2020

MADE EASY WORKBOOK

Detailed Explanations of Try Yourself *Questions*

Civil Engineering

Strength of Materials

MADE EASY
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Publications

2 Simple Stress-Strain & Elastic Constants

Detailed Explanation of Try Yourself Questions

T1 : Solution

Initially, due to load 'W' each steel bar will carry $\frac{W}{4}$ load

$\therefore$ Expansion of steel bars $= \frac{WL}{4AE}$

$\therefore$ The temperature 't°' should be raised such that a expansion of '$\delta + \frac{WL}{4AE}$' is observed in the steel rod

$$\Rightarrow \quad (L-\delta)\,\alpha\, t = \delta + \frac{WL}{4AE}$$

but $\quad L >> \delta$

$$\Rightarrow \quad L\,\alpha\, t = \delta + \frac{WL}{4AE} \quad .....(i)$$

Now, the steel rod gets attached at the centre of steel plate, the force in each steel bar gets reduced by 20%.

Therefore, The load caried by each steel bar $= \frac{W}{5}$

Remaining load will be carried by steel rod $= W - \frac{4W}{5} = \frac{W}{5}$

After connection the steel rod will not get detached from square plate, so the length of four steel bars and steel rod will be same in the end

$$\Rightarrow \quad L + \frac{WL}{5AE} = L - \delta + \frac{W(L-\delta)}{5aE}$$

but $\quad L >> \delta$

$$\Rightarrow \qquad \frac{WL}{5AE} = -\delta + \frac{WL}{5aE}$$

$$\Rightarrow \qquad \delta = \frac{WL}{5E}\left(\frac{1}{a} - \frac{1}{A}\right) \qquad .....(ii)$$

from (i) and (ii), we get

$$t = \frac{W}{5E\alpha}\left(\frac{1}{a} + \frac{1}{4A}\right)$$

T2 : Solution

Area of cross-section of AB (A_{AB}) = 100 mm^2

Area of cross-section of BC (A_{BC}) = 200 mm^2

Modulus of elasticity (E) = 200 kN/mm^2

Axial tensile load (P) = 50 kN

Length of portion AB (L_{AB}) = 1 m

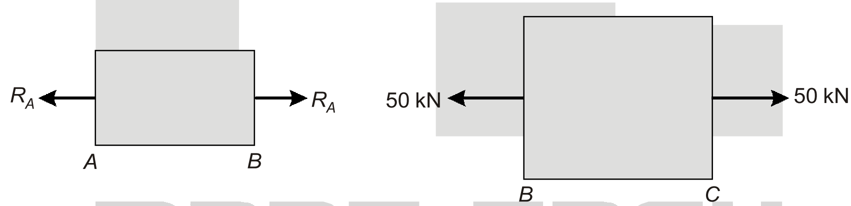
Length of portion BC (L_{BC}) = 1 m

Let R_A be the reaction at fixed end A

By drawing the FBD of the given figure we came to know that to maintain equilibrium

$$R_A - 50 = 0$$

$$R_A = 50 \text{ kN}$$



Portion AB:

$$R_A = P_{AB} = 50 \text{ kN} \quad \text{(Tensile)}$$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{50 \times 10^3}{100} = 500 \text{ N/mm}^2 \quad \text{(Tensile)}$$

$$\Delta_{AB} = \frac{P_{AB}\,L_{AB}}{A_{AB} \cdot E} = \frac{50 \times 10^3 \times 1000}{100 \times 200 \times 10^3}$$

$$= 2.5 \text{ mm} \quad \text{(elongation)}$$

Portion BC:

$$P_{BC} = 50 \text{ kN} \quad \text{(Tensile)}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{50 \times 10^3}{200} = 250 \text{ N/mm}^2 \quad \text{(Tensile)}$$

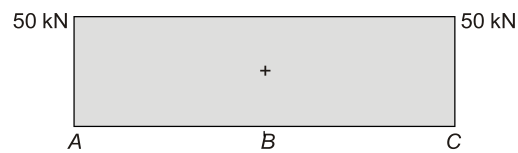
$$\Delta_{BC} = \frac{P_{BC} \cdot L_{BC}}{A_{BC} \cdot E} = \frac{50 \times 10^3 \times 1000}{200 \times 200 \times 10^3}$$

$$\Delta_{BC} = 1.25 \text{ mm} \quad \text{(elongation)}$$

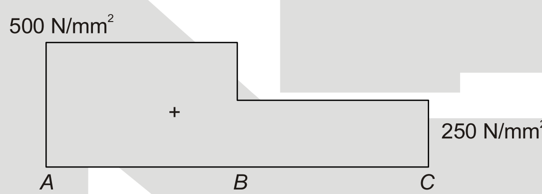
Assuming:

Tensile force = Positive

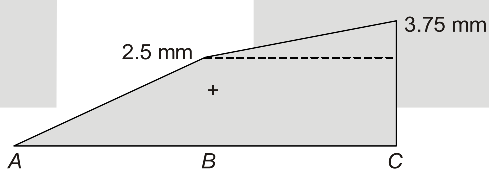
Tensile elongation = Positive



(*i*) Normal force diagram



(*ii*) Normal stress diagram



(*iii*) Elongation/Displacement diagram

T3 : Solution

(*i*) Since the cubes are confined in x-direction, hence

$$\Delta_{xA} + \Delta_{xB} + \Delta_{xC} = 0$$

where,

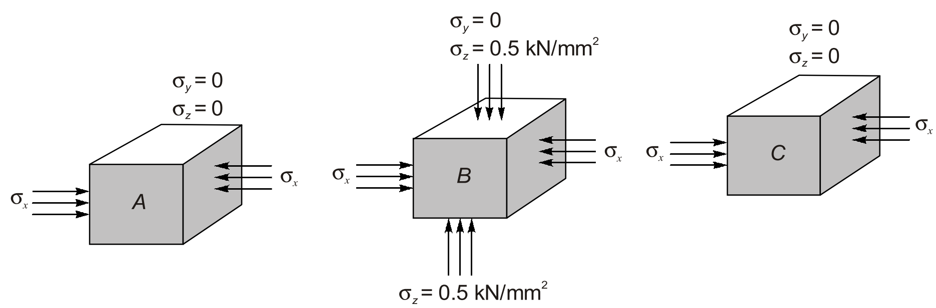
Δ_{xA} is change in length of cube A in x-direction

Δ_{xB} is change in length of cube B in x-direction

Δ_{xC} is change in length of cube C in x-direction

$$\frac{\Delta_{xA}}{L} + \frac{\Delta_{xB}}{L} + \frac{\Delta_{xC}}{L} = 0$$

$$\in_{xA} + \in_{xB} + \in_{xC} = 0 \qquad ...(i)$$



From above FBD, we have,

$$\epsilon_{xA} = -\frac{\sigma_x}{E_A}; \quad \epsilon_{xB} = -\frac{\sigma_x}{E_B} + \mu_B\frac{\sigma_z}{E_B}; \quad \epsilon_{xC} = -\frac{\sigma_x}{E_C}$$

Putting these values in (*i*), we get

$$\left[\frac{-\sigma_x}{E_A}\right] + \left[\frac{-\sigma_x}{E_B} + \mu_B\frac{\sigma_z}{E_B}\right] + \left[\frac{-\sigma_x}{E_C}\right] = 0 \qquad [\because E_A = E_C]$$

$$-\frac{2\sigma_x}{E_A} - \frac{\sigma_x}{E_B} + \mu_B\frac{\sigma_z}{E_B} = 0$$

$$\sigma_x\left[\frac{2}{E_A} + \frac{1}{E_B}\right] = \frac{\mu_B\sigma_z}{E_B}$$

$$\sigma_x = \frac{\mu_B\sigma_z}{2\cdot\frac{E_B}{E_A} + 1} = \frac{0.3\times 0.5}{\left(2\times\frac{200}{150}\right) + 1} = 0.041 \text{ kN/mm}^2$$

(*ii*)

$$\epsilon_{xB} = -\frac{\sigma_x}{E_B} + \frac{\mu_B\sigma_z}{E_B} = \frac{-0.041}{E_B} + \frac{0.3\times 0.5}{E_B} = 5.45\times 10^{-4}$$

$$\epsilon_{yB} = \frac{+\mu_B\sigma_x}{E_B} + \frac{\mu_B\sigma_z}{E_B} = \frac{0.3\times 0.041}{E_B} + \frac{0.3\times 0.5}{E_B} = 8.11\times 10^{-4}$$

$$\epsilon_{zB} = -\frac{\sigma_z}{E_B} + \frac{\mu\sigma_x}{B} = \frac{-0.5}{E_B} + \frac{0.3\times 0.041}{E_B} = -2.44\times 10^{-3}$$

(*iii*) Volumetric strain,

$$\epsilon_V = \epsilon_{xB} + \epsilon_{yB} + \epsilon_{zB}$$
$$= 5.45\times 10^{-4} + 8.11\times 10^{-4} - 2.44\times 10^{-3} = -1.084\times 10^{-3}$$

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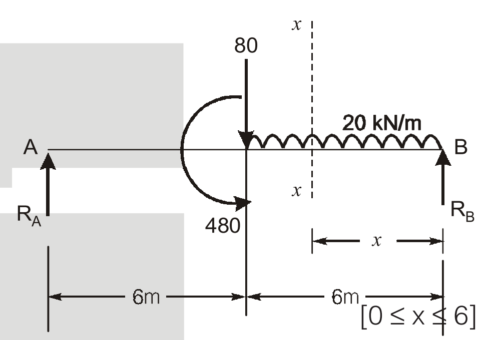
3 Shear Force and Bending Moment

Detailed Explanation of Try Yourself Questions

T1 : Solution



$$R_A + R_B = 200$$

$$\sum M_A = 0$$

$$\Rightarrow 12R_B - 120 \times 9 - 80 \times 6 + 480 = 0$$

$$R_B = 90 \text{ kN}$$

$$R_A = 110 \text{ kN}$$

Taking x from left of support B,

$$M_x = \left[R_B x - \frac{wx^2}{2}\right] \qquad [0 \le x \le 6]$$

$$= 90x - 10x^2$$

Now, for maxima or minima,

$$\frac{dM}{dx} = (90 - 10 \times 2x) = 0$$

$$x = 4.5 \text{ m}$$

$$\frac{d^2M}{dx^2} = -20,$$

$$\frac{d^2M}{dx^2} < 0$$

Hence, bending moment is maximum at x = 4.5 m

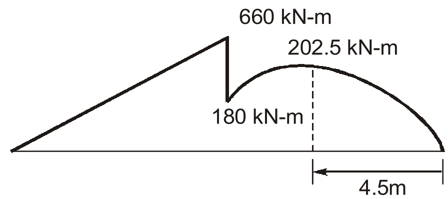
$$M_{max} = 90 \times 4.5 - 10 \times (4.5)^2 = 202.5 \text{ kN-m}$$

Taking x from right of support A,

$$M_x = R_A.x \qquad [0 \le x \le 6]$$

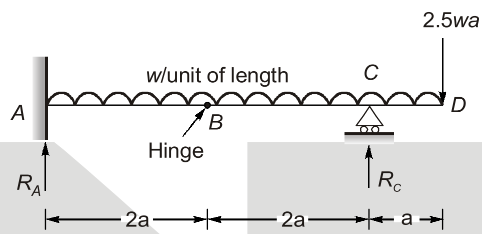
$$M = 110 \times 6 \qquad \text{at } x = 6$$

$$= 660 \text{ kN-m}$$



T2 : Solution

Let R_A and R_C are the vertical reactions at supports A and C respectively. Since there is no horizontal load so horizontal reaction at fixed support A will be zero.



Reactions:

$$\Sigma F_y = 0$$
$$R_A + R_C = 5wa + 2.5wa$$
$$R_A + R_C = 7.5wa \quad \text{...(i)}$$

Since there is an internal hinge at B, therefore moment at B either from right or left is zero. Taking moment about B from right, we get

$$\Sigma M_B = 0$$
$$R_C \times 2a - (2.5wa \times 3a) - (w \times 3a \times 1.5a) = 0$$
$$\Rightarrow \quad R_C = 6\,wa$$

Putting value of R_C in (i), we get

$$R_A = 7.5wa - 6wa$$
$$R_A = 1.5wa$$

For SFD:

Portion DC :

$$S_x(\text{x from D}) = 2.5wa + wx \quad [0 \le x < a]$$

at x = 0, $S_D = 2.5wa$

at x = a, S_C (just right of C) $= 2.5wa + wa = 3.5wa$

Portion CA:

$$S_x(\text{x from D}) = 2.5wa + wx - 6wa \quad [a < x < 5a]$$

at x = a, S_C (just left of C) $= 2.5wa + wa - 6wa = -2.5wa$

at x= 5a, S_A (at support A) $= 2.5wa + 5wa - 6wa = +1.5wa$

If $S_x = 0$, then we have (in CA)

$$2.5wa + wx - 6wa = 0$$
$$3.5wa = wx$$
$$x = 3.5a \text{ from D or } 1.5a \text{ from A}$$

For BMD:

Portion DC:

$$M_x(\text{x from D}) = -2.5wax - \frac{wx^2}{2} \qquad [0 \le x \le a]$$

$$M_x = -2.5wax - \frac{wx^2}{2} \qquad \text{(Parabola)}$$

at x = 0, $M_D = 0$

at x = a, $M_C = -2.5wa^2 - \frac{wa^2}{2} = -3wa^2$

Portion CA:

$$M_x(\text{x from D}) = -2.5wax - \frac{wx^2}{2} + R_C(x-a) \qquad [a \le x \le 5a]$$

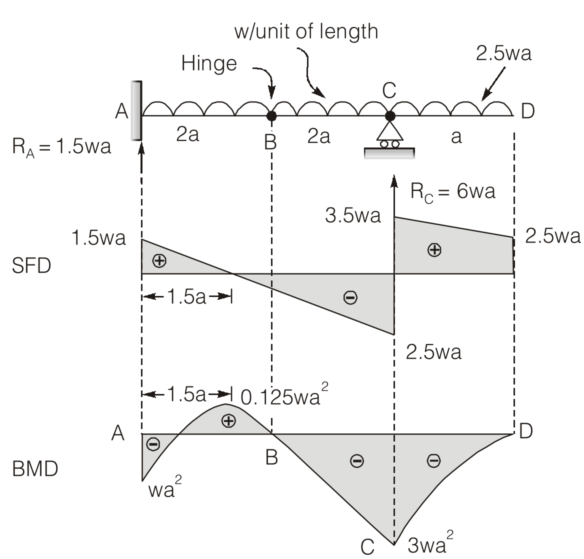
$$M_x = -2.5wax - \frac{wx^2}{2} + 6wa(x-a) \qquad \text{(Parabola)}$$

at x = a, $M_C = -2.5wa^2 - \frac{wa^2}{2} = -3wa^2$

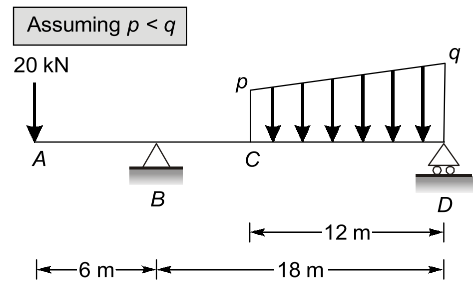
at x = 5a.
$$M_A = -2.5wa(5a) - \frac{w}{2}(5a)^2 + 6wa(5a-a)$$
$$= -12.5wa^2 - 12.5wa^2 + 24wa^2$$
$$= -wa^2$$

For M_{max},
$$\frac{dM_x}{dx} = 0$$
$$-2.5\,wa - wx + 6wa = 0$$
$$x = 3.5a$$
$$M_{max}(\text{ at } x = 3.5a) = -2.5wa(3.5a) - \frac{w}{2}(3.5a)^2 + 6wa(3.5a-a)$$
$$= (-8.75 - 6.125 + 15)\,wa^2 = 0.125wa^2$$

There are two points of contraflexure and they are D_1 and D_2 at a distance 'a' and '2a' respectively from fixed end.



T3 : Solution

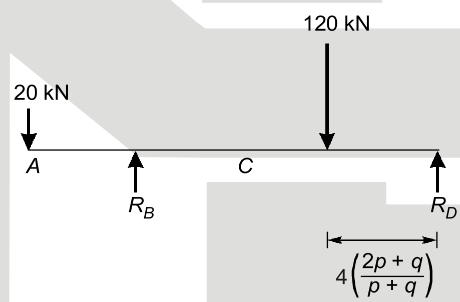


$$\text{Total distributed load} = \frac{12}{2}(p+q) = 120$$

$$\Rightarrow \quad p + q = 20 \text{ kN/m}$$

Distance of centroid of distributed load from support D = $\frac{12}{3}\left(\frac{2p+q}{p+q}\right) = 4\left(\frac{2p+q}{p+q}\right)$

Assuming R_B and R_D to be the support reaction at B and D respectively



$$\sum M_D = 0$$

$$\Rightarrow \quad R_B \times 18 - 20 \times 24 - 120 \times 4\left[\frac{2p+q}{p+q}\right] = 0$$

$$\text{As, } p + q = 20$$

$$\Rightarrow \quad R_B = \frac{1}{18}\left[480 + 480\left(\frac{p+20}{20}\right)\right] \quad \text{.....(i)}$$

$$\sum M_A = 0$$

$$\Rightarrow \quad R_B \times 6 + R_D \times 24 - 120\left[24 - 4\left(\frac{2p+q}{p+q}\right)\right] = 0$$

$$R_B = R_D \quad \text{(given)}$$

$$\text{As, } p + q = 20$$

$$\Rightarrow \quad R_B = \frac{120}{30}\left[24 - 4\left(\frac{p+20}{20}\right)\right] \quad \text{.....(ii)}$$

Equating value of 'R_B' from (i) and (ii), we get

$$\Rightarrow \quad \frac{1}{18}\left[480+480\left(\frac{p+20}{20}\right)\right] = \frac{120}{30}\left[24-4\left(\frac{p+20}{20}\right)\right]$$

$$\Rightarrow \quad 480 + 24(p+20) = 1728 - \frac{72}{5}(p+20)$$

$$\Rightarrow \quad 120p + 2400 + 2400 = 8640 - 72p - 1440$$

$$\Rightarrow \quad p = 12.5 \text{ kN/m}$$

$$\text{and, } p + q = 20$$

$$\Rightarrow \quad q = 7.5 \text{ kN/m}$$

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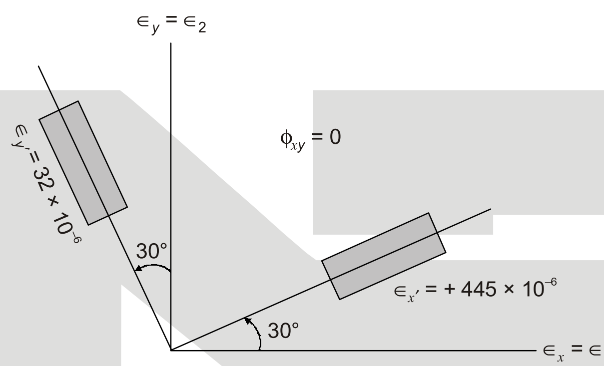
4 Principal Stress-Strain & Theories of Failure

Detailed Explanation of Try Yourself Questions

T1 : Solution



Let normal principal stress in x and y direction are ϵ_1 and ϵ_2 respectively.

$$\epsilon_x' = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_1 + \frac{\phi_{xy}}{2}\sin 2\theta_1$$

Here, $\theta_1 = 30°$ and $\epsilon_x = \epsilon_1, \epsilon_y = \epsilon_2$

$$445 \times 10^{-6} = \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2}\cos(2 \times 30°) + 0 \qquad [\because \phi_{xy} = 0]$$

$$3\epsilon_1 + \epsilon_2 = 1780 \times 10^{-6} \qquad \text{...(i)}$$

We know that

$$\epsilon_1 + \epsilon_2 = \epsilon_x' + \epsilon_y'$$

$$\epsilon_1 + \epsilon_2 = +445 \times 10^{-6} - 32 \times 10^{-6}$$

$$\epsilon_1 + \epsilon_2 = +413 \times 10^{-6} \qquad \text{...(ii)}$$

From eq. (i) and (ii)

$$\epsilon_1 = +683.5 \times 10^{-6}$$

$$\epsilon_2 = -270.5 \times 10^{-6}$$

Principal stresses may be calculated as

$$\sigma_1 = \frac{E}{1-\mu^2}(\epsilon_1 + \mu \epsilon_2) = \frac{2.1\times10^5}{1-(0.3)^2}\left[683.5\times10^{-6} + 0.3\times(-270.5\times10^{-6})\right]$$

$$= 139.00 \text{ N/mm}^2$$

and

$$\sigma_2 = \frac{E}{1-\mu^2}(\epsilon_2 + \mu \epsilon_2) = \frac{2.1\times10^5}{1-(0.3)^2}\left[-270.5\times10^{-6} + 0.3\times683.5\times10^{-6}\right]$$

$$= -15.10 \text{ N/mm}^2$$

T2 : Solution

When a shaft is subjected to combined bending and torsion, the magnitude of principal stresses is given by

$$\sigma_1/\sigma_2 = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2+T^2}\right]$$

where, σ_1 = principal stress
σ_2 = minor principal stress
M = bending moment = 20 kNm
T = Torque = 40 kNm

$$\therefore \quad \sigma_1 = \frac{16}{\pi D^3}\left[M + \sqrt{M^2+T^2}\right]$$

and

$$\sigma_1 = \frac{16}{\pi D^3}\left[20 + \sqrt{20^2+40^2}\right]\times10^6$$

$$\sigma_1 = \frac{329.62\times10^6}{D^3}$$

and

$$\sigma_2 = \frac{16}{\pi D^3}\left[M - \sqrt{M^2+T^2}\right] = \frac{16}{\pi D^3}\left[20 - \sqrt{20^2+40^2}\right]\times10^6$$

$$= -\frac{125.90\times10^6}{D^3}$$

(i) According the maximum shear stress theory

$$\tau_{max} \le \frac{(\sigma_y/\text{FOS})}{2}$$

$$\frac{\sigma_1 - \sigma_2}{2} \le \frac{(\sigma_y/\text{FOS})}{2}$$

$$\therefore \quad \left[\frac{329.62+125.90}{D^3}\right]\times10^6 \le \left(\frac{250}{2}\right)$$

$$D^3 \ge 3644160$$

$$\therefore \quad D \ge 153.88 \text{ mm}$$

Hence minimum diameter of shaft

$$D = 153.88 \text{ mm}$$

(ii) According to maximum strain energy theory

$$U \le \frac{(\sigma_y/FOS)^2}{2E} \qquad \ldots(i)$$

$$U = \frac{1}{2E}\left(\sigma_1^2+\sigma_2^2-2\mu\sigma_1\sigma_2\right)$$

$$= \frac{1}{2E}\left[\left(\frac{329.62\times10^6}{D^3}\right)^2+\left(\frac{-125.90\times10^6}{D^3}\right)^2+\frac{2\times0.3\times329.62\times125.90\times10^{12}}{D^6}\right]$$

$$= \frac{1}{2E}\left[\frac{149399.65\times10^{12}}{D^6}\right]$$

$$\frac{1}{2E}\left[\frac{149399.65\times10^{12}}{D^6}\right] \le \frac{1}{2E}\left(\frac{250}{2}\right)^2$$

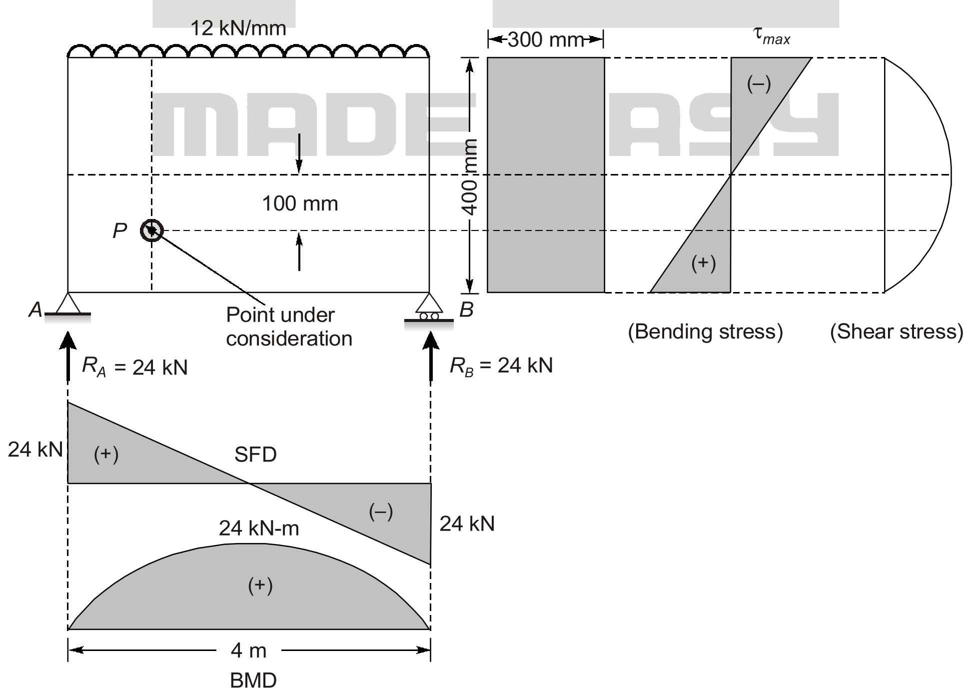
$$\frac{149399.65\times10^{12}}{D^6} \le \left(\frac{250}{2}\right)^2$$

$$D^6 \ge \frac{149399.65\times10^{12}}{(125)^2}$$

$$D \ge 145.69 \text{ mm}$$

Hence minimum diameter of shaft D is 145.69 mm.

T3 : Solution



Shear force in beam AB is given by

$$S_x(x \text{ from } A) = R_A - wx$$
$$= 24 - 12x$$

$$\text{Shear force at } (x = 1\text{ m}) = 24 - 12 \times 1$$
$$= 12\text{ kN}$$

Bending moment in beam AB is given by

$$M_x(x \text{ from } A) = R_A x - \frac{wx^2}{12}$$

$$= 24x - \frac{12x^2}{2} = 24x - 6x^2$$

M_x at $(x = 1.0\text{ m})$

$$M = 24 \times 1 - 6 \times 1^2$$
$$= 18\text{ kNm}$$

and MOI about neutral axis is given by

$$I = \frac{bd^3}{12} = \frac{300 \times 400^3}{12}$$

$$I = 1.6 \times 10^9\text{ mm}^4$$

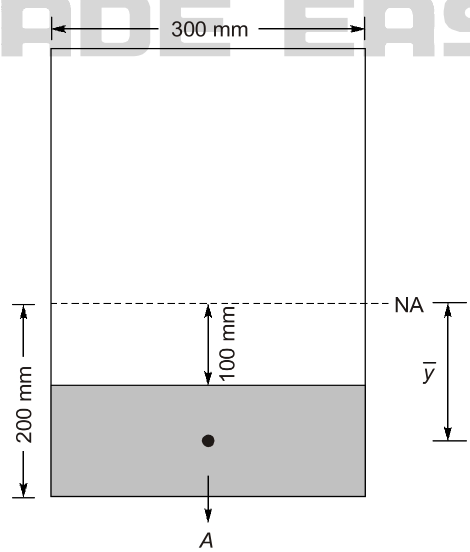
Now we have $M = 18$ kN, $S = 12$ kN and $I = 1.6 \times 10^9\text{ mm}^4$. Hence normal and shear stresses at $y = 100$ below NA can be calculated as

Bending stresses may be given as

$$\sigma = \frac{M}{I}y = \frac{18 \times 10^6 \times 100}{1.6 \times 10^9} = 1.125\text{ N/mm}^2\text{ (tensile)}$$

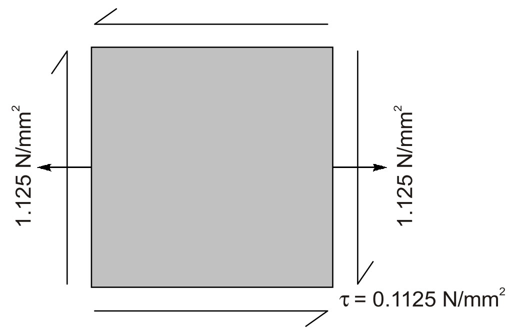
and shear stress τ at a point P ($y = 100$ mm) below NA may be given as

$$\tau = \frac{SA\bar{y}}{Ib} = \frac{12 \times 10^3 \times 100 \times 300 \times 150}{1.6 \times 10^9 \times 300} = -0.125\text{ N/mm}^2$$



Note: *Positive shear will produce negative shear stress element*

Now point can be considered as stress element shown below.



Here we have

$$\sigma_x = 1.125 \text{ N/mm}^2$$
$$\sigma_y = 0$$
$$\tau = -0.1125 \text{ N/mm}^2$$

The principal stresses may be given as

$$\sigma_1/\sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{1.125 + 0}{2} \pm \sqrt{\left(\frac{1.125 - 0}{2}\right)^2 + (-0.1125)^2}$$
$$= 0.5625 \pm 0.5736$$

$\therefore$ $$\sigma_1 = 1.136 \text{ N/mm}^2$$

and $$\sigma_2 = -0.011 \text{ N/mm}^2$$

For direction of principal stresses, we have

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-2 \times 0.1125}{1.125 - 0} = -0.2$$
$$\theta_{P_1} = -5.65° \text{ (clockwise)}$$

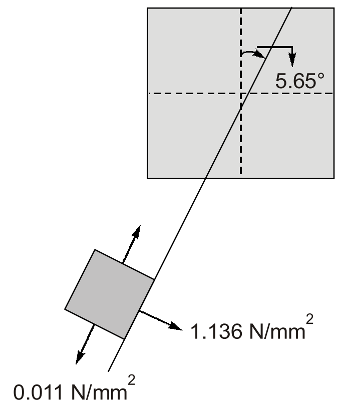
and $$\theta_{P_2} = 90° + \theta_{P_1}$$
$$= -90° - 5.65°$$
$$= -95.65° \text{ (clockwise)}$$

Check: Principal stress normal to $\theta_{P_1} = -5.65°$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_{P_1} + \tau_{xy} \sin 2\theta_{P_1}$$
$$= \frac{1.125 + 0}{2} + \frac{1.125 - 0}{2}\cos(2 \times -5.65°)$$
$$-0.1125 \sin(2 \times -5.65°)$$
$$= 0.5625 + 0.5516 + 0.0220$$
$$\sigma_x' = 1.136 \text{ N/mm}^2 = \sigma_1$$

and $$\sigma_2 = -0.011 \text{ N/mm}^2$$

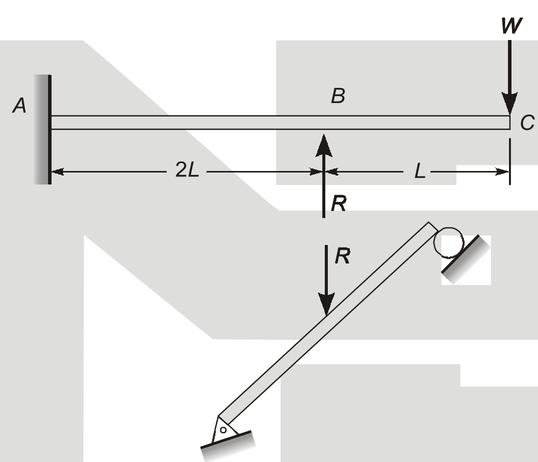
Hence major principal plane is 5.65° clockwise and minor principal plane is 95.65° clockwise from the vertical



5 Deflection of Beams

Detailed Explanation of Try Yourself Questions

T1 : Solution



According to Maxwell's reciprocal theorem,

Deflection at B due to load W at C = Deflection at C due to load W at B

$$\therefore \text{Deflection of B in beam ABC} = \delta_1 = \frac{W(2L)^3}{3EI} + \frac{W(2L)^2}{2EI} \times L - \frac{R(2L)^3}{3\,EI}$$

$$\delta_2 = \frac{R(2L)^3}{48\,EI}$$

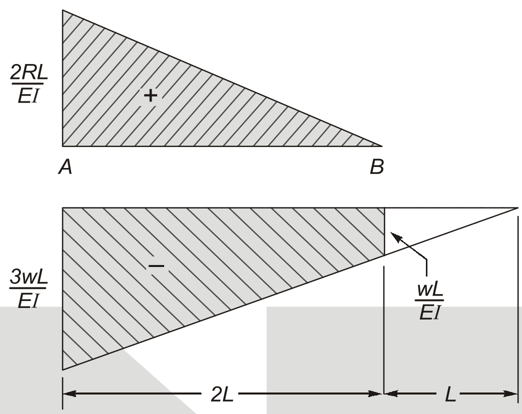
$$\delta_1 = \delta_2$$

$$\frac{W(2L)^3}{3EI} + \frac{W(2L)^2}{2EI} \times L - \frac{R(2L)^3}{3\,EI} = \frac{R(2L)^3}{48\,EI}$$

$$R = \frac{28}{17} W$$

$$\delta_2 = \delta_1 = \delta_B = \left[\frac{28}{17}W\right]\frac{(8L^3)}{48EI} = \frac{14}{51}\frac{WL^3}{EI}$$

By area-moment method, considering,hogging bending moment to be negative & sagging bending moment to be positive



$$\theta_B = \left[\frac{1}{2}\times\frac{2RL}{EI}\times 2L\right] - \frac{1}{2}\left[\frac{(3wL+wL)}{EI}\times 2L\right]$$

$$= \frac{56}{17}\frac{wL^2}{EI} - \frac{4wL^2}{EI} = -\frac{12wL^2}{17EI}$$

– ve sign ⇒ closckwise

T2 : Solution

Using strain energy method, we can obtain the final distance between *AA*.

Since no load is acts at *AA*. Hence apply pseudo load *Q* at ends *A*.

Taking outer face as reference.

Total strain energy, $U = 2\,U_{AC} + 2\,U_{BC} + U_{CC}$

$$U_{AC} = \int_0^a \frac{(Qy)^2 dy}{2EI} = \left[\frac{Q^2y^3}{6EI}\right]_0^a$$

$$U_{AC} = \frac{Q^2a^3}{6EI}$$

$$U_{BC} = \int_0^{2a} \frac{(-Py)^2 dy}{2EI} = \left[\frac{P^2y^3}{6EI}\right]_0^{2a}$$

$$U_{BC} = \frac{4P^2a^3}{3EI}$$

$$U_{CC} = \int_0^a \frac{[-(Qa+2Pa)]^2 dx}{2EI} = \frac{(Qa+2Pa)^2}{2EI}[x]_0^a$$

$$U_{CC} = \frac{(Qa + 2Pa)^2}{2EI} \cdot a$$

∴ Total strain energy,

$$U = 2\,U_{AC} + 2\,U_{BC} + U_{CC}$$

$$= \frac{2 \times Q^2 a^3}{6EI} + \frac{2 \times 4P^2 a^3}{3EI} + \frac{(Qa + 2Pa)^2 a}{2EI}$$

$$= \frac{2Q^2 a^3}{6EI} + \frac{16P^2 a^3}{6EI} + \frac{3a(Q^2 a^2 + 4P^2 a^2 + 4PQa^2)}{6EI}$$

$$U = \frac{28P^2 a^3 + 5Q^2 a^3 + 12PQa^3}{6EI}$$

$$\frac{\partial U}{\partial Q} = \frac{10Qa^3 + 12Pa^3}{6EI}$$

$$\left(\frac{\partial U}{\partial Q}\right)_{Q=0} = \frac{12Pa^3}{6EI} = \frac{2Pa^3}{EI}$$

Thus, final distance by which AA moves away from each other is $\frac{2Pa^3}{EI}$.

T3 : Solution

Let 'S_x' be the shear force at a distance 'x' from the fixed end

$$\frac{dS_x}{dx} = -w = -q_0\left(\frac{L^2 - x^2}{L^2}\right)$$

$$\Rightarrow \quad S_x = \frac{-q_0}{L^2}\left[L^2 x - \frac{x^3}{3}\right] + C_1$$

At free end, i.e., $x = L$

Shear force is 0

$$\Rightarrow \quad C_1 = \frac{2}{3} q_0 L$$

$$\Rightarrow \quad S_x = -q_0 x + q_0 \frac{x^3}{3L^2} + \frac{2}{3} q_0 L$$

Let 'M_x' be the moment at a distance 'x' from the fixed end

$$\Rightarrow \quad \frac{dM_x}{dx} = S_x = -q_0 x + q_0 \frac{x^3}{3L^2} + \frac{2}{3} q_0 L$$

$$\Rightarrow \quad M_x = \frac{-q_0 x^2}{2} + q_0 \frac{x^4}{12L^2} + \frac{2}{3} q_0 L x + C_2$$

At free end, i.e., $x = L$
Moment is 0

$$\Rightarrow \quad C_2 = -\frac{q_0 L^2}{4}$$

$$\Rightarrow \quad M_x = \frac{q_0 x^4}{12L^2} - \frac{q_0 x^2}{2} + \frac{2}{3} q_0 L x - \frac{q_0 L^2}{4}$$

Applying double integration method

$$EI\frac{d^2y}{dx^2} = M_x, \quad \text{where y is the deflection of beam with x distance}$$

$$\Rightarrow \quad EI\frac{dy}{dx} = \frac{q_0 x^5}{60L^2} - \frac{q_0 x^3}{6} + \frac{q_0 L x^2}{3} - \frac{q_0 L^2 x}{4} + C_3$$

$\frac{dy}{dx} = 0$ at fixed end, i.e, $x = 0$ $\Rightarrow C_3 = 0$

$$\Rightarrow \quad EI\,(y) = \frac{q_0 x^6}{360L^2} - \frac{q_0 x^4}{24} + \frac{q_0 L x^3}{9} - \frac{q_0 L^2 x^2}{8} + C_4$$

$y = 0$ at fixed end, i.e., $x = 0$ $\Rightarrow C_4 = 0$

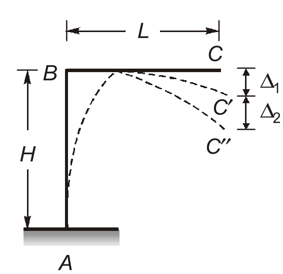
$$\therefore \quad (EI)\,y = \frac{q_0 x^6}{360L^2} - \frac{q_0 x^4}{24} + \frac{q_0 L x^3}{9} - \frac{q_0 L^2 x^2}{8}$$

Deflection at free end. Put $x = L$

$$\Rightarrow \quad y = -\frac{19}{360}\frac{q_0 L^4}{EI}$$

T4 : Solution

The beam will deflect as



Vertical deflection at C, $\Delta = \Delta_1 + \Delta_2$

Δ_1 = Deflection due to moment in *BC*

$$\Delta_1 = \frac{ML^2}{2EI} = \frac{\mu L^2}{2EI}$$

Δ_2 = Deflection due to moment in *AB*

$$\Delta_2 = \frac{MH}{EI} \times L = \frac{\mu L H}{EI}$$

$$\Delta = \Delta_1 + \Delta_2$$

$$\Rightarrow \quad \Delta = \frac{\mu L^2}{2EI} + \frac{\mu L H}{EI}$$

$$\Rightarrow \quad \Delta = \frac{\mu L}{EI}\left(\frac{L}{2} + H\right)$$

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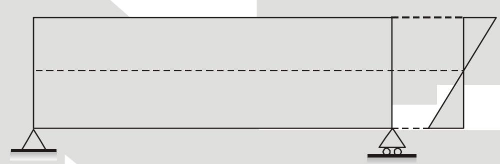
6 Bending & Shear Stresses in Beams

Detailed Explanation *of* Try Yourself Questions

T1 : Solution

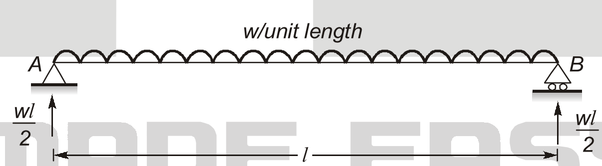
The condition given in the above problem is that the beam has same value of maximum bending stress for both UDL and concentrated load.



$$\because \qquad \sigma_{max} = \frac{M_{max}}{I} \times y_{max}$$

Now for a particular beam σ_{max} is directly proportional to M_{max}

Case 1. For UDL

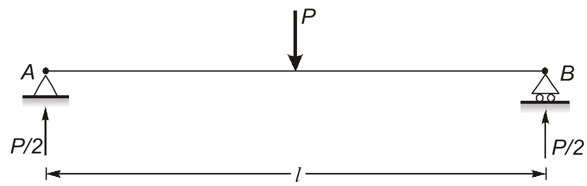


$$M_x = \frac{wl}{2}x - \frac{wx^2}{2}$$

BM will be maximum at $\qquad x = \frac{l}{2}$

$$M_{max} = \frac{wl}{2} \times \frac{l}{2} - \frac{wl^2}{8} = \frac{wl^2}{8}$$

Case 2. For concentrated load P



$$M_x = \frac{P}{2}x$$

BM will be maximum at $x = \frac{l}{2}$

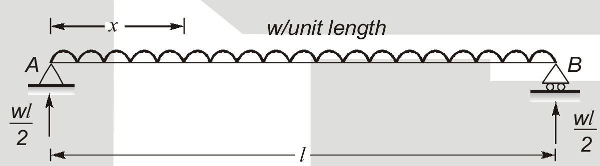
$$\therefore \quad M_{max} = \frac{P}{2} \times \frac{l}{2} = \frac{Pl}{4}$$

Now for same bending steress, M_{max} for both cases should be equal.

$$\therefore \quad \frac{wl^2}{8} = \frac{Pl}{4}$$

$$\Rightarrow \quad P = \frac{wl}{2} \qquad \ldots (1)$$

Strain energy for UDL case



$$U = \int \frac{M^2 dx}{2EI}$$

$$U_{AB} = \int_0^l \frac{\left(\frac{wl}{2}x - \frac{wx^2}{2}\right)^2 dx}{2EI}$$

$$U_{AB} = \int_0^l \frac{\frac{w^2}{4}\left(lx - x^2\right)^2 dx}{2EI}$$

$$U_{AB} = \frac{w^2}{8EI}\int_0^l \left(l^2x^2 + x^4 - 2lx^3\right) dx$$

$$U_{AB} = \frac{w^2}{8EI}\left(\frac{l^2x^3}{3} + \frac{x^5}{5} - \frac{2lx^4}{4}\right)_0^l$$

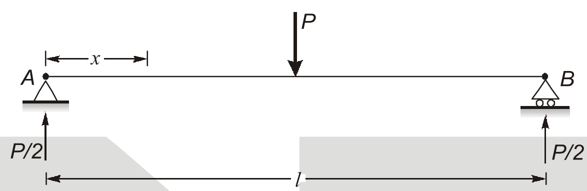
$$U_{AB} = \frac{w^2}{8EI}\left(\frac{l^5}{3} + \frac{l^5}{5} - \frac{2l^5}{4}\right)$$

$$U_{AB} = \frac{w^2}{8EI}\left[\frac{20l^5 + 12l^5 - 30l^5}{60}\right]$$

$$U_{AB} = \frac{w^2}{8EI} \times \frac{2}{60} l^5$$

$$U_{AB} = \frac{w^2 l^5}{240EI} \quad \text{... (2)}$$

Strain energy for concentrated load case



$$U_{AB} = U_{AC} + U_{CB} \quad [\because \text{ loading is symmetrical } U_{AC} = U_{CB}]$$

$$\Rightarrow \quad U_{AB} = U_{AC} + U_{AC}$$

$$\Rightarrow \quad U_{AB} = 2U_{AC}$$

$$\Rightarrow \quad U_{AB} = 2\int_0^{l/2} \frac{\left(\frac{P}{2}x\right)^2 dx}{2EI}$$

$$\Rightarrow \quad U_{AB} = 2\int_0^{l/2} \frac{P^2 x^2}{8EI} dx$$

$$\Rightarrow \quad U_{AB} = \frac{P^2}{4EI}\int_0^{l/2} x^2 dx$$

$$\Rightarrow \quad U_{AB} = \frac{P^2}{4EI}\left[\frac{x^3}{3}\right]_0^{l/2}$$

$$\Rightarrow \quad U_{AB} = \frac{P^2}{4EI} \times \frac{l^3}{24} \quad \left[\text{from (1) we have } P = \frac{wl}{2}\right]$$

$$\Rightarrow \quad U_{AB} = \frac{w^2 l^2}{4 \times 4EI} \times \frac{l^3}{24}$$

$$\Rightarrow \quad U_{AB} = \frac{w^2 l^5}{384EI} \quad \text{... (3)}$$

$$\frac{[U_{AB}]_{UDL}}{[U_{AB}]_{CL}} = \frac{w^2 l^5/240EI}{w^2 l^5/384EI} = \frac{384}{240} = \frac{8}{5} = 1.6$$

$\therefore$Strain energy when beam is loaded with UDL is 1.6 times the strain energy when beam is loaded with concentrated load.

T2 : Solution

$$I = \frac{1}{12}\left[100(200)^3 - 90\times(180)^3\right] = 22.93 \times 10^6 \text{ mm}^4$$

$$F = \frac{\Delta M}{\Delta x} = \frac{80\,\text{kN-m}}{1\text{m}} = 80\,\text{kN}$$

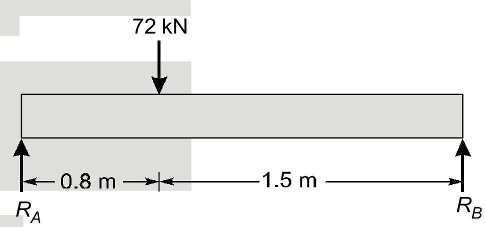
The maximum shear stress occurs at the 'Neutral axis'.

$$q_{max} = \frac{F}{Ib}A\bar{y}$$

$$= \frac{80\times10^3}{22.93\times10^6\times(2\times5)}\times[100\times10\times95+2\times90\times5\times45]$$

$$= 47.27 \text{ N/mm}^2$$

T3 : Solution



$$\sum M_B = 0$$

$\Rightarrow$ $R_A(2.3) - 72(1.5) = 0$

$\Rightarrow$ $R_A = 46.96$ kN

$\therefore$ Shear force at section x – x $= 46.96$ kN

For a constant width of section, shear stress increases as the section moves closer to centre of gravity of the section. (Top or bottom of section)

So, from extremes the shear stress will increase from 0 to some value till the width increases suddenly, which will result in a sudden drop.

After that the shear stress will start increasing again and will again witness a sudden drop with sudden increase in width.

So, we need to find shear stress at three depths, when width changes suddenly,

$$\text{Shear Stress, } \tau = \frac{s(A\bar{y})}{I.B}$$, B is the width of cross-section at the point of consideration

Calculating shear stress at a depth of 60 mm from the top fibre

Area above 60 mm depth till top,

$$A = 30\times20 + 60\times20 + 90\times20 = 3600 \text{ mm}^2$$

Centroid of above area considered from mid-point,

$$\bar{y} = \frac{600\times50+1200\times30+1800\times10}{3600} = 23.33 \text{ mm}$$

$$\tau = \frac{46.96\times10^3(3600\times23.33)}{5.76\times10^6\times90} = 7.6 \text{ N/mm}^2$$

Calculating shear stress at a depth of 40 mm from the top fibre

Area above 40 mm depth till top,

$$A = 30\times20 + 60\times20 = 1800 \text{ mm}^2$$

Centroid of above area considered from mid-point,

$$\bar{y} = \frac{600 \times 50 + 1200 \times 30}{1800} = 36.67 \text{ mm}$$

$$\tau = \frac{46.96 \times 10^3 (1800 \times 36.67)}{5.76 \times 10^6 \times 60} = 8.97 \text{ N/mm}^2$$

Calculating shear stress at a depth of 20 mm from the top fibre

Area above 20 mm depth till top,

$$A = 30 \times 20 = 600 \text{ mm}^2$$

Centroid of above area considered from mid-point,

$$\bar{y} = \frac{600 \times 50}{600} = 50 \text{ mm}$$

$$\tau = \frac{46.96 \times 10^3 (600 \times 50)}{5.76 \times 10^6 \times 30} = 8.15 \text{ N/mm}^2$$

Hence, maximum shear stress on beam at section-xx is 8.97 N/mm^2.

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8 Torsion in Shafts & Springs

Detailed Explanation of Try Yourself Questions

T1 : Solution

We know that
$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

$$\theta_{AB} = \theta_{AC} + \theta_{CB}$$

$$\theta = \frac{TL}{GJ} \qquad \theta_{CB} = \theta_1 = \theta_3$$

$$T_1 + T_3 = T.$$

$$\frac{T_1 L}{G_1 J_1} = \frac{T_3 L}{G_3 J_3} \qquad \Rightarrow T_1 = 2T_3$$

$$T_1 = \frac{2T}{3} \qquad T_3 = \frac{T}{3}$$

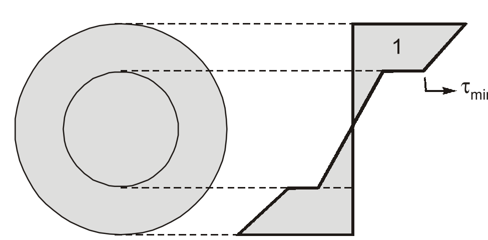
$$\theta_{CB} = \left(\frac{2TL}{3G_1 J_1}\right)$$

$$\theta_{AC} = \frac{T(2L)}{G_2 J_2} = \frac{2TL}{2G_1 J_1}$$

$$\theta_{AB} = \left(\frac{2TL}{3G_1 J_1}\right) + \frac{2TL}{2G_1 J_1} = \frac{5TL}{3G_1 J_1}$$

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau_{min} = \frac{T_1}{J_1} \times \left(\frac{D}{2}\right)$$



$$= \frac{2T}{3J_1} \times \frac{D}{2} = \frac{TD}{3J_1}$$

Alternative:

$$\tau_{min} = \frac{T_3}{J_3}\left(\frac{D}{2}\right) \times \frac{G_1}{G_3} = \frac{TD}{6J_3G_3} \times G_1$$

$$= \frac{TD}{6\left(\frac{G_1J_1}{2}\right)} \times G_1 = \left(\frac{TD}{3J_1}\right)$$

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9 Theory of Columns

Detailed Explanation of Try Yourself Questions

T1 : Solution

$$A = \frac{\pi}{4}\left(100^2 - 75^2\right) = 3434.37 \text{ mm}^2$$

Section modulus, $$Z = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right) = 67.07 \times 10^3 \text{ mm}^2$$

Direct stress $$= \frac{P}{A} = \frac{50 \times 10^3}{3434.37} = 14.55 \text{ N/mm}^2$$

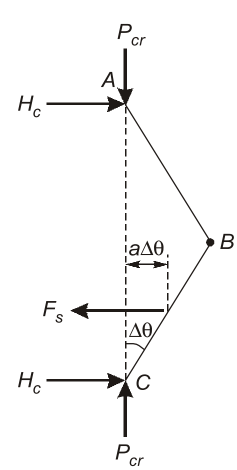
Bending stress $$= \pm\frac{M}{Z} = \pm\frac{50 \times 10^3 \times 100}{67.07 \times 10^3} = \pm 74.55 \text{ N/mm}^2$$

∴ Maximum stress intensity

$$= 74.55 + 14.55 = 89.10 \text{ N/mm}^2$$

T2 : Solution

A free body diagram of the entire system of two rigid bars is shown below



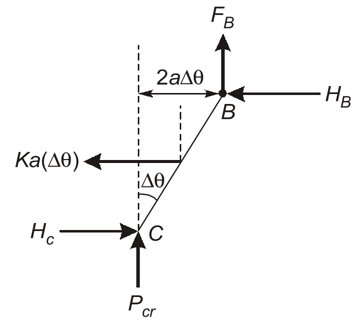
Take, $$\Sigma M_A = 0$$

$$\Rightarrow \quad H_c \times 4a - F_s \times 3a = 0$$

$$\Rightarrow \quad H_c \times 4a - ka(\Delta\theta) \times 3a = 0$$

$$\therefore \quad H_c = \frac{3\,ka(\Delta\theta)}{4}$$

Now, for the calculation of critical load, consider the free body diagram of lower bar *BC*, shown below



Take, $\Sigma M_B = 0$

$\Rightarrow \quad H_c \times 2a - P_{cr} \times 2a(\Delta\theta) - ka(\Delta\theta) \times a = 0$

$$\Rightarrow \quad P_{cr} = \frac{ka}{4}$$

T3 : Solution

Area, $A = \frac{\pi}{4}(20^2 - 16^2) = 113.097 \text{ cm}^2$

Moment of Inertia, $I = \frac{\pi}{64}(20^4 - 16^4) = 4637 \text{ cm}^2$

Radius of Gyration, $k = \sqrt{\frac{I}{A}} = \sqrt{41} \text{ cm}$

Effective length, $l_e = \frac{l}{2} = 2.25 \text{ m}$

Rankine's Critical load

$$P = \frac{\sigma_c A}{1 + \alpha\left(\frac{l_e^2}{k^2}\right)} = \frac{550 \times 10^6 \times 113.097 \times 10^{-4}}{1 + \frac{1}{1600} \times \left(\frac{2.25^2}{41 \times 10^{-4}}\right)}$$

$\Rightarrow \quad P = 3510896 \text{ N}$

Euler's critical load,

$$P_e = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 94 \times 10^9 \times 4637 \times 10^{-8}}{2.25^2}$$

$\Rightarrow \quad P_e = 8497666 \text{ N}$

$$\Rightarrow \quad \frac{P_e}{P} = \frac{8497666}{3510896} = 2.42$$

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