ESE 2025

Main Exam Detailed Solutions

Electrical Engineering

PAPER-II

EXAM DATE : 10-08-2025 | 02:00 PM to 05:00 PM

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ANALYSIS

Electrical Engineering Paper-II
ESE 2025 Main Examination

Sl.	Subjects	Marks
1.	Analog and Digital Electronics	92
2.	Power Systems	84
3.	Systems & Signal Processing	72
4.	Control Systems	64
5.	Electrical Machines	84
6.	Power Electronics	84
		Total 480

Scroll down for detailed solutions

SECTION : A

Q.1 (a) Give the circuit diagram of a negative peak clamper circuit using op-amp, and

(i) Considering V_{ref} = +2 V, sketch the output waveform for an input signal $v_i = 2 \sin(1000t)$.

(ii) Provide conditions to achieve precision clamping and explain how will you protect op-amp against excessive discharge currents.

(iii) State how will you modify your circuit to achieve positive peak clamping.

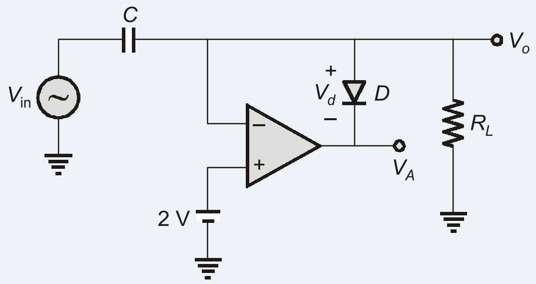
[12 marks : 2025]

Solution:

(i) Given negative peak clamper using op-amp.

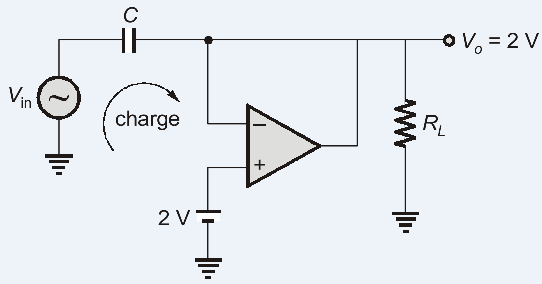
Consider $V_{REF} = 2$ V

$V_{in} = 2 \sin(1000t)$



Initially capacitor is uncharged.

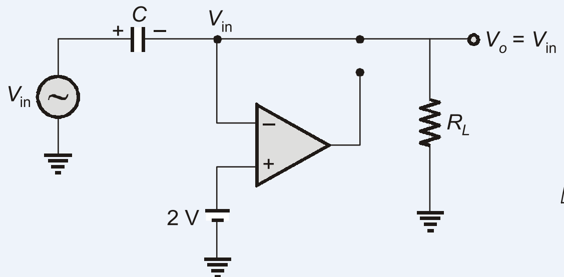
$V_{in} > 2$ V, V_d (–ve) $V_A = -V_s$ at 0 is ON

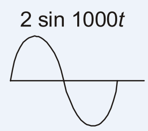


But in the given question

$V_{in} = 2 \sin(1000t)$

That means, $V_{in} \leq 2$ V, D is OFF

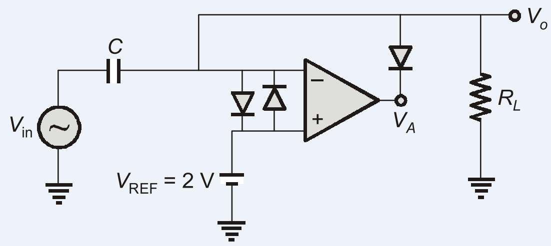




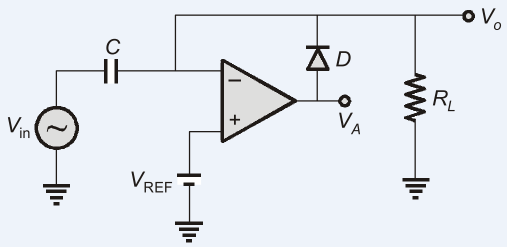
(ii) To achieve precision clamping

$$V_{in} > V_{REF}$$

To protect op-amp against excessive discharge amount, we have to take two diodes in between inverting and non-inverting terminals.



(iii) To modify negative clamper to positive clamper



End of Solution

Q.1 (b) A 7.5 kW, 440 V, 3-phase, star-connected, 50 Hz, 4-pole squirrel cage induction motor develops full load torque at a slip of 3% when operated at rated voltage and frequency. The leakage reactances of stator and rotor windings are five times the respective stator and rotor resistances. The ratio of stator to rotor winding is 3 : 5. Determine the percentage increase in stator reactance to limit starting current to 2.5 times the full load current. Assume R_1 and R_2 are equal and of requisite amount; and motor has negligible magnetising reactance and core losses.

[12 marks : 2025]

Solution:

Given Data : 7.5 kW, 440 V, 4 pole, 50 Hz

$$\text{Full load slip} = 0.03$$

$$R_1 = R_2$$

$$X_1 = 5R_1; \quad X_2 = 5R_2$$

$$\text{Stator to rotor turns ratio} = 3 : 5$$

But given $R_1 = R_2$ which implies stator to rotor turns ratio is 1.

Question is also not framed for ratio 3 : 5. After solving for a ratio 3 : 5, it is considered as ratio 1, as per question and given hints.

$\therefore$ For this question solution, stator to rotor turns ratio is 1.

Given :

$$R_1 = R_2$$

$$R_1 = R'_2$$

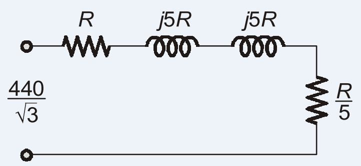
$$X_1 = X'_1$$

Let $$R_1 = R_2 = R$$

$$X_1 = 5R_1 = 5R$$

$$X_2' = 5R_2 = 5R$$

Equivalent Circuit :



At full load, $S = 0.03$

By substituting $S = 0.03$

Finding Z_f and I_f.

$$Z_f = \sqrt{\left(R + \frac{R}{0.03}\right)^2 + (5R + 5R)^2} = 35.76R$$

$$I_f = \frac{V}{Z_f} = \frac{440/\sqrt{3}}{35.76R} = \frac{7.1}{R}$$

Similarly, find starting current without adding reactance.

By substituting $S = 1$

$$Z_{st} = \sqrt{(R + R)^2 + (5R + 5R)^2} = 10.198R$$

$$I_{st} = \frac{V}{Z_{st}} = \frac{440/\sqrt{3}}{10.198R} = \frac{24.91}{R}$$

Ratio of starting current to full load current (without adding any reactance)

$$\frac{I_{st}}{I_f} = \frac{24.91/R}{7.1/R} = 3.5$$

This ratio must be equal to 2.5 by additing additional reactance.

Starting impedance with added reactance :

$$Z'_{st} = \sqrt{(R + R)^2 + (X_1 + X_2' + X_{add})} \qquad (X'_{add} = X_{add})$$

$$= \sqrt{(R + R)^2 + (10R + X_{add})^2}$$

$$I'_{st} = \frac{V}{Z'_{st}}$$

$$I_f = \frac{V}{Z_f}$$

$$\frac{I'_{st}}{I_f} = \frac{Z_f}{Z'_{st}} = 2.5$$

$$\frac{35.76R}{\sqrt{4R^2 + (10R + X_{\text{add}})^2}} = 2.5$$

Dividing both sides by R and squaring on both sides

$$\frac{X_{\text{add}}}{R} \cong 4.158 \quad \text{or} \quad X_{\text{add}} = 4.158R$$

Original stator resistance, $X_1 = 5R$

$$\% \text{ increase} = \frac{X_{\text{add}}}{X_1} \times 100 = \frac{4.158R}{5R} \times 100 = 83.16\%$$

$\therefore$ Stator reactance must be increased by 83.16% to limit starting current to 2.5 times I_f.

End of Solution

Q.1 (c) In a short-circuit test on a 3-pole, 110 kV circuit breaker, power factor of the fault was 0.4, the recovery voltage was 0-95 times full line value. The breaking current was symmetrical. The frequency of oscillation of restriking voltage was 15,000 cycles/sec. Estimate the average rate of rise of restriking voltage. The neutral is grounded and fault involves earth.

[12 marks : 2025]

Solution:

$$\text{RRRV Avg} = \frac{\text{Maximum value of restriking voltage}}{\text{Time taken to reach that value}}$$

$$\text{Time taken, } f = \frac{\pi}{\omega_n} = \frac{\pi}{2\pi d_n} = \frac{1}{2f_n} = \frac{1}{2 \times 15 \times 10^3} = \frac{1}{3 \times 10^4}$$

$$\text{Maximum value restriking voltage} = 2V_{ar}$$

$$V_{ar} = K_1 K_2 K_3 V_{ar}$$

$$V_m = \sqrt{2} \times \frac{1100}{\sqrt{3}}$$

$$\cos\phi = 0.4$$

$$\phi = \cos^{-1}(0.4) = 66.02$$

$$K_1 = \sin\phi = \sin 66.02 = 0.9165$$

$$K_2 = 0.95$$

$$K_3 = 1.0 \text{ as fault is grounded.}$$

$$V_{ar} = (0.9165)(0.95)(1.0)\sqrt{2} \times \frac{110}{\sqrt{3}} = 78.2 \text{ kV}$$

$$V_{\text{rms}} = 2V_{ar} = 2 \times 78.2 = 156.4 \text{ kV}$$

$$(RRRV)_{\text{Avg}} = \frac{156.4 \times 10^3}{\dfrac{1}{3 \times 10^4}} = 4.7 \text{ kV/μsec}$$

End of Solution

Q.1 (d) The standstill impedances of the inner and outer cages of a double cage 3-ϕ induction motor rotor are given as $Z_{ic} = (0.01 + j0.5)\Omega$ and $Z_{oc} = (0.05 + j0.1)\Omega$ respectively. Assuming the stator impedance to be negligible, determine the approximate ratio of the torques produced by the outer cage (T_{oc}) to the torque produced by the inner cage (T_{ic}) at a slip of $s = 0.05$? Also determine the net torque developed as a function of T_{oc} and comment on performance as compared to single cage motor.

[12 marks : 2025]

Solution:

Given : Inner cage $\quad Z_i = 0.01 + j0.5\ \Omega$

Outer cage $\quad Z_o = 0.05 + j0.1\ \Omega$

Torque, $$T = \frac{3\times 60}{2\pi N_s}\cdot\frac{SE_2^2 R_2}{R_2^2 + (SX_2)^2}\ \text{N-m}$$

Torque developed by outer cage to inner cage at start $s = 1$

$$\frac{T_{oc}}{T_{ic}} = \frac{\dfrac{3\times 60}{2\pi N_s}\times\dfrac{E_2^2\times 0.05}{0.05^2+(0.1)^2}}{\dfrac{3\times 60}{2\pi N_s}\times\dfrac{E_2^2\times 0.01}{0.01^2+0.5^2}} = \frac{\dfrac{0.05}{0.0125}}{\dfrac{0.01}{0.2501}} = \frac{4}{0.04} = 100$$

Outer cage develops starting torque 100 times inner cage.

Torque ratio of outer cage to inner at $s = 0.05$.

$$\frac{T_{oc}}{T_{ic}} = \frac{\dfrac{0.05\times 0.05}{0.05^2+(0.05\times 0.1)^2}}{\dfrac{0.05\times 0.01}{(0.01)^2+(0.05\times 0.5)^2}} = \frac{\dfrac{0.0025}{0.002525}}{\dfrac{0.0005}{0.000725}} = \frac{0.99}{0.689}$$

$$= 1.4355$$

At $s = 0.05$, $\quad T_{oc} = 1.435\,T_{ic}$

During running, torque developed by outer cage,

$$T_{oc} \propto 0.99$$

Torque developed by inner cage,

$$T_{ic} \propto 0.689$$

$$\text{Total/Net torque} = T_{oc} + T_{ic} = 0.99 + 0.689 = 1.679$$

$$\frac{T_{\text{net}}}{T_{oc}} = \frac{1.679}{0.99} = 1.695$$

or $\quad T_{\text{net}} = 1.695\,T_{oc}$

$$\frac{T_{\text{net}}}{T_{ic}} = \frac{1.679}{0.689}$$

$$T_{\text{net}} = 2.437\,T_{ic}$$

Comments : At start $s = 1$, outer cage (high resistance) produce large torque, 100 times inner cage.

∴ Double cage rotor provides very high starting torque. Best suitable for high inertia or higher starting torque loads/hard to start loads.

At running condition, inner cage (low R and X) at low slip becomes relatively more effective and produce good torque in low slip region.

Compared with single cage rotor, double cage rotor gives higher starting torque and better torque at running at low slip.

Double cage has better and improved starting torque without sacrificing running efficiency.

End of Solution

Q.1 (e) A 10 MVA, 13.8 kV turbo-generator having $X_d'' = X_2 = 15\%$ and $X_0 = 5\%$ is about to be connected to power system. The generator has current limiting reactor of 0.7 Ω in the neutral. Before the generator is connected to the system, its voltage is adjusted to 13.2 kV. When a double line to ground fault develops at terminal 'b' and 'c' find the initial symmetrical r.m.s. currents in the ground and in line 'b'.

[12 marks : 2025]

Solution:

Let the base MVA be 10 and base kV be 13.8

Base current, $$I_B = \frac{\text{MVA}\times 1000}{\sqrt{3}\times 13.8} = \frac{10000}{\sqrt{3}\times 13.8} = 418.37 \text{ A}$$

Now, $$Z_n = j0.7\ \Omega$$

$$Z_{n\,(\text{p.u.})} = j0.7\times\frac{10}{(13.8)^2} = j0.03675 \text{ p.u.}$$

$$Z_0 = Z_{g0} + 3Z_n$$
$$= j0.05 + 3\times j0.03675 = j0.16 \text{ p.u.}$$

Also, $$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_2 Z_0}{Z_2+Z_0}}$$

$$E_a = \frac{13.2}{13.8}\text{p.u.} = 0.9565 \text{ p.u.}$$

$$I_{a1} = \frac{0.9565}{j0.15 + \dfrac{j0.15\times j0.16}{j0.15+j0.16}} = -j4.206 \text{ p.u.}$$

Now, $$V_{a0} = V_{a2} = V_{a1} = E_a - I_{a1}Z_1$$
$$= 0.9565 - (-j4.206)\times j0.15 = 0.3256 \text{ p.u.}$$

$$I_{a2} = \frac{-V_{a2}}{Z_2} = \frac{-0.3256}{j0.15} = j2.171 \text{ p.u.}$$

$$I_{a0} = \frac{-V_{a0}}{Z_0} = \frac{-0.3256}{j0.16} = j2.035 \text{ p.u.}$$

Initial symmetrical rms current in line b,

$$I_b = a^2 I_{a1} + aI_{a2} + I_{a0}$$

$$= -j4.206(-0.5 - j0.866) + j2.171(-0.5 + j0.866) + j2.035$$
$$= -5.52 + j3.05250 = 6.3\angle 151.06 \text{ p.u.}$$
$$= 2635.73\angle 151.06° \text{ A}$$

Initial symmetrical rms current in ground wire

$$= 3I_{a0} = 3 \times j2.035 \times 418.37$$
$$= 2554.15\angle 90° \text{ A}$$

End of Solution

Q.2 (a) A 15 km long 3-phase overhead line delivers 5 MW at 11 kV at a power factor of 0.8 lagging. Line loss is 12% of the power delivered. Line inductance is 1.1 mH per km per phase.

Calculate:

(i) Sending end voltage and voltage regulation.

(ii) Power factor of the load to make voltage regulation zero.

[15 + 5 marks : 2025]

Solution:

$$l = 15 \text{ km},\ P_R = 5 \text{ MW},\ V_R = 11 \text{ kV},\ \cos V_R = 0.8 \text{ lag}$$
$$L = 1.1 \text{ mH/km},\ P_L = 12\% \text{ of load}$$
$$P_L = \frac{12}{100} \times 5 = 0.6 \text{ MW}$$
$$I_R = \frac{5 \times 10^3}{\sqrt{3} \times 1180.8} = 328 \text{ A}$$
$$P_L = 3I^2R$$
$$R = \frac{0.6 \times 10^6}{3(328)^2} = 1.86\ \Omega$$
$$X_L = 2\pi \times 650 \times 1.1 \times 10^{-3} \times 15 = 5.18\ \Omega$$
$$Z = 1.86 + 855.18 = 5.5\angle 70.2°$$
$$V_R|_{Ph} = \frac{11000}{\sqrt{3}} = 6351 \text{ Volt}$$
$$V_S = V_R + ZI_R = 6351\angle 0° + 5.5\angle 70.2° \times 328\angle -36.86°$$
$$= 7920\angle 7.2° \text{ Volt}$$

(i)
$$V_s(l - l) = \sqrt{3} \times 7920 = 13.7 \text{ kV}$$
$$\%R_{eq} = \frac{V_S - V_R}{V_R} \times 100 = \frac{13.7 - 11}{10} \times 100 = 24.54\%$$

(ii) At zero regulation the condition is $\theta + \phi = 90°$

$$\phi = 90 - \theta = 90 - 70.2 = 19.8°$$
$$\text{P.F.} = \cos 19.8 = 0.94 \text{ lead}$$

End of Solution

Q.2 (b) A 2200/220 V, single phase transformer has maximum possible voltage regulation of 6% and it occurs at a power factor of 0.3 lag. Find the load voltage at full load at a power factor of 0.8 lead.

[20 marks : 2025]

Solution:

The p.u. voltage regulation of transformer is given as,

$$\text{V.R.}_{(pu)} = Z_{pu}\cos(\theta_{eq} - \phi) \text{ pu}$$

Maximum voltage regulation occurs at $\phi = \theta_{eq}$

$$\phi = \theta_{eq} = \cos^{-1}(0.3) = 72.54°$$

Maximum voltage regulation,

$$\text{V.R.}_{(max)} = Z_{pu} = 0.06 \text{ pu}$$

Load voltage at full load and 0.8 pf leading is given as

$$V_L = V_1 - IZ$$

$$V_L = 1\angle 0° - (1\angle 36.86°) \times (0.06\angle 72.54°)$$

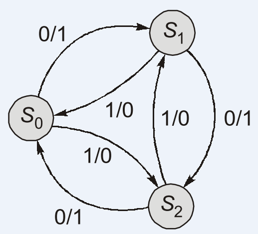
$$V_L = 1.0214\angle -3.17° \text{ pu}$$

$$V_{L(actual)} = 1.0214 \times 220 = 224.73 \text{ Volts}$$

Hence, load voltage at full load and 0.8 pf leading is 224.73 Volts.

End of Solution

Q.2 (c) (i) For the state diagram shown below, design the circuit using D-flip flops. Assume S_0 : 00, S_1 : 10 and S_2 : 01.



Realize the signal circuits with minimum number of NAND gates (More than two input NAND gates are allowed).

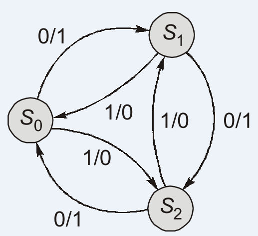
[10 marks : 2025]

Solution:

S_0 : 00

S_1 : 10

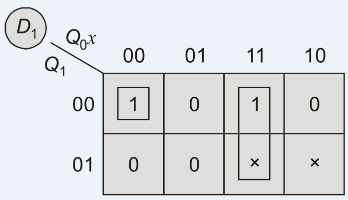
S_2 : 01



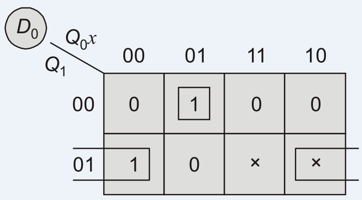
D-flip flop

State transition table:

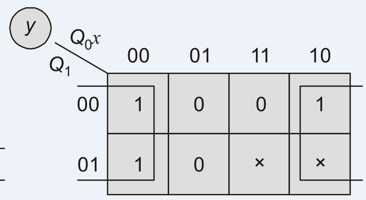
Present state		Input	Next state		Output	Flip flop Inputs	
Q_1	Q_0	x	Q_1^+	Q_0^+	y	D_1	D_0
0	0	0	1	0	1	1	0
0	0	1	0	1	0	0	1
0	1	0	0	0	1	0	0
0	1	1	1	0	0	1	0
1	0	0	0	1	1	0	1
1	0	1	0	0	0	0	0
1	1	0	×	×	×	×	×
1	1	1	×	×	×	×	×



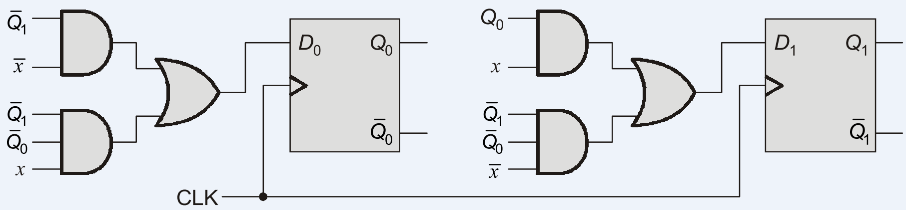
$D_1 = \bar{Q}_1\bar{Q}_0\bar{x} + Q_0 x$

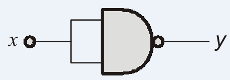


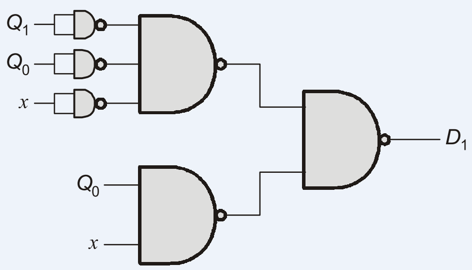
$D_0 = Q_1\bar{x} + \bar{Q}_1\bar{Q}_0 x$

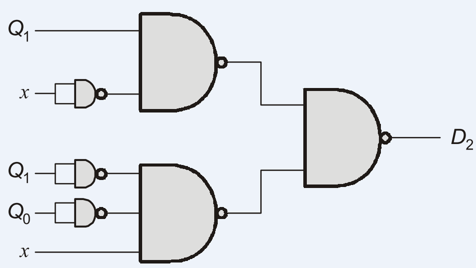


$y = \bar{x}$









End of Solution

Q.2 (c) (ii) An angle modulated signal is given as $X(t) = 20\cos(12000)t$ for $|t| \le 1$. If the carrier wave frequency ω_c = 10000 rad/sec, determine

(A) Modulation index $m(t)$, if $X(t)$ were a PM (phase modulated) signal with $K_P = 500$ over $|t| \le 1$.

(B) Modulation index $m(t)$, if $X(t)$ were a PM (phase modulated) signal with $K_f = 500$ over $|t| \le 1$.

[10 marks : 2025]

Solution:

Given angle modulated signal

$$X(t) = 20\cos(12000)t\,;\ |t| \le 1$$

(A) By considerin given as PM signal

$$SPM(t) = A_c\cos\left[\omega_c t + K_p m(t)\right]$$

Given,

$$\omega_c = 10000 \text{ rad/sec}$$

$$K_P = 500 \text{ rad/volt}$$

$$10000t + 500\,m(t) = 12000t$$

$$m(t) = 4t\,;\ |t| \le 1$$

(B)

$$S_{FM}(t) = A_c\cos\left[\omega_c t + K_f\int m(t)dt\right]$$

Given

$$K_f = 500 \text{ rad/sec volt}$$

$$10000 + K_f\int m(t)dt = 12000t$$

$$K_f\int m(t)dt = 2000t$$

$$K_f\,m(t) = 2000 \rightarrow 500\,m(t) = 2000$$

$$m(t) = 4\,;\ |t| \le 1$$

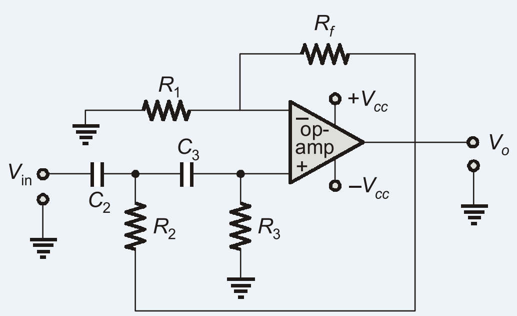
End of Solution

Q.3 (a) Give the circuit diagram of a second-order highpass Butterworth filter circuit using op-amp. Evaluate the component values such that the filter has a lower cutoff frequency of 5 kHz and a passband gain $A_f = 2$. Also, give the expression for voltage gain magnitude and sketch its frequency response.

[20 marks : 2025]

Solution:

The second order high pass Butterworth filter circuit using op-amp



The voltage gain magnitude equation for the second order high pass filter is

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_F}{\sqrt{1+\left(\frac{f_L}{f}\right)^4}}$$

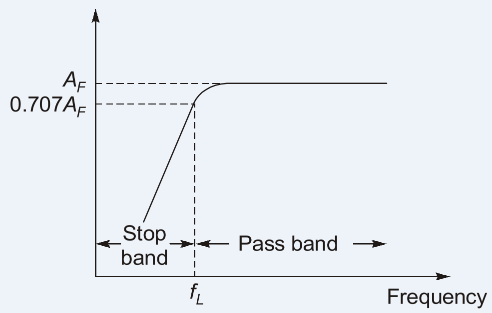
where, f = Input frequency in Hz

f_L = Lower cut-off frequency in Hz = $\dfrac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$

For $R_2 = R_3 = R$ and $C_2 = C_3 = C$,

$$f_L = \frac{1}{2\pi RC}$$

The frequency response of this filter



For cut-off frequency, f_o = 5 kHz

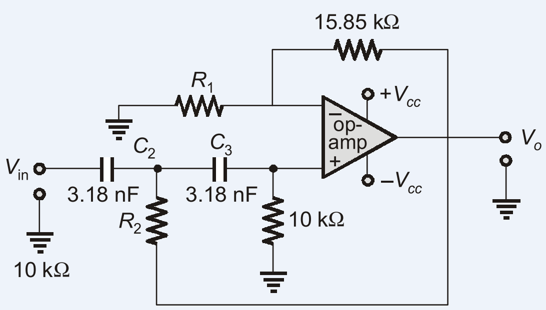
Choose R = 10 kΩ,

$$f_o = \frac{1}{2\pi RC}$$

$$C = \frac{1}{2\pi\times10\times10^3\times5\times10^3} = 3.18 \text{ nF}$$

Given : $\dfrac{R_f}{R_1} = 2$

Hence, R_f = 15.85 kΩ

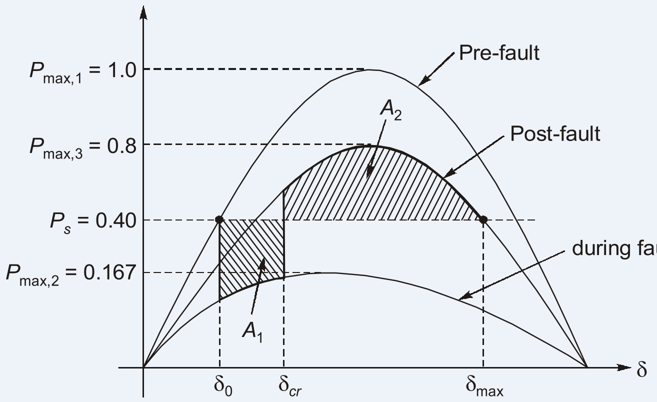


End of Solution

Q.3 (b) A 50 Hz alternator is supplying 40% of the power that it is capable of delivering through a transmission line to an infinite bus. A fault occurs that increases the reactances between the generator and the infinite bus to 600% of the value before the fault. When the fault is isolated, the maximum power that can be delivered is 80% of the original maximum value. Find the critical clearing angle.

[20 marks : 2025]

Solution:



Prefault Fault :

$$P_{max1} = 1.0 \text{ pu}, \; P_s = 0.40 \text{ pu}$$

$$\delta_o = \sin^{-1}\left(\frac{P_s}{P_{max1}}\right) = \sin^{-1}\left(\frac{0.4}{1}\right) = 23.57° \text{ or } 0.411 \text{ rad}$$

Post Fault :

$$P_{max3} = 0.8 \text{ pu}, \; P_s = 0.40 \text{ pu}$$

$$\delta_{max} = 180° - \sin^{-1}\left(\frac{0.4}{0.8}\right) = 150° \text{ or } 2.618 \text{ rad}$$

For system to operate in stable region,

$$\text{Area, } A_1 > \text{Area, } A_2$$

or Acceleration area > Deceleration area

At critical condition,

$$\text{Area, } A_1 = \text{Area, } A_2$$

$$\int_{\delta_o}^{\delta_{cr}} (P_s - P_{max2}\sin\delta)d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{max3}\sin\delta - P_s)d\delta$$

$$P_s(\delta_{cr} - \delta_o) + P_{max2}(\cos\delta_{cr} - \cos\delta_o) = P_{max3}(\cos\delta_{cr} - \cos\delta_{max}) - P_s(\delta_{max} - \delta_{cr})$$

$$(P_{max3} - P_{max2})\cos\delta_{cr} = P_s(\delta_{max} - \delta_o) - P_{max2}\cos\delta_o + P_{max3}\cos\delta_{max}$$

$$\cos\delta_{cr} = \frac{P_s(\delta_{max} - \delta_o) - P_{max2}\cos\delta_o + P_{max3}\cos\delta_{max}}{P_{max3} - P_{max2}}$$

Now, on putting the respective values

$$\cos\delta_{cr} = \frac{0.4(2.618-0.411)-0.167\cos(23.57°)+0.8\cos(150°)}{0.8-0.169}$$

$$\cos\delta_{cr} = 0.05831$$

Thus, critical clearing angle is

$$\delta_{cr} = \cos^{-1}(0.05831) = 86.65°$$

End of Solution

Q.3 (c) A 240 V DC series motor takes 40 A when giving its rated output at 1500 rpm. Its resistance is 0.3 Ω. Calculate the value of resistance that must be added to obtain the rated torque:

(i) During starting and

(ii) at 1000 rpm.

[20 marks : 2025]

Solution:

(i) Additional resistance value to obtain rated torque at starting :

At start $N = 0$, $E_b = 0$.

To obtain rated torque, $I_a = 40$ A

$$I_a = \frac{V-E_b}{R_a+R_{ext}}$$

$$40 = \frac{240-0}{0.3+R_{ext}}$$

$$R_{ext} = 6 - 0.3 = 5.7\ \Omega$$

$\therefore$ If 5.7Ω additional resistance is added in series with motor at start, motor start with rated torque.

(ii) E_b at 1500 rpm $\Rightarrow$ $E_b = 240 - 40(0.3) = 228$ V

$$E_b \propto \phi N$$

For series motor, $E_b \propto I_a N$

$\because$ $\phi \propto I_a$

$\therefore$ $E_b = KI_aN \Rightarrow K = \dfrac{E_b}{I_aN}$

Constant $K = \dfrac{228}{40\times 1500} = 0.0038$

At $N = 1000$ rpm and $I_a = 40$ A

$$E_b = KI_aN = 0.0038 \times 40 \times 1000 = 152 \text{ V}$$

R_{ext} for new $E_b = 152$ V :

$$E_b = V - I_a(R_a + R_{ext})$$

$$152 = 240 - 40(0.3 + R_{ext})$$

$$R_{ext} = 1.9\ \Omega$$

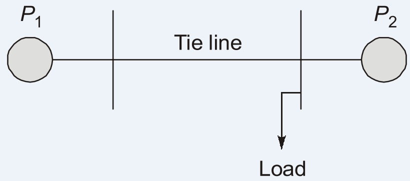
$\therefore$ By adding a resistance of 1.9 Ω in series with motor, rated torque is developed at 1000 rpm by taking $I_a = 40$ A.

End of Solution

Q.4 (a) The figure shows a two-bus system. If a power of 125 MW is transferred from Plant 1 to the load, a power loss of 15.625 MW occurs. Find the generation schedule and load demand if the cost of received power is ₹24 per MWh. The incremental production costs are:

$$\frac{dF_1}{dP_1} = 0.025P_1 + 15\ ; \qquad \frac{dF_2}{dP_2} = 0.05P_2 + 20$$

Assume the penalty factor of the 2nd generator = 1.



[20 marks : 2025]

Solution:

Power loss equation is given as

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

∵ load is concentrated on bus-2 of plant-2

∵ $$B_{12} = 0 \text{ and } B_{22} = 0$$

Therefore, $$P_L = B_{11}P_1^2$$

Now, as per data

$$P_L = 15.625 = B_{11} \times (125)^2$$

$$\Rightarrow \quad B_{11} = 1 \times 10^{-3}$$

Now, penalty factor for plant (1)

$$L_1 = \frac{1}{1-\dfrac{\partial P_L}{\partial P_1}} = \frac{1}{1-2B_{11}P_1}$$

Also, peanlty factor for plant (2)

$$L_2 = 1.0$$

For economic power generation

$$IC_1L_1 = IC_2L_2 = \lambda$$

$$\frac{0.025P_1 + 15}{1-2\times1\times10^{-3}P_1} = 24$$

$$0.025P_1 + 15 = 24 - 48 \times 10^{-3}P_1$$

$$P_1(0.025 + 0.048) = 9$$

$$\Rightarrow \quad P_1 = 123.30 \text{ MW}$$

Also, $$IC_2L_2 = \lambda$$

$$0.05P_2 + 20 = 24$$

$$P_2 = 80 \text{ MW}$$

Now, generation schedule for

Plant-1, $P_1 = 123.30$ MW

Plant-2, $P_2 = 80$ MW

Load demand, $P_D = P_1 + P_2 - P_L$

$$P_D = 123.30 + 80 - 1 \times 10^{-3} \times (123.30)^2$$

$$P_D = 188.1 \text{ MW}$$

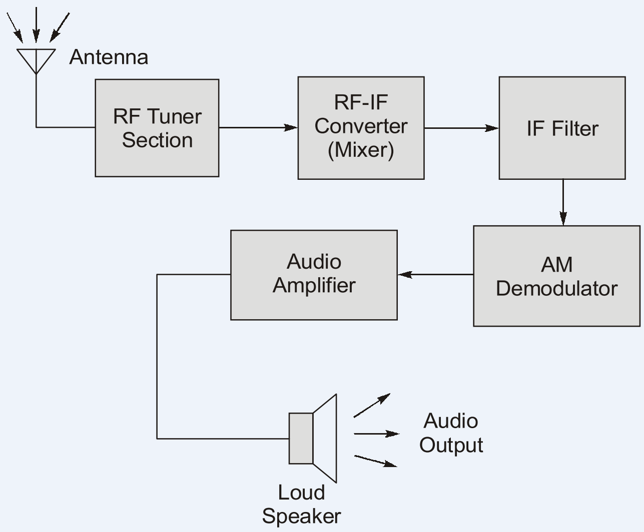
End of Solution

Q.4 (b) (i) Give the block diagram of an AM Receiver and an FM Receiver. Also, explain each block.

[10 marks : 2025]

Solution:

AM Receiver: The AM super heterodyne receiver takes the amplitude modulated wave as an input and produces the original audio signal as an output. Selectivity is the ability of selecting a particular signal, while rejecting the others. Sensitivity is the capacity of detecting RF signal and demodulating it, while at the lowest power level. Radio amateurs are the initial radio receivers. However, they have drawbacks such as poor sensitivity and selectivity. To overcome these drawbacks, super heterodyne receiver was invented. The block diagram of AM receiver is shown in the following figure.



RF Tuner Section: The amplitude modulated wave received by the antenna is first passed to the tuner circuit through a transformer. The tuner circuit is nothing but a LC circuit, which is also called as resonant or tank circuit. It selects the frequency, desired by the AM receiver. It also tunes the local oscillator and the RF filter at the same time.

RF Mixer: The signal from the tuner output is sent to the RF-IF converter, which acts as a mixer. It has a local oscillator, which produces a constant frequency. The mixing process is done here, having the received signal as one input and the local oscillator frequency as the other input. The resultant output is a mixture of two frequencies $[(f_1 + f_2), (f_1 - f_2)]$ produced by the mixer, which is called as intermediate frequencies (IF).

Intermediate Frequency (IF).

The production of IF helps in the demodulation of any station signal having any carrier frequency. Hence, all signals are translated to a fixed carrier frequency for adequate selectivity.

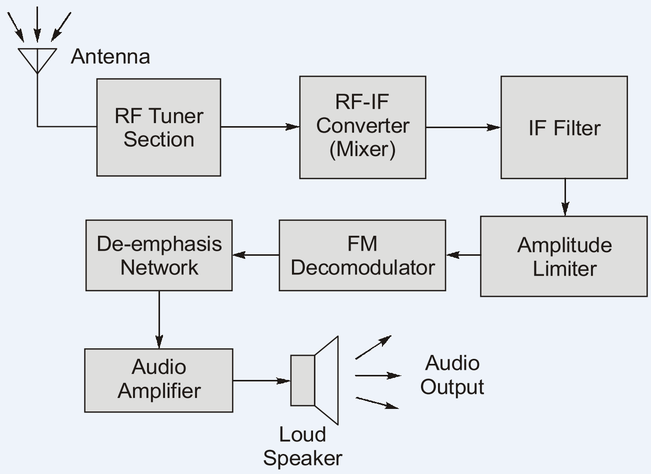
IF Filter:Intermediate frequency filter is a band pass filter, which passes the desired frequency. It eliminates all other unwanted frequency components present in it. This is the advantage of IF filter, which allows only IF frequency.

AM Demodulator:The received AM wave is now demodulated using AM demodulator. This demodulator uses the envelope detection process to receive the modulating signal.

Audio Amplifier:

This is the power amplifier stage, which is used to amplify the detected audio signal. The processed signal is strengthened to be effective. This signal is passed on to the loudspeaker to get the original sound signal.

FM Receiver: The block diagram of FM receiver is shown in the following figure.



This block diagram of FM receiver is similar to the block diagram of AM receiver. The two blocks Amplitude limiter and De-emphasis network are included before and after FM demodulator. The operation of the remaining blocks is the same as that of AM receiver.

We know that in FM modulation, the amplitude of FM wave remains constant. However, if some noise is added with FM wave in the channel, due to that the amplitude of FM wave may vary. Thus, with the help of amplitude limiter we can maintain the amplitude of FM wave as constant by removing the unwanted peaks of the noise signal.

In FM transmitter, we have seen the pre-emphasis network (High pass filter), which is present before FM modulator. This is used to improve the SNR of high frequency audio signal. The reverse process of pre-emphasis is known as de-emphasis. Thus, in this FM receiver, the de-emphasis network (Low pass filter) is included after FM demodulator. This signal is passed to the audio amplifier to increase the power level. Finally, we get the original sound sigle from the loudspeaker.

End of Solution

Q.4 (b) (ii) What is the largest value of output voltage from an 8-bit DAC that produces 2.0 V for a digital input of 01110010?

[10 marks : 2025]

Solution:

$$\text{Digital input} = 0111\,0010 \text{ (Base 2)}$$

Convert to decimal :

$$0111\,0010_2 = 64 + 32 + 16 + 2 = 114$$

So, for digital code 114, DAC output = 2.0 V

For an n-bit DAC (n = 8 here), output voltage is :

$$V_{\text{out}} = \frac{D}{2^n - 1} \times V_{FS}$$

$$V_{\text{out}} = \frac{D}{255} \times V_{FS}$$

$$2.0 = \frac{114}{255} \times V_{FS}$$

$$V_{FS} = \frac{2.0 \times 255}{114} = 4.47 \text{ V}$$

$$\text{Largest possible digital input} = 255(1111\,1111)_2$$

$$V_{\max} = \frac{255}{255} \times V_{FS} = V_{FS} \approx 4.47 \text{ V}$$

End of Solution

Q.4 (c) A 3-phase, 6-pole, 500 kVA, 6600 V, 50 Hz star-connected synchronous motor having synchronous impedance of $j80\ \Omega$ per phase operates at unity power factor at rated conditions.

(i) Determine the mechanical torque driving capability for this motor at rated conditions, neglecting all mechanical losses.

(ii) At this rated torque, what are the required deviations from rated armature current and excitation (in terms of $E_f \angle \delta$) to produce a maximum torque of 1.26 times the maximum rated torque for a leading power factor operation of the motor?

(iii) Determine the value of the leading power factor for motor operation as stated in (ii) above.

[20 marks : 2025]

Solution:

(i) Find Torque :
$$T = \frac{60}{2\pi N_s} P \text{ N-m}$$

500 kVA motor operating at U.P.F., rated conditions means $P_{\text{out}} = 500$ kW.

$$N_S = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$T = \frac{60}{2\pi(1000)} \times 500 \times 10^3$$

$$T = 4777.07 \text{ N-m (Rated torque)}$$

(ii) Deviations required in terms of $E_f\angle\delta$ to produce maximum torque of 1.26. Rated torque for leading P.f operation of motor.

Given : $R_a = 0$, Neglect mech. loss

$$\therefore \quad P_{i/p} = P_{o/p}$$

$$T \propto P \propto E$$

For a given V and X_s,

$$P \propto E$$

$$T \propto E$$

$$P = \frac{EV}{X_S}\sin\delta$$

Maximum P or maximum T occurs at $\sin \delta = 1$

$$\delta = 90°$$

$$V = \frac{6600}{\sqrt{3}} = 3810.512 \text{ V}$$

Rated current,

$$I_a = \frac{500 \times 10^3}{\sqrt{3} \times 6600} = 43.7387 \text{ A}$$

At rated unity P.f. operation

$$E_b\angle{-\delta} = V\angle 0 - I_a\angle\phi\,(Z_s\angle\theta)$$

$$= 3810.512 - 43.73\angle 0°\,(j80)$$

$$= 5173.36\angle{-42.56°}$$

Mentioned maximum torque occurs at $\delta = 90°$, $\sin \delta = 1$

$$T \propto P \propto E$$

$$T_{max} = 1.26T_{rated}$$

$$\therefore \quad E_{b,new} = 1.26E_{b,rated} = 1.26 \times 5173.36$$

$$= 6518.43 \text{ V}$$

For the same real power output, but for new $E_b = 6518.43$, same V and X_S,

$$\delta_{new} = -32.47°$$

As per mentioned case deviation in $E_b\angle\delta$

New $E_b = 6518.43$	Old $E_b = 5173.36$
New $\delta = 32.47°$	Old $\delta = 42.56°$

(iii) Value of leading p.f. as stated in (ii).

For new E_b and δ_i findign new current $V = 3810.512$

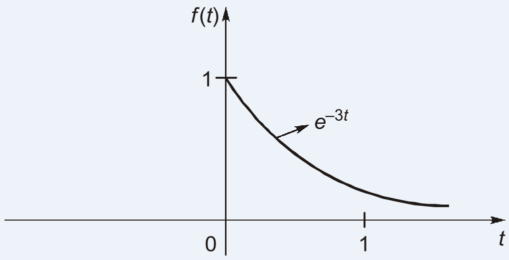
$$I_{a2}\angle\phi_2 = \frac{V\angle 0° - E_{b2}\angle - \delta_2}{jX_s}$$

By solving $I_{a2} = 48.568$ A and $\cos \phi = 0.9$ leading.

End of Solution

SECTION : B

Q.5 (a) An exponential function $f(t) = e^{-3t}\, u(t)$ as shown in the following figure is delayed by 1 sec. Sketch and describe mathematically the delayed function. Also, repeat the same if $f(t)$ is advanced by 1 second.



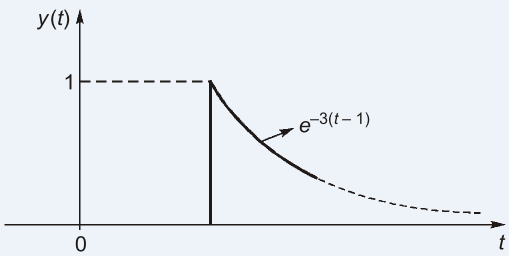
[12 marks : 2025]

Solution:

Given function is, $$f(t) = e^{-3t}\, u(t)$$

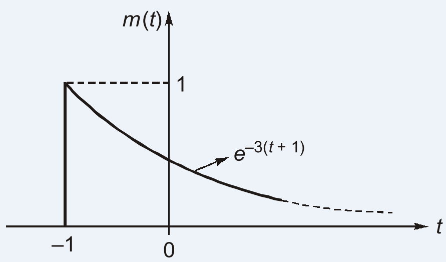
Let the delayed function of $f(t)$ by '1' sec is $y(t)$ then

$$y(t) = f(t-1)$$
$$= e^{-3(t-1)} \cdot u(t-1)$$
$$= \begin{cases} 0, & t < 1 \\ e^{-3(t-1)}, & t > 1 \end{cases}$$



Let the advanced function of $f(t)$ by '1' sec is $m(t)$ then

$$m(t) = f(t+1) = e^{-3(t-1)} \cdot u(t-1)$$
$$= \begin{cases} 0, & t < -1 \\ e^{-3(t+1)}, & t > -1 \end{cases}$$



End of Solution

Q.5 (b) A step-down DC-DC converter is feeding an RLE load with a freewheeling diode across the load. Assuming a ripple-free load current, derive the expression for the maximum duty cycle in terms of supply voltage V_S and back emf of the load E for which the RMS current through the freewheeling diode has a maximum value.

[12 marks : 2025]

Solution:

For chopper with RLE load, average load current I_0 is

$$I_0 = \frac{V_0 - E}{R} = \frac{\alpha V_s - E}{R}$$

Freewheeling-diode current flows during the period T_{off}. Therefore, rms value of freewheeling-diode current, when I_0 is ripple free, is given by

$$I_{fdr} = \sqrt{\frac{T_{\text{off}}}{T}} \cdot I_0 = \left[\frac{T - T_{\text{on}}}{T}\right]^{1/2}\left[\frac{\alpha V_s - E}{R}\right]$$

$$= \frac{1}{R}(\alpha V_s - E)\left(\sqrt{1-\alpha}\right) \qquad \text{...(i)}$$

$$= \frac{1}{R}\left[\alpha\sqrt{1-\alpha}\cdot V_s - \sqrt{1-\alpha}\cdot E\right]$$

or $$I_{fdr} = \frac{1}{R}\left[\sqrt{\alpha^2 - \alpha^3}\cdot V_s - \sqrt{1-\alpha}\cdot E\right]$$

This current I_{fdr} will have maximum value when

$$\frac{dI_{fdr}}{d\alpha} = \frac{1}{R}\left[\frac{1}{2}\frac{(2\alpha - 3\alpha^2)V_s}{\sqrt{\alpha^2 - \alpha^3}} + \frac{1}{2}\frac{E}{\sqrt{1-\alpha}}\right] = 0$$

or $$\frac{(2\alpha - 3\alpha^2)V_s}{\sqrt{\alpha^2-\alpha^3}} = -\frac{E}{\sqrt{1-\alpha}}$$

or $$\frac{(2\alpha - 3\alpha^2)\sqrt{1-\alpha}}{\alpha\sqrt{1-\alpha}} = -\frac{E}{V_s}$$

or $$3\alpha - 2 = \frac{E}{V_s} \quad \text{or} \quad \alpha = \frac{1}{3}\left(2 + \frac{E}{V_s}\right)$$

Substituting this value of a in Eq. (i), we get maximum value of rms current rating $I_{fd.rm}$ of freewheeling diode as under :

$$I_{fd.rm} = \frac{1}{R}\left[\frac{1}{3}\left(2 + \frac{E}{V_s}\right)V_s - E\right]\left[1 - \frac{1}{3}\left(2 + \frac{E}{V_s}\right)\right]^{1/2}$$

$$= \frac{1}{R}\left[\frac{1}{3}\left(\frac{2V_s + E}{V_s}\right)V_s - E\right]\left[1 - \frac{2V_s + E}{3V_s}\right]^{1/2}$$

$$= \frac{1}{R}\left[\frac{2V_s + E - 3E}{3}\right]\left[\frac{3V_s - 2V_s - E}{3V_s}\right]^{1/2}$$

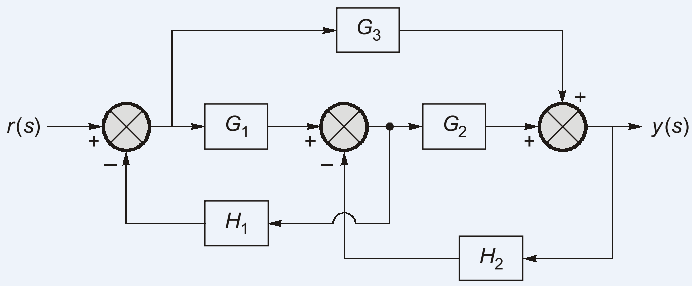
$$= \frac{1}{R}\left[\frac{2V_s - 2E}{3}\right]\left[\frac{V_s - E}{3V_s}\right]^{1/2} = \frac{1}{R}\cdot\frac{2}{3}[V_s - E]^{3/2}\cdot\frac{1}{\sqrt{3V_s}}$$

$$= \frac{1}{R}\cdot\frac{2}{3}\frac{V_s\cdot\sqrt{V_s}}{\sqrt{3V_s}}\left[1 - \frac{E}{V_s}\right]^{3/2} = \frac{2}{3\sqrt{3}}\cdot\frac{V_s}{R}\left[1 - \frac{E}{V_s}\right]^{3/2}$$

$$= 0.3849\frac{V_s}{R}\left[1 - \frac{E}{V_s}\right]^{3/2}$$

End of Solution

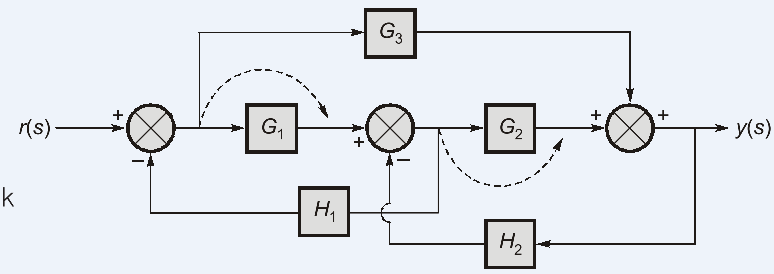
Q.5 (c) The block diagram of a system is as shown below. Find the overall transfer function of the system using block diagram reduction technique.



[12 marks : 2025]

Solution:

Step 1 :



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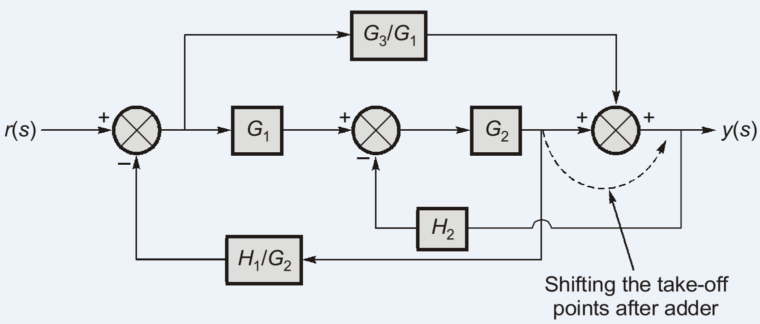
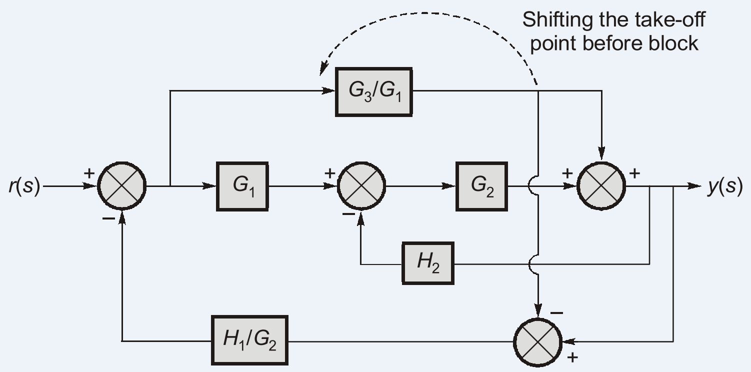
Shifting the take-off points after block

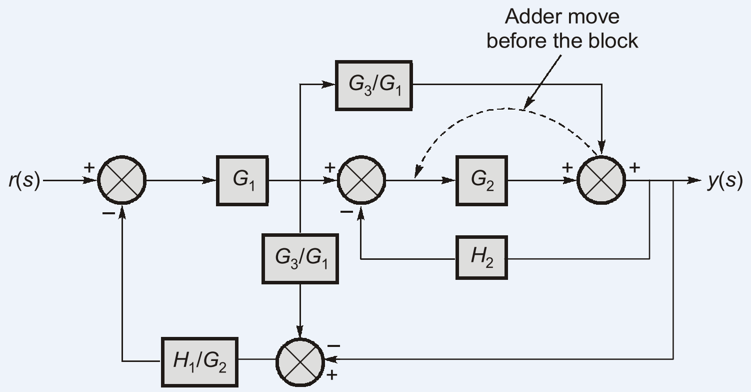
Step 2 :



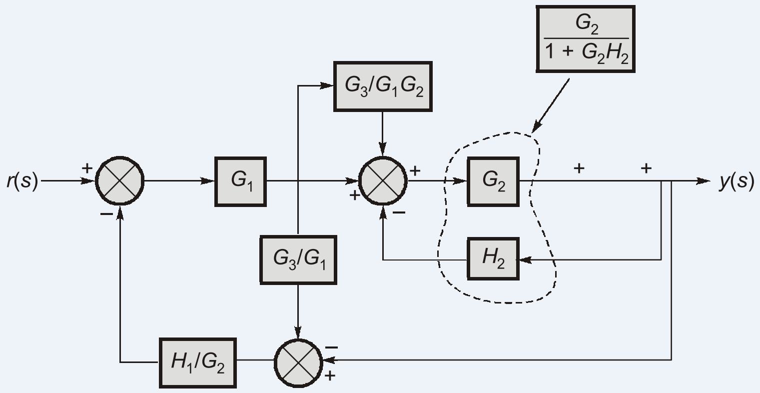
Step 3 :



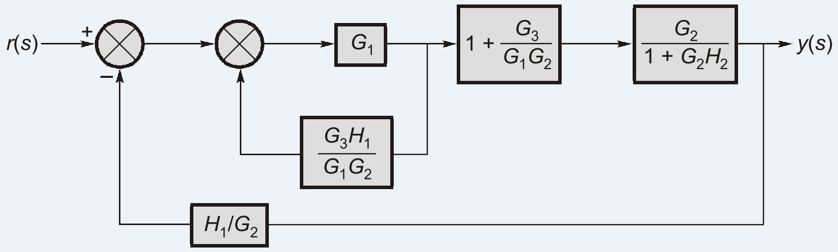
Step 4 :



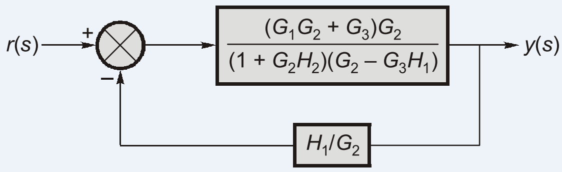
Step 5 :



Step 6 :



Step 7 :



$$\frac{y(s)}{r(s)} = \frac{(G_1G_2+G_3)G_2}{(1+G_2H_2)(G_2-G_3H_1)+(G_1G_2+G_3)H_1}$$

$$= \frac{(G_1G_2+G_3)G_2}{G_2-G_3H_1+G_2^2H_2-G_2G_3H_1H_2+G_1G_2H_1+G_3H_1}$$

$$\frac{y(s)}{r(s)} = \frac{G_1G_2+G_3}{1+G_1H_1+G_2H_2-G_3H_1H_2}$$

End of Solution

Q.5 (d) (i) Draw the silicon cross-section view of an IGBT and identify the distinguishing feature from MOSFET with reference to conductivity modulation. Also, state its impact on IGBT operation and performance.

[8 marks : 2025]

Solution:

IGBT has been developed by combining into it the best qualities of both BJT and PMOSFET. Thus an IGBT possesses high input impedance like a PMOSFET and has low on-state power loss as in a BJT. Further, IGBT is free from second breakdown problem present in BJT. All these merits have made IGBT very popular amongst power-electronics engineers. IGBT is also known as metal oxide insulated gate transistor (MOSIGT), conductively-modulated field effect transistor (COMPET) or gain-modulated FET (GEMFET). It was also initially called insulated gate transistor (IGT).

Basic structure:

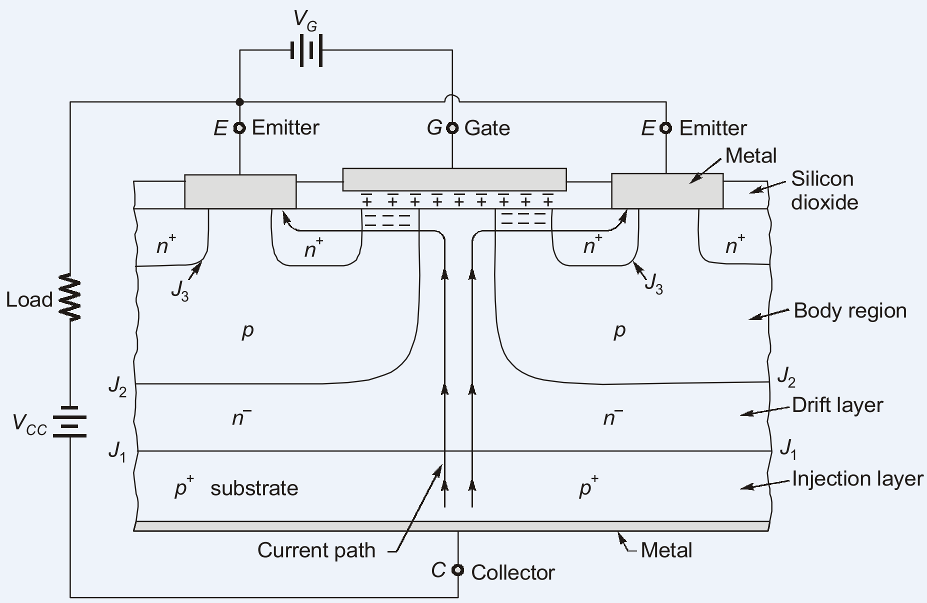


Figure above illustrates the basic structure of an IGBT. It is constructed virtually in the same manner as a power MOSFET. There is, however, a major difference in the substrate. The n^+ layer substrate at the drain in a PMOSFET is now substituted in the IGBT by a p^+ layer substrate called collector C. Like a power MOSFET, an IGBT has also thousands of basic structure cells connected appropriately on a single chip of silicon.

In IGBT, p^+ substrate is called injection layer because it injects holes into n^- layer. The n^- layer is called drift region. As in other semiconductor devices, thickness of n^- layer determines the voltage blocking capability of IGBT. The p layer is called body of IGBT. The n^+ layer in between p^+ and p regions serves to accommodate the depletion layer of pn^- junction, i.e. junction J_2.

Comparison of BJT, IGBT, and MOSFET as lower elements		
	MOSFET	**IGBT**
Drive type	Voltage drive	Voltage drive
Power for driving	High	Low
On state voltage	Tends to increase in proportion to breakdown voltage	Low
Switching speed	High	Moderate
Temperature stability	Good	Good
Difficulty of achieving high breakdown voltage	High	Easy with conductivity modulation

1. Type of Device:
   - IGBT: Combines features of both MOSFET and bipolar transistor. It has a voltage-controlled gate like a MOSFET and a bipolar-like current-carrying capability.
   - MOSFET: A voltage-controlled transistor that relies on the voltage applied to the gate to control the flow of current between the source and drain terminals.
2. Voltage Rating:
   - IGBT: Generally suitable for higher voltage applications (hundreds to thousands of volts).
   - MOSFET: Typically used in lower to medium voltage applications (tens to hundreds of volts).
3. Current Handling:
   - IGBT: Well-suited for high current applications. It combines the voltage control of a MOSFET with the current-carrying capability of a bipolar transistor.
   - MOSFET: Generally used for lower to moderate current applications.
4. Switching Speed:
   - IGBT: Slower switching speed compared to MOSFETs. Suitable for applications where switching speed is not the primary concern.
   - MOSFET: Faster switching speed, making them suitable for applications that require high-frequency operation.

5. Efficiency:
   - IGBT: Lower conduction losses compared to a MOSFET at high voltages and currents. Suitable for high-power applications like motor drives and power inverters.
   - MOSFET: More efficient at low voltages and currents. Often used in applications where efficiency and fast switching are critical, such as power supplies and certain amplifiers.
6. Applications:
   - IGBT: Commonly used in high-power applications such as motor drives, power inverters, and induction heating systems."
   - MOSFET: Widely used in applications where fast switching and efficiency at lower power levels are crucial, such as voltage regulators and electronic switching circuits.
7. Gate Drive Requirements:
   - IGBT: Requires a positive voltage on the gate relative to the emitter for turn-on, but the turn-off can be controlled by reducing the gate voltage.
   - MOSFET: Requires a positive voltage on the gate relative to the source for both turn-on and turn-off.

End of Solution

Q.5 (d) (ii) Draw 2-transistor and simplified equivalent circuits with proper labels and their significance.

[4 marks : 2025]

Solution:

The principle of thyristor operation can be explained with the use of its two-transistor model (or two-transistor analogy). Fig. (a) shows schematic diagram of a thyristor. From this figure, two-transistor model is obtained by bisecting the two middle layers, along the dotted line, in two separate halves as shown in Fig. (b). In this figure, junctions $J_1 - J_2$ and $J_2 - J_3$ can be considered to constitute *pnp* and *npn* transistors separately. The circuit representation of the two-transistor model of a thyristor is shown in Fig. (c).

In the off-state of a transistor, collector current I_C is related to emitter current I_E as

$$I_C = \alpha I_E + I_{CBO}$$

where α is the common-base current gain and I_{CBO} is the common-base leakage current of collector-base junction of a transistor.

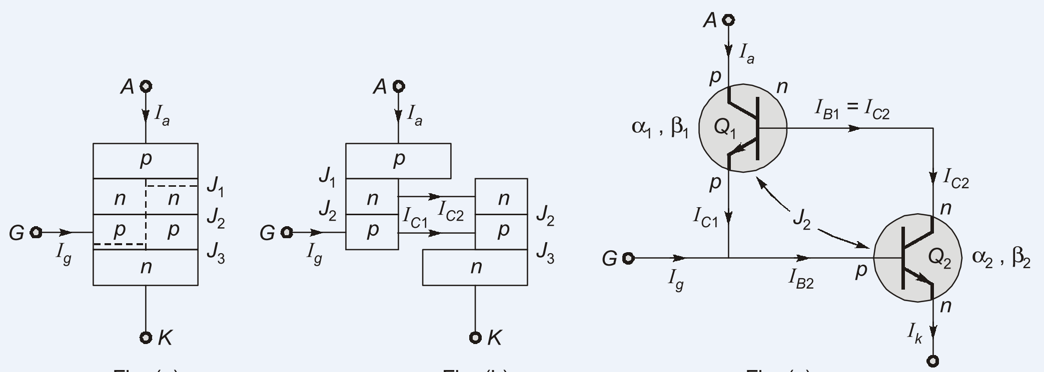


Fig. (a) Fig. (b) Fig. (c)

For transistor Q_1 in figure (c), emitter current I_E = anode current I_a and I_C = collector current I_{C1}. Therefore, for Q_1,

$$I_{C1} = \alpha_1 I_a + I_{CBO1}$$

where, α_1 = common base current gain of Q_1 ...(i)

and I_{CBO1} = common base leakage current of Q_1

Similarly, for transistor Q_2, the collector current I_{C2} is given by

$$I_{C2} = \alpha_2 I_k + I_{CBO2} \quad \text{...(ii)}$$

where α_2 = common base current gain of Q_2

I_{CBO2} = common base leakage current of Q_2

and I_k = emitter current of Q_2

The sum of two collector currents given by equation (i) and (ii), is equal to the external circuit current I_a entering at anode terminal A.

$\therefore$ $$I_a = I_{C1} + I_{C2}$$

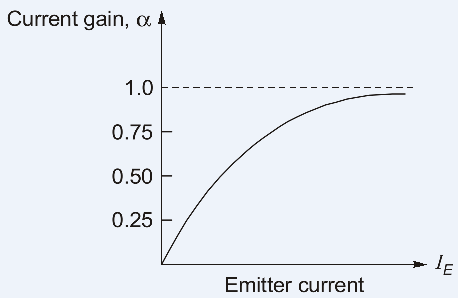
or $$I_a = \alpha I_1 + I_{CBO1} + \alpha I_2 + I_{CBO2} \quad \text{...(iii)}$$

when gate current is applied, then $I_k = I_a + I_g$

Substituting this value of I_k in equation (iii), gives

$$I_\alpha = \alpha_1 I_a + I_{CBO1} + \alpha_2(I_a + I_g) + I_{CBO2}$$

or $$I_\alpha = \frac{\alpha_2 I_g + I_{CBO1} + I_{CBO2}}{1-(\alpha_1+\alpha_2)} \quad \text{...(iv)}$$



For a silicon transistor, current gain α is very low at low emitter current. With an increase in emitter current, α builds up rapidly as shown in figure. With gate current I_g = 0 and with thyristor forward biased, $(\alpha_1 + \alpha_2)$ is very low as per figure. Under these conditions, equation (iv) shows that forward leakage current somewhat more than $(I_{CBO1} + I_{CBO2})$ flows. If, by some means, the emitter current of two component transistors can be increased so that $\alpha_1 + \alpha_2$ approaches unity, then as per equation (iv), I_a would tend to become infinity thereby turning-on the device. Actually, external load limits the anode current to a safe value after the thyristor, in fact, are the methods of making $\alpha_1 + \alpha_2$ to approach unity.

End of Solution

Q.5 (e) The overall transfer function of a unity feedback system is given by:

$$G_{CL}(S) = \frac{Ks+b}{s^2+as+b}.$$

(i) Calculate the open-loop transfer function of the system and its type.

(ii) If the overall system with $K = 0$, admits a unity normalized bandwidth and a settling time of 4 seconds for a 2% tolerance band, compute position, velocity, and acceleration error constants. Assume unity DC gain.

(iii) Compute the sensitivity and complementary sensitivity function value at $\omega = 1$ rad/sec. Consider $K = 1$.

(iv) Discuss the effect of having a non-zero value of K on the behavior of the system in comparison to that with $K = 0$.

[12 marks : 2025]

Solution:

Given : Closed loop system with u.f.s.

$$G_{CL}(s) = \frac{L(s)}{1+L(s)} = \frac{ks+b}{s^2+as+b}$$

(i) We know that

$$G_{CL}(s) = \frac{G(s)}{1+G(s)H(s)}$$

For $H(s) = 0$:

$$G_{CL}(s) = \frac{G(s)}{1+G(s)-G(s)} = \frac{ks+b}{s^2+as+b-ks-b} = \frac{ks+b}{s(s+a-k)}$$

Hence, open loop system is

$$G_{CL}(s) = \frac{ks+b}{s(s+a-k)}$$

Number of poles at the origin = 1. Hence given is type-1 system.

(ii) For $k = 0$:

$$G_{OL}(s) = \frac{b}{s(s+a)}$$

$$T_s = \frac{4}{\xi\omega_n} = 4 \text{ sec}$$

$$\xi\omega_n = 1 \quad \text{and} \quad \omega_n = 1 \text{ rad/sec}$$

$$\Rightarrow \qquad \xi = 1 \;\Rightarrow\; G_{OL}(s) = \frac{1}{s(s+2)}$$

$$a = 2\xi\omega_n = 2, \quad b = \omega_n^2 = 1$$

$$k_p = \lim_{s\to 0} G_{OL}(s) = \lim_{s\to 0}\frac{1}{s(s+2)} = \infty$$

$$k_v = \lim_{s\to 0} sG_{OL}(s) = \lim_{s\to 0} s\cdot\frac{1}{s(s+2)} = 0.5$$

$$k_a = \lim_{s\to 0} s^2 G_{OL}(s) = \lim_{s\to 0} s^2 \cdot \frac{1}{s(s+2)} = 0$$

(iii) For $k = 1$:

$$\omega_n = 1 \text{ rad/sec}$$

$$G_{OL}(s) = \frac{s+1}{s(s+1)} = \frac{1}{s}$$

Sensitivity,
$$S = \frac{1}{1+G_{OL}(s)} = \frac{s}{s+1}$$

Put $s = j\omega$:

$$S = \frac{j\omega}{1+j\omega}$$

$$|S| = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}} \simeq 0.707$$

Complementary sensitivity,

$$S_c = \frac{G_{OL}(s)}{1+G_{OL}(s)}$$

$$= \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$$

Put $s = j\omega$:

$$S_c = \left.\frac{1}{1+j\omega}\right|_{\omega=1}$$

$$|S_c| = \frac{1}{\sqrt{1+1^2}} \simeq 0.707$$

(iv) The characteristics equation $s^2 + as + b$ do not change.

Open Loop Poles/Zeros Change with k : Charging k moves the open-loop poles at $s = k - a$ and moves the zero at $s = \frac{-b}{k}$ (when $k \neq 0$). This affects frequency response, margins and sensitivity.

End of Solution

Q.6 (a) A single-phase AC controller operating on phase control is supplied from a 230 V, 50 Hz AC supply. If the controller is feeding a purely resistive load of 10 Ω at a firing angle of 45°, then determine:

(i) The RMS output voltage $V_{0,\,\text{rmp(phase)}}$ of the phase-controlled AC controller.

(ii) The equivalent duty cycle (K) of an integral cycle AC controller that would produce the same RMS output voltage.

(iii) If the integral cycle controller operates with a total of 100 cycles for one complete operation, determine the number of 'ON' cycles and 'OFF' cycles for the same as in (ii).

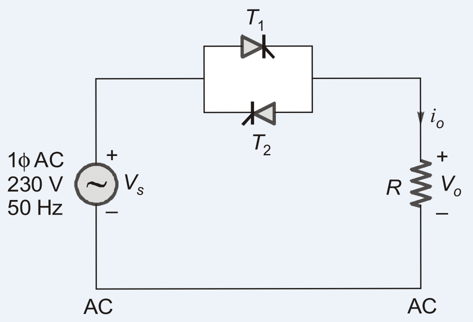
(iv) The input power factor of the integral duty cycle AC controller operating at the equivalent duty cycle.

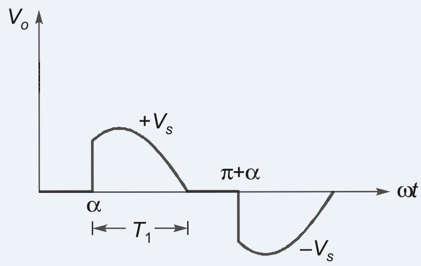
(v) The RMS Thyristor current $I_{T,\,rms}$ for the integral cycle controller operating at this equivalent duty cycle.

Derive the formula used for integral cycle AC controllers as used in the above parts.

[20 marks : 2025]

Solution:





$$V_{or} = \left\{\frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t\, d(\omega t)\right\}^{1/2}$$

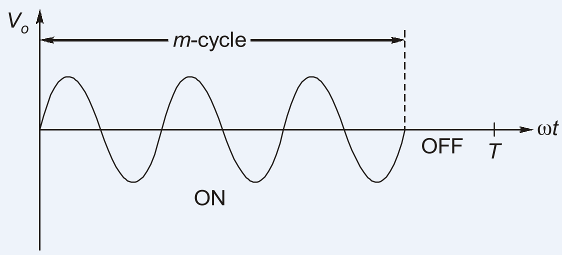
$$V_{or} = \frac{V_m}{\sqrt{2\pi}}\left\{(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right\}^{1/2}$$

$$= \frac{230\sqrt{2}}{\sqrt{2\pi}}\left\{\left(\pi-\frac{\pi}{4}\right)+\frac{1}{2}\sin\frac{\pi}{2}\right\}^{1/2}$$

(i) $$V_{or} = 219.304 \text{ V}$$

$$I_{or} = \frac{V_{or}}{R} = \frac{219.304}{10} = 21.93 \text{ A}$$

(ii) Integral Cycle Control Tech :



For m cycles $\rightarrow$ ON

$\therefore$ $$V_o = V_s$$

For n cycles $\rightarrow$ OFF

$\therefore$ $$V_o = 0$$

$\therefore$ $$V_{or} = V_{sr}\left(\frac{m}{m+n}\right)^{1/2} = \sqrt{K}\cdot V_{sr}$$

where K is the duty cycle of AC voltage constant.

$$V_{or} = \sqrt{K}\cdot V_{sr}$$

$$m + n = 100 \text{ cycles}$$

$$219.304 = \sqrt{K}\cdot 230$$

$$K \text{ (duty check)} = \left(\frac{219.304}{230}\right)^2 = 0.909$$

(iii) $$m + n = 100 \text{ cycles}$$

$$K = 0.909$$

$$\frac{m}{m+n} = 0.909$$

$$m = 0.909 \times 100$$

$$m = 90.9 \text{ cycles}$$

Here, load is connected to supply.

For m-integral cycles and disconnected.

For n-integral cycles.

$\therefore$ $$m \simeq 91 \text{ (integer)}$$

$$m + n = 100 \qquad \text{(ON cycles – 9; OFF cycles – 9)}$$

$$91 + n = 100$$

$$n = 9$$

(iv) $$\text{PF} = \frac{V_{or}}{V_{sr}} \quad \rightarrow \text{R load}$$

$$= \sqrt{K} = \sqrt{\frac{m}{m+n}} = \sqrt{\frac{91}{100}} = 0.9539$$

or $$\text{PF} = \frac{V_{or}}{V_{sr}} = \frac{219.304}{230} = 0.953$$

$$(I_T)_{\text{rms}} = \left\{\frac{1}{2\pi(m+n)}\left[n\int_0^{\pi}\left(\frac{V_m \sin\omega t}{R}\right)^2 \cdot d(\omega t)\right]\right\}^{1/2}$$

$$= \left\{\frac{1}{2\pi\frac{m+n}{n}}\int_0^{\pi}(I_m \sin\omega t)^2 d(\omega t)\right\}^{1/2}$$

$$= \sqrt{\frac{n}{m+n}}\left\{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\omega t\cdot d(\omega t)\right\}^{1/2} = \sqrt{K}\cdot\frac{I_m}{2}$$

$$I_m = \frac{V_m}{R} = \frac{230\sqrt{2}}{10} = 23\sqrt{2} \text{ Amp}$$

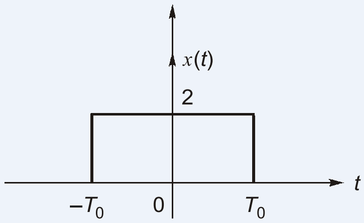
$$= \sqrt{0.909} \times \frac{23\sqrt{2}}{2}$$

$$(I_T)_{rms} = 15.5 \text{ A}$$

End of Solution

Q.6 (b) (i) Derive the even and odd decomposition of a general signal $x(t)$ by applying the definitions of even and odd signals.

(ii) Find the Fourier Transform of $x(t)$, which is given by the following rectangular pulse, as shown in the figure.



[10 + 10 marks : 2025]

Solution:

(i) Let $x_e(t)$ is representing even part of $x(t)$ and $x_0(t)$ is representing odd part of $x(t)$,
Now, we can represent signal $x(t)$ in terms of $x_e(t)$ and $x_0(t)$ in the following form

$$x(t) = x_e(t) + x_0(t) \quad \text{...(i)}$$

Put $t = -t$ in equation (i),

$$x(-t) = x_e(-t) + x_0(-t)$$
$$= x_e(t) - x_0(t) \quad \text{...(ii)}$$

From equation (i) + (ii),

$$x(t) + x(-t) = 2x_e(t)$$

$$\Rightarrow \quad x_e(t) = \frac{x(t) + x(-t)}{2}$$

From equation (i) – (ii),

$$x(t) - x(-t) = 2x_0(t)$$

$$\Rightarrow \quad x_0(t) = \frac{x(t) - x(-t)}{2}$$

Thus we can decomposed $x(t)$ in ferms of $x_e(t)$ and $x_0(t)$ such as

$$x(t) = x_e(t) + x_0(t)$$

where $$x_e(t) = \text{even part of } x(t) = \frac{x(t) + x(-t)}{2}$$

and $$x_0(t) = \text{odd part of } x(t) = \frac{x(t) - x(-t)}{2}$$

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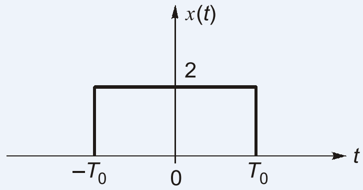
(ii) Given that,



The Fourier transform of $x(t)$ is given by

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-T_0}^{T_0} 2\cdot e^{-j\omega t}dt$$

$$= 2\cdot\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T_0}^{T_0}$$

$$= \frac{2}{-j\omega_0}\left[e^{-j\omega T_0} - e^{j\omega T_0}\right] = \frac{2}{-j\omega_0}[-2j\sin\omega T_0]$$

$$= \frac{4}{\omega_0}\sin\omega T_0 = \frac{4}{T_0}Sa(\omega T_0)$$

$$= \frac{4}{\omega_0}\left[\frac{\sin\omega T_0}{\omega T_0}\right]\omega T_0$$

$$= 4T_0\,\text{Sa}(\omega T_0)$$

$\Rightarrow$ $$X(\omega) = 4T_0\,\text{Sa}(\omega T_0)$$

Where $Sa(\theta)$ is sampling function defined by

$$Sa(\theta) = \frac{\sin\theta}{\theta}$$

End of Solution

Q.6 (c) A unity feedback system has an open-loop transfer function:

$$G(s) = \frac{K\cdot e^{-0.5s}}{(s^2+\alpha s+\beta)}$$

$G(s)$ has a DC gain of $K/16$, and has a decay rate of 2 nepers per second. Using the first-order Pade approximation for the delay, sketch the root locus plot of $G(s)$ and find the range of K for which the unity feedback system remains stable.

[20 marks : 2025]

Solution:

Given : $$G(s) = \frac{Ke^{-0.5s}}{(s^2+\alpha s+\beta)}$$

Using the first order pole approximation for the delay

$$e^{-T_d s} \simeq \frac{1-\frac{T_d s}{2}}{1+\frac{T_d s}{2}} \quad \text{where } T_d = 0.5$$

We get $$e^{-0.5s} \simeq \frac{1-0.25s}{1+0.25s}$$

So, approximated transfer function becomes

$$G(s) \simeq \frac{K(1-0.25s)}{(s^2+\alpha s+\beta)(1+0.25s)}$$

DC gain is $\frac{K}{16}$.

So, $$G(0) = \frac{K \cdot e^0}{0^2+\alpha \cdot 0+\beta} = \frac{K}{\beta} = \frac{K}{16} \quad \Rightarrow \quad \beta = 16$$

Delay rate of 2 neper per second,

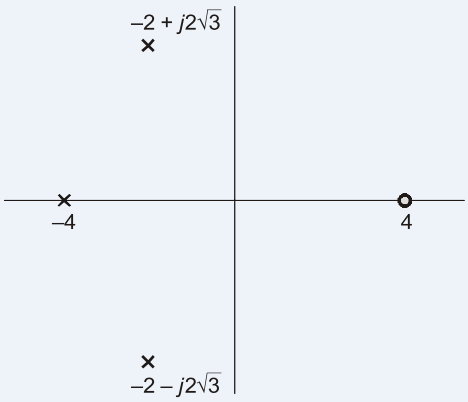
$$\xi\omega_n = 2$$
$$\alpha = 2\xi\omega_n = 4$$

Therefore, $$G(s) = \frac{K(1-0.25s)}{(s^2+4s+16)(1+0.25s)} \simeq \frac{-K(s-4)}{(s+4)(s^2+4s+16)}$$

Poles location, $s_p = -4, -2 \pm j2$

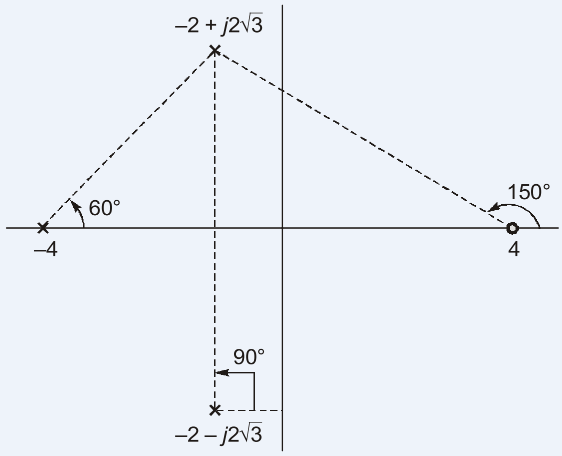
Zero's location, $s_z = 4$

Pole-Zero Plot :



$\because$ Two poles are complex, there will exists angle of departure.

Angle of departure for pole $-2+j2\sqrt{3}$.



$$\theta_{d(-2+j2\sqrt{3})} = 180° + \Sigma\phi_z - \Sigma\phi_p$$

$$\theta_d = 180° + 150° - 90° - 60° = 180°$$

Intersection of Root Locus with Imaginary Axis :

$$1 + G(s) = 0$$

$$(s + 4)(s^2 + 4s + 16) - k(s - 4) = 0$$

$$s^3 + 8s^2 + (32 - k)s + (64 + 4k) = 0$$

Routh Array :

s^3	1	$32-k$
s^2	8	$64+4k$
s^1	$24-1.5k$	0
s^0	$64+4k$	

For stability :

$$64 + 4k > 0 \quad \Rightarrow \quad k > -16$$

$$24 - 1.5k > 0 \quad \Rightarrow \quad k < 16$$

But k is positive. So, stable range is $0 < k < 16$. Root locus will intersect imaginary axis for

$$k = 16$$

Break Points :

$$1 + G(s) = 0$$

$$(s + 4)(s^2 + 4s + 16) - k(s - 4) = 0$$

$$k = \frac{(s^3 + 8s^2 + 32s + 64)}{s - 4}$$

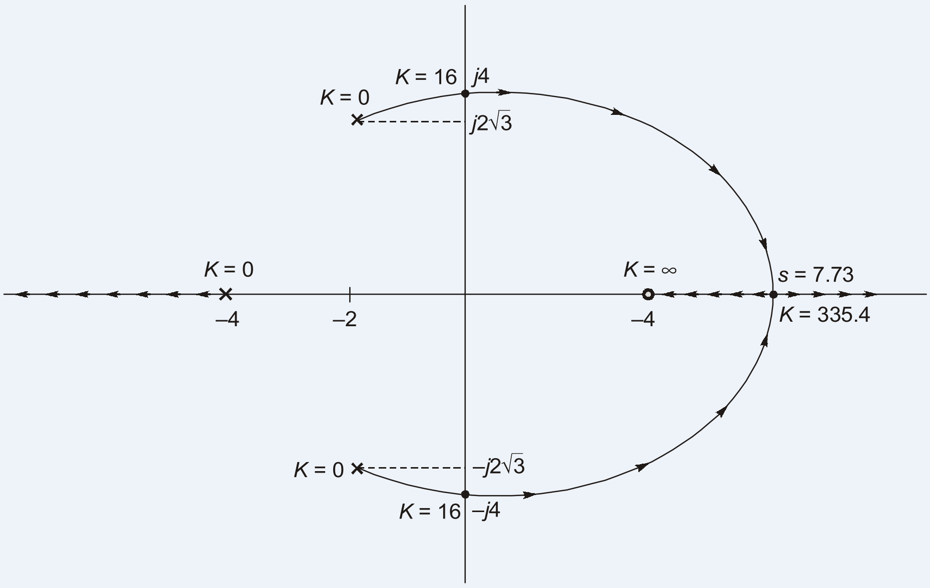
$$\frac{dk}{ds} = \left[\frac{(s-4)(3s^2 + 16s + 32) - (s^3 + 8s^2 + 32s + 64)}{(s-4)^2}\right] = 0$$

$$2s^3 - 4s^2 - 64s - 192 = 0$$

$$s = -2.86 \pm j2.04,\ 7.73$$

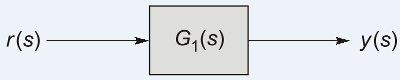
Valid break in point is 7.73.

Root Locus Diagram :



End of Solution

Q.7 (a) A second order system $G_1(s)$ as shown in figure,

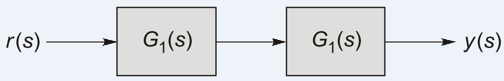


has no zeros and has unity DC gain.

The unit step response of $G_1(s)$ has a decay rate of 2.5 nepers/sec and an undamped natural frequency of $\sqrt{6}$ rad/sec.

(i) Compute the observable canonical state-space representation of $G_1(s)$ and obtain its state transition matrix using the Cayley-Hamilton approach.

(ii) Now, another identical $G_1(s)$ is placed in cascade with the earlier $G_1(s)$, as shown below.



Obtain the state-space representation of the overall cascaded system using the previously computed observable canonical representation of $G_1(s)$.

[20 marks : 2025]

Solution:

(i) The standard second-order transfer functions :

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\xi\omega_n = 2.5$$

$$\omega_n = \sqrt{6} \text{ rad/sec}$$

$$\xi = \frac{2.5}{\sqrt{6}}$$

Thus, C.E. $\quad s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 5s + 6$

$\because$ DC gain is 1.

Therefore, $\quad G_1(s) = \dfrac{6}{s^2 + 5s + 6}$

For the transfer function, $\quad G(s) = \dfrac{b_0}{s^2 + a_1 s + a_0}$

Observable canonical form is :

$$A = \begin{bmatrix} 0 & -a_0 \\ 1 & -a_1 \end{bmatrix}; \quad B = \begin{bmatrix} b_0 \\ 0 \end{bmatrix}, \quad C = [0 \quad 1]; \quad D = 0$$

Hence, $\quad A = \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix}; \quad B = \begin{bmatrix} 6 \\ 0 \end{bmatrix}; \quad C = [0 \quad 1], \quad D = 0$

State transition matrix using Cayley-Hamilton approach.

For A, its characteristics polynomial is

$$|sI - A| = s^2 + 5s + 6$$

So, $\quad A^2 + 5A + 6I = 0$

The state transition matlrix is :

$$\phi(t) = e^{At}$$

By C-H approach, since A^2 can be expressed in terms of A and I, we assume :

$$e^{At} = \alpha(t)I + \beta(t)A$$

To determine $\alpha(t)$, $\beta(t)$, we will use eigen values of A.

C.E. : $\quad s^2 + 5s + 6 = 0$

$$(s + 2)(s + 3) = 0$$

$$\lambda_1 = -2, \quad \lambda_2 = -3$$

So, $\quad e^{At} = \dfrac{(A - \lambda_2 I)e^{\lambda_1 t} - (A - \lambda_1 I)e^{\lambda_2 t}}{\lambda_1 - \lambda_2}$

Substitute the values,

$$e^{At} = \frac{(A + 3I)e^{-2t} - (A + 2I)e^{-3t}}{1}$$

Thus, $\quad \phi(t) = (A + 3I)e^{-2t} - (A + 2I)e^{-3t}$

$$\phi(t) = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix} e^{-2t} - \begin{bmatrix} 2 & -6 \\ 1 & -3 \end{bmatrix} e^{-3t}$$

$$\phi(t) = \begin{bmatrix} e^{-2t} - e^{-3t} & 6(e^{-3t} - e^{-2t}) \\ e^{-2t} - e^{-3t} & 3e^{-3t} - 2e^{-2t} \end{bmatrix} \text{(State transition matrix)}$$

(ii) Cascade of two $G_1(s)$ system :

The cascaded transfer function :

$$G_{overall}(s) = [G_1(s)]^2 = \left(\frac{6}{s^2 + 5s + 6}\right)^2$$

State Space Construction : If the first system is (A, B, C, D) is cascaded with an identical copy, then

$$\dot{x}_1 = Ax_1 + Br$$

$$u = Cx_1$$

$$\dot{x}_2 = Ax_2 + Bu = Ax_2 + B(Cx_1) = BCx_1 + Ax_2$$

$$y = Cx_2$$

Overall combined system states :

$$X = [x_1 \quad x_2]$$

So,
$$\dot{X} = \begin{bmatrix} A & 0 \\ BC & A \end{bmatrix} X + \begin{bmatrix} B \\ 0 \end{bmatrix} r$$

$$y = [0 \quad C]X$$

On substituting the values,

$$A = \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 0 \end{bmatrix}, \quad C = [0 \quad 1]$$

$$BC = \begin{bmatrix} 6 \\ 0 \end{bmatrix} [0 \quad 1] = \begin{bmatrix} 0 & 6 \\ 0 & 0 \end{bmatrix}$$

Thus, overall system becomes

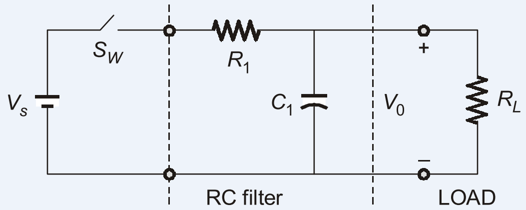
$$A_{overall} = \begin{bmatrix} 0 & -6 & 0 & 0 \\ 1 & -5 & 0 & 0 \\ 0 & 6 & 0 & -6 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$$B_{overall} = \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{overall} = [0 \quad 0 \quad 0 \quad 1]$$

End of Solution

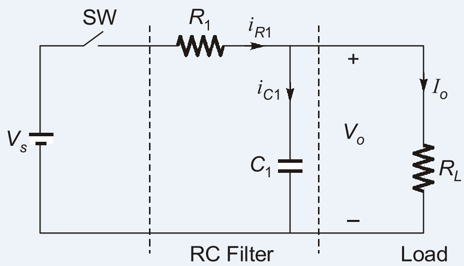
Q.7 (b) A buck converter with an RC filter is shown in the figure below with a load resistance R_L.



The switch S_W is operated with DT_S time ON and $(1 - D)T_S$ time OFF, cyclically with a time period of T_S. Draw the relevant waveforms and derive the expression for output voltage V_0 as a function of duty ratio 'D'. Assume the switching frequency to be high.

[20 marks : 2025]

Solution:

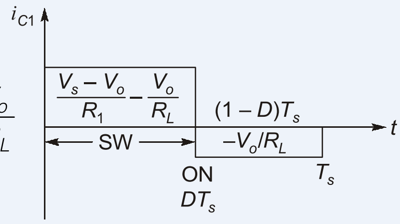


Assuming high switching frequency as per given data, we can neglect the ripple in output voltage V_o.

I. $0 \le t \le DT_s$ $\qquad T_{ON} = D.T_s$

SW → ON

$$i_{R1} = \frac{V_s - V_o}{R_1}, \quad I_o = \frac{V_o}{R_L}$$



∴ KCC : $\qquad i_{R1} = i_{C1} + I_o$

∴ $\qquad i_{C1} = i_{R1} - I_o$

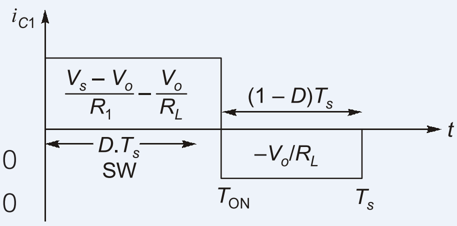
$$i_{C1} = \frac{V_s - V_o}{R_1} - \frac{V_o}{R_L}$$

II. $DT_S \le t \le T_s$:

SW → OFF : $\qquad i_{R1} = 0$

Capacitor will discharge and supply constant load curve :

∴ $\qquad i_{C1} = -\frac{V_o}{R_L}$



After reaching steady state,

$$(i_C)_{Avg} = 0$$

∴ $\qquad$ Area $= 0$

$$\left(\frac{V_s - V_o}{R_1} - \frac{V_o}{R_L}\right) D.T_s - \frac{V_o}{R_L}(1 - D)T_s = 0$$

$$\frac{V_o}{R_1}D + \frac{V_o}{R_L}D + \frac{V_o}{R_L} - \frac{V_o}{R_L}D = D \cdot \frac{V_s}{R_1}$$

$$V_o\left[\frac{D}{R_1} + \frac{1}{R_L}\right] = \frac{D \cdot V_s}{R_1}$$

$$V_o = \frac{DR_L}{R_1 + DR_L} \cdot V_s$$

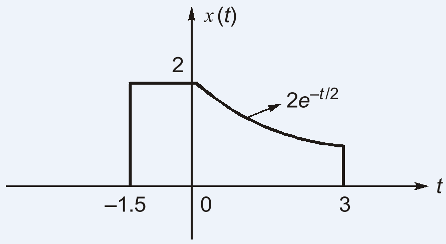
$$\frac{V_o}{V_s} = \frac{DR_L}{R_1 + DR_L} = \frac{D\left(\frac{R_L}{R_1}\right)}{1 + D\left(\frac{R_L}{R_1}\right)}$$

End of Solution

Q.7 (c) Consider a signal $x(t)$ shown in the following figure. Sketch and describe mathematically the signal $x(t)$.

(i) If time-compressed by a factor of 5.

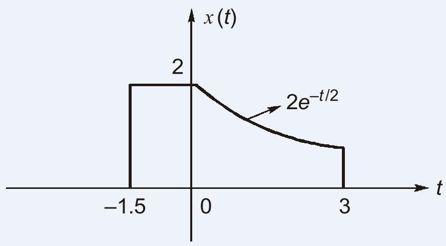
(ii) Repeat the problem for the same signal time-expanded by a factor of 3.



[10 + 10 marks : 2025]

Solution:

Given signal $x(t)$ is



We can write $x(t)$ in the following form

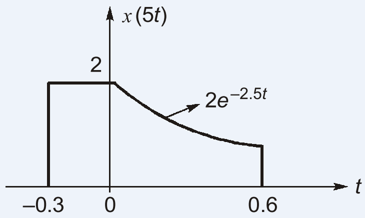
$$x(t) = \begin{cases} 2, & -1.5 < t < 0 \\ 2e^{-t/2}, & 0 < t < 3 \\ 0, & \text{otherwise} \end{cases} \quad \text{...(i)}$$

(i) Put $t = 5t$ in equation (i),

$$x(5t) = \begin{cases} 2, & -1.5 < 5t < 0 \\ 2e^{-5t/2}, & 0 < 5t < 3 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow \qquad x(5t) = \begin{cases} 2, & -0.3 < t < 0 \\ 2e^{-2.5t}, & 0 < t < 0.6 \\ 0, & \text{otherwise} \end{cases}$$

The waveform of time compressed form of $x(t)$ by '5' i.e. $x(5t)$ is shown below

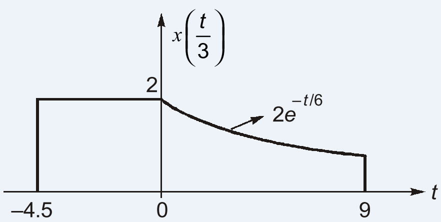


(ii) The time expanded form of $x(t)$ by '3' i.e. $x\left(\frac{t}{3}\right)$ can be obtained by putting $t = \frac{t}{3}$ in equation (i),

$$x\left(\frac{t}{3}\right) = \begin{cases} 2, & -1.5 < \frac{t}{3} < 0 \\ 2e^{-\left(\frac{t}{3}\right)\frac{1}{2}}, & 0 < \frac{t}{3} < 3 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 2, & -4.5 < t < 0 \\ 2e^{-t/6}, & -4.5 < t < 9 \\ 0, & \text{otherwise} \end{cases}$$

The waveform of $x\left(\frac{t}{3}\right)$ is shown below



End of Solution

Q.8 (a) Find the solution of the following second-order differential equation using the Laplace Transform Method: The differential equation is

$$\frac{d^2y(t)}{dt^2}+7\frac{dy(t)}{dt}+12y(t) = \frac{dx(t)}{dt}+x(t)$$

with initial conditions $y(0^-) = 2$ and $\dot{y}(0^-) = 1$ and $x(t) = e^{-5t}\,u(t)$ and $x(0^-) = 0$.

[20 marks : 2025]

Solution:

Given 2nd order differential equation

$$\frac{d^2y(t)}{dt^2}+7\frac{dy}{dt}+12y(t) = \frac{dx(t)}{dt}+x(t) \quad ...(1)$$

and given $y(0^-) = 2,\ \dot{y}(0^-) = 1$

$$x(t) = e^{-5t}u(t) \text{ and } x(0^-) = 0$$

On taking laplace transform of eqn. (1)

$$[s^2Y(s) - sy(0^-) - \dot{y}(0^-)] + 7[s^1Y(s) - y(0^-)] + 12Y(s) = [SX(s) - x(0^-)] + X(s)$$

On putting initial conditions

$$[s^2Y(s) - 2s - 1] + 7[sY(s) - 2] + 12Y(s) = [sX(s) - 0] + X(s)$$

$$Y(s)[s^2 + 7s + 12] - 2s - 1 - 14 = X(s)[s + 1]$$

$$Y(s)[s^2 + 7s + 12] - 2s - 15 = X(s)[s + 1]$$

$$Y(s) = \frac{(s+1)X(s)}{(s^2+7s+12)}+\frac{2s+15}{(s^2+7s+12)} \quad ...(2)$$

Given : $x(t) = e^{-5t}u(t)$

So, $$X(s) = \frac{1}{(s+5)} \quad ...(3)$$

Now, from eqn. (2)

$$Y(s) = \frac{(s+1)}{(s+5)(s^2+7s+12)}+\frac{2s+15}{(s^2+7s+12)}$$

$$= \frac{(s+1)+(2s+15)(s+5)}{(s+5)(s^2+7s+12)}$$

$$= \frac{(s+1)+2s^2+10s+15s+75}{(s+5)(s+3)(s+4)}$$

$$Y(s) = \frac{2s^2+26s+76}{(s+3)(s+4)(s+5)} \quad ...(4)$$

On doing partial fraction

$$\frac{2s^2+26s+76}{(s+3)(s+4)(s+5)} = \frac{A}{(s+3)}+\frac{B}{(s+4)}+\frac{C}{(s+5)}$$

$$(2s^2 + 26s + 76) = A(s+4)(s+5)+B(s+3)(s+5)+C(s+3)(s+4) \quad ...(5)$$

On putting $s = -3$ in eqn. (5)

$$2(-3)^2 + 26(-3) + 76 = A(-3+4)(-3+5) + 0 + 0$$
$$A = 8$$

On putting $s = -4$ in eqn. (5)

$$2(-4)^2 + 26(-4) + 76 = 0 + B(-4+3)(-4+5)$$
$$B = -4$$

On putting $s = -5$ in eqn. (5)

$$2(-5)^2 + 26(-5) + 76 = 0 + 0 + C(-5+3)(-5+4)$$
$$C = -2$$

From eqn. (4) and (5)

$$Y(s) = \frac{8}{(s+3)} - \frac{4}{(s+4)} - \frac{2}{(s+5)}$$

On taking inverse laplace transform

$$y(t) = 8e^{-3t}u(t) - 4e^{-4t}u(t) - 2e^{-5t}u(t)$$
$$y(t) = (8e^{-3t} - 4e^{-4t} - 2e^{-5t})u(t)$$

End of Solution

Q.8 (b) The open-loop transfer function of a unity feedback system is given as:

$$G(s) = \frac{10}{(s-1)(s+5)}$$

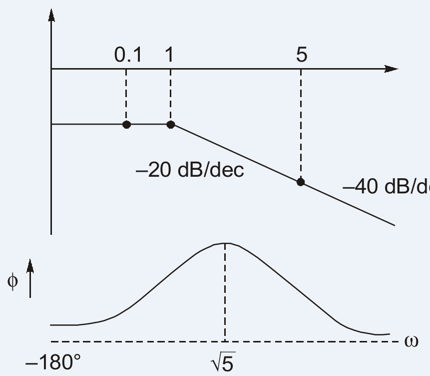
Sketch the Bode plot for the system and calculate Gain and Phase margins.

[20 marks : 2025]

Solution:

$$GH = \frac{10}{(s-1)(s+5)} = \frac{2}{(s-1)\left(\frac{1}{5}s-1\right)}$$

$$GH = \frac{10}{(s-1)(s+5)}$$



$W : 0 \rightarrow \infty$

$M : 2 \rightarrow 0$

$\phi : -180° \rightarrow -180°$

(i) At ω_{pc} $\Rightarrow$ $\phi = -180$

$\therefore$ $\omega_{pc} = 0$

$M_{\text{at}\,\omega_{pc}} = 2$

$\therefore$ $MM = \dfrac{1}{M} = 0.5 = -6 \text{ dB}$

(ii) At ω_{pc} $\Rightarrow$ $M = 1$

$\therefore$ $\omega_{gc} = 1.618$

$$PM = 180 + [-180° + \tan^{-1}(\omega_{gc}) - \tan^{-1}(\omega_{gc}/5)] \quad (\text{at } \omega = \omega_{gc})$$

$$= +40°$$

End of Solution

Q.8 (c) Design a UJT triggering circuit for a 220 V, 50 Hz AC source-fed single-phase half-controlled rectifier using BT 151 - 500 R SCR and 2N2646 UJT having the following parameters:

2N2646 UJT : $\eta = 0.65$, $R_{BB} = 7\ \text{k}\Omega$, $I_P = 5\ \mu\text{A}$,
$V_V = 3$ V, $I_V = 4$ mA

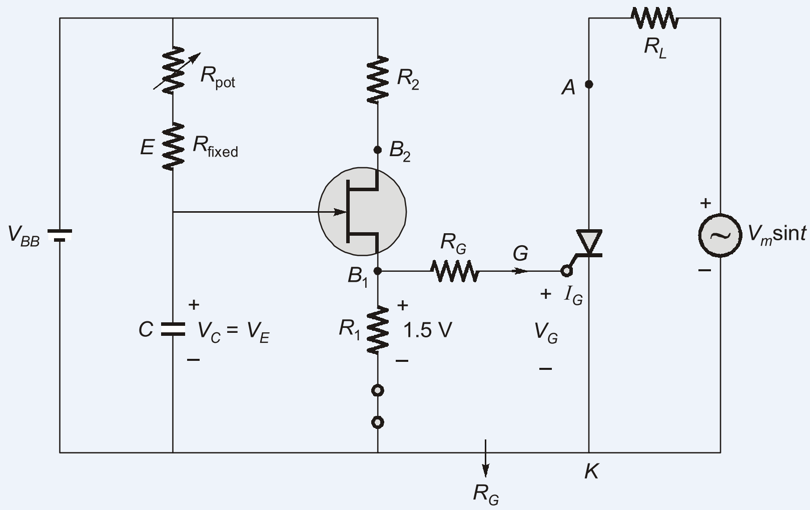
BT 151 - 500 R SCR : $V_{GT} = 0.8$ V (typical), 1.5 V (max)
$I_{GT} = 5$ mA (typical), 15 mA
VDRM = 500 V.

Assume the triggering circuit is fed from a 24 V DC supply.
Take $V_{BB} = 20$ V for design and a pulse width of the triggering pulse of 30 μs.
Draw relevant circuits and show the component values with power ratings.

[20 marks : 2025]

Solution:

UJT working as relaxation oscillator



$$R_G = \frac{1.5\text{ V}}{I_{GT}} = \frac{1.5}{15.10}$$

$$R_h = 100\ \Omega$$

Peak point voltage, $V_p = \eta V_{BB} + V_D$

$n \rightarrow$ Intrinsic stand off ratio

When UJT $\rightarrow$ OFF

Capacitor charger,

$\therefore$ $V_C = V_{BB}\left(1 - e^{-t/RC}\right)$

Where $R = R_{pot} + R_{find}$

UJT : $\eta = 0.65$,

$V_D = 0.7$ V,

$V_V = 3$ V

SCR :

Gate trigger current (For worst case) = 15 mA

$V_{BB} = 20$ V DC

$$\text{Half cycle at 50 Hz} = \frac{1}{2}\left(\frac{1}{T}\right) = 10 \text{ ms}$$

Gate pulse width required, $t_p = 30\ \mu s$

$t_p = 30\ \mu s$

$$V_P = \eta V_{BB} + V_D$$
$$= (0.65 \times 20) + (0.7)$$
$$V_P = 13.7 \text{ V}$$

$\therefore$ UJT fires once capacitor voltage reaches (13.7 V)

When UJT - OFF

Capacitor charges

$\therefore$ $$V_C = V_{BB}\left(1 - e^{-t/RC}\right)$$

When $$V_C = V_P, \qquad \text{UJT} \rightarrow \text{ON}$$

$$V_{BB}\left(1 - e^{-t/RC}\right) = V_P$$

$$V_{BB} - V_{BB} \cdot e^{-t/RC} = V_P$$

$$V_{BB} \cdot e^{-t/RC} = V_{BB} - V_P$$

$$e^{-t/RC} = \frac{V_{BB} - V_P}{V_{BB}}$$

$$e^{t/RC} = \frac{V_{BB}}{V_{BB} - V_P}$$

$$t = RC \cdot \ln\left(\frac{V_{BB}}{V_{BB} - V_P}\right) \qquad \text{...(i)}$$

$\therefore$ Firing depends on product of RC

When UJT - ON,

Capacitor discharger gate pulse across R_1 until value voltage V_V.

$\therefore$ $$\Delta V = V_P - V_V = 13.7 - 3 = 10.7 \text{ V}$$

$$\Delta Q = C \cdot \Delta V_C$$

$$I_{GT} \cdot t_p = C \cdot \Delta V_C \qquad (I_{GT} \geq 15 \text{ mA})$$

$$C = \frac{I_{GT} \cdot t_p}{\Delta V} = \frac{(15.10^{-3}) \times (30.10^{-6})}{10.7}$$

$$C = 0.042\ \mu\text{F}$$

$$C > 0.042\ \mu\text{F} \qquad (\because I_{GT} \geq 15 \text{ mA})$$

Let $$C = 0.047$$

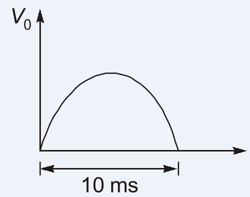
$$I_{GT} = \frac{C \cdot \Delta V_C}{t_p} = \frac{(0.047\ \mu\text{F}) \times (10.7)}{30.10^{-6}}$$

$(I_{GT} = 16.8 \text{ mA}) > 15$ mA

From equation (i),

$$t = RC \cdot \ln\left(\frac{V_{BB}}{V_{BB} - V_P}\right) = RC \cdot \ln\left(\frac{20}{20 - 13.7}\right)$$

$$t = 1.154 \text{ RC}$$



For max firing angle ($\alpha = \pi$),

$$t = 10 \text{ ms}$$

$$\therefore \quad t_{max} = 10 \text{ ms} = 1.154\, R_{max} \cdot C$$

$$R_{max} = \frac{10 \text{ ms}}{1.154 \times RC} = \frac{10 \text{ms}}{1.54 \times 0.047 \mu\text{F}} = 184 \text{ k}\Omega$$

$\therefore$ Let us use,

$$R_{pot} = 200 \text{ k}\Omega$$

$$R_{min} = R_{fixed} = 10 \text{ k}\Omega$$

$$t = 1.154 \text{ RC}$$

$$E_{min} = 1.154 \times R_{min} \cdot C$$

$$= 1.154 \times 10.10^3 \times (0.047 \times 10^{-6})$$

$$= 0.542 \text{ ms}$$

$$\alpha_{min} = wt^{o}{}_{min} = 2\pi \cdot 50 \times (0.542 \times 10^{-3}) \times \frac{180}{\pi} = 9.75^\circ$$

$$\alpha_{max} = 180^\circ$$

$\therefore$ Range of firing angle ($9.75^\circ \le \alpha \le 180^\circ$)

End of Solution

■■■■