MADE EASY

ESE 2025

Main Exam Detailed Solutions

Electrical Engineering

PAPER-I

EXAM DATE : 10-08-2025 | 09:00 AM to 12:00 PM

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ANALYSIS

Electrical Engineering Paper-I
ESE 2025 Main Examination

Sl.	Subjects	Marks
1.	Electric Circuits	84
2.	Electromagnetic Fields	32
3.	Electrical Materials	52
4.	Engineering Mathematics	76
5.	Basic Electronics Engineering	84
6.	Computer Fundamental	60
7.	Electrical and Electronic Measurements	92
		Total 480

Scroll down for detailed solutions

SECTION : A

Q.1 (a) (i) If A is an $n \times n$ diagonalizable matrix and $A^2 = A$, then show that each eigen value of A is 0 or 1.

(ii) Show that all the eigen values of a Hermitian matrix are real.

[6 + 6 = 12 marks : 2025]

Solution:

(i) Let $V \neq 0$ be an eigen vector of A with eigen value λ.

Then, $A^2V = A(AV) = A(\lambda V) = \lambda AV = \lambda^2 V$

But $A^2 = A$

So, $A^2V = AV = \lambda V$

Hence, $\lambda^2 V = \lambda V$

$(\lambda^2 - \lambda)V = 0$

Since, $V \neq 0$, we have $\lambda^2 - \lambda = 0$, i.e., $\lambda(\lambda - 1) = 0$.

Thus, $\lambda = 0$ or $\lambda = 1$.

(ii) Given that matrix A is Hermitian if $A^\theta = A$, i.e.,

where $A^\theta = (\bar{A}')$ or $(\bar{A})'$

Also, $(\lambda A)^\theta = \bar{\lambda} A^\theta = \bar{\lambda} A^\theta$ and $(AB)^\theta = B^\theta A^\theta$

If λ is characteristic root of matrix A then

$$AX = \lambda X \quad ...(1)$$

$\therefore$ $(AX)^\theta = (\lambda X)^\theta$

or $X^\theta A^\theta = \lambda X^\theta$

But A is Hermitian.

$\therefore$ $A^\theta = A$

$X^\theta A = \bar{\lambda} X^\theta$

$\therefore$ $X^\theta AX = \bar{\lambda} X^\theta X \quad ...(2)$

Again from (1), $IX^\theta AX = X^\theta \lambda X = \lambda X^\theta X$

Hence, from (2) and (3), we conclude that $\bar{\lambda} = \lambda$ showing that λ is real.

End of Solution

Q.1 (b) The magnetic field strength in a material is 9×10^5 A/m and its magnetic susceptibility is 0.75×10^{-5}.

(i) Find the flux density and the magnetization in the material.

(ii) Also find its relative permeability.

[12 marks : 2025]

Solution:

Given : Magnetic field strength $\vec{H} = 9 \times 10^5$ A/m

and Magnetic susceptibility, $\chi_m = 0.75 \times 10^{-5}$

(i) Flux density, $\vec{B} = \mu_o \mu_r \vec{H}$

$= \mu_o(1+\chi_m)\vec{H}$

$= 4\pi \times 10^{-7}(1 + 0.75 \times 10^{-5})9 \times 10^5$

$= 1.1309 \text{ Wb/m}^2$

Magnetization, $\vec{M} = \chi_m \vec{H}$

$= 0.75 \times 10^{-5} \times 9 \times 10^5 \text{ A/m}$

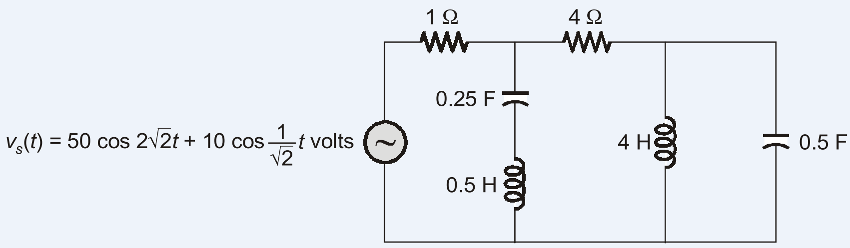
$\vec{M} = 6.75 \text{ A/m}$

(ii) Relative Permeability, $\mu_r = 1 + \chi_m = 1 + 0.75 \times 10^{-5}$

$\mu_r = 1.0000075$

End of Solution

Q.1 (c) For the circuit given in the figure below, find the current through 4 Ω resistor and the total active power delivered by the source. The source voltage $v_s(t) = 50\cos 2\sqrt{2}t + 10\cos\frac{1}{\sqrt{2}}t$ volts :



[12 marks : 2025]

Solution:

Given : $V_s = 50\cos 2\sqrt{2}t + 10\cos\frac{1}{\sqrt{2}}t \text{ V}$

Since circuit is linear and has two source frequencies, we can use superposition theorem.

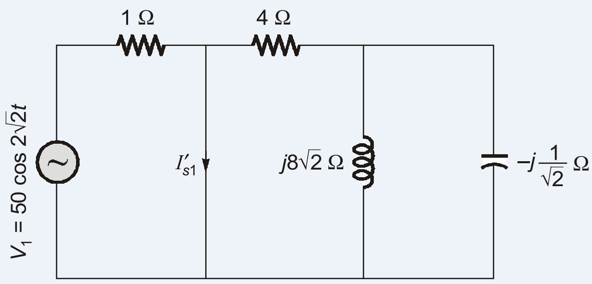
Taking $V_1 = 50\cos 2\sqrt{2}t$ Volt, $\omega_1 = 2\sqrt{2}$ rad/sec

Series L-C branch of circuit where $C = 0.25$ F and $L = 0.5$ H having resonance frequency

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25 \times 0.5}} = 2\sqrt{2} \text{ rad/sec}$$

that is equal to source frequency.

So, this branch will be short circuited.



No current flows through 4 Ω resistor.

$$I'_{4\,\Omega} = 0 \qquad \text{...(1)}$$

Source current,
$$I'_s = \frac{50}{\sqrt{2}\times 1} = \frac{50}{\sqrt{2}}\ \text{A}$$

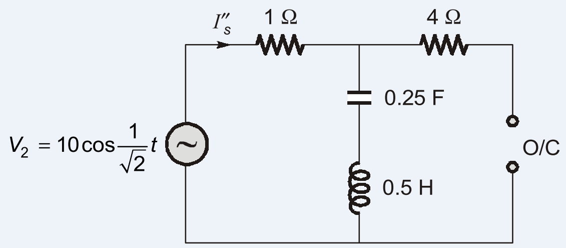
Source power,
$$P_{s1} = (V_1)_{rms}.I'_s.\cos\phi_s$$
$$= \frac{50}{\sqrt{2}}\times\frac{50}{\sqrt{2}} = 1200\ \text{W} \qquad \text{...(2)}$$

Taking source voltage,
$$V_2 = 10\cos\frac{1}{\sqrt{2}}t\ \text{Volt}$$
$$\omega_2 = \frac{1}{\sqrt{2}}\ \text{rad/sec}$$

Here, parallel branch of L-C in the circuit, having L = 4 H and C = 0.5 F having resonance frequency

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times 0.5}} = \frac{1}{\sqrt{2}}\ \text{rad/sec}$$

that is equal to source frequency. So, parallel branches are open circuited.



No current flows through 4 Ω resistor.

$$I''_{4\,\Omega} = 0$$

Source current,
$$I''_s = \frac{10}{\sqrt{2}(Z_{eq})}$$
$$Z_{eq} = R + j(X_L - X_C) = 1 + j\left[\omega L - \frac{1}{\omega C}\right]$$
$$= 1 + j\left[\frac{1}{\sqrt{2}}\times 0.5 - \frac{1}{\frac{1}{\sqrt{2}}\times 0.25}\right]$$
$$= 1 + j\left[\frac{1}{2\sqrt{2}} - 4\sqrt{2}\right]$$
$$= (1 - j5.303)\ \Omega$$
$$I''_s = \frac{10}{\sqrt{2}[1 - j5.303]} = 1.310\angle 79.31°\ \text{A}$$

Power delivered by source,

$$P''_s = (V_2)_{rms} \cdot I''_s \cos\phi_s$$

$$= \frac{10}{\sqrt{2}} \times 1.31\cos 79.31° = 1.718 \text{ W}$$

So, using super position theorem, current through 4 Ω resistor

$$I_{4\,\Omega} = I'_{4\,\Omega} + I''_{4\,\Omega} = 0 \text{ A}$$

Power delivered by source

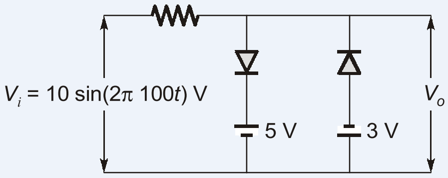
$$P_S = P'_S + P''_S = 1250 + 1.718 = 1251.718 \text{ W}$$

End of Solution

Q.1 (d) Consider the circuit shown in the figure below. Assuming that the diodes are ideal, sketch the following waveforms :

(i) Two cycles of V_i (input) and V_o (output)

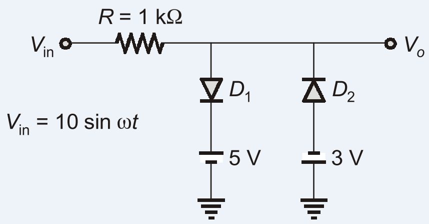
(ii) Transfer characteristics of the circuit, i.e., V_o versus V_i.



[12 marks : 2025]

Solution:

(i)

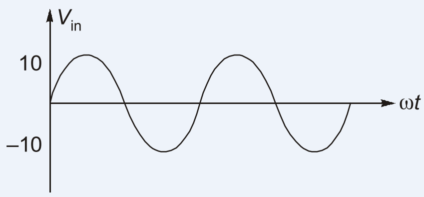


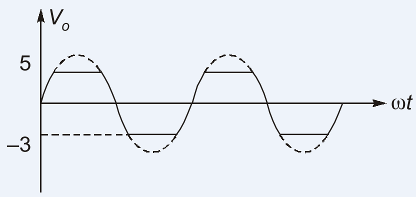
The above circuit is two independent level clipper.

$V_{in} < -3$ V, D_1 OFF, D_2 ON, $V_o = -3$ V

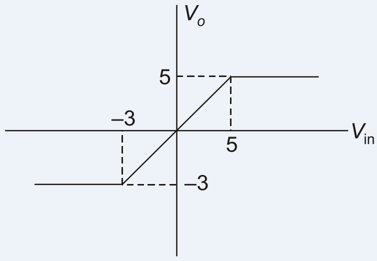
$-3V < V_{in} < 5$ V, D_1 OFF, D_2 OFF, $V_o = V_{in}$

$V_{in} > 5$ V, D_1 ON, D_2 OFF, $V_o = 5$ V





(ii) Transfer characteristics



End of Solution

Q.1 (e) Draw the circuit diagram and explain the process of measurement of low resistance values using Kelvin's double bridge. Derive the expression and mention two conditions which ensure that the unknown resistance can be easily measured in terms of the standard resistance.

[12 marks : 2025]

Solution:

Measurement of Low Resistance using "Kelvin Double Bridge" Method : The Kelvin bridge is a modification of the wheat stone bridge and provides greatly increased accuracy in measurement of low value resistances.

The Kelvin double bridge incorporates the idea of a second set of ratio arms hence the name double bridge and the use of four terminal resistors for the low resistance arms. Figure below shows schematic diagram of the Kelvin double bridge.

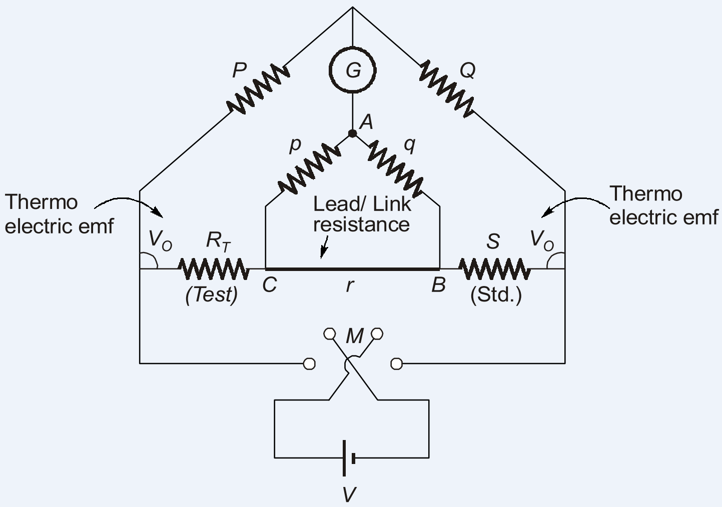
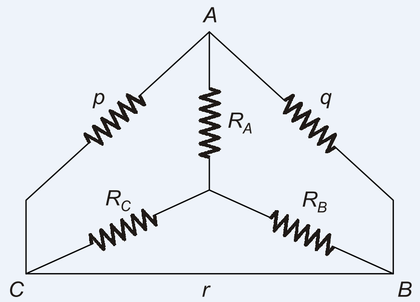


Fig. : Practical Kelvin Double bridge

Here, $\left\{\begin{array}{r l} P,\ Q \rightarrow & \text{Upper/outer ratio arms} \\ p,\ q \rightarrow & \text{Lower/Inner ratio arms} \\ M \rightarrow & \text{Reversable switch} \\ r \rightarrow & \text{Lead/Link resistance} \\ R_T \rightarrow & \text{Test resistance (to be measured)} \\ S \rightarrow & \text{Standard resistance} \end{array}\right\}$

Converting the Δ into *Y* configuration: (*ABC* forms a Δ configuration)



The bridge is simplified as shown below in figure.

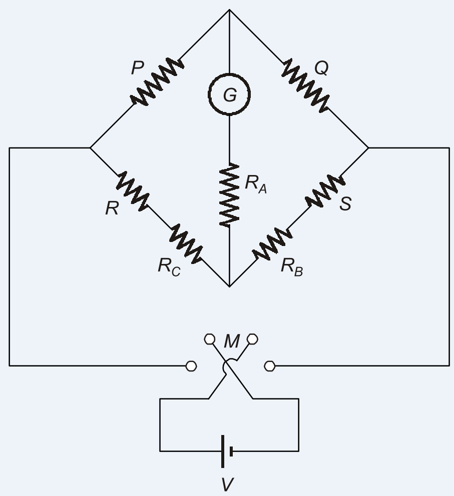


Fig. : Simplified Kelvin Double bridge

Under balance conditions there is no current through the galvanometer (*G*). So, product of opposite arm resistance is equal.

∴ $$P(S + R_B) = Q(R + R_C)$$

or, $$P\left[S + \frac{qr}{p+q+r}\right] = Q\left[R + \frac{pr}{p+q+r}\right] \text{ (Using results for } R_B \text{ and } R_C)$$

or, $$R = \frac{P}{Q}\cdot S + \left(\frac{qr}{p+q+r}\right)\left[\frac{P}{Q} - \frac{p}{q}\right]$$

The value of test resistance, *R* obtained above include lead resistance also. Condition to eliminate lead/Contact resistance :

If $\frac{P}{Q} = \frac{p}{q}$ then, $R = \frac{P}{Q} \cdot S$

Thus, contact resistance is eliminated. By changing the reversible switch, *M*, thermoelectric emfs are eliminated.

Kelvin Double bridge is used to eliminate lead resistance effect. It is suitable to measure low resistance upto micro ohm (μΩ). It is used to measure the resistance of winding coils of electrical motors/Generators, transformers and Earth conductor resistance.

End of Solution

Q.2 (a) (i) What are Lissajous patterns? Explain. Also elaborate what patterns appear on the cathode ray oscilloscope screen, when voltages of different frequencies and phase differences are applied in the horizontal and vertical plates of the scope. Take two examples for each of the above two cases. Explain how the unknown signal frequency is measured accurately with the help of observing the patterns.

(ii) Explain the principle of operation of piezoelectric transducer. Write its advantages, disadvantages and some applications.

[12 + 8 marks : 2025]

Solution:

(i) **Lissajous Pattern :** When a sinusoidal input voltage is applied both to the vertical and horizontal deflecting plate then, the waveform pattern appearing on the screen of CRT are called "***Lissajous Patterns***". Lissajous pattern is used to find the phase angle between two input signals & their frequency ratio. Lissajous pattern can be used for the accurate measurement of frequency. The signal, whose frequency is to be measured is applied to the *y*-plate. At any point of time the beam on the screen is the vector sum of voltages applied to the horizontal and the vertical deflecting plate.

Let $V_x = V_m \sin \omega t$ and $V_y = V_m \sin(\omega t + \phi)$, i.e., both the plates are applied with a sinusoidal input signal having same frequency and a phase angle ϕ between them. The Lissajous pattern is shown below in figure for a phase shift of $\phi°$.

It is clear from the figure that the Lissajous pattern is an ellipse.

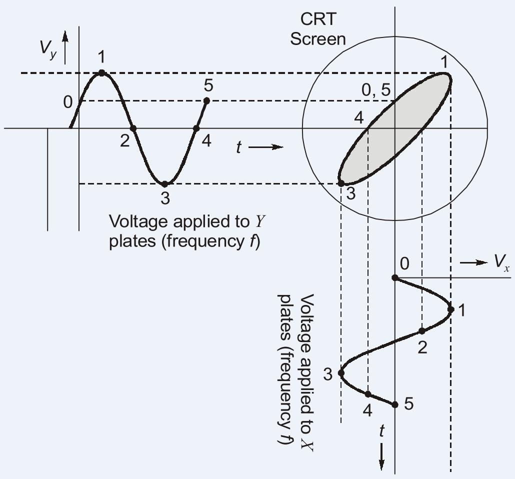


Fig. : Lissajous pattern with two equal voltages of same frequency and phase shift of ϕ

Case-I: When $\phi = 0°$

$$V_x = V_m \sin \omega t \text{ and } V_y = V_m \sin \omega t$$

Points	$\lvert V_x \rvert$	$\lvert V_y \rvert$	$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$	$\sqrt{x^2 + y^2}$
0	0	0	0°	0
1	V_m	V_m	45°	$\sqrt{2}\,V_m$
2	0	0	0°	0
3	$-V_m$	$-V_m$	225°	$\sqrt{2}\,V_m$
4	0	0	0°	0

It is clear from the below figure, if the frequencies of the two input signals are same having zero phase difference between them, the pattern appearing on the screen is a straight line.

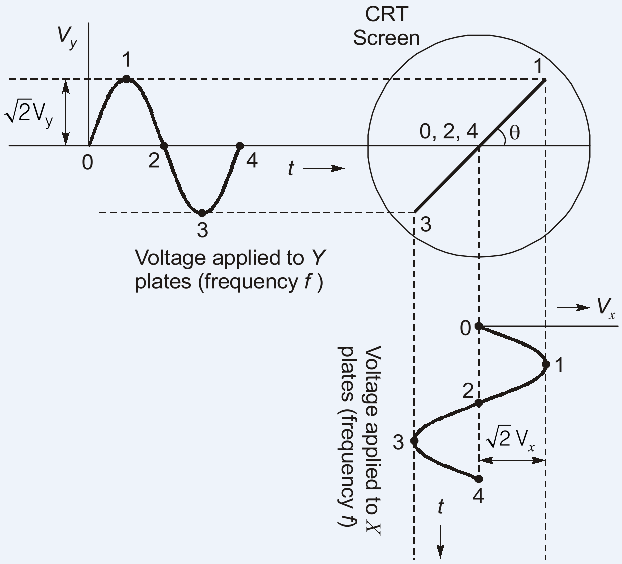


***Fig. :** Lissajous pattern with equal frequency voltages and zero phase shift*

Case-II: $\phi = 90°$

$$V_x = V_m \sin \omega t,\ V_y = V_m \sin(\omega t + 90°) = V_m \cos \omega t$$

Points	$\lvert V_x \rvert$	$\lvert V_y \rvert$	$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$	$\sqrt{x^2 + y^2}$
0	0	V_m	90°	V_m
1	V_m	0	0°	V_m
2	0	$-V_m$	270°	V_m
3	$-V_m$	0	180°	V_m
4	0	V_m	90°	V_m

The Lissajous pattern is shown below in figure.

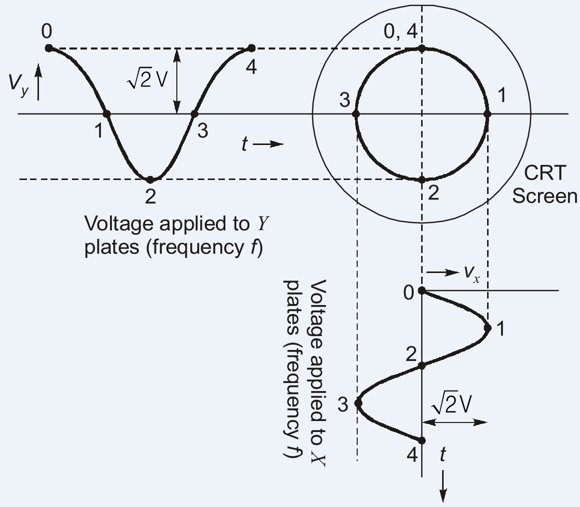


***Fig. :** Lissajous pattern with equal voltages of equal frequency and phase shift of* 90°

Special Case : When the signals applied to the plates have different frequencies. The waveform pattern appearing on the screen is shown below in figure.

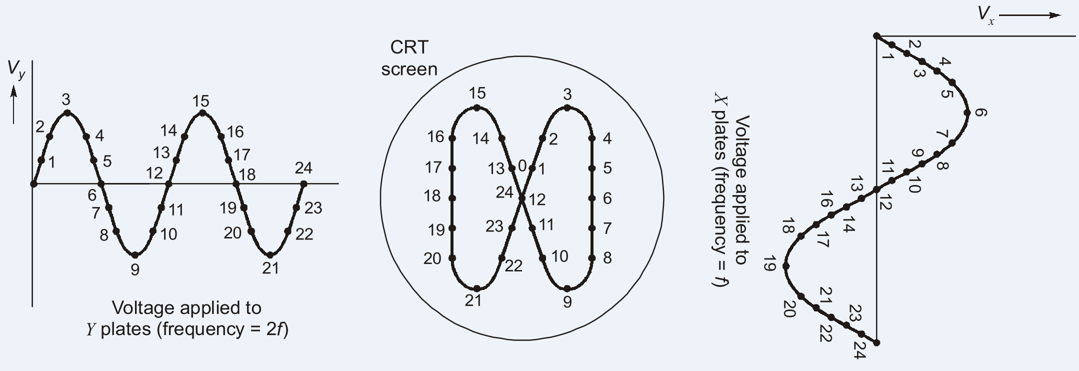


***Fig. :** Lissajous pattern with frequency ratio 2 : 1*

$$V_x = V_m \sin \omega_x t \text{ and } V_y = V_m \sin \omega_y t$$

then, the frequency ratio of the two signals is given by

$$\frac{\omega_y}{\omega_x} = \frac{f_y}{f_x} = \frac{\text{Number of horizontal tangencies}}{\text{Number of vertical tangencies}}$$

(ii) **Piezo-Electric Transducers :** When a varying potential applied to the proper axis of a crystal, there is a change in dimension of the crystal which is known as **"Piezo-electric effect"**. The reverse effect is also true, i.e., if the dimensions of the crystal are changed by the application of a mechanical force, an electric potential appears across certain surface of the crystal due to the displacement of charges. The elements which exhibit piezo-electric qualities are called as **"Electro-resistive elements"**.

Below figure describes the phenomena of piezo-electric effect on a piezo-electric crystal with the application of a force, *F*.

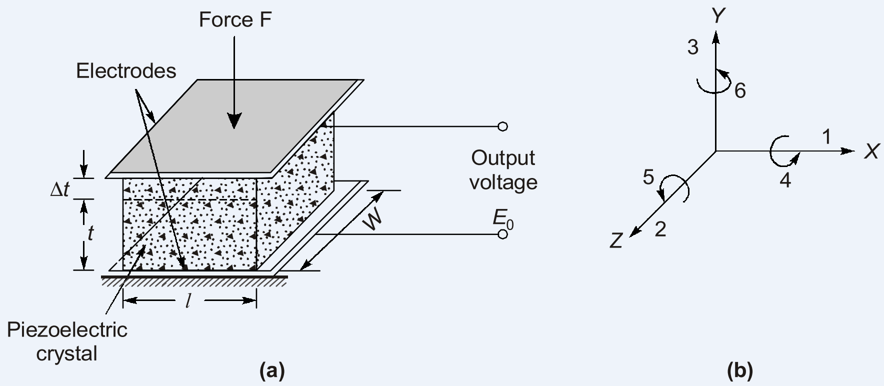


Fig. : Piezo-electric crystal for measuring applied force

Let the applied force on the piezo-electric crystal be *F*.

Q = charge
t = thickness of the crystal
E = electric field intensity
P = pressure applied
g = voltage sensitivity of the crystal

$$Q \propto F$$
$$Q = dF$$

Charge sensitivity, $d = \frac{Q}{F}$ (C/N)

Voltage sensitivity, $g = \frac{E}{P}$ (V-m/N) $\left(P = \frac{F}{A} = \text{pressure or stress in N/mm}^2\right)$

$$g = \frac{V_o}{tP} \quad \left(E = \frac{V_o}{t}\right)$$

and output voltage of the piezoelectric crystal, $\boxed{V_o = gtP}$ Volts

Also, charge sensitivity, $\boxed{d = \varepsilon_0 \varepsilon_r g}$

End of Solution

Q.2 (b) (i) Write a program in C language to print the following full pyramid of numbers :

```
        1
      2 3 2
    3 4 5 4 3
  4 5 6 7 6 5 4
5 6 7 8 9 8 7 6 5
```

(ii) Minimize the four-variable logic function using K-map

$$f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 5, 7, 8, 9, 11, 14)$$

[10 + 10 marks : 2025]

Solution:

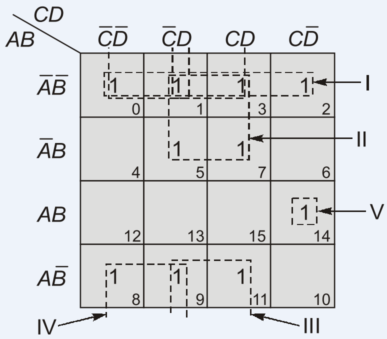
(i)

```
#include <stdio.h>
int main ()
int i, j, k;
int rows=5;
for (i=1; i<=rows; i++)
{
for (j=1; j<rows; j++)
{
printf(" ");
}
for (k=i; k < 2*i; k++)
{
print f("%d", k);
}
for (k=2*i – 2; k > =i; k– –)
{
printf ("%d", k);
}
printf("\n");
}
return 0;
}
```

(ii) Given four variable function

$$f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 5, 7, 8, 9, 11, 14)$$

Drawing K-map for four variables



Group-I : Group of 4 elements at (0, 1, 2, 3)

$$f_1 = \bar{A}\bar{B}$$

Group-II : Group of 4 elements at (1, 3, 5, 7)

$$f_2 = \bar{A}D$$

Group-III : Group of 4 elements at (1, 3, 9, 11)

$$f_3 = \bar{B}D$$

Group-IV : Group of 4 elements at (0, 1, 8, 9)

$$f_4 = \bar{B}\bar{C}$$

Group-V : Single element at (14)

$$f_5 = ABC\bar{D}$$

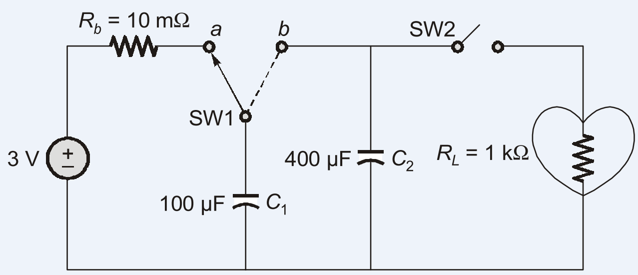
Minimized Function :

$$f = f_1 + f_2 + f_3 + f_4 + f_5$$

$$f = \bar{A}\bar{B} + \bar{A}D + \bar{B}D + \bar{B}\bar{C} + ABC\bar{D}$$

End of Solution

Q.2 (c) A cardiac pacemaker is represented by the circuit given in the figure below. The battery internal resistance R_b is 10 mΩ, whereas the heart equivalent resistance is 1 kΩ. The switch 1 (SW1) is at position *a* initially for a long time when switch 2 (SW2) is OFF. Then SW1 is moved to position *b* at $t = 0$ and SW2 is ON simultaneously for next $t = 10$ ms. At $t = 10$ ms, SW1 moves to position *a* and SW2 is OFF for another 10 ms. Find the voltages of the capacitors C_1 and C_2 at $t = 0$, 10 ms and 20 ms, and sketch the capacitor voltages upto 20 ms. Also calculate the energy dissipated in R_L during the interval 0 to 10 ms when SW2 was ON :



[20 marks : 2025]

Solution:

Battery, $V_s = 3$ Volt

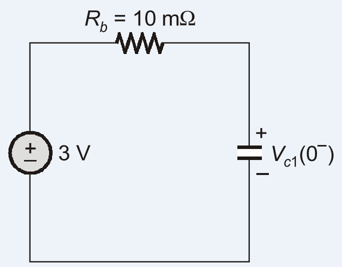
$R_b = 10\ \text{m}\Omega$ (battery internal resistance)

Heat equivalent resistance, $R_L = 1\ \text{k}\Omega$

$C_1 = 100\ \mu\text{F}$ and $C_2 = 400\ \mu\text{F}$

At $t = 0$, SW1 $\rightarrow b$ and SW2 is ON for $0 \le t \le 10$ msec

At $t = 0^-$, SW1 $\rightarrow a$ (just before switching)



$V_{C1} = 3$ V (charged to battery)

$V_{C2} = 0$ V (isolated)

At $t = 0^+$, C_1 and C_2 are connected together and total charge is conserved.

Charged stored by C_1, $Q = C_1V_1 = 100 \times 3 = 300\ \mu\text{C}$

Since charge is conserved, $Q_1 = Q_{2+}$

$$C_1V_1 = C_1V + C_2V$$

(V is the common voltage upto which charge transfer takes place)

$$\frac{300}{(C_1 + C_2)} = V$$

$$V = \frac{300}{500} = 0.6 \text{ Volt}$$

So, both capacitors $V_{c1}(0^+) = V_{c2}(0^+) = 0.6$ Volt ...(1)

For $0 \le t \le 10$ msec (SW2 is ON), discharge of Ceq through R_L.

Voltage (both capacitor), $V(t) = 0.6e^{-t/\tau}$

($\tau = R.C_{eq} = 1 \times 10^3 \times 500 \times 10^{-6} = 0.5$ sec)

$= 0.6e^{-t/0.5} = 0.6e^{-2t}$ Volt

At $t = 10$ msec, $V(t = 10 \text{ msec}) = 0.6e^{-2(10 \times 10^{-3})}$

$= 0.5881$ Volt

So, at $t = 10$ msec, $V_{c1} = V_{c2} = 0.5881$ Volt ...(2)

Now for $10 \text{ ms} \le t \le 20$ msec, SW1 $\rightarrow a$, SW2 – OFF

SW2 is OFF indicates, V_{c2} voltage stays at value $t = 10$ msec.

$V_{c2}(t) = 0.5881$ volt for $10 \text{ msec} \le t \le 20$ msec

SW1 $\rightarrow a$ reconnects C_1 to 3 V source.

$\tau_1 = R_bC_1 \rightarrow$ charging time constant

$= 0.01\ \Omega \times 100 \times 10^{-6} = 1\ \mu\text{sec}$

This is very fast charging compared with msec, so at $t = 20$ msec, C_1 is full charged to 3 V.

At t = 20 msec, $V_{c1} = 3$ V

$V_{c2} = 0.5881$ Volt

So $t = 0^-$: $V_{c1} = 3$ V, $V_{c2} = 0$ V

$t = 0^+$: $V_{c1} = V_{c2} = 0.6$ V

t = 10 msec : $V_{c1} = V_{c2} = 0.5881$ V

t = 20 msec : $V_{c1} = 3$ V, $V_{c2} = 0.5881$ V

Energy dissipated in R_L during $0 \le t \le 10$ msec is equal to loss at stored energy in the capacitors over the interval.

Energy stored in capaacitor at $t = 0^+$

$$E_0 = \frac{1}{2}C_{eq}[V(0^+)]^2 - \frac{1}{2}[500\times10^{-6}][0.6]^2 = 90 \text{ μJ}$$

Energy at t = 10 msec,

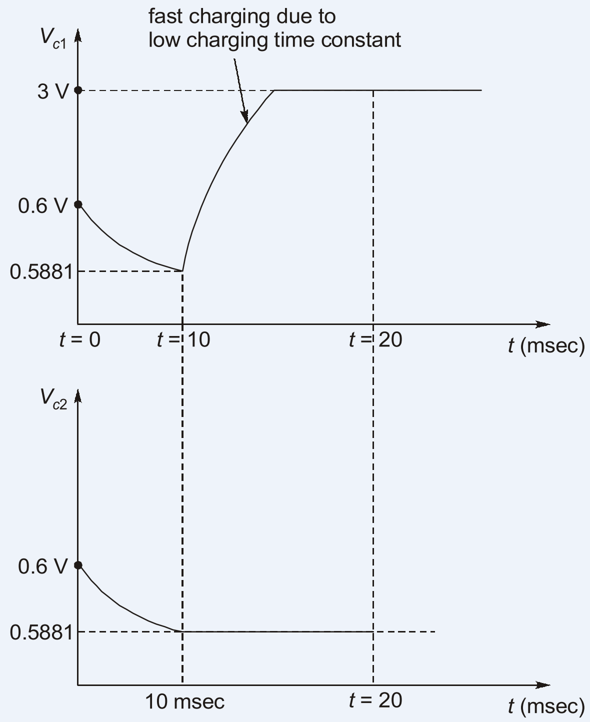
$$E_{10} = \frac{1}{2}C_{eq}[V(10)]^2 = \frac{1}{2}\times(500\times10^{-6})(0.5881)^2$$

$$= 86.471 \text{ μJ}$$

Energy dissipated in R_L during 0-10 msec

$$E_{RL} = E_0 - E_{10} = 90 - 86.471 = 3.529 \text{ μJ}$$

Waveforms of V_{c1} and V_{c2} Vs. Time :



End of Solution

Q.3 (a) (i) Find the singular solution of the partial differential equation

$$6yz - 6pxy - 3qy^2 + pq = 0$$

(ii) Derive the formula by Newton-Raphson method to find next approximation of the root of the equation $f(x) = 0$, if x_0 is an initial approximation. Also perform three iterations to find a root of the equation $x^4 - x - 10 = 0$ which is near to $x = 2$, correct to three decimal places.

[10 + 10 marks : 2025]

Solution:

(i) Let $p = z_x,\ q = z_y$

The PDE is $F(x, y, z, p, q) = 6yz - 6xyp - 3y^2q + pq = 0$

$$F_p = -6xy + q,\ F_q = -3y^2 + p,\ -F_x - pF_z = 0$$

Hence, along characteristics $dp = 0$, so $p = a$.

$$\frac{dy}{F_a} = \frac{dq}{-F_y - qF_z}$$

One obtains (after substitution $p = a$)

$$\frac{dy}{-3y^2 + a} = \frac{dq}{-q(3y^2 - a)y}$$

$$\frac{dq}{q} = \frac{dy}{y}$$

So, $q = Ky$ with constant K.

Substituting $p = a$, $q = Ky$ into $F = 0$ and dividing by y gives

$$6z - 6ax - 3Ky^2 + aK = 0$$

So, $$z = ax + \frac{K}{z}y^2 - \frac{aK}{6}$$

Thus, the general integral (two parameter family) is :

$$z(x, y) = ax + \frac{K}{2}y^2 - \frac{aK}{6}$$, with parameter a, K.

The singular solution is the envelope of this family :

$$\frac{\partial z}{\partial a} = x - \frac{K}{6} = 0$$

$$\frac{\partial z}{\partial K} = \frac{1}{2}y^2 - \frac{a}{6} = 0$$

Hence, $K = 6x$ and $a = 3y^2$. Substituting into z gives the singular solution

$$z = 3xy^2$$

(ii) By this method, we get closer approximation of the root of an equation if we already know its approximate root.

Let the equation be $f(x) = 0$. ...(1)

Let its approximate root be a and better approximate root be $a + h$.

Now, we proceed to find h.

$f(a + h) = 0$ approximately [as $a + h$, is the root of $(x) = 0$] ...(2)

By Taylor's theorem

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2}f''(a) + ...$$

or $$f(a+h) = f(a) + hf'(a)$$

Since h is small, we neglect the h^2 and higher power of h.

From (2) and (3), we have

$$0 = f(a) + hf'(a) \quad \Rightarrow \quad h = \frac{f(a)}{f'(a)}$$

or $$a + h = a - \frac{f(a)}{f'(a)} = a_1 \qquad \text{[First approximate root = } a\text{]}$$

Second approximate root $$a_2 = a_1 - \frac{f(a_1)}{f'(a_1)}$$

Similarly, third approximat root, $$a_3 = a_2 - \frac{f(a_2)}{f'(a_2)}$$

By repeating this operation, we get closer approximation of the root.

Note : (1) In the beginning, we guess two numbers b and c such that $f(b)$ and $f(c)$ are of opposite sign. Then the first approximate root a lies between b and c.

(2) If $f'(x)$ is zero or nearly zero, this method fails.

$$f(x) = x^4 - x - 10$$
$$f'(x) = 4x^3 - 1$$
$$f(2) = 24 - 2 - 10 = 4$$
$$f'(2) = 4 \times 2^3 - 1 = 31$$
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
$$x_1 = 2 - \frac{4}{31} = 1.871$$
$$x_2 = 1.871 - \frac{0.3826}{24.564} = 1.8557$$
$$x_3 = 1.8557 - \frac{0.0048}{24.564} = 1.85558$$
$$x \approx 1.856$$

End of Solution

Q.3 (b) (i) Prove that the susceptibility of a perfectly superconducting material is –1 and its relative permeability is zero.

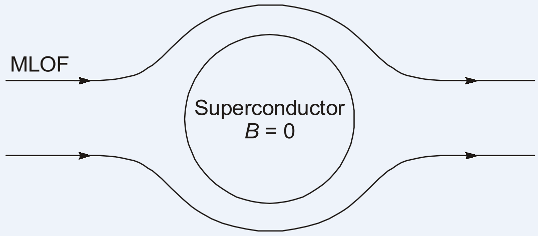
(ii) Find the critical current and critical current density at temperature 4.2 K for a superconducting wire made of lead with a diameter of 2 mm. The critical temperature for lead is 7.2 K and its critical field is $H_0 = 6.5 \times 10^4$ A/m.

[8 + 12 marks : 2025]

Solution:

(i) Superconducting Material : Materials which shows zero resistivity ($\rho = 0$) and infinite conductivity ($\sigma = \infty$) below a certain temperature and magnetic field are called as superconductor.

Superconducting materials behaves as perfect diamagnetic materials means they repel all magnetic line of forces (MLOF) such that magnetic field inside material is zero.



As we know the relation, $B = \mu_o(H + M)$ ...(1)

For superconductor,, $B = 0$

From equation (1), $\mu_o(H + M) = 0$

$M = -H$

On comparing with relationm, $M = \chi_m . H$

So, $\chi_m = -1 \Rightarrow$ magnetic susceptibility

and $\mu_r = 1 + \chi_m$ where μ_r = relative permeability

$= 1 - 1 = 0$

$\mu_r = 0$

(ii) Given : Diameter of lead, $d = 2$ mm

Radius, $r = 1$ mm

Critical temperature for lead, $T_c = 7.2$ K

Critical field, $H_o = 6.5 \times 10^4$ A/m

Finding critical field (H_c) at temperkature ($T = 4.2$ K)

Since $$H_c = H_o\left[1-\left(\frac{\tau}{\tau_c}\right)^2\right]$$

$$= 6.5\times10^4\left[1-\left(\frac{4.2}{7.2}\right)^2\right]$$

$H_c = 4.288 \times 10^4$ A/m

Critical current, $I_c = H_c[2\pi r]$

$= 4.288 \times 10^4 \times 2\pi \times 1 \times 10^{-3}$

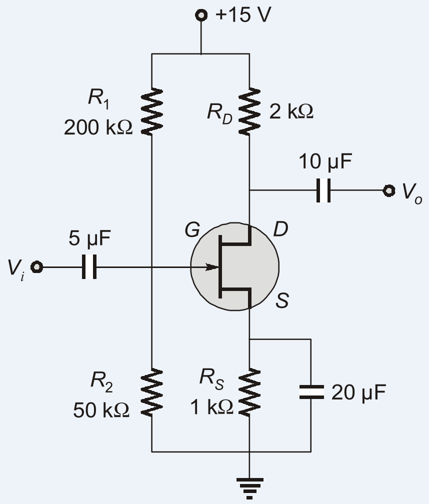
$I_c = 269.435$ A

Critical current density, $$J_c = \frac{I_c}{A} = \frac{I_c}{\pi r^2} = \frac{269.435}{\pi(1\times10^{-3})^2}$$

$J_c = 8.576 \times 10^7$ A/m^2

End of Solution

Q.3 (c) Consider the circuit shown in the figure below :

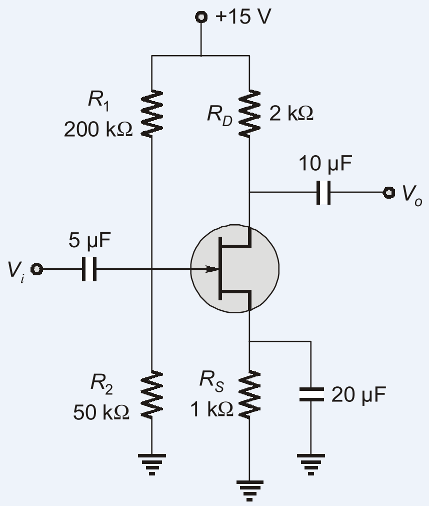


(i) Determine Q-point of the circuit by assuming maximum drain current I_{DSS} = 8 mA and pinch-off voltage V_p = –4 V.

(ii) Plot the transfer characteristics and DC load line, and indicate the Q-point.

[20 marks : 2025]

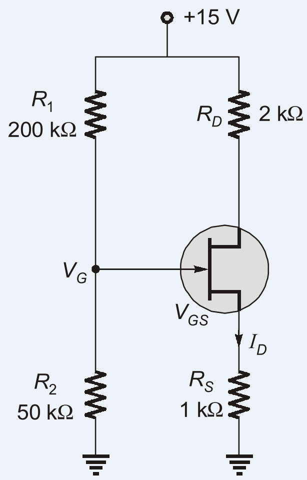
Solution:



(i) Given :

$$I_{DSS} = 8 \text{ mA}$$
$$V_P = -4 \text{ V}$$

DC Mode 1 :



$$I_D = \frac{V_G - V_{GS}}{R_S}$$

$$V_G = \frac{15 \times 50K}{250K} = 3 \text{ V}$$

$$I_D = \frac{3 - V_{GS}}{1} \quad \text{...(1)}$$

$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2$$

$$= 8\left[1 + \frac{V_{GS}}{4}\right]^2 \quad \text{...(2)}$$

Equation (1) in (2)

$$3 - V_{GS} = 8\left[1 + \frac{V_{GS}}{4}\right]^2$$

$$3 - V_{GS} = 8\left[\frac{4 + V_{GS}}{4}\right]^2$$

$$3 - V_{GS} = \frac{8}{16}\left[V_{GS}^2 + 8V_{GS} + 16\right]$$

$$3 - V_{GS} = \frac{1}{2}\left[V_{GS}^2 + 8V_{GS} + 16\right]$$

$$6 - 2V_{GS} = V_{GS}^2 + 8V_{GS} + 16$$

$$V_{GS}^2 + 10V_{GS} + 10 = 0$$

$$V_{GS} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{100 - 40}}{2}$$

$$= \frac{-10 \pm \sqrt{60}}{2} = \frac{-10 \pm 7.74}{2}$$

$$= \frac{-10+7.74}{2}, \frac{-10-7.74}{2}$$

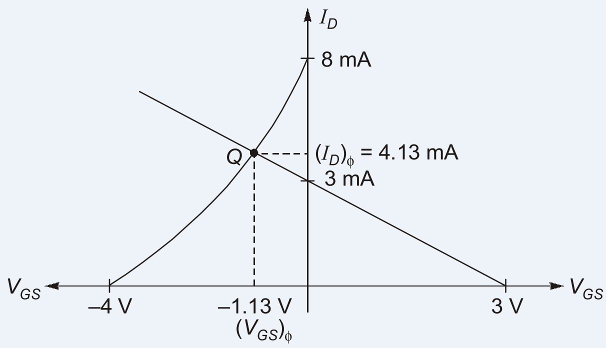
$$= -1.13 \text{ V}, 8.87 \text{ V}$$

$$(V_{GS})_Q = -1.13 \text{ V}$$

$$(I_D)_Q = \frac{V_G - (V_{GS})_Q}{R_S} = \frac{3+1.13}{1K} = \frac{4.13}{1K} = 4.13 \text{ mA}$$

Operating point, $Q((I_D)_Q, (V_{GS})_Q) = Q(4.13 \text{ mA}, -1.13 \text{ V})$

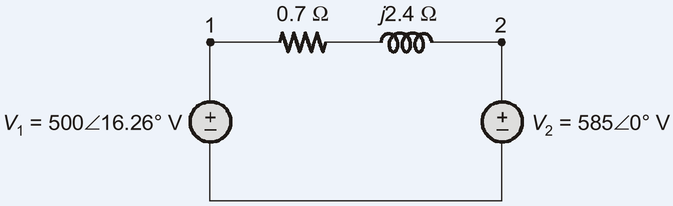
(ii)



End of Solution

Q.4 (a) (i) In a factory, there are following two loads :
Lighting and heating load : 100 kW
Induction motor load : 1000 HP at 0.7 lagging power factor and 85% efficiency
The overall load power factor of the factory has to be raised to 0.95 lagging. A 3-phase synchronous motor is installed for the above purpose. The motor is rated at 300 HP with 100% efficiency. Find the kVA rating of the synchronous motor. Also, find the power factor of the synchronous motor. Given 1 HP (horse power) = 746 watts.

(ii) Two single-phase ideal voltage sources are connected by a line of impedance of (0.7 + j2.4) ohms as shown in the figure below. Given V_1 = 500∠16.26° volts and V_2 = 585∠0° volts. Find the complex power for each source and determine whether they are delivering or receiving real and reactive power. Also, find the real and reactive power losses in the line :



[12 + 8 marks : 2025]

Solution:

(i) Given : Lighting and heating load :

$$P_1 = 100 \text{ kW (PF = 1)}$$
$$Q_1 = 0$$

Induction motor load :

$$\text{Output power} = 746 \times 1000 = 746000 \text{ W}$$

Given : $\eta = 85\%$

So, input power, $P_2 = \dfrac{746000}{0.85} = 877.647 \text{ kW}$

kVA of induction motor, $S_2 = \dfrac{P_2}{P.F} = \dfrac{877.647}{0.7} = 1253.79 \text{ kVA}$

Reactive power, $Q_2 = \sqrt{S^2 - P^2} = 895.388 \text{ kVAR}$

Now, total load before correction

$$P_{Total} = 100 + 877.647 = 977.647 \text{ kW}$$
$$Q_{Total} = 0 + 895.388 = 895.388 \text{ kVAR}$$

Power factor before correction

$$\cos\phi_1 = \frac{P_{Total}}{S_{Total}} = \frac{977.647}{\sqrt{(977.647)^2 + (895.388)^2}} = 0.737 \text{ lag}$$

Desired power factor after correction

$$\cos\phi_2 = 0.95 \text{ lag}$$

Total active power after correction

$$(P_{Total})_{after} = 977.647 + P_{syn} \text{ kW}$$
$$= 977.647 + 300 \times 0.746 = 1201.447 \text{ kW}$$

At Pf = 0.95 lag total apparent power

$$S_{after} = \frac{P_{Total}}{0.95} = \frac{1201.447}{0.95} = 1264.681 \text{ kVA}$$

Total reactive power after correction

$$Q_{Total,\ after} = \sqrt{S_{after}^2 - P_{after}^2} = \sqrt{(1264.681)^2 - (1201.447)^2}$$
$$Q_{Total,\ after} = 394.896 \text{ kVAR}$$

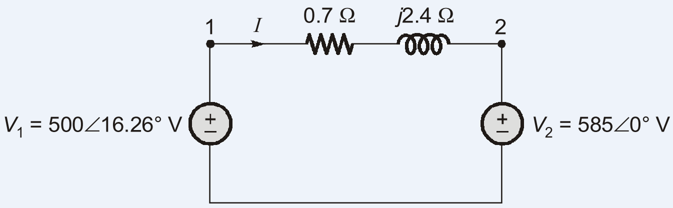
So, $Q_{syn} = (394.896 - 895.388) \text{ kVAR}$

$$= -500.50 \text{ kVAR (leading reactive power supplied)}$$

Power factor of syn. motor

$$(\cos\phi)_{syn} = \frac{P_{syn}}{S_{syn}} = \frac{P_{syn}}{\sqrt{P_{syn}^2 + Q_{syn}^2}}$$
$$= \frac{300 \times 0.746}{\sqrt{(300 \times 0.746)^2 + (500.50)^2}}$$
$$(\cos\phi)_{syn} = 0.408 \text{ (lead)}$$

(ii) Current I flowing from node 1 to node 1 :



$$I = \frac{(500\angle 16.26) - (585\angle 0°)}{0.7 + j2.4}$$

$$I = 70\angle 53.13°\text{ A}$$

Complex power delivered by sourve V_1

$$S_1 = V_1.I^* = (500\angle 16.26°)(70\angle -53.13°)$$
$$= (28000 - j21000)\text{ VA}$$
$$= (28 - j21)\text{ kVA}$$

Complex power absorbed by sourve V_2

$$S_2 = V_2.I^* = 585(70\angle -53.13°)$$
$$= (24570 - j32759.95)\text{ VA}$$
$$= (24.57 - j32.76)\text{ kVA}$$

Active power loss in transmission line

$$P_L = I^2R = \frac{(70)^2 \times 0.7}{1000} = 3.430\text{ kW}$$

Active power delivered by V_1 voltage source = 28 kW
Active power absorbed by V_2 voltage source = 24.57 kW
Active power loss in transmission line = 3.430 kW
Reactive power loss in transmission line

$$Q_L = I^2X_L = \frac{(70)^2 \times 2.4}{1000}$$
$$= 11.760\text{ kVAR}$$

Lagging kVAR absorbed by source V_1 = 21 kVAR
Lagging kVAR delivered by source V_2 = 32.76 kVAR
Reactive power loss in transmission line (Q_L) = 11.76 kVAR

End of Solution

Q4 (b) A priority encoder truth table is given below :

Inputs				Outputs		
I_0	I_1	I_2	I_3	x	y	z
1	×	×	×	0	0	1
0	1	×	×	0	1	1
0	0	1	×	1	0	1
0	0	0	1	1	1	1
0	0	0	0	×	×	0

Obtain the minimized Boolean expressions for x, y and z outputs. Design a combinational circuit for the minimized Boolean expressions of x, y and z. Consider that × is don't care.

[20 marks : 2025]

Solution:

Given : Truth table of a priority encoder

Inputs				Outputs		
I_0	I_1	I_2	I_3	x	y	z
1	×	×	×	0	0	1
0	1	×	×	0	1	1
0	0	1	×	1	0	1
0	0	0	1	1	1	1
0	0	0	0	×	×	0

I_0	I_1	I_2	I_3	x	y	z
0	0	0	0	×	×	0
0	0	0	1	1	1	1
0	0	1	0	1	0	1
0	0	1	1	1	0	1
0	1	0	0	0	1	1
0	1	0	1	0	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	0	1
1	0	1	1	0	0	1
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	0	1

Expressions for outputs using K-maps :

x: I_0I_1 \ I_2I_3	00	01	11	10
00	×	1	1	1
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$x = \bar{I}_0 \cdot \bar{I}_1$

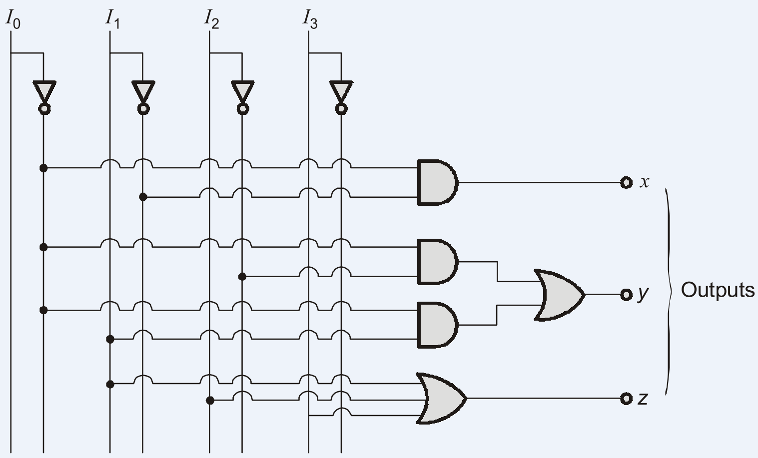
y: I_0I_1 \ I_2I_3	00	01	11	10
00	×	1	0	0
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$y = \bar{I}_0 \cdot \bar{I}_2 + \bar{I}_0 I_1$

z: I_0I_1 \ I_2I_3	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$z = I_1 + I_3 + I_2$

Combinational Circuits :



End of Solution

Q.4 (c) (i) A uniform volume charge density of 0.8 μC/m^3 is present throughout the spherical shell extending from r = 3 cm to r = 5 cm. If the volume charge density is zero elsewhere, find the total charge present throughout the shell. If the half of the total charge is located in the region where the radius varies as 3 cm < r < r_1, find the value of r_1 in cm.

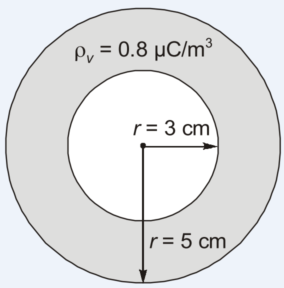
(ii) A current filament on the z-axis carries a current of 7 mA in the a_z direction and current sheets of $0.5a_z$ A/m and $-0.2a_z$ A/m are located at ρ = 1 cm and ρ = 0.5 cm respectively. What is the value of H at ρ = 4 cm? What value of current sheet should be located at ρ = 4 cm so that H = 0 for all ρ > 4 cm? Given : H : magnetic field intensity; ρ : radius variable of cylindrical coordinates.

[10 + 10 marks : 2024]

Solution:

(i) Given :

$$\rho_v = \begin{cases} 0.8\ \mu C/m^3 & 3\text{ cm} \le r \le 5\text{ cm} \\ 0 & \text{elsewhere} \end{cases}$$



Total change enclosed in the shell :

$$Q_{enc} = \iiint_V \rho_V dV$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0.03\text{ m}}^{r=0.05\text{ m}} \rho_v r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 0.8\times10^{-6}\times\frac{4\pi}{3}r^3\Big|_{0.03}^{0.05}$$

$$Q_{enc} = 3.24\times10^{-10}\text{ C}$$

Now half of the changed is enclosed between 3 cm < r < r_1.

Then, $$\frac{3.24\times10^{-10}}{2} = \iiint_V \rho_V dV$$

$$1.64\times10^{-10} = \rho_V\frac{4}{3}\pi r^3\Big|_{r=0.03}^{r_1}$$

$$(r_1^3 - 0.03^3) = \frac{1.64\times10^{-10}\times3}{4\pi\times0.8\times10^{-6}} = 4.894\times10^{-5}$$

$$r_1 = \sqrt[3]{0.03^3 + 4.894\times10^{-5}} = 0.0423\text{ m or }4.23\text{ cm}$$

Hence, $r_1 = 4.23$ cm

(ii) $\bar{H}/\rho = 4$ cm

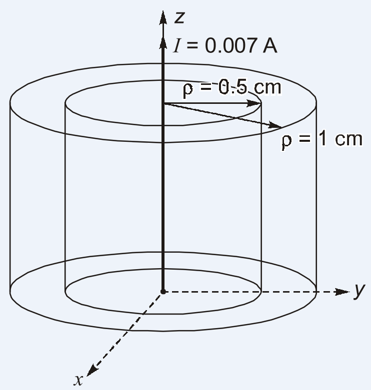
Applying Ampere's law

$$\oint \bar{H}\cdot\bar{dl} = I_{enc}$$

$$I_{enc} = I|_{\text{line current}} + I|_{\text{sheet at }\rho = 0.5\text{ cm}} + I|_{\text{sheet at }\rho = 1\text{ cm}}$$

$$\therefore \quad I_{enc} = 0.007 + (-0.2)(2\pi\times0.5\times10^{-2}) + (0.5)(2\pi\times1\times10^{-2}) = 0.0321\text{ A}$$

$$\oint \bar{H}\cdot\bar{dl} = H\times2\pi(4\times10^{-2}) = 0.251\text{ H}$$



$\therefore \quad 0.251H = 0.0321$

$\Rightarrow \quad H = 0.1279$ A/m

K value at $\rho > 4$ cm so that $H = 0$.

From Ampere's law, $I_{enc} = 0$ to make $H = 0$.

$\therefore \quad I_{enc} = I_{net} + I|_{\text{sheet at 4 cm}} = 0$

$\Rightarrow 0.0321 + I|_{\text{sheet at 4 cm}} = 0$

$\Rightarrow \quad I|_{\text{sheet at 4 cm}} = -0.0321$

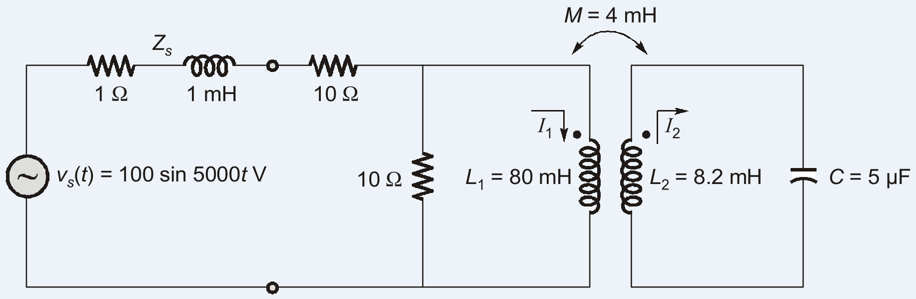
$$\Rightarrow \quad K = \frac{I|_{\text{sheet at 4 cm}}}{2\pi(4\times10^{-2})} = \frac{-0.0321}{0.251}$$

$\Rightarrow \quad K = -0.128$ A/m

End of Solution

SECTION : B

Q.5 (a) For the circuit shown in the figure below, the two magnetically coupled coils have mutual inductance $M = 4$ mH. The self-inductances are $L_1 = 80$ mH and $L_2 = 8.2$ mH respectively. The source voltage is $v_s(t) = 100\sin 5000t$ volts with a source resistance of 1 Ω and inductance of 1 mH. Find the power delivered by the source and the corresponding source power factor when the connected load with the second coil is a capacitor C of 5 μF as shown in the figure :

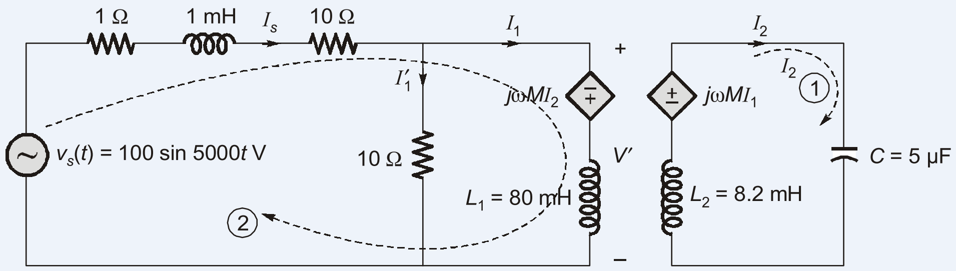


[12 marks : 2025]

Solution:

Drawing the circuit showing the effect of mutual inductance.

Given :
$$\omega = 5000 \text{ rad/sec}$$
$$M = 4 \text{ mH}$$



Applying KVL in loop (1)

$$\frac{1}{j\omega C}I_2 + j\omega L_2 I_2 - j\omega M I_1 = 0$$

$$I_2\left[\frac{1}{j\times 5000\times 5\times 10^{-6}} + j5000\times 8.2\times 10^{-3}\right] = j5000\times 4\times 10^{-3} I_1$$

$$jI_2[41-40] = j20I_1$$

$$I_2 = 20I_1 \qquad \text{...(1)}$$

Finding V' :

$$V' = -j\omega M I_2 + j\omega L_1 I_1$$
$$= j[\omega L_1 I_1 - \omega M I_2]$$

$$V' = j[5000\times 80\times 10^{-3} I_1 - 5000\times 4\times 10^{-3}\times 20I_1] \qquad [I_2 = 20I_1]$$

$$V' = j[400-400]I_1 = 0 \text{ Volt}$$

Current $$I' = \frac{V'}{10} = 0 \text{ A}$$

Now source current $I_s = I_1$

Applying KVL in loop (2)

$$-V_s + (1 + j\omega\times 1\times 10^{-3} + 10)I_s + 0 = 0$$

$$V_s = (1 + j\omega\times 10^{-3} + 10)I_s$$

$$I_s = \frac{V_s}{(11 + j5000\times 10^{-3})} = \frac{100\angle 0^\circ}{\sqrt{2}(11+j5)}$$

$$I_s = 5.852\angle{-24.44°}\text{ A}$$

Power delivered by source

$$P_s = V_{s,rms}.I_{s,rms}.\cos\phi_s$$

$$= \frac{100}{\sqrt{2}} \times 5.582\cos 24.44$$

$$P_s = 376.72\text{ W}$$

Source power factor, $\cos\phi_s = \cos(24.44) = 0.91$ lag

End of Solution

Q.5 (b) Given : $\mu = 3 \times 10^{-5}$ H/m, $\varepsilon = 1.2 \times 10^{-10}$ F/m and $\sigma = 0$ everywhere. If $H = 2\cos(10^{10}t - \beta x)a_z$ A/m, use Maxwell's equation to obtain the expressions for B, D, E and β.

Given : μ : Permeability; ε : Permittivity; B : Flux density; H : Magnetic field intensity; E : Electric field intensity; D : Electric flux density; β : Phase constant

[12 marks : 2025]

Solution:

Given : $\bar{H} = 2\cos(10^{10}t - \beta x)\hat{a}_z$ A/m

$\because$ $v = \frac{\omega}{\beta}$

$\therefore$ $\beta = \frac{\omega}{v}$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{3\times10^{-5}\times1.2\times10^{-10}}} = 1.67 \times 10^7$$

$\Rightarrow$ $\beta = \frac{10^{10}}{1.67\times10^7} = 598.8$ rad/m

$\bar{B}$: $\bar{B} = \mu\bar{H} = 3\times10^{-5}\times2\cos(10^{10}t - 598.8x)\hat{a}_z$

$\Rightarrow$ $\bar{B} = 60\cos(10^{10}t - 598.8x)\hat{a}_z$ μWb/m²

$\bar{D}$: $\nabla\times\bar{H} = \bar{I} + \frac{\partial\bar{D}}{\partial t} = \frac{\partial\bar{D}}{\partial t}$ $[\because \sigma = 0]$

$$\Rightarrow \begin{vmatrix} \hat{a}_x & \hat{a}_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 2\cos(10^{10}t - \beta x) \end{vmatrix} = \frac{\partial\bar{D}}{\partial t}$$

$$\Rightarrow 0\hat{a}_x - [2\sin(10^{10}t - \beta x)\times -\beta]\hat{a}_y = \frac{\partial\bar{D}}{\partial t}$$

$$\Rightarrow 2\beta\sin(10^{10}t - \beta x)\hat{a}_y = \frac{\partial\bar{D}}{\partial t}$$

$$\Rightarrow \bar{D} = 2\beta\int\sin(10^{10}t - \beta x)dt\,\hat{a}_y$$

$$\Rightarrow \quad \bar{D} = \frac{2\beta\cos(10^{10}t - \beta x)}{10^{10}}\hat{a}_y$$

$$\Rightarrow \quad \bar{D} = \frac{2 \times 598.8}{10^{10}}\cos(10^{10}t - 598.8x)\hat{a}_y$$

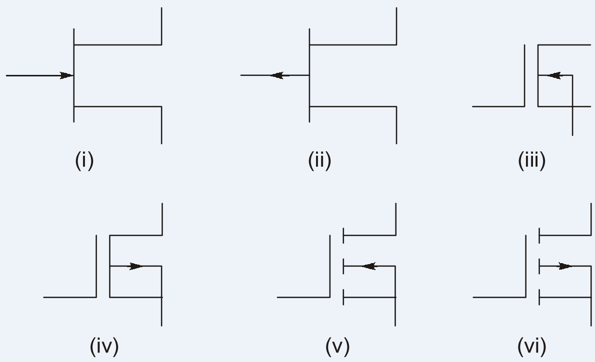
$$\Rightarrow \quad \bar{D} = 0.120\cos(10^{10}t - 598.8x)\hat{a}_y \ \mu\text{c/m}^2$$

Now,
$$\bar{E} = \frac{\bar{D}}{\epsilon} = \frac{0.120}{1.2 \times 10^{-10}}\cos(10^{10}t - 598.8x)\hat{a}_y \times 10^{-6}$$

$$\Rightarrow \quad \bar{E} = 1000\cos(10^{10}t - 598.8x)\hat{a}_y \ \text{V/m}$$

End of Solution

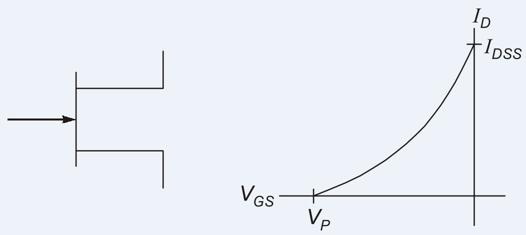
Q.5 (c) Identify the names of the following electronic devices, mark their terminals and plot their transfer characteristics :



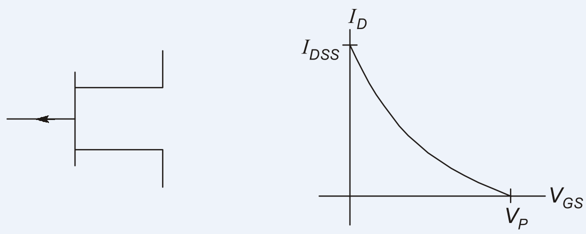
[12 marks : 2025]

Solution:

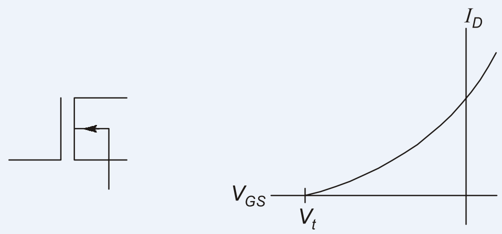
(i) n-channel JFET



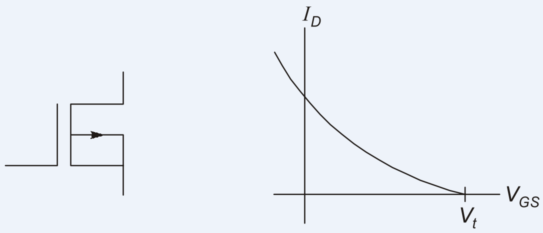
(ii) p-channel JFET



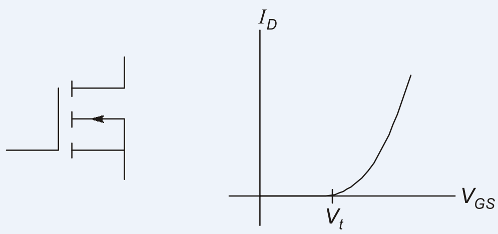
(iii) Depletion NMOS



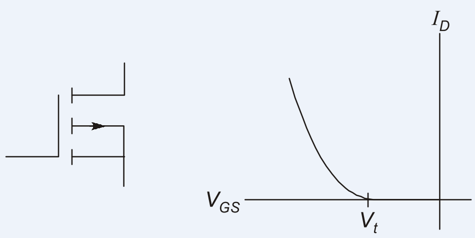
(iv) Depletion pMOS



(v) Enhancement NMOS



(vi) Enhancement pMOS



End of Solution

Q.5 (d) If the probability of a bad reaction from certain injection is 0.001, determine the chance that out of 2000 persons, more than two will get a bad reaction.

[12 marks : 2025]

Solution:

Given probability of getting a bad reaction = 0.001 and number of people = 2000.

$\because$ $n \to$ large and $p \to$ least.

So that $\lambda = np = 0.001 \times 2000 = 2$ out of 2000 individuals

By Poisson Distribution :

Let X = Number of people get affected by bad reaction out of 2000 individuals

P.M.F. : $$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

We need to find probability of $X = 2$.

$\therefore$ $$P(X > 2) = 1 - P(X \le 2)$$
$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$
$$= 1 - \left[\frac{e^{-2}2^0}{\angle 0} + \frac{e^{-2}2^1}{\angle 1} + \frac{e^{-2}2^2}{\angle 2}\right] = 1 - 5e^{-2}$$

End of Solution

Q.5 (e) (i) Determine the possible base of the number in the operation mentioned below :

$$23 + 44 + 14 + 32 = 223$$

(ii) Find the number of divisors and sum of divisors of 4900.

[6 + 6 = 12 marks : 2025]

Solution:

(i) Mentioned operation : 23 + 44 + 14 + 32 = 223

Let base of the given operation is x.

Expanding each number in base weightage form

$$\underbrace{2x^1 + 3x^0}_{\downarrow\, 23} + \underbrace{4x^1 + 4x^0}_{\downarrow\, 44} + \underbrace{1x^1 + 4x^0}_{\downarrow\, 14} + \underbrace{3x^1 + 2x^0}_{\downarrow\, 32} = \underbrace{2x^2 + 2x^1 + 3x^0}_{\downarrow\, 223}$$

On solving

$$2x + 3 + 4x + 4 + x + 4 + 3x + 2 = 2x^2 + 2x + 3$$
$$10x + 13 = 2x^2 + 2x + 3$$
$$2x^2 - 8x - 10 = 0$$

On solving, $x = 5$

So, base of above operation, $x = 5$

(ii) If $n = P_1^{a_1} \times P_2^{a_2} \times P_3^{a_3}$

$$\text{Number of divisors} = (a_1 + 1)(a_2 + 1)(a_3 + 1)$$
$$4900 = 2^2 \times 5^2 \times 7^2$$

$\therefore$ $$\text{Number of divisors} = (2 + 1)(2 + 1)(2 + 1) = 27$$

Sum of divisors, $$6n = \frac{P_1^{a_1+1} - 1}{P_1 - 1} \times \frac{P_2^{a_2+1} - 1}{P_2 - 1} \times \frac{P_3^{a_3+1} - 1}{P_3 - 1}$$
$$= \frac{2^3 - 1}{2 - 1} \times \frac{5^3 - 1}{5 - 1} \times \frac{7^3 - 1}{7 - 1}$$
$$= 7 \times 31 \times 57 = 12369$$

End of Solution

Q.6 (a) (i) Explain the principle on which a Q-meter works. Describe briefly the direct connection, series connection and parallel connection of using the Q-meter. Also, mention for which types of loads, these connections are used.

(ii) A power transformer was tested to determine losses and efficiency. The input power was measured as 3650 watts and the delivered output power

was 3385 watts, with each reading in doubt by ±10 watts. Calculate (1) the percentage uncertainty in losses of the transformer and (2) the percentage uncertainty in the efficiency of the transformer, as determined by the difference in input and output power readings.

[12 + 8 marks : 2025]

Solution:

(i) A quality factor meter is an instrument which is used to measure the value of storage factor Q directly and measuring the characteristics of coils and capacitors. It is used in laboratories for testing radio frequency coils.

Principle of Operation :

- Q-meter works on the principle of series resonance.
- Figure given below shows a series RLC circuit.

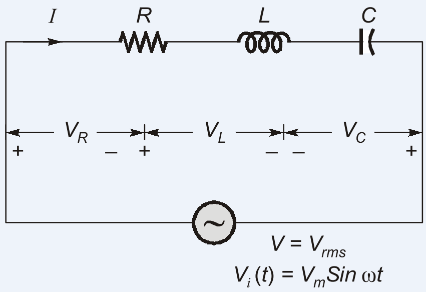
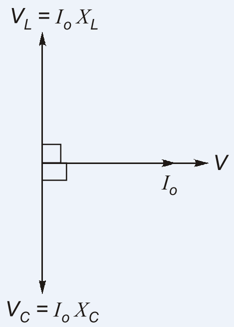


Fig. : Series RLC circuit

- At resonance, $X_L = X_C$ or $\omega_0 L = \dfrac{1}{\omega_0 C}$

  i.e., Inductive reactance = Capacitive reactance and current through the circuit is

$$I_0 = \frac{V}{R}$$



(Phaser diagram)

Voltage drop across inductor, $V_L = I_0 X_L$
and voltage drop across capacitor $V_C = I_0 X_C$

For inductor, $Q = \dfrac{X_L}{R}$ and for capacitor, $Q = \dfrac{X_C}{R}$

Now, $V_C = I_0 X_C = \dfrac{V}{R} \cdot X_C = \left(\dfrac{X_C}{R}\right) V$

or, $\boxed{V_C = Q \cdot V}$ or $\boxed{V_C \propto Q}$ (If V = constant)

Therefore, if the input voltage is constant, voltage across the capacitor is magnified Q times and a voltmeter can be connected across the capacitor which can be calibrated to read the value of Q directly.

Practical Circuit of Q-Meter : Below figure shows a practical Q-meter

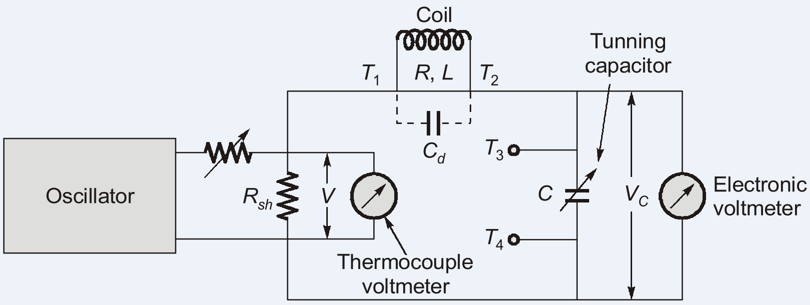


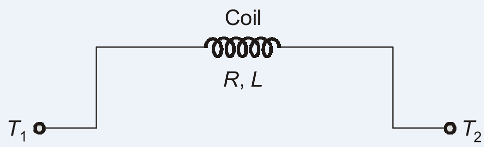
***Fig. :** Practical Q-meter*

It consists of a variable radio frequency oscillator which delivers a current to a low shunt resistance, R_{sh} which is order of 0.02 Ω (typical value). The voltage is measured by a thermocouple voltmeter. A calibrated standard variable capacitor C is used for getting the series resonance condition. An electronic voltmeter is connected across this capacitor C. The coil under test is connected between the terminals T_1 and T_2.

Applications of Q-meter :

1. **Measurement of Quality Factor of Test Coil :** The test coil is connected between terminals T_1 and T_2. The oscillator is set to the desired frequency and then, the capacitor, C is adjusted to get the series resonance condition.

Let the coil resistance and inductance be R and L respectively,



∴ True value of quality factor of the coil, $Q_T = \dfrac{\omega L}{R}$ and measured value of quality factor

(storage factor),

$$Q_m = \left[\frac{\omega L}{R + R_{sh}}\right],$$

$$\Rightarrow \qquad Q_m = \frac{\frac{\omega L}{R}}{1 + \frac{R_{sh}}{R}} = \frac{Q_T}{1 + \frac{R_{sh}}{R}}$$

$$\Rightarrow \qquad \frac{Q_T}{Q_m} = 1 + \frac{R_{sh}}{R}$$

To get minimum error, the shunt resistance should be very small. As R_{sh} of the order of mΩ therefore, $Q_T \approx Q_m$. Therefore, the true value of quality factor of the coil is obtained when R_{sh} is maintained in mΩ range (can be neglected).

2. **Measurement of Unknown Capacitance :**
   - For measurement of unknown capacitance, the test capacitance C_T is connected in parallel with the known value of the capacitance, C between the terminals T_3 and T_4.
   - By adjusting the capacitor C to C_1, the circuit is resonated to a frequency, f_0.

$$f_0 = \frac{1}{2\pi\sqrt{L(C_1 + C_T)}} \quad ...(i)$$

   - Now, the test capacitance across T_3 and T_4 is removed and the capacitor C is readjusted to C_2 to get the same value of resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC_2}} \quad ...(ii)$$

   As the frequencies are same in both cases

$$\therefore \quad \frac{1}{2\pi\sqrt{L(C_1 + C_T)}} = \frac{1}{2\pi\sqrt{LC_2}}$$

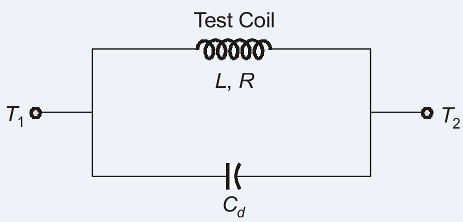
   or, $C_T = (C_2 - C_1)$ = Unknown capacitance

3. **Measurement of Unknown Inductance :** We have,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

   or, $L = \dfrac{1}{4\pi^2 f_0^2 C}$ = unknown inductance

4. **Measurement of Self Capacitance :** Let C_d be the self capacitance. The capacitor is set to a high value and the circuit is resonanced by adjusting the oscillator frequency. Let the value of tuning capacitor be C_1 and frequency be f_1.



then, $$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_d)}} \quad ...(i)$$

Now, the frequency is increased to twice its initial value and circuit is again resonated with the tuning capacitor value C_2 and frequency f_2

$$\therefore \quad f_2 = \frac{1}{2\pi\sqrt{L(C_2 + C_d)}} \quad ...(ii)$$

if $f_2 = nf_1$, then $$\frac{1}{2\pi\sqrt{L(C_2 + C_d)}} = \frac{n}{2\pi\sqrt{L(C_1 + C_d)}}$$

$$C_d = \frac{C_1 - n^2 C_2}{n^2 - 1}$$

For example, $f_2 = 2f_1$

$$\therefore \quad \frac{1}{2\pi\sqrt{L(C_2+C_d)}} = 2\times\frac{1}{2\pi\sqrt{L(C_1+C_d)}} \quad \text{or, } C_d = \frac{C_1-4C_2}{3}$$

$\therefore$ Distributed/self capacitance, $C_d = \left(\frac{C_1}{3}-\frac{4}{3}C_2\right)$

(ii)
Input power = 3650 Watts ± 10
Output power = 3385 Watts ± 10
Losses, $L = P_i - P_o = 3650 - 3385 = 265$ Watts

1. **Percentage uncertainity in losses :** Formula for uncertainity in subtraction

$$\Delta L = \sqrt{(\Delta P_i)^2+(\Delta P_o)^2}$$

$$\Delta L = \sqrt{(10)^2+(10)^2} = 14.14 \text{ Watts}$$

$$\% \text{ uncertainity} = \frac{\Delta L}{L}\times 100 = \frac{14.14}{265}\times 100 = 5.34\%$$

2. **Percentage uncertainity in efficiency :**

$$\text{Efficiency} = \frac{P_o}{P_i}\times 100 = \frac{3385}{3650}\times 100 = 92.73\%$$

Uncertainity formula for division :

$$\frac{\Delta\eta}{\eta} = \sqrt{\left(\frac{\Delta P_o}{P_o}\right)^2+\left(\frac{\Delta P_i}{P_i}\right)^2}$$

$$= \sqrt{\left(\frac{10}{3385}\right)^2+\left(\frac{10}{3650}\right)^2} = 0.00403$$

$$\% \text{ uncertainity in efficiency} = 0.00403 \times 100 = 0.403\%$$

End of Solution

Q.6 (b) (i) An electric field in x-y plane is given by $f(x, y) = 3x^2y - y^3$. Find the stream function $g(x, y)$ such that the complex potential $w = f + ig$ is an analytic function.

(ii) Find the mass of the surface of the cone $z = 2+\sqrt{x^2+y^2}$, $2 \le z \le 7$ in the first octant, if the density $\rho(x, y, z)$ at any point of the surface is proportional to its distance from x-y plane.

[10 + 10 marks : 2025]

Solution:

(i)
$$f(x, y) = 3x^2y - y^3$$

We know that, by C-R theorem

$$\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y} = 6xy$$

and
$$\frac{\partial f}{\partial y} = \frac{-\partial g}{\partial x} = 3x^2 - 3y^2$$

By total derivative method,

$$dg = \frac{\partial g}{\partial x}\cdot dx + \frac{\partial g}{\partial y}\cdot dy$$

$$dg = \left(\frac{\partial f}{\partial x}\right)\cdot dx + \left(\frac{-\partial f}{\partial y}\right)\cdot dy$$

$$dg = 6xy.dx + (3y^2 - 3x^2)dy$$

Now integrating both sides,

$$\int dg = \int \underbrace{6xy\cdot dx}_{y\text{-constant}} + \int \underbrace{(3y^2 - 3x^2)dy}_{x\text{-free term}}$$

$$g(x, y) = 3x^2y + y^3$$

(ii) Given : $\rho(x, y, z) \propto$ distance from xy-plane to point on 'S'.

[distance between $(x, y, 0)$ to (x, y, z)]

$$\rho \propto z$$

$\therefore$ $\rho = Kz, \quad K > 0$

Let $K = 1$: $\rho = z$

Given surface S : cone in the 1st octant

$$z = 2 + \sqrt{x^2 + y^2}, \quad 2 \le z \le 7 \text{ and } x \ge 0,\ y \ge 0,\ z \ge 0$$

$$(z - 2)^2 = x^2 + y^2$$

$2 \le z \le 7$ forming a right circular cone at point (0, 0, 2) open upwards.

We have mass of surface 'S', $M = \iint\limits_S \rho\, ds$

where ds = surface area elemental

$$ds = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\ dxdy$$

Converting into polar coordinate system by $x = r\cos\theta$, $y = r\sin\theta$

Then $S : z = 2 + r \Rightarrow r = z - 2 \Rightarrow 0 \le r \le 5$

$\because$ It is in iff octant $\theta : 0$ to $\frac{\pi}{2}$.

Now, $$ds = \sqrt{1 + \left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)}\ r\,dr\,d\theta = \sqrt{1 + \left(\frac{x^2}{r^2} + \frac{y^2}{r^2}\right)}\ r\,dr\,d\theta$$

$$ds = \sqrt{2}r\,dr\,d\theta$$

$\therefore$ $$M = \int\limits_{\theta:0}^{\pi/2}\int\limits_{r=0}^{5}(2 + r)\sqrt{2}r\,dr\,d\theta = \int\limits_{0}^{\pi/2}\sqrt{2}\left(r^2 + \frac{r^3}{3}\right)_0^5 d\theta$$

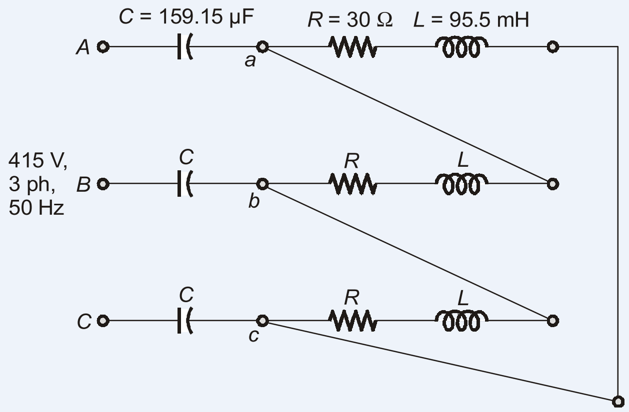
$$= \sqrt{2}\left(25 + \frac{125}{3}\right)(\theta)_0^{\pi/2} = \sqrt{2}\times\frac{200}{3}\times\frac{\pi}{2}$$

$\therefore$ for $K = 1$, $$M = \frac{100\pi\sqrt{2}}{3}$$

In general solution is $M = \dfrac{100\pi K\sqrt{2}}{3}$.

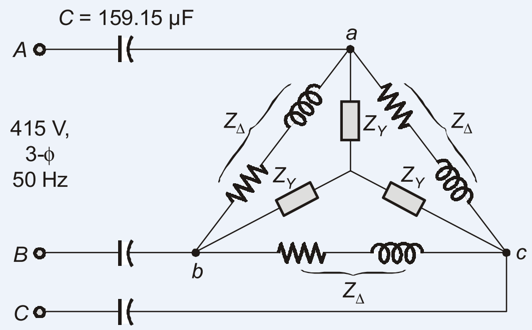
End of Solution

Q.6 (c) A balanced load is shown in the figure below, where R = 30 Ω, C = 159.15 μF and L = 95.5 mH. The r.m.s. value of the balanced input supply voltage is 415 V (L-L), 50 Hz. Now find (i) the magnitude of the voltage Vab, (ii) the phase of V_{ab} with respect to V_{AB} and (iii) the total power supplied to the load and corresponding power factor calculated from source side :



[20 marks : 2025]

Solution:

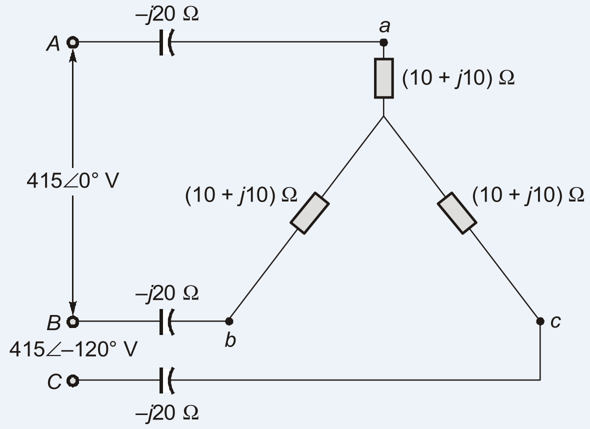


$$Z_\Delta = R + j\omega L$$
$$= (30 + j30)\ \Omega$$
$$X_C = 20\ \Omega$$

Converting the delta load into Y

$$Z_Y = \frac{Z_\Delta}{3} = (10 + j10)\ \Omega$$

Now, the circuit becomes



$$V_a = \frac{(415\angle-30°)}{\sqrt{3}} \times \frac{10+j10}{10+j10-j20}$$

$$= 239.60\angle 60° \text{ Volts}$$

$$V_b = \left(\frac{415}{\sqrt{3}}\angle-150°\right) \times \frac{10+j10}{10+j10-j20}$$

$$= 239.60\angle-60° \text{ Volts}$$

(i) Now $|V_{ab}| = |V_a - V_b| = 415$ Volts

(ii) $V_{ab} = V_a - V_b$

$$= (239.60\angle 60°) - (239.60\angle-60°) = 415\angle 90° \text{ Volts}$$

$$\angle V_{ab} = 90°$$

The phase of V_{ab} w.r.t. V_{AB} is

$$\angle\phi = \angle V_{ab} - \angle V_{AB}$$

$$\angle\phi = 90° - 0° = 90°$$

(iii) Phase current : $\vec{I}_a = \left(\frac{415\angle-30°}{\sqrt{3}}\right) \times \frac{1}{10-j10} = 16.94\angle 15°$ Amp

Since load is balanced. So, total complex power supplied to load

$$\vec{S}_L = 3V_a \cdot I_a^*$$

$$\vec{S}_L = 3 \times 239.60\angle 60° \times 16.94\angle-15°$$

$$\vec{S}_L = (8610 + j8610)V_A$$

Active power supplied to load

$$P_L = 8610 \text{ Watts}$$

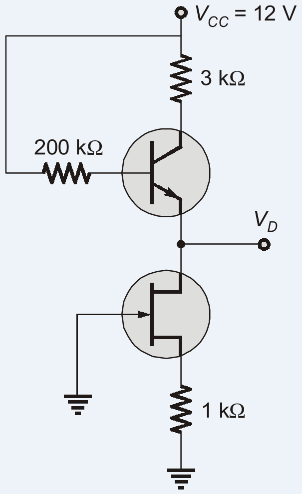
Load power factor calculated from source side,

$$\vec{Z}_{in} = 10 + j10 - j20 = (10 - j10)\ \Omega$$

$$\cos\phi_s = \cos\left[\tan^{-1}\left(\frac{-10}{10}\right)\right] = 0.707 \text{ leading}$$

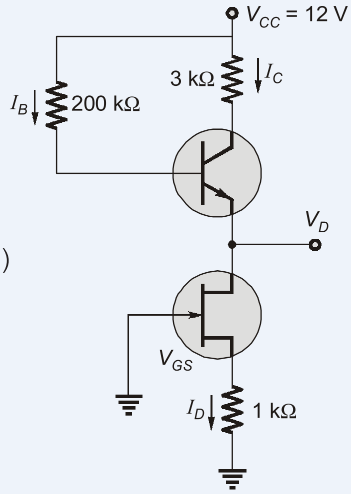
End of Solution

Q.7 (a) Consider the silicon transistor circuit shown in the figure below. The data pertaining to transistors are as follows : (i) β = 100; (ii) maximum drain current I_{DSS} = 6 mA; (iii) pinch-off voltage V_p = –2 V. Determine the voltage V_D.



[20 marks : 2025]

Solution:



Given :

$$\beta = 100$$
$$I_{DSS} = 6 \text{ mA}$$
$$V_P = -2 \text{ V}$$

Assume

$$V_{BE} = 0.7 \text{ V}$$
$$V_{GS} = -I_D \times 1 \text{ K}$$
$$I_D = -\frac{V_{GS}}{1} \quad ...(1)$$
$$I_D = 6 \text{ mA}\left[1+\frac{V_{GS}}{2}\right]^2$$

Eqn. (1) in (2),

$$-V_{GS} = \frac{6}{4}[2+V_{GS}]^2$$
$$-V_{GS} = \frac{3}{2}[V_{GS}^2+4V_{GS}+4]$$
$$-\frac{2}{3}V_{GS} = V_{GS}^2 + 4V_{GS} + 4$$
$$-0.66V_{GS} = V_{GS}^2 + 4V_{GS} + 4$$
$$V_{GS}^2 + 4.66V_{GS} + 4 = 0$$
$$V_{GS} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-4.66\pm\sqrt{21.71-16}}{2}$$

$$= \frac{-4.66 \pm \sqrt{5.71}}{2} = \frac{-4.66 \pm 2.38}{2}$$

$$= -1.14, -3.52$$

$$I_D = \frac{-V_{GS}}{1K} = \frac{1.14}{1K} = 1.14 \text{ mA} \approx I_C$$

$$I_C = 1.14 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1.14}{100}\text{mA} = 11.4 \text{ μA}$$

KVA at i/P :

$$V_{CC} = I_B \times 200K + V_{BE} + V_D$$

$$12 = 11.4 \text{ μA} \times 200K + 0.7 + V_D$$

$$12 - 2.28 - 0.7 = V_D$$

$$V_D = 9.02 \text{ V}$$

End of Solution

Q.7 (b) A conducting wire has resistivity of 1.57×10^{-8} Ω-m at room temperature. There are 5.85×10^{28} number of conducting electrons per m^3 for the material at room temperature. For an electric field of 1.1 V/cm along the wire, calculate the (i) drift velocity, (ii) relaxation time, (iii) mobility and (iv) mean free path for the conducting electrons in the material.
(Assume charge of electron = 1.609×10^{-19} C, mass of electron = 9.11×10^{-31} kg, velocity of electrons $v = 3 \times 10^8$ m/s and isotropic scattering).

[20 marks : 2025]

Solution:

Given :

Resistivity of wire, $\rho = 1.57 \times 10^{-8}$ Ω-m

Conducitivty, $\sigma = \frac{1}{\rho} = 6.369 \times 10^7$ (Ω-m)$^{-1}$

Number of electrons, $n = 5.85 \times 10^{28}$ m^3

Electric field, $E = 1.1$ V/cm $= 1.1 \times 10^2$ V/m

(i) Drift velocity (V_d) :

$$\text{Conductivity, } \sigma = ne\left(\frac{V_d}{E}\right)$$

$$V_d = \frac{\sigma E}{ne} = \frac{6.369 \times 10^7 \times 1.1 \times 10^2}{5.85 \times 10^{28} \times 1.609 \times 10^{-19}}$$

$$V_d = 0.7474 \text{ m/sec} \quad ...(1)$$

(ii) Relaxation Time (τ_c) :

Relation : $V_d = \left(\frac{q.E}{m_e}\right) \cdot \tau_c$

$$\tau_c = \frac{V_d \cdot m_e}{q.E} = \frac{V_d \cdot m_e}{e.E}$$

On putting the values, $\tau_c = \dfrac{0.7474 \times 9.11 \times 10^{-31}}{1.609 \times 10^{-19} \times 1.1 \times 10^{2}}$

$\tau_c = 3.863 \times 10^{-14}$ sec ...(2)

(iii) Mobility (μ) :

$$\mu = \frac{V_d}{E}$$

On putting the values from eqn. (1)

$$\mu = \frac{0.7474}{1.1 \times 10^2} = 6.794 \times 10^{-3}$$

$$\mu = 6.794 \times 10^{-3} \text{ m}^2\text{/V-sec}$$

(iv) Mean Free Path (l_e) : $l_e = V_d \cdot \tau_c$

On putting the values from eqn. (1) and (2)

$$l_e = 0.7474 \times 3.863 \times 10^{-14}$$

$$l_e = 2.887 \times 10^{-14} \text{ meter}$$

End of Solution

Q.7 (c) (i) Differentiate between isolated I/O and memory-mapped I/O with their advantages and disadvantages.

(ii) Represent the following numbers and arithmetic operations given in the table :

Numbers/ Operations	8-bit signed magnitude	1's complement (8-bit)	2's complement
+68			
–83			
(+68) + (–83)			
(–68) + (+83)			

[8 + 12 marks : 2025]

Solution:

(i)
- I/O addressing can be done in two types, i.e., isolated and memory mapped I/O.
- It is used when the application to be designed is large, i.e., more memory space is required for storing data, programs, temporary data and ISR's.
- So, a separate 8-bit port address is allocated for I/O devices.
- $2^8 = 256$, i.e., 256 i/p and 256 o/p devices can be possible.
- Instructions used are IN 8-bit port address and OUT 8-bit port address.
- Relevant control signals are $\overline{MEMR}$ and $\overline{IOR}$, $\overline{IOW}$.

Advantages :
- More memory space can be utilized for programs and data.
- I/O is separately addressed by IN and OUT instructions.
- Data accessing speed does not depend on memory delay when accessed from I/O device.
- Less hardware.

Disadvantages :
- I/O addresses are limited as address is 8-bit.
- 256 addresses should be stored between I/P and O/P devices.

Memory Mapped I/O :
- It is used when the application is small, i.e., less memory space is required for storing programs, data, temporary data (ISR) (Interrupt Service Routines).
- As address of I/O is same as memory address (e.g., 16 bits in 8085, 20 bits in 8086).
- I/O also has same length of address for memory. Total address space "2 address bits", is shared among memory and I/O.
- All instructions valid for memory are valid for I/O also.
- Relevant control signals are $\overline{MEMR}$ and $\overline{MEMW}$.

Advantages :
- Processor treats memory or I/O address as similar one, as they share address space.
- I/O is selected as memory, identity by $IO/\overline{M}$ = 0; w.r.t. 1 = 8085 cpu.

Disadvantages :
- Hardware required is more in order to differentiate between memory and I/O address.
- Program and data space is limited if I/O is one more.

(ii)

	8-bit sign magnitude	1's comp. (8-bit)	2's comp. (8-bit)
+68 :	01000100	01000100	01000100
–83 :	11010011	10101100	10101101
(+68) + (–83)			
–15 :	10001111	11110000	11110001
(–68) + (+83)			
+15 :	00001111	00001111	00001111

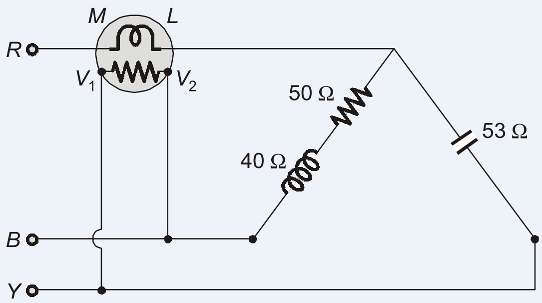
Weights :	128	64	32	16	8	4	2	1	
	0	1	0	0	0	1	0	0	: +68
	0	1	0	1	0	0	1	1	: +83

83 : 01010011
↓
–83 : 10101100 : 1's comp.
+ 1
–83 : 10101101 : 2's comp.

15 : 00001111
↓
–15 : 11110000 : 1's comp.
+ 1
–15 : 10101101 : 2's comp.

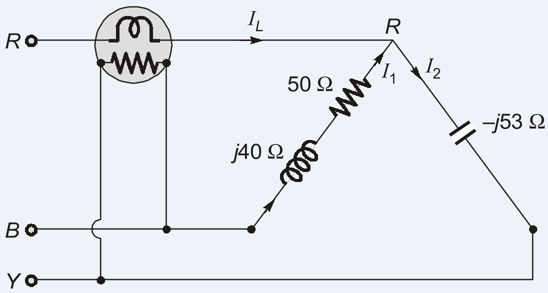
End of Solution

Q.8 (a) Find the reading of the wattmeter when the network shown is connected to a symmetrical 440 V, 3-phase supply. Neglect all losses in the instrument. The phase sequence is RYB. Also, draw the phasor diagram of the network.



[20 marks : 2025]

Solution:



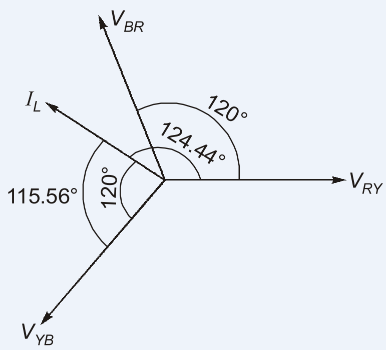
$$V_{supply} = 440 \text{ V}$$

Wattmeter reading, $V_{YB} . I_{line} \cos\phi$

V_{YB} I_{line}

$$I_2 = \frac{V_{RY}}{-j53} = \frac{440\angle 0^\circ}{-j53} = j8.3018$$

$$I_1 = \frac{V_{BR}}{50+j40} = \frac{440\angle -240^\circ}{50+j40} = 6.8716\angle 81.34$$



$$I_L + I_1 = I_2$$

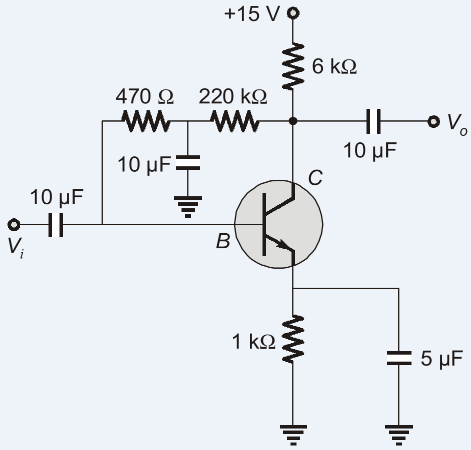
$$I_L = I_2 - I_1 = 1.8292\angle 124.44$$

$$W = (440) \times 1.892 \times \cos 115.56$$

$$W = -359.1785 \text{ Watts}$$

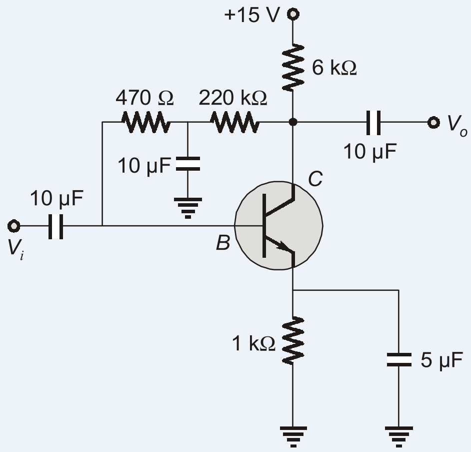
End of Solution

Q.8 (b) Determine the collector voltage V_C of the silicon transistor circuit shown in the figure below, if (i) β = 100 and (ii) β = 50 :



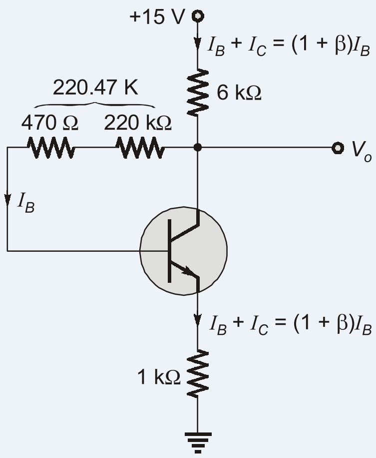
[20 marks : 2025]

Solution:



(i) $\beta = 100$

DC Mode 1 :



KVL at i/P

$$15 = (1+\beta)I_B \times 6K + 220.47K \times I_B + 0.7 + (1+\beta)I_B \times 1K$$

$$I_B = \frac{15-0.7}{220.47K + (1+100)\times 6K + (1+100)\times 1K}$$

$$= \frac{14.3}{220.47K + 606K + 101K}$$

$$= \frac{14.3}{927.47K} = 15.41\ \mu A$$

$$V_c = 15 - 6(1+100) \times 15.41 \times 10^{-3}$$

$$V_c = 5.66 \text{ Volts}$$

(ii) $\beta = 50$

$$I_B = \frac{15-0.7}{220.47K + (1+50)\times 6K + (1+50)\times 1K}$$

$$= \frac{14.3}{220.47K + 306K + 51K}$$

$$= \frac{14.3}{577.47K} = 24.76\ \mu A$$

$$I_B + I_C = (1+\beta)I_B = (1+50) \times 24.76\ \mu A = 1.26 \text{ mA}$$

$$V_C = V_{CC} - (1+\beta)I_B \times R_C$$

$$= 15 - 1.26 \text{ mA} \times 6K$$

$$= 15 - 7.56 = 7.44 \text{ V}$$

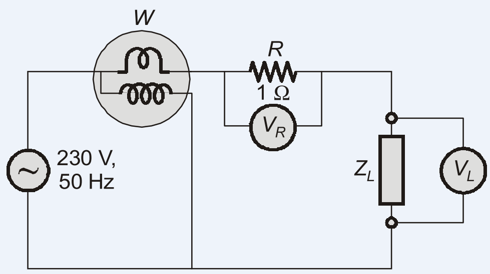
End of Solution

Q.8 (c) Voltmeters are connected across the resistance $R = 1\ \Omega$ and load impedance Z_L and a wattmeter is connected at the input side of the circuit as shown in the figure below. The source voltage is 230 V, 50 Hz and the voltmeters read V_R = 10 V, V_L = 225 V.

(i) Find the wattmeter reading, source current and input power factor with the same supply voltage and frequency.

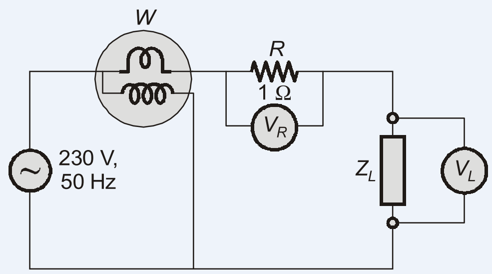
(ii) Find the voltmeter and wattmeter readings when the supply frequency is changed to 60 Hz at same supply voltage of 230 V.

(iii) Draw the phasor diagram of voltage and currents for (i) above.



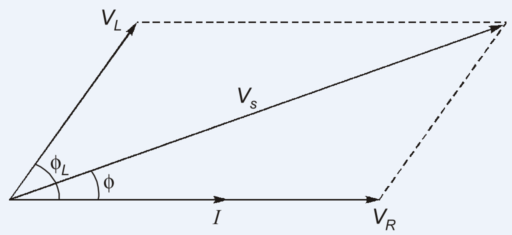
[20 marks : 2025]

Solution:



$$V_R = 10 \text{ V}, \; V_L = 225 \text{ V}, \; V_S = 230 \text{ V}, \; f = 50 \text{ Hz}$$

Phasor Diagram :



(i)
$$V_S^2 = V_R^2 + V_L^2 + 2V_R V_L \cos\phi_L$$

$$\cos\phi_L = \frac{V_S^2 - V_R^2 - V_L^2}{2V_R V_L} = \frac{230^2 - 10^2 - 225^2}{2 \times 10 \times 225} = 0.4833$$

$$\theta_L = 61.096$$

$$I = \frac{V_R}{R} = \frac{10}{1} = 10 \text{ A}$$

Wattmeter reading,
$$\omega = V.I \cos\phi$$

$$|Z_L| = \frac{V_L}{I} = \frac{225}{10} = 22.5$$

$$Z_L = 22.5\angle 61.096°$$

$$Z_L = 22.5 + 561.096 + j22.5 \sin 61.096°$$

$$Z_L = 10.3875 + j19.697$$

$$Z_T = R + 10.875 + j19.697$$

$$Z_T = 1 + 10.875 + j19.697$$

$$Z_T = 11.875 + j19.697$$

$$\phi_S = \tan^{-1}\frac{19.697}{11.875} = 58.915°$$

Wattmeter reading,
$$\omega = V_S I_L \cos\phi$$
$$= 230 \times 10 \times \cos 58.915°$$
$$\omega = 1187.514 \text{ Watts}$$

Input-Power Factor : $\cos(58.915) = 0.5163$ lag

(ii)

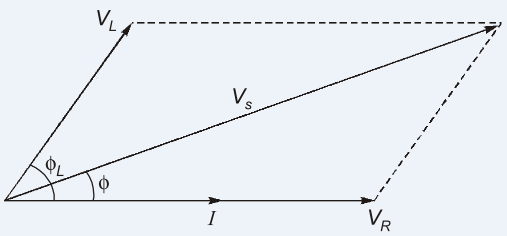
$$Z_L = 10.875 + j19.697$$
$$X_L = 19.697$$
$$L = \frac{19.697}{2\pi(50)} = 0.0627 \text{ H}$$

Frequency is 60 Hz.

$$X_L = 2\pi(60)(0.0627) = 23.636\ \Omega$$
$$Z_T = 1 + 10.875 + j23.636$$
$$Z_T = 11.875 + j23.636$$
$$I = \frac{V_S}{Z_T} = \frac{230\angle 0^\circ}{11.875 + j23.636}$$
$$I = 8.6952\angle{-63.32} \text{ A}$$

$$\text{Wattmeter reading} = 230 \times 8.6952 \times \cos 63.32^\circ$$
$$= 897.957 \text{ Watts}$$
$$\text{Voltmeter reading} = 8.6952 \times 1 = 8.6952 \text{ V}$$

(iii) Phasor diagram :



End of Solution

■■■■