



MADE EASY
Leading Institute for ESE, GATE & PSUs

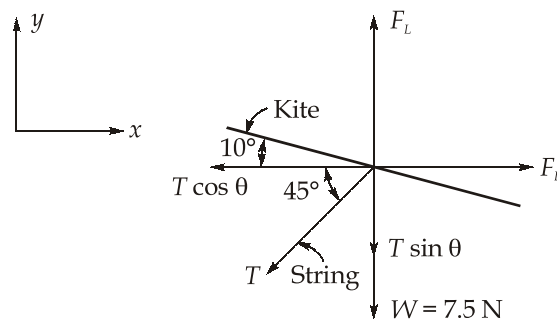
Detailed Solutions

**ESE-2025
Mains Test Series**

**Civil Engineering
Test No : 12**

Section - A

Q.1 (a) Solution:



Given:

$$C_D = 0.6, C_L = 0.8$$

For equilibrium

$$\Sigma F_x = 0 \Rightarrow F_D = T \cdot \cos 45^\circ \quad \dots(i)$$

$$\Sigma F_y = 0 \Rightarrow F_L = W + T \cdot \sin 45^\circ \quad \dots(ii)$$

\therefore

$$F_D = \frac{C_D}{2} \rho_{air} A V_0^2$$

$$F_L = \frac{C_L}{2} \rho_{air} A V_0^2$$

Now,

$$\frac{C_D}{2} \rho_{air} A V_0^2 = T \cdot \cos 45^\circ$$

$$\Rightarrow \frac{0.6}{2} \times 1.2 \times (0.9 \times 0.9) \times V_0^2 = T \cos 45^\circ$$

$$\Rightarrow V_0^2 = 2.4249 T \quad \dots(iii)$$

$$\frac{C_L}{2} \rho_{air} A V_0^2 = W + T \sin 45^\circ$$

$$\Rightarrow \frac{0.8}{2} \times 1.2 \times (0.9 \times 0.9) V_0^2 = 7.5 + T \sin 45^\circ$$

$$\Rightarrow 0.3888 \times 2.4249 T = 7.5 + \frac{T}{\sqrt{2}}$$

$$\Rightarrow 0.23569 T = 7.5$$

$$\Rightarrow T = 31.82 \text{ N}$$

From equation (iii)

$$V_0^2 = 2.4249 \times 31.82$$

$$\Rightarrow V_0^2 = 8.784 \text{ m/sec} = 31.62 \text{ kN/m}$$

Q.1 (b) Solution:

Given:

Depth of flow, $y = 1.5 \text{ m}$

Bed slope, $S = 0.0009$

Width of channel, $B = 3 \text{ m}$

Manning's coefficient, $n = 0.015$

Hydraulic mean radius, $R = \frac{A}{P} = \frac{By}{B+2y}$

$$\Rightarrow R = \frac{3 \times 1.5}{3 + 2 \times 1.5} = 0.75 \text{ m}$$

Velocity,

$$v = \frac{1}{n} R^{2/3} S^{1/2}$$

$$\Rightarrow v = \frac{1}{0.015} (0.75)^{2/3} (0.0009)^{1/2} = 1.65 \text{ m/sec}$$

Discharge per unit width

$$q = \frac{Q}{B} = yv = 1.5 \times 1.65 = 2.475 \text{ m}^3/\text{sec}/\text{m}$$

Critical depth,

$$y_c = \left(\frac{q^2}{g} \right)^{\frac{1}{3}} = \left(\frac{2.475^2}{9.81} \right)^{\frac{1}{3}} = 0.855 \text{ m}$$

Maximum height of hump,

$$z_{\max} = E_1 - E_2 = E_1 - E_C$$

$$\Rightarrow z_{\max} = \left(y_1 + \frac{v_1^2}{2g} \right) - \frac{3}{2} y_c$$

$$\Rightarrow z_{\max} = \left(1.5 + \frac{1.65^2}{2 \times 9.81} \right) - \frac{3}{2} \times 0.855$$

$$\Rightarrow z_{\max} = 0.3563 \text{ m}$$

Q.1 (c) Solution:

(i) Static Head, $H_s = 35 \text{ m}$

$$\text{Frictional head loss, } H_f = \frac{f_s L_s V_s^2}{2g d_s} + \frac{f_d L_d V_d^2}{2g d_d}$$

$$\Rightarrow H_f = \frac{0.025 \times 5 \times \left[\frac{Q}{\frac{\pi}{4} \times 0.30^2} \right]^2}{2 \times 9.81 \times 0.30} + \frac{0.026 \times 1500 \times \left[\frac{Q}{\frac{\pi}{4} \times 0.22^2} \right]^2}{2 \times 9.81 \times 0.22}$$

$$\Rightarrow H_f = 4.25035 Q^2 + 6252.769 Q^2$$

$$\Rightarrow H_f = 6257.019 Q^2$$

At the operating point,

$$\text{Pump Head} = \text{System Head}$$

$$\Rightarrow 100 - 5500 Q^2 = 35 + 6257.019 Q^2$$

$$\Rightarrow Q = 0.07435 \text{ m}^3/\text{sec} \simeq 74.35 \text{ l/sec}$$

(ii) Operating Head, $H = 100 - 5500 Q^2 = 100 - 5500 (0.07435)^2$
 $= 69.596 \text{ m} \simeq 69.6 \text{ m}$

(iii) Power required, $P_s = \frac{1}{\eta_0} \times \rho g Q H$
 $= \frac{1}{0.80} \times 9.81 \times 10^3 \times 0.07435 \times 69.6 \text{ W}$
 $= 63.45 \text{ kW}$

Q.1 (d) Solution:

Given:

$$\text{BOD} = 300 \text{ mg/l}$$

$$Q_o = 5000 \text{ m}^3/\text{day}$$

$$R_1 = R_2 = 1$$

$$f = 0.9 \text{ (treatability factor)}$$

$$V_1 = 7500 \text{ m}^3$$

$$V_2 = 5000 \text{ m}^3$$

$$\text{Total BOD load in first stage} = \frac{300 \times 10^3 \times 5000}{10^6} = 1500 \text{ kg/day}$$

$$\text{Recirculation factor, } F_1 = \frac{1 + R_1}{[1 + (1 - f_1)R_1]^2} = \frac{1 + 1}{[1 + (0.1)1]^2} = 1.653$$

$$\text{Efficiency of first stage, } \eta_1 = \frac{100}{1 + 0.44 \sqrt{\frac{\text{BOD}_1}{V_1 \times F_1}}}$$

$$\Rightarrow \eta_1 = \frac{100}{1 + 0.44 \sqrt{\frac{1500}{7500 \times 1.653}}}$$

$$\Rightarrow \eta_1 = 86.73\%$$

$$\text{Total BOD load in second stage} = 1500 \times (1 - 0.8673) = 199.05 \text{ kg/day}$$

$$\therefore R_1 = R_2 = 1 \text{ (Recirculation ratio)}$$

$$\therefore F_1 = F_2 = 1.653 \text{ (Recirculation factor)}$$

Organic loading rate on second stage filter,

$$\text{OLR}_2 = \frac{\text{BOD in kg/day}}{\text{Volume}} = \frac{199.05}{5000} = 0.0398 \text{ kg/m}^3/\text{d}$$

$$\therefore \text{Efficiency of second stage, } \eta_2 = \frac{100}{1 + \frac{0.44}{(1 - 0.8673)} \sqrt{\frac{0.0398}{1.653}}}$$

$$\Rightarrow \eta_2 = 66.03\%$$

Overall efficiency

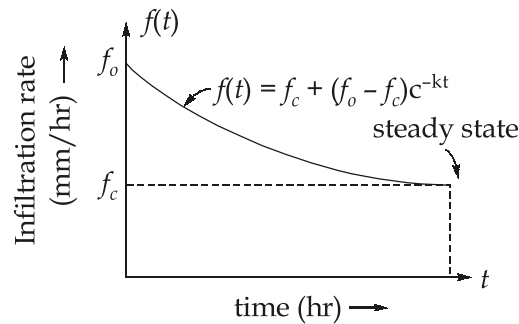
$$1 - \eta_{\text{overall}} = (1 - \eta_1)(1 - \eta_2)$$

$$\Rightarrow 1 - \eta_{\text{overall}} = (1 - 0.8673)(1 - 0.6603)$$

$$\Rightarrow \eta_{\text{overall}} = 0.9549 \simeq 95.5\%$$

Q.1 (e) Solution:

Given: Horton's infiltration equation: $f(t) = 5 + 12 e^{-2.5 t}$... (i)



Infiltration rate at any time t is given by $f(t) = f_c + (f_o - f_c)e^{-kt}$... (ii)

Comparing equation (i) and (ii)

$$f_c = 5 \text{ mm/hr}$$

$$k = 2.5 \text{ per hr}$$

$$f_o - f_c = 12$$

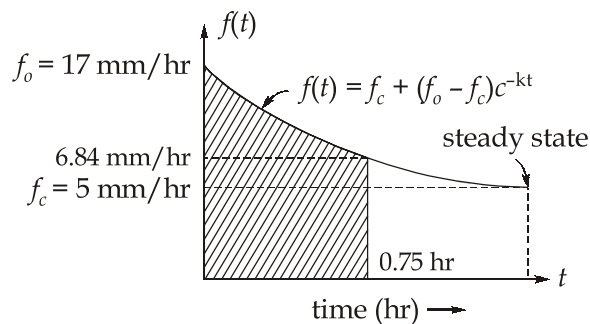
$$\Rightarrow f_o = 12 + f_c = 12 + 5 = 17 \text{ mm/hr}$$

Infiltration rate at first 45 min i.e. 0.75 hr is,

$$f(t = 0.75 \text{ hr}) = 5 + 12 e^{-2.5 \times 0.75} = 6.84 \text{ mm/hr}$$

$$\therefore f(t = 0.75 \text{ hr}) < f_o$$

\therefore Steady state is not attained



Depth of infiltration for first 45 min is,

$$I_{45 \text{ min}} = \int_0^{0.75} (5 + 12 e^{-2.5t}) dt$$

$$\Rightarrow I_{45 \text{ min}} = \left(5t - \frac{12 e^{-2.5t}}{2.5} \right)_0^{0.75}$$

$$\Rightarrow I_{45 \text{ min}} = 5 \times 0.75 - \frac{12}{2.5} \left(e^{-2.5 \times 0.75} - 1 \right)$$

$$\Rightarrow I_{45 \text{ min}} = 7.81 \text{ mm}$$

Ans.

Average infiltration rate for first 75 min i.e. 1.25 hr

$$\begin{aligned} (i_{\text{avg}})_{1.25 \text{ hr}} &= \frac{\int_0^{1.25} f(t) dt}{\int_0^{1.25} 1 dt} = \frac{\int_0^{1.25} (5 + 12e^{-2.5t}) dt}{1.25} \\ &= \frac{1}{1.25} \left(5t - \frac{12}{2.5} e^{-2.5t} \right)_0^{1.25} \\ &= \frac{1}{1.25} \left[5 \times 1.25 - \frac{12}{2.5} (e^{-2.5 \times 1.25} - 1) \right] \\ &= 8.67 \text{ mm/hr} \end{aligned}$$

Ans.

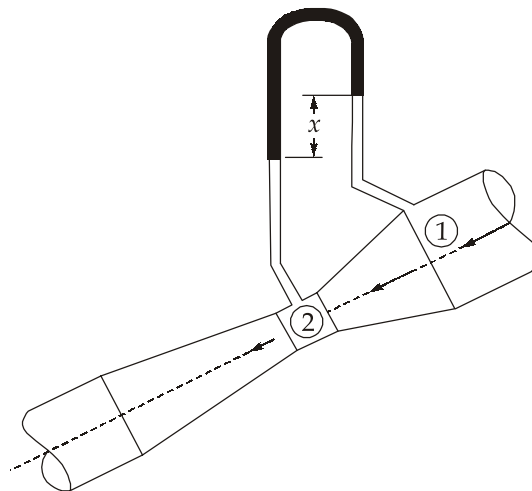
Q.2 (a) Solution:

(i)

Given: Diameter at inlet; $D = 200 \text{ mm} = 0.2 \text{ m}$

$$\therefore \text{Cross-sectional area at inlet, } A = \frac{\pi}{4} D^2 = \frac{3.14}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$$

$$\therefore \text{Cross-sectional area at throat, } a = \frac{\pi}{4} d^2 = \frac{3.14}{4} \times (0.1)^2 = 0.00785 \text{ m}^2$$

Specific gravity of liquid used in manometer, $S = 0.75$

∴ Density of manometer liquid, $\rho = 5 \times 1000 = 0.75 \times 1000 = 750 \text{ kg/m}^3$

Manometer reading, $x = 300 \text{ mm of manometer liquid}$

$= 0.3 \text{ m of manometer liquid}$

$$\text{Difference of pressure head: } h = x \left[1 - \frac{\rho_{\text{mano}}}{\rho_{\text{pipe}}} \right] \quad \text{for } \rho_{\text{pipe}} > \rho_{\text{mano}}$$

where

ρ_{mano} = Density of liquid used in manometer

ρ_{pipe} = Density of liquid flowing through pipe

$$\therefore h = 0.3 \left[1 - \frac{750}{1000} \right] = 0.3 \times 0.25 = 0.075 \text{ m of water}$$

$$\text{Also} \quad h = \left(\frac{p_1}{\rho g} + z_1 \right) - \left(\frac{p_2}{\rho g} + z_2 \right)$$

Loss of head: $h_L = 0.3 \times \text{Kinetic head of the pipe}$

$$= 0.3 \times \frac{V_1^2}{2g}$$

Now applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\Rightarrow \left(\frac{p_1}{\rho g} + z_1 \right) - \left(\frac{p_2}{\rho g} + z_2 \right) + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + 0.3 \frac{V_1^2}{2g}$$

$$\Rightarrow h + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + 0.3 \frac{V_1^2}{2g}$$

$$\Rightarrow 0.075 + \frac{V_1^2}{2g} - 0.3 \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 0$$

$$\Rightarrow 0.075 + 0.7 \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 0 \quad \dots(i)$$

Applying the continuity equation at sections (i) and (ii), we get

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow \frac{\pi}{4} (0.2)^2 V_1 = \frac{\pi}{4} (0.1)^2 V_2$$

$$\Rightarrow V_2 = 4 V_1$$

Putting the value of V_2 in equation (i), we get

$$0.075 + 0.7 \frac{V_1^2}{2g} - \frac{(4V_1)^2}{2g} = 0$$

$$\Rightarrow 0.075 + 0.7 \frac{V_1^2}{2g} - \frac{16V_1^2}{2g} = 0$$

$$\Rightarrow 0.075 - 15.3 \frac{V_1^2}{2g} = 0$$

$$\Rightarrow 0.075 = 15.3 \frac{V_1^2}{2g}$$

$$\Rightarrow \frac{0.075 \times 9.81 \times 2}{15.3} = V_1^2$$

$$\Rightarrow V_1^2 = 0.096176$$

$$\Rightarrow V_1 = 0.310 \text{ m/s}$$

$$\therefore \text{Discharge, } Q = AV_1$$

$$Q = 0.0314 \times 0.310 = 9.734 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$\Rightarrow Q = 9.734 \text{ litre/sec}$$

(ii)

Item	Open Channel Flow	Pipe Flow
Pressure	Liquid flowing through open channel is always at atmospheric pressure.	Liquid flowing through pipe is always at a higher pressure than atmospheric pressure.
Slope	For open channel flow liquid flows under gravity, thus slope has to be provided.	The pipe flow does not require slope.
Velocity	For open channel flow maximum velocity occurs little below the free surface.	For pipe flow maximum velocity occurs at centre of pipe.
Hydraulic gradient line	Hydraulic gradient line coincides with water surface	Hydraulic gradient line does not coincide with water surface.
Shape of channel	The shape of open channel may be circular, trapezoidal, rectangular, triangular etc.	Generally the shape of a pipe is circular.
Surface roughness	Hydraulic roughness depends upon channel perimeter, depth of flow.	Roughness coefficient depends upon material of pipe.

Q.2 (b) Solution:

Dissolved oxygen content of river, $C_R = 0.95 \times 8.53 = 8.1035 \text{ mg/l} \simeq 8.1 \text{ mg/l}$

Given, dissolved oxygen content of waste water, $C_s = 0$

Waste water (sewage) flow, $Q_s = 80 \text{ MLD} = 0.926 \text{ m}^3/\text{s}$

River flow, $Q_R = 8 \text{ m}^3/\text{s}$

\therefore Dissolved oxygen content in river just below the sewer outfall is,

$$C = \frac{C_s Q_s + C_R Q_R}{Q_s + Q_R} = \frac{0(0.926) + 8.1(8)}{0.926 + 8} = 7.26 \text{ mg/l}$$

At the point of sewer outfall in river, initial dissolved oxygen deficit is,

$$D_0 = 8.53 - 7.26 = 1.27 \text{ mg/l}$$

Given, re-oxygenation constant at 20°C is, $R_{20} = 0.25 \text{ day}^{-1}$

$$\begin{aligned} \therefore \text{Re-oxygenation constant at } 23^\circ\text{C}, R_{23} &= R_{20}(1.016)^{T-20} \\ &= 0.25(1.016)^{23-20} \\ &= 0.262 \text{ day}^{-1} \end{aligned}$$

De-oxygenation constant at 23°C , $K_{23} = 0.12 \text{ day}^{-1}$

Since a minimum dissolved oxygen of 4 mg/l need to be maintained after mixing of waste water and river water i.e. critical dissolved oxygen deficit (D_C) must not exceed $(8.53 - 4) = 4.53 \text{ mg/l}$

But critical oxygen deficit is given by,

$$D_C = \frac{K}{R} L_0 (10)^{-K t_c} \quad \dots(i)$$

Where t_c = Time taken to reach critical oxygen deficit after mixing of waste water into the river water and is given as

$$t_c = \frac{1}{R - K} \log_{10} \left[\frac{R}{K} \left(\frac{K L_0 - (R - K) D_0}{K L_0} \right) \right] \quad \dots(ii)$$

In eqs. (i) and (ii), L_0 and t_c are unknown which need to be determined using trial and error.

Let

$$L_0 = 20 \text{ mg/l at } 23^\circ\text{C}$$

\therefore

$$\begin{aligned} t_c &= \frac{1}{R_{23} - K_{23}} \log_{10} \left[\frac{R_{23}}{K_{23}} \left(\frac{K_{23} L_0 - (R_{23} - K_{23}) D_0}{K_{23} L_0} \right) \right] \\ &= \frac{1}{0.262 - 0.12} \log_{10} \left[\frac{0.262}{0.12} \left(\frac{0.12(20) - (0.262 - 0.12) 4.53}{0.12(20)} \right) \right] \\ &= 1.43 \text{ days} \end{aligned}$$

\therefore

$$D_C = \frac{K_{23}}{R_{23}} L_0 (10)^{-K_{23} t_c} = \frac{0.12}{0.262} (20) (10)^{-0.12 \times 1.43} = 6.17 \text{ mg/l} > 4.53 \text{ mg/l}$$

Let

$$L_0 = 18 \text{ mg/l at } 23^\circ\text{C}$$

$$\begin{aligned}\therefore t_c &= \frac{1}{R_{23} - K_{23}} \log_{10} \left[\frac{R_{23}}{K_{23}} \left(\frac{K_{23}L_o - (R_{23} - K_{23})D_o}{K_{23}L_o} \right) \right] \\ &= \frac{1}{0.262 - 0.12} \log_{10} \left[\frac{0.262}{0.12} \left(\frac{0.12(18) - (0.262 - 0.12)4.53}{0.12(18)} \right) \right] \\ &= 1.307 \text{ days}\end{aligned}$$

$$\therefore D_C = \frac{K_{23}}{R_{23}} L_o (10)^{-K_{23}t_c} = \frac{0.12}{0.262} (18)(10)^{-0.12 \times 1.307} = 5.75 \text{ mg/l} > 4.53 \text{ mg/l}$$

Let $L_o = 10 \text{ mg/l at } 23^\circ\text{C}$

$$\begin{aligned}\therefore t_c &= \frac{1}{R_{23} - K_{23}} \log_{10} \left[\frac{R_{23}}{K_{23}} \left(\frac{K_{23}L_o - (R_{23} - K_{23})D_o}{K_{23}L_o} \right) \right] \\ &= \frac{1}{0.262 - 0.12} \log_{10} \left[\frac{0.262}{0.12} \left(\frac{0.12(10) - (0.262 - 0.12)4.53}{0.12(10)} \right) \right] \\ &= 0.04 \text{ days}\end{aligned}$$

$$\therefore D_C = \frac{K_{23}}{R_{23}} L_o (10)^{-K_{23}t_c} = \frac{0.12}{0.262} (10)(10)^{-0.12 \times 0.04} = 4.53 \text{ mg/l}$$

Thus, $L_o = 10 \text{ mg/l at } 23^\circ\text{C}$

Now, $L = L_o(1 - 10^{-Kt})$

Given, $K_{23} = 0.12 \text{ day}^{-1}$

$$\therefore K_{23} = K_{20}(1.047)^{T-20}$$

$$\Rightarrow 0.12 = K_{20}(1.047)^{23-20}$$

$$\Rightarrow K_{20} = 0.105 \text{ day}^{-1}$$

$$\therefore L = 10(1 - 10^{-0.105 \times 5}) = 7.01 \text{ mg/l}$$

Thus, $L = C = 7.01 \text{ mg/l}$

But

$$C = \frac{C_s Q_s + C_R Q_R}{Q_s + Q_R}$$

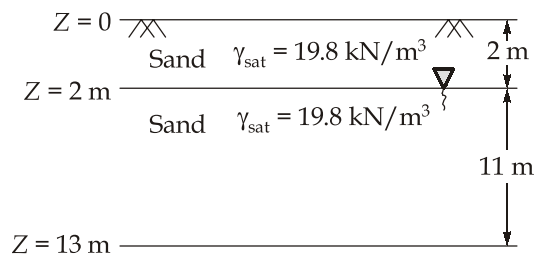
$$\Rightarrow 7.01 = \frac{C_s(0.926) + 0(8)}{0.926 + 8}$$

$$\Rightarrow C_s = 67.57 \text{ mg/l}$$

Thus, maximum allowable 5-day BOD of waste water is 67.57 mg/l at 20°C.

Q.2 (c) Solution:

(i)

Effective stress at $Z = 13 \text{ m}$ is,

$$\bar{\sigma} = \gamma_{\text{sat}} Z_1 + (\gamma_{\text{sat}} - \gamma_w) Z_2$$

$$\Rightarrow \bar{\sigma} = 19.8 \times 2 + (19.8 - 9.81) \times 11$$

$$\Rightarrow \bar{\sigma} = 149.49 \text{ kN/m}^2$$

Correction factor for overburden pressure

$$C_N = 0.9 \quad \text{(From graph given in question)}$$

SPT value at $Z = 13 \text{ m}$, $N = 26$ \therefore Corrected SPT for over burden pressure

$$N_1 = C_N \times N = 0.9 \times 26 = 23.4 > 15$$

 $\therefore N_1 > 15$, we have to also take into consideration of dilatancy correction

Now,

Corrected SPT value, after water table correction

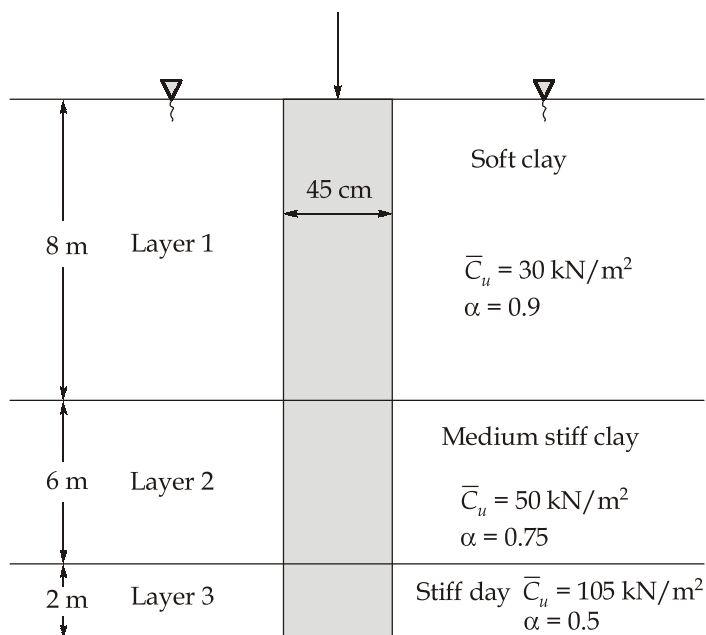
$$N_2 = 15 + \left(\frac{N_1 - 15}{2} \right)$$

$$\Rightarrow N_2 = 15 + \left(\frac{23.4 - 15}{2} \right)$$

$$\Rightarrow N_2 = 19.2 \simeq 19 \text{ (say)}$$

(ii)

Here, the pile is driven through clay soils of different consistencies.



$$Q_u = Q_{\text{end}} + Q_{\text{skin}}$$

$$\Rightarrow Q_u = A_b C N_c + \Sigma \pi d l \alpha C$$

$$\Rightarrow Q_u = \frac{\pi}{4} \times 0.45^2 \times 105 \times 9 + \pi \times 0.45 \times [8 \times 30 \times 0.9 + 6 \times 50 \times 0.75 + 2 \times 105 \times 0.5]$$

$$\Rightarrow Q_u = 150.296 + 771.889 = 922.185 \text{ kN}$$

$$Q_{\text{allowable}} = \frac{Q_u}{\text{FOS}} = \frac{922.185}{2.5} = 368.87 \text{ kN}$$

Q.3 (a) Solution:

(i)

(a) Given: For wheat:

$$\text{During base period, Duty} = 8.64 \frac{B}{\Delta} = 8.64 \times \frac{140}{39} \times 100 = 3101.54 \text{ ha/cumec}$$

$$\text{Discharge} = \frac{A}{D} = \frac{3900}{3101.54} = 1.257 \text{ cumec}$$

$$\text{During kor period Duty} = 8.64 \frac{B}{\Delta} = 8.64 \times \frac{4 \times 7}{13} \times 100 = 1860.92 \text{ ha/cumec}$$

$$\text{Discharge} = \frac{3900}{1860.92} = 2.096 \text{ cumec}$$

(b) For rice:

$$\text{During base period, Duty} = 8.64 \frac{B}{\Delta} = 8.64 \times \frac{120}{117} \times 100 = 886.15 \text{ ha/cumec}$$

$$\text{Discharge} = \frac{A}{D} = \frac{2200}{886.15} = 2.483 \text{ m}^3/\text{s}$$

$$\text{During kor period, Duty} = 8.64 \frac{B}{\Delta} = 8.64 \times \frac{2.5 \times 7}{18} \times 100 = 840 \text{ ha/cumec}$$

$$\text{Discharge} = \frac{A}{D} = \frac{2200}{840} = 2.62 \text{ m}^3/\text{s}$$

Peak discharge required will be maximum of above four values i.e.

$$Q_p = 2.62 \text{ m}^3/\text{s}$$

The kor watering comes within base period watering

$$\begin{aligned} \therefore \text{Total water required} &= V_{\text{wheat}} + V_{\text{rice}} = \{3900 \times 10^4 \times 39 \times 10^{-2} + 2200 \times 10^4 \times 117 \times 10^{-2}\} \\ &= 40.95 \times 10^6 \text{ m}^3 \end{aligned}$$

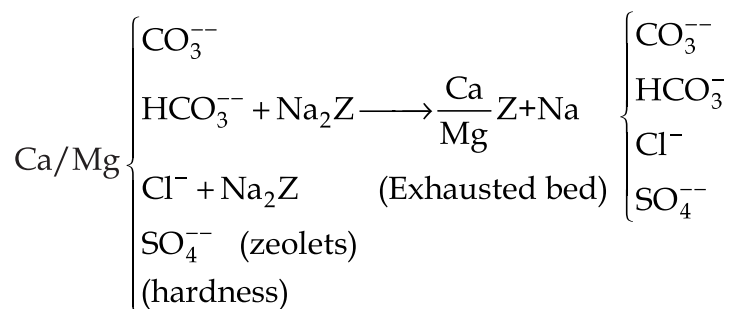
$$\begin{aligned} \therefore Q_{\text{avg}} &= \frac{\text{Total volume of water}}{\text{Total base period}} \\ &= \frac{40.95 \times 10^6}{(140 + 120) \times 24 \times 3600} \\ &= 1.823 \text{ m}^3/\text{s} \end{aligned}$$

(ii)

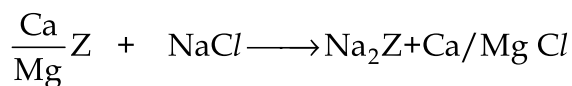
Softening of water is the process of removal of hardness causing salts from water. It is done to counter the problems like scaling of boilers, reduced soap lathering etc. caused by hardness in water.

Zeolite or ion exchange process is one of the methods of treating hardness in water.

Zeolite softener (or a cation exchange unit) resembles a sand filter in which the filtering medium is a zeolite rather than sand. The hard water enters through the top, and is evenly distributed on the entire zeolite bed. The softened water is collected through one strainers at the base. When a significant portion of the sodium in the zeolite has been replaced by calcium and magnesium, it is regenerated by first washing it with water by reversing the flow, and then treating it with 10 per cent solution of brine (NaCl). The excess brine solution retained in the zeolite after the treatment is removed by again washing it with good water. The regenerated zeolite can be used afresh for softening.



On, regeneration,



(Exhausted bed)

The advantages and disadvantages of this method are:

Advantages:

1. Water of zero hardness can be obtained, and hence, useful for specific uses in textile industries, boilers, etc.
2. The plant is compact, automatic and easy to operate.
3. No sludge is formed, and hence, there is no problem of sludge disposal.
4. The *RMO* (Running, maintenance and operation) cost is quite less.
5. It also removes ferrousions and manganese from water.
6. There is no difficulty in treating water of varying quality.
7. There is no problem of incrustation of pipes of the distribution system, as is there in the lime soda process.

Disadvantages:

1. This process is not suitable for treating highly turbid waters, because the suspended impurities get deposited around the zeolite particles, and thus cause obstruction to the working of the zeolite.
2. The process leaves sodium bicarbonate in water, which causes priming and foaming in industrial or boiler feed waters.
3. The zeolite process is costlier and unsuitable for treating waters containing iron and manganese. This is because of the fact that the iron zeolite or manganese zeolite formed during the chemical reactions, cannot be regenerated into sodium zeolite. The zeolite is thus wasted, although the iron and manganese are removed from the water.

Q.3 (b) Solution:

If H is the net head acting on the turbine then the work done by the runner per unit weight of water may be expressed as

$$\frac{V_{w1}u_1 - V_{w2}u_2}{g} = H - \text{Losses}$$

The losses may be combinedly expressed as

$$(k_1 + k_2 + k_3 + k_4) \frac{V_f^2}{2g} = \text{Losses}$$

Also $V_{f1} = V_{f2} = V_f = \text{Constant}$

Thus

$$\Rightarrow \frac{V_{w1}u_1 - V_{w2}u_2}{g} = H - (k_1 + k_2 + k_3 + k_4) \frac{V_f^2}{2g} \quad \dots(i)$$

From inlet and outlet velocity triangles

$$V_{w1} = V_f \cot \alpha$$

$$\frac{V_f}{V_{w1} - u_1} = \tan \theta$$

$$\Rightarrow \frac{V_f}{V_f \cot \alpha - u_1} = \tan \theta$$

$$\therefore u_1 = V_f (\cot \alpha - \cot \theta)$$

$$\frac{V_{f1}}{V_{w2} + u_2} = \tan \phi$$

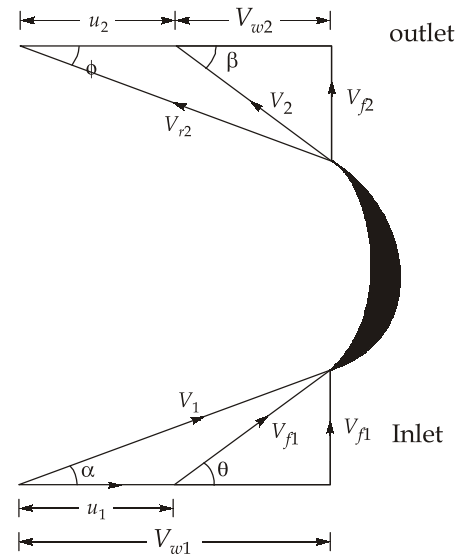
$$\Rightarrow \frac{V_f}{V_{w2} + u_2} = \tan \phi$$

Also $u_1 = \frac{\pi D_1 N}{60}; u_2 = \frac{\pi D_2 N}{60}$

$$\therefore u_2 = \left(\frac{D_2}{D_1} \right) u_1 = n u_1 = n V_f (\cot \alpha - \cot \theta)$$

Substituting this value of u_2 , we get

$$\frac{V_f}{V_{w2} + n V_f (\cot \alpha - \cot \theta)} = \tan \phi = \frac{1}{\cot \phi}$$



$$\Rightarrow V_{w_2} = V_f [\cot \phi - n(\cot \alpha - \cot \theta)]$$

Hydraulic efficiency is given by

$$\eta_h = \frac{V_{w_1} u_1 - V_{w_2} u_2}{gH}$$

From Eq. (i), we have

$$H = (V_{w_1} u_1 - V_{w_2} u_2) + (k_1 + k_2 + k_3 + k_4) \frac{V_f^2}{2}$$

Thus

$$\begin{aligned} \eta_h &= \frac{(V_{w_1} u_1 - V_{w_2} u_2)}{(V_{w_1} u_1 - V_{w_2} u_2) + (k_1 + k_2 + k_3 + k_4) \frac{V_f^2}{2}} \\ &= \frac{2}{2 + \frac{(k_1 + k_2 + k_3 + k_4) V_f^2}{(V_{w_1} u_1 - V_{w_2} u_2)}} \\ &= \frac{2}{2 + \frac{(k_1 + k_2 + k_3 + k_4) V_f^2}{[V_f \cot \alpha \times V_f (\cot \alpha - \cot \theta)] - [V_f \cot \phi - n V_f (\cot \alpha - \cot \theta)] \times n V_f (\cot \alpha - \cot \theta)}} \\ &= \frac{2}{2 + \frac{(k_1 + k_2 + k_3 + k_4) V_f^2}{V_f^2 [\cot \alpha (\cot \alpha - \cot \theta) - n (\cot \alpha - \cot \theta) (\cot \phi - n (\cot \alpha - \cot \theta))]} \\ \therefore \eta_h &= \frac{2}{2 + \frac{(k_1 + k_2 + k_3 + k_4)}{(\cot \alpha - \cot \theta) \left\{ \cot \alpha (1 + n^2) - n (\cot \phi + n \cot \theta) \right\}}} \end{aligned}$$

Q.3 (c) Solution:

(i)

The mechanisms of coagulation which are thought to occur are as follows:

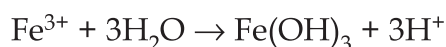
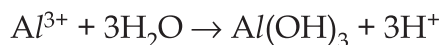
1. Ionic layer compression:

- The quantity of ions in the water surrounding a colloid has an effect on the decay function of the electrostatic potential.
- A high ionic concentration compresses the layers composed predominantly of counter ions towards the surface of colloid.

- If this layer is sufficiently compressed, then the Van-der Waal's force will be predominant across the entire area of influence, so that the net force will be attractive and no energy barriers will exist.
- Although coagulants such as aluminium and ferric salts used in water treatment ionize, at the concentration commonly used they would not increase the ionic concentration sufficiently to affect ionic layer compression.

2. Adsorption and charge neutralization:

- The nature rather than quantity of the ions is of prime importance in the theory of adsorption and charge neutralization.
- The ionization of $Al_2(SO_4)_3$ and $FeCl_3$ in water produces SO_4^{2-} and Cl^- ions along with Al^{3+} and Fe^{3+} ions. The Al^{3+} and Fe^{3+} cations react immediately with water to form a variety of aquometallic ions and hydrogen.



- The aquometallic ions thus formed become part of the ionic cloud surrounding colloid and because they have a great affinity for surfaces they are adsorbed onto the surface on the colloid where they neutralize the surface charge.
- Once the surface charge has been neutralized, the ionic cloud dissipates and the electrostatic potential disappears so that contact occurs freely.

3. Sweep coagulation:

- The last product formed in the hydrolysis of alum is $Al(OH)_3$.
- The $Al(OH)_3$ formed is amorphous and gelatinous flocs that are heavier than water and settle by gravity.
- Colloids may become entrapped in a floc as it is formed, or they may become enmeshed by its sticky surface as the flocs settle.
- The process by which colloids are swept from suspension in this manner is known as sweep coagulation.

4. Interparticle bridging:

- Large molecules may be formed when aluminium or ferric salts dissociate in water.
- Synthetic polymers may also be used instead of, or in addition to, metallic salts which may be linear or branched or grafted and are highly surface reactive.
- Several colloids may get attached to one polymer and several of the polymer-colloid groups may become enmeshed resulting in a settleable mass.
- In addition to the adsorption forces, charges on the polymer may assist in the coagulation process.

- Metallic polymers formed by addition of aluminium or ferric salts are positively charged while synthetic polymers may carry positive or negative charges or may be neutral.
- Judicious choice or appropriate charges may do much to enhance the effectiveness of coagulation.

(ii)

Given:

$$\text{Discharge, } Q = 19000 \text{ m}^3/\text{d}$$

$$\text{Overflow rate, } V_s = 19 \text{ m/d}$$

$$\text{Depth, } D = 3.2 \text{ m}$$

Now, required plane area of tank,

$$A = \frac{Q}{V_s} = \frac{19000}{19} = 1000 \text{ m}^2$$

$$\text{Area of each tank} = \frac{1000}{4} = 250 \text{ m}^2$$

Assuming length to width ratio as 4 : 1

$$\therefore L \times B = 250$$

$$\Rightarrow 4B^2 = 250$$

$$\Rightarrow B = 7.9 \text{ m}$$

$$\therefore \text{Length, } L = 31.6 \text{ m, } B = 7.90 \text{ m}$$

$$\text{Retention time, } t_R = \frac{D}{V_s} = \frac{3.2}{19} \times 24 = 4.04 \text{ hours}$$

$$\text{Horizontal velocity, } V_f = \frac{L}{t_R} = \frac{31.6}{4.04} \times 24 = 187.72 \text{ m/day}$$

When the weir occupies the width of the tank,

$$V_o = \frac{Q}{B} = \frac{19000}{7.9} = 2405.06 \text{ m}^3/\text{m/day}$$

Q.4 (a) Solution:

Given:

$$P_1 = 3.5 \text{ cm, } P_2 = 4.5 \text{ cm, } P_3 = 1.8 \text{ m}$$

$$\phi - \text{index} = 5 \text{ mm/hr} = 0.5 \text{ cm/hr}$$

$$t_1 = 2 \text{ hr, } t_2 = 2 \text{ hr, } t_3 = 2 \text{ hr}$$

 \therefore

$$R_1 = P_1 - \phi t_1$$

$$\therefore \begin{aligned} R_1 &= 3.5 - 0.5 \times 2 = 2.5 \text{ cm} \\ R_2 &= 4.5 - 0.5 \times 2 = 3.5 \text{ cm} \\ R_3 &= 1.8 - 0.5 \times 2 = 0.8 \text{ cm} \end{aligned}$$

The values of R_1 , R_2 and R_3 are multiplied with the ordinates of unit hydrograph and they are added with a lag of 2 hr

The calculations are tabulated below:

Time (hr) (1)	Runoff (m ³ /s) (2)	$R_1 U$ (3)	$R_2 U$ (4)	$R_3 U$ (5)	DRH = $R_1 U + R_2 U + R_3 U$ (6) = (3) + (4) + (5)
0	0	0	-	-	0
1	2.0	5	-	-	5
2	2.5	6.25	0	-	6.25
3	3.2	8	7	-	15
4	3.6	9	8.75	0	17.75
5	4.0	10	11.2	1.6	22.8
6	4.2	10.5	12.6	2	25.1
7	3.8	9.5	14	2.56	26.06
8	3.4	8.5	14.7	2.88	26.08
9	3.0	7.5	13.3	3.2	24
10	2.4	6	11.9	3.36	21.26
11	2.0	5	10.5	3.04	18.54
12	1.5	3.75	8.4	2.72	14.87
13	1.1	2.75	7	2.4	12.15
14	0.6	1.5	5.25	1.92	8.67
15	0.1	0.15	3.85	1.6	5.6
16	0	0	1.5	1.2	2.7
17			0.35	0.88	1.23
18			0	0.48	0.48
19				0.08	0.08
20				0	0

Q.4 (b) Solution:

Total hardness to be left = 78 mg/l

Carbonate hardness to be left = 30 mg/l

So, non carbonate hardness to be left = 78 - 30 = 48 mg/l

Non carbonate hardness present in raw water = 84 mg/l

So, non carbonate hardness of raw water to be removed = (84 - 48) = 36 mg/l.

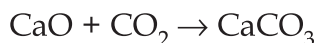
Now, lime removes carbonate hardness, free carbon dioxide and magnesium from water while soda removes non-carbonate hardness.

Requirement of lime

(a) For free CO_2

Molecular mass of $\text{CO}_2 = 44 \text{ gm}$

Molecular mass of $\text{CaO} = 56 \text{ gm}$



$$\begin{aligned} \text{So, } 4 \text{ mg/l of } \text{CO}_2 \text{ requires} &= \frac{56}{44} \times 4 \\ &= 5.091 \text{ mg/l of CaO} \end{aligned}$$

$$\begin{aligned} \text{So, CaO required for 1 Million litres of water} \\ &= 5.091 \times 10^{-6} \times 1 \times 10^6 \\ &= 5.091 \text{ kg} \end{aligned}$$

(b) For carbonate hardness which is equal to alkalinity.

Molecular mass of $\text{CaCO}_3 = 100 \text{ gm}$

Molecular mass of $\text{CaO} = 56 \text{ gm}$

Carbonate hardness of 100 mg/l requires $= 56 \text{ mg/l}$ of CaO

$$\begin{aligned} \text{Carbonate hardness of } 65 \text{ mg/l requires} &= \frac{56}{100} \times 65 \\ &= 36.40 \text{ mg/l of CaO} \end{aligned}$$

$$\begin{aligned} \therefore \text{CaO required for 1 million litre of water} &= 36.40 \times 10^{-6} \times 1 \times 10^6 \\ &= 36.40 \text{ kg} \end{aligned}$$

(c) For magnesium

24 mg/l of magnesium requires $= 56 \text{ mg/l}$ of CaO

$$\begin{aligned} \therefore 13 \text{ mg/l of magnesium requires} &= \frac{56}{24} \times 13 \text{ of CaO} \\ &= 30.33 \text{ mg/l of CaO} \end{aligned}$$

$$\begin{aligned} \therefore \text{CaO required for 1 million litre of water} &= 30.33 \times 10^{-6} \times 1 \times 10^6 \\ &= 30.33 \text{ kg} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total CaO required} &= 5.091 + 36.40 + 30.33 \\ &= 71.821 \text{ kg} \end{aligned}$$

Now, 56 gm of pure lime (CaO) is equivalent to 74 gm of hydrated lime (Ca(OH)_2)

Hence, hydrated lime required per million litre of raw water

$$= 71.821 \times \frac{74}{56} = 94.91 \text{ kg}$$

Requirement of soda

Non carbonate hardness of 100 mg/l of CaCO_3 requires = 106 mg/l of Na_2CO_3

$$\begin{aligned} \text{Non carbonate hardness of 42 mg/l of } \text{CaCO}_3 \text{ requires} &= \frac{106}{100} \times 42 \\ &= 44.52 \text{ mg/l of } \text{Na}_2\text{CO}_3 \end{aligned}$$

$$\begin{aligned} \therefore \text{ Soda required for 1 mL of water} &= 44.52 \times 10^{-6} \times 1 \times 10^6 \\ &= 44.52 \text{ kg} \end{aligned}$$

Q.4 (c) Solution:

Given:

$$\mu_1 = \mu_2 = (\rho\nu) = (0.95 \times 1 \times 9) \text{ g/cm.s} = 8.55 \text{ g/cm.s};$$

$$L_1 = L_2 = 1200 \text{ m}; D_1 = 10 \text{ cm} = 0.10 \text{ m}; D_2 = 12 \text{ cm} = 0.12 \text{ m};$$

$$\gamma = 0.95 \times 9810 \text{ N/m}^3 = 9319.5 \text{ N/m}^3$$

Assuming the flow in each pipe to be laminar, the head loss is given by

$$h_f = \frac{32\mu VL}{\gamma D^2} = \frac{128\mu QL}{\gamma \pi D^4}$$

Let Q_1 and Q_2 be the flow rates through each of the two pipes, which being in parallel, we have

$$h_{f_1} = h_{f_2}$$

$$\Rightarrow \frac{128\mu_1 Q_1 L_1}{\gamma \pi D^2} = \frac{128\mu_2 Q_2 L_2}{\gamma \pi D^2}$$

$$\Rightarrow \frac{128 \times 8.55 \times Q_1 \times 1200}{9319.5 \times \pi \times (0.10)^4} = \frac{128 \times 8.55 \times Q_2 \times 1200}{9319.5 \times \pi \times (0.12)^4}$$

$$\Rightarrow \frac{Q_1}{(0.10)^4} = \frac{Q_2}{(0.12)^4}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \left(\frac{0.10}{0.12} \right)^4 = 0.4823$$

$$\Rightarrow Q_1 = 0.4823 Q_2$$

$$\text{But } Q_1 + Q_2 = 15 \text{ l/s}$$

\therefore $Q_1 = 4.881 \text{ l/s}$; and $Q_2 = 10.119 \text{ l/s}$ Ans.
 Reynolds number is given by

$$R_e = \frac{VD}{\nu}$$

For pipe of diameter 10 cm,

$$V = \frac{4.881 \times 10^3}{(\pi/4)(10)^2} = 62.147 \text{ cm/s}$$

$$\therefore R_e = \frac{62.146 \times 10}{9} = 69.051$$

Similarly for pipe of diameter 12 cm,

$$V = \frac{10.119 \times 10^3}{(\pi/4)(12)^2} = 89.472 \text{ cm/s}$$

$$\therefore R_e = \frac{89.472 \times 12}{9} = 119.296$$

Thus the assumption of laminar flow in both the pipes is correct.

$$\therefore \text{Head loss} \quad h_f = \frac{128\mu QL}{\omega\pi D^4} = \frac{128\mu QL}{\rho g\pi D^4} = \frac{128\nu QL}{g\pi D^4}$$

$$h_f = \frac{128 \times 9 \times 4.881 \times 1200 \times 10^5}{981 \times \pi \times (10)^4}$$

$$= 2.1893 \times 10^4 \text{ cm} = 218.93 \text{ m}$$

Power of the pump required, $P = \gamma Q h_f$

$$\Rightarrow P = 0.95 \times 1000 \times 9.81 \times 15 \times 10^{-3} \times 218.93 \text{ W}$$

$$\Rightarrow P = 30.6 \text{ kW} \quad \text{Ans.}$$

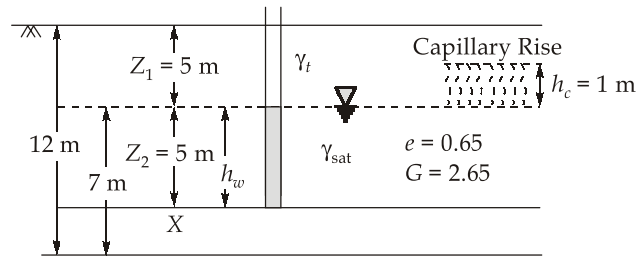
Section - B

Q.5 (a) Solution:

Height of sand layer above water table = $Z_1 = 5 \text{ m}$

Height of saturated layer = $12 - 5 = 7 \text{ m}$

Depth of point X, where pressure is to be computed = 10 m



Height of saturated layer above $X = Z_2 = 10 - 5 = 5 \text{ m}$

Now,
$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.65}$$

$$\Rightarrow \gamma_d = 15.755 \text{ kN/m}^3$$

For sand above water table, $eS = Gw$

$$\Rightarrow 0.65 \times 0.5 = 2.65w$$

$$\Rightarrow w = 0.1226$$

$$\therefore \gamma_1 = \gamma_d(1+w)$$

$$= 15.755(1+0.1226) = 17.687 \text{ kN/m}^3$$

For saturated sand below water table

$$eS = Gw$$

$$\Rightarrow 0.65 \times 1 = 2.65w_{sat}$$

$$\Rightarrow w_{sat} = 0.245$$

$$\gamma_{2 \text{ sat}} = \gamma_d(1+w_{sat})$$

$$= 15.755(1+0.245) = 19.615 \text{ kN/m}^3$$

$$\gamma_{2 \text{ sub}} = 19.615 - 9.81 = 9.805 \text{ kN/m}^3$$

Total stress at X

$$\sigma = Z_1\gamma_1 + Z_2\gamma_{2 \text{ sat}} = 5 \times 17.687 + 5 \times 19.615$$

$$\Rightarrow \sigma = 186.51 \text{ kN/m}^2$$

Pore water pressure, $u = h_w\gamma_w = 5 \times 9.81 = 49.05 \text{ kN/m}^2$

Effective stress, $\sigma' = \sigma - u = 186.51 - 49.05 = 137.46 \text{ kN/m}^2$

Effective stress at X after capillary rise

$$(\sigma')_{hc} = 4\gamma_1 + (5+1)\gamma_{2 \text{ sub}} + h_c\gamma_w$$

$$= 4 \times 17.687 + 6 \times 9.805 + 1 \times 9.81$$

$$(\sigma')_{hc} = 139.388 \text{ kN/m}^2$$

Increase in effective stress $= 139.388 - 137.46 = 1.928 \text{ kN/m}^2$

Q.5 (b) Solution:**(i)**

Given:

 $A = \text{Area of cross-section of rail} = 60 \text{ cm}^2$ $E = \text{Modulus of elasticity of steel}$

$$= 2.15 \times 10^6 \text{ kg/cm}^2$$

$$\alpha = 1.12 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$$

Rise in temperature, $\Delta T = 35^\circ\text{C}^{-1}$

Resistance to track movement = 710 kg/km

 \therefore Force required to prevent expansion due to temperature is given by

$$F = \sigma_{\text{thermal}} \times A = E\alpha\Delta TA$$

$$\Rightarrow F = 2.15 \times 10^6 \times 1.12 \times 10^{-5} \times 35 \times 60$$

$$\Rightarrow F = 50568 \text{ kg}$$

Length of track required to overcome temperature stress;

$$L_t = \frac{50568}{710} = 71.22 \text{ km}$$

To prevent creep for equilibrium, the length of welded track required

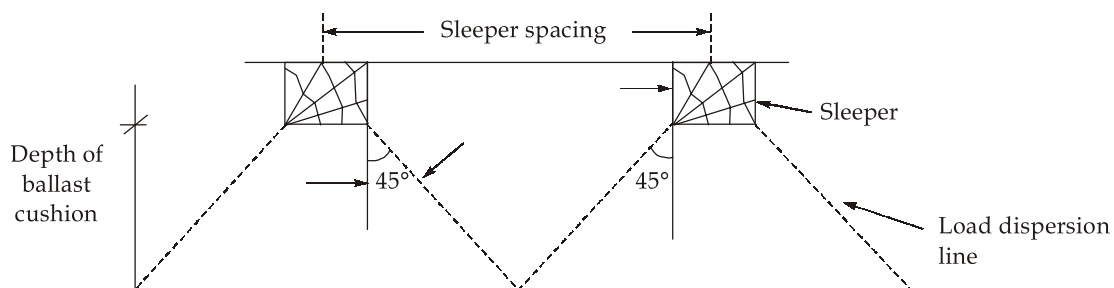
$$= 2 \times L_t = 2 \times 71.22 = 142.44 \text{ km}$$

(ii)**Minimum depth of ballast:**

- For the even distribution of load on the formation, depth of the ballast is determined by the following formulae:

$$\text{Sleeper spacing} = \text{Width of the sleeper} + 2 \times \text{Depth of ballast}$$

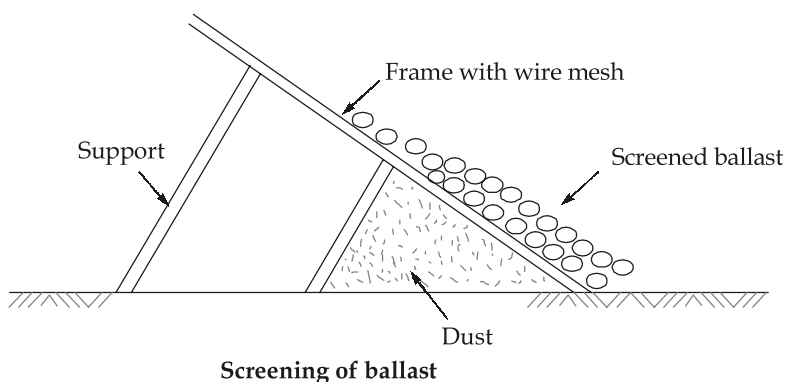
$$\therefore \text{Depth of ballast} = \frac{\text{Sleeper spacing} - \text{Width of sleeper}}{2}$$



- Load dispersion lines should not overlap each other to ensure the safe transmission of load to the formation, the depth of the ballast should be accordingly chosen.
- A minimum cushion of 15-20 cm of ballast below the sleeper bed is normally prescribed in Indian Railways.

Screening of ballast:

1. The ballast used on the railway track is to be renewed from time to time due to the following reasons:
 - Ballast gets powdered due to hammering of wheels and fills voids in the ballast layer forming impermeability.
 - The ballast under the sleepers is constantly pressing the formation. This reduces the quantity of the ballast and also the elasticity of the railway track is affected.
2. In order to remove these defects, the ballast is cleaned at regular intervals by means of screening. The process of screening is carried out as follow:
 - The surface ballast is generally clean and hence it is removed by means of ballast forks. It is then carefully stored separately at suitable pace.
 - The dirty ballast is then made loose by means of equipment such as picks.
 - The frames, about 150 cm × 120 cm with expanded metal mesh, are put at convenient angle as shown in figure.
 - The dirty ballast is thrown on to the mesh to separate the dirt and aggregate ballast.



The required quantity of additional ballast is added to the screened ballast so as to make up the deficiency and then the railway track is packed with this ballast as before.

Q.5 (c) Solution:

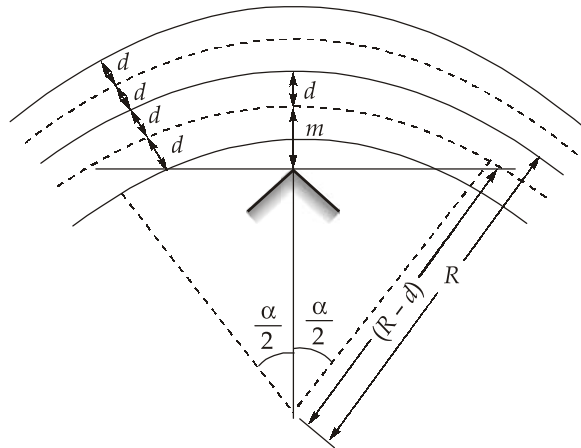
Given:

Radius curve, $R = 160$ m

Number of lanes, $n = 2, f = 0.40$

Width of lane = 3.5 m

$$d = \frac{3.5}{2} = 1.75 \text{ m}$$



Set back distance from centerline of inner lane

$$m = 10 - d = 10 - 1.75 = 8.25 \text{ m}$$

Set back distance from center line of inner lane

$$m = (R - d) - (R - d) \cos\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow 8.25 = (160 - 1.75) - (160 - 1.75) \cos\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow 158.25 \cos\left(\frac{\alpha}{2}\right) = 150$$

$$\Rightarrow \left(\frac{\alpha}{2}\right) = \cos^{-1}(0.947867)$$

$$\Rightarrow \left(\frac{\alpha}{2}\right) = 18.582^\circ$$

$$\Rightarrow \left(\frac{\alpha}{2}\right) = 18.582^\circ \times \frac{\pi}{180} \text{ radian}$$

$$\Rightarrow \left(\frac{\alpha}{2}\right) = 0.32432 \text{ radian}$$

$$\Rightarrow \frac{S}{2(R - d)} = 0.32432 \text{ radian}$$

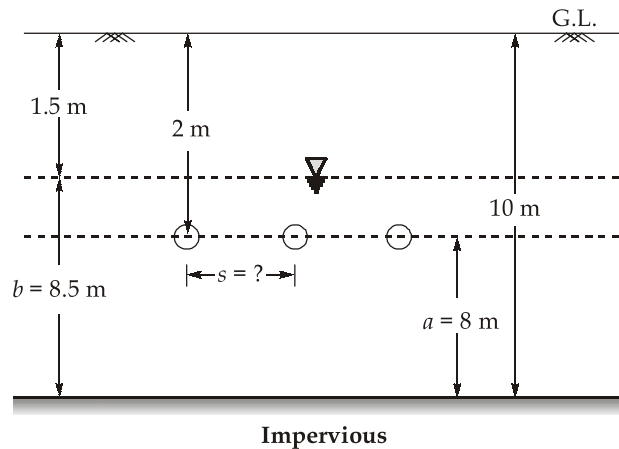
$$\Rightarrow S = 102.647 \text{ m}$$

$$\text{Now, } SSD = V \times 2.5 + \frac{V^2}{2 \times 9.81 \times 0.40}$$

$$\Rightarrow 0.1274V^2 + 2.5V - 102.647 = 0$$

$$\therefore V = 20.22 \text{ m/sec} = 72.792 \text{ km/hr}$$

Q.5 (d) Solution:



Here,

$$a = 10 - 2 = 8 \text{ m}$$

$$b = 10 - 1.5 = 8.5 \text{ m}$$

$$k = 6 \times 10^{-6} \text{ m/sec}$$

$$P_{\text{avg annual}} = 96 \text{ cm} = 0.96 \text{ m}$$

The infiltration discharge, into the ground which should be removed by the drains (in m^3/sec) is given by

$$q = \frac{4k}{S}(b^2 - a^2) \quad \dots(1)$$

where

S = Spacing of the drain pipes

Given that 1% of the average annual rainfall is removed by the drains in 24 hr.

$$q = \frac{P_{\text{avg}} \times 1\%}{24 \times 60 \times 60} \times (S \times 1) \text{ m}^3/\text{sec/m}$$

$$\Rightarrow q = \frac{0.01 \times 0.96}{24 \times 60 \times 60} \times S$$

From equation (i)

$$\frac{0.01 \times 0.96}{24 \times 60 \times 60} \times S = \frac{4 \times 6 \times 10^{-6}}{S} (8.5^2 - 8^2)$$

$$\Rightarrow S^2 = 1782$$

$$\Rightarrow S = 42.21 \text{ m c/c}$$

Q.5 (e) Solution:

Given: Focal length of camera, $f = 30 \text{ mm}$

Flying height, $H = 1200 \text{ m}$

$$\text{Now, } X_A = \left(\frac{H - h_a}{f} \right) x_a = \left(\frac{1200 - 250}{30} \right) \times (20.5)$$

$$\Rightarrow X_A = 649.17 \text{ m}$$

$$Y_A = \left(\frac{H - h_a}{f} \right) y_a = \left(\frac{1200 - 250}{30} \right) \times (15.5)$$

$$\Rightarrow Y_A = 490.83 \text{ m}$$

$$X_B = \left(\frac{H - h_b}{f} \right) x_b = \left(\frac{1200 - 210}{30} \right) \times (-15.5)$$

$$\Rightarrow X_B = -511.5 \text{ m}$$

$$Y_B = \left(\frac{H - h_b}{f} \right) y_b = \left(\frac{1200 - 210}{30} \right) \times (-20.5)$$

$$Y_B = -676.5 \text{ m}$$

Now, horizontal distance between points A and B on ground is

$$AB = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}$$

$$\Rightarrow AB = \sqrt{(649.17 + 511.5)^2 + (490.83 + 676.5)^2}$$

$$\Rightarrow AB = 1646.15 \text{ m}$$

$$\begin{aligned} \text{Average scale} &= \frac{f}{H - h_{avg}} \\ &= \frac{30 \times 10^{-3} \text{ m}}{\left[1200 - \left(\frac{250 + 210}{2} \right) \right] \text{ m}} = \frac{1}{32333.33} \end{aligned}$$

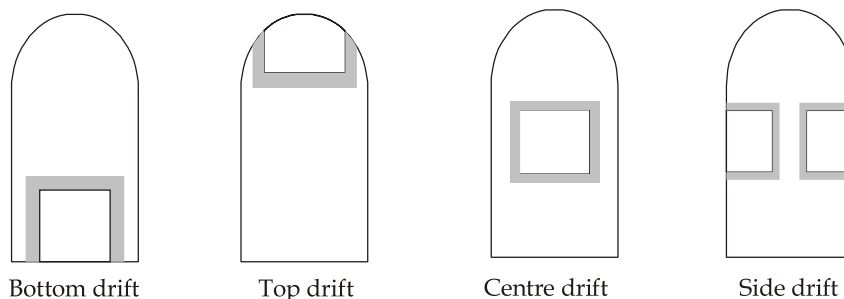
Q.6 (a) Solution:

(i)

Drift method of tunnelling: It consists of driving a small sized heading, centrally at top or bottom of the face, which is enlarged, by widening and fencing. The top drift method is popular and the main operations involved are:

- Boring or blasting the top centre heading or drift, end to end
- Widening and enlarging the drift.
- Benching in stages.

The sequence is illustrated below:



(ii)

Break water: The protective barrier constructed to enclose harbours and to keep the harbour waters undisturbed by the effect of heavy and strong seas is called breakwater. Such a construction makes it possible to use the area enclosed as a safe anchorage for ships and to facilitate loading of cargo in comparatively calm waters. Sometimes the inner side of a breakwater is constructed as a quay and cargo handling and is known as a 'Mole'.

Classification of Breakwaters: Breakwaters are mainly classified as

1. Heap or Mound breakwater
2. Mound with super structure
3. Upright wall breakwater.

(iii)

Basic Runway Length: The length of runway based on the following assumptions is known as the basic runway length:

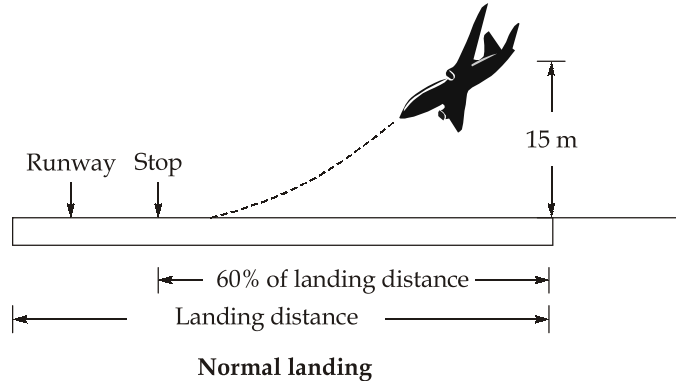
- No wind is blowing on the runway.
- The aircraft is loaded to its full loading capacity.
- The airport is situated at sea level.
- There is no wind blowing on the way to the destination.
- The runway is leveled in the longitudinal direction or in other words, it has zero effective gradient.
- The standard temperature is maintained along the way.
- The standard temperature of 15°C exists at the airport.

Following are the three cases to be considered:

1. Normal Landing:

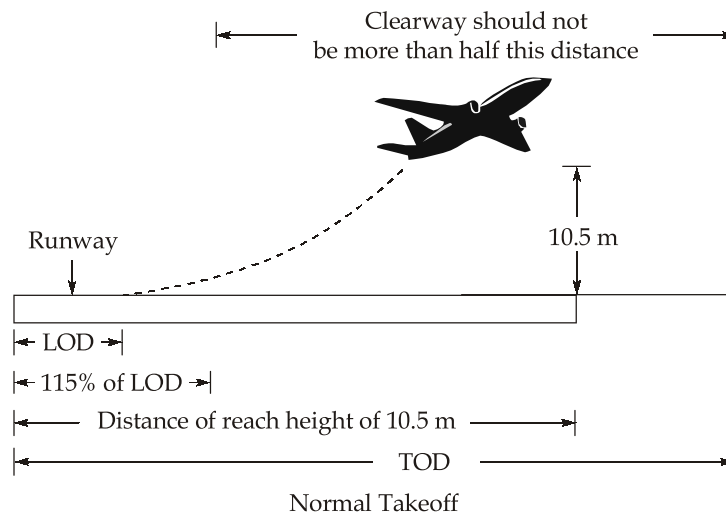
- The aircraft should come to a stop within 60 percent of the landing distance assuming that the pilot makes an approach at the proper speed and crosses the threshold of the runway at a height of 15 m.

- The beginning of the runway portion to be used as landing is known as the threshold.
- The runway of full strength pavement is provided for the entire landing distance.



2. Normal Takeoff:

- The takeoff distance (TOD) must be, for a specific weight of aircraft, 115 percent of the actual distance the aircraft uses to reach a height of 10.5 m.
- The distance to reach the height of 10.5 m should be equal to 115 percent of the lift-off distance (LOD).
- The normal takeoff requires a clearway which is defined as an area beyond the runway not less than 150 m wide, centrally located about the extended centre line of the runway and under the control of the airport authorities.
- It is expressed in terms of a clearway plane extending from the end of the runway with an upward slope not exceeding 1.25 percent.
- It is to be seen that the clearway is free from any obstruction. The clearway should not be more than one half the distance between 115 percent of the LOD and TOD.



$$\Rightarrow \frac{\sin(90^\circ + \alpha + \theta)}{\sin\left[90^\circ - \left(\alpha + \frac{\Delta}{2}\right)\right]} = \frac{R \sec \frac{\Delta}{2}}{R}$$

$$\Rightarrow \frac{\cos(\alpha + \theta)}{\cos\left(\alpha + \frac{\Delta}{2}\right)} = \frac{R \sec \frac{\Delta}{2}}{R}$$

On substituting values, of α and s ,

$$\frac{\cos(34^\circ 36' + \theta)}{\cos(34^\circ 36' + 30^\circ 06')} = \sec(30^\circ 06')$$

$$\Rightarrow \cos(34^\circ 36' + \theta) = 0.49397$$

$$\Rightarrow \theta = 25.7982^\circ = 25^\circ 47' 53.6''$$

From figure,

$$DD_1 = MT_1 = BD \sin \alpha$$

Also,

$$MT_1 = OT_1 - OM$$

{In DMO, $OM = R \cos \theta$ }

$$\Rightarrow MT_1 = R - R \cos \theta$$

$$\Rightarrow R = \frac{MT_1}{1 - \cos \theta}$$

$$\therefore R = \frac{BD \sin \alpha}{1 - \cos \theta}$$

On substituting values of α and θ ,

$$\begin{aligned} R &= \frac{79.44 \times \sin(34^\circ 36')}{1 - \cos(25^\circ 47' 53.6'')} \\ &= 452.599 \text{ m} \simeq 452.60 \text{ m} \end{aligned}$$

Q.6 (c) Solution:

(i)

Given:

For 4 m thick clay stratum,

$$\%U = 40\%$$

$$t = 1 \text{ year}$$

$$\Delta h = 6 \text{ cm}$$

We know, $\%U = \frac{\Delta h}{\Delta H} \times 100$

where ΔH is total settlement at 100% consolidation.

$$\therefore 40 = \frac{6}{\Delta H} \times 100$$

$$\Rightarrow \Delta H = \frac{6 \times 100}{40} = 15 \text{ cm}$$

Now, since final settlement is directly proportional to initial thickness,

\therefore For 20 m thick clay stratum,

$$\frac{\Delta H_{20}}{20} = \frac{15}{4}$$

$$\Rightarrow \Delta H_{20} = \frac{20 \times 15}{4} = 75 \text{ cm}$$

We know that, $T_v = C_v \frac{t}{d^2}$

where T_v and C_v are time factor coefficient of consolidation respectively,
and

t = time for settlement

d = length of drainage path

For the same clay and identical loading,

$$\frac{t_1}{H_1^2} = \frac{t_2}{H_2^2}$$

For 40% consolidation,

$$\frac{1}{5^2} = \frac{t_2}{20^2}$$

$$\Rightarrow t_2 = \frac{20^2}{5^2} = 16 \text{ years}$$

Also, $T_v \propto t$

and

$$T_v \propto U^2$$

$$\left[\because T_v = \frac{\pi}{4} (U^2) \text{ for } U < 60\% \right]$$

$$\therefore \frac{t_2}{t_3} = \frac{U_2^2}{U_3^2}$$

For $t_2 = 16$ years, $U_2 = 40\% = 0.4$

Therefore, for $t_3 = 1$ year

$$\frac{16}{1} = \frac{(0.4)^2}{U_3^2}$$

$$\Rightarrow U_3^2 = 0.01$$

$$\Rightarrow U_3 = 0.1 = 10\%$$

$$\Delta h = \frac{\Delta H_{20} \times U_3}{100} = \frac{75 \times 10}{100} = 7.5 \text{ cm}$$

For $t_3 = 5$ years

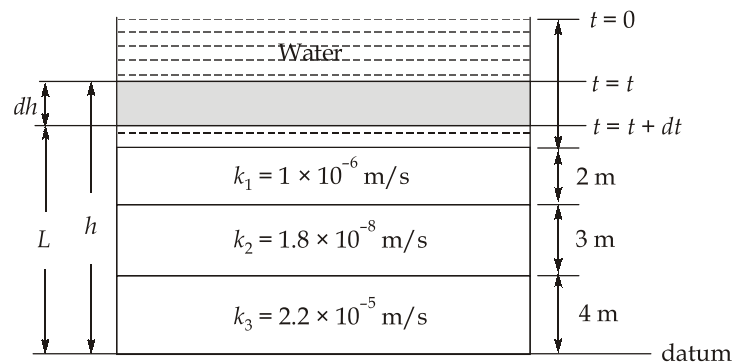
$$\frac{16}{5} = \frac{(0.4)^2}{U_3^2}$$

$$\Rightarrow U_3^2 = 0.05$$

$$\Rightarrow U_3 = 0.2236 = 22.36\%$$

$$\Delta h = \frac{\Delta H_{20} \times U_3}{100} = \frac{75 \times 22.36}{100} = 16.77 \text{ cm}$$

(ii)



Equivalent permeability in normal direction is,

$$k_{eq} = \frac{\frac{z_1 + z_2 + z_3}{\frac{z_1}{k_1} + \frac{z_2}{k_2} + \frac{z_3}{k_3}}}{\frac{z_1}{k_1} + \frac{z_2}{k_2} + \frac{z_3}{k_3}}$$

$$\Rightarrow k_{eq} = \frac{(2+3+4)}{\left(\frac{2}{1 \times 10^{-6}} + \frac{3}{1.8 \times 10^{-8}} + \frac{4}{2.2 \times 10^{-5}}\right)}$$

$$\Rightarrow k_{eq} = 5.33 \times 10^{-8} \text{ m/sec}$$

For time required to drain the water of height, $H = 0.4 \text{ m}$

Taking area same for all layer as 'A'. This is a case of falling head method

$$dq = a \left(-\frac{dh}{dt} \right) = kiA$$

Here, $a = A$

$$\therefore -\frac{dh}{dt} = k_{eq} i$$

$$\Rightarrow -\frac{dh}{dt} = \frac{k_{eq} \times h}{L}$$

$$\therefore \int_{9.4}^9 -\frac{dh}{h} = \int_{t=0}^{t=T} \frac{k_{eq}}{L} dt$$

$$\Rightarrow -[\ln h]_{9.4}^9 = \left(\frac{k_{eq}}{L} \right) T = \frac{5.33 \times 10^{-8}}{9} (T)$$

$$\Rightarrow T = \frac{(\ln 9.4 - \ln 9) \times 9}{5.33 \times 10^{-8}} \text{ sec}$$

$$\Rightarrow T = 84.985 \text{ days}$$

Q.7 (a) Solution:

(i)

For Reynolds number ≤ 0.5 (less than unity), a case of discrete settling can be assumed. When a discrete particle settles down in water, its downward settlement is opposed by the drag force offered by the water. The effective weight of the particle (actual weight – buoyancy) causes the particle to accelerate in the beginning, till it attains a sufficient velocity (v_s) at which the drag force becomes equal to the effective weight of the particle. After attaining that velocity (v_s), the particle falls down with constant velocity.

$$\text{Effective weight of the particle} = \text{Total weight} - \text{Buoyancy}$$

$$= \frac{4}{3}\pi r^3 \times \gamma_s - \frac{4}{3}\pi r^3 \times \gamma_w$$

(Where 'r' is radius of particle, γ_s is unit weight of particle and γ_w is unit weight water)

Also,
$$\text{Drag force} = C_D \times A \times \rho_w \times \frac{V^2}{2}$$

(Where C_D is coefficient of drag, A is projected area of practice, ρ_w is density of water and V = velocity of fall.) Now, when V becomes equal to V_s , the drag force becomes equal to the effective weight of the particle.

$$\therefore C_D \times A \times \rho_w \times \frac{V_s^2}{2} = \frac{4}{3}\pi r^3 (\gamma_s - \gamma_w) \quad [\because A = \pi r^2]$$

$$\Rightarrow C_D \pi r^2 \cdot \rho_w \times \frac{V_s^2}{2} = \frac{4}{3}\pi r^3 (\gamma_s - \gamma_w)$$

$$\Rightarrow V_s^2 = \frac{4}{3} \times \frac{2r(\gamma_s - \gamma_w)}{\rho_w \cdot C_D}$$

$$\Rightarrow V_s^2 = \frac{4}{3} \times \frac{d(\gamma_s - \gamma_w)}{\rho_w \cdot C_D} \quad [\because \gamma_s = \rho_s \times g \text{ and } \gamma_w = \rho_w \times g]$$

$$\Rightarrow V_s^2 = \frac{4}{3} \times \frac{d(\rho_s g - \rho_w g)}{\rho_w \cdot C_D}$$

$$\Rightarrow V_s^2 = \frac{4}{3} \times d \times \rho_w g \left(\frac{\rho_s}{\rho_w} - 1 \right) \times \frac{1}{\rho_w \cdot C_D}$$

$$\Rightarrow V_s^2 = \frac{4}{3} \times g \times d \times (G_s - 1) \times \frac{1}{C_D}$$

For $Re = 0.5 (< 1)$
$$C_D = \frac{24}{Re} = \frac{24v}{V_s d} \quad [\text{where } v \text{ is kinematic velocity}]$$

$$V_s^2 = \frac{4}{3} \times g \times d \times (G_s - 1) \times \frac{V_s d}{24v}$$

$$\Rightarrow V_s = \frac{g}{18} (G_s - 1) \times \frac{d^2}{v}$$

$$\Rightarrow V_s = \frac{\rho g (G_s - 1) d^2}{18\mu} \quad \left(\because v = \frac{\mu}{\rho} \right) \quad \text{Hence proved.}$$

(ii)

Given: Angle of switch $\beta = 1^\circ 30' 00'' = 1.5^\circ$

$$N = 8.5$$

Heel divergence, $d = 11.50 \text{ cm} = 0.115 \text{ m}$

$$\therefore N = \cot \alpha$$

$$\Rightarrow \alpha = \cot^{-1}(8.5) = \tan^{-1}\left(\frac{1}{8.5}\right)$$

$$\Rightarrow \alpha = 6.709^\circ$$

$$G = 1.676 \text{ m} \quad (\text{For BG track})$$

Crossing lead

$$L = (G - d) \cot\left(\frac{\alpha + \beta}{2}\right)$$

$$\Rightarrow L = (1.676 - 0.115) \cot\left(\frac{6.709 + 1.5}{2}\right)$$

$$\Rightarrow L = 21.75 \text{ m}$$

$$\text{Radius of curvature, } R_o = \frac{(G - d)}{\cos \beta - \cos \alpha}$$

$$\Rightarrow R_o = \frac{(1.676 - 0.115)}{\cos(1.5^\circ) - \cos(6.709^\circ)} = 239.969 \text{ m}$$

Q.7 (b) Solution:

(i)

Ultimate load carrying capacity of circular footing is,

$$q_f = 1.3cN_c + \gamma D_f N_q + \frac{0.6\gamma B N_\gamma}{2}$$

$$\Rightarrow q_f = 1.3 \times 9 \times 30.2 + 20 \times D_f \times 19.2 + \frac{0.6 \times 20 \times 4 \times 23}{2}$$

$$\Rightarrow q_f = 353.34 + 384D_f + 552$$

$$\Rightarrow q_f = 384D_f + 905.34$$

$$q_a = \frac{Q_a}{A} = \frac{5400}{\frac{\pi}{4} \times 3^2} = 763.944 \text{ kN/m}^2$$

$$\text{Factor of safety, FOS} = 2.5 = \frac{q_{nf}}{q_{na}} = \frac{q_f - \gamma D_f}{q_a - \gamma D_f} = \frac{q_f - 20D_f}{q_a - 20D_f}$$

$$\begin{aligned}
 & 2.5(q_a - 20D_f) = q_f - 20D_f \\
 \Rightarrow & 2.5(763.944 - 20D_f) = 384D_f + 905.34 - 20D_f \\
 \Rightarrow & 1909.86 - 50D_f = 364D_f + 905.34 \\
 \Rightarrow & 364D_f + 50D_f = 1909.86 - 905.34 \\
 \Rightarrow & 414D_f = 1004.52 \\
 \Rightarrow & D_f = 2.426 \text{ m}
 \end{aligned}$$

(ii)

Liquefaction: It is a phenomenon in which a saturated or partially saturated soil substantially loses its strength and stiffness in response to an applied stress, usually due to earthquake shaking or any sudden change in stress condition.

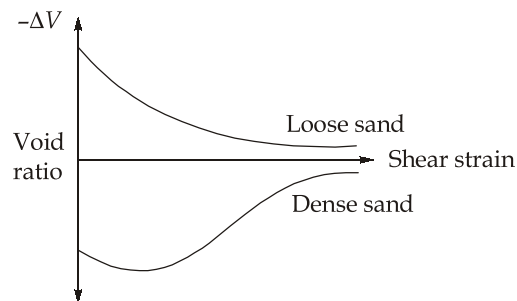
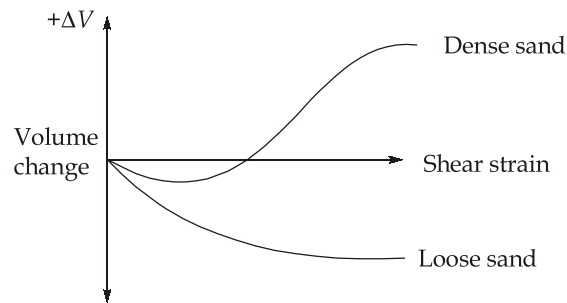
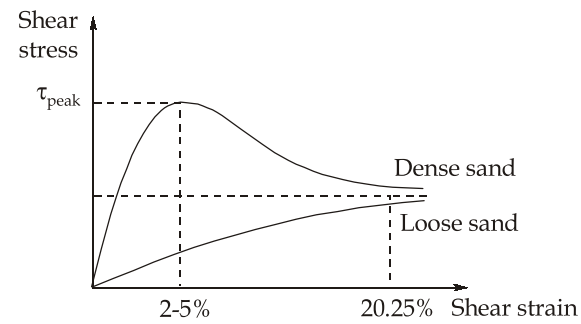
- During liquefaction, the pore water pressure in the soil mass increases to such an extent that the effective stress becomes nearly zero, causing the soil particles to behave like a liquid.
- This condition mostly occurs in loose, saturated, cohesionless soils such as silty sands and fine sands.
- When liquefaction occurs, the soil loses its load carrying capacity and behaves as a heavy fluid, which can lead to severe damage to structures, ground failure resulting in settlement, lateral spreading, tilting or sinking of buildings.

Quicksand condition: It is a phenomenon that occurs when upward flowing water exerts enough hydraulic pressure to reduce the effective stress in loose, saturated sand to nearly zero.

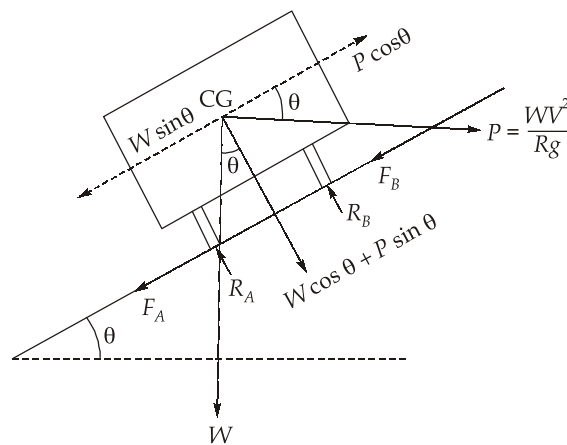
- This usually happens when water flows upward through sand due to artesian pressure or seepage forces.
- In this state, the sand-water mixture behaves like a fluid and objects can easily sink in because the sand particles are suspended and have no shear strength.
- In quicksand condition, the soil loses its shear strength locally, creating unsafe conditions for excavation and construction.

$$\bar{\sigma} = \gamma_{sub}z - i\gamma_w = 0$$

Typical stress-strain and volume change curves for sands:



Q.7 (c) Solution:



Analysis of super-elevation on horizontal curve on highway

The forces acting on the vehicle while moving on a circular curve of radius R (meters) at a speed of V (m/sec) are;

- Centrifugal force, $P = \frac{WV^2}{Rg}$ acting horizontally outwards through the centre of gravity.
- Weight W of the vehicle acting vertically downwards through CG.
- Frictional force developed between wheels and pavement transversely along pavement surface towards centre of curve.

For equilibrium condition,

$$P \cos \theta = W \sin \theta + F_A + F_B$$

The limiting equilibrium is reached when full values of friction forces are developed and then

$$F_A = fR_A; \quad F_B = fR_B$$

where f is the coefficient of lateral friction

$$\begin{aligned} \therefore P \cos \theta &= W \sin \theta + f(R_A + R_B) \\ &= W \sin \theta + f(W \cos \theta + P \sin \theta) \end{aligned}$$

$$\Rightarrow P (\cos \theta - f \sin \theta) = W \sin \theta + fW \cos \theta$$

$$\Rightarrow \frac{P}{W} (1 - f \tan \theta) = \tan \theta + f$$

$$\Rightarrow \frac{P}{W} = \frac{\tan \theta + f}{1 - f \tan \theta}$$

\therefore The maximum value of f is taken as 0.15 and maximum value of $\tan \theta$ i.e. e is taken as 0.07

$$1 - f \tan \theta \simeq 1$$

$$\therefore \frac{P}{W} = f + \tan \theta$$

$$\Rightarrow \frac{P}{W} = e + f$$

But $\frac{P}{W} = \frac{V^2}{gR}$

$$\therefore e + f = \frac{V^2}{gR}$$

If V is represented in kmph,

$$e + f = \frac{(0.278V)^2}{9.81} = \frac{V^2}{127R}$$

$$\therefore e + f = \frac{V^2}{127R}$$

where, V = Speed of vehicle in kmph, R = Radius of horizontal curve in metres.

For ruling design speed,

$$R_{\text{ruling}} = \frac{V_{\text{ruling}}^2}{127(e + f)}$$

$$\text{Now, } R_{\text{ruling}} = \frac{(100)^2}{127(0.07 + 0.15)} = 357.9 \simeq 360 \text{ m (say)}$$

Number of lanes, $n = 2$

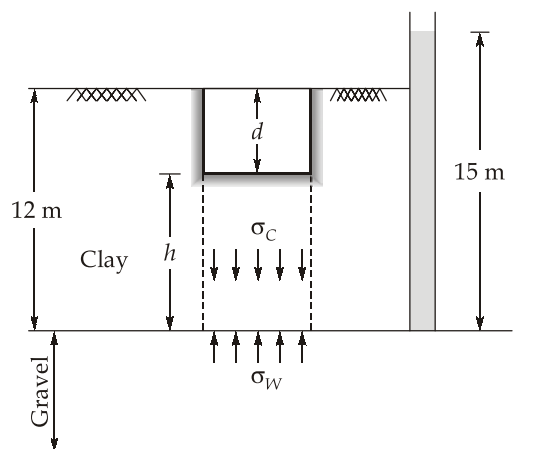
Wheel base of the vehicle, $l = 6.0 \text{ m}$

$$\begin{aligned} \text{Extra widening, } W_e &= \frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}} \\ &= \frac{2 \times 6^2}{2 \times 360} + \frac{100}{9.5\sqrt{360}} = 0.1 + 0.555 = 0.655 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{So, total pavement width on curve} &= W + W_e \\ &= 7 + 0.655 = 7.655 \text{ m} \end{aligned}$$

Q.8 (a) Solution:

(i)



1. At the top of the gravel stratum

$$\text{Total stress, } \sigma_c = 12 \times 17.36 = 208.32 \text{ kN/m}^2$$

The pore water pressure at the top of gravel is

$$u_w = 15 \times 9.81 = 147.15 \text{ kN/m}^2$$

Effective stress at the top of the gravel is;

$$\bar{\sigma} = (\sigma_c - u_w) = (208.32 - 147.15) = 61.17 \text{ kN/m}^2$$

2. If an excavation is made into the clay stratum as shown above, the depth must be such that

$$\sigma_c \leq u_w$$

Let the bottom of excavation be h metre above the top of gravel layer. Now, the downward pressure acting at the top of the gravel layer is

$$\sigma_c = \gamma h = 17.36h$$

$$u_w = 147.15 \text{ kN/m}^2$$

$$\Rightarrow 17.36h = 147.15$$

$$\Rightarrow h = \frac{147.15}{17.36} = 8.476 \text{ m}$$

$$\therefore \text{Depth of excavation} = (12 - 8.476) = 3.524 \text{ m}$$

If FOS = 1.10

$$\therefore \text{Safe depth of excavation} = \frac{3.524}{1.10} = 3.204 \text{ m}$$

(ii)

According to Rankine's theory,

$$k_a = \cos \beta \left(\frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} \right)$$

$$= \cos 9^\circ \left(\frac{\cos 9^\circ - \sqrt{\cos^2 9^\circ - \cos^2 27^\circ}}{\cos 9^\circ + \sqrt{\cos^2 9^\circ - \cos^2 27^\circ}} \right)$$

$$= 0.988 \times 0.397 = 0.392$$

$$k_p = \cos \beta \left(\frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}} \right) = 0.988 \times \frac{1}{0.397}$$

$$= 2.488$$

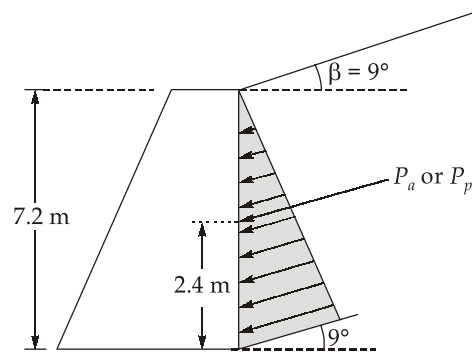
Total active thrust per metre run of the wall

$$P_a = \frac{1}{2} \gamma H^2 k_a = \frac{1}{2} \times 20 \times (7.2)^2 \times 0.392 = 203.2 \text{ kN}$$

Total passive resistance per metre of run of the wall,

$$P_p = \frac{1}{2} \gamma H^2 k_p = \frac{1}{2} \times 20 \times (7.2)^2 \times 2.488 = 1289.8 \text{ kN}$$

The pressure is considered to act parallel to the surface of the backfill soil and the distribution is triangular for both cases. This resultant thrust acts at a height of $\left(\frac{1}{3}H\right)$ or 2.4 m above the base at 9° to horizontal as shown below.



Retaining wall with inclined surcharge and pressure distribution

Q.8 (b) Solution:

(i)

Deviation angle,
$$N = -\frac{1}{25} - \frac{1}{20} = -0.09$$

For national highway in plain country, the design speed is 100 km/hr.

Criterion-1: Comfort condition

$$\begin{aligned} \text{Length of the valley curve} &= 0.38(NV^3)^{1/2} \\ &= 0.38(0.09 \times (100)^3)^{1/2} \\ &= 0.38 \times 300 = 114 \text{ m} \end{aligned}$$

Criterion-2: Head light sight distance condition, let's take $L > \text{HSD}$

For safe driving at night

$$V = 100 \text{ km/hr}$$

Assuming coefficient of friction and reaction time as,

$$f = 0.35$$

$$t = 2.5 \text{ seconds}$$

∴ Stopping sight distance can be calculated as,

$$\text{SSD} = 0.278Vt + \frac{V^2}{254f}$$

$$\Rightarrow \text{SSD} = 0.278 \times 100 \times 2.5 + \frac{100^2}{254 \times 0.35} = 181.986 \text{ m}$$

$$\simeq 182 \text{ m (say)}$$

Assuming $L > \text{SSD}$ (here L is length of valley curve)

$$L = \frac{NS^2}{2h_1 + 2 \times (S) \times \tan \alpha}$$

$$N = 0.09$$

$$\text{SSD} = 182 \text{ m}$$

$$h_1 = \text{Avg. height of the head light} = 0.75 \text{ m (as per IRC)}$$

$$\alpha \text{ (Beam angle)} = 1^\circ \quad \text{(as per IRC)}$$

$$\therefore L = \frac{0.09 \times (182)^2}{2 \times (0.75) + 2 \times 182 \times \tan 1^\circ}$$

$$= 379.589 \text{ m} \simeq 380 \text{ m} > \text{SSD} \quad (\text{OK})$$

The valley curve length based on head light sight distance being higher than that based on comfort condition, the design length of the valley curve is 380 m.

Check for impact factor:

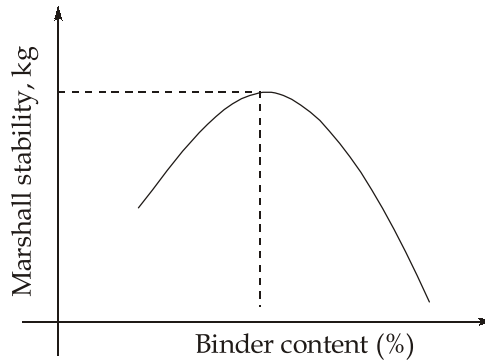
IRC suggested that the impact factor should not exceed 17 percent.

$$\text{Impact factor} = \frac{1.59NV^2}{L}$$

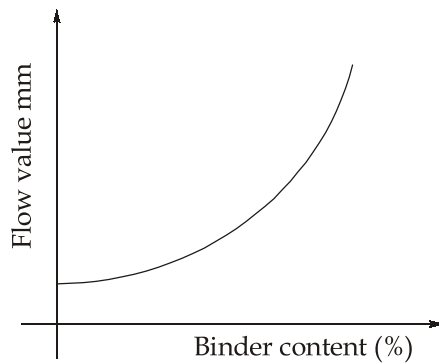
$$= \frac{1.59 \times 0.09 \times 100^2}{380} = 3.766\% \neq 17\% \quad (\text{OK})$$

(ii)

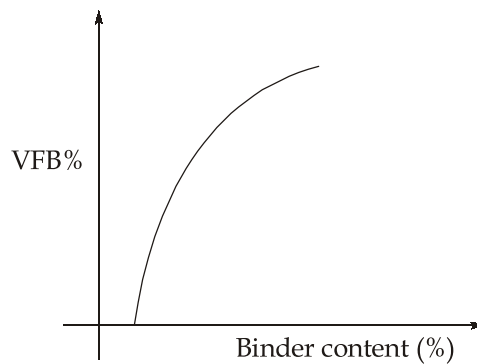
1. Marshall stability vs binder content



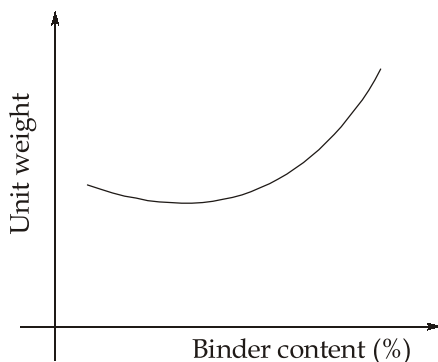
2. Flow value vs binder content



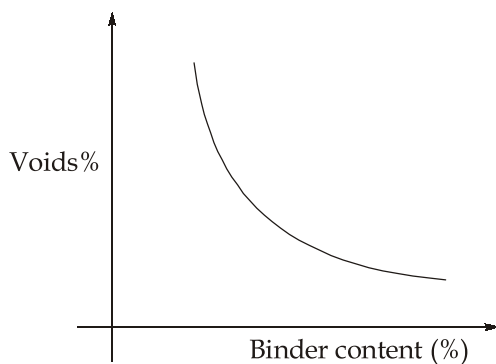
3. Voids filled with bitumen vs binder content



4. Voids in mineral aggregate vs binder content



5. Air voids vs binder content

**Q.8 (c) Solution:**

Since,

In staff readings at both X and Y,

$$2.60 - 1.85 = 1.85 - 1.10 = 0.75 \text{ m}$$

and

$$2.50 - 1.91 = 1.91 - 1.32 = 0.59 \text{ m}$$

Therefore, the line of sight is perpendicular to the staff.

Observations from R to X:

Horizontal distance,

$$D_{RX} = kS_1 \cos \theta_1 + C \cos \theta_1 + S_{m1} \sin \theta_1$$

Since the lens is anallatic,

$$\therefore k = 100, C = 0$$

and

$$S_1 = 2.60 - 1.10 = 1.50 \text{ m}$$

\therefore

$$\begin{aligned} D_{RX} &= (100 \times 1.5 \cos 8^\circ 9') + 0 + (1.85 \times \sin 8^\circ 9') \\ &= 148.75 \text{ m} \end{aligned}$$

$$\text{Vertical distance, } V_1 = kS_1 \sin\theta_1 + C\sin\theta_1$$

$$\Rightarrow V_1 = 100 \times 1.5 \times \sin 8^\circ 9' + 0$$

$$= 21.265 \text{ m}$$

$$\text{R.L of } X = \text{R.L of } R + HI + V_1 - S_{m1}\cos\theta_1$$

$$\text{R.L of } X = 1020.60 + 1.50 + 21.265 - (1.85 \times \cos 8^\circ 9')$$

$$= 1041.534 \text{ m}$$

$$\text{Latitude of } RX = 148.75 \cos 15^\circ 14' = 143.5235 \text{ m}$$

$$\text{Departure of } RX = 148.75 \sin 15^\circ 14' = 39.084 \text{ m}$$

$$\text{Coordinates of } X, \quad D_X = \text{Easting of } R + \text{Departure of } RX$$

$$= 1800 + 39.084 = 1839.084 \text{ m}$$

$$L_X = \text{Northing of } R + \text{Latitude of } RX$$

$$= 800 + 143.5235 = 943.5235 \text{ m}$$

Observations from S to Y:

$$\text{Horizontal distance } SY, D_{SY} = kS_2\cos\theta_2 + C\cos\theta_2 + S_{m2} \sin\theta_2$$

$$\Rightarrow D_{SY} = [100 \times (2.50 - 1.32) \times \cos 2^\circ 3'] + 0 + 1.91 \times \sin 2^\circ 3'$$

$$= 117.993 \text{ m}$$

$$\text{Vertical distance, } V_2 = kS_2\sin\theta_2 + C\sin\theta_2$$

$$\Rightarrow V_2 = 100 \times (2.50 - 1.32) \times \sin 2^\circ 3' + 0$$

$$= 4.221 \text{ m}$$

$$\text{R.L. of } Y = \text{R.L. of } S + HI + V_2 - S_{m2}\cos\theta_2$$

$$\text{R.L. of } Y = 1021.21 + 1.53 + 4.221 - (1.91 \times \cos 2^\circ 3')$$

$$= 1025.05 \text{ m}$$

$$\text{Latitude of } SY = 117.993 \cos 340^\circ 18' = 111.087 \text{ m}$$

$$\text{Departure of } SY = 117.993 \sin 340^\circ 18' = -39.775 \text{ m}$$

Coordinates of Y,

$$D_Y = \text{Easting of } S + \text{Departure of } SY$$

$$= 2500 + (-39.775) = 2460.225 \text{ m}$$

$$L_Y = \text{Northing of } S + \text{Latitude of } SY$$

$$= 950 + 111.087 = 1061.087 \text{ m}$$

For line XY,

$$\Delta L = L_Y - L_X = 1061.087 - 943.5235 = 117.5635 \text{ m}$$

$$\Delta D = D_Y - D_X = 2460.225 - 1839.084 = 621.141 \text{ m}$$

Distance between the points X and Y,

$$D_{XY} = \sqrt{\Delta L^2 + \Delta D^2}$$

$$\Rightarrow D_{XY} = \sqrt{(117.5635)^2 + (621.141)^2}$$

$$\Rightarrow D_{XY} = 632.17 \text{ m}$$

Bearing of line XY,

$$\begin{aligned}\theta_{XY} &= \tan^{-1}\left(\frac{\Delta D}{\Delta L}\right) = \tan^{-1}\left(\frac{621.141}{117.5635}\right) \\ &= 79.2824^\circ \\ &= 79^\circ 16' 56.64''\end{aligned}$$

Gradient from X and Y,

$$\begin{aligned}G_{XY} &= \frac{R.L.\text{of } X - R.L.\text{ of } Y}{D_{XY}} \\ &= \frac{1041.534 - 1025.05}{632.17} \\ &= 0.0261 = \frac{1}{38.3142} \\ &= 1 \text{ in } 38.3142\end{aligned}$$

