MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

E & T Engineering Test No : 11

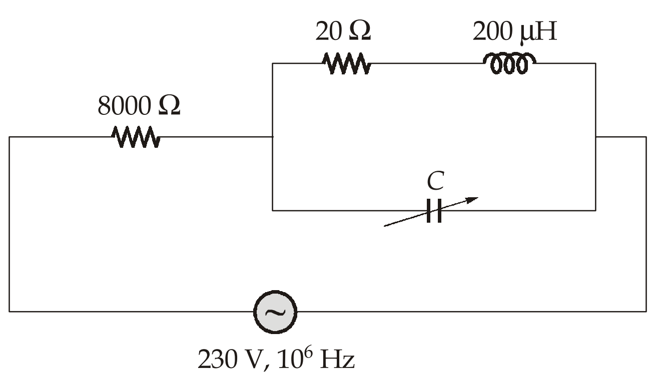
Section A

Q.1 (a) Solution:

(i) Given,
$$R = 20\ \Omega$$
$$L = 200\ \mu H$$
$$R_S = 8000\ \Omega$$
$$V_S = 230\ V$$
$$f = 10^6\ Hz$$

Inductive reactance of coil, $X_L = 2\pi f L$

$$= 2\pi \times 10^6 \times 200 \times 10^{-6} = 1256.6\ \Omega$$



The impedance of the coil is,

$$Z = \frac{(R + j\omega L) \times \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$Z = \frac{R + j\omega L}{(1-\omega^2 LC) + j\omega RC} \times \frac{(1-\omega^2 LC) - j\omega RC}{(1-\omega^2 LC) - j\omega RC}$$

$$Z = \frac{R}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2} + j\frac{\omega L(1-\omega^2 LC) - j\omega R^2 C}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, the impedance is purely real. Equating the imaginary part to zero,

$$L(1-\omega_0^2 LC) = R^2 C$$

$$\omega_0^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

Thus, at resonance, we get the impedance of the coil as,

$$Z = \frac{R}{(1-\omega_0^2 LC)^2 + \omega_0^2 R^2 C^2} = \frac{L}{CR}$$

1. The resonant frequency of the circuit is,

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{200 \times 10^{-6} \times C} - \frac{(20)^2}{(200 \times 10^{-6})^2}}$$

$\Rightarrow$ $C = 126.65 \times 10^{-12}$ F = 126.65 pF

2. Quality factor of the coil,

$$Q_0 = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 10^6 \times 200 \times 10^{-6}}{20} = 62.83$$

3. Dynamic Impedance of the coil,

$$Z = \frac{L}{CR} = \frac{200 \times 10^{-6}}{126.65 \times 10^{-12} \times 20}$$

$$= 78957.7576 \simeq 78958\ \Omega$$

4. Total equivalent impedance of the circuit at resonance is,

$$Z_{\text{equ}} = 78958 + R_S$$

$$= 78958 + 8000 = 86958\ \Omega$$

$\therefore$ Total circuit current,

$$I_S = \frac{V_S}{Z_{equ}} = \frac{230}{86958}$$

$$I_S = 2.645 \times 10^{-3}$$

$$\Rightarrow \quad I_S = 2.645\ \text{mA}$$

Q.1 (b) Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000\ \text{rpm}$$

Slip of the induction motor, $s = \frac{f_r}{f} = \frac{1.5}{50} = 0.03 \text{ or } 3\%$

Rotor speed, $N_r = (1 - s)N_s = (1 - 0.03) \times 1000 = 970$ rpm

$$\omega_r = 2\pi N_r = \frac{2\pi \times 970}{60} = 101.58\ \text{rad/s}$$

Shaft power output, $P_0 = \tau_0 \omega_r$

$= 150 \times 101.58 = 15236$ W $= 15.236$ kW

Gross Torque, $\tau_g = \tau_0$ + Friction Loss

Mechanical power developed,

$$P_{md} = (150 + 10) \times 101.58 = 16252\ \text{W} = 16.252\ \text{kW}$$

(i) Rotor copper loss $p_c = \left(\frac{s}{1-s}\right) P_{md}$

$$= \frac{0.03}{1 - 0.03} \times 16252 = 0.5026\ \text{kW}$$

(ii) Input to motor, $P_i = P_{md} + p_c + p_i$

$$= 16.252 + 0.5026 + 0.700 = 17.4546 \text{ kW}$$

(iii) $$\text{Efficiency} = \frac{P_0}{P_i} = \frac{15.236}{17.4546} = 0.8729 \text{ pu} = 87.29\%$$

Q.1 (c) Solution:

Data: $\beta_{dc} = 150$, $I_{CO} = 2.5$ mA, $I_{CQ} = 500$ mA

Procedure to be followed:

Step 1: Calculate R_B by using the equation, $I_C = \beta I_B + (1 + \beta)I_{CO}$.

Step 2: Calculate the stability factor S.

Step 3: Calculate the value of θ.

Calculate I_B:

$$I_C = \beta I_B + (1 + \beta)I_{CO}$$

$$\therefore \quad 500 = 150 I_B + (151 \times 2.5)$$

$$\therefore \quad I_B = 0.816 \text{ mA}$$

Calculate R_B:

Apply KVL to the base emitter circuit

$$-V_{CC} = I_B R_B - V_{BE} + I_E R_E \qquad \text{[For Ge Transistor, } V_{BE} = -0.3\text{V]} \qquad ...(1)$$

$$20 = 0.816 \times 10^{-3} R_B + 0.3 + (I_C + I_B)R_E$$

$$\therefore \quad R_B = \frac{20 - 0.3 - (500.816 \times 10^{-3} \times 5)}{0.816 \times 10^{-3}} = 21.073 \text{ k}\Omega$$

Calculate V_{CE}:

The load resistance referred to the primary side of the transformer is given by

$$R_C = \left(\frac{N_1}{N_2}\right)^2 \times R_L = \frac{1 \times 10^3}{100} = 10\ \Omega$$

$$V_{CC} = I_C \times 10 + V_{CE} + I_E \times 5$$

$$\therefore \quad V_{CE} = V_{CC} - 10 I_C - 5 I_E = 20 - (10 \times 0.5) - (5 \times 0.500816)$$

$$V_{CE} = 12.49 \text{ V}$$

As $|V_{CE}| > |V_{CC}/2|$, the circuit is not inherently stable.

Calculate the stability factor (S):

The stability factor for the circuit is given by

$$S = \frac{\partial I_C}{\partial I_{CO}} = \frac{1+\beta}{1-\beta\dfrac{\partial I_B}{\partial I_C}}$$

From equation (i), $-V_{CC} = I_B(R_B + R_E) - V_{BE} + I_C R_E$

Differentiating with respect to I_C, we get

$$\frac{\partial I_B}{\partial I_C} = -\frac{R_E}{R_B + R_E}$$

Thus,

$$S = \frac{1+\beta}{1+\beta\dfrac{R_E}{R_B+R_E}}$$

$$S = (1+\beta)\frac{1+(R_B/R_E)}{1+\beta+(R_B/R_E)}$$

$$= 151\frac{1+(21.073\times10^3/5)}{151+(21.073\times10^3/5)} = 145.8$$

Calculate θ:

For thermal stability,

$$[V_{CC} - 2I_C(R_E + R_C)](S)(0.07 I_{CO}) < \frac{1}{\theta}$$

$$\therefore \quad [20 - 2 \times 0.5(5 + 10)](145.8 \times 0.07 \times 2.5 \times 10^{-3}) < \frac{1}{\theta}$$

$$\therefore \quad 0.1275 < \frac{1}{\theta}$$

$$\therefore \quad \theta < 7.83°\text{C/W}$$

∴ For thermally stable circuit,

$$\theta_{max} = 7.83°\text{C/W}$$

Q.1 (d) Solution:

The distance between two consecutives slits = $\dfrac{1}{\text{No. of slits per mm}}$

$$= \frac{1}{500}\times10^{-3}\,\text{m}$$

$$d = 2 \times 10^{-6}\ \text{m}$$

Using the grating equation $d \sin\theta = n\lambda$, the diffraction angle for red light can be calculated as,

$$\sin\theta = \frac{n\lambda}{d}$$

for $n = 1$ (first order diffraction) and $\lambda = 7 \times 10^{-7}$ m

$$\sin\theta = \frac{1\times 7\times 10^{-7}}{2\times 10^{-6}} = 0.35$$

$$\theta = 20.48°$$

The diffraction angle for green light can be calculated as,

$$\sin\theta = \frac{n\lambda}{d}$$

for ($n = 1$) first order diffraction and $\lambda = 5.38 \times 10^{-7}$ m

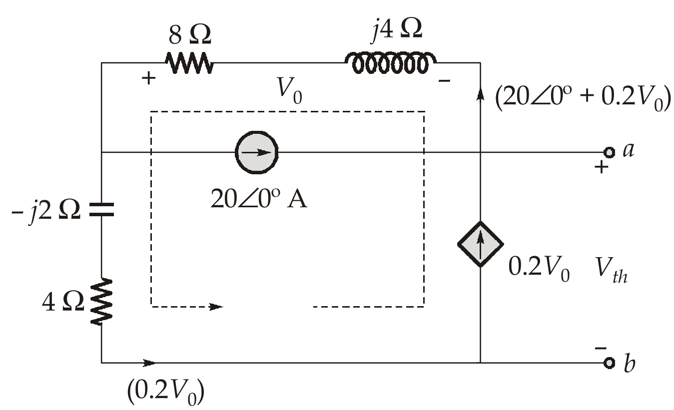
$$\therefore \quad \sin\theta = \frac{1\times 5.38\times 10^{-7}}{2\times 10^{-6}} = 0.269$$

$$\theta = 15.6°$$

The diffraction angle for different wavelengths are not the same. Hence, it is possible to examine the contents of an incident wave by looking at the different angles of deviation produced by the different components of a beam.

Q.1 (e) Solution:

Given circuit,



Step 1: To find V_{th}:

Applying KVL around the loop,

$$-V_{th} - V_0 + (4 - j2)(0.2\,V_0) = 0$$

$$\Rightarrow \quad V_{th} = -V_0 + 0.8V_0 - j0.4V_0$$

$$\Rightarrow \quad V_{th} = -0.2V_0 - j0.4V_0 = -(0.2 + j0.4)V_0 \quad \text{...(i)}$$

We have, $V_0 = -[8 + j4] \cdot [20\angle 0° + 0.2V_0]$

$$V_0 = -[160 + 80j + 1.6V_0 + 0.8jV_0]$$

$$\Rightarrow \quad (2.6 + 0.8j)V_0 = -(160 + 80j)$$

$$\Rightarrow \quad V_0 = \frac{-(160+80j)}{(2.6+0.8j)}$$

$$\Rightarrow \quad V_0 = 65.76\angle -170.538° \text{ V}$$

Using equation (i),

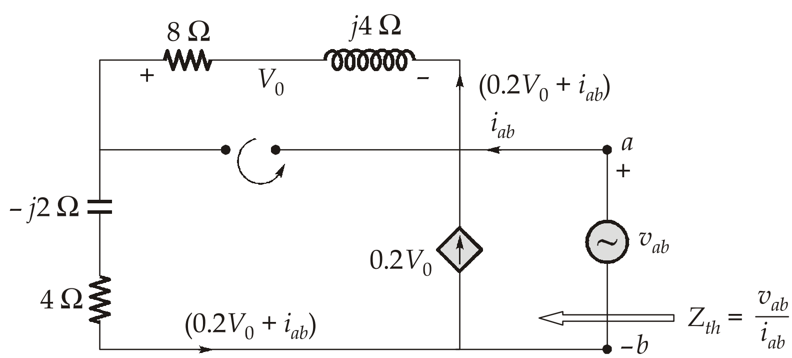
$$V_{th} = -(0.2 + j0.4) \times (65.76\angle -170.538°)$$

$$\Rightarrow \quad V_{th} = 8.65 + j28.11$$

$$\Rightarrow \quad V_{th} = 29.409\angle 72.897° \text{ V}$$

Step 2: To find Z_{th},

Deactivate all the independent sources present in the given circuit and connect a voltage source v_{ab} across the terminals a-b.



Applying KVL around the loop,

$$-v_{ab} - V_0 + (4 - j2) \cdot (0.2\, V_0 + i_{ab}) = 0$$

$$v_{ab} = -V_0 + (0.8 - 0.4j)\, V_0 + (4 - j2)i_{ab}$$

$$v_{ab} = (-0.2 - 0.4j)V_0 + (4 - j2)\, i_{ab} \quad \text{...(ii)}$$

But, $V_0 = -(8 + j4) \times (0.2\, V_0 + i_{ab})$

$$= (-1.6 - 0.8\, j)\, V_0 - (8 + j4)\, i_{ab}$$

$$\Rightarrow \quad V_0 = \frac{-(8+j4)i_{ab}}{(2.6+0.8j)}$$

$$\Rightarrow \quad V_0 = (3.288\angle -170.538°)i_{ab} \quad \text{...(iii)}$$

Substitute equation (iii) in equation (ii),

$$v_{ab} = (-0.2 - 0.4j)\, [3.288\angle -170.538°)i_{ab} + (4 - j2)i_{ab}$$

$$v_{ab} = (4.432443 - 0.5946j)i_{ab}$$
$$= (4.47215 \angle -7.64031°)i_{ab}$$
$$\Rightarrow \quad Z_{th} = \frac{v_{ab}}{i_{ab}} = (4.47215 \angle -7.64031°)\ \Omega$$

Q.2 (a) Solution:

(i) Given, $R = 500\ \Omega$

Lower cut-off frequency, $f_1 = 100$ Hz

Upper cut-off frequency, $f_2 = 10$ kHz

1. $\text{BW} = f_2 - f_1 = (10 \times 10^3) - (100) = 9900$ Hz

Assuming f_0 as the resonant frequency, we have

$$f_1 = f_0 - \frac{\text{BW}}{2} \quad \text{...(i)}$$

$$f_2 = f_0 + \frac{\text{BW}}{2} \quad \text{...(ii)}$$

Adding equations (i) and (ii),

$$f_1 + f_2 = 2f_0$$

$$\Rightarrow \quad f_0 = \frac{f_1 + f_2}{2} = \frac{100 + 10000}{2} = 5050 \text{ Hz}$$

2. For a RLC circuit, $\text{BW} = \frac{R}{2\pi L}$

$$9900 = \frac{500}{2\pi L}$$

$$\Rightarrow \quad L = 8.038 \text{ mH}$$

$$X_{L_0} = 2\pi f_0 L = 2\pi \times 5050 \times 8.038 \times 10^{-3} = 255.05\ \Omega$$

At resonance, $X_{L_0} = X_{C_0} = 255.05\ \Omega$

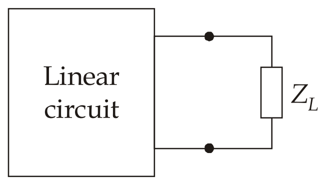
$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$255.05 = \frac{1}{2\pi \times 5050 \times C}$$

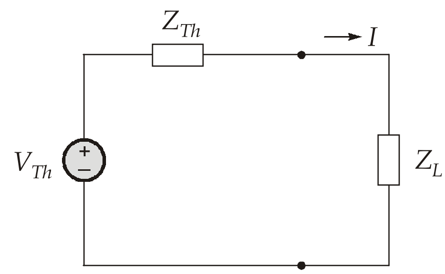
$$\Rightarrow \quad C = 0.12\ \mu\text{F}$$

3. $$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{500}\sqrt{\frac{8.038 \times 10^{-3}}{0.12 \times 10^{-6}}} = 0.5176$$

(ii) Consider the circuit, where an ac circuit is connected to load Z_L and its Thevenin equivalent as shown below.



Circuit with a load



Thevenin equivalent

The thevenin impedance Z_{Th} and the load impedance Z_L are,

$$Z_{Th} = R_{Th} + jX_{Th}$$

$$Z_L = R_L + jX_L$$

The phasor current through the load is,

$$I = \frac{V_{Th}}{Z_{Th} + Z_L}$$

$$= \frac{V_{Th}}{(R_{Th} + jX_{Th}) + (R_L + jX_L)} \qquad \text{where, } I = I_m\angle\theta_i$$

The average power delivered to the load is,

$$P = \frac{1}{2}|I|^2 R_L$$

$$= \frac{\frac{1}{2}|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} \qquad \text{...(i)}$$

Our objective is to adjust the load parameters R_L and X_L So that P is maximum.

Hence, set $\frac{\partial P}{\partial R_L}$ and $\frac{\partial P}{\partial X_L}$ equal to zero.

From equation (i), we obtain

$$\frac{\partial P}{\partial X_L} = -\frac{|V_{Th}|^2 R_L (X_{Th} + X_L)}{\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]^2}$$

and

$$\frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2 \left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2 - 2R_L (R_{Th} + R_L)\right]}{2\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]^2}$$

Setting $\dfrac{\partial P}{\partial X_L}$ to zero gives

$$X_L = -X_{\text{Th}} \quad \text{...(ii)}$$

and setting $\dfrac{\partial P}{\partial R_L}$ to zero results in

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2} \quad \text{...(iii)}$$

Combining equations (ii) and (iii) leads to the conclusion that for maximum average power transfer, Z_L must be selected such that $X_L = -X_{\text{Th}}$ and $R_L = R_{Th}$, i.e.,

$$Z_L = R_L + jX_L = R_{Th} - jX_{Th} = Z_{Th}^* \quad \text{...(iv)}$$

$\therefore$ 'For maximum average power transfer, the load impedance Z_L must be equal to the complex conjugate of the Thevenin impedance Z_{Th}'.

This result is known as the maximum average power transfer theorem for the a.c. circuits,

Setting $R_L = R_{Th}$ and $X_L = -X_{Th}$ in equation (i) gives us the maximum average power as,

$$P_{\max} = \frac{|V_{Th}|^2}{8R_{Th}}$$

If the load is purely resistive, the condition for maximum power transfer is obtained from equation (iii) by setting $X_L = 0$; that is

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = |Z_{Th}|$$

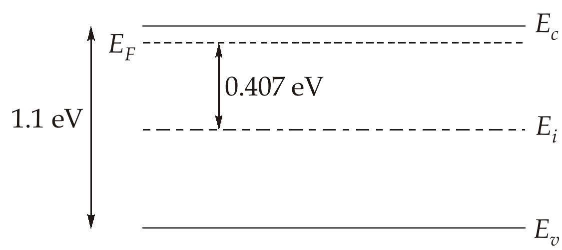
Q.2 (b) Solution:

(i) Since $N_d >> n_i$, we can approximate $n_0 = N_d$ and

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.25\times10^{20}}{10^{17}} = 2.25\times10^3\,\text{cm}^{-3}$$

We have,
$$E_F - E_i = kT\ln\frac{n_0}{n_i} = 0.0259\ln\frac{10^{17}}{1.5\times10^{10}} = 0.407\text{ eV}$$

The resulting band diagram is:



(ii) We have,

$$D_p = \frac{kT}{q}\mu_p = 0.0259\times 500 = 12.95\ \text{cm}^2/\text{s}$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{12.95\times 10^{-10}} = 3.6\times 10^{-5}\ \text{cm}$$

The steady-state hole concentration in the semiconductor is given by

$$p(x) = p_0 + \Delta p e^{-\frac{x}{L_p}} = 10^{17} + 5\times 10^{16} e^{-\frac{10^{-5}}{3.6\times 10^{-5}}} \quad (\text{at } x = 1000\text{Å})$$

$$= 1.379 \times 10^{17}\ \text{cm}^{-3}$$

$$= n_i e^{(E_i - F_p)/kT} = (1.5\times 10^{10}\ \text{cm}^{-3})e^{(E_i - F_p)/kT}$$

$$E_i - F_p = \left(\ln\frac{1.379\times 10^{17}}{1.5\times 10^{10}}\right)\times 0.0259 = 0.415\ \text{eV}$$

$$E_c - F_p = (E_i + E_g/2) - F_p = \frac{1.1}{2}\ \text{eV} + 0.415\ \text{eV} = 0.965\ \text{eV}$$

We can calculate the hole current,

$$I_p = -qAD_p\frac{dp}{dx} = qA\frac{D_p}{L_p}(\Delta p)e^{-\frac{x}{L_p}}$$

At $x = 1000$ Å $= 10^{-5}$ cm,

$$= 1.6\times 10^{-19}\times 0.5\times\frac{12.95}{3.6\times 10^{-5}}\times 5\times 10^{16} e^{-\frac{10^{-5}}{3.6\times 10^{-5}}}$$

$$= 1.09 \times 10^3\ \text{A}$$

Excess stored hole charge, $Q_p = qA(\Delta p)L_p$

$$= 1.6 \times 10^{-19}(0.5)(5 \times 10^{16})(3.6 \times 10^{-5})$$

$$= 1.44 \times 10^{-7}\ \text{C} = 144\ \text{nC}$$

Q.2 (c) Solution:

(i) At rated load,

$$S_{3\phi} = \sqrt{3}V_L I_L$$

$$50 \times 10^3 = \sqrt{3}\times 440 I_L$$

$$I_L = \frac{50\times 10^3}{\sqrt{3}\times 440} = 65.6\text{A} = I_a$$

Let V_p be taken as reference phasor.

$$V_p = V_p\angle 0° = \frac{440}{\sqrt{3}}\angle 0° = 254\angle 0° \text{ V}$$

At unity power factor,

$$I_a = I_a\angle 0° = 65.6\angle 0° = 65.6 + j0$$

1. Leakage impedance

$$Z_L = R_a + jX_L = 0.25 + j0.5 = 0.559\angle 63.4° \ \Omega$$

Internal emf

$$\begin{aligned} E_{p\ \text{exc}} &= V_p + I_a Z_L \\ &= 254\angle 0° + (65.6\angle 0°)(0.559\ \angle 63.4°) \\ &= 254 + j0 + 36.67\angle 63.4° \\ &= 254 + 16.42 + j32.79 = 272.4\angle 6.91° \text{ V} \end{aligned}$$

Line value of internal emf,

$$E_{L\ \text{exc}} = \sqrt{3}\times 272.4 = 471.8 \text{ V}$$

2. Synchronous impedance

$$Z_s = R_a + jX_s = 0.25 + j3.2 = 3.21\angle 85.53° \ \Omega$$

No-load emf per phase,

$$\begin{aligned} E_0 &= V_p + I_a Z_s \\ &= 254\angle 0° + (65.6\angle 0°)(3.21\angle 85.53°) \\ &= 254 + j0 + 210.6\ \angle 85.53° \\ &= 254 + 16.4 + j210 = 342.37\angle 37.83° \text{ V} \end{aligned}$$

Line value of no-load emf,

$$E_0 = \sqrt{3}\,E_{ap} = \sqrt{3}\times 342.37 = 593 \text{ V}$$

3. Voltage regulation

$$\begin{aligned} &= \frac{E_{0p} - V_p}{V_p} = \frac{342.37 - 254}{254} \\ &= 0.3479 \text{ pu} = 34.79\% \end{aligned}$$

4. $$X_S = X_L + X_{AR}$$

$\therefore$ $$X_{AR} = X_S - X_L = 3.2 - 0.5 = 2.7 \ \Omega$$

(ii) The maximum voltage regulation occurs for lagging loads. Percentage voltage regulation at lagging p.f;

$$V.R = \frac{E_2 - V_2}{E_2} = \left(\frac{I_2 r_{e2}}{E_2}\cos\theta_2 + \frac{I_2 X_{e2}}{E_2}\sin\theta_2\right)\times 100$$

$$\frac{d(V.R)}{d\theta} = \left[\frac{-I_2 r_{e2}}{E_2}\sin\theta_2 + \frac{I_2 X_{e2}}{E_2}\cos\theta_2\right]\times 100$$

For maximum voltage regulation, $\frac{d(V.R)}{d\theta} = 0$

$$\frac{I_2 X_{e2}}{E_2}\cos\theta_2 = \frac{I_2 r_{e2}}{E_2}\sin\theta_2$$

$$\tan\theta_2 = \frac{X_{e_2}}{r_{e_2}} \quad ...(i)$$

Since, $\cos\theta_2 = 0.3$

$$\theta_2 = 72.54°$$

Thus, $\tan 72.54° = \frac{X_{e_2}}{r_{e_2}}$

$$3.2 = \frac{X_{e_2}}{r_{e_2}}$$

$$X_{e_2} = 3.2 r_{e_2} \quad ...(ii)$$

Given, maximum% Regulation = 5%. Thus,

$$5 = \left[\frac{I_2 r_{e2}}{E_2}\cos\theta_2 + \frac{I_2 X_{e2}}{E_2}\sin\theta_2\right]\times 100$$

$$5 = \left[\frac{I_2 r_{e2}}{220}\cos(72.54°) + \frac{I_2(3.2 r_{e_2})}{220}\sin(72.54°)\right]\times 100$$

$$5 = \left[I_2 r_{e_2}(1.36\times 10^{-3}) + I_2 r_{e_2} 0.014\right]\times 100$$

$$5 = (I_2 r_{e_2})[0.015]\times 100$$

$$I_2 r_{e_2} = 3.33$$

At leading p.f. of 0.8, $V_2 = E_2 - \left[I_2 r_{e_2} \cos\theta - I_2 X_{e_2} \sin\theta\right]$

$= 220 - [3.33 \times 0.8 - 3.2 \times 3.33 \times 0.6]$

$[\because \cos\theta = 0.8; \quad \sin\theta = 0.6]$

$V_2 = 223.73$ Volt

Q.3 (a) Solution:

(i) The given circuit works as a 8 to 1 multiplexer.

Truth table:

For the given circuit,

S_2	S_1	Y (output)
0	0	$D_0\overline{S}_0 + D_1 S_0$
0	1	$D_2\overline{S}_0 + D_3 S_0$
1	0	$D_4\overline{S}_0 + D_5 S_0$
1	1	$D_6\overline{S}_0 + D_7 S_0$

Thus, the truth table is obtained as below,

S_0	S_2	S_1	Y
0	0	0	D_0
1	0	0	D_1
0	0	1	D_2
1	0	1	D_3
0	1	0	D_4
1	1	0	D_5
0	1	1	D_6
1	1	1	D_7

The logic function for the circuit is given by,

$$Y = \overline{S}_2\overline{S}_1(D_0\overline{S}_0 + D_1 S_0) + \overline{S}_2 S_1(D_2\overline{S}_0 + D_3 S_0)$$

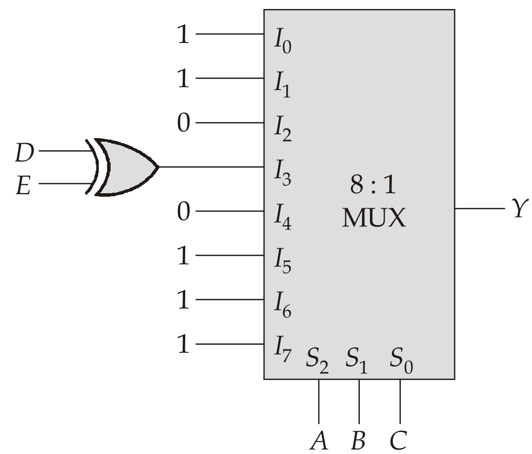
$$+ S_2\overline{S}_1(D_4\overline{S}_0 + D_5 S_0) + S_2 S_1(D_6\overline{S}_0 + D_7 S_0)$$

(ii) The truth table for the given function is obtained as below,

	A	B	C	D	E	Y
(0)	0	0	0	0	0	1
(1)	0	0	0	0	1	1
(2)	0	0	0	1	0	1
(3)	0	0	0	1	1	1
(4)	0	0	1	0	0	1
(5)	0	0	1	0	1	1
(6)	0	0	1	1	0	1
(7)	0	0	1	1	1	1
(8)	0	1	0	0	0	0
(9)	0	1	0	0	1	0
(10)	0	1	0	1	0	0
(11)	0	1	0	1	1	0
(12)	0	1	1	0	0	0
(13)	0	1	1	0	1	1
(14)	0	1	1	1	0	1
(15)	0	1	1	1	1	0
(16)	1	0	0	0	0	0
(17)	1	0	0	0	1	0
(18)	1	0	0	1	0	0
(19)	1	0	0	1	1	0
(20)	1	0	1	0	0	1
(21)	1	0	1	0	1	1
(22)	1	0	1	1	0	1
(23)	1	0	1	1	1	1
(24)	1	1	0	0	0	1
(25)	1	1	0	0	1	1
(26)	1	1	0	1	0	1
(27)	1	1	0	1	1	1
(28)	1	1	1	0	0	1
(29)	1	1	1	0	1	1
(30)	1	1	1	1	0	1
(31)	1	1	1	1	1	1

Assume A, B and C inputs are connected to the select lines $S_2S_1S_0$ of the 8 : 1 MUX. The excitation for the input lines can be obtained as below,

ABC	I_0 000	I_1 001	I_2 010	I_3 011	I_4 100	I_5 101	I_6 110	I_7 111
$\overline{D}\overline{E}/00$	(0)	(4)	8	12	16	(20)	(24)	(28)
$\overline{D}E/01$	(1)	(5)	9	(13)	17	(21)	(25)	(29)
$D\overline{E}/10$	(2)	(6)	10	(14)	18	(22)	(26)	(30)
$DE/11$	(3)	(7)	11	15	19	(23)	(27)	(31)
	1	1	0	$\overline{D}E + D\overline{E} = D \oplus E$	0	1	1	1



Q.3 (b) Solution:

(i) Full-load condition:

$$\text{Armature current } I_{a1} = I - I_{sh} = 41 - \frac{250}{250} = 40 \text{ A}$$

Back emf on full-load,

$$E_1 = V - I_{a1} R_a = 250 - 40 \times 0.2 = 242 \text{ V}$$

When a resistance of 2 Ω is placed in series with the armature, the back emf is

$$E_2 = V - I_{a1}(0.2 + 2) = 250 - 40(2.2) = 162 \text{ V}$$

For the DC motor, $E_b \propto N\phi$. Since the flux remains constant,

$$N_2 = \frac{E_2}{E_1} N_1 = \frac{162}{242} \times 800 = 535.54 \text{ rpm}$$

(ii) Double full-load condition:

At double full-load torque, the armature current

$$I_{a2} = 40 \times 2 = 80 \text{ A}$$

With 2 Ω resistance in the armature circuit, the back emf at double full-load torque

$$E_3 = V - I_{a2}(0.2 + 2) = 250 - 80(2.2) = 74 \text{ V}$$

Thus,
$$N_3 = \frac{E_3}{E_1} N_1 = \frac{74}{242} \times 800 = 244.6 \text{ rpm}$$

(iii) Stalling torque:

Under stalling conditions, the speed is zero and therefore, the back emf is zero.

Let I_{a0} be the armature current taken by the motor under stalling conditions.

$\therefore$
$$E_{b0} = V - I_{a0}(0.2 + 2)$$
$$0 = 250 - 2.2 I_{a0}$$

$$I_{a0} = \frac{250}{2.2} = 113.64 \text{ A}$$

For DC motor, $\tau \propto \phi I_a \Rightarrow \tau \propto I_a$ $(\because \phi \text{ is constant})$

Thus, Stalling torque ∝ stalling current

and Full-load torque ∝ full-load armature current

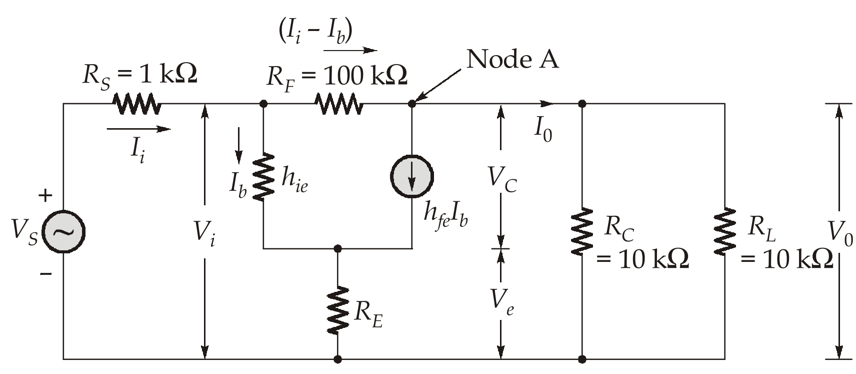
$$\therefore \quad \frac{\text{stalling torque}}{\text{full-load torque}} = \frac{\text{stalling current}}{\text{full-load armature current}}$$

$$= \frac{113.64}{40} = 2.84$$

$\therefore$ stalling torque = 2.84 × full-load torque

Q.3 (c) Solution:

Step 1 : Draw the h-parameter equivalent of the circuit,



$R'_L = (R_C \parallel R_L) = (10\text{ k} \parallel 10\text{ k}) = 5\text{ k}\Omega$

Step 2: Apply KVL to the output loop,

$$V_C = V_0 - V_e$$

where, $V_e = (I_i - I_0)\,R_E$

and $V_0 = I_0 R'_L = 5\times10^3 I_0$

$$\therefore \quad V_C = 5 \times 10^3 I_0 - (I_i - I_0)\,R_E$$

$$= 5 \times 10^3 I_0 - 50I_i + 50I_0 = 5050I_0 - 50I_i.$$

Looking at figure above and applying KCL at node A we can write that,

$$I_0 = (I_i - I_b) - h_{fe} I_b \quad \text{...(1)}$$

$$= I_i - (1 + h_{fe})I_b = I_i - 101I_b$$

$$\therefore \quad I_0 = I_i - 101I_b \quad \text{...(2)}$$

Apply KVL at the input

$$V_i = R_F(I_i - I_b) + V_0 \quad ...(3)$$

and

$$V_i = h_{ie}I_b + (I_i - I_0)R_E \quad ...(4)$$

Now subtract equation (3) from equation (4)

$$0 = h_{ie}I_b + (I_i - I_0)R_E - R_F(I_i - I_b) - V_0$$

Substituting the values we get,

$$0 = 1\,k\,I_b + 50\,I_i - 50\,I_0 - 100\,k\,I_i + 100\,k\,I_b - 5\,k\,I_0$$

$$0 = I_b(1\,k + 100\,k) + I_i(50 - 100\,k) + I_0(-50 - 5\,k)$$

$$\therefore \quad 101\,k\,I_b = 99.95\,k\,I_i + 5.05\,I_0 \Rightarrow I_b = 0.9896I_i + 0.05I_0 \quad ...(5)$$

Now substitute this value of I_b in equation (2) to get,

$$I_0 = I_i - 101[0.9896I_i + 0.05I_0]$$

$$I_0 = -98.9496I_i - 5.05I_0$$

$$6.05I_0 = -98.9496I_i$$

$$\therefore \quad \text{The current gain } A_I = \frac{I_0}{I_i} = \frac{-98.9496}{6.05} = -16.35$$

Substitute this in equation (5) to get,

$$I_b = (0.9896 \times I_i) + 0.05 \times (-16.35I_i) = 0.1721I_i \quad ...(6)$$

Substitute the values of I_0 and I_b into equation (3) to get,

$$V_i = 100\,k(I_i - 0.1721I_i) + 5\,k\,I_0$$

$$= 100\,k(I_i - 0.1721I_i) + 5\,k(-16.35I_i) = 1040I_i$$

$$\therefore \quad \frac{V_i}{I_i} = R_i = 1040\ \Omega$$

$$\text{Voltage gain, } A_V = \frac{I_0R'_L}{I_iR_i} = \frac{A_iR'_L}{R_i} = \frac{-16.35\times5\times10^3}{1040} = -78.6$$

$$A_{VS} = \frac{A_VR_i}{R_i + R_S} = \frac{-78.6\times1040}{1040+1000} = -40.07$$

Using Miller's Theorem, the feedback resistance R_F at the output is given by

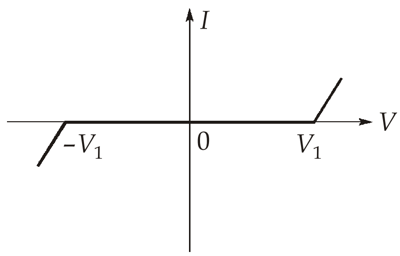
$$R'_F = \frac{R_F}{1 - \dfrac{1}{A_V}} \approx R_F$$

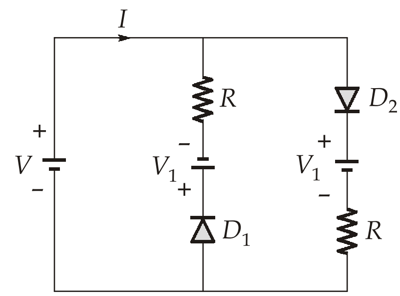
Output resistance is given by,

$$R'_0 = R_L \parallel R_C \parallel R_F = (10\text{ k} \parallel 10\text{ k} \parallel 100\,k) = 4.76\text{ k}\Omega$$

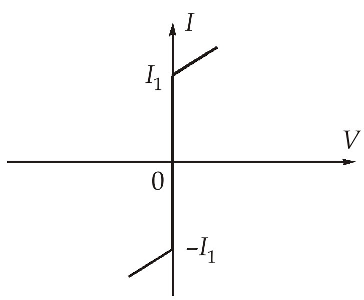
Q.4 (a) Solution:

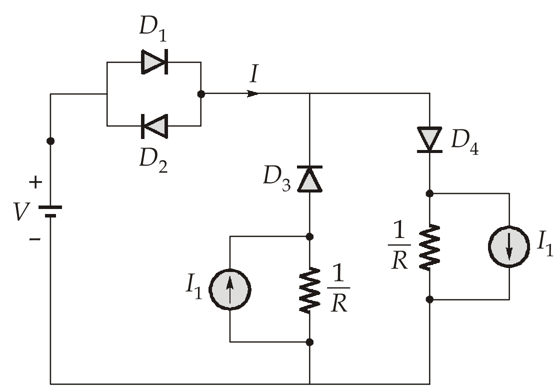
(i) 1. To achieve the given characteristics, we need two diodes D_1 and D_2 connected in parallel. The diode D_1 should be forward biased when $V < -V_1$ and diode D_2 should be reverse biased when $V > V_1$. The circuit can thus be drawn as below,





2. To achieve the given characteristics, we need two diodes D_1 and D_2 connected in parallel. The diode D_1 is forward biased when $V > 0$ and diode D_2 is forward biased when $V < 0$. Since the current is non-zero for $V = 0$. Thus, we place the current sources as shown in the figure below.





(ii) Given, A_0 (dB) = 34 dB $\Rightarrow 20 \log_{10} A_0 = 34 \Rightarrow A_0 = 50.118$

$$Q\text{-factor, } Q = \frac{f_c}{BW} = \frac{160}{16} = 10$$

Given,
$$C_1 = C_2 = C = 0.1\ \mu\text{F}$$

$$R_1 = \frac{Q}{2\pi f_c C A_0} = \frac{10}{2\pi \times 160 \times 0.1 \times 10^{-6} \times 50.118}$$

$$= 1984.752\ \Omega$$

$$R_2 = \frac{Q}{2\pi f_c C(2Q^2 - A_0)}$$

$$= \frac{10}{2\pi\times160\times0.1\times10^{-6}(2\times10^{2}-50.118)} = 663.632\ \Omega$$

$$R_3 = \frac{Q}{\pi f_c C} = \frac{10}{\pi\times160\times0.1\times10^{-6}} = 198943.68\ \Omega$$

Q.4 (b) Solution:

(i) From the expression of voltage across the capacitor, we get ω = 500 rad/sec. The impedances of the circuit elements at ω = 500 rad/sec are given by,

$$X_{L_1} = j\omega L_1 = j500(3\times10^{-3}) = j1.5\ \Omega$$

$$X_{L_2} = j\omega L_2 = j500(4\times10^{-3}) = j2\ \Omega$$

$$X_C = \frac{1}{j\omega C} = -j\frac{1}{500\times1000\times10^{-6}} = -j2\ \Omega$$

Thus, the admittance of R–L_2 circuit branch is given by,

$$Y_{R-L_2} = \frac{1}{2+j2} = 0.354\angle-45^\circ$$

$$= (0.25 - j\,0.25)\text{ mho}$$

Since $Y_C = \frac{1}{X_C} = j0.5$, the equivalent admitance of Y_C and Y_{R-L_2} is given by,

$$Y = Y_C + Y_{R-L_2}$$

$$= j0.5 + 0.25 - j\,0.25 = 0.25 + j\,0.25$$

$$= 0.354\angle45^\circ\text{ mho}$$

From this, $$I = YV_C = 0.354\angle45^\circ\times\frac{100}{\sqrt{2}}\angle45^\circ$$

$$= 25\angle90^\circ\ V$$

⇒ $$i = 25\sqrt{2}\cos(500t+90^\circ)$$

$$= -35.4\sin(500t)\text{ A}$$

Now, drop across L_1 is obtained as

$$V_{drop\ (L_1)} = IX_{L_1} = 25\angle90^\circ\times1.5\angle90^\circ$$

$$= 37.5\angle180^\circ\text{ V}$$

Finally, the supply voltage is obtained as

$$V_S = IX_{L_1} + V_C$$

$$= 37.5\angle 180° + \frac{100}{\sqrt{2}}\angle 45°$$

$$= -37.5 + j0 + 70.72 \times \frac{1}{\sqrt{2}} + j\,70.72 \times \frac{1}{\sqrt{2}}$$

$$= -37.5 + 50 + j50 = 12.5 + j50$$

$$= 51.54\angle 76°\ V$$

This gives,

$$v_s(t) = \sqrt{2} \times 51.54 \cos(500t + 76°)$$

$$= 72.88 \cos(500t + 76°)\,V$$

(ii) The equations for *ABCD* parameters are given by,

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Substituting the given *ABCD* parameters into the above equations, we obtain

$$V_1 = 5V_2 - 10I_2 \qquad \text{...(i)}$$

$$I_1 = 0.4V_2 - I_2 \qquad \text{...(ii)}$$

Applying KVL at input port,

$$-14\angle 0° + 2I_1 + V_1 = 0$$

$$\Rightarrow \quad V_1 = 14\angle 0° - 2I_1 \qquad \text{...(iii)}$$

At the output port,

$$V_2 = -10I_2 \qquad \text{...(iv)}$$

Substitute equations (iii) and (iv) in equations (i) and (ii) and solve for I_1 and I_2

From equation (i),

$$14\angle 0° - 2I_1 = 5(-10I_2) - 10I_2$$

$$-2I_1 + 60I_2 = -14\angle 0° \qquad \text{...(v)}$$

From equation (ii),

$$I_1 = 0.4(-10I_2) - I_2$$

$$I_1 + 5I_2 = 0 \qquad \text{...(vi)}$$

On solving equation (vi) and equation (v),

We get, $I_1 = 1\text{ A}$

$$I_2 = -0.2\text{ A}$$

Q.4 (c) Solution:

Full scale deflection = 90° = 100 scale divisions

$\therefore$ 1 scale division = 0.9° = 0.0157 rad.

(i) Restoring torque, $T_c = 98.1 \times 10^{-6} \times 100 \times 10^{-3}$

$= 9.81 \times 10^{-6}$ Nm

Deflection θ = 35 div.

= 35 × 0.0157 rad = 0.5495 rad.

$$\therefore \quad \text{Spring constant, } K = \frac{T_c}{\theta} = \frac{9.81 \times 10^{-6}}{0.5495}$$

$= 17.85 \times 10^{-6}$ Nm/rad

(ii) The deflecting torque $T_d = GI$ causes the motion, while the inertia torque $T_j = J\dfrac{d^2\theta}{dt^2}$, damping torque $T_D = D\dfrac{d\theta}{dt}$ and controlling torque $T_c = K\theta$ opposing the motion.

The equation of motion is given as

$$T_j + T_D + T_c = T_d$$

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = GI$$

Taking Laplace Transform, we obtain

$$\theta(s) = \frac{GI}{s(Js^2 + Ds + K)} = \frac{GI/J}{s\left(s^2 + \frac{D}{J}s + \frac{K}{J}\right)}$$

Comparing denominator with the second-order characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0,$$

we get $$\omega_n = \sqrt{\frac{K}{J}}$$

And $$\xi = \frac{D}{2\sqrt{JK}}$$

Period of free oscillations,

$$T_0 = \frac{2\pi}{\omega_n} = 2\pi\sqrt{J/K}$$

∴ Moment of Inertia, $J = \dfrac{KT_0^2}{4\pi^2}$

$$= \frac{17.85\times10^{-6}\times(0.55)^2}{4\pi^2} = 0.137\times10^{-6}\ \text{kg-m}^2$$

(iii) Control torque for a deflection of 35 scale divisions is 9.81×10^{-6} Nm.

∴ Control torque for full scale deflection (100 div.)

$$T_c = 9.81\times10^{-6}\times\frac{100}{35} = 28\times10^{-6}\,\text{Nm}$$

∴ Deflecting torque at full scale

$$T_d = NB\,ld\,I$$

$$= N \times 0.24 \times 15 \times 10^{-3} \times 14 \times 10^{-3} \times 1 \times 10^{-3}$$

$$= 50.4\ N \times 10^{-9}\ \text{Nm}$$

At equilibrium,

$$T_d = T_c \quad \text{or} \quad 50.4\,N \times 10^{-9} = 28 \times 10^{-6}$$

∴ Number of turns, $N = \dfrac{28\times10^{-6}}{50.4\times10^{-9}} = 556$

(iv) First maximum deflection

$$\theta_1 = 106 \text{ divisions} = 1.06\,\theta_F$$

We can write, $\theta_1 = \theta_F\left[1+\exp\left(-\pi\xi/\sqrt{1-\xi^2}\right)\right]$

or $1.06 = 1+\exp(-\pi\xi/\sqrt{1-\xi^2})$

$$\pi\xi/\sqrt{1-\xi^2} = 2.81$$

On solving, we get, damping ratio, $\xi = 0.667$

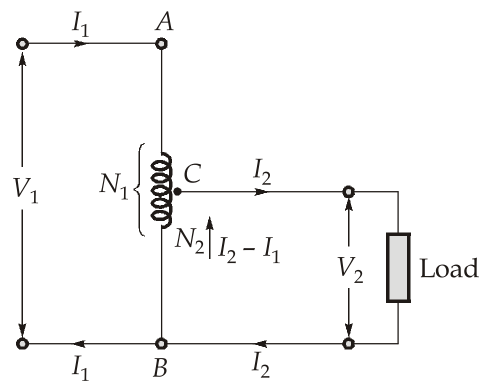
(v) Damping constant, $D = \xi\times2\sqrt{KJ}$

$$= 0.667\times2\times\sqrt{17.85\times10^{-6}\times0.137\times10^{-6}}$$

$$= 2.09 \times 10^{-6}\ \text{Nm/rad s}^{-1}$$

Section B

Q.5 (a) Solution:



(i) Transformation ratio of the autotransformer

$$K = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

where V_2 and V_1 are the secondary and primary voltages respectively.

$$\therefore \quad K = \frac{115}{230} = 0.5$$

(ii) Power output, $P_L = V_L I_L \cos\phi$

$\therefore$ Secondary current,

$$I_2 = I_L = \frac{P_L}{V_L \cos\phi} = \frac{5\times1000}{115\times1} = 43.48 \text{ A}$$

(iii) We have $K = \dfrac{I_1}{I_2}$

$\therefore$ Primary current, $I_1 = K \times I_2 = 0.5 \times 43.48 = 21.74$ A

(iv) Number of turns in the secondary winding,

$$N_2 = K \times N_1 = 0.5 \times 400 = 200$$

(v) Power transformed

$$P_{\text{trans}} = (1-K)\times\text{power output} = \left(1-\frac{1}{2}\right)\times 5 = 2.5 \text{ kW}$$

(vi) Power conducted

$$P_{\text{contd.}} = K\times\text{power output} = \frac{1}{2}\times 5 = 2.5 \text{ kW}$$

Q.5 (b) Solution:

$$\text{Oxide capacitance, } C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{10^{-6}} = 3.45\times10^{-7}\,\text{F/cm}^2$$

For $V_G = 5$ V, $V_D = 0.1$ V and $V_{DS} < (V_{GS} - V_T)$, hence the MOSFET is operating in the linear region. Thus,

$$I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_{GS}-V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

[Since source is grounded, $V_{GS} = V_G$ and $V_{DS} = V_D$]

$$= \frac{25}{1}(200)(3.45\times10^{-7})\left[(5-0.6)\times0.1 - \frac{1}{2}(0.1)^2\right]$$

$$= 7.51 \times 10^{-4}\ \text{A}$$

For $V_G = 3$ V, $V_D = 5$ V, $V_{DS} > V_{GS} - V_T$. Thus, the MOSFET is operating in the saturation region. We have,

$$I_D = \mu_n C_{ox}\frac{W}{2L}(V_{GS}-V_T)^2$$

$$= \frac{25}{2}(200)(3.45\times10^{-7})[(3-0.6)^2]$$

$$= 4.97 \times 10^{-3}\ \text{A}$$

For $V_D = 7$ V and $V_G = 3$ V, I_D will not increase, because MOSFET is in the saturation region.

Q.5 (c) Solution:

Multiplying power of shunt

$$m = I/I_m = 50/5 = 10$$

$$\therefore \quad \text{Resistance of shunt, } R_{sh} = \frac{R}{m-1} = \frac{0.09}{10-1} = 0.01\ \Omega$$

In order that the meter may read correctly at all frequencies, the time constants of meter and shunt circuits should be equal. Thus,

$$\frac{L}{R} = \frac{L_{sh}}{R_{sh}}$$

∴ Inductance of shunt,

$$L_{sh} = \frac{L}{R}R_{sh} = \frac{90}{0.09}\times0.01 = 10\ \mu\text{H}$$

With d.c., the current through the meter for a full-scale current of 50 A is,

$$I_m = \frac{R_{sh}}{R+R_{sh}} \times I = \frac{0.01}{0.09+0.01} \times 50 = 5.0 \text{ A}$$

With 50 Hz, the current through the meter for a full-scale current of 50 A is, (Assuming the shunt is non-inductive)

$$I_m = \frac{R_{sh}}{\sqrt{(R+R_{sh})^2 + \omega^2 L^2}} \times I$$

$$= \frac{0.01}{\sqrt{(0.09+0.01)^2 + (2\pi \times 50 \times 90 \times 10^{-6})^2}} \times 50$$

$$= 4.81 \text{ A}$$

Since the meter reading is proportional to the current,

$$\text{Error} = \frac{4.81-5}{5} \times 100$$

$$= -3.8\% \text{ or the meter reads } 3.8\% \text{ low.}$$

Q.5 (d) Solution:

Writing in signed 10's complement form

$$+9286 = 009286$$

$$+801 = 000801$$

$$-9286 = \begin{array}{r} 999999 \\ -9286 \\ \hline 990713 \\ +1 \\ \hline 990714 \\ \hline \end{array} \quad \begin{array}{l} \\ \\ \Rightarrow \text{9's complement form} \\ \\ \Rightarrow \text{10's complement form} \end{array}$$

$$-801 = \begin{array}{r} 999999 \\ -801 \\ \hline 999198 \\ +1 \\ \hline 999199 \\ \hline \end{array} \quad \begin{array}{l} \\ \\ \Rightarrow \text{9's complement form} \\ \\ \Rightarrow \text{10's complement form} \end{array}$$

(i) $(+9286) + (+801) = 009286 + 000801$

$= 010087$

Hexadecimal equivalent of result

$= (2767)_{16}$

16	10087	
16	630	7
16	39	6
	2	7

(ii) $(+9286) + (-801) = 009286 + 999199$

$= \textcircled{1}008485 = 008485$

(1 → Discard)

16	8485	
16	530	5
16	33	2
	2	1

Hexadecimal equivalent of result

$= (2125)_{16}$

(iii) $(-9286) + (+801) = 990714 + 000801$

$= 991515$

16	991515	
16	61969	B
16	3873	1
16	242	1
16	15 (F)	2

Hexadecimal equivalent of result

$= (F211B)_{16}$

(iv) $(-9286) + (-801) = 990714 + 999199$

$= \textcircled{1}989913 = 989913$

(1 → Discard)

16	989913	
16	61869	9
16	3866	D
16	241	A
	15 (F)	1

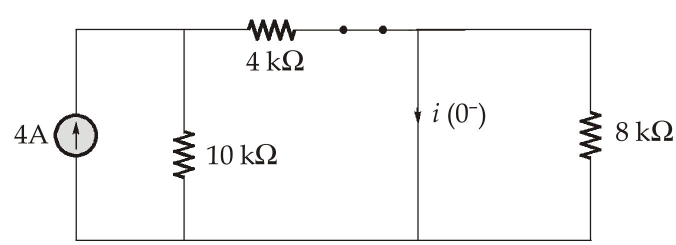
Hexadecimal equivalent of result = $(F1AD9)_{16}$

Q.5 (e) Solution:

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as short circuit. Thus,

$$i(0^-) = 4\left[\frac{10k}{10k+4k}\right] = 4\left[\frac{10}{14}\right]$$

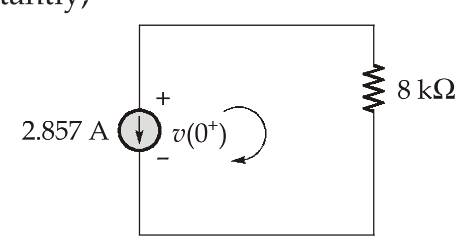
$$\Rightarrow \quad i(0^-) = 2.857 \text{ A}$$



At $t = 0^+$: Since the inductor current cannot change instantly,

$$i(0^-) = i(0^+) = 2.857 \text{ A}$$

Apply KVL around the loop:



$$v(0^+) + 8000\,(2.857) = 0$$

$$\Rightarrow \quad v(0^+) = -22856 \text{ V}$$

$$\therefore \quad \frac{di(0^+)}{dt} = \frac{v(0^+)}{L} = \frac{-22856}{4\times10^{-3}} = -5714\times10^{3} \text{ A/s}$$

For $t \geq 0$:

$\rightarrow$ A source-free RL circuit.

$$i(t) = i(0^+)e^{-t/\tau};\ t \geq 0$$

$$\text{Time-constant, } \tau = \frac{L}{R} = \frac{4\times10^{-3}}{8\times10^{3}} = \frac{1}{2}\times10^{-6}\text{ sec}$$

$\therefore$
$$i(t) = 2.857\ e^{-t/\left(\frac{1}{2}\times10^{-6}\right)}$$

$\Rightarrow$
$$i(t) = 2.857\ e^{-2t\times10^6} = 2.857\ e^{-(2\times10^6)t}\text{ A};\ t \geq 0$$

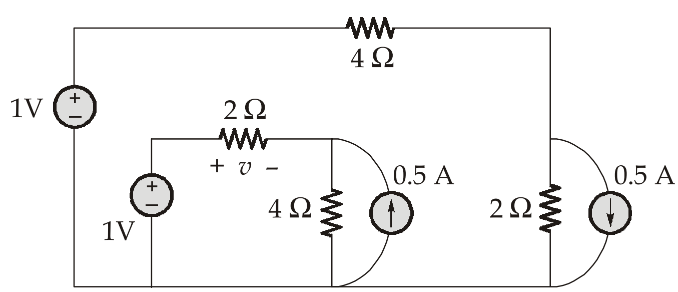
At $t = 1\ \mu s$:

$$i(1\ \mu s) = 2.857\,e^{-(2\times10^6)\times1\times10^{-6}} = 2.857\ e^{-2}$$

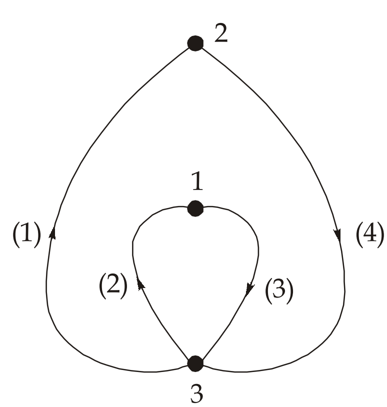
$\Rightarrow$
$$i(1\ \mu s) = 0.3867\text{ A}$$

Q.6 (a) Solution:

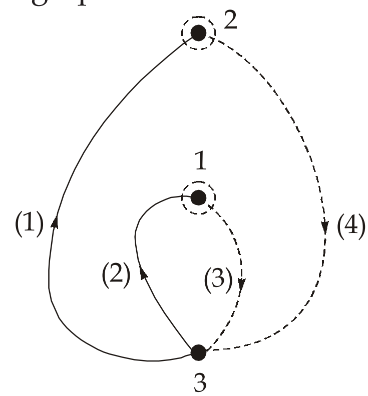
The voltage and current sources are converted into accompanied sources by source-shifting method.



The oriented graph is as shown below.



The selected tree for the oriented graph is shown below.



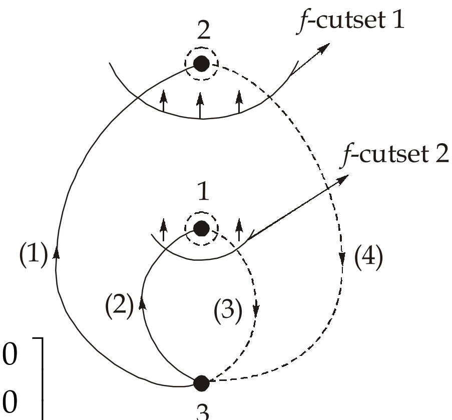
Twigs : {1, 2}

f-cutset 1 : {1, 4}

f-cutset 2 : {2, 3}

The f-cutset matrix is,

$$Q = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \end{array}$$



The KCL equation in the matrix form is given by,

$QY_bQ^TV_t = QI_S - QY_bV_S$

where Admitttance Matrix, $$Y_b = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$Q^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$I_s = \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix}$$

$$V_s = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$QY_b = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.25 & 0 & 0 & -0.5 \\ 0 & 0.5 & -0.25 & 0 \end{bmatrix}$$

$$QY_bQ^T = \begin{bmatrix} 0.25 & 0 & 0 & -0.5 \\ 0 & 0.5 & -0.25 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \end{bmatrix}$$

$$QI_s = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$$

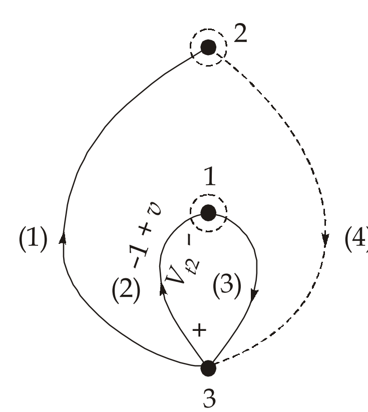
$$QY_bV_s = \begin{bmatrix} 0.25 & 0 & 0 & -0.5 \\ 0 & 0.5 & -0.25 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}$$

$$QI_s - QY_bV_s = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.25 \\ -1 \end{bmatrix}$$

Hence, the KCL equation can be written as,

$$QY_bQ^TV_t = QI_s - QY_bV_s$$

$$\begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \end{bmatrix}\begin{bmatrix} v_{t_1} \\ v_{t_2} \end{bmatrix} = \begin{bmatrix} 0.25 \\ -1 \end{bmatrix}$$



On solving the matrix equations,

$$0.75\, v_{t_1} = 0.25$$

$$\Rightarrow \quad v_{t_1} = \frac{0.25}{0.75} = 0.333 \text{ V}$$

$$0.75\, v_{t_2} = -1$$

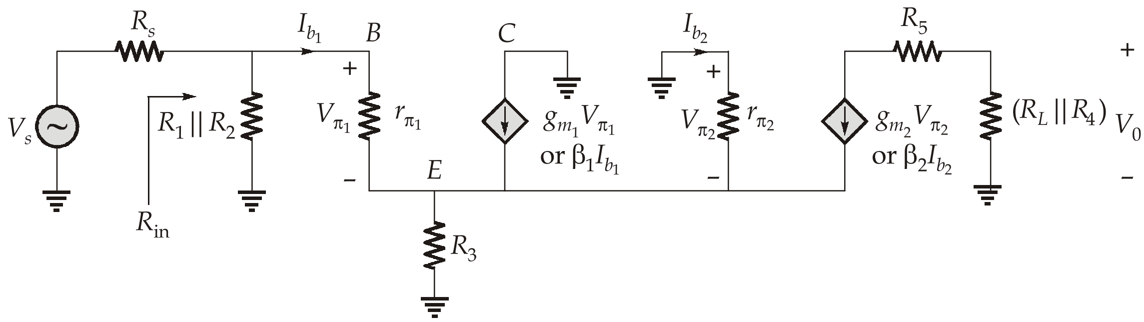
$$\Rightarrow \quad v_{t_2} = \frac{-1}{0.75} = -1.333 \text{ V}$$

From the figure,

$$v = 1 + v_{t_2} = 1 - 1.333 = -0.333 \text{ V}$$

Q.6 (b) Solution:

(i) Drawing small signal equivalent circuit,



(ii) Input resistance $R_{in} = \left(\dfrac{V_{in}}{I_{b_1}}\right) || (R_1 || R_2)$

Now from the circuit,

$$V_{in} = V_{\pi 1} - V_{\pi 2} \quad ...(1)$$

Applying KVL at Node E,

$$R_3(1 + \beta_1)I_{b1} + (1 + \beta_2)I_{b_2}R_3 = -V_{\pi 2}$$

$$\frac{R_3 V_{\pi_1}}{r_{\pi_1}}(1+\beta_1) = -V_{\pi_2}\left[1+\frac{R_3(1+\beta_2)}{r_{\pi_2}}\right] \quad \left[\because I_{b_2} = \frac{V_{\pi_2}}{r_{\pi_2}}\right]$$

We know that $r_\pi = (1 + \beta)r_e$

$$\therefore \quad \frac{R_3}{r_{e_1}}V_{\pi_1} = -V_{\pi_2}\left[\frac{r_{e_2}+R_3}{r_{e_2}}\right]$$

$$V_{\pi_2} = \frac{-R_3 r_{e_2}}{\left[r_{e_2}+R_3\right]} \times \frac{V_{\pi_1}}{r_{e_1}} \quad ...(2)$$

Put this value in equation (1)

$$V_{in} = V_{\pi_1} + \frac{R_3 r_{e_2} V_{\pi_1}}{r_{e_1}(r_{e_2}+R_3)}$$

$$= V_{\pi_1}\left[\frac{r_{e_1}+R_3 || r_{e_2}}{r_{e_1}}\right] \quad ...(3)$$

$$= \frac{r_{\pi_1} \cdot I_{b_1}}{r_{e_1}}\left[r_{e_1}+R_3 || r_{e_2}\right]$$

$$\frac{V_{in}}{I_{b_1}} = (1+\beta_1)\left(r_{e_1} + R_3 \| r_{e_2}\right)$$

$\therefore$ Input resistance, $R_{\text{in}} = [R_1 \| R_2] \| \left[(1+\beta_1)[r_{e_1} + (R_3 \| r_{e_2})]\right]$

(iii) Mid band voltage gain, $A_V = \dfrac{V_0}{V_s}$

$$V_0 = -g_{m_2} V_{\pi 2} (R_4 \| R_L)$$

Substitute the value of $V_{\pi 2}$ from equation (2)

$$V_0 = \frac{g_{m_2} V_{\pi 1}}{r_{e_1}} [R_4 \| R_L]\left(R_3 \| r_{e_2}\right) \quad \text{...(4)}$$

$$V_{\text{in}} = \frac{V_s R_{in}}{[R_{in} + R_s]}$$

$$V_s = V_{in} \frac{(R_{in} + R_s)}{R_{in}} \quad \text{...(5)}$$

From equation (3), $V_{\text{in}} = \dfrac{V_{\pi 1}\left[r_{e_1} + R_3 \| r_{e_2}\right]}{r_{e_1}}$

Substitute the value of V_{in} in equation (5)

$$V_s = V_{\pi 1} \left[\frac{R_{in} + R_s}{R_{in}}\right] \frac{\left(r_{e_1} + R_3 \| r_{e_2}\right)}{r_{e_1}} \quad \text{...(6)}$$

From equation (4) and (6)

$$\frac{V_0}{V_s} = \frac{g_{m_2} V_{\pi_1} (R_4 \| R_L)(R_3 \| r_{e_2}) R_{in} \times r_{e_1}}{r_{e_1} \times V_{\pi_1} (R_{in} + R_s) \times (r_{e_1} + R_3 \| r_{e_2})}$$

$$\frac{V_0}{V_s} = \frac{g_{m_2} (R_4 \| R_L)(R_3 \| r_{e_2})}{\left[r_{e_1} + R_3 \| r_{e_2}\right]} \times \frac{R_{in}}{(R_{in} + R_s)} = A_V$$

(iv) Lower cut-off frequency associated with C_1

$$\omega_{L_1} = \frac{1}{(R_s + R_{in}) C_1}$$

$$f_{L_1} = \frac{1}{2\pi (R_s + R_{in}) C_1}$$

where, $R_{\text{in}} = (R_1 \| R_2) \| \left[[1+\beta_1]\left[r_{e_1} + (R_3 \| r_{e_2}\right]\right]$

Lower cut-off frequency associated with C_2

$$\omega_{L_2} = \frac{1}{(R_L + R_4)C_2}$$

$$f_{L_2} = \frac{1}{2\pi(R_L + R_4)C_2}$$

Q.6 (c) Solution:

R.M.S value of supply voltage,

$$V = 100 \text{ V}$$

Maximum value of voltage, $V_m = \sqrt{2}\times 100$ V.

Instantaneous value of voltage,

$$v = \sqrt{2}\times 100 \sin\theta \text{ V}$$

Resistance in the forward direction

$$R_f = 50 + 50 = 100\ \Omega$$

Resistance in the reverse direction

$$R_f = 250 + 50 = 300\ \Omega$$

Instantaneous value of current in the forward direction

$$i_f = \frac{V_m}{R_f} = \frac{\sqrt{2}\times 100}{100}\sin\theta = \sqrt{2}\sin\theta$$

Instantaneous value of current in the reverse direction

$$i_r = \frac{V_m}{R_f} = \frac{\sqrt{2}\times 100}{300}\sin\theta = \frac{\sqrt{2}}{3}\sin\theta \text{ A}$$

The moving coil ammeter reads the average value of the current and dynamometer type ammeter reads the R.M.S value of the current.

For half of the sinusoidal cycle, the current flows in the forward direction and in the other half of the sinusoidal cycle, the current flows in the reverse direction. Thus, R.M.S value of current,

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi}(\sqrt{2}\sin\theta)^2 d\theta + \int_{\pi}^{2\pi}\left(\frac{\sqrt{2}}{3}\sin\theta\right)^2 d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi}(1-\cos 2\theta)d\theta + \frac{1}{9}\int_{\pi}^{2\pi}(1-\cos 2\theta)d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi} + \frac{1}{9}\left(\theta - \frac{\sin 2\theta}{2}\right)_{\pi}^{2\pi}\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left(\pi + \frac{\pi}{9}\right)} = \frac{\sqrt{5}}{3}$$

$$= 0.745 \text{ A}$$

Average value of current, $$I_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} \sqrt{2}\sin\theta d\theta + \int_{\pi}^{2\pi} \frac{\sqrt{2}}{3}\sin\theta d\theta\right]$$

$$= \frac{1}{2\pi}\left[\sqrt{2}(-\cos\theta)_0^{\pi} + \frac{\sqrt{2}}{3}(-\cos\theta)_{\pi}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[2\sqrt{2} - \frac{2\sqrt{2}}{3}\right] = \frac{2\sqrt{2}}{3\pi}$$

$$= 0.3 \text{ A}$$

∴ Reading of dynamometer ammeter = 0.745 A

Reading of moving coil ammeter = 0.3 A

Power taken from the mains = Power supplied in the forward half cycle

+ Power supplied in the backward half cycle

$$= \frac{1}{2}\left(\frac{V^2}{R_f} + \frac{V^2}{R_r}\right) = \frac{1}{2}\left[\frac{(100)^2}{100} + \frac{(100)^2}{300}\right] = 66.7 \text{ W}$$

Power consumed in 50 ohm resistor = $I_{rms}^2 R$ = $(0.745)^2 \times 50 = 27.7$ W

Power dissipated in rectifier = Total power supplied – Power consumed in resistor

= 66.7 – 27.7 = 39 W

Q.7 (a) Solution:

(i) Let L = Length of the plot = 150 m,

B = Width of the plot = 50 m

and A = area of the plot

$= L \times B = 150 \times 50 = 7500 \text{ m}^2$.

Now $A = LB$

∴ $$\frac{\partial A}{\partial L} = B \text{ and } \frac{\partial A}{\partial B} = L$$

Uncertainty of area, $w_A = \pm\sqrt{\left(\frac{\partial A}{\partial L}\right)^2 w_L^2 + \left(\frac{\partial A}{\partial B}\right)^2 w_B^2} = \pm\sqrt{B^2 w_L^2 + L^2 w_B^2}$

Case I. When there is no uncertainty in measurement of L, $w_L = 0$.

Thus, uncertainty in measurement of area

$$w_A = \pm\sqrt{B^2 w_L^2 + L^2 w_B^2} = \pm\sqrt{L^2 w_B^2}$$

$$= \pm L w_B = \pm 150 \times 0.01 = \pm 1.5 \text{ m}^2$$

Case II. When there is uncertainty in measurement of L.

The uncertainty in area is not to exceed

$$1.5 \times 1.5 = \pm 2.25 \text{ m}^2$$

We have, $w_A = \pm\sqrt{B^2 w_L^2 + L^2 w_B^2}$

or $2.25 = \pm\sqrt{(50)^2 w_L^2 + (150)^2 (0.01)^2} \Rightarrow w_L = \pm 0.0335 \text{ m}$

Hence, uncertainty in measurement of L is $w_L = \pm 0.0335$ m

(ii) Excitation table for conversion of D Flip Flop to SR Flip Flop:

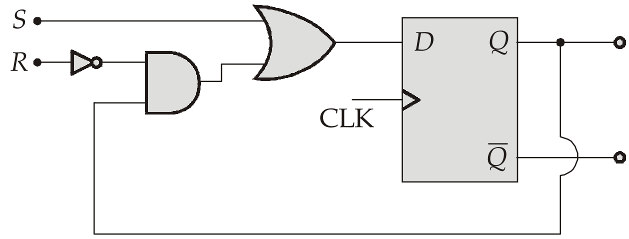
S	R	Q	$D = Q^+$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	X
1	1	1	X

} As $S = R = 1$ is an invalid input

K-map:

S \ RQ	00	01	11	10
0	0	1	0	0
1	1	1	X	X

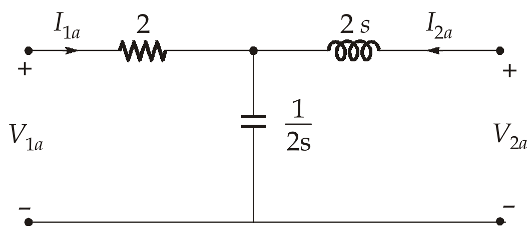
$$D = S + \bar{R}Q$$



Q.7 (b) Solution:

(i) The given network can be considered as a cascade connection of two networks N_1 and N_2.

For the network N_1: Drawing the circuit in s-domain,



Applying KVL to mesh 1,

$$V_{1a} = \left(2+\frac{1}{2s}\right)I_{1a}+\frac{1}{2s}I_{2a} \quad \text{...(i)}$$

Applying KVL to mesh 2,

$$V_{2a} = \frac{1}{2s}I_{1a}+\left(2s+\frac{1}{2s}\right)I_{2a} \quad \text{...(ii)}$$

From the equation (ii),

$$I_{1a} = 2sV_{2a} - (4s^2+1)\, I_{2a} \quad \text{...(iii)}$$

Substituting the equation (iii) in equation (i),

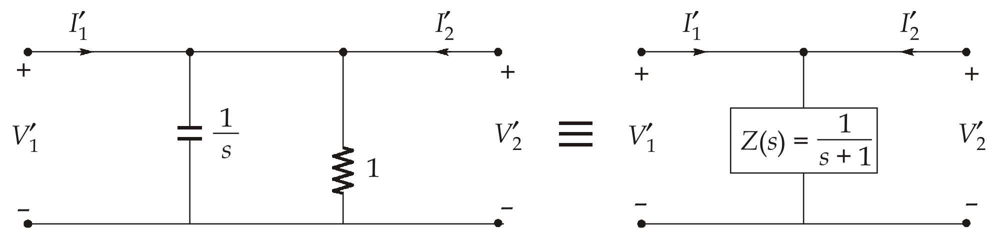
$$V_{1a} = \left(2+\frac{1}{2s}\right)\left[2sV_{2a}-\left(4s^2+1\right)I_{2a}\right]+\frac{1}{2s}I_{2a}$$

$$V_{1a} = (4s+1)V_{2a}-\left(8s^2+2s+2\right)I_{2a} \quad \text{...(iv)}$$

Comparing equations (iv) and (iii) with *ABCD* parameter equations, we get

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 4s+1 & 8s^2+2s+2 \\ 2s & 4s^2+1 \end{bmatrix}$$

For the network N_2 : Drawing the circuit in s-domain,



Applying KVL to mesh 1,

$$V_1' = \frac{1}{s+1} I_1' + \frac{1}{s+1} I_2' \quad \text{...(v)}$$

Applying KVL to mesh 2,

$$V_2' = \frac{1}{s+1} I_1' + \frac{1}{s+1} I_2' \quad \text{...(vi)}$$

From the equation (vi),

$$I_1' = (s+1)\, V_2' - I_2' \quad \text{...(vii)}$$

Also, $$V_1' = V_2' \quad \text{...(viii)}$$

Comparing equations (viii) and (vii) with *ABCD* parameter equations, we get

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

The equivalent *ABCD* parameters for the cascade network is the product of the individual *ABCD* parameters matrix. Hence, overall *ABCD* parameters are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$= \begin{bmatrix} 4s+1 & 8s^2+2s+2 \\ 2s & 4s^2+1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8s^3+10s^2+8s+3 & 8s^2+2s+2 \\ 4s^3+4s^2+3s+1 & 4s^2+1 \end{bmatrix}$$

(ii) The load impedance (16 – j24) Ω can be reflected to the primary side and we get,

$$Z_L' = \frac{(16-j24)}{n^2} = \frac{(16-j24)}{\left(\frac{4}{1}\right)^2} = \frac{(16-j24)}{16} = (1 - j1.5)\ \Omega$$

Thus, $$Z_{in} = 2 + Z_L' = 2 + (1 - j1.5) = (3 - j1.5)\ \Omega$$

$$I_1 = \frac{240\angle 0°}{Z_{in}} = \frac{240\angle 0°}{(3 - j1.5)} = (64 + j32)A$$

The complex power supplied by the source is,

$$S = V_S I_1^*$$

$$= (240\angle 0°) \times (64 + j32)^*$$

$$= (240\angle 0°) \times (64 - j32)$$

$$= (15360 - j7680)$$

$$= 17173\angle -26.57° \text{ VA}$$

$\Rightarrow$ $$S = 17.173\angle -26.57° \text{ kVA}$$

Q.7 (c) Solution:

(i) The solution of this problem involves three steps:

1. To obtain the Q point at 30°C.
2. To obtain the Q point at 80°C.
3. To calculate the percent change in Q point values.

Step 1: To obtain the Q point at 30°C:

Obtaining Q point values means to calculate V_{CEQ} and I_{CQ}. At 30°C, $\beta_{dc} = 100$.

Applying KVL to the base circuit, we get,

$$V_{CC} - I_B R_B - V_{BE} = 0$$

$\therefore$ $$I_B = \frac{V_{CC} - V_{BE}}{R_B} \quad ...(1)$$

Substituting the values we get,

$$I_B = \frac{12 - 0.7}{100 \times 10^3} = 113\ \mu\text{A} \quad ...(2)$$

Now, neglecting I_{CEO}, we can write

$$I_{CQ} = \beta_{dc} I_B$$

Substituting the values we get,

$$I_{CQ} = 100 \times 113 \times 10^{-6} = 11.3 \text{ mA} \quad ...(3)$$

Apply KVL to the collector circuit, we get

$$V_{CC} - V_{CEQ} - I_{CQ} R_C = 0$$

$\therefore$ $$V_{CEQ} = V_{CC} - I_{CQ} R_C$$

Substituting the values we get,

$$V_{CEQ} = 12 - (11.3 \times 10^{-3} \times 500) = 6.35 \text{ V} \quad ...(4)$$

Thus, the Q point values at 30°C are:

$$Q \text{ point at } 30°C = (V_{CEQ}, I_{CQ}) = (6.35 \text{ V}, 11.3 \text{ mA})$$

Step 2: To obtain the Q point at 80°C:

At 30°C, β_{dc} = 120°. Now from equation (2), I_B = 113 μA.

Therefore the new value of I_{CQ} at 80°C is given by,

$$I_{CQ}(80°C) = \beta_{dc} \times I_B = 120 \times 113 \times 10^{-6}$$

$$\therefore \quad I_{CQ}(80°C) = 13.56 \text{ mA} \quad ...(5)$$

Applying KVL to collector circuit, we get,

$$V_{CEQ}(80°C) = V_{CC} - I_{CQ}R_C = 12 - (13.56 \times 10^{-3} \times 500)$$

$$\therefore \quad V_{CEQ}(80°C) = 5.22 \text{ V} \quad ...(6)$$

Thus, the Q point values at 80°C are:

$$Q \text{ point at } 80°C = (V_{CEQ}, I_{CQ}) = (5.22 \text{ V}, 13.56 \text{ mA})$$

Step 3: To calculate percent change in Q point values:

$$\text{Percent change in } I_{CQ} = \frac{I_{CQ}(80°C) - I_{CQ}(30°C)}{I_{CQ}(30°C)} \times 100\%$$

$$= \frac{13.56 - 11.3}{11.3} \times 100\%$$

$$\therefore \quad \text{Percent change in } I_{CQ} = 20\% \text{ (increase)}$$

$$\text{Percent change in } V_{CEQ} = \frac{V_{CEQ}(80°C) - V_{CEQ}(30°C)}{V_{CEQ}(30°C)} \times 100\%$$

$$= \frac{5.22 - 6.35}{6.35} \times 100\%$$

$$\therefore \quad \text{Percent change in } V_{CEQ} = -17.795\% \text{ (decrease)}$$

(ii) The stability factor S' is defined as:

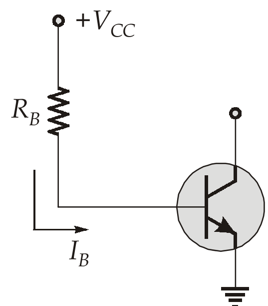
$$S' = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{I_{CO} \text{ and } \beta_{dc} \text{ constant}}$$

For a common emitter configuration, we know that,

$$I_C = \beta_{dc} I_B + (1 + \beta_{dc})I_{CO}$$

$$\therefore \quad I_B = \frac{I_C - (1+\beta_{dc})I_{CO}}{\beta_{dc}} \quad \text{...(i)}$$

Now consider the base circuit of the fixed bias circuit and applying KVL to it, we get:



$$V_{CC} - I_B R_B - V_{BE} = 0$$

$$\therefore \quad V_{CC} = V_{BE} + I_B R_B$$

Substitute the value of I_B from equation (i), we get,

$$V_{CC} = V_{BE} + R_B\left[\frac{I_C - (1+\beta_{dc})I_{CO}}{\beta_{dc}}\right] \quad \text{...(ii)}$$

Assuming β_{dc} and I_{CO} to be constants, differentiate equation (ii) w.r.t I_C,

$$0 = \frac{\partial V_{BE}}{\partial I_C} + R_B\left[\frac{1}{\beta_{dc}}\right]$$

But $\frac{\partial V_{BE}}{\partial I_C} = \frac{1}{S'}$

$$\therefore \quad 0 = \frac{1}{S'} + R_B\left[\frac{1}{\beta_{dc}}\right]$$

$$\therefore \quad S' = \frac{-\beta_{dc}}{R_B} \quad \text{...(iii)}$$

Relation between S and S':

The stability factor 'S' is defined as:

$$S = \left.\frac{\partial I_C}{\partial I_{CO}}\right|_{V_{BE},\beta_{dc} = \text{constant}}$$

We have, $I_C = \beta_{dc} + (1 + \beta_{dc})I_{CO}$

Differentiating w.r.t I_{CO}, we get

$$S = 1 + \beta_{dc}$$

Now, from equation (iii), substitute $\beta_{dc} = -S'R_B$ to get,

$$S = 1 - S'R_B \quad \Rightarrow \quad S' = \frac{(1-S)}{R_B}$$

Q.8 (a) Solution:

(i) Advantages of Autotransformers:

1. An autotransformer uses less winding material than a 2-winding transformer. The saving is large if the transformation ratio is small.
2. An autotransformer is smaller in size and cheaper than the two winding transformer of the same output.
3. Since there is a reduction in conductor and core materials, the ohmic losses in conductor and the core losses are smaller. Thus, an autotransformer has higher efficiency than the equivalent 2-winding transformer.
4. Since one winding has been completely eliminated, the resistance and leakage flux of this winding are zero. Hence the voltage regulation of the autotransformer is superior because of reduced voltage drops in the resistance and reactance.
5. An autotransformer has variable output voltage when a sliding contact is used for the secondary.

Disadvantages of Autotransformers:

1. There is a direct connection between the high-voltage and low-voltage sides. In case of an open circuit in the common winding, the full primary voltage would be applied to the load on the secondary. This high voltage may burn out or seriously damage equipment connected to the secondary side.
2. The effective per-unit impedance of an autotransformer is smaller compared to a 2-winding transformer. The reduced internal impedance results in a larger short-circuit (fault) current.
3. In an autotransformer, there is a loss of isolation between input and output circuits. This is particularly important in three-phase transformers where one may wish to use a different winding and earthing arrangement on each side of the transformer.

(ii) Given Impedance drop, $\dfrac{I_2 Z_{e2}}{V_2}\times 100 = 10\%$

$$\Rightarrow \qquad I_2 Z_{e2} = \frac{10V_2}{100} = \frac{10\times 200}{100} = 20 \text{ V}$$

Resistance Drop, $\dfrac{I_2 R_{e2}}{V_2}\times 100 = 5$

$$I_2 R_{e2} = \frac{5V_2}{100} = \frac{5\times 200}{100} = 10 \text{ V}$$

$$I_2 X_{e2} = \sqrt{(I_2 Z_{e2})^2 - (I_2 R_{e2})^2} = \sqrt{20^2 - 10^2} = 17.32 \text{ V}$$

1. Approximate voltage regulation at lagging power factor

$$= \frac{I_2 R_{e2}\cos\phi_2 + I_2 X_{e2}\sin\phi_2}{V_2}$$

[Given, $\cos\phi_2 = 0.8 \Rightarrow \sin\phi_2 = 0.6$]

$$= \frac{10\times 0.8 + 17.32\times 0.6}{200}$$

$$= 0.0919 \text{ pu} = 0.0919 \times 100\% = 9.19\%$$

2. For zero regulation, the power factor must be leading

$$\therefore \quad \frac{I_2 R_{e2}\cos\phi_2 - I_2 X_{e2}\sin\phi_2}{V_2} = 0$$

or $$\tan\phi_2 = \frac{I_2 R_{e2}}{I_2 X_{e2}} = \frac{10}{17.32} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$\therefore$ power factor $\cos\phi_2 = \cos 30^\circ = 0.866$ (leading)

Q.8 (b) Solution:

(i) Using Clausius-Mossotti relation:

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{n\alpha}{3\varepsilon_0},$$

where 'n' represents the number of atoms per unit volume and α is the polarizability.

We have, $$n = \frac{N_A \rho}{M}$$

where N_A = Avogadro's number, ρ = density and M = molecular weight.

Substituting, we get:

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{\alpha N_A \rho}{3\varepsilon_0 M}$$

Substituting, the values, we get:

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{6.023\times10^{23}\times2.08\times10^{6}\times3.28\times10^{-40}}{3\times8.854\times10^{-12}\times32} = 0.483$$

On solving, we get $\varepsilon_r = 3.8$.

(ii) Let susceptibility and magnetisation in copper and Fe_2O_3 be χ_{m1}, M_1 and χ_{m2}, M_2 respectively. We have:

$$\chi_{m1} = \frac{M_1}{H} \text{ and } \chi_{m2} = \frac{M_2}{H} \text{ or } M_1 = \chi_{m1} H \text{ and } M_2 = \chi_{m2}H$$

Substituting the given values,

$$\chi_{m1} = -0.5\times10^{-5},\ \chi_{m2} = 1.4\times10^{-3} \text{ and } H = 10^6 \text{ Am}^{-1}$$

We get, $M_1 = -0.5\times10^{-5}\times10^6 = -5\ Am^{-1}$

and $M_2 = 1.4\times10^{-3}\times10^6 = 1.4\times10^3 = 1400\ Am^{-1}$

Similarly, let the flux densities in copper and Fe_2O_3 be B_1 and B_2 respectively, we have:

$$B_1 = \mu_0 H + \mu_0 M_1 = \mu_0(H + M_1)$$

and

$$B_2 = \mu_0 H + \mu_0 M_2 = \mu_0(H + M_2)$$

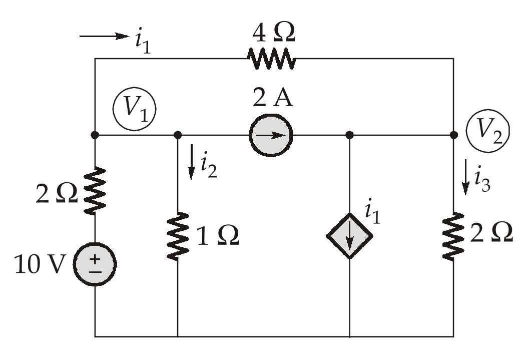
Substituting the values, we have:

$$B_1 = 4\pi\times10^{-7}(10^6 - 5) = 1.256 \text{ Wbm}^{-2}$$

$$B_2 = 4\pi\times10^{-7}(10^6 + 1400) = 1.258 \text{ Wbm}^{-2}$$

Q.8 (c) Solution:

(i) Let V_1 and V_2 are node voltages.



Apply KCL at node V_1:

$$\frac{V_1 - 10}{2} + \frac{V_1}{1} + 2 + \frac{V_1 - V_2}{4} = 0$$

$$2V_1 - 20 + 4V_1 + 8 + V_1 - V_2 = 0$$

$$7V_1 - V_2 = 12 \quad \text{...(i)}$$

Apply KCL at node V_2:

$$-2 + i_1 + \frac{V_2}{2} + \frac{V_2 - V_1}{4} = 0$$

$$-8 + 4i_1 + 2V_2 + V_2 - V_1 = 0$$

$$-V_1 + 3V_2 + 4i_1 = 8$$

$$-V_1 + 3V_2 + 4\left[\frac{V_1 - V_2}{4}\right] = 8 \qquad \left\{\text{We have, } i_1 = \frac{V_1 - V_2}{4}\right\}$$

$$\Rightarrow \quad 2V_2 = 8$$

$$\Rightarrow \quad V_2 = 4 \text{ V}$$

From equation (i),

$$7V_1 - 4 = 12$$

$$\Rightarrow \quad V_1 = \frac{16}{7} \text{V}$$

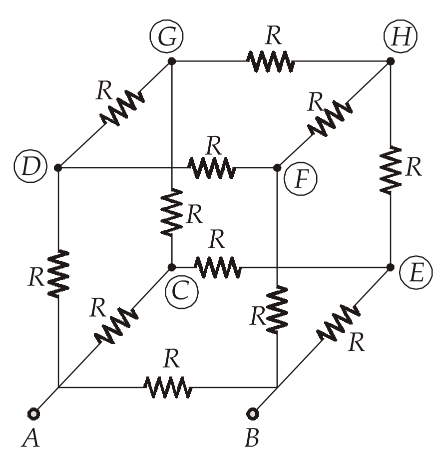
Thus,

$$i_1 = \frac{V_1 - V_2}{4} = \frac{\frac{16}{7} - 4}{4} = -0.429 \text{ A}$$

$$i_2 = \frac{V_1}{1} = \frac{16}{7} = 2.29 \text{A}$$

$$i_3 = \frac{V_2}{2} = \frac{4}{2} = 2 \text{A}$$

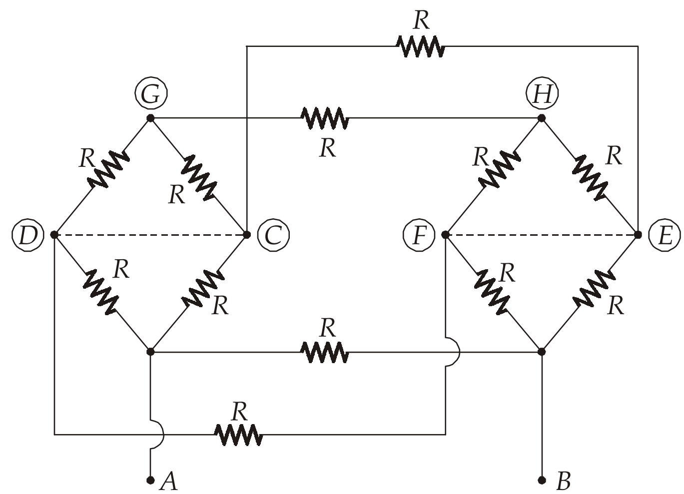
(ii) Observe the equipotential points and convert them into single points.



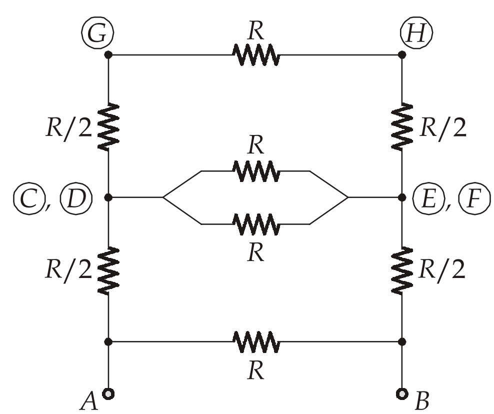
From the above figure, (C) and (D) are equipotential points.

(E) and (F) are equipotential points.

The given circuit can be redrawn as follows:



After combining the equipotential points.



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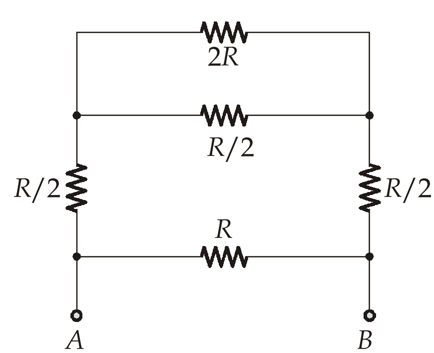

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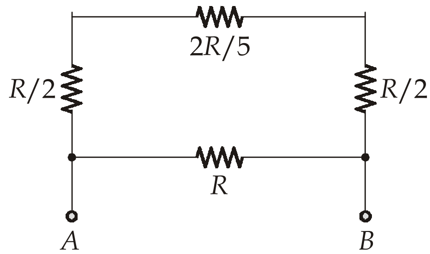

∴ $$R_{AB} = \left[\frac{R}{2}+\frac{2R}{5}+\frac{R}{2}\right] \Big\| R = \frac{7R}{5} \Big\| R = \frac{7R}{12}\,\Omega$$

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