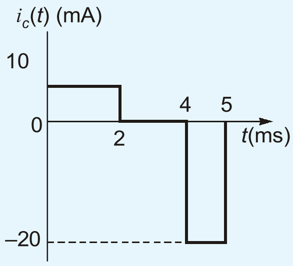


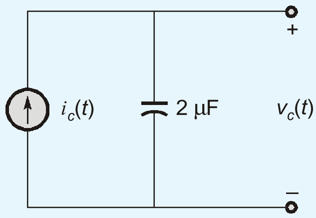


[20 marks : 2025]

Solution:

Given, $C = 2\ \mu F$





$$i_c(t) = 10\text{ mA};\ 0 < t < 2\text{ ms}$$
$$= 0;\quad 2\text{ ms} < t < 4\text{ ms}$$
$$= -20\text{ mA};\ 4\text{ ms} < t < 5\text{ ms}$$

We know, voltage across the capacitor is given by

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt$$

For $0 < t < 2$ ms:

$$v_c(t) = \frac{1}{2\times10^{-6}}\int_0^t (10\times10^{-3})dt = 5000t$$

For 2 ms $< t <$ 4ms:

$$v_c(t) = \frac{1}{2\times10^{-6}}\int_{2\text{ms}}^{t} (0)dt + v_c(2\text{ms}) = 0 + 10 = 10\text{ V}$$

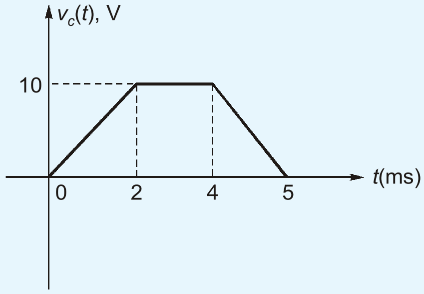
For 4 ms $< t <$ 5 ms:

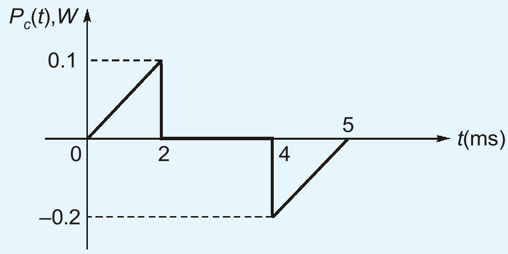
$$v_c(t) = \frac{1}{2\times10^{-6}}\int_{4\text{ms}}^{t} (-20\times10^{-3})dt + v_c(4\text{ ms})$$
$$= -10000(t - 0.004) + 10 = -10000t + 50$$

$$\therefore \quad v_c(t) = 5000t;\quad 0 < t < 2\text{ ms}$$
$$= 10;\quad 2\text{ ms} < t < 4\text{ ms}$$
$$= -10000t + 50;\quad 4\text{ ms} < t < 5\text{ ms}$$

$$\text{Power, } p(t) = v_c(t)i_c(t) = 50t; \quad 0 < t < 2 \text{ ms}$$
$$= 0; \quad 2 \text{ ms} < t < 4 \text{ ms}$$
$$= 200\,t - 1; \; 4 \text{ ms} < t < 5 \text{ ms}$$

The voltage and power waveform with respect to the time is as shown below:





End of Solution

Q.7 (c) (i) A piezoelectric ceramic disc of thickness $t = 2$ mm and area = 1.5×10^{-4} m^2 is subjected to a compressive force of $F = 50$ N applied perpendicular to its faces. The material has the following properties:

- Piezoelectric coefficient = 300×10^{-12} C/N
- Relative permittivity = 1200
- Volume permittivity = 8.854×10^{-12} F/m

Determine the following :

(1) Charge generated on electrodes due to applied force

(2) Capacitance of the piezoelectric disc

(3) Voltage generated across the ceramic disc

[10 marks : 2025]

(ii) For a 2-port network, express Z-parameters in terms of inverse hybrid parameters.

[10 marks : 2025]

Solution:

(i) Given,

Thickness, t = 2 mm = 0.002 m

Area, A = 1.5×10^{-4} m^2

Applied Force, F = 50 N

Charge sensitivity, d = 300×10^{-12} C/N

Relative permittivity, ε_r = 1200

ε_0 = 8.854×10^{-12} F/m

Charge generated (Q)

$$Q = d.F = 300 \times 10^{-12} \times 50 = 15 \times 10^{-9} \text{ C}$$

Capacitance of the piezoelectric disc,

$$C = \frac{\varepsilon_0 \varepsilon_r A}{t} = \frac{8.854 \times 10^{-12} \times 1200 \times 1.5 \times 10^{-4}}{0.002}$$

$$C = 7.97 \times 10^{-10} \text{ F}$$

Voltage generated across the ceramic disc,

$$V = \frac{Q}{C} = \frac{15\times10^{-9}}{7.97\times10^{-10}} \simeq 18.8 \text{ V}$$

(ii) The z-parameter equations are given by,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{...(i)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{...(ii)}$$

and inverse hybrid parameters,

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \text{...(iii)}$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad \text{...(iv)}$$

For $I_2 = 0$:

1. $Z_{11} = \frac{V_1}{I_1}$ and $\frac{I_1}{V_1} = g_{11}$

Thus,

$$\frac{V_1}{I_1} = \frac{1}{g_{11}} \quad \text{...(v)}$$

$$Z_{11} = \frac{1}{g_{11}}$$

2. $\frac{V_2}{I_1} = Z_{21}$ and $\frac{V_2}{V_1} = g_{21}$ $\Rightarrow$ $V_1 = \frac{V_2}{g_{21}}$

$$I_1 = g_{11}\left(\frac{V_2}{g_{21}}\right) \quad \text{...using equation (v)}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{g_{21}}{g_{11}}$$

For $I_1 = 0$

1. $\frac{V_1}{I_2} = Z_{12}$

$$-g_{11}V_1 = g_{12}I_2$$

$$\frac{V_1}{I_2} = \frac{-g_{12}}{g_{11}} = Z_{12}$$

2. $\frac{V_2}{I_2} = Z_{22}$ and $-g_{11}V_1 = g_{12}I_2$

$$V_1 = \left[\frac{-g_{12}I_2}{g_{11}}\right]$$

Using equation (iv),

$$V_2 = g_{21}\left(\frac{-g_{12}I_2}{g_{11}}\right) + g_{22}I_2$$

$$\frac{V_2}{I_2} = \left[\frac{-g_{21}g_{12}}{g_{11}} + g_{22}\right] = Z_{22}$$

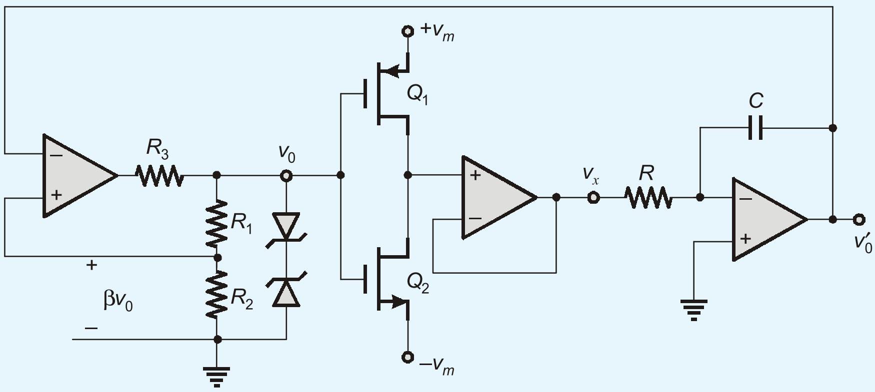
$$\therefore \quad \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{-g_{12}g_{21}}{g_{11}} + g_{22} \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta g}{g_{11}} \end{bmatrix}$$

where $\Delta g = g_{22}g_{11} - g_{12}g_{21}$

End of Solution

Q.8 (a) Explain the working of each stage of the voltage-controlled oscillator circuit shown below :



Evaluate the effect of change in modulating voltage v_m to the output frequency.

[20 marks : 2025]

Solution:

The given circuit has three stages:

Stage 1: Schmitt Trigger

Stage-1 is a Schmitt Trigger having the output as a square wave with outputs, $(V_z + V_{D(ON)})$ or $(V_z - V_{D(ON)})$. Assuming the diode to be an ideal diode, $V_{D(ON)} = 0$. Thus, the square wave generated at the output of Schmitt Trigger has levels i.e. $v_0 = +V_z$ or $-V_z$.

The upper and lower threshold voltage of the Schmitt trigger is given by

$$V_{UT} = \left(\frac{R_2}{R_1 + R_2}\right)V_z = \beta V_z$$

$$V_{LT} = \left(\frac{R_2}{R_1 + R_2}\right)(-V_z) = -\beta V_z$$

We have,

$$v_0 = \begin{cases} V_z; \text{if } v_0' > \beta v_0 \\ -V_z; \text{if } v_0' < \beta v_0 \end{cases}$$

Stage 2: Voltage Follower

The MOS Transistors Q_1 and Q_2 forms CMOS inverter and the op-amp circuit acts as a voltage follower. Thus,

If $v_0 = V_z$, then Q_1 : OFF and Q_2 : ON, we get $v_x = -v_m$

If $v_0 = -V_z$, then Q_1 : ON and Q_2 : OFF, we get $v_x = v_m$

The op-amp prevents loading effect and provides isolation between the stages.

Stage 3: Integrator

The third stage is an integrator and converts square wave into triangular wave.

For $v_x = v_m$,

$$v_0' = -\frac{1}{RC}\int v_m dt = \left(-\frac{v_m}{RC}\right)t$$

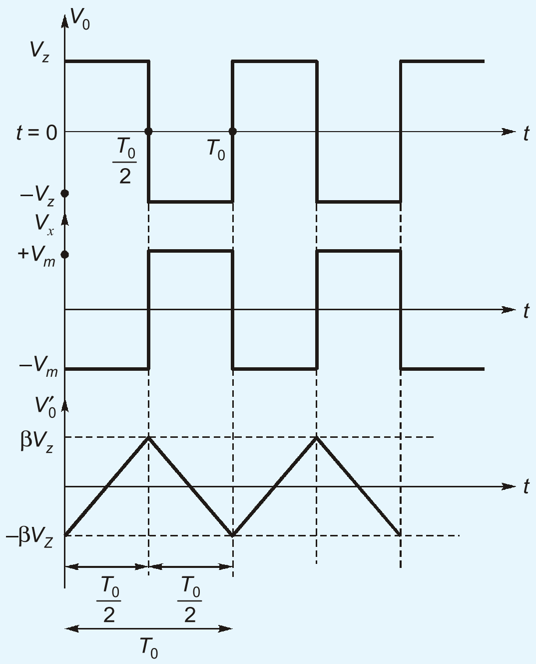
It generates a negative ramp until $v_0' < \beta v_0$ and thereafter, v_x switches to $-v_m$

For $v_x = -v_m$,

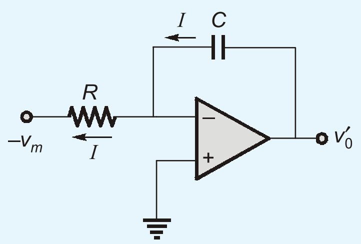
$$v_0' = -\frac{1}{RC}\int -v_m dt = \left(\frac{v_m}{RC}\right)t$$

It generates a positive ramp until $v_0' > \beta v_0$ and thereafter, v_x switches to v_m

The waveform at the various stages of the oscillator is shown below,



Consider $0 \le t \le \frac{T_0}{2}$: Assume $v_0 = V_z$; $v_x = -v_m$



$$I = \frac{0 + v_m}{R} = \frac{v_m}{R}$$

The output is a positive going ramp and v_0' changes from $-\beta V_z$ to $+\beta V_z$. Thus,

$$\Delta V_c = \Delta v_0' = 2\beta V_z$$

$$I \times \Delta t = C \times \Delta V_c$$

$$\frac{v_m}{R} \times \frac{T_0}{2} = C \times 2\beta V_z$$

$$T_0 = 4RC \times \frac{\beta V_z}{v_m}$$

Oscillation frequency, $f_0 = \dfrac{1}{T_0} = \dfrac{1}{4RC} \times \dfrac{v_m}{\beta V_z}$

If v_m increases, then f_0 increases. Thus, f_0 can be controlled through v_m.

End of Solution

Q.8 (b) Use a decoder to design a binary-to-hexadecimal character generator. The outputs of the character generator are to be connected via current limiting resistors to a common anode seven-segment display. Assume that the inputs are positive logic signals.

[20 marks : 2025]

Solution:

Designing of a binary-to-hexadecimal character generator:

1. Inputs:
   - Binary to hexadecimal means we need 4 input binary bits (A, B, C, D).
   - These 4 inputs represent hexadecimal digits 0-F.
2. Decoder (Driver IC):
   A 4-to-16 line decoder is used to convert the 4-bit binary input into 16 individual output lines, one for each hexadecimal digit (0-F).
3. Display Type: Common anode display.
   Segments turn ON when a logic 0 (low) is applied to their cathode pins through current-limiting resistors.
4. Character Generator Design:
   - The outputs of the decoder are given to the character generator which generates an active low signal for the corresponding segments of the seven-segment display (a-g) to display the hexadecimal character.
   - The outputs of the character generator are connected to the 7-segment display through current limiting resistors.
   - The common anode is tied to $+V_{CC}$.

5. **Truth table:** The truth table for the character generator is as below,

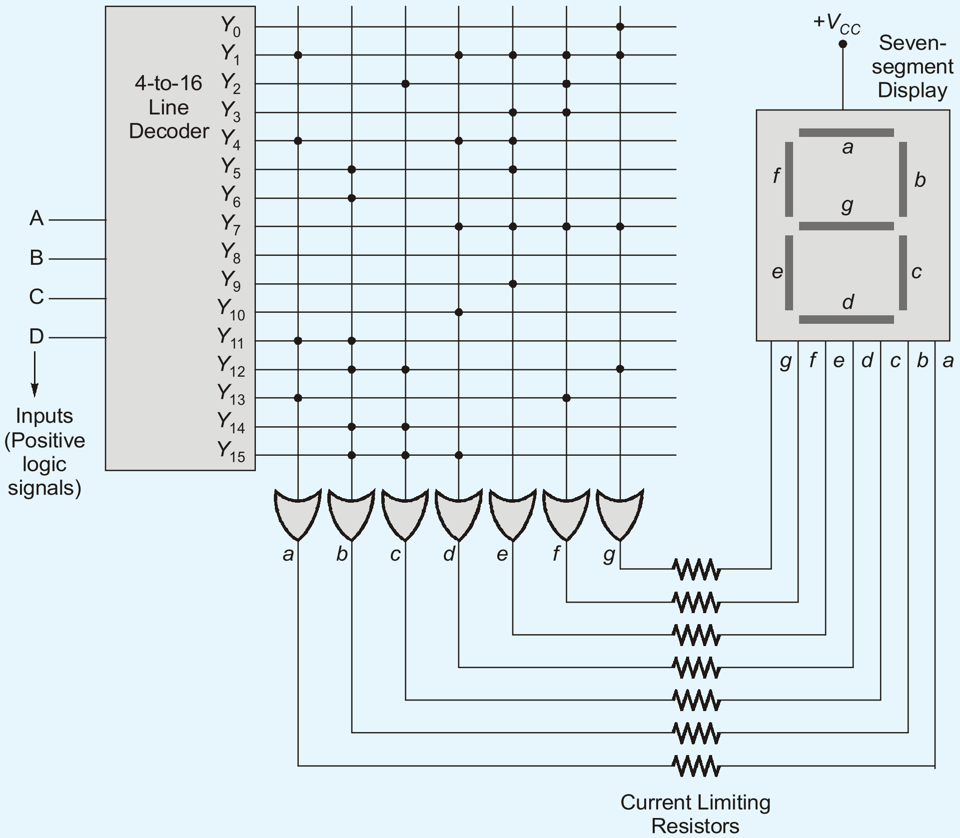
Binary Inputs				Hexadecimal Character	7 segment display						
A	*B*	*C*	*D*		*a*	*b*	*c*	*d*	*e*	*f*	*g*
0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	1	1	0	0	1	1	1	1
0	0	1	0	2	0	0	1	0	0	1	0
0	0	1	1	3	0	0	0	0	1	1	0
0	1	0	0	4	1	0	0	1	1	0	0
0	1	0	1	5	0	1	0	0	1	0	0
0	1	1	0	6	0	1	0	0	0	0	0
0	1	1	1	7	0	0	0	1	1	1	1
1	0	0	0	8	0	0	0	0	0	0	0
1	0	0	1	9	0	0	0	0	1	0	0
1	0	1	0	A	0	0	0	1	0	0	0
1	0	1	1	b	1	1	0	0	0	0	0
1	1	0	0	C	0	1	1	0	0	0	1
1	1	0	1	d	1	0	0	0	0	1	0
1	1	1	0	E	0	1	1	0	0	0	0
1	1	1	1	F	0	1	1	1	0	0	0

6. **Design of character generator:**

From the truth table, the outputs of the character generator which drive the common-anode seven-segment display are obtained as below:

$$a = \Sigma m(1, 4, 11, 13)$$
$$b = \Sigma m(5, 6, 11, 12, 14, 15)$$
$$c = \Sigma m(2, 12, 14, 15)$$
$$d = \Sigma m(1, 4, 7, 10, 15)$$
$$e = \Sigma m(1, 3, 4, 5, 7, 9)$$
$$f = \Sigma m(1, 2, 3, 7, 13)$$
$$g = \Sigma m(0, 1, 7, 12)$$

Using above expressions, we obtain the interface circuit as below,

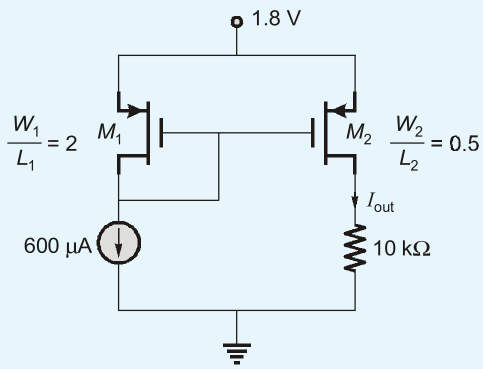


End of Solution

Q.8 (c) (i) A voltage 50 + 25 sin ωt volts is applied to a series R-L circuit having a resistance of 10 Ω and inductance of 0.1 H. A wattmeter is connected in the circuit to measure power. Calculate the reading of the wattmeter if $\omega = 100\pi$ rad/s.

[10 marks : 2025]

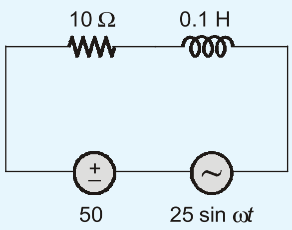
(ii) For the circuit shown in the figure, find I_{out}. Assume $\mu_n C_{ox} = 250\ \mu A/V^2$ and $V_{tn} = 0.4$ V for both the MOSFETs:



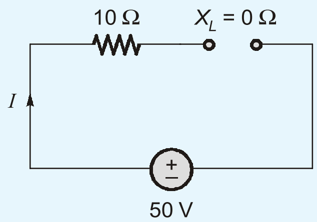
[10 marks : 2025]

Solution:

(i)



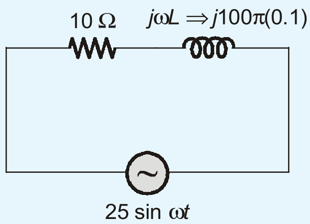
50 V source is applied to the circuit



$$I = \frac{50}{10} = 5\text{ A}$$

$$P_{dc} = I^2R = 5^2(10) = 250\text{ watts}$$

25 sin ωt source is applied to the circuit, where $\omega = 100\pi$.



$$X_L = \omega L \Rightarrow (100\pi)(0.1)$$

$$X_L = 31.416\ \Omega$$

$$I_{rms} = \frac{25/\sqrt{2}}{10 + j31.416} \Rightarrow 0.5355\angle -72.366°\text{ A}$$

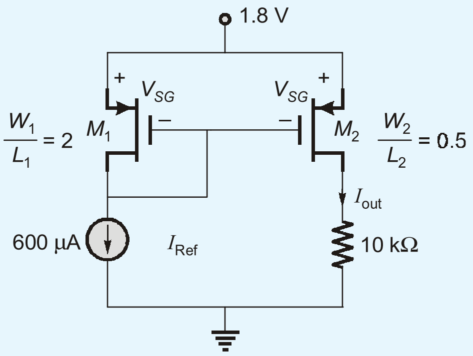
$$P_{AC} = (0.5355)^2 \times 10 \Rightarrow 2.8676\text{ watts}$$

$$P_{Total} = P_{DC} + P_{AC}$$

$$= 250 + 2.8676$$

$$= 252.8676\text{ watts}$$

(ii)



$$\frac{I_{out}}{I_{Ref}} = \frac{\frac{\mu_N C_{ox}}{2} \times \left(\frac{W}{L}\right)_2 \times (V_{SG} - |V_T|)^2}{\frac{\mu_N C_{ox}}{2} \times \left(\frac{W}{L}\right)_1 \times (V_{SG} - |V_T|)^2} = \frac{(W/L)_2}{(W/L)_1}$$

$$\frac{I_{out}}{600\ \mu A} = \frac{0.5}{2}$$

$$I_{out} = 150\ \mu A$$

End of Solution

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