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ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-9 : Full Syllabus Test (Paper-I)

Name :

Roll No :

Test Centres

Delhi ☒ Bhopal ☐ Jaipur ☐
Pune ☐ Kolkata ☐ Hyderabad ☐

Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	49
Q.2	52
Q.3	
Q.4	
Section-B	
Q.5	39
Q.6	39
Q.7	
Q.8	41
Total Marks Obtained	220

Signature of Evaluator

Cross Checked by

Sourabh
kumar

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section-A

- (a) (i) Consider the circuits shown in the following figures (a) and (b):

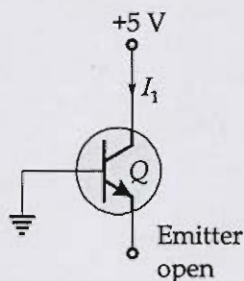


Figure (a)

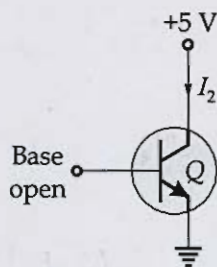
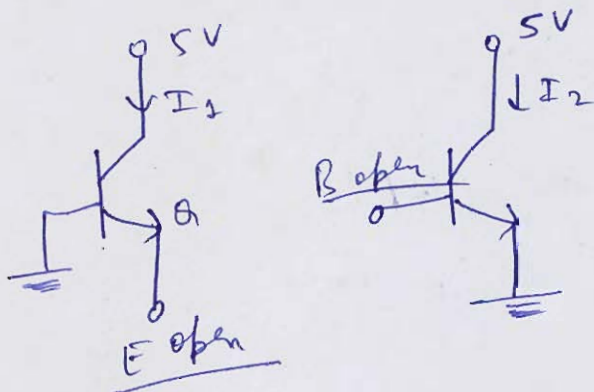


Figure (b)

If the transistors in both the circuits are identical with same value of " β ", then prove that the current I_2 is $(1 + \beta)$ times of the current I_1 .

- (ii) A transistor operating in CB configuration has $I_C = 2.98$ mA, $I_E = 3$ mA and $I_{CO} = 0.01$ mA. If the same transistor is rebiased to get CE configuration with a base current of $30 \mu\text{A}$, then find the collector current in the modified circuit.

[8 + 4 marks]



In fig (a) when emitter is open

$$I_C = \beta I_B + (1 + \beta) I_{CO} \rightarrow (1)$$

In fig (b) when base is open

$$I_{CEO} = (1 + \beta) I_{CO} \rightarrow (2)$$

So,

$$I_1 = \beta I_B + (1 + \beta) I_{CO} \approx (1 + \beta) I_{CO}$$

$$I_2 = \beta I_B + I_{CEO} \approx I_{CEO}$$

from these both equations, it is clear

$$(1 + \beta) I_1 = I_2$$

(ii) Given $I_C = 2.98 \text{ mA}$

$$I_E = 3 \text{ mA}$$

$$I_{CO} = 0.01 \text{ mA}$$

CB

$$I_C = \beta I_B + (1 + \beta) I_{CO}$$

$$2.98 = 0.02 \beta + 0.01(1 + \beta)$$

$$0.03 \beta = 2.97$$

$$\beta = 99$$

In CE

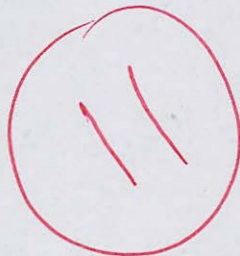
$$I_B = 30 \text{ } \mu\text{A}$$

$$I_C = \beta I_B + I_{CO}(1 + \beta)$$

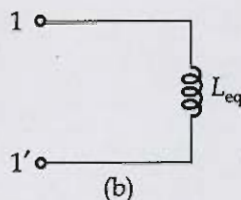
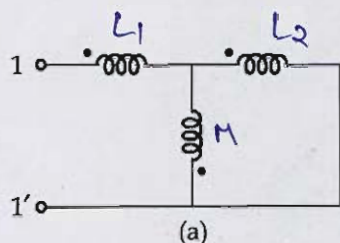
$$I_C = 99 \times 30 \times 10^{-3} + 0.01 \times 100$$

$$I_C = 3.97 \text{ mA}$$

Good
Approach



- (b) In the network of (a) of the given figures, all self inductance values are 1 H, and mutual inductance values are $\frac{1}{2}$ H. Find L_{eq} , the equivalent inductance, shown in (b) of the figure.

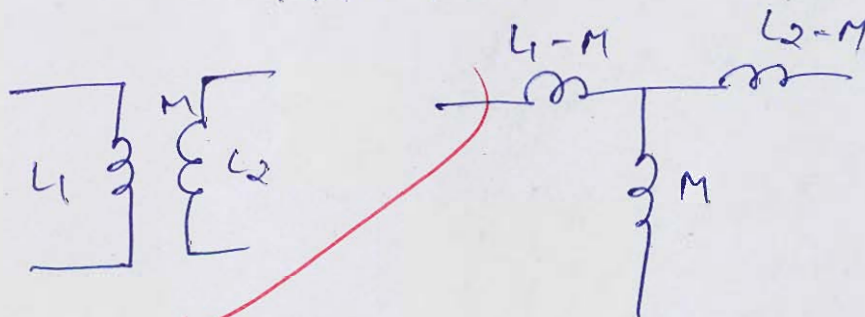


$$L = 1 \text{ H} \quad M = \frac{1}{2} \text{ H}$$

[12 marks]

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$L_{eq} = \frac{1 - \frac{1}{4}}{1 + 1 - 1} = 0.75 \text{ H}$$



$$M = 0.5$$

$$L_2 - M = 0.5$$

$$L_{eq} = L_1 - M + M \parallel L_2 - M$$

$$L_{eq} = \frac{(L_2 - M)M}{L_2} + L_1 - M$$

$$= 0.5 \times 0.5 + 1 - 0.5$$

$$= 0.75$$

10

- (c) Find the solution of $(D^2 - 1)y = x \sin x + (1 + x^2)e^x$.

[12 marks]

Complementary function

$$m^2 - 1 = 0 \Rightarrow m = \pm 1$$

$$y_{cf} = C_1 e^x + C_2 e^{-x} \rightarrow \text{①}$$

Particular Integral

$$PI = \frac{1}{D^2 - 1} x \sin x + (1 + x^2)e^x$$

$$PI = PI_1 + PI_2$$

$$PI_1 = \frac{1}{D^2 - 1} x \sin x = x \frac{1}{D^2 - 1} \sin x - \frac{2D}{(D^2 - 1)^2} \sin x$$

$$= \frac{x \sin x}{-2} - \frac{2D}{4} \sin x$$

$$= -\frac{1}{2} x \sin x - \frac{1}{2} \cos x$$

$$PI_2 = \frac{1}{D^2 - 1} (1 + x^2)e^x = \frac{1}{D^2 - 1} e^x + \frac{1}{D^2 - 1} x^2 e^x$$

$$= \frac{x}{2D} e^x + e^x \frac{1}{(D+1)^2 - 1} x^2$$

$$= \frac{x}{2} e^x + e^x \frac{1}{2D(1 + \frac{D}{2})} x^2$$

$$= \frac{x}{2} e^x + \frac{e^x}{2} \frac{1}{D} \left(1 - \frac{D}{2}\right)^{-1} x^2$$

$$= \frac{x}{2} e^x + \frac{e^x}{2D} \left(1 + \frac{D}{2} + \frac{D^2}{4}\right) x^2$$

$$= \frac{x}{2} e^x + \frac{e^x}{2D} \left(x^2 + x + \frac{1}{2}\right)$$

$$PI_2 = \frac{x}{2}e^x + \frac{e^x}{2}\left(\frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{2}\right) + C$$

$$PI = PI_1 + PI_2$$

$$PI = -\frac{1}{2}x \sin x - \frac{1}{2} \cos x + \frac{x}{2}e^x + \frac{e^x}{2}\left(\frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{2}\right)$$

$$y = y_{cf} + y_{PI}$$

$$y = C_1 e^x + C_2 e^{-x} - \frac{1}{2}x \sin x - \frac{1}{2} \cos x + \frac{x}{2}e^x$$

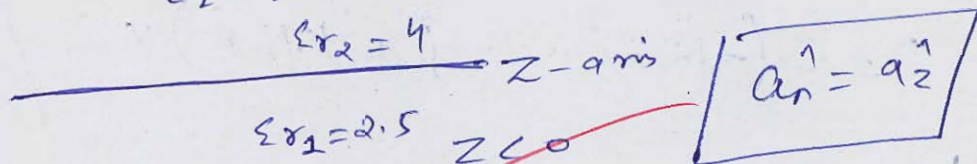
$$+ \frac{e^x}{2}\left(\frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{2}\right) + C$$

10

- (d) A boundary exists at $z = 0$ between two dielectrics $\epsilon_{r1} = 2.5$ in the region $z < 0$, and $\epsilon_{r2} = 4$ in region $z > 0$. The field in region of ϵ_{r1} is $\vec{E}_1 = -30\hat{i} + 50\hat{j} + 70\hat{k}$ V/m. Find the electric displacement vector in the second medium. Also, find the angle between electric field intensity in the second medium and the normal to the boundary surface.

[12 marks]

$$E_2 = ?$$



$$E_1 = -30\hat{i} + 50\hat{j} + 70\hat{k}$$

$$E_{t1} = E_{t2} \quad [\text{tangential field components are equal}]$$

$$E_{t1} = -30\hat{i} + 50\hat{j} = E_{t2}$$

$$D_{n1} = D_{n2} \Rightarrow \epsilon_{r1} E_{n1} = \epsilon_{r2} E_{n2}$$

~~$$E_{n1} = \frac{D_{n1}}{\epsilon_{r1}} = \frac{70\hat{k}}{2.5}$$~~

$$E_{n2} = \frac{\epsilon_{r1} E_{n1}}{\epsilon_{r2}} = \frac{2.5 \times 70\hat{k}}{4}$$

$$E_{n2} = 43.75\hat{k}$$

$$E_2 = E_{t2} + E_{n2}$$

$$E_2 = -30\hat{i} + 50\hat{j} + 43.75\hat{k}$$

angle between E_2 and normal

$$\cos \theta_2 = \frac{\vec{E}_2 \cdot \vec{a}_n}{|\vec{E}_2| |\vec{a}_n|} = \frac{43.75}{\sqrt{900 + 2500 + (43.75)^2}}$$

$$\theta_2 = 53.11^\circ$$

Good
Approach

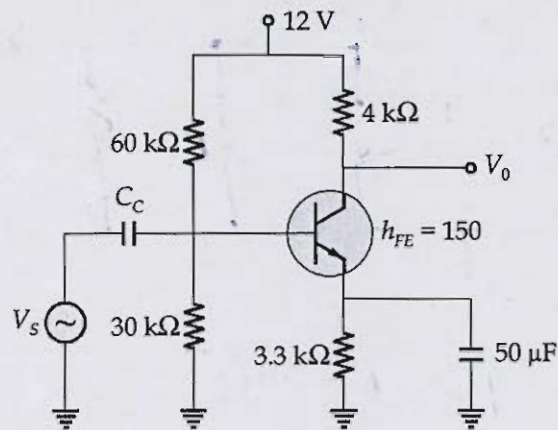
Electric displacement Vector in med. ②

$$D_2 = \epsilon_0 \epsilon_r E_2$$

$$D_2 = \epsilon_0 \times 4 \times (-30\hat{i} + 50\hat{j} + 42.75\hat{k})$$

$$D_2 = (-120\hat{i} + 200\hat{j} + 175\hat{k}) \times \epsilon_0 \text{ C/m}^2$$

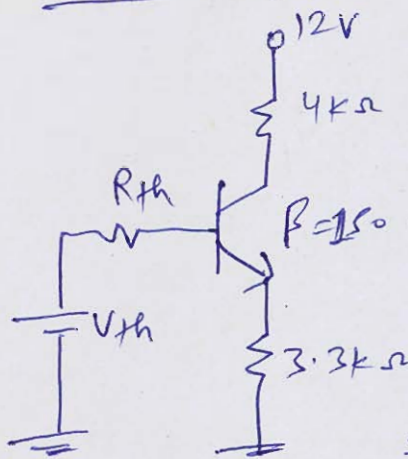
- (e) An amplifier circuit is shown in the given figure:



Find the voltage gain $\frac{V_0}{V_s}$. (Neglect the base current of transistor)

DC analysis

[12 marks]



$$V_{th} = 12 \times \frac{30}{90} = 4V$$

$$R_{th} = \frac{30 \times 60}{30 + 60} = 20k\Omega$$

$$-V_{th} + I_B R_{th} + V_{BE} + I_E R_E = 0$$

$$\Rightarrow 20I_B + 1 \times 2.3I_B = 4 - 0.7$$

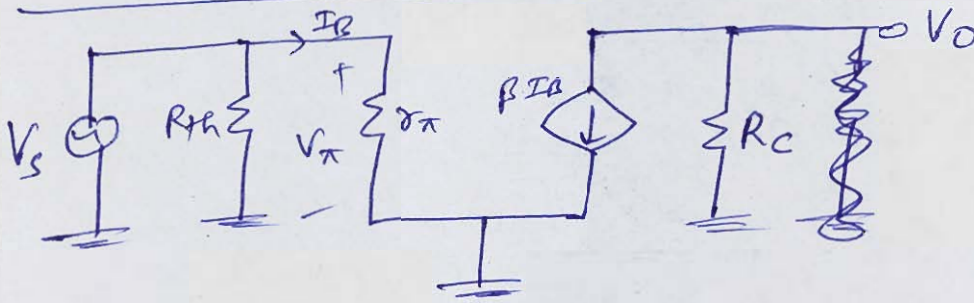
$$I_B = 6.366 \times 10^{-3} \text{ mA}$$

$$r_{\pi} = \frac{V_T}{I_B} = \frac{25 \text{ mV}}{6.366 \times 10^{-3}} = 3.92 k\Omega$$

$$I_C = \beta I_B = 0.955 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.955}{25} = 0.0382$$

Small signal model



$$V_o = -\beta I_B R_C \rightarrow (1)$$

$$V_{\pi} = V_s = I_B r_{\pi}$$

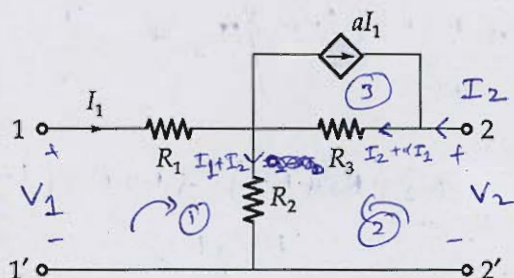
$$A_v = \frac{V_o}{V_s} = \frac{-\beta I_B R_C}{I_B r_{\pi}}$$

$$A_v = -\frac{\beta R_C}{r_{\pi}} = \frac{-150 \times 4}{3.92}$$

$$A_v \equiv -153$$



- 2 (a) The network of the following figure represents a certain transistor over a given range of frequencies. For this network, determine
- the h -parameters and
 - the g -parameters.



[20 marks]

(i) h -parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

KVL in loop ①

$$V_1 = I_1 R_1 + (I_1 + I_2) R_2$$

$$V_1 = I_1 (R_1 + R_2) + I_2 R_2 \rightarrow \textcircled{1}$$

KVL in loop ②

$$V_2 = R_3 (I_2 + \alpha I_1) + (I_1 + I_2) R_2$$

$$V_2 = I_1 (\alpha R_3 + R_2) + I_2 (R_2 + R_3) \rightarrow \textcircled{2}$$

from ① & ②

$$I_2 = \frac{V_2 - I_1 (\alpha R_3 + R_2)}{R_2 + R_3} \rightarrow \textcircled{5}$$

Putting in ①

$$V_1 = I_1 (R_1 + R_2) + \frac{R_2}{R_2 + R_3} (V_2 - I_1 (\alpha R_3 + R_2))$$

$$V_1 = I_1 (R_1 + R_2) + \frac{R_2}{R_2 + R_3} V_2 - I_1 \frac{(\alpha R_3 + R_2) R_2}{R_2 + R_3}$$

$$V_1 = I_1 \left(\frac{(R_1 + R_2)(R_2 + R_3) - \alpha R_3 R_2 - R_2^2}{R_2 + R_3} \right) + \frac{R_2}{R_2 + R_3} V_2$$

$$V_1 = I_1 \left(\frac{R_1 R_2 + R_1 R_3 + R_2^2 + R_2 R_3 - \alpha R_2 R_3 - R_2^2}{R_2 + R_3} \right) + \frac{R_2}{R_2 + R_3} V_2$$

$$V_1 = \left[\frac{R_2(R_2 + R_3) + R_2 R_3(1 - \alpha)}{R_2 + R_3} \right] I_1 + \frac{R_2}{R_2 + R_3} V_2 \rightarrow (3)$$

$$h_{11} = \frac{R_2(R_2 + R_3) + R_2 R_3(1 - \alpha)}{R_2 + R_3}$$

$$h_{12} = \frac{R_2}{R_2 + R_3}$$

Also, from (5)

$$I_2 = -\frac{I_1(\alpha R_3 + R_2)}{R_2 + R_3} + \frac{V_2}{R_2 + R_3} \rightarrow (4)$$

$$h_{21} = \frac{-(\alpha R_3 + R_2)}{R_2 + R_3}$$

$$h_{22} = \frac{1}{R_2 + R_3}$$

(ii) g-parameters

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_1 = g_{21} V_1 + g_{22} I_2$$

from eqn (4)

$$I_1 = \frac{V_1 - I_2 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V_1}{R_1 + R_2} + \left(\frac{-R_2}{R_1 + R_2} \right) I_2$$

$$g_{11} = \frac{1}{R_1 + R_2}$$

$$g_{12} = \frac{-R_2}{R_1 + R_2}$$

KVL

$$-V_2 + R_3(I_2 + \alpha I_1) - R_2 I_1 + V_1 = 0$$

$$V_2 = V_1 + I_1(\alpha R_3 - R_2) + I_2 R_3$$

$$V_2 = V_1 + I_2 R_3 + \frac{(V_1 - I_2 R_2)}{R_1 + R_2} (\alpha R_3 - R_2)$$

$$V_2 = V_1 + \frac{V_1(\alpha R_3 - R_2)}{R_1 + R_2} + I_2 R_3 - \frac{I_2 R_2(\alpha R_3 - R_2)}{R_1 + R_2}$$

$$V_2 = V_1 \left[\frac{R_1 + R_2 + \alpha R_3 - R_2}{R_1 + R_2} \right] + I_2 \left[\frac{R_3 R_1 + R_3 R_2 - \alpha R_2 R_3 + R_2 R_2}{R_1 + R_2} \right]$$

$$V_2 = V_1 \left(\frac{R_2 + \alpha R_3}{R_1 + R_2} \right) + I_2 \left(\frac{R_1(R_2 + R_3) + R_2 R_3(1 - \alpha)}{R_1 + R_2} \right)$$

$$g_{21} = \frac{R_2 + \alpha R_3}{R_1 + R_2}$$

$$g_{22} = \frac{R_1(R_2 + R_3) + R_2 R_3(1 - \alpha)}{R_1 + R_2}$$

18

Good
Approach

Q.2 (b) (i) Find the value of $\int_{|z|=1} \frac{\cosh z}{4z^2+1} dz$.

[10 marks]

$$\int \frac{\cosh z}{4z^2+1} \quad ; |z|=1 \rightarrow \text{Contour}$$

$$\text{Pole} \Rightarrow 4z^2+1=0$$

$$z^2 = -\frac{1}{4} \Rightarrow z = \pm \frac{i}{2} = \pm 0.5i$$

$$\int \frac{\cosh z}{(2z+i)(2z-i)}$$

$$\text{for } z = i/2$$

$$R_1 = \lim_{z \rightarrow i/2} \frac{\cosh z}{(2z+i)} = \frac{\cosh i/2}{2i}$$

$$\text{for } z = -i/2$$

$$R_2 = \lim_{z \rightarrow -i/2} \frac{\cosh z}{(2z-i)} = \frac{\cosh i/2}{-2i}$$

$$\begin{aligned} \int \frac{\cosh z}{4z^2+1} &= 2\pi i (R_1 + R_2) \\ &= 2\pi i \left(\frac{\cosh i/2}{2i} - \frac{\cosh i/2}{2i} \right) \\ &= \underline{\underline{0}} \end{aligned}$$

Good Approach

Q.2(b) (ii) The matrix $A = \begin{bmatrix} a & h \\ -h & b \end{bmatrix}$ is transformed to the diagonal form $D = T^{-1}AT$, where

$T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$. Find the value of θ , which give this diagonal transformation.

[10 marks]

$$A = \begin{bmatrix} a & h \\ -h & b \end{bmatrix}$$

$$D = T^{-1}AT$$

$$T^{-1} = \frac{\text{adj } T}{|T|} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$T^{-1}AT = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} a & h \\ -h & b \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$D = \begin{bmatrix} a \cos \theta + h \sin \theta & h \cos \theta - b \sin \theta \\ a \sin \theta - h \cos \theta & h \sin \theta + b \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$D = \begin{bmatrix} a \cos^2 \theta + h \sin \theta \cos \theta - h \sin \theta \cos \theta + b \sin^2 \theta & a \sin \theta \cos \theta + h \sin \theta \cos \theta + h \sin \theta \cos \theta - b \sin \theta \cos \theta \\ a \sin \theta \cos \theta + h \sin \theta \cos \theta + h \sin \theta \cos \theta - b \sin \theta \cos \theta & a \sin^2 \theta - h \sin \theta \cos \theta + h \sin^2 \theta + b \cos^2 \theta \end{bmatrix}$$

$$D = \begin{bmatrix} a \cos^2 \theta + b \sin^2 \theta & (a-b) \sin \theta \cos \theta + 2h \sin \theta \cos \theta \\ (a-b) \sin \theta \cos \theta + 2h \sin \theta \cos \theta & (a+h) \sin^2 \theta + b \cos^2 \theta - h \sin \theta \cos \theta \end{bmatrix}$$

For diagonalization

$$D = T^{-1}AT = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$$

$$\text{So, } (a-b) \sin \theta \cos \theta + 2h \sin \theta \cos \theta = 0$$

$$a-b = -2h$$

$$h = \frac{b-a}{2}$$

and $\sin 2\theta = 0 = n\pi$

$$\theta = \pm \frac{n\pi}{2}$$



- Q.2 (c) Electron drift mobility in indium (In) has been measured to be $6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$. The room temperature (27°C) resistivity of In is $8.37 \times 10^{-8} \Omega\text{-m}$, and its atomic mass and density are $114.82 \text{ g mol}^{-1}$ and 7.31 g cm^{-3} respectively.
- Based on the resistivity value, determine how many free electrons are donated by each In atom in the crystal.
 - If the mean speed of conduction electrons in In is $1.74 \times 10^8 \text{ cm s}^{-1}$, what is the mean free path?
 - Calculate the thermal conductivity of In at room temperature.

[20 marks]

Given, $\mu_e = \frac{6 \text{ cm}^2}{\text{V sec}}$ $T = 300 \text{ K}$

$\rho = 8.37 \times 10^{-8} \Omega\text{-m}$ $A = 114.82 \text{ g/mol}$
density = 7.31 g/cm^3

(i) We know, $\tau = ne\mu$

$$\rho = \frac{1}{ne\mu}$$

$$n = \frac{1}{\rho \times e \times \mu} = \frac{1}{8.37 \times 10^{-8} \times 1.6 \times 10^{-19} \times 6 \times 10^{-4}}$$

$$n = \frac{1}{80.352 \times 10^{-31}}$$

$$n = 12.44 \times 10^{28} \text{ /m}^3$$

↳ no. of free electrons

(ii) Given, $v = 1.74 \times 10^8 \frac{\text{cm}}{\text{sec}}$
 $= 1.74 \times 10^6 \text{ m/sec}$

mobility is given by

$$\mu = \frac{e\tau}{m}$$

$$\tau = \frac{m\mu}{e} = \frac{6 \times 10^{-4} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}$$

$$\tau = 34.125 \times 10^{-16} \text{ sec.} \rightarrow \text{relaxation time}$$

mean free path

$$\lambda = v \times \tau$$

$$\lambda = 1.74 \times 10^6 \times 34.125 \times 10^{-16}$$

$$\lambda = 59.37 \times 10^{-10} \text{ m}$$

(iii) Thermal conductivity at room temp.

By Lorentz equation

$$\frac{K}{\sigma T} = L = 2.45 \times 10^{-8} \rightarrow \text{Lorentz number}$$

$K \rightarrow$ thermal conductivity

$$K = 2.45 \times 10^{-8} \times \sigma T$$

$$K = 2.45 \times 10^{-8} \times \frac{1}{\rho} \times T$$

$$K = 2.45 \times 10^{-8} \times \frac{1}{8.27 \times 10^{-8} \times 10^2} \times 300$$

$$K = 87.82 \times 10^{-2}$$

18

Good
Approach

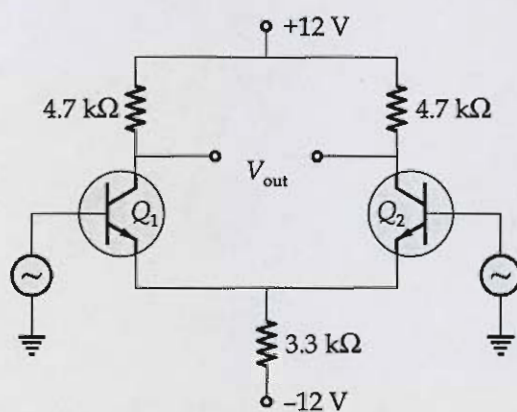
- 3 (a) (i) The parameters of a crystal oscillator equivalent circuit are given as:

$$L_s = 0.8 \text{ H}, C_s = 0.08 \text{ pF}, R_s = 5.5 \text{ k}\Omega \text{ and } C_p = 1.0 \text{ pF}.$$

Find the series resonant frequency and parallel resonant frequency.

[10 marks]

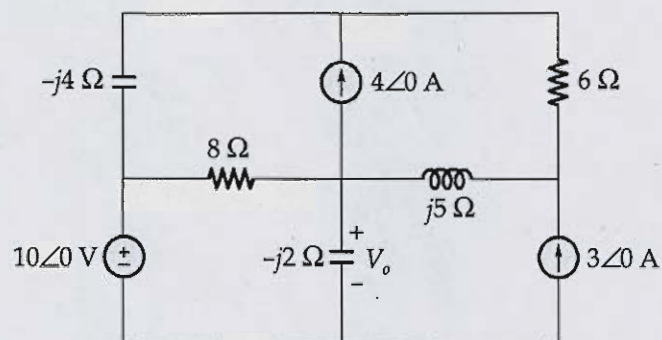
- 3 (a) (ii) Determine the operating point values for the circuit shown in figure:



[10 marks]

3 (b) Name the different types of CROs and mention their applications. Find the velocity of electrons that have been accelerated through a potential of 2000 V in a CRO.
[20 marks]

Q.3 (c) Solve for V_o in the circuit of figure using mesh analysis.



[20 marks]

- 4 (a) (i) Predict the crystal structure and compute the theoretical density for FeO.

Given:

Ionic radius of $\text{Fe}^{++} = 0.77 \text{ nm}$;

Ionic radius of $\text{O}^{-} = 0.140 \text{ nm}$;

Atomic weight of Fe = 55.845 g/mole ;

Atomic weight of O = 16 g/mole ;

Avogadro's number = 6.022×10^{23} /mole

[10 marks]

4 (a) (ii) How are ceramic products fabricated? Explain the role of powder pressing and sintering in the fabrication of ceramic products.

[10 marks]

- 4 (b) A current transformer has a bar primary and 200 secondary winding turns. The secondary winding burden is an ammeter of resistance 1.2Ω and reactance 0.5Ω , the secondary winding has a resistance of 0.2Ω and reactance 0.3Ω . The core requires the equivalent of an mmf of 100 A for magnetization and 50 A for core losses.
- (i) Find the primary winding current and ratio error when the ammeter in the secondary winding circuit indicates 5 A.
 - (ii) How many turns could be reduced in the secondary winding in order that the ratio error to be zero for this condition?

[20 marks]

- Q.4 (c) (i) Find the value of surface integral $\oiint_s (\vec{A} \cdot \vec{n}) ds$ where, $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

[10 marks]

Q.4 (c) (ii) The two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$.

Find:

1. Mean of x .
2. Mean of y .
3. The correlation coefficient between x and y .

[10 marks]

Section-B

Q.5 (a)

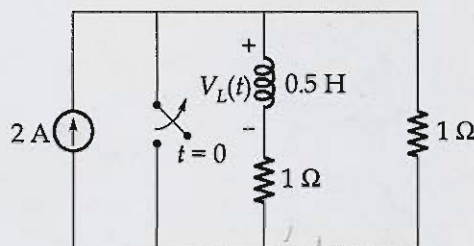
A computer system has a level-1 instruction cache (I-cache), a level-1 data cache (D-cache) and a level-2 cache (L2-cache) with the following specifications:

	Capacity	Mapping method	Block size
I-cache	4K words	Direct mapping	4 Words
D-cache	4K words	2-way set-associative mapping	4 Words
L2-cache	64K words	4-way set-associative mapping	16 Words

Capacity mapping method block size I-cache 4K words direct mapping 4 Words D-cache 4K words 2-way set-associative mapping 4 Words L2-cache 64K words 4-way set-associative mapping 16 Words. The length of the physical address of a word in the main memory is 30 bits. Find the capacity of the tag memory in the I-cache, D-cache and L2-cache.

[12 marks]

- Q.5 (b) For the network shown in figure below, the switch is closed for a long time and at $t = 0$, the switch is opened.



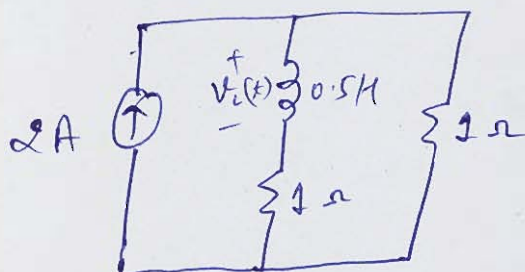
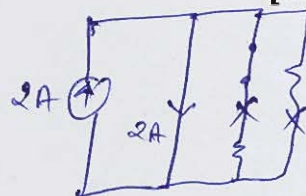
Determine the voltage across inductor for $t > 0$.

$t < 0$ Switch is closed

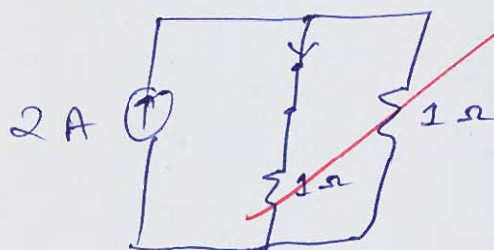
$$i_L(0^-) = 0$$

$t > 0$ switch is opened

[12 marks]



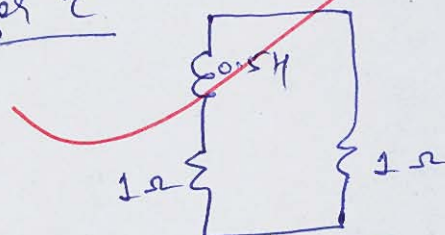
at steady state ($t = \infty$)



$$i_L(\infty) = 1A$$

$$i_L(t) = i_L(\infty) + (i_L(0^-) - i_L(\infty))e^{-t/\tau}$$

for τ



$$\tau = \frac{L}{R} = \frac{0.5}{2}$$

$$\tau = 0.25 \text{ sec.}$$

$$i_L(t) = 1 - e^{-4t} \text{ A}$$

$$V_L(t) = L \frac{di}{dt}$$

$$V_L(t) = 0.5 \times (e^{-4t} \times 4)$$

$$\boxed{V_L(t) = 2e^{-4t} \text{ Volts}}$$

11

Good
Approach

- Q.5 (c) The law of deflection of a moving iron ammeter is given by $I = 4\theta^n$ ampere where θ is deflection in radian and n is a constant. The self-inductance when the meter current is zero is 10 mH. The spring constant is 0.16 N-m/rad.
- (i) Determine an expression for self-inductance of the meter as a function of θ and n .
- (ii) With $n = 0.75$, calculate the meter current and the deflection that corresponds to a self-inductance of 60 mH.

[12 marks]

Given, MI ammeter $I = 4\theta^n$

$$I = 0 \quad L = 10 \text{ mH}$$

$$K = 0.16 \text{ N-m/rad}$$

(i) for moving iron ammeter

$$T = \frac{1}{2} I^2 \frac{dL}{d\theta} = K\theta$$

$$\frac{dL}{d\theta} = \frac{2K\theta}{I^2} = \frac{2 \times 0.16 \times \theta}{16\theta^{2n}}$$

$$\frac{dL}{d\theta} = 0.02 \theta^{1-2n}$$

Integrating both side

$$L = 0.02 \frac{\theta^{1-2n+1}}{1-2n+1} + C$$

$$L = \frac{0.02}{2-2n} \theta^{2-2n} + C$$

→ ①

(ii) $n = 0.75$ $I = 4\theta^{0.75}$

$$L = 60 \text{ mH}$$

from eqn ①

$$I = 0 \quad L = 10 \text{ mH}$$

$$\theta = 0$$

$$C = 10 \times 10^{-3} = 10^{-2}$$

$$L = \frac{0.02}{2-2n} \theta^{2-2n} + 0.01$$

$$\text{at } n = 0.75$$

$$L = 60 \times 10^{-3} = \frac{0.02}{2-2n} \theta^{2-1.5} + 0.01$$

$$\frac{0.05 \times (2-1.5)}{0.02} = \theta^{0.5}$$

$$\theta^{0.5} = 1.25$$

$$\theta = 1.5625 \text{ rad}$$

$$I = 4 \theta^{0.75}$$

$$I = 5.6 \text{ A}$$

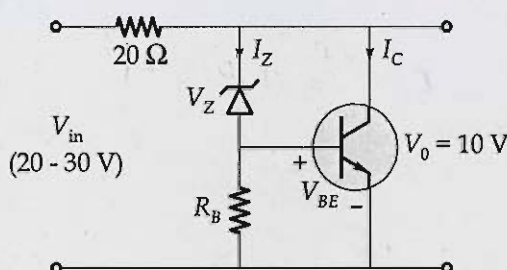


Good Approach

- Q.5 (d) The transistor shunt regulator shown in the figure below has a regulated output voltage of 10 V, when the input varies from 20 V to 30 V. The relevant parameters for the zener diode and the transistor are;

$$V_Z = 9.5 \text{ V}, \quad V_{BE} = 0.3 \text{ V}, \quad \beta = 99$$

Neglect the current through R_B . Find maximum power dissipated in the zener diode (P_Z) and the transistor P_T .

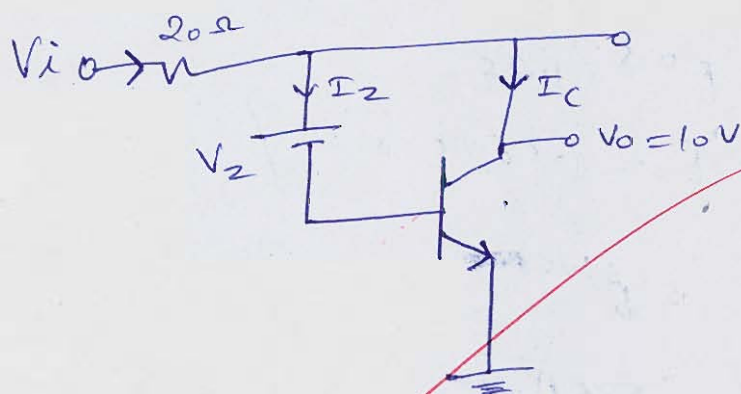


Given

$$V_o = 10 \text{ V} \quad V_i \Rightarrow 20 \text{ V to } 30 \text{ V}$$

$$V_Z = 9.5 \text{ V} \quad V_{BE} = 0.3 \text{ V} \quad \beta = 99$$

$$I_{R_B} = 0$$



Applying KVL

(Considering $V_i = 20 \text{ V}$)

$$-V_i + 20(I_Z + I_C) + V_Z + V_{BE} = 0$$

$$20(I_Z + I_C) = V_i - V_Z - V_{BE}$$

$$I_Z + I_C = \frac{30 - 9.5 - 0.3}{20}$$

$$I_Z + I_C = 1.01 \rightarrow (1)$$

Also,

$$\Rightarrow -V_i + 20(I_Z + I_C) + V_o = 0$$

$$I_Z + I_C = 20$$

$$I_Z = I_B = \frac{I_C}{\beta} \Rightarrow I_C = 99 I_Z$$

from eqn ①

$$I_B + 99 I_B = 1.01 \Rightarrow I_B = 0.0101 \text{ A}$$

~~Power dissipated in~~ $P_Z = V_Z I_Z$

Zener diode

$$P_Z = 9.5 \times 0.0101$$

$$P_Z = 0.0959 \text{ W}$$

~~Power dissipated~~
~~in transistor~~

$$\Rightarrow P_T = V_O I_C$$

$$P_T = 10 \times 0.0101 \times 99$$

$$P_T = 9.99 \text{ W}$$



Good
Approach

Q.5 (e) A conducting bar of $20\ \mu\text{m}$ length, $2\ \mu\text{m}$ wide and $1\ \mu\text{m}$ thick is taken. Find the resistance of the bar if it is

(i) n -doped Silicon with $N_D = 10^8/\text{cm}^3$.

(ii) p -doped Silicon with $N_A = 10^{10}/\text{cm}^3$.

Take $\mu_n = 2.5\ \mu_p = 1200\ \text{cm}^2/\text{Vs}$ and n_i for Silicon is $1.5 \times 10^{10}/\text{cm}^3$.

[12 marks]

Given, $l = 20\ \mu\text{m}$
 $w = 2\ \mu\text{m}$
 $t = 1\ \mu\text{m}$

(i) n -doped $N_D = 10^8/\text{cm}^3$

$$\mu_n = 2.5\ \mu_p = 1200\ \frac{\text{cm}^2}{\text{Vs}}$$

$$n_i = 1.5 \times 10^{10}/\text{cm}^3$$

$$\sigma = N_D e \mu_n \rightarrow \text{for } n\text{-doped Si}$$

$$n_i^2 = np$$

$$N_A = \frac{(1.5 \times 10^{10})^2}{10^8} = 2.25 \times 10^{12}$$

~~$$\sigma = (N_D \times N_A) e \mu_n$$~~

$$\sigma = 10^8 \times 1.6 \times 10^{-19} \times 1200$$

$$\sigma = 1920 \times 10^{-11}$$

$$\rho = 5.208 \times 10^7\ \Omega\text{-cm}$$

and p -doped $N_A = 10^{10}/\text{cm}^3$

$$\sigma = N_A e \mu_p$$

$$\sigma = 10^{10} \times 1.6 \times 10^{-19} \times \frac{1200}{2.5}$$

$$\sigma = 768 \times 10^{-9}$$

$$\rho = \frac{1}{\sigma} \Rightarrow \rho = 1.302 \times 10^6 = 0.13 \times 10^7 \Omega\text{-cm.}$$

$$\rho_T = (5.208 + 0.13) \times 10^7 = 5.338 \times 10^7 \Omega\text{-cm.}$$

$$R = \frac{\rho L}{A}$$

$$R = \frac{5.338 \times 10^7 \times 20 \times 10^{-4}}{2 \times 1 \times 10^{-8}} \quad (A = wt)$$

$$R = 53.38 \times 10^{11} \Omega$$

⑥

- Q.6 (a) (i) The diameter of an electric cable is assumed to be continuous random variate with probability density function:

$$f(x) = 6x(1-x), 0 \leq x \leq 1$$

1. Verify that above is a p.d.f.
2. Find the mean and variance.

[10 marks]

Given, $f(x) = 6x(1-x)$

1) for Pdf

$$\int_{-\infty}^{\infty} 6x(1-x) dx = \int_0^1 6x(1-x) dx$$

$$= 6 \int_0^1 x - x^2 dx$$

$$= 6 \left(\frac{x^2}{2} - \frac{x^3}{3} \right)_0^1$$

$$= 6 \left(\frac{1}{2} - \frac{1}{3} \right) = 6 \times \frac{1}{6} = \underline{\underline{1}}$$

hence proved

$$2) \text{ Mean} = \int_0^1 x f(x) dx = \int_0^1 (6x^2 - 6x^3) dx$$

$$\text{Mean} = 6 \left(\frac{x^3}{3} - \frac{x^4}{4} \right)_0^1$$

$$= 6 \left(\frac{1}{3} - \frac{1}{4} \right) = 6 \times \frac{1}{12} = \underline{\underline{0.5}}$$

$$\text{Variance} = E(X^2) - [E(X)]^2$$

$$\Rightarrow 6 \int_0^1 (n^3 - n^4) dn = E(X^2)$$

$$\Rightarrow E(X^2) = 6 \left(\frac{n^4}{4} - \frac{n^5}{5} \right)_0^1$$

$$= 6 \left(\frac{1}{4} - \frac{1}{5} \right) = 6 \times \frac{1}{20} = \frac{3}{10} = 0.3$$

$$\text{Variance} = 0.3 - (0.5)^2$$

$$= 0.3 - 0.25$$

$$= \underline{\underline{0.05}}$$

9

Good
Approach

- Q.6 (a) (ii) Five thousand candidates appeared in a certain examination carrying a maximum of 100 marks. It was found that the marks were normally distributed with mean 39.5 and with standard deviation 12.5. Determine approximately the number of candidates who secured a first class for which a minimum of 60 marks is necessary. You may see the table given below (x denotes the deviation from the mean). The proportion A of the whole area of the normal curve lying to the left of the ordinate at the deviation $\frac{x}{\sigma}$ is:

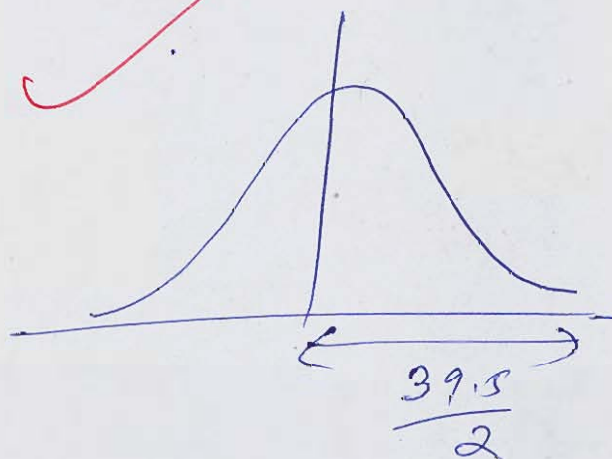
$\frac{x}{\sigma}$	1.5	1.6	1.7	1.8
A	0.93319	0.94520	0.95543	0.96407

[10 marks]

$$\text{Mean} = 39.5 = \mu$$

$$\sigma = 12.5$$

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



②

$$Z = \frac{x - \mu}{\sigma} = \frac{60 - 39.5}{12.5}$$

Incomplete
solution

$$Z = 8$$

Q.6 (b) (i) Perform the following operation using 2's complement method:

1. $(18)_{10} - (33)_{10}$ 2. $-(14)_{10} - (26)_{10}$

(ii) A memory system contains a cache, a main memory and a virtual memory. The access time of the cache is 8 nsec and it has an 85% hit rate. The access time of main memory is 125 nsec and it has a 9.5% hit rate. The access time of virtual memory of 15 msec. Determine the average access time of the hierarchy.

[12 + 8 marks]

$$(i) \quad (18)_{10} - (33)_{10} \\ = (18)_{10} + (-33)_{10}$$

$$(18)_{10} = (0010010)_2 \\ (-33)_{10} = (1100001)_2$$

$$\begin{array}{r} 0010010 \\ 1100001 \\ \hline 1110011 \end{array}$$

No. is in 2's complement form so we have to do its 2's complement again to find actual number

$$(18)_{10} - (33)_{10} = (1001111)_2$$

$$(ii) \quad (-14)_{10} - (26)_{10} = - (14)_{10} + (-26)_{10}$$

$$- (14)_{10} = (1001110)_2$$

$$- (26)_{10} = (1011010)_2$$

$$\begin{array}{r} 1001110 \\ + 1011010 \\ \hline 1011000 \end{array}$$

taking 2's complement to find actual number

$$-(14)_{10} - (26)_{10} = \underline{\underline{(1101000)_2}}$$

(ii) Given

$$\text{Cache} \rightarrow t_c = 8 \text{ nsec} \quad H_c = 0.85$$

$$\text{MM} \rightarrow t_m = 125 \text{ nsec} \quad H_m = 0.095$$

$$\text{VM} \rightarrow t_v = 15 \text{ msec}$$

Hit ratio of virtual memory

$$H_v = 1 - 0.85 - 0.095$$

$$H_v = 0.055 = 5.5\%$$

$$(t_{\text{avg}}) = H_c t_c + H_m t_m + H_v t_v$$

$$= 0.85 \times 8 + 0.095 \times 125 + 0.055 \times 15 \times 10^6$$

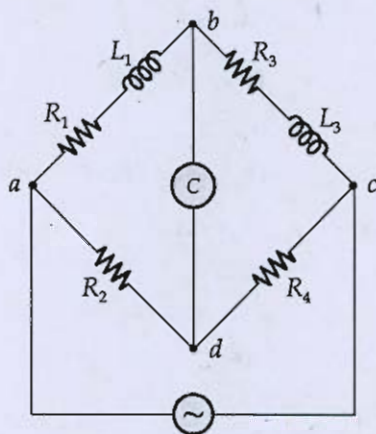
$$= 0.825 \times 10^{-3} \text{ sec}$$

$$= 0.825 \text{ msec}$$

↳ avg. access time
of hierarchy

18

- 2.6 (c) (i) An inductance of 0.22 H and $20\ \Omega$ resistance is measured by comparison with a fixed standard inductance of 0.1 H and $40\ \Omega$ resistance. They are connected as shown in figure below. The unknown inductance is in arm ab and the standard inductance is in arm bc , a resistance of $750\ \Omega$ is connected in arm cd and a resistance whose amount is not known is in arm da .
Find the resistance of arm da and show any necessary and practical additions required to achieve both resistive and inductive balance.



Given,

$$L_1 = 0.22\text{ H} \quad R_1 = 20\ \Omega$$

$$L_3 = 0.1\text{ H} \quad R_3 = 40\ \Omega$$

$$R_4 = 750\ \Omega \quad R_2 = ?$$

[10 marks]

Under bridge balance condition

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1) R_4 = R_2 (R_3 + j\omega L_3)$$

$$R_1 R_4 + j\omega R_4 L_1 = R_2 R_3 + j\omega R_2 L_3$$

Separating Real & Imaginary parts

$$R_1 R_4 = R_2 R_3$$

$$R_2 = \frac{R_1 R_4}{R_3} = \frac{20 \times 750}{40}$$

$$R_2 = 375\ \Omega$$

→ Resistance of arm da

Also,

$$R_4 L_1 = R_2 L_3$$

$$\frac{L_1}{L_3} = \frac{R_2}{R_4}$$

$$L_1 = \frac{R_2 L_3}{R_4}$$

→ unknown inductance in arm ab
on balance condition

from given values, L_1 is coming out as

$$L_1 = \frac{375 \times 0.1}{750} = 0.05 \text{ H}$$

And the measured value given is 0.22 H

So R_4 need to be variable in the given bridge to obtain balance.

3

- 2.6 (c) (ii) A sheet of bakelite 4.5 mm thick is tested at 50 Hz between electrodes 0.12 m in diameter. The Schering bridge employs a standard air capacitor C_2 of 106 pF capacitance, a non-reactive resistance R_4 of $1000/\pi \Omega$ in parallel with a variable capacitor $C_4 = 0.5 \mu\text{F}$, and a non-reactive variable resistance R_3 .

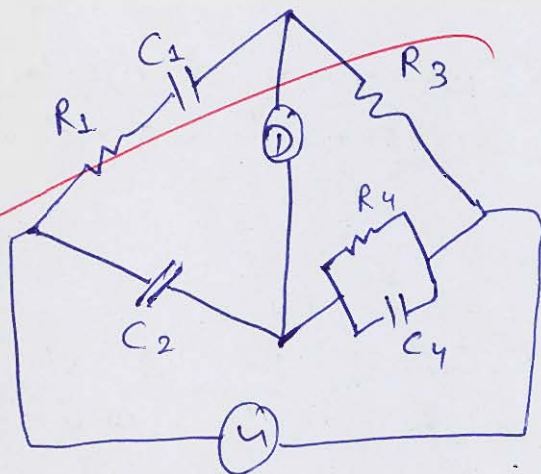
Balance is obtain with $C_4 = 0.5 \mu\text{F}$ and $R_3 = 260 \Omega$

Calculate the capacitance, power factor and relative permittivity of sheet.

[10 marks]

Given, $t = 4.5 \text{ mm}$ $r = \frac{0.12}{2} = 0.06 \text{ m}$

Schering bridge



$C_2 = 106 \text{ pF}$ $C_4 = 0.5 \mu\text{F}$ $R_4 = \frac{1000}{\pi} \Omega$ $R_3 = 260 \Omega$

Under bridge balance condition

$$\left(R_1 + \frac{1}{j\omega C_1}\right) \left(\frac{R_4}{1 + j\omega R_4 C_4}\right) = R_3 \times \frac{1}{j\omega C_2}$$

$$\left(R_1 - \frac{j}{\omega C_1}\right) \left(\frac{R_4 (1 - j\omega R_4 C_4)}{1 + (\omega R_4 C_4)^2}\right) = \frac{-jR_3}{\omega C_2}$$

$$\frac{R_3 R_4}{1 + (\omega R_4 C_4)^2} - \frac{\omega R_4^2 C_4}{\omega C_1 (1 + (\omega R_4 C_4)^2)} = 0 \quad \left(\text{Equating Real parts}\right)$$

$$R_1 R_4 = \frac{R_4^2 C_4}{C_1}$$

$$\boxed{R_1 C_1 = R_4 C_4}$$

on equating imaginary parts

$$\frac{C_1}{C_2} = \frac{R_4}{R_3}$$

$$C_1 = C_2 \frac{R_4}{R_3} = 106 \times \frac{1000}{\pi \times 260}$$

$$C_1 = 129.75 \text{ pF}$$

$$C_1 = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$\epsilon_r = \frac{C_1 d}{\epsilon_0 A} = \frac{129.75 \times 10^{-12} \times 4.5 \times 10^{-3}}{8.854 \times 10^{-12} \times \pi \times (0.06)^2}$$

$$\epsilon_r \approx 5.82$$

Also, $R_1 = \frac{R_4 C_4}{C_1} = \frac{1000}{\pi} \times \frac{0.5 \times 10^{-6}}{129.75 \times 10^{-12}}$

$$R_1 = 1.226 \times 10^6 \Omega$$

Power factor = $\cos \phi$

$$= \cos \left(\tan^{-1} \frac{1}{\omega R_1 C_1} \right)$$

$$= \cos \left[\tan^{-1} \left(\frac{1}{2\pi \times 50 \times 1.226 \times 10^6 \times 129.75 \times 10^{-12}} \right) \right]$$

$$= 0.05 \text{ lag}$$



- 2.7 (a)
- (i) Explain the two sources of magnetic moments for electrons.
 - (ii) Briefly describe the phenomenon of magnetic hysteresis and why it occurs for ferromagnetic and ferrimagnetic materials?
 - (iii) A ferromagnetic material has a remanence of 1.0 Tesla and a coercivity of 15000 A/m. Saturation is achieved at a magnetic field strength of 25000 A/m, at which the flux density is 1.25 Teslas. Sketch the hysteresis curve and from the plot, find the energy loss per cycle of the material.

[20 marks]

Q.7 (b) (i) Consider a hypothetical CPU which supports 16 bit instruction, 64 registers and 1 KB memory space. If there exist 12 2-address instruction which uses register reference and 12 1-address memory reference instructions, how many 0-address instructions are possible?

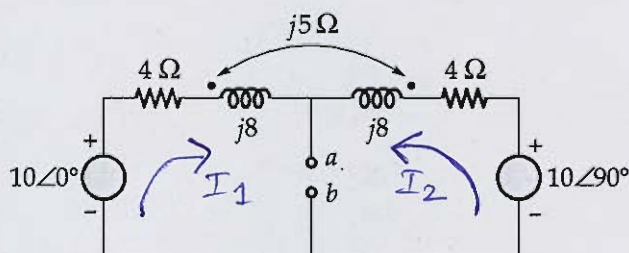
(ii) What are deadlock characteristics? Write the prevention techniques for deadlock.

[10 + 10 marks]

- 7 (c) (i) Three point charges of ' q ' are placed in air at the vertices of an equilateral triangle of side ' d '. Determine the magnitude and direction of the force on one charge due to other charges.
- (ii) Using $\nabla \cdot \vec{D} = \rho$, ohm's law, and the equation of continuity, show that if at any instant a charge density ρ existed with in conductor, it would decrease to $\frac{1}{e}$ times this value in a time $\frac{\epsilon}{\sigma}$ second. Calculate this time for a copper conductor.

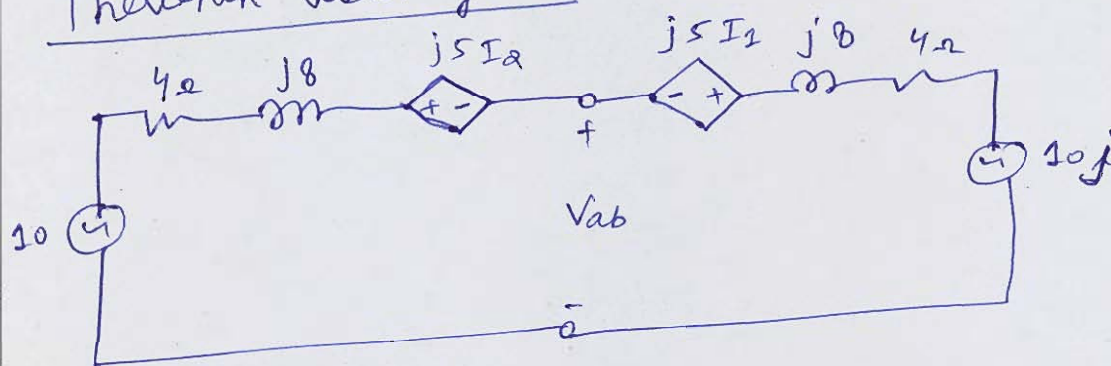
[10 + 10 marks]

- Q.8 (a) Obtain the Thevenin and Norton equivalent circuit at terminals ab of the coupled circuit shown in figure below,



[20 marks]

Thevenin Voltage



Since $I_1 = -I_2$

~~Apply~~ KVL

$$-10 + (4 + j8)I_1 + j5I_2 - j5I_1 + (4 + j8)I_1 + 10j = 0$$

$$I_1 [8 + j16 - j5 - j5] = 10 - 10j$$

$$I_1 = \frac{10(1 - j)}{8 + j6} = 9.43 \angle -4.86^\circ$$

So, KVL

$$-10 + (4 + j8 - j5)I_1 + V_{ab} = 0$$

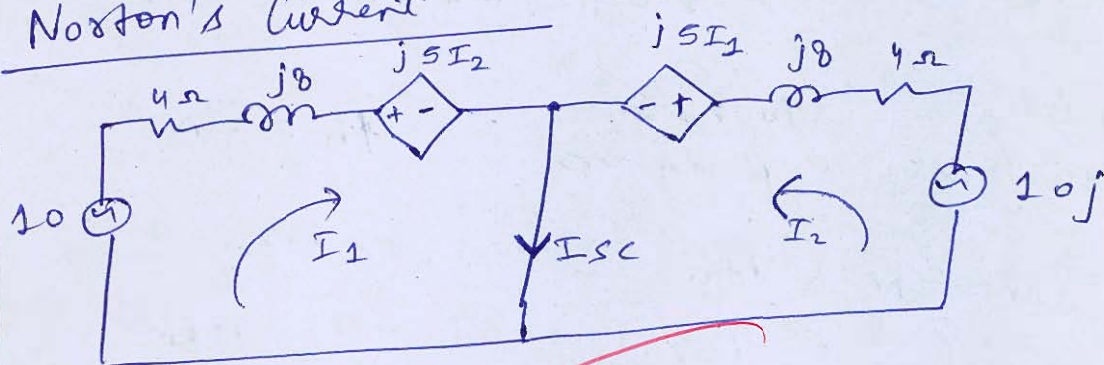
$$V_{ab} = 10 - (4 + j3)I_1$$

$$V_{ab} = 10 - (4 + j3) \times 9.43 \angle -4.86^\circ$$

$$V_{ab} = -30 - 25j$$

$$V_{ab} = 39.05 \angle -140.2^\circ \text{ V}$$

Norton's Current



$$I_{SC} = I_1 + I_2$$

KVL in loop ①

$$-10 + (4 + j8)I_1 + j5I_2 = 0$$

$$I_1(4 + j8) + j5I_2 = 10 \rightarrow \textcircled{1}$$

KVL in loop ②

$$-10j + (4 + j8)I_2 + j5I_1 = 0$$

$$\Rightarrow (4 + j8)I_2 + j5I_1 = 10j$$

$$\Rightarrow j5I_1 + (4 + j8)I_2 = 10j \rightarrow \textcircled{2}$$

Using Cramer's Rule

$$\begin{bmatrix} 4 + j8 & j5 \\ j5 & 4 + j8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10j \end{bmatrix}$$

$$\Delta = (4 + j8)^2 + 25 = 68 \angle 109.76^\circ$$

$$\Delta_1 = \begin{vmatrix} 10 & j5 \\ 10j & 4 + j8 \end{vmatrix} = 120.41 \angle 41.63^\circ$$

$$\Delta_2 = \begin{vmatrix} 4 + j8 & 10 \\ j5 & 10j \end{vmatrix} = 80.62 \angle -172.27^\circ$$

$$I_1 = \frac{\Delta_1}{\Delta} = 1.77 \angle -68.13^\circ \text{ A} \quad I_2 = \frac{\Delta_2}{\Delta} = 1.18 \angle 77.27^\circ \text{ A}$$

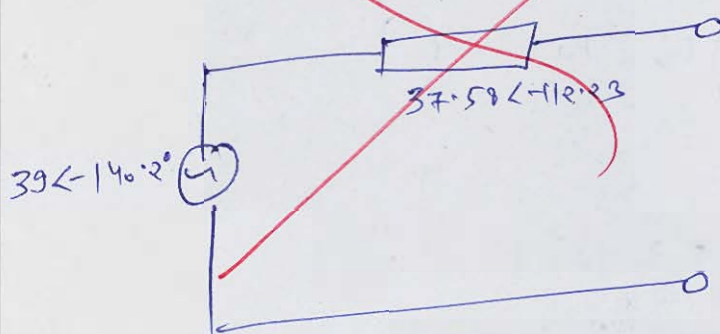
$$I_{sc} = I_1 + I_2 = 1.04 \angle -27.86^\circ \text{ A}$$

$$\text{Thevenin Resistance} = \text{Norton Resistance} = Z_{th} = Z_N$$

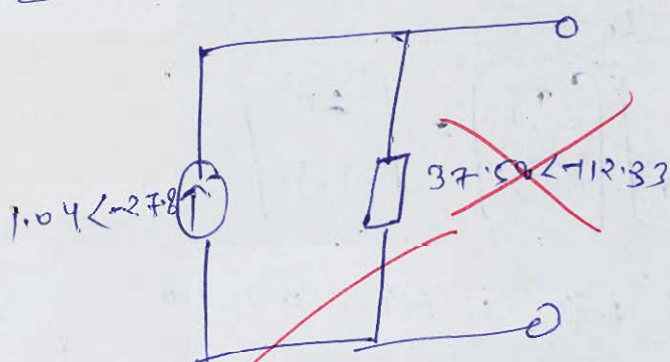
$$Z_{th} = Z_N = \frac{V_{th}}{I_{sc}} = \frac{39.05 \angle -140.2^\circ}{1.04 \angle -27.86^\circ}$$

$$Z_{th} = Z_N = 37.58 \angle -112.33^\circ \Omega$$

Thevenin equivalent



Norton equivalent

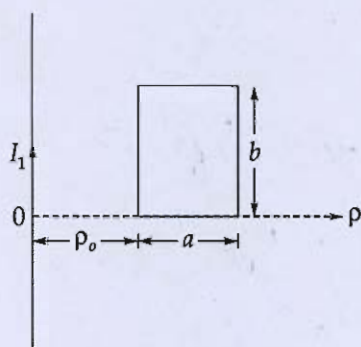


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8 (b) (i) The cross-section of a toroid is 12 cm^2 and is made of material with $\mu_r = 200$. If the mean radius of toroid is 50 cm , calculate the number of turns needed to obtain an inductance of 2.5 H .

(ii) Show that the mutual inductance between the rectangular loop and the infinite line

current shown in the figure below is $M = \frac{\mu b}{2\pi} \ln\left(\frac{a + \rho_0}{\rho}\right)$.



Also calculate the mutual inductance between wire and loop when $a = b = \rho_0 = 1 \text{ m}$.

[8 + 12 marks]

(i) Given $A = 12 \times 10^{-4} \text{ m}^2$
 $\mu_r = 200$
 $r = 50 \times 10^{-2} = 0.5 \text{ m}$
 $L = 2.5 \text{ H}$

for toroid

$$L = \frac{\mu_0 \mu_r N^2 A}{2\pi r}$$

$$N^2 = \frac{L \times 2\pi r}{\mu_0 \mu_r \times A}$$

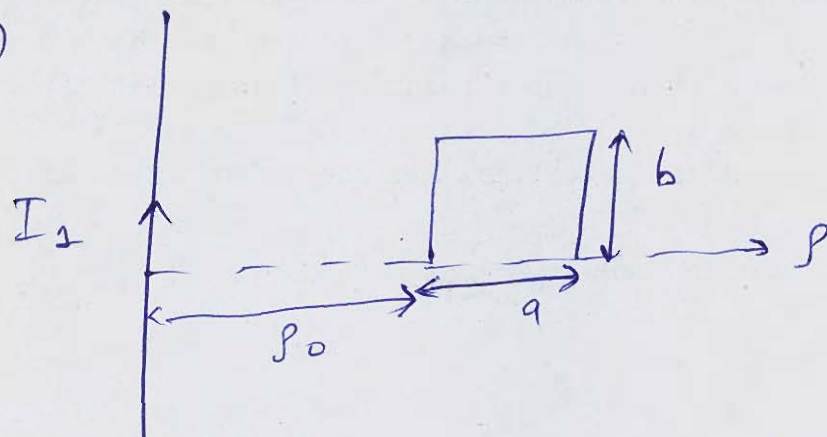
$$N^2 = \frac{2.5 \times 2\pi \times 0.5}{4\pi \times 10^{-7} \times 200 \times 12 \times 10^{-4}}$$

$$N^2 = 2.6 \times 10^7$$

$$N \approx 5101$$

↳ Number of turns

(ii)



Magnetic field due to infinite wire carrying current I is given by

$$B_1 = \frac{\mu_0 I}{2\pi r}$$

Rectangular loop is placed from r_0 to $r_0 + a$

$$\text{Flux density } \phi = B \times A$$

$$\text{and } \phi = LI$$

$$LI = BA$$

$$L = \frac{\mu_0 I ab}{2\pi I}$$

$$L = \frac{\mu_0 b}{2\pi} \int_{r_0}^{r_0+a} \frac{1}{r} dr$$

[Infinite small area taken as dr on r axis $dr = r$]

on integrating

$$L = \frac{\mu_0 b}{2\pi} \int_{r_0}^{r_0+a} \frac{1}{r} dr$$

$$L = \frac{\mu_0 b}{2\pi} \left[\ln p \right]_{p_0}^{a+p_0}$$

$$L = \frac{\mu_0 b}{2\pi} [\ln(a+p_0) - \ln p_0]$$

$$L = \frac{\mu_0 b}{2\pi} \ln \left(\frac{a+p_0}{p_0} \right)$$

↳ mutual inductance b/w wire & loop

Given $a = b = p_0 = 1 \text{ m}$

$$L = \frac{4\pi \times 10^{-7} \times 1}{2\pi} \ln \left(\frac{2}{1} \right)$$

$$L = 2 \times 10^{-7} \times 0.693$$

$$L = 0.138 \mu\text{H}$$

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Q.8 (c) An electrodynamic wattmeter is used for measurement of power in a single phase circuit. The load voltage is 100 V and the load current is 9 A at lagging power factor of 0.1. The wattmeter voltage circuit has a resistance of 3000 Ω and an inductance of 30 mH. Estimate the percentage error in the wattmeter reading when the pressure coil is connected

(i) on the load side, and

(ii) on the supply side.

The current coil has a resistance of 0.1 Ω and negligible inductance. The frequency is 50 Hz. Comment upon the result.

[20 marks]

$$V_L = 100 \text{ V} \quad I_L = 9 \text{ A} \quad \text{pf} = 0.1 \text{ lag}$$

$$P_T = VI \cos \phi = 100 \times 9 \times 0.1 = 90 \text{ W}$$

Pressure coil $\rightarrow R_P = 3000 \Omega$

$$X_P = j\omega L = 2\pi \times 50 \times 30 \times 10^{-3}$$

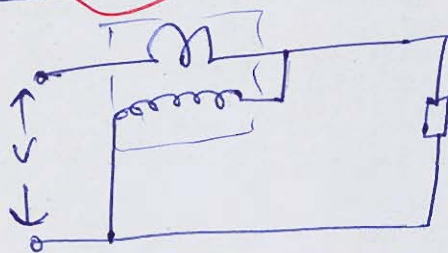
$$X_P = 9.426 \Omega$$

$$\beta = \tan^{-1} \left(\frac{X_P}{R_P} \right) = 0.18 \text{ deg.}$$

or $3.142 \times 10^{-2} \text{ radian}$

Current coil $\rightarrow r_c = 0.1 \Omega$

(i) Pressure coil on load side



Error due to inductance of PC

$$P_e = P_T (1 + \tan \phi \tan \beta)$$

$$P_e = 90 (1 + \tan(84.26) \tan(0.18))$$

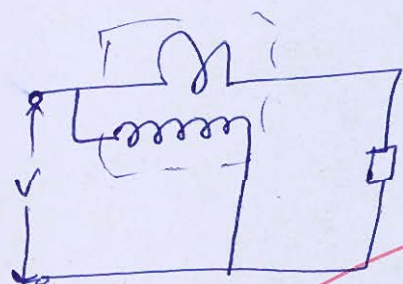
$$P_e = 92.81 \text{ W}$$

Error due to $R_P = \frac{V^2}{R_P} = \frac{(100)^2}{3000} = 3.33 \text{ W}$

$$\text{Total Wattmeter reading} = 92.81 + 3.23 \\ = 96.14 \text{ W}$$

$$\% \text{ error in Wattmeter reading} = \frac{96.14 - 90}{90} \times 100 \\ = 6.82\%$$

(ii) Pressure coil on supply side



$$\text{Total reading} = I^2 R + 90 \text{ W} \\ = 81 \times 0.1 + 90 = 98.1 \text{ W}$$

$$\text{Load impedance, } Z = \frac{V}{I} = \frac{100}{9} = 11.11 \Omega$$

$$R = Z \cos \phi = 11.11 \times 0.1 = 1.11 \Omega$$

$$X = Z \sin \phi = 11.054 \Omega$$

$$\text{Resistance of current coil} = 0.1 \Omega$$

$$\text{Total resistance} = 1.11 + 0.1 = 1.21$$

$$\text{New pf} = \tan^{-1} \left(\frac{11.054}{1.21} \right) = 82.753^\circ$$

angle

$$\text{New pf} = 0.1088$$

$$\text{Total Wattmeter reading} = 98.1 (1 + \tan \phi \tan \beta) \\ = 98.1 (1 + \tan(82.75) \tan(0.18)) \\ = 100.31 \text{ W}$$

$$\% \text{ error in Wattmeter reading} = \frac{100.92 - 90}{90} \times 100$$

$$= \underline{\underline{12.12\%}}$$

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Good
Approach

Space for Rough Work

Space for Rough Work
