MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Mechanical Engineering Test No : 12

Section : A

1. (a) Solution:

$$\text{Speed of flywheel shaft} = \text{Hole capacity} \times \text{Gear ratio}$$
$$= 30 \times 8 = 240 \text{ rpm}$$
$$\text{Force required} = \text{Hole shear area} \times \text{Shear strength}$$
$$= \pi d t \times \tau = \pi \times 20 \times 16 \times 360$$
$$= 361.9 \text{ kN}$$
$$\text{Work required} = \text{Mean force} \times \text{Distance}$$
$$= \frac{0 + 361.9}{2} \times \frac{16}{1000} = 2.8952 \text{ kNm}$$
$$\text{Work required at flywheel} = \frac{2.8952}{\eta_{mech}} = \frac{2.8952}{0.8} = 3.619 \text{ kNm}$$

(i)
$$\text{Power, } P = \frac{\text{Work required} \times \text{Number of holes/min}}{60}$$
$$= \frac{3.619 \times 30}{60} = 1.81 \text{ kW}$$ **Ans.**

$$\text{Punching period} = \frac{36°}{360°} = 0.1$$

(ii) Energy supplied to flywheel or fluctuation of energy

$$\Delta E = 0.9 \times 3.619 = 3.257 \text{ kNm}$$ **Ans.**

(iii) Mass of flywheel, $m = \dfrac{\Delta E}{k^2\omega^2 C_s}$

where, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 240}{60} = 25.13$ rad/s

$$m = \frac{3.257 \times 10^3}{0.375^2 \times 25.13^2 \times 0.1} = 366.75 \text{ kg}$$

As, $m_r = 0.9 \times \text{mass of flywheel} = 0.9 \times 366.75 = 330.0$ kg

$m_r = \pi d \times b \times h \times \rho$

$330 = \pi \times 0.75 \times b \times h \times 7100$

or $b \times h = 0.01972\ \text{m}^2$

Since, $\dfrac{b}{h} = 1.5$

Width: $b = 172$ mm **Ans.**

Thickness: $h = 114.66$ mm **Ans.**

1. (b) Solution:

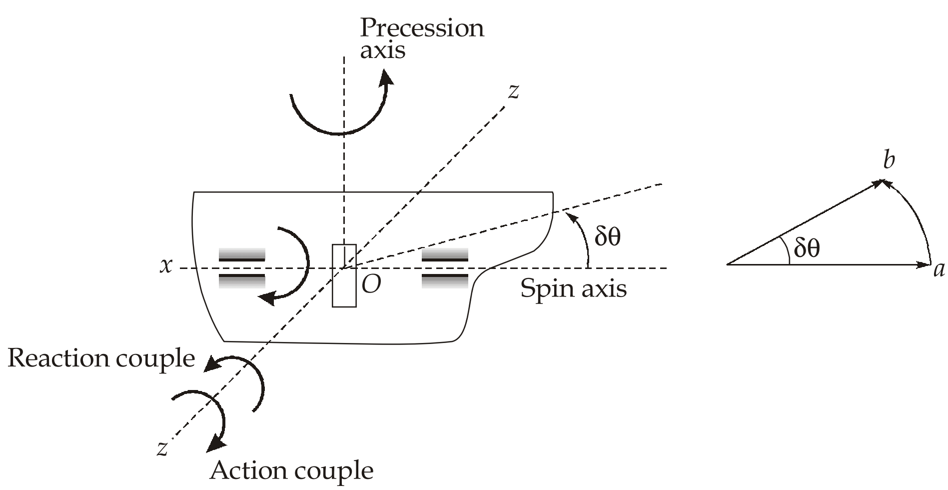
Given : $V = 22.32$ kmph

$$= \frac{22.32 \times 1000}{3600} = 6.2 \text{ m/s}$$

Angular velocity of the rotor:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60} = 209.44 \text{ rad/s}$$

(i) When ship steers to the left, the reaction gyroscopic couple action is in anticlockwise direction and the bow of the ship is raised and stern is lowered, as shown in figure.



Precession velocity: $\omega_p = \dfrac{V}{R} = \dfrac{6.2}{70} = 0.08857$ rad/s

Moment of inertia, $I = mk^2 = 3500 \times 0.5^2 = 875\ \text{kgm}^2$

Gyroscopic couple : $C = I w w_p$

$= 875 \times 209.44 \times 0.08857$

$= 16231.34\ \text{Nm}$ **Ans.**

(ii) Amplitude of swing, $A = \dfrac{6° \times 2\pi}{360°} = 0.1047\ \text{rad}$

$$\theta = A \sin \omega_o t$$

Angular velocity of precession,

$$\omega_p = \frac{d\theta}{dt} = A\omega_0 \cos \omega_o t$$

Maximum angular velocity of precession:

$$\omega_{p,\max} = \omega_0 A$$

where, $\omega_0 = \dfrac{2\pi}{\text{Time period of oscillation}} = \dfrac{2\pi}{30}$

$= 0.2094\ \text{rad/s}$

$$\omega_{p,\max} = 0.2094 \times 0.1047 = 0.022\ \text{rad/s}$$

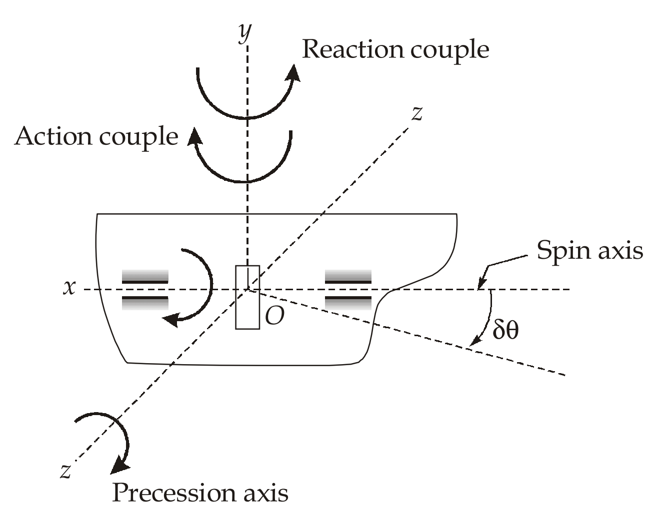
Maximum couple for pitching:

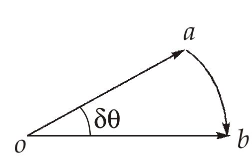
$$C_{\max} = I w w_{p,max}$$

$= 875 \times 209.44 \times 0.022$

$= 4031.72\ \text{Nm}$ **Ans.**

The gyroscopic effect during pitching is shown in figure. The reaction gyroscopic couple will act in anticlockwise direction during the bow descending and will turn ship towards the left side.





(iii) Angular velocity of precession while the ship rolls is

$$\omega_p = 0.05 \text{ rad/s}$$

and gyroscopic couple: $C = I\omega\omega_p$

$$= 875 \times 209.44 \times 0.05$$

$$= 9163 \text{ Nm}$$ **Ans.**

Since the ship rolls in the same plane as the plane of spin, there is no gyroscopic effect.

Angular velocity of precess during pitching is

$$\omega_p = \frac{d\theta}{dt} = A\omega_0 \cos \omega_0 t$$

Therefore, angular acceleration,

$$\alpha = \frac{d^2\theta}{dt^2} = -A\omega_0^2 \sin \omega_0 t$$

Maximum angular acceleration,

$$\alpha_{max} = -A\omega_0^2$$

$$= -0.1047 \times 0.2094^2$$

$$= -0.00459 \text{ rad/s}^2$$ **Ans.**

1. (c) Solution:

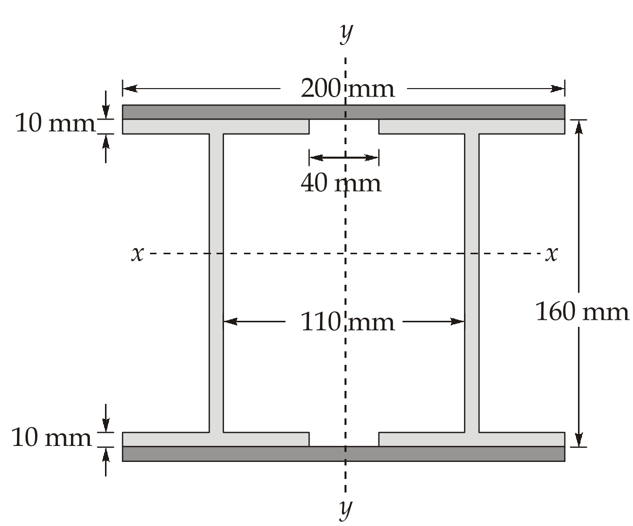
Given data : I-sections = 160 mm × 80 mm × 10 mm, plates 200 mm × 10 mm,

length = 4 m; FOS = 3, $a = \dfrac{1}{7500}$ and $\sigma_c = 320 \text{ MN/m}^2$.

Area of cross-section of column,

$$A = 2[80 \times 10 \times 2 + 140 \times 10] + 2 \times 200 \times 10$$

$$= 10000 \text{ mm}^2$$



As the section is symmetrical, the centre of gravity will lie at the point of intersection of two axes of symmetry.

$$\text{i.e. } I_{xx} = 4\left[\frac{8\times1^3}{12}+8\times1\times(8-0.5)^2\right]+2\left[\frac{1\times14^3}{12}\right]+2\left[\frac{20\times1^3}{12}+20\times1\times(8+0.5)^2\right]$$

$$I_{xx} = 5153.333 \text{ cm}^4$$

$$I_{xx} = 5153333 \text{ mm}^4$$

$$I_{yy} \text{ for one I-section} = 2\left[\frac{1\times8^3}{12}\right]+\frac{14\times1^3}{12}$$

$$= 86.49 \text{ cm}^4$$

$$= 864900 \text{ mm}^4$$

Moment of inertia for the whole section about y-y axis

$$I_{yy} = 2\left[8649+30\times6^2\right]+2\times\frac{1\times20^3}{12}$$

$$= 3666.31 \text{ cm}^4$$

As $I_{yy} < I_{xx}$, the column will tend to buckle in the y-y direction.

As the end condition is both ends hinged

$$l_e = l = 4 \text{ m}$$

$$k^2 = \frac{I}{A} = \frac{3666.31\times10^4}{10000} = 3666 \text{ mm}^2$$

Using Rankine's formula,

$$P = \frac{\sigma_c A}{1+a(l_e/k)^2} = \frac{320\times10^6\times100\times10^{-4}}{1+\frac{1}{7500}\left(\frac{4^2}{3666}\right)\times10^6}$$

$$= 2022 \times 10^3 \text{ N}$$

$$\text{Safe load} = \frac{2022\times10^3}{3} = 674.284 \text{ N}$$

$$= 674.284 \text{ kN}$$

1. (d) Solution:

Given : θ = –25° [Anticlockwise rotation is taken as +ve]

$$\text{Strain components, } \varepsilon_x = \frac{-6.4\times10^{-6}\,\text{m}}{10\times10^{-3}\,\text{m}} = -640\times10^{-6}$$

$$\varepsilon_y = \frac{3.6\times10^{-6}\,\text{m}}{10\times10^{-3}\,\text{m}} = 360\times10^{-6}$$

$$\gamma_{xy} = -160\times10^{-6}$$

$$\varepsilon_{x'} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \frac{-640+360}{2\times10^6} - \frac{-640-360}{2\times10^6}\cos(-50°) + \frac{-160}{2\times10^6}\sin(-50°)$$

$$= -522.677\times10^{-6}$$

Similarly normal strain in y' direction,

$$\varepsilon_{y'} = \frac{\varepsilon_x+\varepsilon_y}{2} - \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \frac{-640+360}{2\times10^6} - \frac{-640-360}{2\times10^6}\cos(-50°) + \frac{-160}{2\times10^6}\sin(-50°)$$

$$= 242.677\times10^{-6}$$

Hence, strain in $x'y'$ direction is given by

$$\frac{\gamma_{x'y'}}{2} = \frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$= \frac{-640-360}{2\times10^6}\sin(-50°) + \frac{-160}{2\times10^6}\cos(-50°)$$

$$= 663.2\times10^{-6}$$

1. (e) Solution:

Given: P = 12 MPa, d = 800 mm, t = 20 mm, L = 1600 mm, μ = 0.3, E = 205 GPa, K = 2080 MPa

$$\sigma_c\text{ (Hoop stress)} = \frac{Pd}{2t} = \frac{12\times800}{2\times20} = 240\text{ MPa}$$

$$\sigma_l\text{ (Longitudinal stress)} = \frac{Pd}{4t} = \frac{12\times800}{4\times20} = 120\text{ MPa}$$

$$\text{Hoop strain, } \varepsilon_c = \frac{1}{E}(\sigma_c - \mu\sigma_l) = \frac{1}{E}(240 - 0.3 \times 120)$$

$$\varepsilon_c = \frac{204}{E}$$

$$\text{Longitudinal strain, } \varepsilon_l = \frac{1}{E}(\sigma_l - \mu\sigma_h) = \frac{1}{E}(120 - 0.3 \times 240)$$

$$\varepsilon_l = \frac{48}{E}$$

$$\text{Increase in capacity} = (2\varepsilon_c + \varepsilon_l)V_c = \frac{(2 \times 204 + 48)}{205 \times 10^3} \times \frac{\pi}{4} \times 800^2 \times 1600$$

$$= 1.789 \times 10^6 \text{ mm}^3$$

For two hemispherical ends:

$$\text{Hoop strain, } \varepsilon_c = \frac{204}{E} \qquad \text{[Same as for cylinder]}$$

$$\text{Increase in capacity} = 3\varepsilon_c V_s = \frac{(3 \times 204)}{205 \times 10^3} \times \frac{\pi}{6} \times 800^3 = 0.8 \times 10^6 \text{ mm}^3$$

$$\text{Decrease in volume of water} = \frac{p}{K} \times \text{Volume}_{\text{total}}$$

$$= \frac{12}{2080} \times \left[\frac{\pi}{4} \times 800^2 \times 1600 + \frac{\pi}{6} \times 800^3\right]$$

$$= 6.187 \times 10^6 \text{ mm}^3$$

Additional volume of water in the drum

$$= 1.789 \times 10^6 + 0.8 \times 10^6 + 6.187 \times 10^6$$

$$= 8.776 \times 10^6 \text{ mm}^3$$

2. (a) Solution:

From the figure, the coordinates of points A, B, C and D are

$A(-1, -3, 0)$, $B(-3, 3, 0)$, $C(1, 2, 0)$ and $D\ (0, 0, -5)$

Unit vectors along DA, DB and DC are

$$\overrightarrow{DA} = \overrightarrow{OA} - \overrightarrow{OD}$$

$$= -\bar{i} - 3\bar{j} + 5\bar{k}$$

$$\therefore \qquad \hat{n}_{DA} = \frac{-\bar{i} - 3\bar{j} + 5\bar{k}}{\sqrt{(-1)^2 + (-3)^2 + (5)^2}} = \frac{-\bar{i} - 3\bar{j} + 5\bar{k}}{\sqrt{35}}$$

$$= -0.169\bar{i} - 0.507\bar{j} + 0.845\bar{k}$$

$$\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD}$$

$$= -3\bar{i} + 3\bar{j} + 5\bar{k}$$

$$\therefore \quad \hat{n}_{DB} = \frac{-3\bar{i} + 3\bar{j} + 5\bar{k}}{\sqrt{(-3)^2 + (3)^2 + (5)^2}} = \frac{-3\bar{i} + 3\bar{j} + 5\bar{k}}{\sqrt{43}}$$

$$= -0.457\bar{i} + 0.457\bar{j} + 0.762\bar{k}$$

$$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$$

$$= \bar{i} + 2\bar{j} + 5\bar{k}$$

$$\therefore \quad \hat{n}_{DC} = \frac{\bar{i} + 2\bar{j} + 5\bar{k}}{\sqrt{(1)^2 + (2)^2 + (5)^2}} = \frac{\bar{i} + 2\bar{j} + 5\bar{k}}{\sqrt{30}}$$

$$= 0.183\bar{i} + 0.365\bar{j} + 0.913\bar{k}$$

Let T_{DA}, T_{DB} and T_{DC} be the tensions in the cables DA, DB and DC respectively. Then the tension vectors can be represented as

$$\bar{T}_{DA} = T_{DA}\hat{n}_{DA}$$

$$= T_{DA}\left[-0.169\bar{i} - 0.507\bar{j} + 0.845\bar{k}\right]$$

$$\bar{T}_{DB} = T_{DB}\hat{n}_{DB}$$

$$= T_{DB}\left[-0.457\bar{i} - 0.457\bar{j} + 0.762\bar{k}\right]$$

$$\bar{T}_{DC} = T_{DC}\hat{n}_{DC}$$

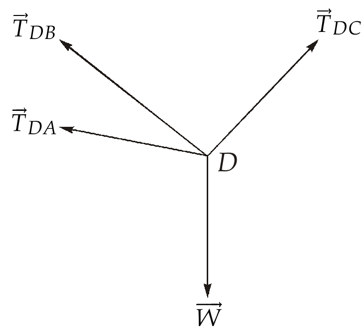
$$= T_{DC}\left[-0.183\bar{i} + 0.365\bar{j} + 0.913\bar{k}\right]$$

As the mass of the box is 200 kg, its weight can be represented in vector form as

$$\bar{W} = 200 \times 9.81\left[-\bar{k}\right]$$

$$= -1962\bar{k}$$

Note that the weight is directed along the negative Z-axis. Since the tension and weight vectors are concurrent at point D, the resultant of the system of forces is given as the vector addition of individual forces.



$$\vec{R} = \vec{T}_{DA} + \vec{T}_{DB} + \vec{T}_{DC} + \vec{W}$$

$$= [-0.169T_{DA} - 0.457T_{DB} + 0.183T_{DC}]\vec{i}$$

$$+ [-0.507T_{DA} + 0.457T_{DB} + 0.365T_{DC}]\vec{j}$$

$$+ [0.845T_{DA} + 0.762T_{DB} + 0.913T_{DC} - 1962]\vec{k}$$

Applying the condition of equilibrium, $\vec{R} = \vec{O}$, we get three independent simultaneous equations:

$$-0.169T_{DA} - 0.457T_{DB} + 0.183T_{DC} = 0 \quad \text{...(i)}$$

$$-0.507T_{DA} + 0.457T_{DB} + 0.365T_{DC} = 0 \quad \text{...(ii)}$$

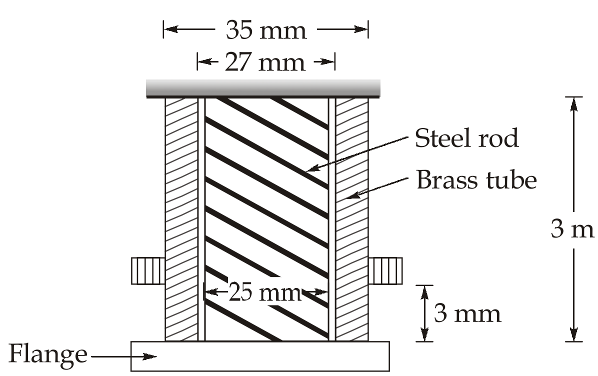
$$0.845T_{DA} + 0.762T_{DB} + 0.913T_{DC} = 1962 \quad \text{...(iii)}$$

Solving the equation (i), (ii) and (iii) simultaneously, we get

$$T_{DA} = 950.02 \text{ N}, T_{DB} = 117.26 \text{ N and } T_{DC} = 1171.92 \text{ N}$$

2. (b) Solution:

Given : P = 15 kN; L = 3 m = 3000 mm; E_s = 200 GPa, E_b = 1.0 × 10^5 N/mm^2



$$A_s = \frac{\pi}{4}(25)^2 = 490.625 \text{ mm}^2$$

$$A_b = \frac{\pi}{4}(35^2 - 27^2) = \frac{\pi}{4} \times (1225 - 729) = 389.36 \text{ mm}^2$$

Since both the ends are fixed together,

Strain in steel rod = Strain in brass tube

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\sigma_s = \frac{\sigma_b}{E_b} \times E_s = \sigma_b \times \frac{200000}{100000}$$

$$\sigma_s = 2\sigma_b \quad \text{...(i)}$$

Volume of steel rod, V_s = Area × Length

= 490.625 × 3000 = 1471875 mm^3

Volume of brass tube = 389.36 × 3000

= 1168080 mm^3

Strain energy in steel rod, $U_s = \frac{\sigma_s^2}{2E_s} \times \text{Volume}$

$$= \frac{(2\sigma_b)^2}{2 \times 200000} \times 1471875 = 14.72\,\sigma_b^2$$

Strain energy in brass tube, $U_b = \frac{\sigma_b^2}{2E_b} \times \text{Volume} = \frac{\sigma_b^2 \times 1168080}{2 \times 100000} = 5.840\sigma_b^2$

Total strain energy stored in compound bar,

$$U = U_s + U_b$$

$$= 14.72\sigma_b^2 + 5.840\sigma_b^2 = 20.56\sigma_b^2 \quad \text{...(ii)}$$

Work done by falling load = $P(h + x)$

$= 15000(3 + x)$ ...(iii)

Strain in brass rod = $\frac{\sigma_b}{E_b}$

or $$\frac{x}{l} = \frac{\sigma_b}{1 \times 10^5}$$

$$x = \frac{\sigma_b \times 3000}{1 \times 10^5} = 0.03\sigma_b$$

Substituting this value of x in equation (iii),

$$P(h + x) = 15000(3 + 0.03\sigma_b)$$

Now equating the work done by the falling weight to the total strain energy.

$$15000(3 + 0.03\sigma_b) = 20.56\sigma_b^2 \qquad \text{(from equation (ii))}$$

$$45000 + 450\sigma_b = 20.56\sigma_b^2$$

$$20.56\sigma_b^2 - 450\sigma_b = 45000 = 0$$

$$\sigma_b^2 - 21.89\sigma_b - 2188.7 = 0$$

This is a quadratic equation,

$$\therefore \qquad \sigma_b = \frac{21.89 \pm \sqrt{(21.89)^2 + 4 \times 2188.7}}{2}$$

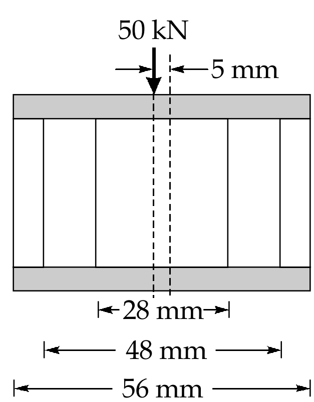
$$= \frac{21.89 \pm \sqrt{(479.17)^2 + 8754.8}}{2}$$

$$= \frac{21.89 + 96.1}{2} = 59 \text{ N/mm}^2$$ **Ans.**

$$\sigma_s = 2\sigma_b$$

$$= 2 \times 59 = 118 \text{ N/mm}^2$$ **Ans.**

2. (c) Solution:



Let F_s = Direct load at the axis of the steel rod.

F_b = Direct load at the axis of the brass tube.

M_s = Bending moment on steel rod.

M_b = Bending moment on brass tube.

For equilibrium :

$$F_s + F_b = 50000 \quad \text{...(i)}$$

$$M_s + M_b = F_s \times 5 \quad \text{...(ii)}$$

$$A_s = \frac{\pi}{4} \times 28^2 = 615.8 \text{ mm}^2$$

$$A_b = \frac{\pi}{4} \times (56^2 - 48^2) = 653.5 \text{ mm}^2$$

$$I_s = \frac{\pi}{64} \times 28^4 = 30172 \text{ mm}^4$$

$$= \frac{\pi}{64} \times (56^4 - 48^4) = 222173 \text{ mm}^4$$

As the ends plates are rigid, the tube and rod can be assumed to bend together with same radius of curvature, thus

$$R = \frac{E_S I_S}{M_s} = \frac{E_b I_b}{M_b}$$

$$\frac{210000 \times 30172}{M_s} = \frac{95000 \times 222173}{M_b}$$

$$M_b = 3.331\, M_s$$

From equation (ii),

$$M_s + 3.331\, M_s = F_s \times 5$$

or

$$M_s = 1.154\, F_s \text{ Nmm}$$

The compatibility equation

Reduction in length of rod = Reduction in length of tube due to compressive stress in rod + Strain due to bending moment in rod = Strain due to compressive stress in the tube

$$\frac{F_s}{E_s A_s} + \frac{M_s y}{E_s I_s} = \frac{F_b}{E_b A_b} \quad \text{[As } l \text{ is same]}$$

$$\frac{F_s}{210 \times 10^3 \times 615.8} + \frac{1.154 F_s \times 5}{210 \times 10^3 \times 30172} = \frac{50000 - F_S}{95 \times 10^3 \times 653.5}$$

$$F_s + 0.1178\, F_s = 104150 - 2.083\, F_s$$

$$F_s = 32539 \text{ N}$$

$\Rightarrow$

$$F_b = 50000 - F_s = 17461 \text{ N}$$

$$M_s = 1.154 \times 32539 = 37550 \text{ N.mm}$$

$$M_b = 3.331 \times 37550 = 125079 \text{ N.mm}$$

$$\text{Maximum stress in steel rod} = \frac{F_s}{A_s} + \frac{M_s y}{I_s}$$

$$= \frac{32539}{615.8} + \frac{37550 \times \left(\frac{28}{2}\right)}{30172} = 70.26 \text{ MPa}$$

$$\text{Minimum stress in steel rod} = \frac{F_s}{A_s} - \frac{M_s y}{I_s}$$

$$= \frac{32539}{615.8} - \frac{37550 \times \left(\frac{28}{2}\right)}{30172} = 35.42 \text{ MPa}$$

$$\text{Maximum stress in brass tube} = \frac{F_b}{A_b} + \frac{M_b y}{I_b}$$

$$= \frac{17461}{653.8} + \frac{125079 \times \left(\frac{56}{2}\right)}{222173}$$

$$= 26.71 + 15.76 = 42.47 \text{ MPa}$$

$$\text{Minimum stress in brass tube} = \frac{F_b}{A_b} - \frac{M_b y}{I_b}$$

$$= \frac{17461}{653.8} - \frac{125079 \times \left(\frac{56}{2}\right)}{222173}$$

$$= 26.71 - 15.76 = 10.95 \text{ MPa}$$

3. (a) Solution:

$$\text{Shear stress} = \frac{\text{Shearing force}}{\text{Area of weld group}}$$

Let t be the throat thickness and s be the size of weld.

Hence, $t = 0.707s$

Area of weld group, $A = 2 \times 25t = 50t$

Therefore, $$\tau = \frac{15000}{50t} = \frac{300}{t} \text{N/mm}^2 \quad \text{...(i)}$$

$$I_{xx} = 2\left[\frac{25t^3}{12} + \frac{25t \times 75^2}{4}\right]$$

As t is a small quantities as compared to other quantities hence $\frac{25t^3}{12}$ can be neglected.

$$\therefore \quad I_{xx} = \frac{25t \times (75)^2}{2} = 70312.5t\,\text{mm}^4$$

Bending moment, $M = 15000 \times 50 = 750000$ Nmm

$$\therefore \quad \sigma = \frac{M}{I_{xx}} \times \frac{75}{2} = \frac{750000}{70312.5t} \times \frac{75}{2} = \frac{400}{t} \quad \text{...(ii)}$$

The maximum stress (shearing) on the plane of throat,

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{200}{t}\right)^2 + \left(\frac{300}{t}\right)^2}$$

$$= \frac{1}{t}\sqrt{40000 + 90000} = \frac{360.6}{t}\,\text{N/mm}^2$$

As, permissible stress is 150 N/mm^2

$$\therefore \quad 150 = \frac{360.6}{t}$$

or
$$t = \frac{360.6}{150} = 2.4 \text{ mm}$$

$$\therefore \quad s = \frac{2.4}{0.707} = 3.39 \simeq 3.4 \text{ mm}$$ **Ans.**

3. (b) Solution:

Given: $\phi = 20°$, $P = 250$ kW, $N = 1300$ rpm, $T_1 = 30$, $T_2 = 60$, $T_3 = 25$ and $T_4 = 50$, $m = 8$ mm.

(i)

Now,
$$\omega = \frac{2\pi \times 1300}{60} = 136.135 \text{ rad/s}$$

Power, $P = T \times \omega = f_{t1} \times r_1 \times \omega_1$

$$P = f_{t1} \times \frac{mT_1}{2} \times \omega_1$$

$$250 \times 10^3 = f_{t1} \times \frac{8 \times 30}{2000} \times 136.135$$

$$f_{t1} = 15303.43 \text{ N}$$ **Answer**

and
$$f_{r1} = f_{t1} \tan\phi = 15303.43 \times \tan 20$$

$$= 5570 \text{ N}$$ **Answer**

$$f_{t1} = f_{t2} \text{ and } f_{r1} = f_{r2}$$

For gear 3 and gear 4, $P = T_3 \times \omega_3$

$$\frac{\omega_2}{\omega_1} = \frac{\omega_3}{\omega_1} = \frac{T_1}{T_2} \qquad [\because \omega_2 = \omega_3]$$

$\therefore$
$$\omega_3 = \omega_1 \times \frac{30}{60} = 136.135 \times \frac{1}{2} = 68.06 \text{ rad/s}$$

$$P = f_{t3} \times r_3 \times \omega_3$$

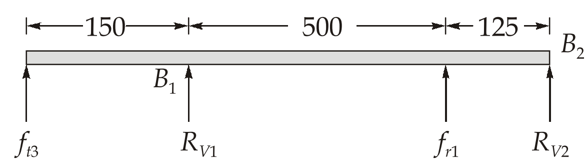
$$250 \times 10^3 = f_{t3} \times \frac{8 \times 25}{2000} \times 68.06$$

$\therefore$
$$f_{t3} = 36732.29 \text{ N} = f_{t4}$$ **Answer**

and
$$f_{r3} = f_{r4} = f_{t3} \tan\phi$$
$$= 36732.29 \times \tan 20$$
$$f_{r3} = f_{r4} = 13369.46 \text{ N}$$ **Answer**

(ii) For reactions at B_1 and B_2, consider the gear 2 and 3 which are mounted on shaft and calculate the reactions in vertical and horizontal plane.

For vertical plane, load diagram is shown below:



Moment about B_1: $F_{t3} \times 150 = F_{r1} \times 500 + R_{v2} \times 625$

$$36732.29 \times 150 = 5570 \times 500 + R_{v2} \times 625$$

$\therefore$
$$R_{v2} = 4359.75 \text{ N}$$

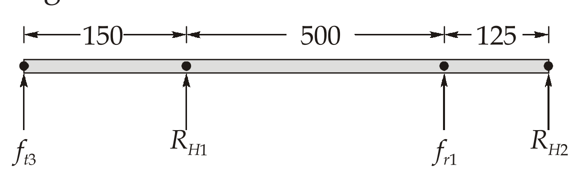
Moment about B_2:

$F_{t3} \times 775 + R_{v1} \times 625 + F_{r1} \times 125 = 0$

$36732.29 \times 775 + R_{v1} \times 625 + 5570 \times 125 = 0$

$\therefore$
$$R_{v1} = -46662.04 \text{ N}$$

Now, Horizontal load diagram:



Moment about B_1:

$$R_{H2} \times 625 + F_{t1} \times 500 = F_{r3} \times 150$$

$$R_{H2} \times 625 + 15303.43 \times 500 = 13369.46 \times 150$$

$\therefore$
$$R_{H2} = -9034.07 \text{ N}$$

Moment about B_2:

$R_{H1} \times 625 + F_{r3} \times 775 + F_{t1} \times 125 = 0$

$R_{H1} \times 625 + 13369.46 \times 775 + 15303.43 \times 125 = 0$

$\therefore \quad R_{H1} = -19638.81 \text{ N}$

Resultant reactions:

At B_1, $R_{B1} = \sqrt{(R_{v_1})^2 + (R_{H_1})^2} = \sqrt{(-46662.04)^2 + (-19638.81)^2}$

$= 50626.36 \text{ N}$ **Answer**

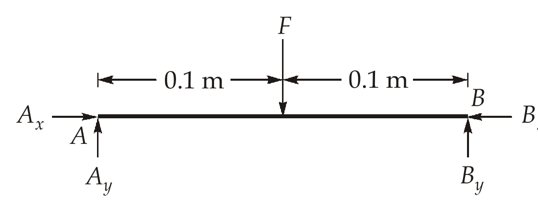
At B_2, $R_{B2} = \sqrt{(R_{v_2})^2 + (R_{H_2})^2} = \sqrt{(4359.75)^2 + (-9034.07)^2}$

$= 10031.04 \text{ N}$ **Answer**

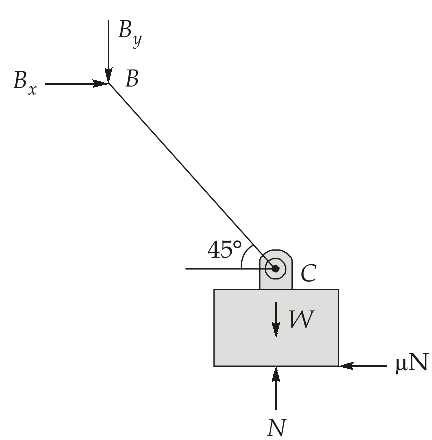
3. (c) Solution

(i)

FBD of AB:



$$\Sigma M_A = 0$$
$$75 \times 0.1 = 0.2\, B_y$$
$$B_y = 37.5 \text{ N}$$
$$\Sigma F_V = 0$$
$$A_y = 75 - B_y$$
$$= 75 - 37.5 = 37.5 \text{ kN}$$
$$\Sigma F_H = 0$$
$$A_X = B_X \quad \text{...(i)}$$



$\Sigma M_C = 0$ (Torque of μN is zero about C because thickness of box is neglected)

$$\Rightarrow \quad B_y (BC)\cos 45° = B_x (BC)\sin 45°$$

$$\Rightarrow \quad B_x = B_y = 37.5$$

$$\Sigma F_V = 0$$

$$B_y + 20 = N$$

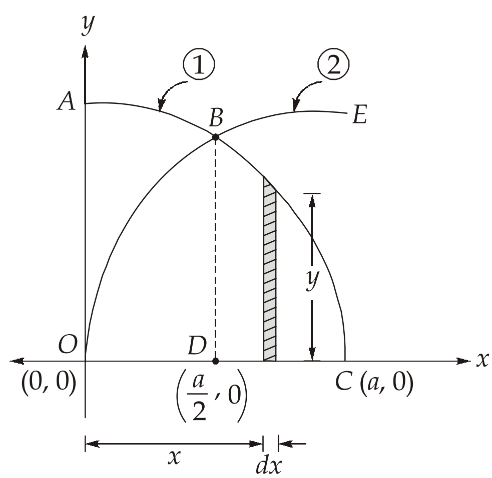
$$N = 37.5 + 20 = 57.5 \text{ N}$$

$$\Sigma F_H = 0$$

$$B_x = \mu N$$

$$\mu = \frac{B_x}{N} = \frac{37.5}{57.5} = 0.6522 \simeq 0.66$$

(ii)



Equation of circle-1: $x^2 + y^2 = a^2$

Equation of circle-2: $(x - a)^2 + y^2 = a^2$

Area moment of inertia of portion BDC about x-axis.

$$I_{BDC} = \int_{a/2}^{a} \frac{dx \times y^3}{3}$$

$$\Rightarrow \quad I_{BDC} = \frac{1}{3}\int_{a/2}^{a} \left(a^2 - x^2\right)^{3/2} dx \qquad \because \; y = \sqrt{a^2 - x^2}$$

Put

$$x = a\sin\theta$$

$$dx = a\cos d\theta$$

$$\Rightarrow \quad I_{BDC} = \frac{1}{3}\int_{\pi/6}^{\pi/2} a^3\cos^3\theta \times a\cos\theta d\theta = \frac{a^4}{3}\int_{\pi/6}^{\pi/2} \cos^4\theta d\theta$$

$$\Rightarrow \quad I_{BDC} = \frac{a^4}{3}\int_{\pi/6}^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$

$$\Rightarrow \quad I_{BDC} = \frac{a^4}{3\times 4}\int_{\pi/6}^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$

$$\Rightarrow \quad I_{BDC} = \frac{a^4}{12}\int_{\pi/6}^{\pi/2}\left[1+\left(\frac{1+\cos 4\theta}{2}\right)+2\cos 2\theta\right]d\theta$$

$$\Rightarrow \quad I_{BDC} = \frac{a^4}{12\times 2}\int_{\pi/6}^{\pi/2}[3+4\cos 2\theta+\cos 4\theta]\,d\theta$$

$$= \frac{a^4}{24}\left[3\left(\frac{\pi}{2}-\frac{\pi}{6}\right)+\frac{4}{2}(\sin 2\theta)_{\pi/6}^{\pi/2}+\frac{1}{4}(\sin 4\theta)_{\pi/6}^{\pi/2}\right]$$

$$= \frac{a^4}{24}\left[3\left(\frac{\pi}{2}-\frac{\pi}{6}\right)+\frac{2}{2}(0-\sin 60^\circ)+\frac{1}{4}(0-\sin 120^\circ)\right]$$

$$I_{BDC} = 0.0497098a^4$$

$$\because \quad I_{OBCD} = 2I_{BDC}$$

$$= 0.0994196a^4$$

Now area moment of inertia of portion OAB

$$I_{OAB} = \frac{1}{4}\left(\frac{\pi}{4}a^4\right) - I_{OBCD}$$

$$\Rightarrow \quad I_{OAB} = \left(\frac{\pi}{16}-0.0994196\right)a^4$$

$$I_{OAB} = 0.096929\, a^4$$

4. (a) Solution:

Given : Diameter = 30 cm, thickness = 15 mm, diameter of steel wire 5 mm, $\sigma_w = 75$ N/mm^2, water pressure = 3.5 N/mm^2 , E_{c_1} = 0.96 × 10^5 N/mm^2, E_s = 2.02 × 10^5 N/mm^2 and μ = 0.3.

(i) Before filling with water

Let us consider 1 cm length of pipe.

$$\text{Number of turns per cm length of wire} = \frac{\text{Length of pipe}}{\text{Diameter of wire}} = \frac{1}{0.5} = 2$$

Compressive force due to one turn of wire on the cylinder = 2 × (Area for cross section of wire) σ_w

$$= 2\times\frac{\pi}{4}\times 5^2\times 75$$

Total compressive force exerted by wire per cm length of pipe

$$= 2\left(2\times\frac{\pi}{4}\times 5^2\times 75\right) = 5890.48 \text{ N}$$

Cross-sectional area of the cylinder over which this force acts

$$= 2(l \times t)$$
$$= 2(1 \times 10 \times 15)$$
$$= 300 \text{ mm}^2$$

Initial compressive stress in the wall of the cylinder = $-\frac{5890.48}{300} = -19.63 \text{ N/mm}^2$

Hence, before allowing water in the pipe, the stresses in the wall of pipe and wire are 19.63 N/mm^2 (C) and 75 N/mm^2 (T)

(ii) After filling: Stresses due to fluid pressure alone

Let σ_{pf} be stress in pipe due to fluid pressure and σ_{wf} stress in wire due to fluid pressure.

Again considering 1 cm length of pipe, force of fluid pressure causing to burst the pipe along longitudinal section

$$= pd \times 1$$
$$= 3.5 \times 300 \times 1 \times 10$$
$$= 10500 \text{ N} \quad \text{...(i)}$$

The force is combinedly resisted by the pipe and the wire

Resisting force of pipe = Stress in the pipe × Area of pipe resisting

Resisting force of pipe = $\sigma_{pf}(2 \times 1 \times t)$

$$= \sigma_{pf} \times 2 \times 1 \times 10 \times 15 = 300\,\sigma_{pf} \quad \text{...(ii)}$$

Resisting force of wire = Number of turns × Area of wire resisting × Stress in wire due to fluid pressure

$$= 2\times\left(2\times\frac{\pi}{4}\times 5^2\right)\sigma_{wf}$$
$$= 78.5\,\sigma_{wf} \quad \text{...(iii)}$$

∴ Total resisting force = $300\,\sigma_{pf} + 78.5\,\sigma_{wf}$

Equating the resisting and longitudinal bursting force [(Equation (i) – (iii)]

$$300\ \sigma_{pf} + 78.5\ \sigma_{wf} = 10500 \quad ...(iv)$$

The hoop strain in the pipe is equal to the strain in the wire.

$$\text{Hoop strain in pipe} = \frac{(\text{Hoop stress})}{E_p} - \frac{(\text{Longitudinal stress})}{mE_p}$$

$$= \frac{\sigma_{pf}}{E_p} - \left(\frac{p \times d}{4t}\right)\frac{1}{mE_p}$$

$$= \frac{1}{E_p}\left(\sigma_{pf} - \frac{3.5 \times 300}{4 \times 15} \times 0.3\right) = \frac{1}{E_p}\left(\sigma_{pf} - 5.25\right) \quad ...(v)$$

$$\text{Strain in the wire} = \frac{\sigma_{wf}}{E_s} \quad ...(vi)$$

Equating the strains [Equations (v) and (vi)]

$$\frac{1}{E_p}\left(\sigma_{pf} - 5.25\right) = \frac{\sigma_{wf}}{E_s}$$

i.e.,
$$\sigma_{wf} = \frac{E_s}{E_p}\left(\sigma_{pf} - 5.25\right) = \frac{2.02 \times 10^5}{0.96 \times 10^5}\left(\sigma_{pf} - 5.25\right)$$

i.e.,
$$\sigma_{wf} = 2.1(\sigma_{pf} - 5.25)$$

Substituting for σ_{wf} in equation (iv)

$$300\sigma_{pf} + 78.5 \times 2.1(\sigma_{pf} - 5.25) = 10500$$

i.e.
$$300\sigma_{pf} + 164.85\,\sigma_{pf} = 10500 + 865.46$$

Solving,
$$\sigma_{pf} = 24.45 \text{ N/mm}^2 \text{ (tensile)}$$

Substituting for σ_{pf} in Equation (vii)

$$\sigma_{wf} = 2.1(24.45 - 5.25) = 40.32 \text{ N/mm}^2 \text{ tensile}$$

After filling: Resultant stress in pipe and wire

Resultant stress in pipe = (Initial stress in pipe) + (Stress in pipe due to fluid pressure alone)

$$= 19.63 \text{ N/mm}^2 \text{ (compressive)} + 24.49 \text{ N/mm}^2 \text{ (tensile)}$$

$$= 4.86 \text{ N/mm}^2 \text{ (tensile)}$$

Resultant stress in wire = (Initial stress in pipe) + (Stress in wire due to fluid pressure alone)

$$= 75.0 \text{ N/mm}^2 \text{ (tensile)} + 42.55 \text{ N/mm}^2 \text{ (tensile)}$$

$$= 117.6 \text{ N/mm}^2$$

4. (b) Solution:

Given : $T_D = 100$; $T_A = 40$; $T_B = T_C = 30$

The gears B and C are in mesh with gear A. Also the gears B and C are attached to the arm D. The number of teeth on gears B and C are equal. Hence revolution of gears B and C will be equal.

S.No.	Operations	Revolution of			
		Arm E	Gear A	Gear B or Gear C	Gear D
1.	Arm fixed, Gear A rotates through +1 revolution.	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B}\times\frac{T_B}{T_D}=-\frac{T_A}{T_D}$
2.	Arm fixed, Gear A rotates through $+x$ revolution (Multiply by x to all	0	$+x$	$-x\times\frac{T_A}{T_B}$	$-x\times\frac{T_A}{T_D}$
3.	Add $+y$ to all	$+y$	$+y$	$+y$	$+y$
4	Resultant motion	y	$x+y$	$y-x\times\frac{T_A}{T_B}$	$y-x\times\frac{T_A}{T_D}$

Take +ve sign for anti-clockwise and –ve sign for clockwise

(i) Gear A makes one revolution clockwise and D makes half a revolution anticlockwise.

Given conditions are: $N_A = -1$(–ve sign due to clockwise)

$$N_D = +\frac{1}{2} \text{ (+ve sign due to anticlockwise)}$$

But from Table, the resultant motion of gear A is $(x + y)$ or $N_A = (x + y)$

Hence, equating the motion of gear A (i.e. two values of N_A), we get

$$x + y = -1 \quad \text{...(i)}$$

Again from Table, the resultant motion of gear D is $\left(y-x\times\frac{T_A}{T_D}\right)$

or
$$N_D = \left(y-x\times\frac{T_A}{T_D}\right)$$

Equating the two values of N_D, we get

$$y-x\times\frac{T_A}{T_D} = \frac{1}{2}$$

or
$$y-x\times\frac{40}{100} = \frac{1}{2} \quad (\because T_A = 40 \text{ and } T_D = 100)$$

or $$y - 0.4x = \frac{1}{2}$$

We have to find the revolution of arm. But the revolution of the arm from Table is given by 'y'. Hence let us find the value of 'y' from equations (i) and (ii)

From equation (i), $x = -1 - y$

Substituting this value of x in equation (ii), we get

$$y - 0.4(-1 - y) = \frac{1}{2} \text{ or } 0.5$$

or $$y + 0.4 + 0.4y = 0.5$$

or $$1.4y = 0.5 - 0.4 = 0.1$$

or $$y = \frac{0.1}{1.4} = \frac{1}{14} = 0.0714 \text{ revolution}$$

(+ve sign means the revolution will be anticlockwise)

∴ Speed of arm, y = 0.0714 revolution anti-clockwise **Ans.**

(ii) Gear A makes one revolution clockwise and gear D is stationary.

∴ Given conditions are : $N_A = -1$

$N_D = 0$

But from Table resultant motions of gears A and D are,

$$N_A = (x + y) \text{ and } N_D = \left(y - x\frac{T_A}{T_D}\right)$$

∴ $$N_A = (x + y) = -1$$

and $$N_D = \left(y - x\frac{T_A}{T_D}\right) = 0$$

or $$x + y = -1 \qquad ...(iii)$$

and $$y - x\frac{T_A}{T_D} = 0$$

or $$y - x\frac{40}{100} = 0 \qquad (\because \; T_A = 40; T_D = 100)$$

or $$y - 0.4x = 0 \qquad ...(iv)$$

From equation (iii), $x = -1 - y$

Substituting this value of x in equation (iv), we get

$$y - 0.4(-1 - y) = 0$$

or $$y + 0.4 + 0.4y = 0$$

or $$1.4y = -0.4$$

or $$y = -\frac{0.4}{1.4} = -0.286 \text{ revolutions}$$

(–ve sign means the revolution will be clockwise)

Speed of arm, $y = 0.286$ revolution clockwise **Ans.**

4. (c) Solution:

The equation of damped free vibrating system is

$$m\ddot{x} + c\dot{x} + kx = 0$$

For which normal frequency is

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\therefore \quad \omega_n = \sqrt{\frac{343}{3.43}} = 10 \text{ rad/s}$$

(i) When damping coefficient, $c = 137.2$ N/m

$$\text{Damping factor, } \xi = \frac{c}{2m\omega_n} = \frac{137.2}{2 \times 3.43 \times 10} = 2.0$$

Therefore, the system is overdamped and its solution equation is

$$x = Ae^{\left[-\xi+\sqrt{\xi^2-1}\right]\omega_n t} + Be^{\left[-\xi-\sqrt{\xi^2-1}\right]\omega_n t}$$

$$= Ae^{\left[-2+\sqrt{2^2-1}\right]10t} + Be^{\left[-2-\sqrt{2^2-1}\right]10t}$$

or $$x = Ae^{-2.68t} + Be^{-37.32t} \quad \text{...(i)}$$

Differentiating equation (i) with respect to time, we get

$$\dot{x} = -2.68Ae^{-2.68t} - 37.32Be^{-37.32t} \quad \text{...(ii)}$$

For initial conditions:

at $t = 0$, $x = 0.02$ m

and at $t = 0$, $\dot{x} = 0$

Substituting initial condition in equation (i) and (ii), we get

$$0.02 = A + B \quad \text{...(iii)}$$

and $$0 = -2.68A - 37.32B$$

Solving the above equations for constants A and B, we get

$$A = 0.02155$$

and $$B = -0.00155$$

The equation of motion is

$$x = 0.02155e^{-2.68t} - 0.00155e^{-37.32t}$$

(ii) When damping coefficient, $c = 13.72$ Ns/m

Natural frequency: $$\omega_n = 10 \text{ rad/}$$

Damping factor: $$\xi = \frac{c}{2m\omega_n} = \frac{13.72}{2 \times 3.43 \times 10} = 0.2$$

Therefore, it is underdamped system

The solution of equation of motion for underdamped system is

$$x = Ce^{-\xi\omega_n t}\sin(\omega_d t + \phi)$$

where, $$\omega_d = \omega_n \times \sqrt{1-\xi^2} = 10 \times \sqrt{1-0.2^2} = 9.8$$

or $$x = Ce^{-0.2\times10\times t}\sin(9.8t + \phi)$$

or $$x = Ce^{-2t}\sin(9.8t + \phi)$$

Differentiating the above equation, we get

$$\dot{x} = -2Ce^{-2t}\sin(9.8t+\phi) + 9.8 \times Ce^{-2t} \times \cos(9.8t+\phi)$$

Substituting initial conditions,

at $t = 0$, $x = 0.02$ m

and $t = 0$, $\dot{x} = 0$,

$$0.02 = C \times \sin\phi \quad \text{...(v)}$$

$$0 = -2C \times \sin\phi + 9.8C \times \cos\phi \quad \text{...(vi)}$$

Solving equation (v) and (vi), we get

$$\phi = 78.46° \text{ and } C = 0.0204$$

Therefore, the equation for underdamped system is

$$x = 0.0204e^{-2t}\sin(9.8t + 78.46°)$$

Section : B

5. (a) Solution:

For the whole section, $I = \frac{200 \times 300^3}{12} = 45000 \times 10^4 \text{ mm}^4$

$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{150 \times 10^6}{45000 \times 10^4} \times 150 = 50 \text{ N/mm}^2$$

(i) Compressive force on the shaded area,

$$P_C = \frac{\sigma_{max}}{y_{max}} \times A\bar{y}$$

$$= \frac{50}{150} \times (200 \times 100)(50 + 50)$$

$$= 666.667 \text{ kN}$$ **Answer**

Moment of this force about N. A. is

$$M_C = \frac{\sigma_{max}}{y_{max}} \times I_A$$

I_A = Moment of inertia of the shaded area about the NA

$$I_A = \frac{200 \times 100^3}{12} + 200 \times 100(100)^2$$

$$= 21667 \times 10^4 \text{ mm}^4$$

$\therefore$ $$M_C = \frac{50}{150} \times 21667 \times 10^4$$

$$= 72.222 \text{ kN-m}$$ **Answer**

(ii) Tensile force on the cross hatched area,

$$P_T = \frac{\sigma_{max}}{y_{max}} \times A\bar{y} = \frac{50}{150} \times (80 \times 120) \times 60$$

$$= 192 \text{ kN}$$ **Answer**

$$I_A = \frac{80 \times 120^3}{3} = 4608 \times 10^4 \text{ mm}^4$$

$$M_T = \frac{\sigma_{max}}{y_{max}} \times I_A = \frac{50}{150} \times 4608 \times 10^4$$

$$= 15.36 \text{ kNm}$$ **Answer**

5. (b) Solution:

S.No.	Monitoring methods	Detection of problem by	Determination of nature of problem by analysis of measurement
1.	Visual monitoring	Overall appearance	Colouring/shape/texture
2.	Performance monitoring	Rate of output	Uniform quality level/rate of output/uniformity
3.	Vibration/Noise level monitoring	Overall noise/Vibration level	Frequency of noise level/signal waveform/signal statistics
4.	Wear monitoring	Amount of Debris/ Friction colour	Shape/size/size distribution of debris/chemical composition
5.	Corrosion monitoring	Colour/Chemical analysis	Variation in coating thickness/ chemical composition

Wear Debris Monitoring : This works on the principle that the working surfaces of a machine are washed by the lubricating oil, and any damage to them should be detectable from particles of wear debris in the oil. If the debris consists of relatively large ferrous lumps such as those generated by the fatigue of rolling element bearings and gears or the pitting of cams and taproots, these can be picked up by removable magnetic plugs inserted in the oil return lines. For small debris particles, spectrographic analysis or microscopic examination of oil samples after magnetic separation are commonly used techniques. Another popular technique is SOAP analysis for debris monitoring.

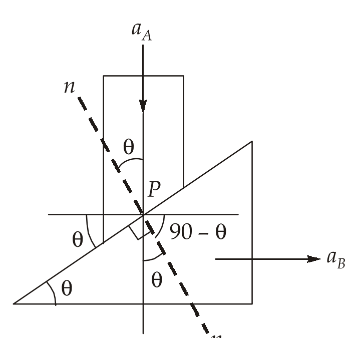
5. (c) Solution:

Given : $\frac{m_B}{m_A} = 0.5$

$\therefore \quad m_A = 2\, m_B$

Let acceleration of A and B are a_A and a_B respectively

Applying constraint motion at contact point P i.e. the component of acceleration of P along n - n must be same



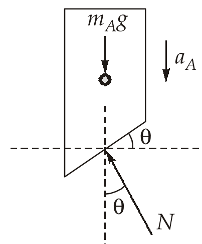
$$a_A = a_B \tan\theta$$

$$a_A \cos\theta = a_B \cos(90 - \theta)$$

$$a_A = \frac{1}{\sqrt{3}} a_B$$

$$a_B = \sqrt{3}\, a_A \quad \text{...(i)}$$

Considering FBD of rod



$$\Sigma F_v = ma$$

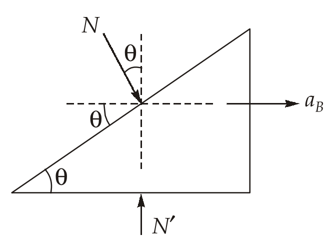
$$m_A g - N\cos\theta = m_A \cdot a_A \quad \text{...(ii)}$$

From equation (i) and (ii),

$$2m_B g - \frac{N \cdot \sqrt{3}}{2} = 2m_B \frac{a_B}{\sqrt{3}}$$

$$\frac{N}{2} = \frac{2}{\sqrt{3}} m_B \left(g - \frac{a_B}{\sqrt{3}} \right) \quad \text{...(iii)}$$

Consider FBD of wedge



$$\Sigma F_H = ma$$

$$N\sin\theta = m_B a_B$$

$$\frac{N}{2} = m_B a_B \quad \text{...(iv)}$$

From equation (iii) and (iv),

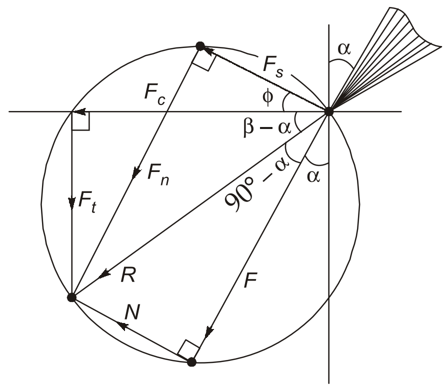
$$m_B a_B = \frac{2}{\sqrt{3}} m_B \left(g - \frac{a_B}{\sqrt{3}} \right)$$

$$a_B \left(1 + \frac{2}{3} \right) = \frac{2}{\sqrt{3}} g$$

$$a_B = \frac{6}{5\sqrt{3}} g$$

$$a_A = \frac{a_B}{\sqrt{3}} = \frac{2}{5} g$$

5. (d) Solution



Given: d = 2.5 mm, $t = f$ = 0.3 mm ($\because \lambda = 90°$), V = 250 m/min, t_c = 0.4 mm, F_c = 1500 N, F_t = 750 N

From tool designation: α = 12°, λ = 90°

$$r = \frac{t}{t_c} = \frac{0.3}{0.4} = 0.75$$

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} = \frac{0.75\cos 12°}{1 - 0.75\sin 12°} = 0.8691$$

$\Rightarrow$ $\phi = 40.995° \approx 41°$

(i) Shear force, $F_s = F_c \cos\phi - F_t \sin\phi$

$= 1500 \cos 41° - 750 \sin 41° = 640.02$ N **Ans.**

(ii) Normal force at shear plane,

$F_n = F_c \sin\phi + F_t \cos\phi$

$= 1500 \sin 41° + 750 \cos 41° = 1550.12$ N **Ans.**

(iii) Friction force, $F = F_c \sin\alpha + F_t \cos\alpha$

$= 1500 \sin 12° + 750 \cos 12° = 1045.478$ N **Ans.**

(iv) Kinetic coefficient of friction,

$$\mu = \frac{F_c\sin\alpha + F_t\cos\alpha}{F_c\cos\alpha - F_t\sin\alpha} = \frac{1500\sin 12° + 750\cos 12°}{1500\cos 12° - 750\sin 12°}$$

$= 0.797$ **Ans.**

(v) Specific cutting energy, $e = \frac{F_c}{bt} = \frac{1500}{fd} = \frac{1500}{0.3 \times 2.5} = 2000 \text{ N/mm}^2$ **Ans.**

5. (e) Solution:

Advantages of SCARA robot are as follows:

1. Very high precision
2. High compliance in horizontal plane
3. High stiffness to the arm in vertical direction

4. High speed of motion and operation
5. High stiffness of height axis
6. Large work area for floor space
7. Moderately easy to program
8. Excellent repeatability
9. It enables Sophisticated motion control with full programmability

Disadvantages of SCARA robot:

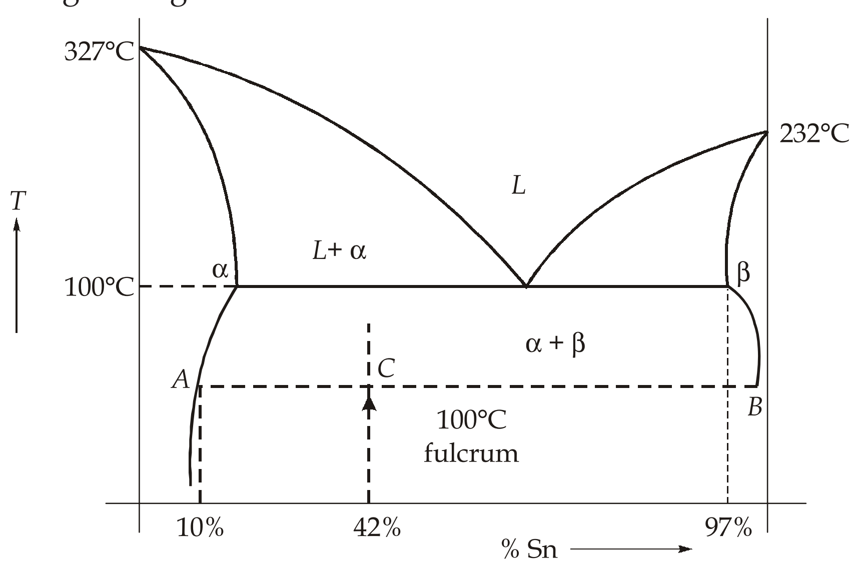
1. It has limited applications
2. Difficult to program off-line
3. Two ways to reach the desired point
4. It has highly complex arm.

Major applications of SCARA robot:

1. Pick and place operations of parts
2. Application of seal out.
3. Efficient handling of machine tools
4. Assembly operations

6. (a) Solution:

(i) From the given figure



$$m_\alpha = \frac{97-42}{97-10} = 0.6322 \quad \textbf{Ans.}$$

$$m_\beta = 1 - m_\alpha = 0.3678 \quad \textbf{Ans.}$$

The densities of α and β can be calculated as:

$$\frac{100}{\rho_\alpha} = \frac{10}{\rho_{Sn}} + \frac{90}{\rho_{Pb}} = \frac{10}{7.25} + \frac{90}{11.25}$$

$$\rho_\alpha = 10.66 \text{ g/cc}$$

$$\frac{100}{\rho_\beta} = \frac{97}{\rho_{Sn}} + \frac{3}{\rho_{Pb}} = \frac{97}{7.25} + \frac{3}{11.25}$$

$$\rho_\beta = 7.328 \text{ g/cc}$$

Volume fraction of α and β,

$$V_\alpha = \frac{\frac{m_\alpha}{\rho_\alpha}}{\frac{m_\alpha}{\rho_\alpha} + \frac{m_\beta}{\rho_\beta}} = \frac{\frac{0.6322}{10.66}}{\frac{0.6322}{10.66} + \frac{0.3678}{7.328}} = 0.5416$$ **Ans.**

$$V_\beta = 1 - V_\alpha = 1 - 0.5416 = 0.4584$$ **Ans.**

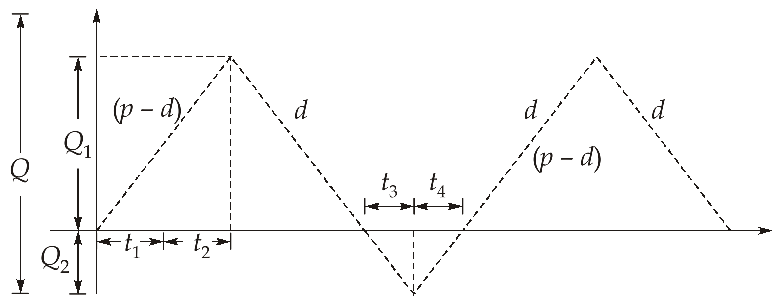
(ii)

Comparison of NDT Techniques

S.No.	Testing method	When to use	Where to use	Advantages	Limitations
1.	Magnetic particle testing	To detect surface or shallow substance flaws, cracks, porosity, non-metallic inclusions and weld defects.	Only for Ferro magnetic materials, parts of any size, shape, composition or heat treatment	Economical; simple in principle; easy to perform; portable (for field-testing); fast for production testing.	Limited to magnetic materials, needs to de-magnetise after test; needs power source; needs to be cleaned before test.
2.	Eddy current inspection	To find variation in coating thickness; longitudinal seam/crack in tubing; defects due to HT and composition.	Tubing and bar stock, parts of uniform geometry, flat stock or sheets and wire.	High speed; non-contact; automatic.	False indications result from many variables; only good for conductive materials; limited depth of penetration.
3.	Radio graphy: X-rays	To detect internal flaws and defects; to find weld flaws, cracks, seams, porosity, inclusions, holes; to measure thickness variation.	Assemblies of electronic parts, casting, welded vessels; field testing of welds; corrosion surveys, components of nonmetallic materials.	Provides permanent record on film; works on thin sections; high sensitivity; fluoroscopy techniques available; adjustable energy level.	High initial costs; power source required; radiation hazard, trained technicians needed.
4.	Radio graphy: Gamma-rays	Detecting internal flaws, cracks, seams, holes, inclusion, weld defects; measuring thickness variations.	Forging, coatings, tubing, welded vessels; field testing welded pipes; corrosion surveys.	Detect variety of flaws; provide a permanent record; portable; low initial cost; source is small, good for inside shots; make panoramic exposure	One energy level per source; radiation hazard; needs trained personnel; source looses strength continuously.

6. (b) Solution:

Given : $p = 3000 \times 12 = 36000$ units/annum; $d = 18000$ units/annum; C_o = ₹500/setup, C_h = ₹(0.15 × 12) = ₹1.8/unit/year; C_s = ₹20/unit/year



(i) Economic order quantity (EOQ),

$$Q^* = \sqrt{\frac{2dC_o}{C_h\left(1-\frac{d}{p}\right)}\left(\frac{C_h+C_s}{C_s}\right)}$$

$$Q^* = \sqrt{\frac{2\times500\times18000}{1.8\left(1-\frac{18000}{36000}\right)}\times\left(\frac{1.8+20}{20}\right)}$$

$$Q^* = 4669.047 \simeq 4669 \text{ units}$$

(ii) Maximum inventory, $Q_1^* = \sqrt{2dC_o \times \left(\frac{p-d}{p}\right)\left(\frac{C_s}{C_s+C_h}\right)}$

$$= \sqrt{\frac{2\times18000\times500(36000-18000)}{1.8\times36000}\times\left(\frac{20}{20+1.8}\right)}$$

$$= 2141.76 \simeq 2142 \text{ units}$$

(iii) Maximum stockout, $Q_2^* = \sqrt{\frac{2\times C_0 C_h}{C_s(C_s+C_h)}\times\frac{d(p-d)}{p}}$

$$Q_2^* = \sqrt{\frac{2\times500\times1.8}{20(20+1.8)}\times\frac{18000(36000-18000)}{36000}}$$

$$Q_2^* = 192.76 \simeq 193 \text{ units}$$

(iv) Cycle time $(t) = \frac{Q^*}{d} = \frac{4669}{18000}\times365 = 94.67 \simeq 95$ days

$$t_1 = \frac{Q_1^*}{p-d} = \frac{2142}{(36000-18000)} \times 365 = 43.44 \simeq 43.5 \text{ days}$$

$$t_2 = \frac{Q_1^*}{d} = \frac{2142}{18000} \times 365 = 43.5 \text{days}$$

$$t_3 = \frac{Q_2^*}{d} = \frac{193}{18000} \times 365 = 3.914 \simeq 4 \text{ days}$$

$$t_4 = \frac{Q_2^*}{p-d} = \frac{193}{(36000-18000)} \times 365 = 4 \text{days}$$

6. (c) Solution

Given: $\phi = 20°$, $Z_p = 24$, $Z_g = 60$, $m = 5$ mm, $b = 50$ mm, $N_p = 600$ rpm, $(S_{ut})_p = (S_{ut})_g = 650$ N/mm^2. BHN = 230, $k_a = 1.6$, FOS = 2, $k_m = 1$.

(i) Beam strength: When pinion and gear both are made of same material at that time pinion is weaker than the gear in bending. Hence, it is necessary to find beam strength of pinion.

$$F_b = (bmY\sigma_b)_p$$

$$Y_p = 0.48 - \frac{2.8}{Z_p} = 0.48 - \frac{2.8}{24} = 0.3633$$

$\therefore$ $$F_b = 50 \times 5 \times 0.3633 \times \frac{650}{3} = 19678.75 \text{ N}$$

$= 19.678$ kN **Ans.**

(ii) Wear strength:

$$F_W = D_P bQk$$

$$D_p = m \times Z_p = 5 \times 24 = 120 \text{ mm}$$

$$Q = \frac{2Z_G}{Z_G + Z_P} = \frac{2 \times 60}{60 + 24} = 1.4285$$

$$k = 0.16 \times \left(\frac{230}{100}\right)^2 = 0.8464 \text{ N/mm}^2$$

• $\therefore$ $$F_W = 120 \times 50 \times 1.4285 \times 0.8464$$

$= 7254.49$ N $= 7.254$ kN **Ans.**

(iii) In this case, wear strength is lower than beam strength so wear strength is the criteria of design.

$\therefore$ $$F_W = (\text{FOS}) \times F_{\text{eff.}}$$

$$7254.49 = 2 \times F_{eff.}$$

$$\Rightarrow \quad F_{eff.} = 3627.2472 \text{ N}$$

Now, $$F_{eff.} = \frac{k_a k_m F_t}{C_V} \quad \text{... (i)}$$

$$V = \frac{\pi D_P N_P}{60} = \frac{\pi \times 120 \times 600}{60 \times 1000} = 3.77 \text{ m/s}$$

$$C_V = \frac{3}{3+V} = \frac{3}{3+3.77} = 0.443$$

∴ From equation (i)

$$3627.2472 = \frac{1.6 \times 1 \times F_t}{0.443}$$

$$\Rightarrow \quad F_t = 1004.605 \text{ N}$$ **Ans.**

(iv) Power transmitted by gear pair:

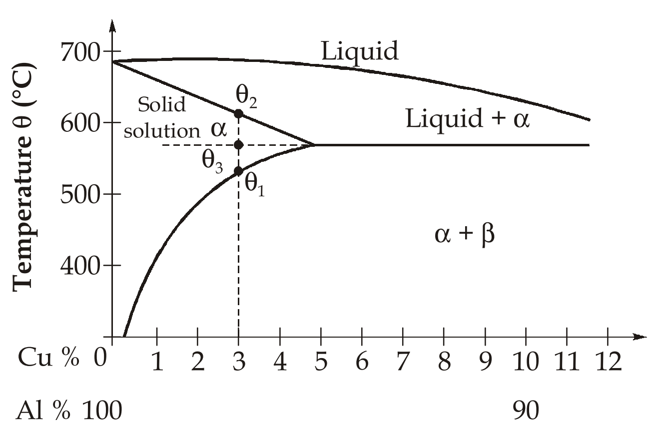
$$P = F_t \times V = 1004.605 \times 3.77$$

$$= 3787.36 \text{ W} = 3.787 \text{ kW}$$ **Ans.**

7. (a) Solution:

(i)

Age hardening and precipitation hardening: This process of hardening is applicable only for those alloys that exist as a two-phase material at the room temperature and can be heated up to a single phase. The phase diagram of one such alloy is shown in figure below. Assuming that the composition is 3% Cu and 97% Al, the alloy exists as a two-phase material (α + β) below the temperature θ_1.



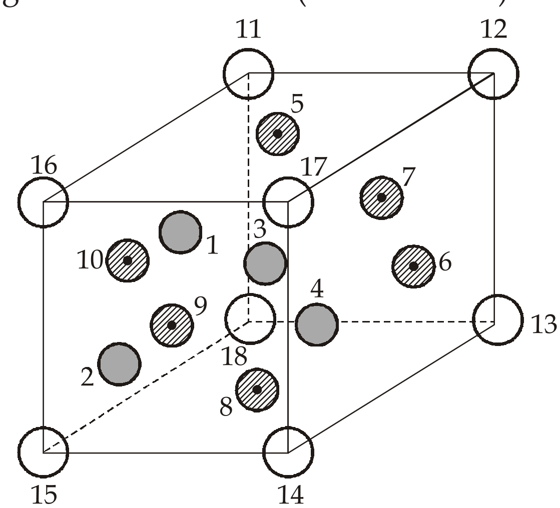
Principle of precipitation hardening

However, between θ_1 and θ_2, it exists as a single-phase solid solution (α). A solution heat treatment process consists in heating the alloy to a temperature between θ_1 and θ_3.

Also, a sufficient time is given at this temperature for the material to homogenize. A subsequent rapid quenching does not allow all the β-phase to separate out. Thus, the solution becomes supersaturated. This supersaturated β-phase precipitates slowly, the rate being dependent on the final temperature after quenching. The precipitation takes place at the grain boundaries and crystallographic planes, making the slippage of atomic layers more difficult. Thus, the alloy becomes harder and stronger. If the precipitation takes place at the room temperature, a longer time is necessary for the completion of precipitation, and this process is referred to as age hardening. On the other hand, higher than the room temperature, the process then is called precipitation hardening. Depending on the precipitation temperature, the hardness (at the room temperature), instead of increasing continuously, may attain a maximum before it starts decreasing. The optimal properties are very sensitive to both the temperature (at which the precipitation takes place) and the time elapsed after quenching.

(ii)

There are four atoms (within the unit cell) marked 1-4, six atoms at a centre of six faces (marked 5-10) and eight atoms at corners (marked 11-18).



Diamond-Atomic structure

Effective number of atoms in a diamond cubic unit cell

$$= \frac{1}{8}\times 8 + \frac{1}{2}\times 6 + 4 = 8$$

Volume of each spherical atom = $\frac{4}{3}\pi r^3$

where, r is the atomic radius of carbon atom.

Radius of atom, $r = \frac{a\sqrt{3}}{8}$, where a is the lattice parameter.

$$\text{Volume of atoms in unit cell} = 8 \times \frac{4}{3}\pi r^3 = \frac{32}{3}\pi r^3 = \frac{32}{3} \times \pi \left(\frac{a\sqrt{3}}{8}\right)^3$$

$$= \frac{32}{3} \times \frac{\pi \times 3\sqrt{3}}{512} \times a^3 = 0.340a^3$$

Volume of unit cell = a^3

$$\text{Packing factor} = \frac{0.34a^3}{a^3} = 0.340$$

$$\text{Density} = \frac{\text{Mass of atoms in unit cell}}{\text{Volume of unit cell}}$$

Mass of one carbon atom = 12 amu = $12 \times 1.66 \times 10^{-27}$ kg

= 1.992×10^{-26} kg

Volume of the lattice = $a^3 = (0.365 \times 10^{-9})^3 \text{ m}^3 = 4.8627 \times 10^{-29} \text{ m}^3$

$$\text{Density} = \frac{\text{Mass of average no. of atoms}}{\text{Volume of the unit cell}}$$

$$= \frac{8 \times 1.992 \times 10^{-26}}{4.8627 \times 10^{-29}} = 3277.183 \text{ kg/m}^3$$ **Ans.**

7. (b) Solution:

(i) Subtracting all cell elements from the largest element which is 60.

	I	II	III	IV	V
P	20	20	25	35	10
Q	18	30	44	35	33
R	10	12	20	0	10
S	40	41	40	42	35
T	2	0	1	5	7

	I	II	III	IV	V
P	10	10	15	25	0
Q	0	12	26	17	15
R	10	12	20	0	10
S	5	6	5	7	0
T	2	0	1	5	7

Matrix after row operation
(Contains zero in each row)

	I	II	III	IV	V
P	10	10	14	25	0
Q	0	12	25	17	15
R	10	12	19	0	10
S	5	6	4	7	0
T	2	0	0	5	7

Matrix after column operation

(Contains zero in every row and column)

	I	II	III	IV	V
P	10	10	14	25	[0]
Q	[0]	12	25	17	15
R	10	12	19	[0]	10
S	5	6	4	7	~~0~~
T	2	[0]	~~0~~	5	7

Initial basic feasible solution

(Contains no assignment in 4^{th} row and 3^{rd} column)

	I	II	III	IV	V
P	6	6	10	21	[0]
Q	[0]	12	25	17	19
R	10	12	19	[0]	14
S	1	2	[0]	3	~~0~~
T	2	[0]	~~0~~	5	11

Second basic feasible solution

(Optimal solution)

Optimal assignment policy is

Batsman	Batting position	Number of runs
P	V	50
Q	I	42
R	IV	60
S	III	20
T	II	60
		232

7. (c)

Given; Initial thickness, h_i = 30 mm, Final thickness, h_f = 26 mm,
Roll diameter, D = 600 mm, Flow stress, σ_o = 120 MPa

$$\Delta h = 2R(1 - \cos\alpha)$$

$$\Rightarrow \quad (30 - 26) = 600\,(1 - \cos\alpha)$$

$$\Rightarrow \quad \alpha = 6.62°$$ **Ans.**

$$\mu = \tan\alpha = \tan 6.62° = 0.1160$$ **Ans.**

$$H_o = 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\cdot\alpha\right)$$

$$= 2\sqrt{\frac{300}{26}}\tan^{-1}\left(\sqrt{\frac{300}{26}}\times 6.62\times\frac{\pi}{180}\right) = 2.5408$$

$$H_n = \frac{1}{2}\left(H_o - \frac{1}{\mu}\ln\frac{h_i}{h_f}\right) = \frac{1}{2}\left(2.5408 - \frac{1}{0.1160}\ln\frac{30}{26}\right) = 0.6536$$

$$\theta_n = \sqrt{\frac{h_f}{R}}\tan\left(\sqrt{\frac{h_f}{R}}\times\frac{H_n}{2}\right)$$

$$= \sqrt{\frac{26}{300}}\tan\left(\sqrt{\frac{26}{300}}\times\frac{0.6536}{2}\right) = 0.02841 \text{ radians}$$ **Ans.**

$$h_n = h_f + 2R\,(1 - \cos\theta_n) = h_f + R\theta_n^2$$

$$= 26 + 300 \times (0.02841)^2 = 26.242 \text{ mm}$$ **Ans.**

$$\text{Backward slip} = \frac{V_r - V_o}{V_r}\times 100 = \left(1 - \frac{V_o}{V_r}\right)\times 100$$

$$\frac{V_o}{V_r} = \frac{h_n}{h_i} = \frac{26.242}{30}$$

$$\therefore \quad \text{Backward slip} = \left(1 - \frac{26.242}{30}\right)\times 100 = 12.526\%$$ **Ans.**

$$\text{Forward slip} = \left(\frac{V_f - V_r}{V_r}\right)\times 100 = \left(\frac{V_f}{V_r} - 1\right)\times 100$$

$$\frac{V_f}{V_r} = \frac{h_n}{h_f} = \frac{26.242}{26}$$

$$\therefore \quad \text{Forward slip} = \left(\frac{26.242}{26} - 1\right)\times 100 = 0.936\%$$ **Ans.**

$$\text{Maximum pressure, } P_{max} = \sigma_o' \times \frac{h_n}{h_f}e^{\mu H_n}$$

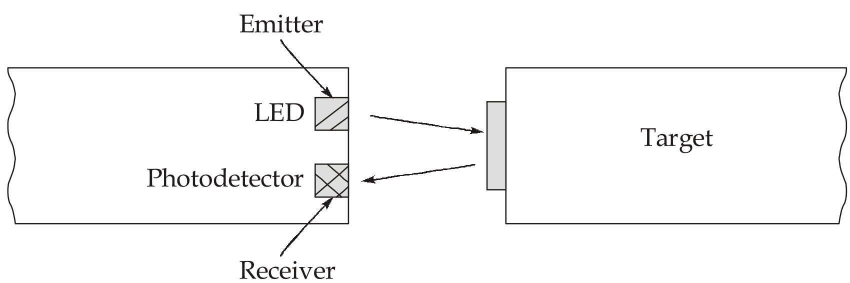
As per Von-Mises theory, $\sigma_o' = \frac{2}{\sqrt{3}}\sigma_o$

$$\therefore \quad P_{max} = \frac{2}{\sqrt{3}} \times 120 \times \frac{26.242}{26} \times e^{0.1160 \times 0.6536} = 150.87 \text{ MPa} \quad \textbf{Ans.}$$

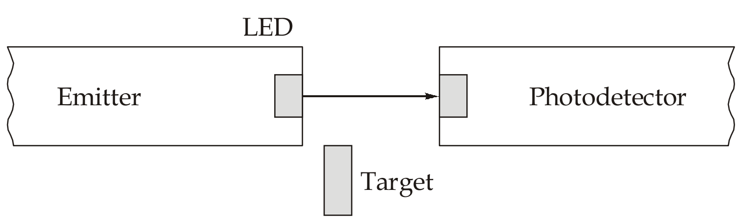
8. (a)

(i)

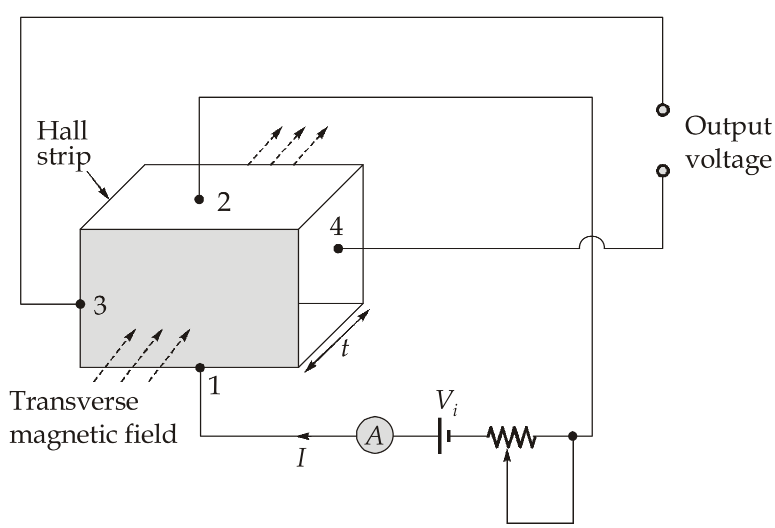
Hall effect sensors : When a beam of charged particles passes through a magnetic field, the beam is deflected from its straight line path due to the forces acting on the particles. A current flowing in a conductor, such as a beam, is deflected by a magnetic field. This effect is called hall effect.



(a) Reflective-type photoelectric sensor



(b) Transmissive-type photoelectric sensor



Hall effect sensor

The working principle of a Hall effect sensor is that if a strip of conducting material carries a current in the presence of a transverse magnetic field as shown in figure above, the difference of potential is produced between the opposite edge of the conductor. The magnitude of the voltage depends upon the current and the magnetic field. The current is passed through leads 1 and 2 of the strip and the output leads 3 and 4 are connected with a Hall strip. When a transverse magnetic field passes through the strip, the voltage difference occurs in the output leads. The Hall effect sensor has the advantage of being able to operate as a switch and it can operate upto 100 kHz.

Application of Hall Effect Sensor

- It is used as a magnetic switch for electric transducer.
- It is used for the measurement of the position, displacement and proximity.
- It is used for measurement of current.
- It is used for measurement of power.

(ii)

$$\text{Sensitivity of LVDT} = \frac{\text{Output current}}{\text{Displacement}}$$

$$= \frac{2 \times 10^{-3}}{0.75} = 2.67 \times 10^{-3}\,\text{A/mm} \quad \text{Ans.}$$

$$\text{Sensitivity of instrument} = \text{Amplification factor} \times \text{Sensitivity of LVDT}$$

$$= 100 \times 2.67 \times 10^{-3} = 0.267\ \text{A/mm} \quad \text{Ans.}$$

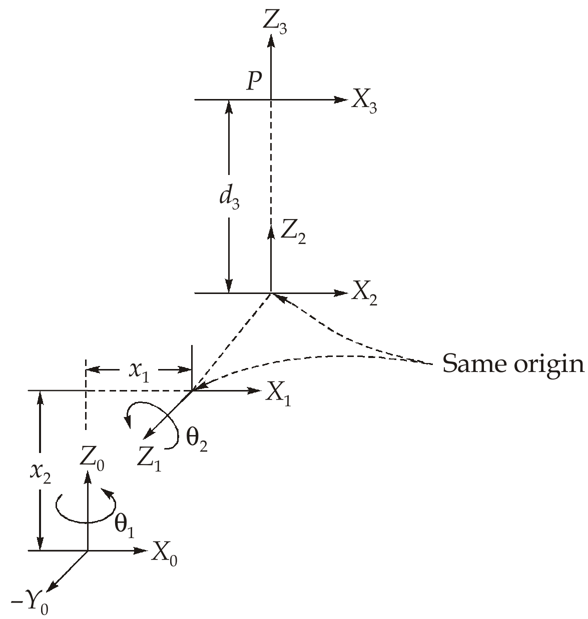
$$1\ \text{scale division} = \frac{12}{100} = 0.12\,A$$

$$\text{Least current that can be measured} = \frac{1}{10} \times 0.12 = 0.012\,A$$

$$\therefore \quad \text{Resolution} = 0.012 \times \frac{1}{0.267} = 0.045\ \text{mm} \quad \text{Ans.}$$

8. **(b) Solution**

For the given home position of 3-DOF RRP Manipulator, the frame assignment using D-H algorithm is shown below.



Frame assignment for home position of arm

The joint link parameter from the frames assigned are tabulated in table:

i	a_i	α_i	d_i	θ_i	q_i	$C\theta_i$	$S\theta_i$	$C\alpha_i$	$S\alpha_i$
1	x_1	+90	x_2	θ_1	θ_1	C_1	S_1	0	1
2	0	–90	0	θ_2	θ_2	C_2	S_2	0	–1
3	0	0	d_3	0	d_3	1	0	1	0

Now we have,

$$^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, $C\theta_i = \cos\theta_i$, $S\theta_i = \sin\theta_i$, $C\alpha_i = \cos\alpha_i$ and $S\alpha_i = \sin\alpha_i$.

The three transformation matrices are thus, obtained as:

$$^{0}T_1 = \begin{bmatrix} C_1 & 0 & S_1 & x_1 C_1 \\ S_1 & 0 & -C_1 & x_1 S_1 \\ 0 & 1 & 0 & x_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the overall transformation of the arm is,

$$T = {}^0T_1 \times {}^1T_2 \times {}^2T_3$$

On substituting and simplifying, we have,

$$T = \begin{bmatrix} C_1C_2 & -S_1 & -C_1S_2 & -C_1S_2 \times d_3 + x_1C_1 \\ S_1C_2 & C_1 & -S_1S_2 & -S_1S_2 \times d_3 + x_1S_1 \\ S_2 & 0 & C_2 & C_2 \times d_3 + x_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The orientation of the end-effector (point P) is described in terms of joint variables by the rotation submatrix of T, that is,

$$R = \begin{bmatrix} C_1C_2 & -S_1 & -C_1S_2 \\ S_1C_2 & C_1 & -S_1S_2 \\ S_2 & 0 & C_2 \end{bmatrix}$$

For $\theta_1 = 90°$ and $\theta_2 = -45°$, the orientation of P is

$$\begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0.707 & 0 & 0.707 \\ -0.707 & 0 & 0.707 \end{bmatrix}$$

and the position of end effector is described by the translation vector D as

$$D = \begin{bmatrix} -C_1S_2d_3 + x_1C_1 \\ -S_1S_2d_3 + x_1S_1 \\ d_3C_2 + x_2 \end{bmatrix}$$

For given joint link parameters and q, the position of P is

$$D = [0 \quad 156.568 \quad 106.568]^T$$ **Ans.**

8. (c) Solution:

Given: W = 3600 N, N = 1510 rpm, l/d = 0.5, P = 1.5 MPa, C_1 = 0.1 mm, Z = 38 cP

(i) Journal diameter and bearing length

$$n_s = \frac{1510}{60} \text{ rev/s}$$

$$\mu = \frac{Z}{10^9} = \frac{38}{10^9}\ \text{N-s/mm}^2$$

$$\text{Bearing pressure } (P) = \frac{W}{ld}$$

$$1.5 = \frac{3.6\times 10^3}{d\times 0.5\times d} \qquad [l = 0.5d]$$

or, $d = 69.282 \simeq 70$ mm

and $l = 0.5d = 70 \times 0.5 = 35$ mm

(ii) Performance parameter, $P = \frac{W}{ld} = \frac{3600}{35\times 70} = 1.46938$ MPa

$$\text{Sommerfeld number } (s) = \left(\frac{r}{C}\right)^2 \frac{\mu n_s}{P} = \left(\frac{35}{0.05}\right)^2 \times \frac{38}{10^9}\times\frac{1510}{60}\times\left(\frac{1}{1.46938}\right)$$

$$= 0.3189$$

So, from table, corresponding to $l/d = 0.5$ and $s = 0.3189$

$$\varepsilon = 0.6,\ \frac{h_o}{C} = 0.4, \left(\frac{r}{C}\right)f = 8.10, \qquad \frac{Q}{rCn_s l} = 4.85$$

$\therefore$ Coefficient of friction, $\left(\frac{r}{C}\right)f = 8.10$

or, $f = 8.1\times\frac{0.05}{35} = 0.011571$

(iii) Power lost in friction, $P = \frac{2\pi n_s f Wr}{10^6} = \frac{2\pi\times 1510\times 0.011571\times 3600\times 35}{10^6\times 60}$

$$= 0.23054 \text{ kW}$$

(iv) Flow requirement, $Q = 4.85 \times r \times C \times n_s \times l$

$$= 4.85 \times 35 \times 0.05 \times \frac{1510}{60}\times 35 = 7476.0729 \text{ mm}^3/\text{s}$$

$$= 7476.0729 \times 60 \times 10^{-6}$$

$$Q = 0.44856 \text{ litre/min}$$

❍❍❍❍