MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

E & T Engineering Test No : 12

Section A

Q.1 (a) Solution:

Given data,

$$\in_c = 16\in_0(1 - j10^{-4}) \quad ...(i)$$

$$\in_c = \in' - j\in'' = \in - j\frac{\sigma}{\omega} \quad ...(ii)$$

Comparing (i) and (ii), we get,

$$\in' = 16\in_0, \ \frac{\sigma}{\omega} = 16\in_0 \times 10^{-4} \quad ...(iii)$$

For TM_{mn} mode, the cut-off frequency is given by

$$f_c = \frac{c}{2\sqrt{\in'}}\left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right]^{1/2}$$

For TM_{21} mode,

$$f_c = \frac{3\times10^{10}}{2\sqrt{16}}\left[\left(\frac{2}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]^{1/2}$$

$$f_c = \frac{3\times10^{10}}{2\times4}\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2} = 2.0963\text{ GHz}$$

Given, f = 10% higher than cut-off frequency

$$f = 1.1f_c = 1.1 \times 2.0963 \times 10^9$$

$$= 2.3059\text{ GHz}$$

From equation (iii),
$$\sigma = 16\epsilon_0\omega \times 10^{-4}$$
$$= 16\times 2\pi\times 2.3059\times 10^9 \times \frac{10^{-9}}{36\pi}\times 10^{-4}$$
$$= 2.05 \times 10^{-4}$$
$$\eta' = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{16}} = 30\pi\ \Omega$$

The attenuation constant due to dielectric loss for a propagating mode in a rectangular waveguide is given by:

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{2.05\times 10^{-4}\times 30\pi}{2\sqrt{1-\left(\frac{1}{1.1}\right)^2}} = 0.0232\ \text{Np/m}$$

The magnitude of the electric field (or voltage) along the waveguide is given by $E(z) = E_0 e^{-\alpha_d z}$. Let '$z$' be the distance at which the magnitude of the field is reduced by 20%. We have,

$$E_0 e^{-\alpha_d z} = 0.8E_0$$
$$z = \frac{1}{\alpha_d}\ln\left(\frac{1}{0.8}\right) = \frac{1}{0.0232}\ln\left(\frac{1}{0.8}\right) = 9.62\ \text{m}$$

Q.1 (b) Solution:

(i) Let 'a' be the primitive root and 'n' be the modulus. Given; $n = 23$; $a = 7$

Private key of A = 3

Private key of B = 5

Step-1: Both the parties calculate the value of their public key and exchange with each other.

Public key of A = $7^{\text{Private key of A}}$ mod 23

= 7^3 mod 23 = 343 mod 23 = 21

Public key of B = $7^{\text{Private key of B}}$ mod 23

= 7^5 mod 23 = 16807 mod 23 = 17

Step 2: Both the parties calculate the value of secret key at their respective side.

Secret key obtained by A = $(21)^{\text{private key of A}}$ mod 23 = $(21)^5$ mod 23

= 4084101 mod 23 = 14

$$\text{Secret key obtained by } B = (17)^{\text{Private key of B}} \text{ mod } 23$$
$$= (17)^3 \text{ mod } 23 = 4913 \text{ mod } 23$$
$$= 14$$

Finally, both the parties obtain the same value of secret key

$\therefore$ The value of common secret key = 14

(ii) Given data:

$$p = 7$$
$$q = 11$$
$$e = 17 \text{ (public key)}$$

1. Mod (n) = $p \times q$ = 11 × 7 = 77

2. $\phi(n) = (p - 1)(q - 1) = 6 \times 10 = 60$

3. The private key (d) must satisfy (exd) mod $\phi(n)$ = 1

i.e. (17d) mod 60 = 1

Thus, 17 d is (multiple of 60 + 1)

Using hit and trial method to obtain the value of d,

60 × 1 + 1 = 61 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 2 + 1 = 121 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 3 + 1 = 181 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 4 + 1 = 241 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 5 + 1 = 301 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 6 + 1 = 361 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 7 + 1 = 421 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 8 + 1 = 481 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 9 + 1 = 541 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 10 + 1 = 601 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 11 + 1 = 661 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 12 + 1 = 721 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 13 + 1 = 781 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 14 + 1 = 841 $\Rightarrow$ 'd' value as fraction (Reject)

60 × 15 + 1 = 901 $\Rightarrow$ d = 53 (Accepted)

Hence, Private Key, d = 53

Q.1 (c) Solution:

In IEEE-754 representation, a floating point number is represented as follows:

Sign(S)	Exponent (E)	Mantissa (M)
1 bit	8 bits	23 bits

The floating point number is given by $N = (-1)^S \times (1.M)_2 \times 2^{E-127}$

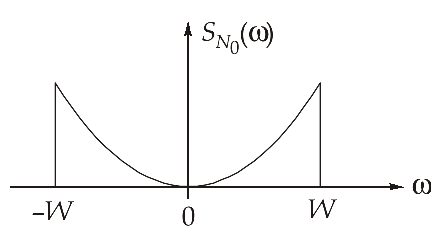
1	10000011	01111110000....0

$$
\begin{aligned}
\text{Exponent, } E &= (10000011)_2 = 131 \\
\text{Actual exponent, } AE &= 131 - \text{bias} \\
&= 131 - 127 \\
&= 4 \\
\text{Mantissa, } M &= 0111111 \\
\text{Sign} &= 1 \text{ (negative)}
\end{aligned}
$$

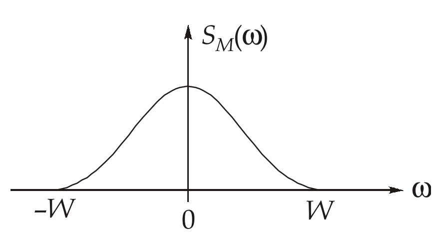
Thus,
$$
\begin{aligned}
N &= (-1)\,(1.0111111)_2 \times 2^4 \\
&= -(10111.111)_2 \\
&= -(23.875)_{10}
\end{aligned}
$$

Q.1 (d) Solution:

Without the use of pre-emphasis and de-emphasis, the power spectral density of noise at FM receiver output is as follows:

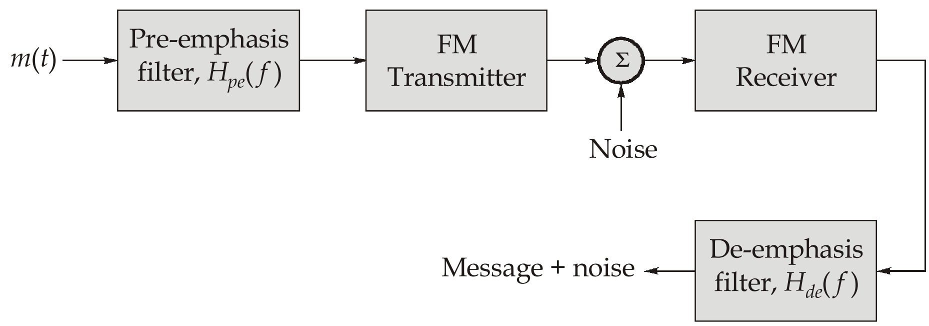


The power-spectral density of the typical message signal is as below:



Thus, the effect of noise is more pronounced at higher frequencies within the FM signal's bandwidth, thereby leading to a poor signal-to-noise ratio (SNR)

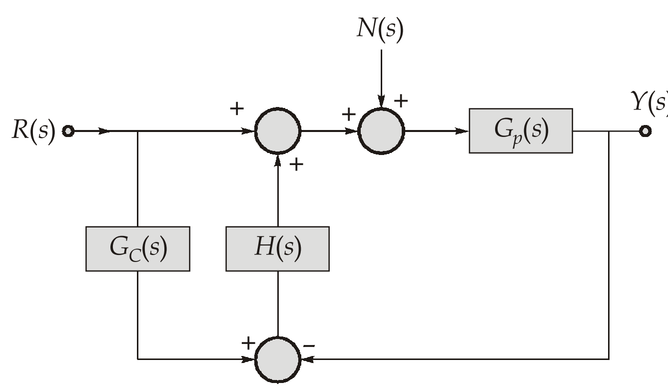
Use of pre-emphasis and de-emphasis in FM system:



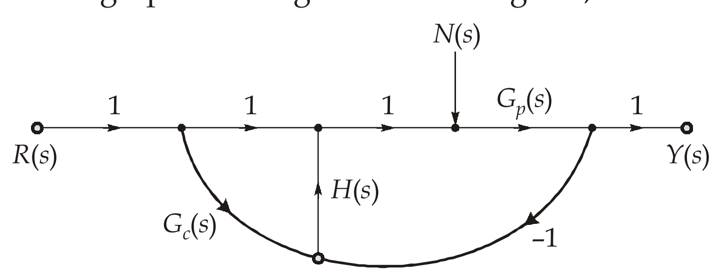
Pre-emphasis artificially emphasis the high frequency component of the message signal prior to modulation in the transmitter before the noise is introduced. In effect, the low frequency and high-frequency portions of the power spectral density of message are equalized in such a way that message fully occupies the frequency band allotted to it.

At the discriminator output in the receiver, inverse operation is performed by de-emphasizing the high frequency components, so as to restore the original signal-power distribution of the message. In this process, the high frequency components of the noise at the discriminator output are also reduced, there by effectively increasing the output signal to noise ratio of the system.

Q.1 (e) Solution:



Drawing the signal flow graph for the given block diagram,



For finding $\dfrac{Y(s)}{R(s)}$ when $N(s) = 0$.

There are two forward paths with gains, $P_1 = G_p(s) \cdot G_c(s)\, H(s)$ and $P_2 = G_p(s)$

$$\text{Loop Gains, } L_1 = -G_p(s)\, H(s)$$

∴ By mason's gain formula,

$$\left.\frac{Y(s)}{R(s)}\right|_{N(s)=0} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_p(s)+G_p(s)G_c(s)H(s)}{1+G_p(s)H(s)} = \frac{G_p(s)(1+G_c(s)H(s))}{1+G_p(s)H(s)}$$

For finding $\frac{Y(s)}{N(s)}$ when $R(s) = 0$

There is only one forward path with gain $P_1 = G_p(s)$

$$\text{Loop gain, } L_1 = -G_p(s)H(s)$$

$$\left.\frac{Y(s)}{N(s)}\right|_{R(s)=0} = \frac{P_1\Delta_1}{\Delta} = \frac{G_p(s)}{1+G_p(s)H(s)}$$

For finding the output response $y(t)$ when $r(t)$ = unit step and $n(t) = 0$, we have

$$\frac{Y(s)}{R(s)} = \frac{G_p(s)(1+G_c(s)H(s))}{1+G_p(s)H(s)} = \frac{\left(\frac{100}{s+1}\right)\left(1+\frac{100}{s+1}\right)}{\left(1+\frac{100}{s+1}\right)}$$

For unit step input, $R(s) = 1/s$,

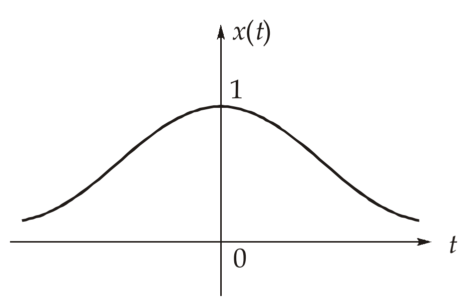
$$\therefore \quad Y(s) = \frac{1}{s}\cdot\frac{100}{s+1} = 100\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

$$\therefore \quad y(t) = 100[1 - e^{-t}]u(t)$$

Q.2 (a) Solution:

Consider the Gaussian pulse signal,

$$x(t) = e^{-\pi t^2}$$



By definition of Fourier transform,

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

$$\Rightarrow \quad X(\omega) = \int_{-\infty}^{+\infty} e^{-\pi t^2} \cdot e^{-j\omega t}dt = \int_{-\infty}^{+\infty} e^{-(\pi t^2 + j\omega t)}dt$$

Substituting $\pi t^2 + j\omega t = \left(\sqrt{\pi}t + \frac{j\omega}{2\sqrt{\pi}}\right)^2 - \left(\frac{j\omega}{2\sqrt{\pi}}\right)^2$ gives

$$\Rightarrow \quad X(\omega) = \int_{-\infty}^{+\infty} e^{-\left[\left(\sqrt{\pi}t+\frac{j\omega}{2\sqrt{\pi}}\right)^2 - \left(\frac{j\omega}{2\sqrt{\pi}}\right)^2\right]}dt$$

$$\Rightarrow \quad X(\omega) = e^{\left(\frac{j\omega}{2\sqrt{\pi}}\right)^2}\int_{-\infty}^{+\infty} e^{-\left(\sqrt{\pi}t+\frac{j\omega}{2\sqrt{\pi}}\right)^2}dt$$

$$\Rightarrow \quad X(\omega) = e^{-\frac{\omega^2}{4\pi}}\int_{-\infty}^{+\infty} e^{-\left(\sqrt{\pi}t+\frac{j\omega}{2\sqrt{\pi}}\right)^2}dt$$

A change of variable is performed by letting $u = \left(\sqrt{\pi}t + \frac{j\omega}{2\sqrt{\pi}}\right)$, which yields $dt = \frac{du}{\sqrt{\pi}}$,

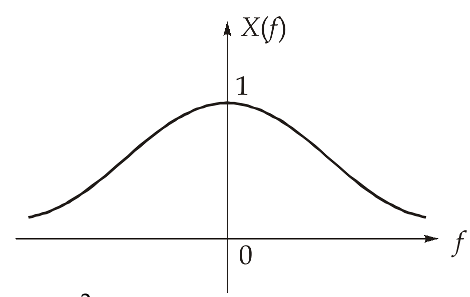
We have, $u \to \infty$ as $t \to \infty$ and $u \to -\infty$ as $t \to -\infty$.

Therefore, $$X(\omega) = e^{-\frac{\omega^2}{4\pi}}\int_{-\infty}^{+\infty} e^{-u^2}\frac{du}{\sqrt{\pi}} = \frac{2}{\sqrt{\pi}}e^{-\frac{\omega^2}{4\pi}}\int_{0}^{+\infty} e^{-u^2}du$$

We know, $$\int_0^{\infty} e^{-u^2}du = \frac{\sqrt{\pi}}{2}$$

$$\Rightarrow \quad X(\omega) = \frac{2}{\sqrt{\pi}}e^{-\frac{\omega^2}{4\pi}} \times \frac{\sqrt{\pi}}{2}$$

$$\Rightarrow \quad X(\omega) = e^{-\frac{\omega^2}{4\pi}} \Rightarrow X(f) = e^{-\pi f^2}$$



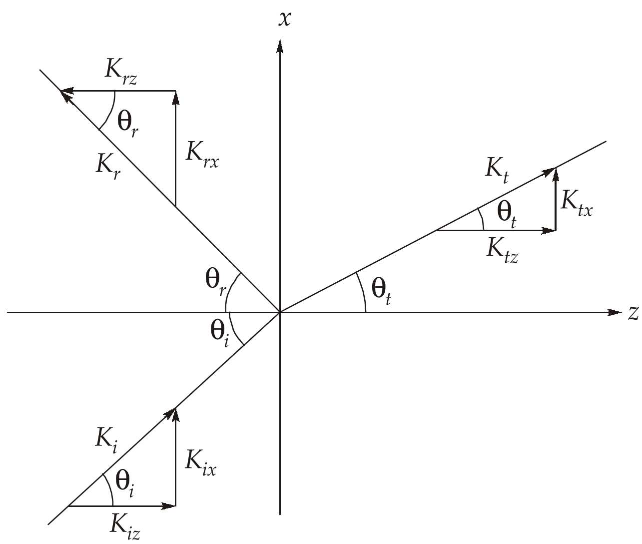
$$\therefore \quad e^{-\pi t^2} \longleftrightarrow e^{-\pi f^2}$$

The Fourier transform $X(\omega)$ of a Gaussian pulse is a Gaussian pulse. Hence proved.

Q.2 (b) Solution:

Given data

$$\vec{H} = 0.2\cos(10^9 t - Kx - K\sqrt{8}z)\hat{a}_y \text{ A/m}$$



(i) From the given expression of $\vec{H}$, we can write

$$\tan\theta_i = \frac{K_{ix}}{K_{iz}} = \frac{1}{\sqrt{8}}$$

$$\theta_i = \theta_r = \tan^{-1}\frac{1}{\sqrt{8}} = 19.47°$$

Using Snell's law,

$$\sin\theta_t = \sin\theta_i\sqrt{\frac{\in_{r1}}{\in_{r2}}} = \frac{1}{3}(3) = 1$$

$$\theta_t = 90°$$

(ii) $$\beta_1 = \sqrt{K_{1x}^2 + K_{1z}^2} = \frac{\omega}{c}\sqrt{\in_{r1}} = \frac{10^9}{3\times10^8}\times 3$$

$\Rightarrow$ $$K\sqrt{1+8} = 10$$

$$K = \frac{10}{3} = 3.333 \text{ m}^{-1}$$

(iii) We have, $$\lambda = \frac{2\lambda}{\beta}$$

In dielectric, $$\lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{10} = 0.6283 \text{ m}$$

From the given expression, $\omega = 10^9$ rad/sec. Thus, in air

$$\beta_2 = \frac{\omega}{c} = \frac{10^9}{3\times10^8} = \frac{10}{3} \Rightarrow \lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi\times3}{10} = 1.885 \text{ m}$$

(iv) The incident electric field is given by

$$\vec{E_i} = \eta_1 \vec{H}_x \times \hat{a}_k$$

where $\hat{a}_k$ is the unit vector in the direction of propagation

$$= 40\pi(0.2)\cos(\omega t - \vec{K}\cdot\vec{r})\hat{a}_y \times \frac{(\hat{a}_x + \sqrt{8}\hat{a}_z)}{3}$$

$$= (23.695\hat{a}_x - 8.377\hat{a}_z)\cos(10^9 t - Kx - K\sqrt{8}z) \text{ V/m}$$

(v) Since E_i is parallel to the plane of incidence, hence it represents parallel polarization. The transmission coefficient for parallel polarization is given by,

$$\tau_{||} = \frac{2\eta_2 \cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Substituting $\theta_t = 90°$, we get

$$\tau_{||} = \frac{2\eta_2}{\eta_1} = \frac{2\times120\pi}{\frac{120\pi}{\sqrt{9}}} = 6$$

$$\Gamma_{||} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = -1$$

The transmitted and reflected electric field is given by,

$$\vec{E}_t = -E_{t0}(\cos\theta_t\hat{a}_x - \sin\theta_t\hat{a}_z)$$
$$\cos(10^9 t - \beta_2 x \sin\theta_t - \beta_2 z\cos\theta_t) \text{ V/m}$$

$$\vec{E}_r = -E_{r0}(\cos\theta_i\,\hat{a}_x + \sin\theta_i\,\hat{a}_z)\cos(10^9 t - \beta_1 x\sin\theta_i$$
$$+ \beta_1 z\cos\theta_i) \text{ V/m}$$

We have, $\sin\theta_t = 1$, $\cos\theta_t = 0$, $\beta_2\sin\theta_t = \frac{10}{3}$

$$E_{t0}\sin\theta_t = \tau_{||}E_{i0} = \tau_{||}\eta_1 H_{i0} = 6\times\frac{120\pi}{\sqrt{9}}\times0.2$$

$$= 150.8$$

Hence, $\vec{E}_t = 150.8\cos(10^9 t - 3.333x)\hat{a}_z \text{ V/m}$

since, $\Gamma = -1$, we have,

$$E_{r0} = -E_{i0} = -40\pi \times 0.2 = -25.13 \text{ V/m and } \theta_r = \theta_i = 19.47°$$

Thus, $\vec{E}_r = (23.695\hat{a}_x + 8.377\hat{a}_z)\cos(10^9 t - Kx + K\sqrt{8}z) \text{ V/m}$

(vi) Brewster angle, $\tan\theta_B = \sqrt{\dfrac{\epsilon_2}{\epsilon_1}} = \sqrt{\dfrac{\epsilon_0}{9\epsilon_0}} = \dfrac{1}{3}$

$$\theta_B = 18.43°$$

Q.2 (c) Solution:

(i) Given data:

$x(t)$ is a stationary process with auto correlation function $R_X(\tau)$ having following values:

$$R_X(0) = 1, \quad R_X(1) = 0.945,$$
$$R_X(2) = 0.72, \quad R_X(3) = 0.56$$

Let a_1, a_2 and a_3 be the coefficients of 3rd order prediction filter.

The predicted value $\hat{x}[n]$ is given by

$$\hat{x}[n] = \sum_{k=1}^{3} a_k x[n-k]$$

The prediction error is given by $e(n) = x(n) - \hat{x}(n)$. The mean square error is,

$$J = E\left[e^2[n]\right] = E\left[x^2[n] - 2x[n]\hat{x}[n] + \hat{x}[n]^2\right]$$

$$J = E[x^2[n]] - 2\sum_{k=1}^{3} a_k E[x[k]x[n-k]] + \sum_{j=1}^{3}\sum_{k=1}^{3} a_j a_k E[x[n-j]x[n-k]]$$

In terms of auto-correlation function, we can write

$$J = \sigma_x^2 - 2\sum_{k=1}^{3} a_k R_X[k] + \sum_{j=1}^{3}\sum_{k=1}^{3} a_j a_k R_X[k-j]$$

For finding the coefficients of optimum linear predictor, differentiate J with respect to a_k. Thus,

$$R_X[k] = R_X[-k] = \sum_{j=1}^{3} a_j R_X[k-j]$$

In matrix form, it can be written as

$$\begin{bmatrix} R_X(1) \\ R_X(2) \\ R_X(3) \end{bmatrix} = \begin{bmatrix} R_X(0) & R_X(1) & R_X(2) \\ R_X(1) & R_X(0) & R_X(1) \\ R_X(2) & R_X(1) & R_X(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} R_X(0) & R_X(1) & R_X(2) \\ R_X(1) & R_X(0) & R_X(1) \\ R_X(2) & R_X(1) & R_X(0) \end{bmatrix}^{-1} \begin{bmatrix} R_X(1) \\ R_X(2) \\ R_X(3) \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & 0.945 & 0.72 \\ 0.945 & 1 & 0.945 \\ 0.72 & 0.945 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.945 \\ 0.72 \\ 0.56 \end{bmatrix}$$

The inverse of a matrix is given by

$$A^{-1} = \frac{1}{|A|}[\text{adj}(A)]$$

Let,
$$A = \begin{bmatrix} 1 & 0.945 & 0.72 \\ 0.945 & 1 & 0.945 \\ 0.72 & 0.945 & 1 \end{bmatrix}$$

$$|A| = 1(1 - 0.945^2) - 0.945(0.945 - 0.72 \times 0.945) + 0.72(0.945^2 - 0.72)$$
$$= 0.1069 - 0.250 + 0.1245$$
$$= 0.2314 - 0.250 = -0.0186$$

$$\text{adj}\,(A) = \begin{bmatrix} 0.1069 & -0.2646 & 0.173 \\ -0.2646 & 0.4816 & -0.2646 \\ 0.173 & -0.2646 & 0.1069 \end{bmatrix}^T$$

$$\text{adj}\,(A) = \begin{bmatrix} 0.1069 & -0.2646 & 0.173 \\ -0.2646 & 0.4816 & -0.2646 \\ 0.173 & -0.2646 & 0.1069 \end{bmatrix}$$

Thus,
$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$= \frac{1}{-0.0186}\begin{bmatrix} 0.1069 & -0.2646 & 0.173 \\ -0.2646 & 0.4816 & -0.2646 \\ 0.173 & -0.2646 & 0.1069 \end{bmatrix}$$

$$= \begin{bmatrix} -5.747 & 14.225 & -9.301 \\ 14.225 & -25.892 & 14.225 \\ -9.301 & 14.225 & -5.747 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -5.747 & 14.225 & -9.301 \\ 14.225 & -25.892 & 14.225 \\ -9.301 & 14.225 & -5.747 \end{bmatrix}\begin{bmatrix} 0.945 \\ 0.72 \\ 0.56 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -0.3974 \\ 2.7663 \\ -1.7657 \end{bmatrix}$$

We get, $a_1 = -0.3974,$

$a_2 = 2.7663,$

$a_3 = -1.7657$

(ii) Variance of the resulting predictor error

$$\min \sigma_e^2 = R_X(0) - [a_1 R_X(1) + a_2 R_X(2) + a_3 R_X(3)]$$

$$= 1 - [-0.3974 \times 0.945 + 2.7663 \times 0.72 + (-1.7657) \times 0.56]$$

$$\min \sigma_e^2 = 0.3725$$

(iii) Processing gain, $G_p = \dfrac{\sigma_m^2}{\sigma_e^2} = \dfrac{R_X(0)}{\min \sigma_e^2} = \dfrac{1}{0.3725} = 2.685$

$$G_p = 2.685$$

$$G_p(\text{in dB}) = 10 \log (2.685) = 4.289 \text{ dB}$$

(iv) Let a_1 be the coefficient for one unit time delay linear predictor filter. Thus,

$$a_1 = \frac{R_X(1)}{R_X(0)} = 0.945$$

Thus, $\min \sigma_e^2 = R_X(0) - a_1 R_X(1)$

$$= 1 - 0.945 \times 0.945$$

$$= 0.107$$

Processing gain, $$G_P = \frac{\sigma_m^2}{\sigma_e^2} = \frac{R(0)}{\min \sigma_e^2} = \frac{1}{0.107} = 9.346$$

$$G_P \text{ (in dB)} = 10 \log (9.346) = 9.706 \text{ dB}$$

Improvement in processing gain,

$$G_P = 9.706 - 4.286 = 5.42 \text{ dB}$$

Q.3 (a) Solution:

We have,

Bandwidth message signal, $f_m = 12$ kHz

Power content of normalised message signal, $P_{mn} = 0.75$ Watt

Power content of carrier signal, $P_c = 220$ Watt

(i) For DSB-SC Modulation:

$\Rightarrow$ $$S_{\text{DSB}}(t) = A_c m(t) \times \cos 2\pi f_c t$$

$$P_t = \frac{A_c^2}{2} P_{mn}$$

$$P_t = P_c \cdot P_{mn} = 220 \times 0.75 = 165 \text{ W}$$

Bandwidth, BW = $2f_m$ = 24 kHz

(ii) For SSB-SC:

$\Rightarrow$ $$P_{SSB} = \frac{P_{DSB}}{2} = \frac{165}{2} = 82.5 \text{ Watt}$$

(iii) For AM:

$\Rightarrow$ $$\begin{aligned} S_{AM}(t) &= A_c[1 + k_a m(t)]\cos 2\pi f_c t \\ &= A_c[1 + k_a |m(t)|_{\max} m_n(t)]\cos 2\pi f_c t \\ &= A_c[1 + \mu m_n(t)]\cos 2\pi f_c t \end{aligned}$$

$$P_t = \frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{2} P_{mn}$$

$$P_t = 220 + 220 \times 0.5^2 \times 0.75$$

$$P_t = 261.25 \text{ Watt}$$

(iv) For FM:

$\Rightarrow$ $$\text{Bandwidth BW} = (\beta + 1)2f_m$$

$\therefore$ $$\beta = \frac{\Delta f}{f_m}$$

$$\Delta f = k_f |m_n(t)|_{max}$$
$$= 50 \times 1 = 50 \text{ kHz}$$

On substituting, $f_m = 12$ kHz and $\Delta f = 50$ kHz

We get,

$$\text{Bandwidth, } BW = \left(\frac{50}{12}+1\right)(2\times 12)$$
$$= 124 \text{ kHz}$$

$$\text{Power} = \frac{A_c^2}{2} = 220 \text{ Watt}$$

Q.3 (b) Solution:

(i)
```
# include <stdio.h>    //Libraries
# include <string.h>
int main ()
{
   char s[100]; //Input array
   int i, f = 1;
   scanf("%s", s); //Reading string without space
   int len = strlen(s);
   for (i = 0; i < len/2; i++)
   if (s[i]! = s[leng – i – 1])
   {
      f = 0; //If mismatch, set flag = 0
      break;
   }
   printf (f? "Palindrome": "Not a Palindrome"); //Printing output with ternary operator
   return 0;
}
```

(ii) According to the power series expansion of e^x, we have

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Therefore,
$$X(z) = e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = \sum_{n=-\infty}^{0} \frac{z^{-n}}{(-n)!}$$

$$\Rightarrow \qquad X(z) = \sum_{n=-\infty}^{+\infty} u[-n]\frac{z^{-n}}{(-n)!} = \sum_{n=-\infty}^{+\infty} \frac{u[-n]}{(-n)!} z^{-n}$$

Comparing the above equation with,

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$$

We get, $$x[n] = \frac{u[-n]}{(-n)!}$$

Q.3 (c) Solution

(i) The total number of modes supported by a graded index fiber can be given by,

$$M = \frac{\alpha}{\alpha+2}\left(\frac{V^2}{2}\right); \ \alpha = 2 \text{ for parabolic index profile}$$

So, $$M = \frac{2}{2+2}\left(\frac{V^2}{2}\right) = \frac{V^2}{4}$$

Given that, $M = 742$

So, $$V = \sqrt{4M} \approx 54.5$$

$$V = \frac{2\pi a}{\lambda}(NA)$$

$$\lambda = \frac{\pi d}{V}(NA) = \frac{\pi(70)}{54.5}(0.30)\ \mu\text{m} = 1.21\ \mu\text{m}$$

For single mode operation,

$$V \le 2.405\sqrt{\frac{\alpha+2}{\alpha}} = 2.405\sqrt{2}$$

So, $$\frac{\pi d}{\lambda}(NA) \le 2.405\sqrt{2}$$

$$d \le \frac{2.405\sqrt{2}\times 1.21}{\pi\times 0.30}\ \mu\text{m}$$

$$d_{\max} = 4.37\ \mu\text{m}$$

(ii) The birefringence (B_f) of a single mode fiber can be given by,

$$B_f = n_y - n_x = \frac{\beta}{k_0}$$

where, $\beta = \dfrac{2\pi}{L_p}$; L_p = fiber beat length

$$k_0 = \frac{2\pi}{\lambda} = \text{free space propagation constant}$$

So, $$B_f = \frac{2\pi/L_p}{2\pi/\lambda} = \frac{\lambda}{L_p} = \frac{1300\times10^{-9}}{8\times10^{-2}} = 1.625\times10^{-5}$$

Q.4 (a) Solution:

(i) 1. $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

The Routh's array is

s^6	1	8	20	16
s^5	2	12	16	X
	1	6	8	X
s^4	2	12	16	X
	1	6	8	X
s^3	0	0	X	X
	1	3	X	X
s^2	3	8	X	X
s^1	1/3	X	X	X
s^0	8	X	X	X

Auxiliary equation of s^4 row

$$s^4 + 6s^2 + 8 = 0$$

Derivative of auxiliary equation

$$4s^3 + 12s = 0$$

or $$s^3 + 3s = 0$$

(Form row s^3 with its coefficients)

The s^3 row has become a zero row because the elements of s^5 and s^4 rows are same. The zero row was replaced by writing the auxiliary equation of the row above the s^3 and then the first derivative of the auxiliary equation was taken as shown with the Routh's array. The s^3 row was then formed by the coefficients of the equation obtained after the derivation of the auxiliary equation.

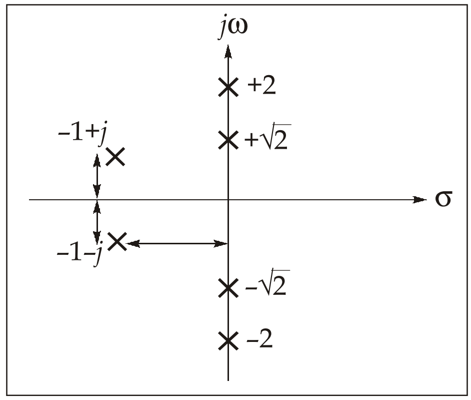
The first column of the Routh's array shows no sign change. Therefore, no roots are on the R.H.S. of the s-plane. However, presence of a zero row indicates presence of symmetrically located roots in the s-plane. In order to find the roots, we proceed as given below:

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = (s^4 + 6s^2 + 8)(s^2 + 2s + 2) = 0$$

Therefore, $(s^4 + 6s^2 + 8) = 0$ or $(s^2 + 4)(s^2 + 2) = 0$

gives $s = \pm 2j;\ s = \pm\sqrt{2}j$ and $s^2 + 2s + 2 = 0$, gives $s = -1 \pm j1$

These roots are shown in figure below:



It can be seen that there are two pairs of imaginary roots and one pair of complex conjugate root having negative real parts. Therefore, the system is marginally stable.

2. $\mathbf{s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4 = 0}$

The Routh's array

s^6	1	–2	–7	–4
s^5	1	–3	–4	X
s^4	1	–3	–4	X
s^3	0	0	X	X
	2	–3	X	X
s^2	–1.5	–4	X	X
s^1	–25/3	X	X	X
s^0	–4	X	X	X

Sign change

Auxiliary equation of s^4 row

$s^4 - 3s^2 - 4 = 0$

Derivative of Auxiliary equation

$4s^3 - 6s = 0$

or $\quad 2s^3 - 3s = 0$

(Form row s^3 with its coefficients)

There is one sign change in the first column of the Routh's array between s^3 and s^2 rows. Therefore, one root is lying on the right hand side of the s-plane. The system is thus unstable. Roots can be ascertained as given below:

$s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4 = 0$

or$(s^4 - 3s^2 - 4)(s^2 + s + 1) = 0$

which means $s^4 - 3s^2 - 4 = 0$ or $(s^2 + 1)(s^2 - 4) = 0$

$\therefore \qquad s = \pm j$ and $s = \pm 2$

and $\qquad (s^2 + s + 1) = 0$, gives $s = \dfrac{-1}{2} \pm j\dfrac{\sqrt{3}}{2}$

Therefore, the roots are $+j, -j, +2, -2, -\dfrac{1}{2} \pm j\dfrac{\sqrt{3}}{2}$

(ii) The characteristic equation is

$$1 + G(s)H(s) = 0$$

or $$1+\frac{K}{s(1+Ts)} = 0$$

or $$s(1 + Ts) + K = 0$$

or $$Ts^2 + s + K = 0$$

or $$s^2+\frac{1}{T}s+\frac{K}{T} = 0$$

To find the roots lying to the left of the line $s = -a$, we replace s by $(s - a)$ and then use the Routh's Hurwitz criteria.

Thus, $$(s-a)^2+\frac{1}{T}(s-a)+\frac{K}{T} = 0$$

or $$s^2+\left(-2a+\frac{1}{T}\right)s+\left(a^2+\frac{K}{T}-\frac{a}{T}\right) = 0$$

The Routh's array is

s^2	1	$a^2+\frac{K}{T}-\frac{a}{T}$
s^1	$\left(-2a+\frac{1}{T}\right)$	0
s^0	$a^2+\frac{K}{T}-\frac{a}{T}$	

If no root is to lie on the right of the line $s = -a$, then the first column of the Routh's array should have no sign change

i.e. $$\left(-2a+\frac{1}{T}\right) > 0 \quad \text{or} \quad \frac{1}{T}>2a \quad \text{or} \quad T<\frac{1}{2a}$$

Also, $$a^2+\frac{K}{T}-\frac{a}{T} > 0$$

or $$\frac{K}{T} > \left(\frac{a}{T}-a^2\right)$$

or $$K > (a - a^2T)$$

or $$K > a(1 - aT)$$

$\therefore$ if $$T = 0, \text{ then } K > a$$

and if $$T = \frac{1}{2a}, \quad \text{then} \quad K>\frac{a}{2}$$

Q.4 (b) Solution:

(i) The distribution function of $X(t)$ is,

$$F_{X(t)}(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}, & 0 \le x \le A \\ 1, & x > A \end{cases}$$

and, the corresponding pdf is,

$$f_{X(t)}(x) = \frac{1}{2}\delta(x) + \frac{1}{2}\delta(x - A)$$

which are depicted as,

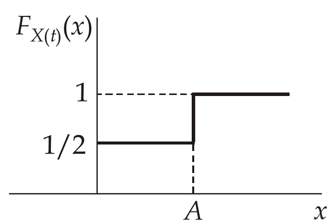


Fig. (a)

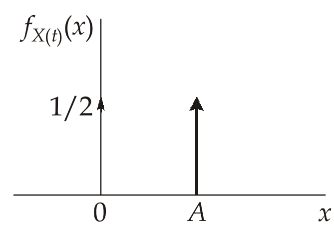


Fig. (b)

(ii) By ensemble-averaging, we have

$$E[X(t)] = \int_{-\infty}^{\infty} x\, f_{X(t)}(x)dx = \int_{-\infty}^{\infty} x\left[\frac{1}{2}\delta(x) + \frac{1}{2}\delta(x - A)\right]dx = \frac{A}{2}$$

The autocorrelation function of $X(t)$ is,

$$R_X(\tau) = E[X(t + \tau)X(t)]$$

Defining the square function $S_{qT_0}(t)$ as the square-wave shown below,

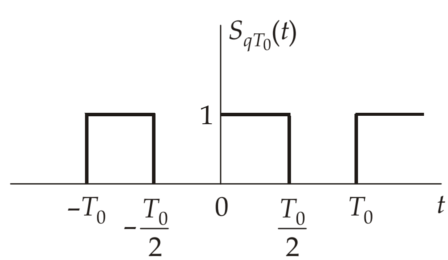


Fig. (c)

Then,

$$R_X(\tau) = E\left[AS_{qT_0}(t - t_d + \tau)\cdot AS_{qT_0}(t - t_d)\right]$$

$$= A^2 \int_{-\infty}^{\infty} S_{qT_0}(t - t_d + \tau)\cdot S_{qT_0}(t - t_d) f_{T_d}(t_d)\cdot dt_d$$

$$= A^2 \int_{-T_0/2}^{T_0/2} S_{qT_0}(t - t_d + \tau) S_{qT_0}(t - t_d) \frac{1}{T_0} dt_d$$

$$= \frac{A^2}{2}\left(1 - \frac{2|\tau|}{T_0}\right); |\tau| \leq \frac{T_0}{2}$$

Since the wave is periodic with period T_0, $R_X(\tau)$ must also be periodic with period T_0.

(iii) On a time-averaging basis, the mean is,

$$<X(t)> = \int_{-T_0/2}^{T_0/2} x(t)dt = \frac{A}{2}$$

Next, the autocorrelation function,

$$<X(t+\tau)\, X(t)> = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t+\tau)x(t)dt$$

We have, $$x(t)x(t+\tau) = \begin{cases} A^2 \begin{pmatrix} \frac{T_0}{2} - |\tau|; & |\tau| \leq \frac{T_0}{2} \\ 0 & ; |\tau| > \frac{T_0}{2} \end{pmatrix} \end{cases}$$

Then, $$<x(t+\tau) \cdot x(t)> = \frac{A^2}{2}\left(1 - 2\frac{|\tau|}{T_0}\right), |\tau| \leq \frac{T_0}{2}$$

Here the autocorrelation function is periodic with period T_0.

(iv) Since, the ensemble-averaging and time-averaging procedures yield the same set of results for the mean and autocorrelation functions. Thus, $X(t)$ is ergodic in both the mean and the autocorrelation function. Since ergodicity implies strict-sense stationarity, it follows that $X(t)$ must be wide-sense stationary.

Q.4 (c) Solution:

Given: hit ratio, $(H_C) = 0.95$; Main memory hit ratio, $(H_M) = 0.99$

Cache access time, $(T_C) = 5$ ns; SSD access time $(T_{SSD}) = 5\ \mu s = 5000$ ns

Let Main memory access time $(T_M) = x$

Formula for effective memory access time

$$= H_C T_C + (1 - H_C)H_M(T_C + T_M) + (1 - H_C)(1 - H_M) H_{SSD}(T_C + T_M + T_{SSD})$$

Since given effective memory access time = 20 ns

So,

$20 = 0.95 \times 5 + 0.05 \times 0.99 \times (5 + x) + 0.05 \times 0.01 \times (5 + x + 5000)$

$20 = 4.75 + 0.0495(5 + x) + 0.0005(5005 + x)$

$$20 - 4.75 = 0.0495x + 0.2475 + 2.5025 + 0.0005x$$

$$15.25 = 2.75 + 0.05x$$

$$15.25 - 2.75 = 0.05x$$

$$12.5 = 0.05x$$

$$x = \frac{12.5}{0.05} = 250 \text{ ns}$$

Capacity of SSD:

Given cost here is cost per KB. Let capacity of SSD is 2^x GB = $2^x \times 2^{30} = 2^{30+x}$ =Bytes

So, $$22000 = \frac{2^{20}}{2^{10}} \times 1 + \frac{127 \times 2^{20}}{2^{10}} \times 0.1 + \frac{2^{30+x}}{2^{10}} \times 0.001$$

$$22000 = 1024 + 13004.8 + 1048.576 \times 2^x$$

$$7971.2 = 2^x \times 1048.576$$

$$2^x = \frac{7971.2}{1048.576} = 7.6$$

We get, $x = 3$

So, Size of SSD $= 2^3$ GB = 8 GB

Section B

Q.5 (a) Solution:

Let $n = 3$

Consider seven-cell reuse pattern in cellular cluster and $N = 7$.

The signal to interference ratio is $\frac{S}{I} = \frac{(D/R)^n}{i_o} = \frac{(\sqrt{3N})^n}{i_o}$, where i_o is the number of co-channel interfering cells.

Co-channel reuse factor $Q = \frac{D}{R} = \sqrt{3N} = \sqrt{3 \times 7} = 4.583$

(i) For $n = 3$, $\frac{S}{I} = \frac{(4.583)^3}{6} = 16.04$

Therefore, $\frac{S}{I}\text{(in dB)} = 10\log(16.04) = 12.05 \text{ dB}$

But, for satisfactory performance, the S/I should be atleast 14 dB as per the requirement of cellular system.

Therefore, $N = 7$ is not suitable since 12.05 dB is less than standard value 14 dB.

The frequency reuse factor is given by $N = i^2 + j^2 + ij$, where $i, j = 0, 1, 3\ldots$ Thus, N can take the values 3, 7, 12, 13, etc.

Considering the frequency reuse factor, $N = 12$. Thus,

$$\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} = \frac{(\sqrt{3\times 12})^3}{6} = 36$$

Therefore, $\frac{S}{I}(dB) = 10 \log_{10} 36 = 15.56 \text{ dB}$

which is greater than the minimum requirement of 14 dB. Hence, a frequency reuse factor of 12 can be used for $n = 3$.

(ii) For $n = 4$,

Now, let us consider a seven cell reuse pattern and $N = 7$.

Co-channel reuse factor $= Q = \frac{D}{R} = 4.583$

Signal to interference ratio $= \frac{S}{I} = \frac{1}{6}(4.583)^4 = 73.52$

Therefore, $\frac{S}{I}\text{(dB)} = 10\log(73.52) = 18.664 \text{ dB}$

The signal to interference is greater than the minimum requirement of 14 dB , hence a frequency reuse factor of 7 can be used for $n = 4$.

Q.5 (b) Solution

RIM instruction is used to read the status of serial input data (SID) line into the accumulator. The most significant bit of the Accumulator after the execution of the RIM instruction provides the serial input data.

Assembly Language Program:

```
        MVI B, 00H   ;  Clear register B
        MVI C, 08H   ;  Preset the counter to 8
NEXT:   RIM          ;  GET the bit through SID line
        ANI 80H      ;  Isolate the bit received through SID
        ORA B        ;  Convert to parallel word
```

```
RRC           ;  Rotate right the content of accumulator
MOV B, A      ;  Save the accumulator content to register B
DCR C         ;  Decrement the counter
JNZ NEXT      ;  Jump to read next bit if counter value is non-zero
RLC           ;  Rotate accumulator left
STA 2500H     ;  Store the parallel data at 2500H location
HLT
```

Q.5 (c) Solution:

A direct-mapped cache is a type of cache memory where each block of main memory has a unique location (cache line) in the cache where it can be stored given by (Main Memory Block Reference) MOD (Number of Cache Lines). Thus, Block '*i*' from memory maps to cache block (*i* mod 4).

Direct mapped cache technique:

4 mod 4 = 0 → 4-miss
5 mod 4 = 1 → 5-miss
7 mod 4 = 3 → 7-miss
12 mod 4 = 0 → 12-miss
4 mod 4 = 0 → 4-miss
5 mod 4 = 1 → 5-Hit
13 mod 4 = 1 → 13-miss
4 mod 4 = 0 → 4-Hit
5 mod 4 = 1 → 5-miss
7 mod 4 = 3 → 7-Hit

0	$\cancel{4}$ $\cancel{12}$ 4
1	$\cancel{5}$ $\cancel{13}$ 5
2	
3	7

Thus, $\text{Hit ratio} = \dfrac{3}{10} = 0.3$

Q.5 (d) Solution

- Let, the concentration of boron in the final solid crystal = N_{Bs}
  and the initial concentration of boron in the melt = N_{Bl}
- So, the equilibrium segregation coefficient for boron can be written as,

$$k_0 = \frac{N_{Bs}}{N_{Bl}}$$

Given that,
$$k_0 = 0.8$$
$$N_{Bs} = 10^{16} \text{ atoms/cm}^3$$

So, $$N_{Bl} = \frac{N_{Bs}}{k_0} = 1.25 \times 10^{16} \text{ atoms/cm}^3$$

Number of boron atoms required = N_{Bl} × (Volume of the melt)

Since the amount of boron concentration is so small, the volume of the melt can be calculated from the weight of silicon.

Given that, the amount of silicon present in the crucible is 60 kg and the density of molten silicon is 2.53 g/cm^3.

So, $$\text{Volume of the melt} = \frac{(60\times1000)}{(2.53)}\text{cm}^3 = 2.37 \times 10^4 \text{ cm}^3$$

Number of boron atoms required = (1.25×10^{16}) (2.37×10^4) atoms = 2.96×10^{20} atoms

$$\text{So, Amount of boron required} = \frac{\text{(Number of boron atoms) (Atomic weight of boron)}}{\text{(Avogadro's number)}}$$

$$= \frac{2.96\times10^{20}\times10.8}{6.02\times10^{23}}\text{ g} = 5.31 \text{ mg}$$

Q.5 (e) Solution:

Given data, $$\vec{E}_s = \frac{\cos 2\theta}{r}e^{-j\beta r}\hat{a}_\theta \text{ V/m}$$

(i) We know that, $|\vec{H}_s| = \dfrac{|\vec{E}_s|}{\eta}$. Thus, we can write, $\vec{H}_s = \dfrac{\cos 2\theta}{\eta r}e^{(-j\beta r)}\hat{a}_H$

We have, $$\hat{a}_E \times \hat{a}_H = \hat{a}_k \rightarrow \hat{a}_\theta \times \hat{a}_H = \hat{a}_r \Rightarrow \hat{a}_H = \hat{a}_\phi$$

$$\vec{H}_s = \frac{\cos 2\theta}{120\pi r}e^{(-j\beta r)}\hat{a}_\phi$$

(ii) $$\vec{P}_{\text{avg}} = \frac{|E_s|^2}{2\eta}\hat{a}_r = \frac{\cos^2(2\theta)}{2\eta r^2}\hat{a}_r$$

$$P_{\text{rad}} = \frac{1}{2\eta}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\frac{\cos^2 2\theta}{r^2}r^2\sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{240\pi}2\pi\int_0^{\pi}\cos^2 2\theta\sin\theta\, d\theta$$

We have, $$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

Let $\cos\theta = u \Rightarrow -\sin\theta\, d\theta = du$. We can write,

$$P_{\text{rad}} = -\frac{1}{120}\int_{\cos 0}^{\cos\pi}(2u^2-1)^2\,du$$

$$= -\frac{1}{120}\int_0^{\pi}(2\cos^2\theta-1)^2\,d(\cos\theta)$$

$$= -\frac{1}{120}\int_0^{\pi}(4\cos^4\theta-4\cos^2\theta+1)\,d(\cos\theta)$$

$$= -\frac{1}{120}\left(\frac{4\cos^5\theta}{5}-\frac{4\cos^3\theta}{3}+\cos\theta\right)\Bigg|_0^{\pi}$$

$$= -\frac{1}{120}\left[-\frac{4}{5}+\frac{4}{3}-1-\frac{4}{5}+\frac{4}{3}-1\right]$$

$$= \frac{1}{120}\times\frac{14}{15} = 7.778\text{ mW}$$

(iii) In the region $60° < \theta < 120°$,

$$P_{\text{rad}} = -\frac{1}{120}\int_{60°}^{120°}(2\cos^2\theta-1)^2\,d(\cos\theta)$$

$$= -\frac{1}{120}\left(\frac{4\cos^5\theta}{5}-\frac{4\cos^3\theta}{3}+\cos\theta\right)\Bigg|_{60°}^{120°}$$

$$= -\frac{1}{120}\left[\frac{4}{5}\left(-\frac{1}{32}\right)-\frac{4}{3}\left(-\frac{1}{8}\right)-\frac{1}{2}-\frac{4}{5}\left(\frac{1}{32}\right)+\frac{4}{3}\left(\frac{1}{8}\right)-\frac{1}{2}\right]$$

$$= \frac{1}{60}\left[\frac{1}{40}+\frac{1}{2}-\frac{1}{6}\right]$$

$$= 5.972\text{ mW}$$

which is $\frac{5.972}{7.778} = 0.7678 \text{ or } 76.78\%$ of the total radiated power.

Q.6 (a) Solution:

(i) Auto correlation function of $X(t)$ is expressed as,

$$R_X(t_1, t_0) = E[X(t_1) \cdot X(t_0)]$$
$$= E\left[\{Z_1(t_1) + 5Z_2(t_1 - \tau)\}\{Z_1(t_0) + 5Z_2(t_0 - \tau)\}\right]$$
$$= E[Z_1(t_1)Z_1(t_0)] + 5E[Z_1(t_1)Z_2(t_0 - \tau)] + E[Z_2(t_1 - \tau)Z_1(t_0)]$$
$$+ 25[Z_2(t_1 - \tau)Z_2(t_0 - \tau)]$$

We know that,

$$E[Z_1(t_1)\, Z_2(t_0 - \tau)] = 0$$

since, $Z_1(t)$ and $Z_2(t)$ are independent processes.

Similarly, $E[Z_2(t - \tau)Z_1(t_0)] = 0$

For white noise $W(k)$,

$$E[W(k)W(k + n)] = \sigma^2\, \delta(n), \text{ where } \sigma^2 \text{ is the variance}$$

$\therefore$ We have,

$$E[Z_1(t_1) \cdot Z_1(t_0)] = 1.5\, \delta(t_1 - t_0)$$

Similarly,

$$E[Z_2(t_1 - \tau) \cdot Z_2(t_0 - \tau)] = 1.5\, \delta(t_1 - t_0)$$

Thus, $$R_X(t_1, t_0) = 1.5\, \delta(t_1 - t_0) + 37.5\, \delta(t_1 - t_0)$$

$$R_X(t_1, t_0) = 39\, \delta(t_1 - t_0) \quad ...(i)$$

Similarly, $$R_Y(t_1, t_0) = 6\, \delta(t_1 - t_0) + 37.5\, \delta(t_1 - t_0)$$

$$R_Y(t_1, t_0) = 43.5\, \delta(t_1 - t_0) \quad ...(ii)$$

From equation (i) and (ii), we see that the random processes $X(t)$ and $Y(t)$ are wide sense stationary (WSS) as the mean is independent of time and the autocorrelation depends only on the time difference.

(ii) **1.** Given data:

The given system is an ASK system with

$$S_1(t) = A\sin\left(\frac{\pi t}{T}\right) \text{ and } S_2(t) = 0$$
$$A = 0.2 \text{ mV}, T = 2\ \mu\text{sec},$$
$$\frac{N_0}{2} = 10^{-15} \text{ W/Hz}$$

Since the symbols are equally probable, for binary signalling scheme,

Probability of error, $P_e = Q\left[\sqrt{\dfrac{d_{\min}^2}{2N_0}}\right]$, where $d_{\min}$ is the minimum distance between the two signalling points in the constellation diagram.

From the constellation diagram for ASK,

$$d_{\min} = \sqrt{E_b} = \sqrt{\frac{A^2 T_b}{2}}$$

$$d_{\min} = \sqrt{\frac{(0.2\times10^{-3})^2\times2\times10^{-6}}{2}}$$

$$d_{\min} = \frac{2\times10^{-4}\times10^{-3}\times\sqrt{2}}{\sqrt{2}} = 2\times10^{-7}$$

Thus,
$$P_e = Q\left(\sqrt{\frac{(2\times10^{-7})^2}{2\times2\times10^{-15}}}\right)$$

$$P_e = Q\left[\frac{2\times10^{-7}\times\sqrt{10}}{2\times10^{-7}}\right] = Q\left(\sqrt{10}\right)$$

2. Given data: Bandwidth of signal is

$$f_m = 4\text{ kHz.}$$

$$\text{Nyquist Rate} = 2f_m = 8\text{ kHz}$$

Hence, $\quad \text{sampling rate} = 1.25\times8\times10^3 = 10^4\,\frac{\text{samples}}{\text{sec}}$

Since, the samples are quantized into 256 equally likely levels. The number of bits per sample is given by

$$H = \log_2(M) = \log_2(256) = 8\text{ bits/sample}$$

$\therefore \quad \text{Information rate} = H \times \text{sampling rate}$

$$(R) = 8\times10^4 = 80\text{ kbps}$$

According to Shannon's Theorem, to ensure for error free transmission for a channel bandwidth $B = 10$ kHz,

$$R \le C = B\log_2\left(1+\frac{S}{N}\right)$$

$$80\times10^3 \le 10\times10^3\log_2\left(1+\frac{S}{N}\right)$$

$$\frac{S}{N} \ge 255$$

$$\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}255 = 24\text{ dB}$$

Q.6 (b) Solution:

(i) SJF (Without pre-emption):

Shortest Job First (SJF) is a scheduling algorithm that selects the waiting process with the shortest execution time to execute next. In non-preemptive SJF, once a process has been allocated to the CPU, it will continue to execute until it finishes, even if a shorter process arrives and enters the ready queue.

Gantt Chart:

P_2	P_3	P_1	P_5	P_4

0 5 8 18 20 40

Process	AT (ms)	BT (ms)	CT (ms)	TAT = CT – AT (ms)	WT = TAT – BT (ms)
P_1	0	10	18	18	8
P_2	0	5	5	5	0
P_3	2	3	8	6	3
P_4	5	20	40	35	15
P_5	10	2	20	10	8

[AT = Arrival Time, BT = Burst Time, CT = Completion Time, TAT = Turnaround Time, WT = Waiting Time]

$\therefore$ Average waiting time $= \dfrac{8+0+3+15+8}{5} = \dfrac{34}{5}$ = 6.8 msec

(ii) SJF (With pre-emption):

SJF with pre-emption is a scheduling algorithm that selects the waiting process with the shortest remaining execution time to execute next. However, unlike non-preemptive SJF, a process can be preempted if a shorter process arrives and enters the ready queue.

Gantt Chart:

P_2	P_3	P_1	P_5	P_1	P_4

0 5 8 10 12 20 40

Process	AT (ms)	BT (ms)	CT (ms)	TAT = CT – AT (ms)	WT = TAT – BT (ms)
P_1	0	10	20	20	10
P_2	0	5	5	5	0
P_3	2	3	8	6	3
P_4	5	20	40	35	15
P_5	10	2	12	2	0

$$\text{Average waiting time} = \frac{10+0+3+15+0}{5} = \frac{28}{5} = 5.6 \text{ msec}$$

(iii) Priority Scheduling (without pre-emption):

Priority scheduling without pre-emption selects the waiting process with the highest priority to execute next. Once a process has been allocated to the CPU, it will continue to execute until it finishes, even if a higher priority process arrives and enters the ready queue.

Gantt Chart:

P_2	P_3	P_4	P_5	P_1

0 5 8 28 30 40

Process	AT (ms)	BT (ms)	CT (ms)	TAT = CT – AT (ms)	WT = TAT – BT (ms)
P_1	0	10	40	40	30
P_2	0	5	5	5	0
P_3	2	3	8	6	3
P_4	5	20	28	23	3
P_5	10	2	30	20	18

$$\therefore \quad \text{Average waiting time} = \frac{30+0+3+3+18}{5} = \frac{54}{5} = 10.8 \text{ msec}$$

(iv) Priority scheduling (with pre-emption):

Priority scheduling with pre-emption is a scheduling algorithm that selects the waiting process with the highest priority to execute next. However, unlike priority scheduling without preemption, a process can be preempted if a higher priority process arrives and enters the ready queue.

Gantt Chart:

P_2	P_3	P_2	P_4	P_5	P_4	P_1

0 2 5 8 10 12 30 40

Process	AT (ms)	BT (ms)	CT (ms)	TAT = CT – AT (ms)	WT = TAT – BT (ms)
P_1	0	10	40	40	30
P_2	0	5	8	8	3
P_3	2	3	5	3	0
P_4	5	20	30	25	5
P_5	10	2	12	2	0

$$\therefore \quad \text{Average waiting time} = \frac{30+3+0+5+0}{5} = \frac{38}{5} = 7.6 \text{ msec}$$

Q.6 (c) Solution

(i) The incident flux density at the earth station can be given by,

$$F = \frac{P_t G_t}{4\pi r^2}$$

$[G_t] = 30 \text{ dB} \Rightarrow G_t = 1000$

So,
$$F = \frac{(20)\,(1000)}{4\pi \times (38000\times 10^3)^2} \text{ W/m}^2 = 1.1\times 10^{-12} \text{ W/m}^2$$

In decilogs,
$$[F] = 10 \log_{10} F = -119.6 \text{ dBW/m}^2$$

(ii) The effective area of the earth station antenna can be given by,

$$A_{\text{eff}} = \eta_A \left(\pi \frac{d^2}{4}\right) = 0.65\times\pi\times\frac{(2)^2}{4} = 2.042 \text{ m}^2$$

So, the power received by the earth station receiver can be given by,

$$P_r = FA_{\text{eff}} = 1.1 \times 10^{-12} \times 2.042 = 2.2462 \times 10^{-12} \text{ W}$$

In decilogs,
$$[P_r] = 10\log_{10}(P_r) = -116.5 \text{ dBW}$$

(iii) The on-axis gain of the earth station antenna with circular aperture can be given by,

$$G_r = \eta_A \left(\frac{\pi d}{\lambda}\right)^2$$

We have, $\eta_A = 0.65$ and $\lambda = \frac{c}{f} = \frac{3\times10^8}{4\times10^9} = 0.075$ m

So, $$G_r = 0.65\times\left(\frac{\pi\times2}{0.075}\right)^2 = 4561.95$$

In decilogs, $$[G_r] = 10\log_{10}(G_r) = 36.6 \text{ dB}$$

(iv) The free space path loss between the satellite and the earth station can be given by,

$$L_p = \left(\frac{4\pi r}{\lambda}\right)^2$$

In decilogs,

$$[L_p] = 10\log_{10}\left(\frac{4\pi r}{\lambda}\right)^2 = 20\log_{10}\left(\frac{4\pi\times38000\times1000}{0.075}\right)\text{dB} = 196.1 \text{ dB}$$

(v) By using the results obtained in parts (iii) and (iv), the power received by the earth station can be given by,

$$[P_r] = [P_t] + [G_t] + [G_r] - [L_p]$$

$P_t = 20$ W $\Rightarrow [P_t] = 10\log_{10}(P_t) = 13$ dBW

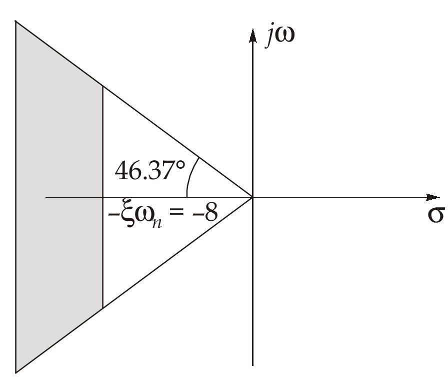
So, $$[P_r] = 13 + 30 + 36.6 - 196.1 = -116.5 \text{ dBW}$$

The power received by the earth station is obtained as –116.5 dBW using both the methods.

Q.7 (a) Solution:

(i) Given, $$M_p \le 0.05$$

Considering $M_p = 0.05$,



$$e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = 0.05$$

$$\frac{-\xi\pi}{\sqrt{1-\xi^2}} = \ln 0.05$$

$$\xi = 0.69$$

$$\Rightarrow \quad \theta = \cos^{-1}(0.69) = 46.37°$$

$\therefore \qquad \xi \geq 0.69 \quad \Rightarrow \quad -46.37° < \theta < 46.37°$

where θ is the angle subtended by the pole at the origin.

Also, $\qquad t_s \leq 500 \text{ ms}$

For 2% tolerance band,

$$\frac{4}{\xi\omega_n} \leq 500 \text{ ms}$$

$$\xi\omega_n \geq 8$$

i.e. the poles of a second order system must lie to the left of the line $s = -8$ in the region as specified in the plot.

(ii) 1. The spectrum of the AM signal $u(t)$ contains the impulses at different frequency components. From the given spectrum plot, we can write frequency spectrum of $u(t)$ as

$$\begin{aligned} U(f) = {} & 0.25[\delta(f-(f_c+1.5k)] + 0.25[\delta(f-(f_c-1.5k)] \\ & + 0.25[\delta(f+(f_c-1.5k)] + 0.25[\delta(f+(f_c+1.5k)] \\ & + 2.5[\delta(f-(f_c+3k)] + 2.5[\delta(f-(f_c-3k)] \\ & + 2.5[\delta(f+(f_c-3k)] + 2.5[\delta(f+(f_c+3k)] \end{aligned}$$

And we know that,

$$A\cos 2\pi f_0 t \xleftrightarrow{\text{F.T.}} \frac{A}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$$

Using the above relation, we can write

$$\begin{aligned} u(t) = {} & 0.5\cos(2\pi(f_c+1500)t) + 0.5\cos(2\pi(f_c-1500)t) \\ & + 5\cos(2\pi(f_c+3000)t) + 5\cos(2\pi(f_c-3000)t) \qquad \text{...(i)} \end{aligned}$$

Since, $\cos(a+b) + \cos(a-b) = 2\cos a\cos b$,

$$u(t) = \cos 3000\pi t \times \cos 2\pi f_c t + 10\cos 6000\pi t \cdot \cos 2\pi f_c t$$

$$u(t) = 10(0.1\cos 3000\pi t + \cos 6000\pi t)\cos 2\pi f_c t \qquad \text{...(ii)}$$

Comparing with the standard expression of DSB-SC AM signal $s(t) = m(t).c(t)$. We get,

$$m(t) = (0.1\cos 3000\,\pi t + \cos 6000\,\pi t)$$

$$c(t) = 10\cos 2\pi f_c t$$

2. From the given spectrum, we observe that the spectrum has both the sidebands, LSB and USB. However, the carrier signal is not present.

Therefore, we conclude that here Double sideband-suppressed carrier modulation scheme (DSB-SC) is used.

Main Advantage of DSB-SC over DSB-FC is that, in DSB-SC power consumption is less than DSB-FC. DSB-SC is an amplitude modulated wave transmission scheme in which only sidebands are transmitted and the Carrier is not transmitted as it gets suppressed. Whereas, in DSB-FC sidebands are transmitted along with the carrier. The carrier does not contain any information and its transmission results in loss of power.

Percentage of power saved in DSB-SC as compared to DSB-FC is given as,

$$\% \text{ Power saved} = \frac{\text{Carrier power}}{\text{Power of DSB-FC signal}}$$

$$= \frac{P_c}{P_c\left[1+\frac{\mu^2}{2}\right]} = \frac{2}{2+\mu^2}\times 100\%$$

From equation (ii), we get

$$\mu_1 = 0.1$$

$$\mu_2 = 1$$

Thus, $$\mu_t = \sqrt{\mu_1^2+\mu_2^2} = \sqrt{(0.1)^2+1^2} \approx 1$$

$\therefore$ $$\text{Power saved} = \frac{2}{2+1} = 0.667$$

% power saved in DSB-SC w.r.t DSB-FC = 66.7%

3. The power content at the frequency, P_{f_c+1500} is the same as the power content at the frequency P_{f_c-1500} and equal to $(0.25)^2 + (0.25)^2 = 0.125$ Watt.

Similarly,

$$P_{f_c+3000} = P_{f_c-3000} = (2.5)^2 + (2.5)^2 = 12.5 \text{ Watt}$$

4. Using equation (i),

Total power dissipated,

$$P_t = \frac{P_c\mu_t^2}{2} = \left(\frac{(0.5)^2}{2}\times 2\right)+\left(\frac{5\times 5}{2}\times 2\right) = 25.25 \text{ Watt}$$

Thus, $$\frac{P_c\mu_t^2}{2} = 25.25$$

$$\mu_t^2 = \frac{50.5}{P_c} \quad \text{...(iii)}$$

Since, $$P_c = \frac{A_c^2}{2} = \frac{100}{2} = 50 \text{ W}$$

From equation (iii), $\mu_t^2 = \frac{50.5}{50} = 1.01$

$$\mu_t \approx 1.005$$

Bandwidth of DSB-SC = $2f_{max}$, where f_{max} is the maximum frequency in the modulating signal

We have, $m_1(t) = 0.1 \cos 3000\pi t \Rightarrow f_{m1} = 1500$ Hz

$m_2(t) = \cos 6000\,\pi t \Rightarrow f_{m2} = 3000$ Hz

Thus, $\text{BW} = 2f_{max} = 2f_{m2}$

$= 2 \times 3000$

$\text{BW} = 6$ kHz

Q.7 (b) Solution:

(i) Let

P_d = Power loss or power dissipated

P_a = Power delivered to the antenna

P_0 = Input power to the guide

Thus, $P_0 = P_d + P_a$

The power in the waveguide decays exponentially as

$$P_a = P_0\, e^{-2\alpha z}$$

Hence, $P_a e^{2\alpha z} = P_d + P_a$

or $P_d = P_a(e^{2\alpha z} - 1)$

Now, we need to determine α from

$$\alpha = \alpha_d + \alpha_c,$$

where $\alpha_d + \alpha_c$ are the attenuation constant due to dielectric loss and conductor loss respectively.

We have, $$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Since the loss tangent

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-17}\times 36\pi\times 10^9}{2\pi\times 5\times 10^9\times 3\times 1} = 1.2\times 10^{-17}$$

$= 1.2 \times 10^{-17} << 1$ (lossless dielectric medium) then $$\eta' = \sqrt{\frac{\mu}{\in}} = \frac{377}{\sqrt{\in_r}} = \frac{377}{\sqrt{3}} = 217.661\ \Omega$$

$$v' = \frac{1}{\sqrt{\mu\in}} = \frac{c}{\sqrt{\in_r}} = \frac{3\times10^8}{\sqrt{3}} = 1.732\times10^8\,\text{m/s}$$

For TE_{10} mode, $$f_c = \frac{v'}{2a} = \frac{1.73\times10^8}{2\times4\times10^{-2}} = 2.162\ \text{GHz}$$

$\therefore$ $$\alpha_d = \frac{10^{-17}\times217.661}{2\sqrt{1-\left(\frac{2.162}{5}\right)^2}} = 1.206\times10^{-15}\,\text{Np/m}$$

For the TE_{10} mode, gives

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[0.5+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

where, $$R_s = \frac{1}{\sigma_c\delta} = \sqrt{\frac{\pi f\mu}{\sigma_c}} = \sqrt{\frac{\pi\times5\times10^9\times4\pi\times10^{-7}}{8\times10^7}}$$

$$= 0.0157 = 15.707\times10^{-3}\ \Omega$$

Hence, $$\alpha_c = \frac{2\times15.707\times10^{-3}\left(0.5+\frac{2.5}{4}\left(\frac{2.162}{5}\right)^2\right)}{2.5\times10^{-2}\times217.661\sqrt{1-\left(\frac{2.162}{5}\right)^2}}$$

$$= 3.949\times10^{-3}\ \text{Np/m}$$

Note that $\alpha_d << \alpha_c$, showing that the loss to the finite conductivity of the guide walls is more important than the loss due to the dielectric medium. Thus,

$$\alpha = \alpha_d + \alpha_c \cong \alpha_c = 3.949\times10^{-3}\ \text{Np/m}$$

Thus, for a guide length of 50 cm, the power dissipated is

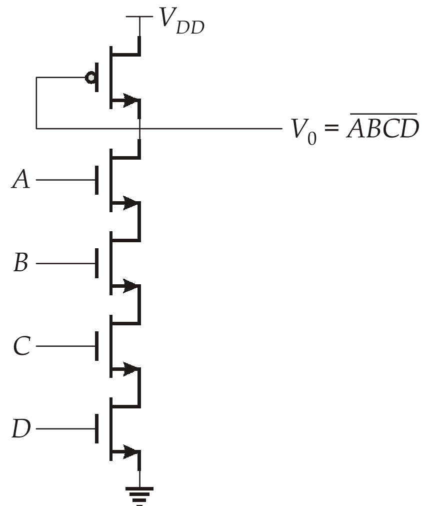
$$P_d = P_a(e^{2\alpha z}-1)$$

$$= 2.4\times10^3(e^{2\times3.949\times10^{-3}\times0.5}-1)$$

$$P_d = 9.496\ \text{W}$$

(ii) A four input NAND gate using depletion load gate is as shown below. When all the transistors are ON, then the total ON resistance becomes $4R_{ON}$. The ON resistance is inversely proportional to the (W/L) ratio of the transistor. So, if the driver of the reference inverter has $(W/L)_{ref} = 2/1$, then the (W/L) of the drivers in the NAND gate will be

$$\left(\frac{W}{L}\right)_{driver} = 4\left(\frac{W}{L}\right)_{ref} = \frac{8}{1}$$



So, the drivers will have the size of $\frac{8}{1}$. When all the four drivers are conducting then $\left(\frac{W}{L}\right)$ ratio of the load devices are exactly same as those in the reference inverter. Thus,

$$\left(\frac{W}{L}\right)_{load} = \left(\frac{W}{L}\right)_{ref} = \frac{2}{1}$$

Q.7 (c) Solution:

(i) Using the Taylor series expansion, $\exp(-x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6}$, we may express the diode current i, normalized with respect to I_0, as

$$\frac{i}{I_0} = \exp\left(-\frac{v}{V_T}\right) - 1 = -\frac{v}{V_T} + \frac{1}{2}\left(\frac{v}{V_T}\right)^2 - \frac{1}{6}\left(\frac{v}{V_T}\right)^3 \quad ...(1)$$

(ii) Given,

$$\frac{v}{V_T} = \frac{0.01}{0.026}[\cos(2\pi f_m t) + \cos(2\pi f_c t)]$$

$$= 0.385\,[\cos(2\pi f_m t) + \cos(2\pi f_c t)] \quad ...(2)$$

We find that substitution of equation (2) in (1) yields

$$\frac{i}{I_0} = -0.385[\cos(2\pi f_m t) + \cos(2\pi f_c t)] + 0.074[\cos(2\pi f_m t) + \cos(2\pi f_c t)]^2 - 0.0095[\cos(2\pi f_m t) + \cos(2\pi f_c t)]^3 \quad ...(3)$$

Noting that

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

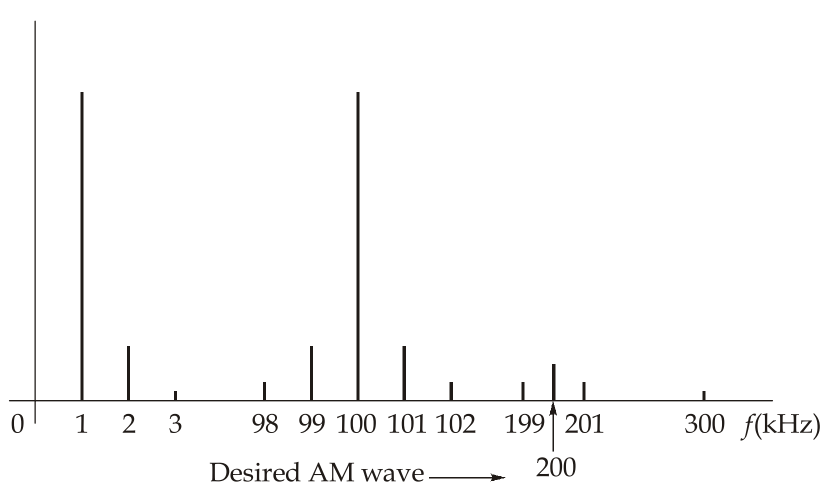
$$\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$$

$$\cos\theta\cos\phi = \frac{1}{2}[\cos(\theta+\phi)+\cos(\theta-\phi)]$$

We may rewrite equation (3) in the form:

$$\frac{i}{I_0} = 0.074 - 0.406[\cos(2\pi f_m t) + \cos(2\pi f_c t)] + 0.037 \{\cos(4\pi f_m t) + \cos(2\pi f_c t) + 2\cos[2\pi(f_c + f_m)t] + 2\cos(2\pi(f_c - f_m)t]\} - 0.0024[\cos(6\pi f_m t) + \cos(6\pi f_c t)] - 0.0071\{\cos[2\pi(f_c + 2f_m)t] + \cos[2\pi(f_c - 2f_m)t] + \cos[2\pi(2f_c + f_m)t] + \cos[2\pi(2f_c - f_m)t]\}$$

For f_m = 1 kHz and f_c = 100 kHz, the discrete amplitude spectrum of the diode current i (for $f \geq 0$) is as shown below:



(iii) From this amplitude spectrum, we note that in order to extract an AM wave, with carrier frequency f_c from the diode current i, we need a band-pass filter that passes only the frequency components: 99, 100 and 101 kHz, corresponding to $f_c - f_m$, f_c and $f_c + f_m$, respectively. We therefore, require a band-pass filter with centre frequency 100 kHz and bandwidth 2 kHz.

(iv) The resulting band-pass filter output is

$$\frac{i}{I_0} \cong -0.406\cos(2\pi f_c t) + 0.148\cos(2\pi f_c t)\cos(2\pi f_m t)$$

$$= -0.406(1 - 0.364\cos(2\pi f_m t))\cos(2\pi f_c t)$$

The percentage modulation is therefore 36.4 percent.

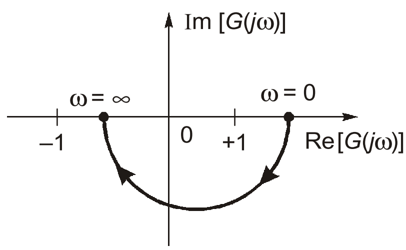
Q.8 (a) Solution:

```
#include<stdio.h>
// Function to print lower triangular matrix
void lower(int matrix[3][3], int row, int col)
{
    int i, j;
    for (i=0; i<row; i++)
    {
            for (j=0; j<col; j++)
                        {
                          if (i<j)
                          {
                            matrix[i][j] = 0;
                          }
                         printf("%d ",matrix[i][j]);
                        }
                printf("\n");
    }
}
// Function to print upper triangular matrix
void upper(int matrix[3][3], int row, int col)
```

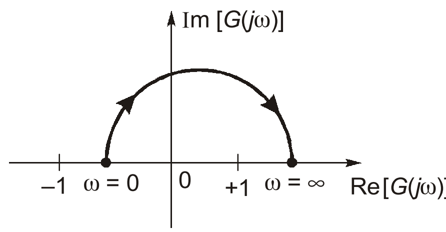
```
{
    int i, j;
    for (i=0; i<row; i++)
    {
            for (j=0; j<col; j++)
                        {
                         if (i>j)
                           {
                              matrix[i][j] = 0;
                           }
                         printf("%d",matrix[i][j]);
                        }
                printf("\n");
    }
}
// Main function
int main()
{
    int matrix[3][3];
    int row = 3, col = 3;
    printf ("Enter the elements of the 3 × 3 matrix:\n");
    for (i = 0; i < row; i++)
    {
      for (j = 0; j < col; j++)
      {
        scanf("%d", &matrix[i][j]);
      }
    }
    printf("Lower triangular matrix: \n");
    lower(matrix, row, col);
    printf("Upper triangular matrix: \n");
    upper(matrix, row, col);
    return 0;
}
```

Q.8 (b) Solution:

(i) The possible polar plots for the given Nyquist plot are,



or



Let, $$G(s) = \frac{(s+b_1)(s+b_2)........(s+b_m)}{(s+a_1)(s+a_2)........(s+b_n)}$$

If $n > m \Rightarrow \quad |G(j\omega)|_{\omega=\infty} = 0$

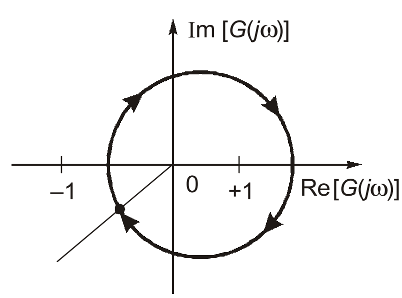
If $n < m \Rightarrow \quad |G(j\omega)|_{\omega=\infty} = \infty$

- But in both the possible cases of polar plots, gain is finite at $\omega = \infty$. This is possible only when $m = n$.
- For a strictly proper transfer function, $n > m$. But here for the given system $n = m$. So, $G(s)$ cannot be a strictly proper transfer function.

(ii) Let us take $N_{(0+j0)}$ as the number of encirclements about the origin of the Nyquist plot in the anti-clockwise direction. Thus,

$N_{(0+j0)}$ = [No. of poles of $G(s)$ on RHS of s-plane] – [No. of zeros of $G(s)$ on RHS of s-plane]

By calculating the number of encirclements about the origin of the given Nyquist plot, we get,



$N_{(0+j0)} = -1$ [Negative sign indicates encirclements in clockwise direction] ...(i)

So,

[No. of poles and $G(s)$ on RHS of s-plane] – [No. of zero of $G(s)$ on RHS of s-plane] = –1

[No. of zeros of $G(s)$ on RHS of s-plane] = [No. of poles of $G(s)$ on RHS of s-plane] + 1

So, [No. of zeros of $G(s)$ on RHS of s-plane] > [No. of poles of $G(s)$ on RHS of s-plane]

(iii) When the open-loop system in stable,

[No. of poles of $G(s)$ on RHS of s-plane] = 0

From equation (i), $N_{(0 + j0)} = -1$

[No. of poles of $G(s)$ on RHS of s-plane] – [No. of zeros of $G(s)$ on RHS of s-plane] = –1

So, when the open-loop system is stable,

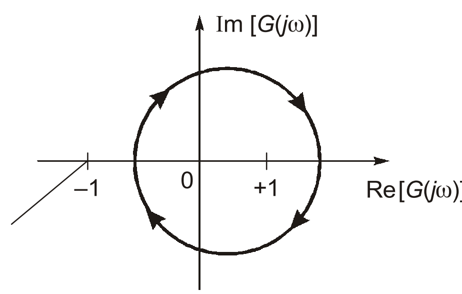
0 – [No. of zeros of $G(s)$ on RHS of s-plane] = –1

[No. of zeros of $G(s)$ on RHS of s-plane] = 1

As there is one zero in the RHS of s-plane, the function $G(s)$ is a non-minimum-phase transfer function when the open-loop system is stable.

(iv) To investigate the stability of a closed-loop system, we have to take the encirclements about the point $(-1 + j0)$ of the Nyquist plot.

By taking encirclements about the point $(-1 + j0)$ of the Nyquist plot, we get,



$$N_{(-1 + j0)} = 0 \quad \text{... (ii)}$$

$N_{(-1 + j0)}$ = [No. of poles of $(1 + G(s))$ on RHS of s-plane]
– [No. of zeros of $(1 + G(s))$ on RHS of s-plane]

We have, Poles of $(1 + G(s))$ = Poles of $G(s)$

Zeros of $(1 + G(s))$ = Closed-loop poles of feedback system

So, $N_{(-1 + j0)}$ = [No. of poles of $G(s)$ on RHS of s-plane] – [No. of closed-loop poles on RHS of s-plane]

When the closed-loop system is stable,

[No. of closed-loop poles on RHS of s-plane] = 0

So, 0 = [No. of poles of $G(s)$ on RHS of s-plane] – 0

[No. of poles of $G(s)$ on RHS of s-plane] = 0

Hence, the open-loop system is also stable.

So, the closed-loop system will be stable only when the open-loop system is stable.

Q.8 (c) Solution:

(i) Given line constant, $R = 2.25\ \Omega/\text{m}$; $L = 1\ \mu\text{H/m}$; $C = 1\ \text{pF/m}$; $G = 0$

Frequency of operation, $f = 0.5$ GHz

The propagation constant γ for lossy transmission line is,

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(2.25 + j2\pi \times 0.5 \times 10^9 \times 10^{-6})(0 + j2\pi \times 0.5 \times 10^9 \times 1 \times 10^{-12})}$$

$$\gamma = \sqrt{7.06 \times 10^{-3} j - 9.86} = \sqrt{9.86\angle 179.95^\circ} = 3.14\angle 89.975^\circ$$

$$\gamma = (1.37 \times 10^{-3}) + j(3.14) = \alpha + j\beta$$

$\therefore$ The attenuation constant, $\alpha = (1.37 \times 10^{-3})/\text{m}$

(ii) 1. To feed equal power to two loads, the input resistance at the junction with the main line looking towards each load must be

$$R_{i_1} = 2R_o = 100\ \Omega$$

$$R_{i_2} = 2R_o = 100\ \Omega$$

Under matched conditions, there are no standing waves on the main transmission line, i.e., VSWR = 1.

Standing wave ratio for matching section 1.

$$\Gamma_{L_1} = \frac{R_{L1} - R'_{01}}{R_{L1} + R'_{01}},$$

where R'_{01} is the characteristic impedance of matching section 1.

The input impedance of a quarter-wave line is given by

$$R_{i1} = \frac{(R'_{01})^2}{R_{L1}}$$

$\therefore$ $$R'_{01} = \sqrt{R_{i1} \times R_{L1}} = \sqrt{100 \times 64} = 80\ \Omega$$

Similarly, $$R'_{02} = \sqrt{R_{i2} \times R_{L2}} = \sqrt{100 \times 25} = 50\ \Omega$$

$\therefore$ $$\Gamma_{L_1} = \frac{64 - 80}{64 + 80} = -0.11$$

$\therefore$ $$S_1 = \frac{1 + |\Gamma_{L_1}|}{1 - |\Gamma_{L_1}|} = \frac{1 + 0.11}{1 - 0.11} = 1.25$$

For matching section 2:

$$\Gamma_{L_2} = \frac{R_{L2} - R'_{02}}{R_{L2} + R'_{02}} = \frac{25-50}{25+50} = -0.33$$

$$\therefore \qquad S_2 = \frac{1+|\Gamma_{L_2}|}{1-|\Gamma_{L_2}|} = \frac{1+0.33}{1-0.33} = 1.99$$

2. Given, $\quad v_p = 0.5c = \frac{c}{2}$

Wavelength, $\quad \lambda = \frac{v_p}{f} \text{ (or) } \frac{2\pi}{\beta}$

$$\therefore \qquad \lambda = \frac{1.5\times10^8}{10\times10^6} = 15 \text{ m}$$

Therefore, the length of the each transmission line section,

$$L = \frac{\lambda}{4} = \frac{15}{4} = 3.75 \text{ m}$$

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