MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Electrical Engineering Test No : 12

Section-A

Q.1 (a) Solution:

A smart grid is an electrical grid which includes a variety of operational and energy measures including smart meters, smart appliances renewable energy resources and energy efficient resources. Electronic power conditioning and control of the production and distribution of electricity are important aspects of the smart grid. The ultimate aim is increasing user-interface for real time system optimization.

Difference of smart grid from conventional grid:

- Encouraging of distributed generation sources like solar, wind, fuel cells biomass plants etc.
- Two way flow of electricity and communication between centralized generation and distribution systems.
- Using the dynamic pricing for electricity.
- Using smart meters in order to make the electricity bill clearly visible for consumers.
- Self healing property of grid.
- Capable to operate in island mode efficiency.

Q.1 (b) Solution:

The delay produced by the given subroutine program can be calculated by determining the total number of T-states required to execute the given program.

Total number of T-states required to execute the program can be determined by analyzing the given program as shown in table.

	Instruction	No. of times executed	No. of T-state for one time execution
Delay	MVI B, 02H	1	7
LOOP 2	MVI C, FFH	1 × 2 = 2	7
LOOP 1	DCR C	255 × 2 = 510	4
	JNZ LOOP 1	254 × 2 = 508 ⇒ True 1 × 2 = 2 ⇒ False	10 ⇒ True 7 ⇒ False
	DCR B	2	4
	JNZ LOOP 2	1 ⇒ True 1 ⇒ False	10 ⇒ True 7 ⇒ False
	RET	1	10

⇒ Total delay produced by the program in terms of T-state can be given by,

$$\begin{aligned}\text{Delay} &= (1 \times 7T) + (2 \times 7T) + (510 \times 4T) + (508 \times 10T) \\ &\quad + (2 \times 7T) + (2 \times 4T) + (10T + 7T) + 10T \\ &= 7T + 14\,T + 2041\,T + 5080\,T + 14T + 8T + 17T + 10T \\ &= 7190\,T\end{aligned}$$

$$\text{Therefore, total delay produced} = 7190 \times \frac{1}{f} = \frac{7190}{2 \times 10^6} = 3595\ \mu\text{sec}$$

Q.1 (c) Solution:

Average power loss during turn on

$$P = \frac{1}{T}\int_0^{t_{on}} v_a i_a \, dt$$

$$P = \frac{1}{(1/100)}\int_0^{t_{on}} \left(-\frac{600}{t_{on}}t + 600\right)\left(\frac{100}{t_{on}}t\right)dt$$

$$P = 6 \times 10^6 \int_0^{t_{on}} \left(\frac{t}{t_{on}} - \frac{t^2}{t_{on}^2}\right)dt$$

$$P = 6 \times 10^6 \left[\frac{t^2}{2t_{on}} - \frac{t^3}{3t_{on}^2}\right]_0^{t_{on}}$$

$$= 6 \times 10^6 \left[\frac{t_{on}^2}{2t_{on}} - \frac{t_{on}^3}{3t_{on}^2}\right]$$

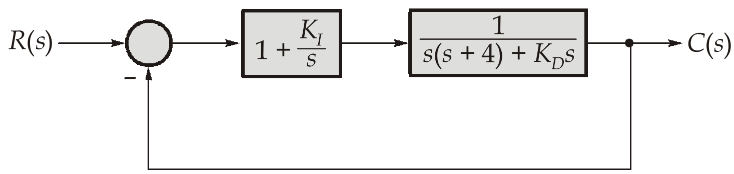
$$P = 6\times10^{6}\left[\frac{t_{on}}{6}\right]$$

$$P = t_{on} \times 10^6 = 5 \times 10^{-6} \times 10^6$$

$$P = 5 \text{ watt}$$

Q.1 (d) Solution:

On solving inner feedback loop



The forward path transfer function,

$$G(s) = \frac{s+K_I}{s}\cdot\frac{1}{s(s+4+K_D)} = \frac{(s+K_I)}{s^2(s+4+K_D)}$$

The system type is type-2.

The characteristics equation,

$$1+G(s)H(s) = 0$$

$$1+\frac{(s+K_I)}{s^2(s+4+K_D)} = 0$$

$$s^3 + (4 + K_D)s^2 + s + K_I = 0$$

Routh Array :

s^3	1	1
s^2	$4+K_D$	K_I
s^1	$\frac{4+K_D-K_I}{4+K_D}$	
s^0	K_I	

The system remains stable if first column of Routh array have same sign.

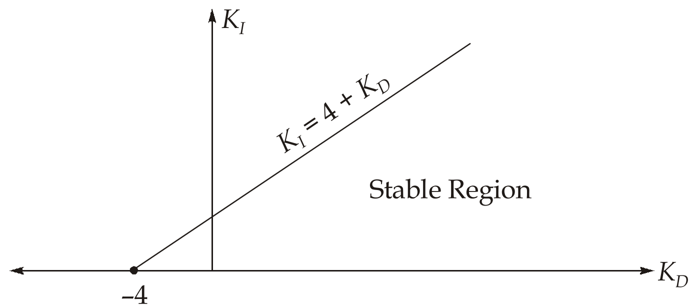
i.e., $4 + K_D > 0,\quad K_I > 0$

$$4 + K_D - K_I > 0$$

$\Rightarrow$ $K_I < 4 + K_D$

$$0 < K_I < 4 + K_D$$

and $K_D > -4$



Q.1 (e) Solution:

(i)

$$Y(x, y, z, w) = \overline{w} + x(y + \overline{z}) + wx(\overline{y} + z)$$
$$= \overline{w} + xy + x\overline{z} + wx\overline{y} + wxz$$

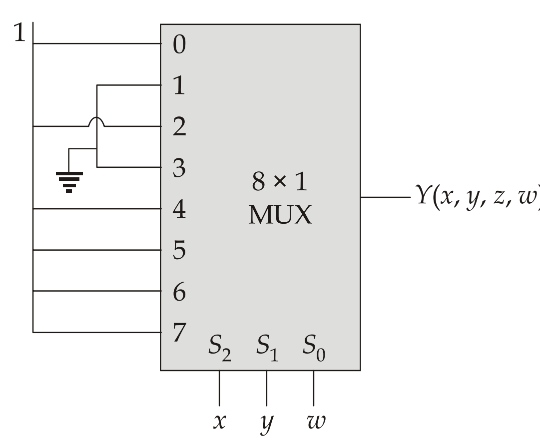
K-map:

xy \ zw	00	01	11	10
00	1			1
01	1			1
11	1	1	1	1
10	1	1	1	1

$$Y(x, y, z, w) = \Sigma m(0, 2, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15)$$

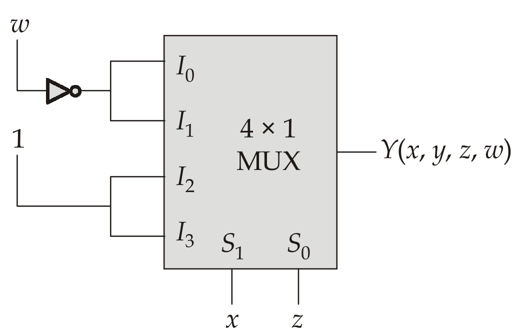
Using 8 : 1 MUX and x, y, w as select line,

	I_0 $\overline{x}\,\overline{y}\,\overline{w}$	I_1 $\overline{x}\,\overline{y}\,w$	I_2 $\overline{x}\,y\,\overline{w}$	I_3 $\overline{x}\,y\,w$	I_4 $x\,\overline{y}\,\overline{w}$	I_5 $x\,\overline{y}\,w$	I_6 $x\,y\,\overline{w}$	I_7 $x\,y\,w$
$\overline{z}$	(0)	1	(4)	5	(8)	(9)	(12)	(13)
z	(2)	3	(6)	7	(10)	(11)	(14)	(15)
	1	0	1	0	1	1	1	1



(ii) Using 4 : 1 MUX and x, z as select lines,

	I_0 $\bar{x}\bar{z}$	I_1 $\bar{x}z$	I_2 $x\bar{z}$	I_3 xz
$\bar{y}\bar{w}$	(0)	(2)	(8)	(10)
$\bar{y}w$	1	3	(9)	(11)
$y\bar{w}$	(4)	(6)	(12)	(14)
yw	5	7	(13)	(15)
	$\bar{w}$	$\bar{w}$	1	1

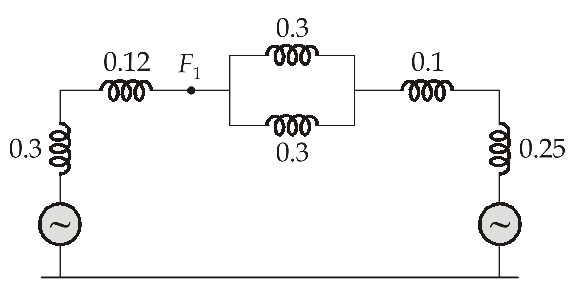


Q.2 (a) Solution:

The sequence networks are shown in figure,

From the positive sequence network of figure (a), the equivalent impedance upto the point of the fault is given by

Location (1)



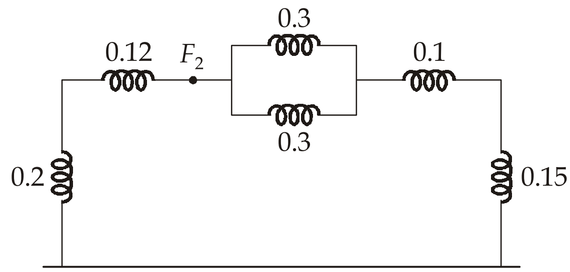
$$Z_1 = j\left[(0.3+0.12) \,||\, \left(0.25+0.1+\frac{0.3}{2}\right)\right]$$

$$= j[(0.42) \,||\, (0.5)]$$

$$= j\frac{0.42 \times 0.5}{0.42+0.5} = j0.22826 \text{ p.u}$$

From the negative-sequence network of figure (b), the equivalent impedance upto F is given by

Location (2)

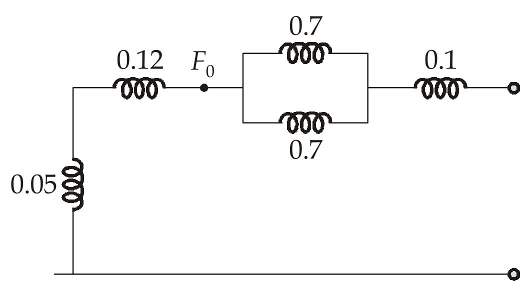


$$Z_2 = j\left[(0.2+0.12)\,\|\left(0.15+0.1+\frac{0.3}{2}\right)\right]$$

$$= j(0.32)\,\|\,(0.4) = j\frac{0.32\times 0.4}{0.32+0.4} = j0.1778 \text{ p.u.}$$

From the zero-sequence network of figure (c), the equivalent impedance upto F is given by

Location (3)



$$Z_0 = j(0.05 + 0.12) = j0.17 \text{ p.u.}$$

(i) If phase a is assumed to be the reference phasor and phases b and c are shorted at the fault, then from equation.

$$I_{a1} = \frac{V_f}{Z_{a1} + \dfrac{Z_{a0}Z_{a2}}{Z_{a0}+Z_{a2}}} = \frac{1\angle 0°}{j\left(0.22826 + \dfrac{0.17\times 0.1778}{0.17+0.1778}\right)}$$

If we put $Z_f = 0$ and $Z_g = 0$ then from figure by current division rule

$$I_{a0} = -I_{a1}\frac{Z_{a2}}{Z_{a0}+Z_{a2}} = j\left(3.1729\times\frac{0.1778}{0.17+0.1778}\right)$$

$$= j1.622 \text{ p.u.} = 1.622\angle 90° \text{ p.u.}$$

$$I_{a2} = -I_{a1}\frac{Z_{a0}}{Z_{a0}+Z_{a2}} = j\left(3.1729\times\frac{0.17}{0.17+0.1778}\right)$$

$$= j1.551 \text{ p.u.} = 1.551\angle 90° \text{ p.u.}$$

Check
$$I_a = I_{a0} + I_{a1} + I_{a2} = j1.622 - j3.1729 + j1.551 \approx 0$$

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$
$$= j1.622 + (1\angle 240°)(3.1729\angle -90°) + 1(\angle 120°)(1.551\angle 90°)$$
$$= j1.622 + 3.1729\angle 150° + 1.551\angle 210°$$
$$= j1.622 - 2.7478 + j1.5864 - 1.3432 - j0.7735$$
$$= -4.091 + j2.4729$$

$$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$$
$$= j1.622 + (1\angle 120°)(3.1729\angle -90°) + 1(\angle 240°)(1.551\angle 90°)$$
$$= j1.622 + 2.7478 + j1.5864 + 1.3432 - j0.7755$$
$$= 4.091 + j2.4729$$

$$|I_b| = |I_c|$$
$$= \sqrt{(4.091)^2 + (2.4729)^2} = 4.78 \text{ p.u.}$$

(ii) LL fault at F

If the line-to-line fault is between phases b and c, then from equation

$$I_{a1} = \frac{V_f}{Z_{a1} + Z_{a2}} = \frac{1\angle 0°}{j(0.22826 + j0.1778)} = -j2.4627 \text{ p.u.}$$
$$= 2.4628\angle -90° \text{ p.u.}$$

The phase a negative sequence current is given by

$$I_a = -I_{a1} = +j2.4627 \text{ p.u.} = 2.4627\angle 90° \text{ p.u.}$$

The phase a fault current,

$$I_a = I_{a0} + I_{a2} = 0 - j2.4627 + j2.4627 = 0$$

The phase b fault current,

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$
$$= 0 + (1\angle 240°)(2.4627\angle -90°) + (1\angle 120°)(2.4627\angle 90°)$$
$$= 2.4627\angle 150° + 2.4627\angle 210°$$
$$= -2.133 + j1.231 - 2.133 - j1.231 = -4.266 \text{ p.u.}$$

The phase c fault current,

$$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$$
$$= 0 + (1\angle 120°)(2.4627\angle -90°) + (1\angle 240°)(2.4627\angle 90°)$$
$$= 2.4627\angle 30° + 2.4627\angle 330°$$

$$= 2.133 + j1.231 + 2.133 - j1.231$$
$$= 4.266 \text{ p.u.}$$

Thus, it is calculated that

$$I_b = -I_c = -4.266 \text{ p.u.}$$

Q.2 (b) Solution:

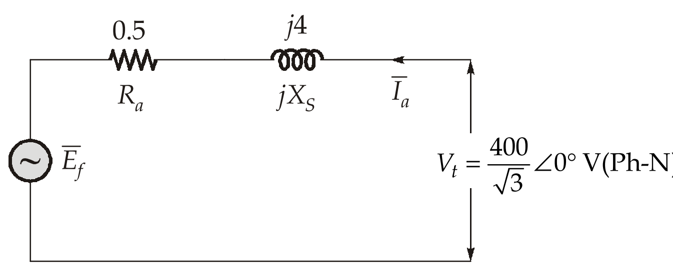
(i) Given that terminal voltage,

$$V_t = 400 \text{ V}$$

Synchronous impedance, $Z_s = (0.5 + j4)\text{ohm}$

The current $\bar{I}_a = 15\angle 0° \text{ Amp}$

The synchronous speed, $N_s = 120 \times \frac{f}{P} = 120 \times \frac{50}{6} = 1000 \text{ rpm}$



So excitation emf, $E_f = V_t - \bar{I}_a Z_s$

$$\bar{E}_f = \frac{400}{\sqrt{3}}\angle 0° - 15\angle 0°(0.5 + j4)$$

$$\bar{E}_f = 231.36 \angle -15.03° \text{ Volt}$$

Now torque is adjusted such that $I_a = 60$ A

Let power factor is ϕ for new current,

$$I_a' = 60$$

So, $I_a' = 60\angle\phi$

So, $\bar{E}_f = 231.36\angle -\delta = \frac{400}{\sqrt{3}}\angle 0° - 60\angle\phi(0.5 + j4)$

$$\bar{E}_f = 231.36\angle -\delta = 230.94 + 241.867\angle(-97.12 + \phi)$$

Let, $\theta = \phi - 97.12$

$\Rightarrow$ $231.36^2 = 230.94^2 + 241.867^2 + 2 \times 230.94 \times 241.867 \times \cos\theta$

$$\Rightarrow \qquad \cos\theta = -0.52192$$

$$\theta = \cos^{-1}(-0.52192) = -121.46°$$

So, $$\theta = \phi - 97.12 = -121.46$$

$$\phi = -24.34°$$

1. The new power factor $= \cos\phi$

$$= \cos(24.34) = 0.911 \text{ lagging}$$

2. The developed power,

$$P_m = \sqrt{3}V_t I_a \cos\phi - P_{loss}$$

$$P_m = \sqrt{3}\times 400\times 60\times 0.911 - 3(60)^2\times 0.5$$

$$P_m = 32.47 \text{ kW}$$

So, Developed Torque,

$$T_d = \frac{P_m}{w_s} = \frac{32.47 \text{ kW}}{\frac{2\pi}{60}\times 1000}$$

$$T_d = 310.10 \text{ N.m}$$

(ii) For a cylindrical rotor synchronous machine,

The real power, $$P = \frac{E_f V_t}{X_s}\sin\delta$$

$$0.5 = \frac{1.4\times 1}{1.2}\sin\delta$$

$$\delta = 25.377°$$

1% increase in torque means 1% increase in real power.

dP = 1% of it's previous value

$$dP = \frac{1}{100}\times 0.5 = 0.005 \text{ p.u.}$$

For cylindrical-rotor machine, the reactive power

$$Q = \frac{E_f V_t}{X_s}\cos\delta - \frac{V_t^2}{X_s}$$

$$\frac{dQ}{d\delta} = \frac{-E_f V_t}{X_s}\sin\delta$$

But, $$\frac{dP}{d\delta} = \frac{E_f V_t}{X_s}\cos\delta$$

$$\frac{dQ}{dP} = -\tan\delta = -\tan(25.377°) = -0.474$$

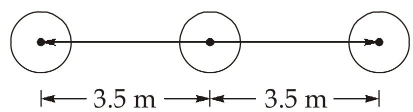
$$dQ = -0.474 \times (1\%) = -0.474\%$$

This shows that with 1% increase in prime-mover torque, the active power P is increased by 1% of it's previous value, but the reactive power Q is decreased by 0.474% of it's previous value.

Q.2 (c) (i) Solution:

The equivalent spacing, $D_{eq} = (3.5 \times 3.5 \times 7)^{1/3}$

$= 4.5$ m



Radius of conductor, $r = \frac{1}{2} \times 1.05 \times 10^{-2} = 5.25 \times 10^{-3}$ m

The capacitance to the neutral,

$$C_n = \frac{2\pi\in}{\ln\left(\frac{D_m}{r}\right)} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln\left(\frac{4.5}{5.25 \times 10^{-3}}\right)}$$

$$= 8.24 \times 10^{-12} \text{ F/m}$$

So capacitive reactance to natural,

$$X_C = \frac{1}{2\pi f C_n} = \frac{1}{2\pi \times 50 \times 8.24 \times 10^{-12}} = 3.86 \times 10^5 \ \Omega/\text{km}$$

So total reactance for 10 km, $X_{CT} = 3.86 \times 10^5 \times 10$

$= 38.6 \times 10^5$ ohm

So reactive volt amperes generated by the line,

$$Q_C = \frac{V_L^2}{X_{CT}} = \frac{(110 \times 10^3)^2}{38.6 \times 10^5} = 3.134 \text{ kVAR}$$

So total reactive volt ampere generated by the line is 3.134 kVAR.

Q.2 (c) Solution:

(ii) **1.** When S_1 and S_2 are ON, inductor stores energy.

Apply KVL, we get

$$-v + L\frac{di_L}{dt} = 0$$

When S_1 and S_2 are OFF, inductor releases energy. Apply KVL, we get

$$L\frac{di_L}{dt} + v_o = 0$$

Apply volt-sec balance equation,

$$\int V_L dt = 0$$

$$V.t_{on} - V_o.t_{off} = 0$$

$$V_o = V \cdot \frac{t_{on}}{t_{off}}$$

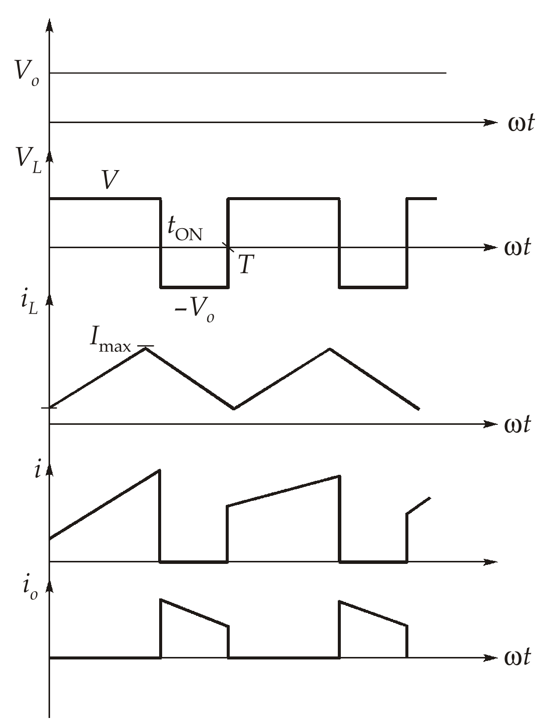
$$V_o = V \cdot \frac{t_{on}}{T - t_{on}} = \frac{\alpha}{1-\alpha} V$$

For $\alpha = 0$,

$$V_o = \frac{0}{1-0} V = 0 \text{ V}$$

α	V_o
0	0
0.5	V
1.0	∞

2.



3. This is buck and boost converter.

- Polarity of V_o is same as that of V.
- Higher conduction losses because two switches conducts at a time. Therefore, efficiency deteriorates.

Q.3 (a) Solution:

$$l_1 = (6 + 0.5 + 1) \times 2 + (4 + 2) = 21 \text{ cm}$$

$$l_2 = (3 + 0.5 + 1) \times 2 + (4 + 2) = 15 \text{ cm}$$

$$l_0 = 4 + 2 = 6 \text{ cm}$$

$$R_1 = \frac{21\times10^{-2}}{4\pi\times10^{-7}\times1600\times2\times2\times10^{-4}} = 0.261 \times 10^6$$

$$R_2 = \frac{15\times10^{-2}}{4\pi\times10^{-7}\times1600\times2\times2\times10^{-4}} = 0.187 \times 10^6$$

$$R_0 = \frac{6\times10^{-2}}{4\pi\times10^{-7}\times1600\times1\times2\times10^{-4}} = 0.0746 \times 10^6$$

(i) Coil 1 excited with 1 A : $R = R_1 + R_0 \parallel R_2 = 0.261 + 0.1871 \parallel 0.0746 = 0.3143 \times 10^6$

$$\phi_1 = \frac{(500\times1)}{(0.3143\times10^6)} = 1.59 \text{ mWb}$$

By flux division (similar to current division) :

$$\phi_{21} = \phi_2 = \frac{0.0746}{(0.187)} = 0.4534 \text{ mWb}$$

$$L_{11} = N_1\phi_1 = 500 \times 1.59 \times 10^{-3} = 0.795 \text{ H}$$

$$M_{21} = N_2\phi_{21} = 1000 \times 0.4534 \times 10^{-3} = 0.4522 \text{ H}$$

(ii) Coil 2 excited with 1 A : $R = R_2 + \frac{(R_0R_1)}{(R_0R_1)}$

$$= 0.187 + (0.0746 \parallel 0.261) = 0.245 \times 10^6$$

$$\phi_2 = \frac{(1000\times1)}{(0.245\times10^6)} = 4.081 \text{ mWb}$$

$$L_{22} = N_2\phi_2 = 1000 \times 4.081 \times 10^{-3} = 4.081 \text{ H}$$

$$M_{12} = M_{21} \text{ (bilateral)} = 0.4522 \text{ H}$$

Q.3 (b) (i) Solution:

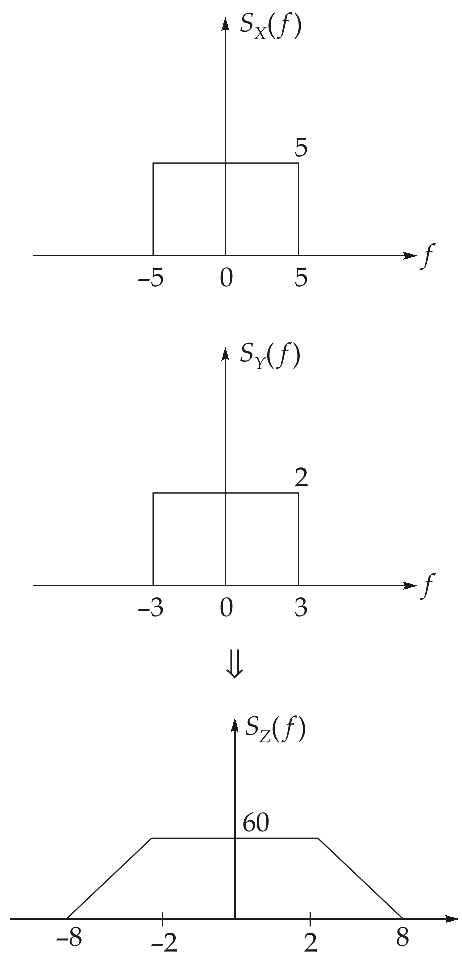
Output $Z(t) = X(t)Y(t)$ has auto-correlation function.

$$R_{zz}(\tau) = R_{xx}(\tau)R_{yy}(\tau)$$

Apply Fourier transform on both sides

$\because$ $$\text{FT[ACF]} = \text{PSD}$$

$$S_z(f) = S_x(f) * S_y(f), \text{ convolution of both PSDs.}$$



Total power, $$P_T = \int_{-\infty}^{\infty} S_z(f)df$$

$$= 2 \times \text{Area of triangle} + \text{Area of rectangle}$$

$$= 2\times\frac{1}{2}\times 6\times 60 + 4\times 60 = 600 \text{ W}$$

Q.3 (b) (ii) Solution:

- At point A, we have

  Carrier frequency, $f_c = 3 \times 10 = 30$ MHz

  $\Delta f = 3 \times 10 = 30$ kHz

  $m_f = 3 \times 5 = 15$

  Minimum frequency, $f_{\text{min}} = 30 \text{ MHz} - 30 \text{ kHz} = 29.97$ MHz

  Maximum frequency, $f_{\text{max}} = 30 \text{ MHz} + 30 \text{ kHz} = 30.03$ MHz

- At point B, we have $f_c = 30 \text{ MHz} + 10 \text{ MHz} = 40$ MHz

  $f_{\text{max}} = 30.03 \text{ MHz} + 10 \text{ MHz} = 40.03$ MHz

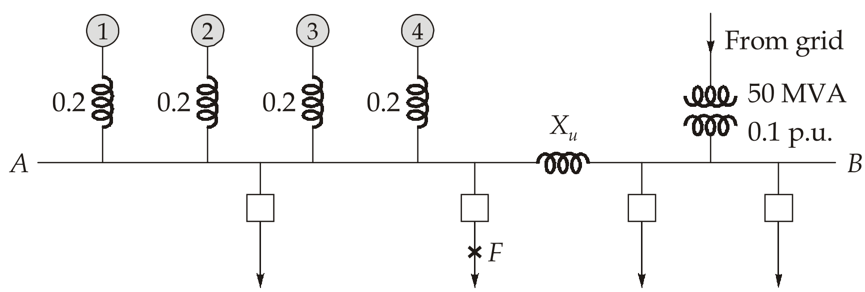
  $f_{\text{min}} = 29.97 \text{ MHz} + 10 \text{ MHz} = 39.97$ MHz

  $\therefore$ Frequency deviation $= f_{\text{max}} - f_c = 40.03 - 40 = 30$ kHz

  As there is no change in deviation due to mixing, the modulation index will remain same i.e., $m_f = 15$.

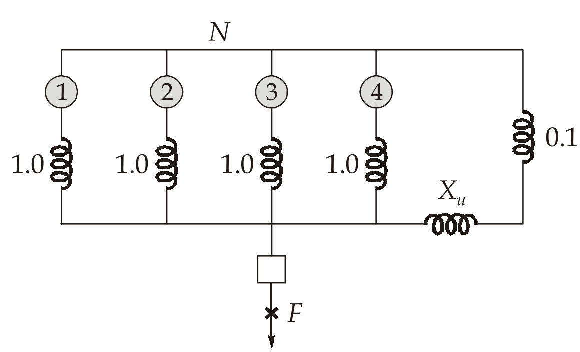
Q.3 (c) Solution:

The system layout is shown in figure below,



Let 50 MVA be taken as the base MVA,

Let the per unit reactance of the reactor be X_u on 50 MVA base. For the symmetrical fault at point F, the equivalent single-phase circuit is shown in figure below,



The four generators are connected in parallel each having a per unit reactance of 1.0.

The combined per unit reactance of the generators $= \dfrac{1.0}{4} = 0.25.$

Combined per unit reactance of transformer and reactor in series = $X_u + 0.1$

Thevenin equivalent per unit reactance at the fault point F is

$$X_T = \frac{0.25(X_u + 0.1)}{0.25 + X_u + 0.1} = \frac{0.25(X_u + 0.1)}{X_u + 0.35}$$

Short circuit MVA fed into the fault at F

$$S_{sc} = \frac{S_b}{X_T} = \frac{50(X_u + 0.35)}{0.25(X_u + 0.1)}$$

If the short circuit MVA is not to exceed 500 MVA, then

$$\frac{50(X_u + 0.35)}{0.25(X_u + 0.1)} = 500\,;$$

$$X_u + 0.35 = \frac{500 \times 0.25}{50}(X_u + 0.1)$$

$$2.5X_u - X_u = 0.35 - 0.25$$

$$X_u = \frac{0.10}{1.5} = \frac{1}{15}\text{ p.u.}$$

Full-load current per phase corresponding to 500 MVA is

$$I_{fl} = \frac{S_b}{\sqrt{3}V_l} = \frac{50 \times 10^6}{\sqrt{3} \times 33 \times 10^3} = 874.8\text{ A}$$

Voltage to neutral,
$$V_n = \frac{V_l}{\sqrt{3}} = \frac{33000}{\sqrt{3}} = 19052.6\text{ V}$$

$$\text{Per unit reactance} = \frac{I_{fl}X_\Omega}{V_n}$$

$$\frac{1}{15} = \frac{874.8\,X_\Omega}{19052.6}$$

$$X_\Omega = \frac{19052.6}{15 \times 874.8} = 1.452\ \Omega$$

Alternate method:

$$Z_{pu} = Z_\Omega \frac{[(\text{MVA})_b]_{3\varphi}}{(\text{kV}_{lb})^2}$$

$$\frac{1}{15} = X_\Omega \frac{50}{(33)^2}$$

$$X_\Omega = \frac{(33)^2}{15 \times 50} = 1.452\ \Omega$$

Q.4 (a) Solution:

(i) The z-transforms of the signals specified in the first piece of information are

$$X_1(z) = \frac{1}{1-\frac{1}{6}z^{-1}}, \qquad |z| > \frac{1}{6}, \qquad \text{...(i)}$$

$$Y_1(z) = \frac{a}{1-\frac{1}{2}z^{-1}} + \frac{10}{1-\frac{1}{3}z^{-1}} \qquad \text{...(ii)}$$

$$= \frac{(a+10)-\left(5+\frac{a}{3}\right)z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}, \qquad |z| > \frac{1}{2},$$

The algebraic expression for the system function is

$$H(z) = \frac{Y_1(z)}{X_1(z)} = \frac{\left[(a+10)-\left(5+\frac{a}{3}\right)z^{-1}\right]\left(1-\frac{1}{6}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} \qquad \text{...(iii)}$$

Furthermore, we know that the response to $x_2[n] = (-1)^n$ must equal $(-1)^n$ multiplied by the system function $H(z)$ evaluated at $z = -1$. Thus from the second piece of information given, we see that

$$\frac{7}{4} = H(-1) = \frac{\left[(a+10)+5+\frac{a}{3}\right]\left(\frac{7}{6}\right)}{\left(\frac{3}{2}\right)\left(\frac{4}{3}\right)} \qquad \text{...(iv)}$$

Solving equation (iv), we find that $a = -9$, so that

$$H(z) = \frac{(1-2z^{-1})\left(1-\frac{1}{6}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} \qquad \text{...(v)}$$

or

$$H(z) = \frac{1-\frac{13}{6}z^{-1}+\frac{1}{3}z^{-2}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \qquad \text{...(vi)}$$

or finally, $$H(z) = \frac{z^2 - \frac{13}{6}z + \frac{1}{3}}{z^2 - \frac{5}{6}z + \frac{1}{6}}$$

Also, from the convolution property, we know that the ROC of $Y_1(z)$ must include at least the intersections of the ROCs of $X_1(z)$ and $H(z)$. Examining the three possible ROCs for $H(z)$. (Namely, $|z| < \frac{1}{3}, \frac{1}{3} < |z| < \frac{1}{2}$ and $|z| > \frac{1}{2}$), we find that the only choice that is consistent with the ROCs of $X_1(z)$ and $Y_1(z)$ is $|z| > \frac{1}{2}$.

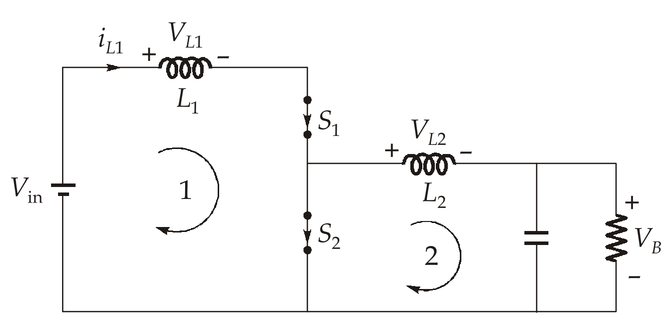
(ii) Since the ROC for the system includes the unit circle, we know that the system is stable. Furthermore, from equation (v) with $H(z)$ viewed as a ratio of polynomials in z, the order of the numerator does not exceed that of the denominator, and thus we can conclude that the LTI system is causal. Also, using equation (vi), we can write the difference equation that, together with the condition of initial rest, characterizes the system:

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n] - \frac{13}{6}x[n-1] + \frac{1}{3}x[n-2]$$

Q.4 (b) Solution:

Case 1 : $0 \leq t \leq \frac{T}{3}$

Switch S_1 and S_2 are ON.



Applying KVL in loop 1

$$V_{L1} = V_{in} = 100 \text{ V}$$

$\therefore$ Inductor L_1 stores energy.

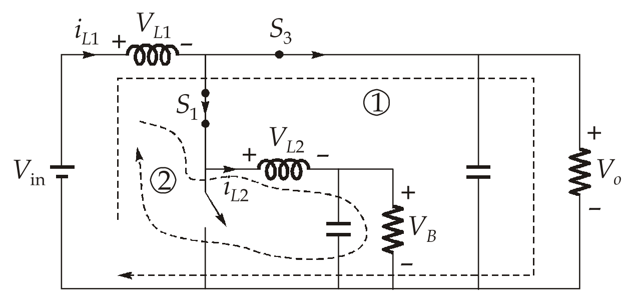
Applying KVL in loop 2

$$V_{L2} + V_B = 0$$
$$V_{L2} = -V_B$$

$\therefore$ Inductor L_2 releases energy.

Case 2 : $\frac{T}{3} \le t \le \frac{2T}{3}$

Switch S_1 and S_3 are ON.



KVL in loop (1)

$$-V_{in} + V_{L1} + V_o = 0$$
$$V_{L1} = V_{in} - V_o$$
$$V_{L1} = -(V_o - V_{in})$$

$\therefore$ Inductor L_1 releases energy.

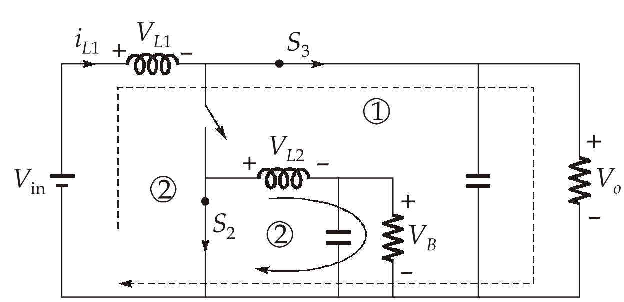
KVL in loop (2)

$$-V_{in} + V_{L1} + V_{L2} + V_B = 0$$
$$V_{L2} = V_{in} - (V_{L1} + V_B)$$
$$V_{L2} = V_{in} - (V_{in} - V_o + V_B)$$
$$V_{L2} = V_o - V_B$$

$\therefore$ Inductor L_2 stores energy

Case 3 : $\frac{2T}{3} \le t \le T$

Switches S_2 and S_3 are ON.



Applying KVL in loop (1)

$$-V_{in} + V_{L1} + V_o = 0$$

$$V_{L1} = V_{in} - V_o$$

$$V_{L1} = -(V_o - V_{in})$$

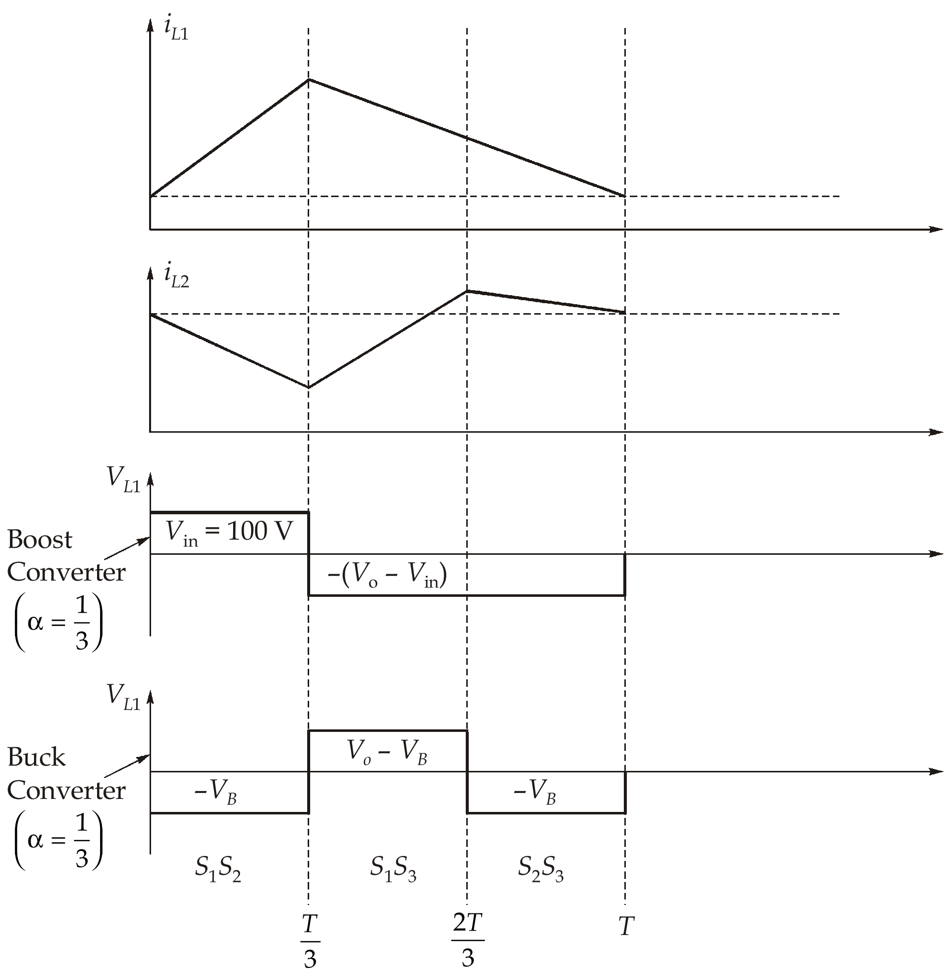
$\therefore$ Inductor L_1 releases energy.

Applying KVL in loop (2)

$$V_{L2} + V_B = 0$$

$$V_{L2} = -V_B$$

$\therefore$ Inductor L_2 also releases energy.



$\because$ Average inductor voltage = 0

$$(V_{L1})_{Avg} = 0$$

$$V_{in} \times \frac{T}{3} - (V_o - V_{in}) \times \frac{2T}{3} = 0$$

$$V_{in} \times \frac{T}{3} + V_{in} \times \frac{2T}{3} = V_o \times \frac{2T}{3}$$

$$V_{in} \times T = V_o \times \frac{2T}{3}$$

$$V_o = \frac{3}{2} V_{in}$$

$$V_o = \frac{3}{2} \times 100$$

$$V_o = 150 \text{ V}$$

For boost converter :

Alternatively,

$$V_o = \frac{V_{in}}{1 - \alpha}$$

$$V_o = \frac{V_{in}}{1 - \frac{1}{3}}$$

$$V_o = \frac{3}{2} V_{in}$$

$$(V_{L2})_{Avg} = 0$$

$$(V_o - V_B) \times \frac{T}{3} - V_B \times \frac{2T}{3} = 0$$

$$V_o \times \frac{T}{3} = V_B \times T$$

$$V_B = \frac{V_o}{3} = \frac{150}{3} = 50 \text{ V}$$

Hence,

$$V_o = 150 \text{ V}$$

$$V_B = 50 \text{ V}$$

For Buck converter :

Alternatively,

$$V_B = \alpha . V_{supply}$$

$$V_B = \alpha . V_o$$

$$V_B = \frac{1}{3} \times 150$$

$$V_B = 50 \text{ V}$$

Q.4 (c) Solution:

(i) The closed loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{\dfrac{K}{s(s+b)}}{1+\dfrac{K}{s(s+b)}} = \frac{K}{s^2+bs+K}$$

Comparing with standard second order transfer function,

$$\omega_n^2 = K$$

or $$\omega_n = \sqrt{K}$$

$$2\xi\omega_n = b$$

$$\xi = \frac{b}{2\omega_n} = \frac{b}{2\sqrt{K}}$$

i.e., $$\xi^2 = \frac{b^2}{4K}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.1$$

$$(1.1)^2 = \frac{1}{4\xi^2(1-\xi^2)} = \frac{1}{\dfrac{b^2}{K}\left(1-\dfrac{b^2}{4K}\right)}$$

and $$\omega_r = \omega_n\sqrt{1-2\xi^2} = 12$$

∴ $$(12)^2 = \omega_n^2\left(1-2\xi^2\right) = K\left(1-\frac{2b^2}{4K}\right)$$

or $$144 = K-\frac{b^2}{2} \quad \text{...(i)}$$

$$b^2 = 2K - 288$$

$$1.21 = \frac{1}{\dfrac{b^2}{K}\left(1-\dfrac{b^2}{4K}\right)}$$

$$\frac{b^2}{K}\left(\frac{4K-b^2}{4K}\right) = \frac{1}{1.21}$$

$$\therefore \qquad b^2(4K - b^2) = \frac{4K^2}{1.21} = 3.305\,K^2 \qquad \text{...(ii)}$$

Substituting value from equation (i) in equation (ii), we get

$$(2K - 288)(4K - 2K + 288) = 3.305\,K^2$$

i.e.,
$$(2K - 288)(2K + 288) = 3.305\,K^2$$

$$0.695\,K^2 = (288)^2$$

$$K = 345.7$$

Therefore,
$$b = \sqrt{2K - 288} = \sqrt{(2)(345.7) - 288} = 20.08$$

$$\omega_n = \sqrt{K} = \sqrt{345.7} = 18.593 \text{ rad/sec}$$

$$\xi = \frac{b}{2\sqrt{K}} = \frac{20.08}{(2)(18.593)} = 0.54$$

(ii) Settling time,
$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{(0.54)(18.543)} = 0.3983 \text{ (2\% criterion)}$$

$$t_s = \frac{3}{\xi\omega_n} = \frac{3}{(0.54)(18.593)} = 0.298s \text{ (5\% criterion)}$$

$$\text{Bandwidth, } \omega_b = \omega_n\sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}$$

$$\omega_b = 18.593\left[1 - 2(0.54)^2 + \sqrt{2 - 4\times 0.54^2 + 4\times 0.54^4}\right]^{1/2}$$

$$= 22.068 \text{ rad/sec}$$

For $M_r = 1.1$ and $\omega_r = 12$ rad/sec

$$K = 345.7,$$

$$b = 20.08$$

$$t_s = 0.398 \text{ sec or } 0.298 \text{ sec}$$

$$\text{bandwidth} = 22.068 \text{ rad/sec}$$

Section-B

Q.5 (a) Solution:

The total number of armature conductor

$$Z = 2C\,N_C$$

N_C = Number of turns per coil

C = Total number of coils = number of slots for two layer winding

Here, $C = 24$

$N_C = 18$

$Z = 2 \times 24 \times 18 = 864$

No. of poles $= 2$

For a wave wound machine number of parallel paths,

$a = 2$

$$\text{The actual pole area} = \frac{2\pi rL}{P}$$

r = radius = 10 cm = 0.1 m

L = length = 20 cm = 0.2 m

$P = 2$

$$A_P = \frac{2\pi \times 0.1 \times 0.2}{2} = 0.063 \text{ m}^2$$

The effective pole area is, $A_e = 0.063 \times 0.8 = 0.05 \text{ m}^2$

Thus, effective flux per pole is

$\phi_P = BA_e$

$B = 1$ T

$A_e = 0.05 \text{ m}^2$

$\phi_P = 1 \times 0.05 = 0.05$ Wb

(i) The machine constant,

$$K_a = \frac{ZP}{2\pi a}$$

$Z = 864,$

$P = 2,$

$a = 2$

$$K_a = \frac{2 \times 864}{2\pi \times 2} = 137.51$$

Induced emf $E_a = K_a \phi_p \, \omega_m$

$K_a = 137.51$

$\phi_P = 0.05$

$\omega_m = 183.2$ rad/s

$E_a = 137.51 \times 0.05 \times 183.2 = 1259.6$ V

(ii) As there are two parallel paths, the number of coils in each path is $\frac{24}{2} = 12$.

Thus, induced emf per coil $= \frac{1259.6}{12} = 104.97$ V

(iii) As there are 18 turns in each coil, the induced emf per turn is

$$E_{turn} = \frac{104.97}{18} = 5.83 \text{ V}$$

(iv) Induced emf per conductor,

$$E_{cond} = \frac{5.83}{2} = 2.916 \text{ V}$$

Q.5 (b) Solution:

(i) Given, $G = 100$ MVA, $H = 10$ MJ/MVA

Kinetic energy stored in rotor $= G \cdot H \times 100 \times 10$ MJ $= 1000$ MJ

(ii) $P_a = P_i - P_e = (60 - 50)$ MW $= 10$ MW

We know, $M = \frac{GH}{180f} = \frac{100 \times 10}{180 \times 60} = \frac{5}{54}$ MJ-sec/ele-deg.

Now, $M \cdot \frac{d^2\delta}{dt^2} = P_a$

$\Rightarrow$ $\frac{5}{54} \cdot \frac{d^2\delta}{dt^2} = 10$

$\therefore$ $\frac{d^2\delta}{dt^2} = \alpha = \frac{10 \times 54}{5} = 108$ elec-deg/sec^2

$$\alpha = 108 \text{ elec-deg/sec}^2$$

$$= 108 \times \frac{2}{P} \text{ mech-deg/sec}^2$$

$$= 108 \times \frac{2}{4} \times \left(\frac{60°}{360°}\right) = 9 \text{ rpm/sec}$$

(iii) 12 cycles is equivalent to $\frac{12}{60} = 0.2$ sec

Change in load angle, $\Delta\delta = \frac{1}{2}\alpha(\Delta t)^2 = \frac{1}{2} \times 108 \times (0.2)^2$

$$\Delta\delta = 2.16 \text{ elec-degree}$$

Now, $\alpha = 108$ elec-degree/sec^2

$$\alpha = 60 \times \frac{108}{360} \times \frac{2}{4} = 9 \text{ rpm/sec}$$

Assuming no accelerating torque, before beginning of 12 cycle period.

$$360° = 1 \text{ rev}$$

$$\Rightarrow \quad 1° = \frac{1}{360} \text{rev}$$

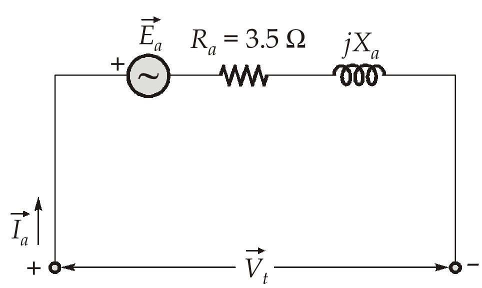
$$\Rightarrow \quad 108° = \frac{108}{360} \text{rev}$$

Thus, rotor speed at the end of 12 cycles

$$= \frac{120f}{P} + \alpha \Delta t$$

$$= \left(\frac{120 \times 60}{4} + 9 \times 0.2\right) \text{rpm} = 1801.8 \text{ rpm}$$

Q.5 (c) Solution:



From operating condition,

$$V_t I_a \cos\phi = 300 \text{ W}$$

$$100 \times 4.6 \cos\phi = 300$$

$$\phi = 49.29°$$

$$\vec{V}_t = 100\angle 49.29° \text{ V}$$

$$\vec{I}_a = 4.6\angle 0° \text{ A}$$

(i) From the circuit,

$$\frac{\vec{V}_t - \vec{E}_a}{R_a + jX_a} = \vec{I}_a$$

In universal motor, armature winding and field winding are in series, i.e., $\vec{E}_a$ in phase with $\vec{I}_a$.

$$\Rightarrow \quad \vec{E}_a = E_a\angle 0°$$

$$\frac{100\angle 49.3° - E_a}{3.5 + jX_a} = 4.6$$

Comparing real and imaginary parts

$$65.2 - E_a = 16.1$$

$$\Rightarrow \quad E_a = 49.1 \text{ V}$$

$$75.8 = 4.6X_a$$

$$\Rightarrow \quad X_a = 16.48\ \Omega$$

(ii)

$$\hat{E}_a = \frac{zNP}{60} \cdot \frac{\phi}{A}$$

$$49.1\sqrt{2} = \frac{(960)(5000)(2)(\phi)}{(60)(2)}$$

$$\phi = 0.868 \text{ mWb}$$

Q.5 (d) Solution:

$x[n]$ can be written in Fourier series form as

$$x[n] = \frac{1}{2}e^{j(2\pi/N)n} + \frac{1}{2}e^{-j(2\pi/N)^n}$$

We know,

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n]e^{-j\omega n}$$

$$\Rightarrow \quad H(e^{j\omega}) = \sum_{n=0}^{\infty} \alpha^n e^{-j\omega_n} = \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^n \quad \text{...(i)}$$

This geometric series will be yielding,

$$H(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}} \quad \text{...(ii)}$$

The Fourier series for the output is,

$$y[n] = \frac{1}{2}H\left(e^{j2\pi/N}\right)e^{j(2\pi/N)n} + \frac{1}{2}H\left(e^{-j2\pi/N}\right)e^{-j(2\pi/N)n}$$

$$= \frac{1}{2}\left(\frac{1}{1-\alpha e^{-j2\pi/N}}\right)e^{j(2\pi/N)n} + \frac{1}{2}\left(\frac{1}{1-\alpha e^{j2\pi/N}}\right)e^{-j(2\pi/N)n}$$

If we write

$$\frac{1}{1-\alpha e^{-j2\pi/N}} = re^{j\theta}$$

then,

$$y[n] = r\cos\left(\frac{2\pi}{N}n + \theta\right)$$

If $N = 4$,

$$\frac{1}{1-\alpha e^{-j2\pi/4}} = \frac{1}{1+\alpha j} = \frac{1}{\sqrt{1+\alpha^2}} e^{j(-\tan^{-1}(\alpha))}$$

and thus, $$y[n] = \frac{1}{\sqrt{1+\alpha^2}} \cos\left(\frac{\pi n}{2} - \tan^{-1}(\alpha)\right)$$

Q.5 (e) Solution:

DC transmission voltage = 200 + 200 = 400 kV

Direct current in the transmission lines,

$$I_d = \frac{1000 \times 10^3}{400} = 2500 \text{ A}$$

It is seen from the working of a 3-phase full converter that each thyristor conducts for 120° for a periodicity of 360°.

$$\therefore \text{ RMS current rating of thyristor } = I_d \sqrt{\frac{120}{360}} = \frac{2500}{\sqrt{3}} = 1443.38 \text{ A}$$

Also, $$\frac{3V_{mL}}{\pi} \cos\alpha = 200 \text{ kV}$$

For extreme case, $$\alpha = 0°$$

$$\frac{3V_{mL}}{\pi} = 200 \text{ kV}$$

$$\Rightarrow \quad V_{mL} = 209.44 \text{ kV}$$

Since, there are two SCRs conducting simultaneously in a six-pulse converter, the peak reverse voltage across each thyristor valve

$$= \frac{209.44}{2} = 104.72 \text{ kV}$$

Q.6 (a) Solution:

(i) With $\alpha = 0$, the forward-path transfer function.

$$G(s) = \frac{16}{s(s+4)}$$

$$\text{Velocity error constant } k_v = \lim_{s\to 0} sG(s) = 4$$

$$e_{ss}\big|_{\text{unit ramp}} = \frac{1}{k_V} = 0.25$$

The characteristic equation of the system is

$$s^2 + 4s + 16 = 0$$

Comparison with the standard characteristic equation.

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

Gives $\omega_n = 4$ and $\xi = 0.5$

Therefore, $M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100 = 16.3\%$

(ii) With rate feedback, the forward-path transfer function is obtained by eliminating the inner loop:

$$G(s) = \frac{\dfrac{16}{s(s+4)}}{1+\dfrac{16\times s}{s(S+4)}} = \frac{16}{s(s+4+16\alpha)}$$

$$k_v = \frac{16}{4+16\alpha} = \frac{4}{1+4\alpha}$$

The characteristic equation now takes the form

$$s^2 + (4 + 16\alpha)s + 16 = 0$$

Corresponding to 1.5% overshoot, the damping ratio ξ is given by

$$\xi^2 = \frac{[\ln(0.015)]^2}{[\ln(0.015)]^2 + \pi^2}$$

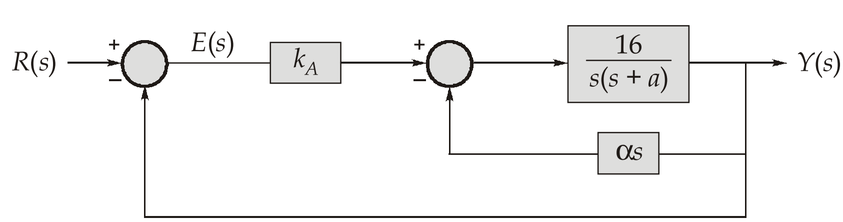
The value of α that results in $\xi = 0.8$ is given by the following equation

$$2 \times 0.8 \times 4 = 4 + 16\alpha$$

Solving, we get $\alpha = 0.15$

For this value of α, $e_{ss}|_{\text{unit ramp}} = \dfrac{1}{K_v} = \dfrac{1+4\alpha}{4} = 0.4$

(iii) The system of given figure may be extended by including an amplifier k_A as shown in figure. The forward-path transfer function now becomes.



$$G(s) = \frac{16k_A}{s(s+4+16\alpha)}$$

$$k_V = \frac{16k_A}{4+16\alpha} = \frac{4k_A}{1+4\alpha}$$

The required k_V = 4, therefore,

$$\frac{4k_A}{1+4\alpha} = 4 \quad \text{...(i)}$$

The characteristic equation of the extended system is

$$s^2 + (4 + 16\alpha)s + 16k_A = 0 \quad \text{...(ii)}$$

The requirement of $\xi = 0.8$ gives rise to the following equation:

$$2\times0.8\times\sqrt{16k_A} = 4 + 16\alpha \quad \text{...(iii)}$$

From equation (i) and (iii), we obtain.

$$k_A = 2.56;\ \alpha = 0.39$$

Q.6 (b) Solution:

Since the load is at the bus of plant-2, therefore, the line loss will not be affected by variation of P_2

Thus, $B_{12} = B_{21} = 0$

and $B_{22} = 0$

For a 2-plant system,

$$P_L = P_1^2 B_{11} + P_2^2 B_{22} + 2P_1 P_2 B_{12}$$

When $P_1 = 125$ MW,

$P_L = 12.5$ MW

We have $12.5 = (125)^2 B_{11} + 0 + 0$

or $B_{11} = \dfrac{12.5}{(125)^2} = 8 \times 10^{-4}\ \text{MW}^{-1}$

Therefore, $P_L = 8 \times 10^{-4}\ P_1^2$

$$\frac{\partial P_L}{\partial P_1} = 16 \times 10^{-4}\ P_1$$

We are also given, $\lambda = 70$ Rs/MWh

(i) Use of coordinates equation:

The coordination equation for plant-1 is given by

$$\frac{dC_1}{dP_1} + \lambda\frac{\partial P_L}{\partial P_1} = \lambda$$

Substituting the values of $\frac{dC_1}{dP_1}$, λ and $\frac{\partial P_L}{\partial P_1}$ we have

$$0.25P_1 + 40 + 70 \times 16 \times 10^{-4} P_1 = 70$$

$$(0.25 + 0.112)P_1 = 30$$

$$P_1 = 82.8729 \text{ MW}$$

The coordination equation for plant-2 is given by

$$\frac{dC_2}{dP_2} + \lambda\frac{\partial P_L}{\partial P_2} = \lambda$$

or
$$0.20P_2 + 50 + 0 = 70,$$

$$P_2 = 100 \text{ MW}$$

The line loss is given by

$$P_L = 8 \times 10^{-4} P_1^2$$

$$= 8 \times 10^{-4} \times (82.8729)^2 = 5.494 \text{ MW}$$

Therefore,
$$\text{the total load, } P_R = P_1 + P_2 - P_L$$

$$= 82.8729 + 100 - 5.494$$

$$= 177.3789 \text{ MW}$$

(ii) Use of penalty factor:

The penalty factor for plant-1 is given by

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{(1 - 16 \times 10^{-4} P_1)}$$

For optimal dispatch,

$$L_1 \frac{dC_1}{dP_1} = \lambda,$$

$$\frac{1}{(1 - 16 \times 10^{-4} P_1)} \times (0.25P_1 + 40) = 70$$

or
$$0.25P_1 + 40 = 70 - 70 \times 16 \times 10^{-4} P_1$$

or
$$P_1 = \frac{30}{0.25 + 0.112} = 82.8729 \text{ MW}$$

Penalty factor for plant-2,

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = \frac{1}{1 - 0} = 1$$

For optimal dispatch,

$$L_2 \frac{dC_2}{dP_2} = \lambda$$

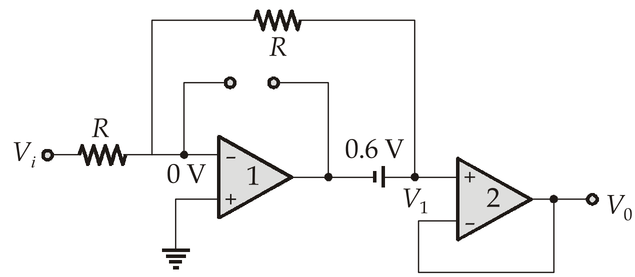
$$1 \times (0.20P_2 + 50) = 70$$

$$P_2 = \frac{20}{0.20} = 100 \text{ MW}$$

It is seen that both the methods give identical results. This due to the fact that the penalty factor has been derived from the coordination equation.

Q.6 (c) Solution:

(i) For positive half of input cycle, D_1 is reverse-biased and D_2 is forward-biased.



From the virtual short concept,

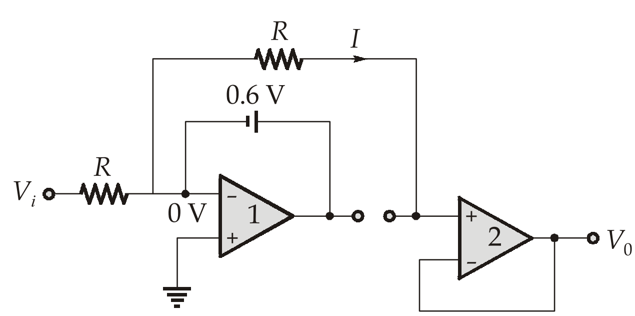
$$V_1 = V_o$$

Applying KCL at inverting terminal of op-amp 1,

$$\frac{V_i - 0}{R} = \frac{0 - V_o}{R}$$

$$\Rightarrow \quad V_0 = -V_i$$

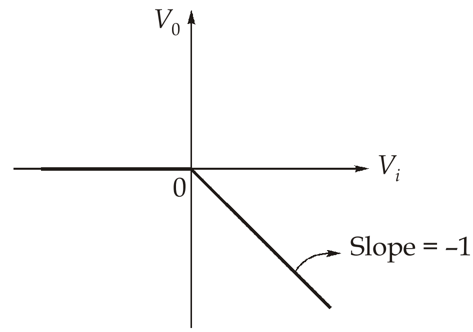
For negative half of the input cycle, D_1 is forward-biased and D_2 is reverse-biased.



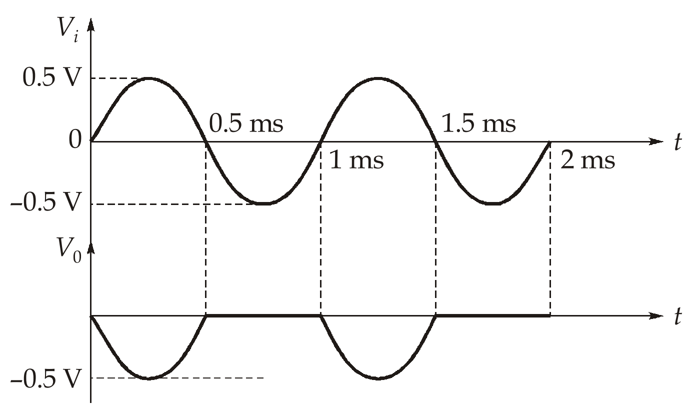
Since, no current flows in the input-terminal of op-amp 2. Hence, $I = 0$

$$\therefore \quad V_0 = 0 \text{ V}$$

Transfer characteristics:



Output waveform:



(ii) If the diode polarities are reversed.

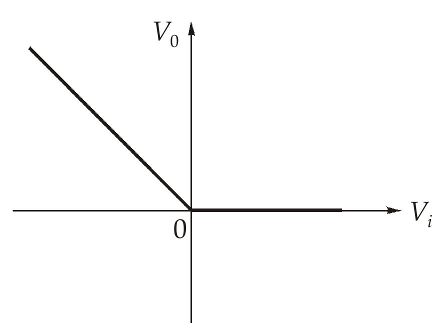
During the input positive half cycle, D_1 is forward-biased and D_2 is reverse biased. Hence,

$$V_0 = 0$$

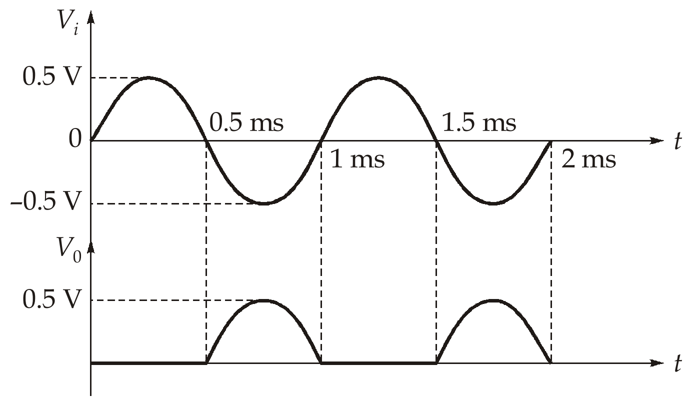
During the input negative half cycle, D_1 is reverse-biased and D_2 is forward-biased. Hence,

$$V_0 = -V_i$$

Transfer characteristics:



Output waveform :



Q.7 (a) Solution:

(i) Let base MVA = 100 MVA and base kV = 115 kV

Given :
$$S_2 = (184.8 + j6.6)\text{ MVA} = (1.848 + j0.066)\text{ pu}$$
$$S_3 = (0 + j20)\text{ MVA} = j0.2\text{ pu}$$
$$V_3 = \frac{115\angle 0^\circ}{115} = 1\angle 0^\circ\text{ pu}$$
$$I_3 = \frac{S_3^*}{V_3^*} = \frac{-j0.2}{1\angle 0^\circ} = -j0.2\text{ pu}$$
$$\because \quad Z_L(\text{pu}) = \frac{j66.125}{(115)^2}\times 100 = j0.5\text{ pu}$$
$$V_2 = V_3 + Z_L(\text{pu})I_3$$
$$= 1\angle 0^\circ + (j0.5)(-j0.2)$$
$$V_2 = 1.1\angle 0^\circ\text{ pu}$$

Therefore, line-to-line voltage at bus-2 is,
$$V_2 = (115)\times(1.1) = 126.5\text{ kV}$$

Similarly,
$$I_2 = \frac{S_2^*}{V_2^*} = \frac{1.848 - j0.066}{1.1\angle 0^\circ}(1.68 - j0.06)\text{pu}$$
$$\vec{I}_{12} = \vec{I}_2 + \vec{I}_3 = (1.68 - j0.06) + (-j0.2)$$
$$= (1.68 - j0.26)\text{ pu}$$

Therefore,
$$V_1 = V_2 + \vec{I}_{12}\times Z_T(\text{pu})$$
$$= 1.1\angle 0^\circ + (j0.2)(1.68 - j0.26)$$
$$= 1.2\angle 16.26^\circ\text{ pu}$$

Hence, the line-to-line voltage at bus-1 is

$$V_1 = (23 \times 1.2) = 27.6\angle 16.26° \text{ kV}$$

(ii)
$$D_{ac} = \sqrt{4^2 + 3^2} = 5 \text{ m}$$

$$D_{ad} = \sqrt{6^2 + 3^2} = 6.7082 \text{ m}$$

Flux linkage in telephase line due to phase a :

$$\lambda_{cd}I_a = \left[0.2 \ln \frac{D_{ad}}{D_{ac}}\right] \times 226 = 13.28 \text{ mWb/km}$$

Similarly due to phase b,

$$\lambda_{cd}I_b = \left[0.2 \ln \frac{D_{bc}}{D_{bd}}\right] \times 226 = 0 \qquad [\because D_{bc} = D_{bd}]$$

Total flux linkage is

$$\Psi_{cd} = 13.28 \text{ mWb/km}$$

The voltage induced in the telephone line per km is

$$V = \omega\psi_{cd} = 2\pi \times 50 \times 13.28 \times 10^{-3}$$
$$= 4.173 \text{ V/km}$$

Q.7 (b) Solution:

(i) In order to find the parallel realisation of the given IIR digital filter, the partial fraction expansion of $\frac{H(z)}{z}$ is determined.

Given,
$$H(z) = \frac{3\left[2z^2 + 5z + 4\right]}{(2z+1)(z+2)}$$

$\therefore$
$$\frac{H(z)}{z} = \frac{\frac{3}{2}\left[2z^2 + 5z + 4\right]}{z\left(z + \frac{1}{2}\right)(z+2)}$$

Now using partial fraction expansion,

$$\frac{H(z)}{z} = \frac{A}{z} + \frac{B}{\left(z + \frac{1}{2}\right)} + \frac{C}{(z+2)}$$

$$A = \left.\frac{\frac{3}{2}(2z^2 + 5z + 4)}{\left(z + \frac{1}{2}\right)(z+2)}\right|_{z=0} = \frac{\frac{3}{2}[4]}{1} = 6$$

$$A = 6$$

$$B = \left.\frac{\frac{3}{2}(2z^2+5z+4)}{(z+2)z}\right|_{z=-\frac{1}{2}} = \frac{\frac{3}{2}\left(2\times\frac{1}{4}-\frac{5}{2}+4\right)}{-0.5\times 1.5}$$

$$B = -4$$

$$C = \left.\frac{\frac{3}{2}(2z^2+5z+4)}{\left(z+\frac{1}{2}\right)z}\right|_{z=-2} = \frac{\frac{3}{2}[2\times 4-5\times 2+4]}{(-2)\times(-1.5)}$$

$$C = 1$$

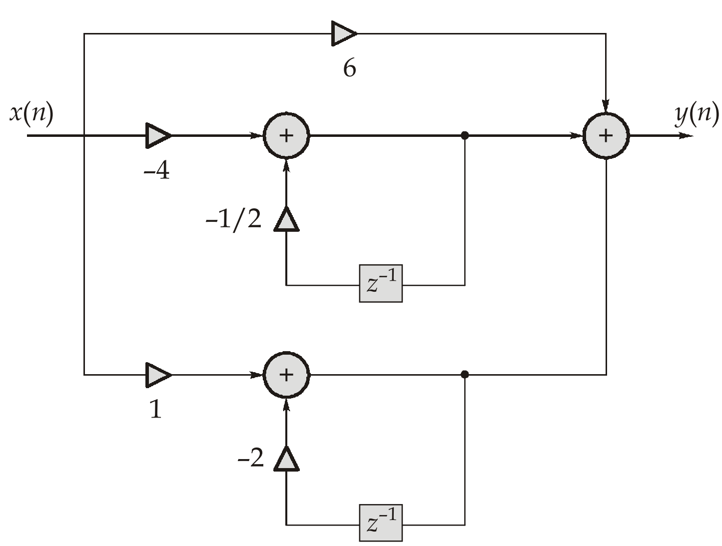
$\therefore$
$$\frac{H(z)}{z} = \frac{6}{z}+\frac{(-4)}{\left(z+\frac{1}{2}\right)}+\frac{1}{(z+2)}$$

$$\frac{H(z)}{z} = \frac{6}{z}-\frac{4}{\left(z+\frac{1}{2}\right)}+\frac{1}{(z+2)}$$

$$H(z) = 6-\frac{4z}{\left(z+\frac{1}{2}\right)}+\frac{z}{(z+2)}$$

$$H(z) = 6-\frac{4}{\left[1+\frac{1}{2}z^{-1}\right]}+\frac{1}{[1+2z^{-1}]}$$

$\therefore$ The parallel realization of $H(z)$ is



(ii) From the system diagram,

$$V(z) = X(z) + H_2(z)Y(z)$$

and
$$Y(z) = H_1(z)V(z)$$

$$\frac{Y(z)}{H_1(z)} = V(z)$$

$$\frac{Y(z)}{H_1(z)} = X(z) + H_2(z)Y(z)$$

$$Y(z) = H_1(z)X(z) + H_1(z)H_2(z)Y(z)$$

$$\frac{Y(z)}{X(z)} = \frac{H_1(z)}{1 - H_1(z)H_2(z)}$$

Therefore, overall transfer function,

$$\frac{Y(z)}{X(z)} = \frac{(10\beta z)/(z-1)}{1 - \frac{10\beta z}{z-1} \times 1}$$

$$= \frac{10\beta z}{z - 1 - 10\beta z} = \frac{10\beta z}{z(1-10\beta) - 1}$$

$$H(z) = \frac{10\beta}{1-10\beta} \times \left[\frac{z}{z - \frac{1}{1-10\beta}}\right]$$

In order to assess the BIBO stability of the system, we need to consider the poles of the system function $H(z)$. From the expression of $H(z)$, we can see that $H(z)$ has a single pole at $\frac{1}{1-10\beta}$. Since the system is causal, the system is BIBO stable if and only if, all of the poles are strictly inside a unit circle. Thus, we have

$$\left|\frac{1}{1-10\beta}\right| < 1$$

$$\Rightarrow \quad |1 - 10\beta| > 1$$

$$1 - 10\beta > 1 \text{ or } 1 - 10\beta < -1$$

$$10\beta < 0 \text{ or } 10\beta > 2$$

$$\beta < 0 \text{ or } \beta > \frac{1}{5}$$

Therefore, system is BIBO stable if and only if,

$$\beta < 0 \text{ OR } \beta > \frac{1}{5}$$

Q.7 (c) Solution:

Given : $Z = 5 + j25 = 25.50\angle 78.7°\ \Omega$

Receiving end power, $S_R = P_D + jQ_D = 15 + j15\tan(36.87°) = (15 + j11.25)$ MVA

As we know,

$$P_R = \frac{V_S V_R}{|Z|}\cos(\theta - \delta) - \frac{V_R^2}{Z_S}\cos(\theta)$$

$$\Rightarrow \quad 15 = \frac{(33)^2}{25.50}\cos(78.7° - \delta) - \frac{(33)^2}{25.50}\cos(78.7°)$$

$$\cos(78.7 - \delta) = \frac{25.5 \times 15}{(33)^2} + \cos(78.7°)$$

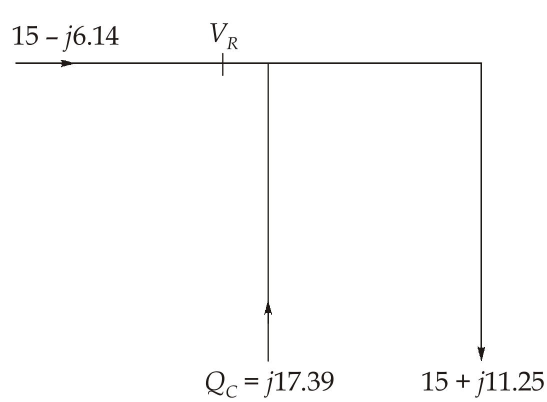
$$\Rightarrow \quad \delta = 21.90°$$

Now, receiving end reactive power,

$$Q_R = \frac{(33)^2}{25.5}\sin(78.7 - \delta) - \frac{(33)^2}{25.50}\sin(78.7°)$$

$$Q_R = \frac{(33)^2}{25.5}\sin(78.7 - 21.9°) - \frac{(33)^2}{25.50}\sin(78.70°)$$

$$= -6.14 \text{ MVAR}$$



Therefore, compensation element rating,

$$Q_C = Q_R - Q_D$$

$$Q_C = -6.14 - 11.25 = 17.39 \text{ MVAR leading}$$

(capacitive compensation)

Now, $|V_R| = 28$ kV

$$P_D + jQ_D = P_D(1 + j\tan\phi)$$
$$= P_D(1 + j\tan 36.87°) = P_D(1 + j0.75)$$
$$P_R + jQ_R = P_D + j(0.75P_D - 17.39)$$
$$P_R = P_D = \frac{33\times 28}{25.5}\cos(78.7° - \delta) - \frac{(28)^2}{25.50}\cos(78.7°)$$

And
$$Q_R = 0.75P_D - 17.39$$
$$= \frac{33\times 28}{25.5}\sin(78.7 - \delta) - \frac{(28)^2}{25.50}\sin(78.7°)$$

or
$$\cos(78.7° - \delta) = \frac{25.50}{33\times 28}P_D + \frac{28}{33}\cos 78.7°$$
$$= 0.0276P_D + 0.1663 \quad ...(1)$$

Also,
$$\sin(78.7 - \delta) = \frac{25.5\times 0.75}{33\times 28}P_D - \frac{25.50\times 17.39}{33\times 28} + \frac{28}{33}\sin(78.7°)$$
$$= 0.0207P_D + 0.352 \quad ...(2)$$

Eqn. $(1)^2$ + Eqn. $(2)^2$

$$1 = 1.19\times 10^{-3}P_D^2 + 23.7\times 10^{-3}P_D + 0.1516$$

On solving,

$$= 18.54 \text{ MW (negative sign is neglected)}$$

$$\text{Extra power transmitted} = 18.54 - 15 = 3.54 \text{ MW}$$

Note : It is assumed that as receiving-end voltage drops, the compensating element draws the same MVAR (leading).

Q.8 (a) Solution:

(i) The overall transfer function is,

$$\frac{\theta_o(s)}{\theta_r(s)} = \frac{K_T}{Js^2 + fs + K_T}$$

Substituting the given values

$$\frac{\theta_o(s)}{\theta_r(s)} = \frac{360}{10s^2 + 60s + 360}$$
$$= \frac{36}{s^2 + 6s + 36}$$

$$\theta_o(s) = \theta_r(s) \cdot \frac{36}{s^2 + 6s + 36}$$

$\because$ $$\theta_r = 50° \text{ or } 50 \times \frac{\pi}{180°} = \frac{5\pi}{18}\text{rad}$$

$$\theta_r(s) = \frac{5\pi}{18} \times \frac{1}{s}$$

Therefore, $$\theta_o(s) = \frac{5\pi}{18} \times \frac{1}{s} \times \frac{36}{s^2 + 6s + 36}$$

In the above expressions,

$$\omega_n = \sqrt{36} = 6 \text{ rad/sec}$$

$$2\xi\omega_n = 6$$

$$\xi = 0.5$$

The time response expression is given by

$$\theta_o(t) = \frac{5\pi}{18}\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\left(\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right)\right]\right]$$

Substituting the values,

$$\theta_o(t) = \frac{5\pi}{18}\left[1 - \frac{e^{-3t}}{0.866}\sin(5.196t + 60°)\right]$$

(ii) On taking Laplace transform of $\dot{x} = Ax + Bu$, we get

$$sX(s) - x(0) = AX(s) + BU(s)$$

$$(sI - A)X(s) = x(0) + BU(s)$$

$$X(s) = (sI - A)^{-1}[x(0) + BU(s)] \quad \text{...(i)}$$

Similarly on taking Laplace transform of $y = Cx + Du$, we get

$$Y(s) = CX(s) + DU(s) \quad \text{...(ii)}$$

On substituting the value of $X(s)$ from equation (i), we get

$$Y(s) = C((sI - A)^{-1}x(0) + (sI - A)^{-1}BU(s)) + DU(s)$$

Now, to get the desired transfer function,

$$T(s) = C(sI - A)^{-1}B + D$$

We have to assume that the system has zero initial condition, i.e.,

$$x(0) = 0$$

We get $$Y(s) = C(sI - A)^{-1}BU(s) + DU(s)$$

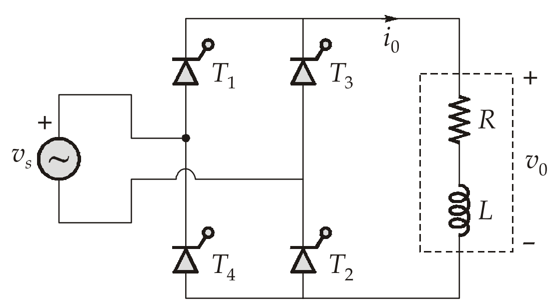
Therefore, $$\frac{Y(s)}{U(s)} = [C(sI - A)^{-1} B + D]$$

Q.8 (b) Solution:

$$V = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$R = 20\ \Omega;$$
$$X = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$$
$$\alpha = 60°$$

When thyristors are in ON-state

applying KVL from source to load,



$$v_s = L\frac{di_0}{dt} + Ri_0$$

$$v_m \sin\omega t = L\frac{di_0}{dt} + Ri_0$$

The general solution to this equation is

$$i_0(t) = \frac{v_m}{|z|}\sin(\omega t - \theta) + Ae^{-t/\tau}$$

Where, $v_m = 230\sqrt{2}$

$$|z| = \sqrt{20^2 + 62.83^2} = 65.94\,\Omega$$

$$\theta = \tan\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{62.83}{20}\right) = 72.34°$$

and $$T = \frac{L}{R} = \frac{0.2}{20} = 0.01$$

$\therefore \quad i_0(t) = \frac{230\sqrt{2}}{65.94}\sin(\omega t - 72.34°) + Ae^{-t/0.01}$

$= 4.933 \sin(\omega t - 72.34°) + Ae^{-100t}$

At $\omega t = \alpha = 60°$,

$i_0 = 0$

$\therefore \quad 0 = 4.933\sin(60° - 72.34°) + Ae^{-100\times\frac{60°}{\omega}\times\frac{\pi}{180°}}$

$0 = -1.054 + Ae^{-104.72/\omega} = -1.054 + Ae^{-104.72/100\pi}$

$1.054 = Ae^{-1/3}$

$A = 1.471$

$\therefore \quad i_0(t) = 4.933\sin(\omega t - 72.34°) + 1.471e^{-100t}$

At $\omega t = \beta$, $i_0 = 0$

$0 = 4.933\sin(\beta - 72.34°) + 1.471e^{\frac{-100\times\beta}{\omega}}$

$0 = 4.933 \sin(\beta - 1.263) + 1.471e^{-0.318\beta}$

[$\because$ 72.34° = 1.263 radian and β is in radian]

Solving this transcedental equation we get

$\beta = 4.473$ rad

$= 256.30°$

(i) It is clear from the values of β that the load current is continous in nature.

(ii) Average output voltage, $V_0 = \frac{2v_m}{\pi}\cos\alpha = \frac{2\times\sqrt{2}\times 230}{\pi}\cos 60° = 103.536$ volts

Average/dc load current, $I_0 = \frac{V_0}{R} = \frac{103.536}{20} = 5.177$ A

(iii) The Fourier series of output voltage will be

$$v_0 = V_0 + \sum_{n=1}^{\infty}[a_n\cos n\omega t + b_n \sin n\omega t]$$

where, $a_n = \frac{2}{T}\int_0^T v_0(\omega t)\cos n\omega t\, d\omega t$

$$= \frac{2}{\pi}\int_{\alpha}^{(\pi+\alpha)} v_0 \sin\omega t \cos n\omega t\, d\omega t$$

$$= \frac{v_m}{\pi} \int_{\alpha}^{(\pi+\alpha)} [\sin(n+1)\omega t - \sin(n-1)\omega t] \cdot d\omega t$$

$$= \frac{v_m}{\pi} \left[\frac{-\cos(n+1)\omega t}{(n+1)} + \frac{\cos(n-1)\omega t}{(n-1)} \right]_{\alpha}^{(\pi+\alpha)}$$

$$= \frac{v_m}{\pi} \left[\left\{ \frac{\cos(n+1)(\pi+\alpha) - \cos(n+1)\alpha}{(n+1)} \right\} + \frac{\cos(n-1)(\pi+\alpha) - \cos(n-1)\alpha}{(n-1)} \right]$$

For n = odd ;

$$\cos(n+1)(\pi+\alpha) = \cos(n+1)\alpha$$

$$\cos(n-1)(\pi+\alpha) = \cos(n-1)\alpha$$

For n = even ;

$$\cos(n+1)(\pi+\alpha) = -\cos(n+1)\alpha$$

$$\cos(n-1)(\pi+\alpha) = -\cos(n-1)\alpha$$

$$\therefore \quad a_n = \frac{v_m}{\pi} \left[-\left\{ \frac{-\cos(n+1)\alpha - \cos(n+1)\alpha}{(n+1)} \right\} + \frac{-\cos(n-1)\alpha - \cos(n-1)\alpha}{(n-1)} \right]$$

.... n is even

$$= \frac{2v_m}{\pi} \left[\frac{\cos(n+1)\alpha}{(n+1)} - \frac{\cos(n-1)\alpha}{(n-1)} \right]$$

Similarly,

$$b_n = \frac{2}{T} \int_0^T v_0(\omega t) \sin n\omega t \, d\omega t$$

$$= \frac{2}{\pi} \int_{\alpha}^{(\pi+\alpha)} v_m \sin \omega t \sin n\omega t \, d\omega t$$

$$= \frac{v_m}{\pi} \int_{\alpha}^{(\pi+\alpha)} [-\cos(n+1)\omega t + \cos(n-1)\omega t] d\omega t$$

$$= \frac{v_m}{\pi} \left[-\frac{\sin(n+1)\omega t}{(n+1)} + \frac{\sin(n-1)\omega t}{(n-1)} \right]_{\alpha}^{(\pi+\alpha)}$$

$$= \frac{v_m}{\pi} \left[-\left\{ \frac{\sin(n+1)(\pi+\alpha) - \sin(n+1)\alpha}{(n+1)} \right\} + \frac{\sin(n-1)(\pi+\alpha) - \sin(n-1)\alpha}{(n-1)} \right]$$

For n = odd;

$$\sin(n+1)(\pi+\alpha) = \sin(n+1)\alpha$$

$$\sin(n-1)(\pi+\alpha) = \sin(n-1)\alpha$$

For n = even ;

$$\sin(n+1)(\pi+\alpha) = -\sin(n+1)\alpha$$

$$\sin(n-1)(\pi+\alpha) = -\sin(n-1)\alpha$$

$$\therefore \quad b_n = \frac{2v_m}{\pi}\left[\frac{\sin(n+1)\alpha}{(n+1)} - \frac{\sin(n-1)\alpha}{(n-1)}\right] \quad \ldots n \text{ is even}$$

Since the dominant harmonic is 2nd harmonic

So,

$$a_2 = \frac{2v_m}{\pi}\left[\frac{\cos 3\alpha}{3} - \frac{\cos\alpha}{1}\right]$$

$$= \frac{2\sqrt{2}\times 230}{\pi}\left[\frac{\cos 3\times 60°}{3} - \cos 60°\right]$$

$$= -172.561 \text{ V}$$

and

$$b_2 = \frac{2v_m}{\pi}\left[\frac{\sin 3\alpha}{3} - \frac{\sin\alpha}{1}\right]$$

$$= \frac{2\times\sqrt{2}\times 230}{\pi}\left[\frac{\sin 3\times 60°}{3} - \sin 60°\right] = -179.33 \text{ V}$$

Peak value of 2nd harmonic voltage

$$\hat{V}_2 = \sqrt{a_2^2 + b_2^2} = \sqrt{(-172.561)^2 + (-179.33)^2}$$

$$= 248.871 \text{ V}$$

Rms 2nd harmonic voltage,

$$V_2 = \frac{\hat{V}_2}{\sqrt{2}} = \frac{248.871}{\sqrt{2}} = 175.978 \text{ V}$$

2nd harmonic impedance,

$$Z_2 = \sqrt{R^2 + [(2\omega)2]^2} = \sqrt{20^2 + [2\times 100\pi\times 0.2]^2} = 127.242\ \Omega$$

Rms 2nd harmonic current,

$$I_2 = \frac{V_2}{Z_2} = \frac{175.798}{127.242} = 1.382 \text{ A}$$

Rms output current,

$$I_{0r} = \sqrt{I_0^2 + I_2^2} = \sqrt{5.177^2 + 1.382^2} = 5.358 \text{ A}$$

∴ Power absorbed by the load is

$$P_0 = I_{0r}^2 R = 5.358^2 \times 20 = 574.163 \text{ W}$$

Q.8 (c) (i) Solution:

For the given system,

$$G(s) = \frac{K(s+1)}{s^3 + as^2 + 2s + 1},$$

$$H(s) = 1$$

Therefore, the characteristic equation is,

$$1 + G(s)H(s) = 0$$

$$s^3 + as^2 + (2 + K)s + (1 + K) = 0$$

The Routh table is formulated as follows:

s^3	1	$2+K$
s^2	a	$1+K$
s^2	$\frac{a(2+K) - 1(1+K)}{a}$	0
s^0	$1+K$	

For stability, all the elements in the first column of the Routh array must be positive:

From the s^0 row,

$$1 + K > 0$$

$$K > -1$$

From the s^1 row,

$$\frac{a(2+K) - 1(1+K)}{a} > 0$$

From s^2 row,

$$a > 0$$

But for the system to oscillate

$$a(2 + K) - (1 + K) = 0$$

i.e. $$a = \frac{1+K}{2+K} \quad ...(i)$$

To obtain the frequency of oscillations, from the auxiliary equation using the elements of the s^2 row i.e.

$$A(s) = as^2 + (1 + K) = 0$$

Given the frequency of oscillation,

$$\omega = 2 \text{ rad/sec}$$

By putting $s = j\omega$ in $A(s)$

$$A(s) = a(-\omega^2) + 1 + K = 0$$

$$-4a = -(1 + K)$$

$$a = \frac{1+K}{4} \quad ...(ii)$$

Comparing equation (i) and (ii), we get

$$a = \frac{1+K}{4} = \frac{1+K}{2+K}$$

$\therefore$ $$2 + K = 4$$

or $$K = 4 - 2 = 2$$

$\therefore$ $$a = \frac{1+K}{2+K} = \frac{1+2}{2+2} = 0.75$$

So the values of K and a so that the system oscillates at a frequency of 2 rad/s is $K = 2$ and $a = 0.75$.

Q.8 (c) (ii) Solution:

The system transfer function is

$$M(s) = \frac{C(s)}{R(s)} = \frac{K}{s+a}$$

For a unit-step input, $$R(s) = \frac{1}{s}$$

Therefore, the output $$C(s) = R(s)\,M(s) = \frac{1}{s}\cdot\frac{K}{s+a}$$

The steady state value of the output is,

$$\lim_{t\to\infty} c(t) = \lim_{t\to\infty} L^{-1}\left[\frac{K}{s(s+a)}\right] = \frac{K}{a}\lim_{t\to\infty} L^{-1}\left[\frac{1}{s} - \frac{1}{s+a}\right]$$

$$= \frac{K}{a}\lim_{t\to\infty}\left(1-e^{-at}\right) = \frac{K}{a}$$

From figure (ii), $c(\infty) = 1$

Therefore, $\frac{K}{a} = 1$

$$K = a$$

The initial slope of the output $= \lim_{t\to 0}\left(\frac{d}{dt}c(t)\right) = \frac{1}{0.1} = 10$

$$L\left(\frac{d}{dt}c(t)\right) = sC(s) = s\left[\frac{K}{s(s+a)}\right] = \frac{K}{s+a}$$

$\therefore$ Slope of the output $= \frac{d}{dt}C(t) = L^{-1}\left[\frac{K}{s+a}\right] = Ke^{-at}$

The initial slope $= \lim_{t\to 0}\left[\frac{d}{dt}C(t)\right] = K$

$\therefore$ $K = 10$

$\therefore$ $a = K = 10$

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