Detailed Solutions

ESE-2025 Mains Test Series

Electrical Engineering Test No : 11

Section-A

Q.1 (a) Solution:

For solving such type of circuit, the direction of current should be considered in the same direction in different loops. Let I_1 and I_2 be the current in mesh-1 and mesh-2 respectively. And their direction as shown in figure for convenient.

Now applying KVL in mesh-1:

$$2(I_1 + I_2) + j5(I_1 + I_2) + j2I_2 = V_{ab}\angle 0°$$

or, $$(2 + j5)I_1 + (2 + j7)I_2 = V_{ab}\angle 0° \quad \text{...(i)}$$

By KVL in mesh-2

$$j_2(I_1 + I_2) + (2 + j5)I_2 + 2(I_1 + I_2) + j5(I_1 + I_2) + j2I_2 = 0$$

$$(2 + j7)I_1 + (4 + j14)I_2 = 0 \quad \text{...(ii)}$$

The above two equations can be written in the matrix form

$$\begin{bmatrix} 2+j5 & 2+j7 \\ 2+j7 & 4+j14 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{ab}\angle 0° \\ 0 \end{bmatrix}$$

Now, $$I_1 = \frac{\begin{vmatrix} V_{ab}\angle 0° & 2+j7 \\ 0 & 4+j14 \end{vmatrix}}{\begin{vmatrix} 2+j5 & 2+j7 \\ 2+j7 & 4+j14 \end{vmatrix}}$$

or, $$I_1 = \frac{(V_{ab}\angle 0°)(14.56\angle 74.05°)}{78.33\angle 142.24 - 53\angle 148.1} = \frac{(V_{ab}\angle 0°)(14.56\angle 74.05°)}{26.24\angle 130.36°}$$

or, $$Z_{in} = \frac{V_{ab}}{I_1}$$

∴ $$Z_{in} = \frac{26.24\angle 130.36°}{14.56\angle 74.05°} = 1.8\angle 56.31°$$

$$Z_{in} = 1 + j1.5\ \Omega$$

Q.1 (b) Solution:

Given : $$\lambda = 0.7\ \text{Å} = 0.7 \times 10^{-10}\ \text{m}$$

$$n = 1,\ (h, k, l) = (3, 0, 2),\ \theta = 35°$$

We know, $$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{a}{\sqrt{3^2 + 0 + 2^2}} = \frac{a}{\sqrt{13}}$$

From Bragg's law,

$$n\lambda = 2d \sin\theta$$

$$d = \frac{\lambda}{2\sin\theta}$$

$$d = \frac{0.7 \times 10^{-10}}{2 \times \sin(35°)}$$

$$d = 0.6102 \times 10^{-10}\ \text{m}$$

But $$d = \frac{a}{\sqrt{13}}$$

$$a = 0.6102 \times 10^{-10} \times \sqrt{13}$$

$$= 2.2 \times 10^{-10}$$

$$a = 2.2\ \text{Å}$$

Q.1 (c) Solution:

Case I: When A plays 3 games against B.

In this case, we have $n = 3$, $p = 0.4$ and $q = 0.6$,

Let X denote the number of wins,

Then $$P(X = r) = {}^3C_r (0.4)^r (0.6)^{3-r}\ ;\ r = 0, 1, 2, 3$$

∴ $$P_1 = \text{Probability of winning the best of 3 games}$$

$$= P(X \geq 2)$$

$$= P(X = 2) + P(X = 3)$$

$$= {}^3C_2(0.4)^2(0.6)^1 + {}^3C_3(0.4)^3(0.6)^0$$

$$= 0.288 + 0.064 = 0.352$$

Case II: When A plays 5 games against B

In this case, we have

$$n = 5, \qquad P = 0.4$$

and $q = 0.6$

Let X denotes the number of wins in 5 games

$$= P(X \geq 3)$$

$$= P(X = 3) + P(X = 4) + P(X = 5)$$

$$= {}^5C_3(0.4)^3(0.6)^2 + {}^5C_4(0.4)^4(0.6) + {}^5C_5(0.4)^5(0.6)$$

$$= 0.2304 + 0.0768 + 0.1024$$

$$= 0.31744$$

Clearly, $P_1 > P_2$. Therefore case-I i.e, best of 3 games has higher probability of winning the match.

Q.1 (d) Solution:

Given : Gain, $A_v = 52 \text{ dB}, \beta = 1\% = 0.01$

$$A_v = 10^{52/20} = 398.107$$

$$f_L = 200 \text{ Hz}, f_U = 18 \text{ kHz}$$

Gain with feedback,

$$A_{vf} = \frac{A_v}{1 + A_v\beta} = \frac{398.107}{1 + 398.107 \times 0.01} = 79.924$$

$$\text{Gain in dB} = 20 \log(79.924)$$

$$= 38.05 \text{ dB}$$

Due to negative feedback lower 3-dB frequency decreases and upper 3-dB frequency increases, i.e.,

$$f_{Lf} = \frac{f_L}{1 + A_v\beta} = \frac{200}{1 + 398.107 \times 0.01}$$

$$f_{Lf} = 40.15 \text{ Hz}$$

$$f_{Uf} = f_U(1 + A_v\beta)$$

$$= 18 \times 10^3(1 + 398.107 \times 0.01)$$

$$f_{Uf} = 89.66 \text{ kHz}$$

Bandwidth with feedback

$$BW_f = f_{Uf} - f_{Lf}$$
$$= 89.66 \times 10^3 - 40.15$$
$$BW_f = 89.61985 \times 10^3$$
$$BW_f \cong 89.62 \text{ kHz}$$

Q.1 (e) Solution:

Voltage across instrument for full scale deflection = 100 mV

Current in instrument for full scale deflection

$$I = \frac{V}{R} = \frac{100 \times 10^{-3}}{20} = 5 \times 10^{-3} \text{ A}$$

Deflecting torque, $T_d = NBl\,dI$

$$= 100 \times B \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 5 \times 10^{-3}$$
$$= 375 \times 10^{-8}\ \Omega\text{m}$$

$\therefore$ Controlling torque for a deflection,

$$\theta = 120°$$
$$T_c = K\theta = 0.375 \times 10^{-6} \times 120$$
$$= 45 \times 10^{-6} \text{ N-m}$$

At final steady position, $T_d = T_c$

or $375 \times 10^{-6} B = 45 \times 10^{-6}$

$\therefore$ Flux density in the airgap, $B = \dfrac{45 \times 10^{-6}}{375 \times 10^{-6}} = 0.12 \text{ Wb/m}^2$

Resistance of coil winding, $R_c = 0.3 \times 20 = 6\ \Omega$

Length of mean turn, $L_{mt} = 2(l + d) = 2(30 + 25) = 110$ mm

Let a be the area of cross-section of wire and ρ be the resistivity

Resistance of coil, $R_c = N\rho \dfrac{L_{mt}}{a}$

$\therefore$ Area of cross-section of wire, $a = \dfrac{100 \times 1.7 \times 10^{-8} \times 110 \times 10^{-3}}{6} \times 10^6$

$$= 31.37 \times 10^{-3} \text{ mm}^2$$

Diameter of wire, $d = \left[\left(\dfrac{4}{\pi}\right)(31.37 \times 10^{-3})\right]^{1/2} = 0.2 \text{ mm}$

Q.2 (a) Solution:

```
# include <stdio.h>
# include <conio.h>
void main ( )
{
   int mat1 [3] [3], mat2 [3] [3], mat3 [3] [3], sum=i, j, k
   printf("Enter the first 3*3 matrix element:  ");
   for (i=0, i<3; i++)
      {
         for (j=0; j<3; j++)
         scanf("%d", & mat1[i][j]);
      }
   printf("Enter the second 3*3 matrix elements: ");
   for (i=0, i<3; i++)
      {
         for (j=0; j<3; j++)
         scanf ("%d", & mat2[i][j]);
      }
   printf ("\n Multiplying two matrices ");
      for (i=0; i<3; i++)
      {
         for (j=0; j<3; j++)
         {
             sum=0;
             for (k=0, k<3; k++)
             sum=sum + mat1[i][k] * mat2[k][j];
             mat3[i][j] = sum;
         }
      }
   printf("\n Multiplication result of two matrix is: \n");
      for (i=0; i<3; i++)
```

```
    {
        for(j=0; j<3; j++)
        printf("%d\t", mat3[i][j]);
        printf("\n");
    }
    getch();
}
```

Q.2 (b) Solution:

$$\vec{B}_1 = 3\hat{a}_x + 1\hat{a}_y \text{ (T)}$$

$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = 4.77\times10^5\hat{a}_x + 1.59\times10^5\hat{a}_y$$

Unit vector normal to the plane $y + z = 1$ is :

$$\hat{n} = \frac{1}{\sqrt{1^2+1^2}}(\hat{a}_y + \hat{a}_z) = \frac{1}{\sqrt{2}}\hat{a}_y + \frac{1}{\sqrt{2}}\hat{a}_z$$

$$\vec{H}_{1t} = \vec{H}_1 - H_{1n}\hat{n}$$

$$H_{1n} = \vec{H}_1\cdot\hat{n} = \left(4.77\times10^5\hat{a}_n + 1.59\times10^5\hat{a}_y\right)\cdot\left(\frac{1}{\sqrt{2}}\hat{a}_y + \frac{1}{\sqrt{2}}\hat{a}_z\right)$$

$\Rightarrow$ $$\vec{H}_{1n} = (1.1243\times10^5)\hat{n} = 1.1243\times10^5\left(\frac{1}{\sqrt{2}}\hat{a}_y + \frac{1}{\sqrt{2}}\hat{a}_z\right)$$

And $$\vec{H}_{1t} = \vec{H}_1 - \vec{H}_{1n}$$

$$\vec{H}_{1t} = \left(4.77\hat{a}_x + 1.59\hat{a}_y - \frac{1.1243}{\sqrt{2}}\hat{a}_y - \frac{1.1243}{\sqrt{2}}\hat{a}_z\right)\times10^5$$

$\Rightarrow$ $$\vec{H}_{1t} = \left(4.77\hat{a}_x + 0.795\hat{a}_y - 0.795\hat{a}_z\right)\times10^5$$

$$B_{1n} = \bar{B}_1\cdot\hat{n} = (3\hat{a}_x + \hat{a}_y)\cdot\left(\frac{\hat{a}_y}{\sqrt{2}} + \frac{\hat{a}_z}{\sqrt{2}}\right)$$

$$B_{1n} = 0.707 = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$\bar{B}_{1n} = 0.707\hat{n} = \frac{1}{\sqrt{2}}\hat{n} = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\hat{a}_y + \frac{1}{\sqrt{2}}\hat{a}_z\right) = \frac{\hat{a}_y + \hat{a}_z}{2}$$

$$\Rightarrow \qquad \bar{B}_{1t} = \bar{B}_1 - \bar{B}_{1n} = 3\hat{a}_n + \hat{a}_y - \frac{\hat{a}_y + \hat{a}_z}{2}$$

$$= 3\hat{a}_x + 0.5\hat{a}_y - 0.5\hat{a}_z$$

Boundary Conditions :

$$\bar{B}_{1n} = \bar{B}_{2n} = 0.5\hat{a}_y + 0.5\hat{a}_z$$

$$\therefore \qquad \vec{H}_{1t} = \vec{H}_{2t} = (4.77\hat{a}_x + 0.795\hat{a}_y - 0.795\hat{a}_z)\times 10^5$$

($\because$ Surface current $\vec{K} = 0$ (by default))

$$\vec{B}_{2t} = \mu_2 \vec{H}_{2t} = (7 \times 4\pi \times 10^{-7}) \times \vec{H}_{2t}$$

$$\vec{B}_{2t} = 4.196\hat{a}_x + 0.7\hat{a}_y - 0.7\hat{a}_z$$

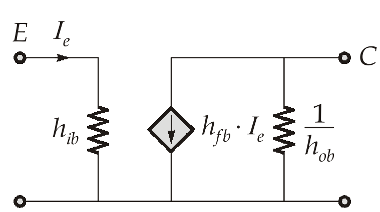
$$\vec{B}_2 = B_{2n} + B_{2t} = (0.5\hat{a}_y + 0.5\hat{a}_z) + (4.196\hat{a}_x + 0.7\hat{a}_y - 0.7\hat{a}_z)$$

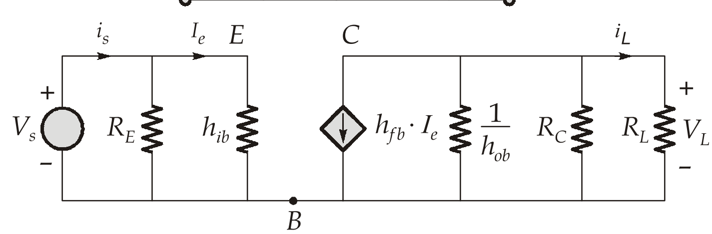
$$\vec{B}_2 = (4.196\hat{a}_x + 1.2\hat{a}_y - 0.2\hat{a}_z) \text{ Wb/m}^2$$

$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} = (4.77\times 10^5\hat{a}_x + 1.364\times 10^5\hat{a}_y - 0.227\times 10^5\hat{a}_z) \text{ A/m}$$

Q.2 (c) Solution:

Using the common-base h-parameter model of BJT, the equivalent circuit can be drawn as below:





Since, the capacitor C_B acts as short-circuit which appears across R_1 and R_2. Hence, no current flows through the resistances R_1 and R_2 and therefore, can be removed from the small signal equivalent circuit.

(i) Current gain ratio, $\qquad A_i = \dfrac{i_L}{i_s}$

Using current division rule at output,

$$i_L = \frac{-h_{fb} \cdot I_e \times \left[\frac{1}{h_{ob}} \| R_C\right]}{\left[\left(\frac{1}{h_{ob}} \| R_C\right) + R_L\right]} \quad \text{...(i)}$$

Using current division rule at input,

$$I_e = \frac{i_s \cdot R_E}{R_E + h_{ib}} \quad \text{...(ii)}$$

From equations (i) and (ii),

$$\frac{i_L}{i_s} = -\frac{h_{fb} \cdot R_E}{R_E + h_{ib}} \times \frac{\left[\frac{1}{h_{ob}} \| R_C\right]}{\left[\left(\frac{1}{h_{ob}} \| R_C\right) + R_L\right]}$$

Substituting the values, we get,

$$A_i = \frac{i_L}{i_s} = \frac{0.99 \times 3.3 \times 10^3}{3.3 \times 10^3 + 25} \times \frac{(10^6 \| 2.2 \times 10^3)}{(10^6 \| 2.2 \times 10^3) + 1.1 \times 10^3}$$

$$A_i = 0.9825 \times \frac{2.195 \times 10^3}{2.195 \times 10^3 + 1.1 \times 10^3}$$

$$= 0.9825 \times 0.666 = 0.6543$$

(ii) Voltage-gain ratio, $A_v = \frac{V_L}{V_s}$

$$A_v = \frac{V_L}{V_s} = \frac{i_L \cdot R_L}{i_s[R_E \| h_{ib}]} = A_i\left[\frac{R_L}{R_E \| h_{ib}}\right]$$

$$A_v = \frac{A_i R_L (R_E + h_{ib})}{R_E \cdot h_{ib}}$$

$$= \frac{0.6543 \times 1.1 \times 10^3 \times (3.3 \times 10^3 + 25)}{3.3 \times 10^3 \times 25}$$

$$A_v = 29.0073$$

Q.3 (a) Solution:

(i) **1.** Permittivity, $\in = \in_r \in_0 = 8 \times (8.85 \times 10^{-12}\ \text{F/m})$

$$= 7.08 \times 10^{-11}\ \text{F/m}$$

Capacitance, $C = \frac{\in A}{d} = 7.08 \times 10^{-11} \times \frac{6.45 \times 10^{-4}}{2 \times 10^{-3}}$

$$= 2.28 \times 10^{-11}\ \text{F or } 22.83\ \text{pF}$$

2. Charge stored in capacitor,

$$Q = CV$$
$$= (2.28 \times 10^{-11}) \times 10$$
$$= 2.28 \times 10^{-10} \text{ C}$$

3. Dielectric displacement,

$$D = \in E = \in \frac{V}{d} = 7.08 \times 10^{-11} \times \frac{10}{2 \times 10^{-3}}$$
$$= 3.54 \times 10^{-7} \text{ C/m}^2$$

4. Polarization, $$P = D - \in_0 E = 3.54 \times 10^{-7} - \frac{8.85 \times 10^{-12} \times 10}{2 \times 10^{-3}}$$
$$= 3.54 \times 10^{-7} - 4.425 \times 10^{-8}$$
$$= 3.0975 \times 10^{-7} \text{ C/m}^2$$

(ii) Electric displacement,

$$D = \in_0 E + P$$
$$\in_0 \in_r E = \in_0 E + P$$

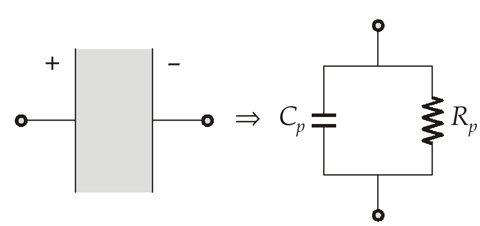
So, $$P = \in_0 E(\in_r - 1)$$

$\because$ $$P = \chi_e \in_0 E$$

So, $$\chi_e \in_0 E = \in_0 E(\in_r - 1)$$
$$\chi_e = \in_r - 1$$

Above expression represents electric susceptibility in terms of dielectric constant.

(iii) For parallel RC representation,



Given, $$\in_r' = 3,$$
$$\omega = 2\pi f = 2 \times 3.14 \times 1.5 \times 10^6$$
$$= 9.42 \times 10^6 \text{ Hz}$$

Loss tangent, $$\tan \delta = \frac{\in_r''}{\in_r'} = 3.8 \times 10^{-4}$$

$$\epsilon_r'' = \epsilon_r' \tan\delta$$
$$= 3 \times 3.8 \times 10^{-4} = 11.4 \times 10^{-4}$$

For parallel circuit,

Equivalent resistance, $$R_p = \frac{d}{\omega \epsilon_r'' \epsilon_0 A}$$

$$= \frac{0.3 \times 10^{-3}}{9.42 \times 10^{6} \times 8.85 \times 10^{-12} \times 11.4 \times 10^{-4} \times 25 \times 10^{-4}}$$

$$= 1.26 \times 10^{6}\ \Omega$$

$$C_p = \frac{\epsilon_0 \epsilon_r' A}{d} = \frac{8.85 \times 10^{-12} \times 3 \times 25 \times 10^{-4}}{0.3 \times 10^{-3}} = 221.25 \text{ pF}$$

For series RC representation,

$$R_s = \frac{1}{\omega^2 C_p^2 R_p}$$

Substituting the values we get

$$R_s = \frac{1}{(9.42 \times 10^{6})^2 \times (221.25)^2 \times 10^{-24} \times 1.26 \times 10^{6}}$$

$$= 0.1827\ \Omega$$

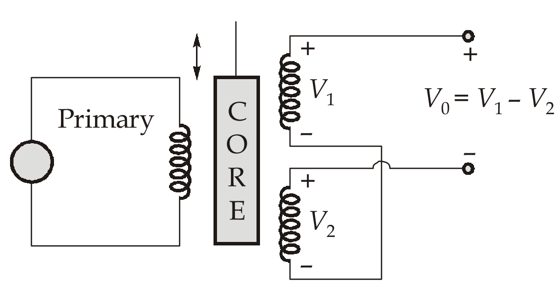
We know for series capacitance,

$$C_s = \frac{1}{\omega\sqrt{R_s R_p}} = \frac{1}{(9.42 \times 10^{6})\sqrt{0.1827 \times 1.26 \times 10^{6}}}$$

$$= 221.25 \text{ pF}$$

Q.3 (b) Solution:

LVDT stands for linear variable differential transformer. If works on the principle of mutual induction and converts displacement into voltage difference.



- Based on the displacement of the core, induced voltage in secondary side will change as flux linked with secondary winding changes.
- The differential output produced is proportional to the displacement of core.

Advantages of LVDT:

(i) High range of displacement: (1.25 mm to 250 mm)

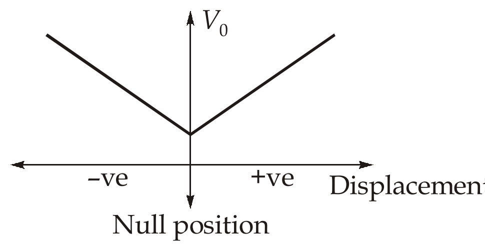
(ii) No frictional losses.

(iii) High input and high sensitivity.

(iv) Low hysteresis.

(v) Low power consumption.

(vi) Direct conversion of electrical signals.



Applications of LVDT:

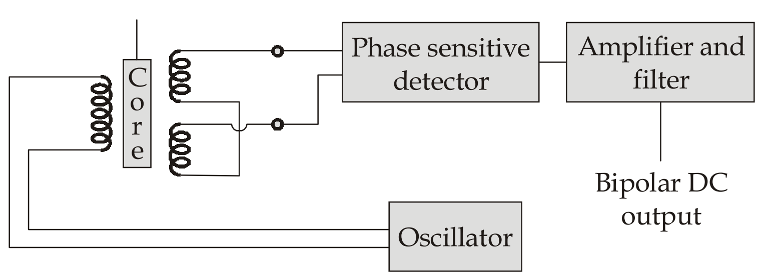
(i) Can be used as primary as well as secondary transducer

Primary: in displacement measurement.

Secondary: in pressure measurement using Bourdon tube.

(ii) Can be used where displacement measurement is very small.

Role of phase sensitive detector used for signal conditioning in LVDT: As the output of LVDT is AC waveform it has no polarity. Magnitude of output of an LVDT increases regardless of direction of movement from electrical zero position. To know in which half core is located one must consider phase as well as magnitude of output as compared to AC excitation source on the primary winding. The signal conditioning device should combine information available on the phase of the output with information on the magnitude of the output so the user can know the direction the core has moved as well as how far from the electrical zero position it has moved.



Phase sensitive detector

The bipolar output would help us in knowing the direction of displacement.

Q.3 (c) Solution:

(i) Characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2 - \lambda)[(2 - \lambda)^2 - 1] + 1[-1(2 - \lambda) + 1] + 1[1 - (2 - \lambda)] = 0$$

$$(2 - \lambda)^3 - (2 - \lambda) - 2 + \lambda + 1 + 1 - 2 + \lambda = 0$$

$$-\lambda^3 + 8 - 12\lambda + 6\lambda^2 - 2 + 2\lambda - 2 + \lambda = 0$$

$$-\lambda^3 + 6\lambda^2 - 9\lambda + 4I = 0$$

$$\lambda^3 - 6\lambda^2 + 9\lambda - 4I = 0$$

By Cayley-Hamilton theorem,

$$A^3 - 6A^2 + 9A - 4I = 0$$

Multiiply by A^{-1}, we get

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

or

$$4A^{-1} = A^2 - 6A + 9I$$

$$4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Also, $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I =$

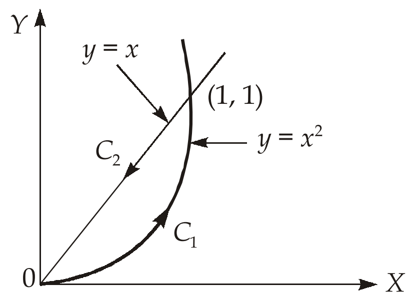
$$A^3(A^3 - 6A^2 + 9A - 4I) + 2(A^3 - 6A^2 + 9A - 4I) + 5A - I$$

$$= 5A - I$$

(ii) Here, $\phi = xy + y^2$ and $\Psi = x^2$

$\therefore$ $$\int_c (\phi\, dx + \Psi dy) = \int_{c_1} + \int_{c_2}$$

Along c_1, $y = x^2$ and x varies from 0 to 1



$$\therefore \quad \int_{c_1} = \int_0^1 [\{x(x)^2 + (x^2)^2\}\, dx + x^2 d(x^2)] = \int_0^1 (3x^3 + x^4)\, dx$$

$$= \left[\frac{3x^4}{4} + \frac{x^5}{5}\right]_0^1 = \frac{3}{4} + \frac{1}{5} = \frac{15+4}{20} = \frac{19}{20}$$

Along c_2, $y = x$ and x varies from 1 to 0

$$\therefore \quad \int_{c_2} = \int_1^0 [\{x(x) + (x)^2\}\, dx + x^2 d(x)]$$

$$= \int_1^0 3x^2 dx = \left[\frac{3x^3}{3}\right]_1^0 = -1$$

Thus, $$\int_c (\phi\, dx + \Psi dy) = \frac{19}{20} - 1 = -\frac{1}{20} \quad ...(i)$$

$$\text{RHS} = \iint_S \left(\frac{\partial \Psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\, dy$$

$$= \iint_S \left[\frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial y}(xy + y^2)\right] dx\, dy$$

$$= \int_0^1 \int_{x^2}^{x} (2x - x - 2y)\, dy\, dx = \int_0^1 [xy - y^2]_{x^2}^{x}\, dx$$

$$= \int_0^1 (x^4 - x^3)\, dx$$

$$= \frac{x^5}{5} - \frac{x^4}{4} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20} \quad ...(ii)$$

Hence, Green theorem is verified from the equality of (i) and (ii).

Q.4 (a) Solution:

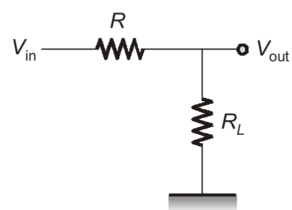
(i) When a MOSFET is operated as an on-switch, it works in the triode or ohmic region when ON. In the ohmic region, the MOSFET acts as a resistor.

Current in the ohmic region,

$$I_{DS} = \frac{\mu_n C_{ox} W}{L}[(V_{GS}-V_{TN})V_{DS} - V_{DS}^2/2] \cong \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TN})V_{DS}$$

$$\frac{V_{DS}}{I_{DS}} = R = \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TN})} \quad \text{...(i)}$$

The small signal equivalent of the circuit can thus be obtained by replacing the MOSFET with the resistance R as shown below :



Given, $V_{out} = 0.95\, v_{in}$

$$\frac{V_{out}}{v_{in}} = 0.95$$

Now, $$V_{out} = \frac{v_{in} \times R_L}{R+R_L} \Rightarrow \frac{V_{out}}{v_{in}} = \frac{100}{R+100} = 0.95$$

$\Rightarrow$ $R = 5.263\ \Omega$

Using equation (i), we can write,

$$\frac{W}{L} = \frac{1}{\mu_n C_{ox}(V_{GS}-V_{TN}) \times R}$$

We have, $V_{GS} = V_G - V_S$, where $V_G = 1.8$ V and $V_S = 0.95\, v_{in}$. Since, v_{in} is very small, hence $V_{GS} = V_G = 1.8$ V. Substituting the values, we get

$$\frac{W}{L} = \frac{1}{200 \times 10^{-6}(1.8-0.4)\times 5.263} \cong 678.6$$

(ii) • For pentavalent impurity, material becomes n-type material,

$$N_D = 10^{16}\ \text{atoms/cm}^3$$

$$n_i = 1.6 \times 10^{10}/\text{cm}^3$$

According to law of mass action,

$$np = n_i^2 = (1.6 \times 10^{10})^2$$

$$= 2.56 \times 10^{20}$$

According to the principle of electrical neutrality

$$n + N_A = p + N_D$$

Being n-type material

where $N_D = 10^{16}$ atom/cm^3

$$n = p + N_D = p + 10^{16}$$

Also $N_D >> p$

$$n \approx 10^{16}$$

Therefore, $p = \dfrac{2.56 \times 10^{20}}{10^{16}} = 25.6 \times 10^3$ atom/cm^3

- Fermi level energy for n-type material is given by

$$n = n_i e^{(E_f - E_i)/KT}$$

or $\dfrac{n}{n_i} = e^{(E_f - E_i)/KT}$

Taking natural log both sides

$$\ln\left(\frac{n}{n_i}\right) = \frac{E_f - E_i}{KT}$$

$\Rightarrow$ $E_f - E_i = KT \ln\left(\dfrac{n}{n_i}\right)$

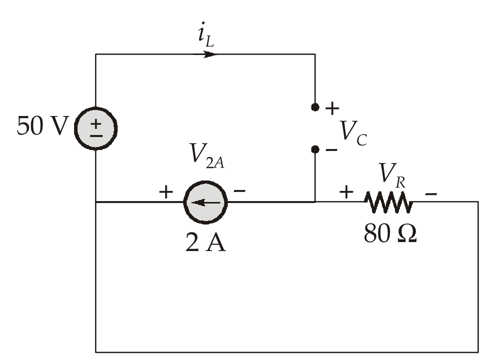
$$E_f - E_i = 8.62 \times 10^{-5} \times 300 \ln\left(\frac{10^{16}}{1.6 \times 10^{10}}\right)$$

$$= 8.62 \times 10^{-5} \times 300 \times 13.345$$

$$= 0.345 \text{ eV}$$

Q.4 (b) Solution:

(i) Fot $t < 0$;



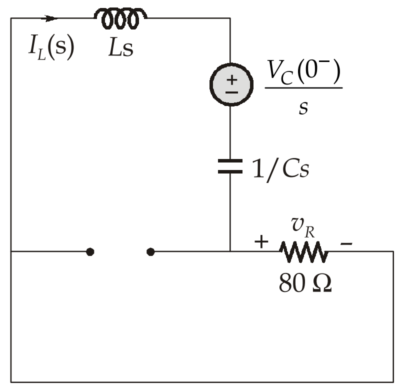
$$i_L(0^-) = i_L(0^+) = 0 \text{ A}$$

$$V_R(0^-) = -2 \times 80$$

$$= -160 \text{ V}$$

$$V_C(0^-) = 50 - V_R = 210 \text{ V}$$

Applying laplace transform



$$I_L(s) = \frac{-210/s}{2s + \frac{10^3}{s} + 80} = \frac{-105}{s^2 + 40s + 500}$$

$$i_L(t) = -10.5\, e^{-20t} \sin 10t \text{ A}$$

$$v_L(t) = 210e^{-20t} [2 \sin 10t - \cos 10t] \text{ V}$$

$$v_R(t) = Ri_L(t) = 80[-10.5\, e^{-20t} \times \sin 10t]$$

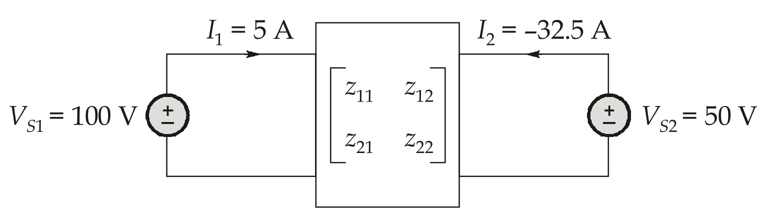
$$= -840e^{-20t} \sin 10t \text{ V}$$

$$v_C(t) = -[V_L(t) + V_R(t)]$$

$$= 210e^{-20t} [2 \sin 10t + \cos 10t] \text{ V}$$

(ii) For z-parameter $$[Z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

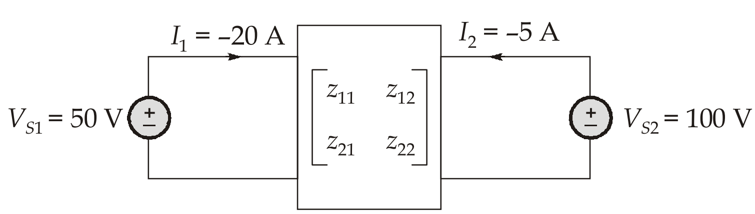
From experimeter-1



$$V_{s1} = z_{11}I_1 + z_{12}I_2$$

$$100 = z_{11} \times 5 + z_{12}(-32.5) \quad \text{...(i)}$$

From experiment-2



$$V_{s1} = z_{11}I_1 + z_{12}I_2$$

$$50 = z_{11}(-20) + z_{12}(-5) \quad \text{...(ii)}$$

On solving equation (i) and (ii)

$$z_{11} = -1.66\ \Omega$$

$$z_{12} = -3.33\ \Omega$$

Similarly, from experiment 1

$$V_{s2} = z_{21}I_1 + z_{22}I_2$$

$$50 = z_{21}(5) + z_{22}(-32.5) \quad \text{...(iii)}$$

From experiment 2

$$V_{s2} = z_{21}I_1 + z_{22}I_2$$

$$100 = z_{21}(-20) + z_{22}(-5) \quad \text{...(iv)}$$

On solving equation (iii) and (iv),

$$z_{21} = -4.44\Omega$$

$$z_{22} = -2.22\Omega$$

Hence, Z-parameter matrix

$$[Z] = \begin{bmatrix} -1.66 & -3.33 \\ -4.44 & -2.22 \end{bmatrix}\Omega$$

Y-parameter matrix, $[Y] = [Z]^{-1}$

$$[Y] = \begin{bmatrix} -1.66 & -3.33 \\ -4.44 & -2.22 \end{bmatrix}^{-1}$$

$$= \frac{\begin{bmatrix} -2.22 & 3.33 \\ 4.44 & -1.66 \end{bmatrix}}{[(1.66\times 2.22)-(3.33\times 4.44)]} = \frac{1}{11.133}\begin{bmatrix} -2.22 & 3.33 \\ 4.44 & -1.66 \end{bmatrix}$$

$$\therefore \quad [Y] = \begin{bmatrix} 0.2 & -0.3 \\ -0.4 & 0.15 \end{bmatrix}\mho$$

Q.4 (c) Solution:

(i)

$$\text{Power factor} = \cos\phi = 0.5 \text{ lagging}$$

$$\text{Phase angle, } \phi = 60°$$

The phase angle between applied voltage and load current is 60°. Under ideal condition, the voltage magnet flux ϕ_p should lag behind the applied voltage V by 90° and the current magnet flux ϕ_s, should be in phase with load current I. This is shown in figure (a),

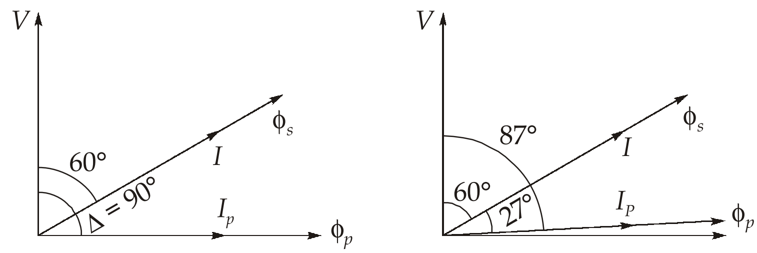


Figure (a) Figure (b)

$$\text{Energy registered} = K\int VI\sin(\Delta - \phi)dt$$

$$\propto \sin(\Delta - \phi)$$

Now under ideal conditions, $\Delta = 90°$

$\therefore$ Energy registered under ideal conditions $\propto \sin(90° - \phi)$

$$\propto \cos\phi$$

$$\propto \cos 60°$$

The phasor diagram for actual working conditions in shown in figure (b)

Now,

$\because$ $\Delta - \phi = 27°$

$\therefore$ $\Delta = 27° + 60° = 87°$

$\therefore$ Energy registered under actual working conditions

$$\propto \sin(\Delta - \phi) \propto \sin(87° - 60°) \propto \sin 27°$$

It is clear from the above conclusion that the energy registered under actual working condition is not the same as under ideal condition. Therefore, the meter does not read correctly

$$\text{Error} = \frac{(\sin 27° - \cos 60°)}{\cos 60°} \times 100$$

$$= \frac{0.454 - 0.5}{0.5} \times 100 = -9.2\%$$

(ii) Angle of shear, $\theta = \dfrac{2T}{\pi G r^3}$

where, G is the shaft shear modulus

r is the radius of the shaft

T is the applied torque

An area of the shaft surface, originally square with the sides of unit length, is deformed by stain to a parallelogram. The original length of the diagonal is $\sqrt{2}$. If the angle of shear, θ is small, the length of the diagonal of the parallelogram is longer than the diagonal of the square. The difference in lengths is $\frac{\theta}{\sqrt{2}}$. Therefore, the longitudinal strain is:

$$\varepsilon = \frac{\Delta L}{L} = \frac{\theta/\sqrt{2}}{\sqrt{2}} = \frac{\theta}{2}$$

But $$\frac{\Delta R}{R} = G_f \varepsilon$$

$$= 2 \times \frac{\theta}{2} = \theta$$

or $$\theta = \frac{\Delta R}{R} = \frac{0.24}{120} = 2 \times 10^{-3} \text{ rad}$$

∴ $$\text{Torque, } T = \frac{\pi G r^3}{2}\theta = \frac{\pi \times 80 \times 10^9 \times (15 \times 10^{-3})^3}{2} \times 2 \times 10^{-3}$$

$$= 848.23 \text{ Nm}$$

Section-B

Q.5 (a) Solution:

Types of error:

1. Gross errors
2. Systematic errors
3. Random errors

1. **Gross error:** Mainly due to human error in reading instruments, recording and calculating measurement results e.g. reading current of 31.1 A as 37.1 A.
2. **Systematic error:** This error is subdivided as follows:
   - **Instrumental errors:** It arises due to 3 reason: (i) Inherent shortcoming of instrument, (ii) Misuse of instruments, (iii) Loading effect of instruments
     - May be due to construction, calibration or ageing of instrument e.g. if spring of an instrument becomes weak.
     - A good instrument used in unintelligent way may give erroneous results e.g. failure to adjust zero, use load of high resistance etc.
     - Use of voltmeter for measuring voltage across a high resistance circuit.
   - **Environmental error:** Error due to conditions external to device e.g. temperature, humidity, pressure, dust.
   - **Observational error:** eg. parallex error,

3. **Random error:** Error due to multitude of factors which change or fluctuate from one measurement to another. **Example:** error due to noise

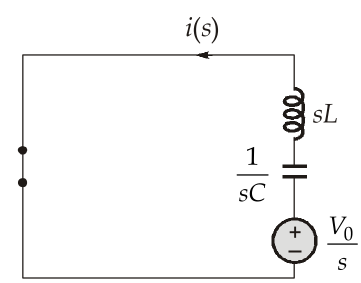
Ways to minimise systematic errors:

- Calibration of instruments against standard.
- Correction factor should be applied after determining the error.
- The error due to loading effect should be considered and corrections for these effects should be made e.g. when measuring a low resistance by ammeter-voltmeter method, voltmeter with high resistance should be made.
- Conditions for measurement should be kept same.
- Using of equipment immune to these effects. e.g. variation of resistance with temperature could be minimised by using resistive material with low temperature coefficient of resistance.
- Electrostatic and magnetic shield may be provided.
- Parallex error can be eliminated by having a pointer and scale in same plane.

Q.5 (b) Solution:

(i) $V_C(t = 0) > 0 \rightarrow$ Diode is F.B.

Circuit in s-domain,



$$I(s) = \frac{\frac{V_0}{s}}{sL + \frac{1}{sC}} = \frac{V_0}{L} \cdot \frac{1}{s^2 + \frac{1}{LC}}$$

$$= \frac{V_0}{L} \cdot \frac{\frac{\sqrt{LC}}{\sqrt{LC}}}{s^2 + \left(\frac{1}{\sqrt{LC}}\right)^2}$$

Taking inverse Laplace transformer

$$i(t) = V_0\sqrt{\frac{C}{L}} \sin \omega_0 t$$

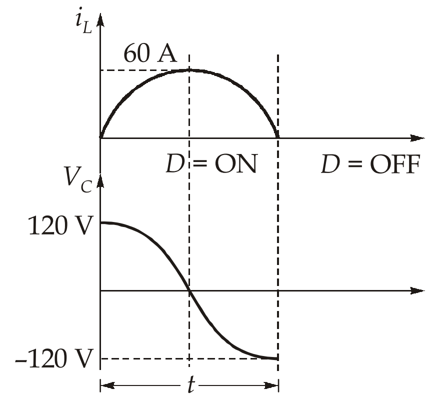
Where $V_0 = 120$ V;

$$C = 12\ \mu F, \qquad L = 48\ \mu H$$

$$i(t) = 120\sqrt{\frac{12}{48}}\sin(\omega_0 t)A = 60\sin\omega_0 t\ A$$

Peak value of current, $i = 60$ A

(ii)



$$\omega t = \pi$$

$$t = \frac{\pi}{\omega} = \frac{\pi}{\frac{1}{\sqrt{LC}}} = \pi\sqrt{LC}\ \text{sec} = \pi\sqrt{12\times10^{-6}\times48\times10^{-6}}$$

$$t = 75.39\ \mu s = \text{conduction time of diode}$$

Q.5 (c) Solution:

Inductive reactance of the pressure coil

$$= 2\pi \times 100 \times 10 \times 10^{-3} = 6.28\ \Omega$$

Resistance of pressure coil current = 2000 Ω

Let phase angle of pressure coil circuit β

$$\tan\beta = \frac{6.28}{2000} \quad \text{or} \quad \beta = 0.179°$$

For inductive load : reading of wattmeter ∝ cos β cos (ϕ – β)

True power is proportional to cos ϕ

$$\text{True power} = \frac{\cos\phi}{\cos\beta\cos(\phi-\beta)} \times \text{reading of wattmeter} \qquad \text{...(i)}$$

$$\text{True power} = I^2R$$

$$= (4.5)^2R = 20.25Z\cos\phi$$

Where R is resistance of load,

Impedance of load, $\vec{Z} = \frac{V}{I} = \frac{240}{4.5} = 53.33\ \Omega$

True power, $(4.5)^2 \times 53.33 \cos\phi$

Reading of wattmeter $= 23$ W

Substituting values in equation-(i)

$$(4.5)^2 \times 53.33 \cos\phi = \frac{\cos\phi}{\cos\beta\cos(\phi-\beta)} \times 23$$

$$\cos\beta\cos(\phi-\beta) = 0.0213$$

$$\cos(0.18)\cos(\phi-0.18) = 0.0213$$

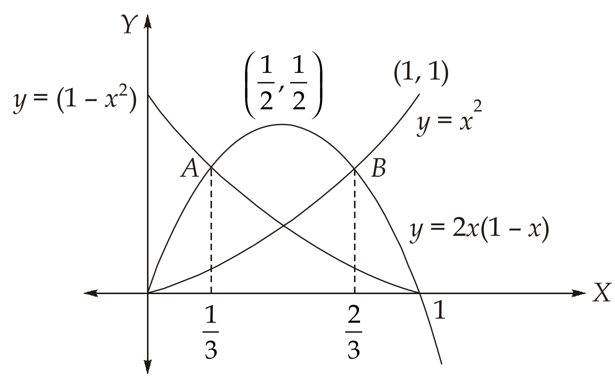
$$\cos(\phi-0.18) = \frac{0.0213}{0.999}$$

$$\phi - 0.18 = 88.77$$

$$\Rightarrow \quad \phi = 88.96°$$

Q.5 (d) Solution:

We can draw the graph of $y = x^2$, $y = (1 - x^2)$ and $y = 2x(1 - x)$ in following figure,



Now, to get the point of intersection of $y = x^2$ and $y = 2x(1 - x)$, we get

$$x^2 = 2x(1-x)$$

$$x^2 = 2x - 2x^2$$

$$3x^2 = 2x$$

$$x(3x-2) = 0$$

$$x = 0, \frac{2}{3}$$

Similarly, we can find the coordinate of the points of intersection of

$$y = (1 - x^2)$$

and $\qquad y = 2x(1-x)$ are $x = \frac{1}{3}$ and $x = 1$

From the figure, it is clear that,

$$f(x) = \begin{cases} (1-x^2), & \text{if } 0 \le x \le \frac{1}{3} \\ 2x(1-x) & \text{if } \frac{1}{3} \le x \le x\frac{2}{3} \\ x^2, & \text{if } \frac{2}{3} \le x \le 1 \end{cases}$$

∴ The required area

$$A = \int_0^1 f(x)\,dx$$

$$= \int_0^{1/3} (1-x)^2\,dx + \int_{1/3}^{2/3} 2x(1-x)dx + \int_{2/3}^{1} x^2 dx$$

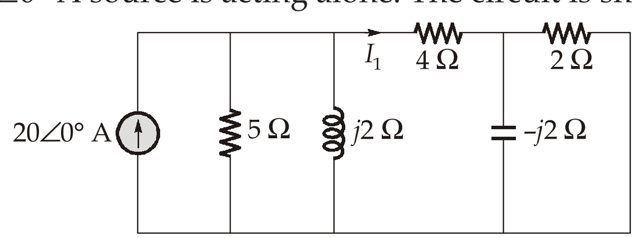
$$= \left[-\frac{1}{3}(1-x)^3\right]_0^{1/3} + \left[x^2 - \frac{2x^2}{3}\right]_{1/3}^{2/3} + \left[\frac{1}{3}x^3\right]_{2/3}^{1}$$

$$= \left[-\frac{1}{3}\left(\frac{2}{3}\right)^3 + \frac{1}{3}\right] + \left[\left(\frac{2}{3}\right)^2 - \frac{2}{3}\left(\frac{2}{3}\right)^3 - \left(\frac{1}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^3\right] + \left[\frac{1}{3}(1) - \frac{1}{3}\left(\frac{2}{3}\right)^3\right]$$

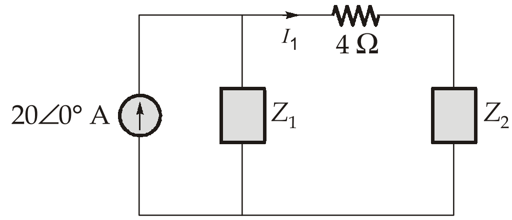
$$= \frac{19}{81} + \frac{13}{81} + \frac{19}{81} = \frac{17}{27} \text{ square unit}$$

Q.5 (e) Solution:

Case I : When the 20∠0° A source is acting alone. The circuit is shown in figure.



Reducing the parallel combination, the simplified circuit is shown in figure.



$$Z_1 = \frac{5 \times j2}{5 + j2} = 1.857\angle 68.2° = (0.69 + j1.72)\ \Omega$$

$$Z_2 = \frac{2 \times (-j2)}{2 - j2}$$

$$= (1 - j1)\ \Omega = 1.414\angle -45°\ \Omega$$

By current division rule, the current through the 4 Ω resistor is

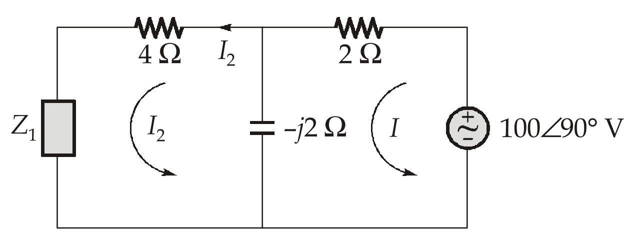
$$I_1 = 20\angle 0° \times \frac{Z_1}{Z_1 + 4 + Z_2}$$

$$= 20\angle 0° \times \frac{1.857\angle 68.2°}{0.69 + j1.72 + 4 + 1 - j1}$$

$$= 6.48\angle 61° = (3.14 + j5.66)\ \text{A}$$

Case II : When the 100∠90° V source is acting alone.

Here, the current source in open-circuited. Combining the parallel connection of 5 Ω and $j2$ Ω, the simplified circuit is shown in below figure.



Applying KVL for the above two loops, we get

$$(4 + 0.69 + j1.72 - j2)I_2 + j2I = 0$$

$$(4.69 - j0.28)I_2 + j2I = 0 \quad \text{...(i)}$$

and, $$j2I_2 + (2 - j2)I = 100\angle 90° = 100j \quad \text{...(ii)}$$

Solving (i) and (ii), we get

$$I_2 = \frac{\begin{vmatrix} 0 & j2 \\ j100 & (2-j2) \end{vmatrix}}{\begin{vmatrix} (4.69 - j0.28) & j2 \\ j2 & (2-j2) \end{vmatrix}} = \frac{200}{12.82 - j9.94}$$

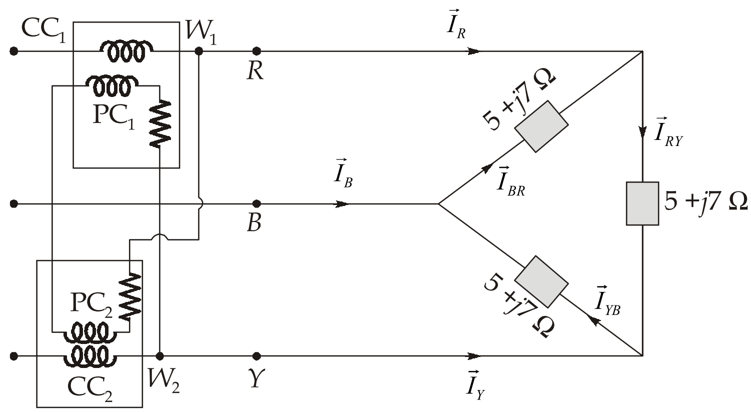
$$= 12.33\angle 37.75°\ \text{A} = (9.75 + j7.55)\ \text{A}$$

From superposition theorem, when both the sources are acting simultaneously, the current through the 4 Ω resistor is

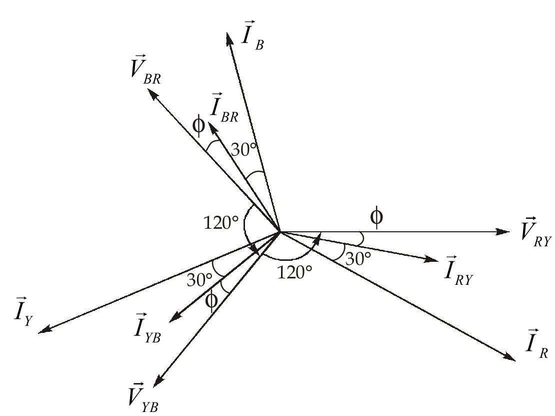
$$I = I_1 - I_2$$
$$= (3.14 + j5.66) - (9.75 + j7.55)$$
$$= -6.61 - j1.9 = 6.89\angle{-163.67°}\text{ A}$$

Q.6 (a) Solution:

Watthour meters W_1 and W_2 connection diagram:



$$Z_{RY} = Z_{YB} = Z_{BR} \text{ (Balanced load)}$$



$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{7}{5}\right) = 54.46°$$

Watt hour meter reading of W_1 $= V_{YB}I_R \cos(90° - \phi) \times t$ hr

$= tV_{YB}I_R \sin\phi = tV_LI_L \sin\phi$ Wh

Watt hour meter reading of W_2 $= V_{BR}I_Y \cos(90° - \phi) \times t$ hr

$= tV_{BR}I_Y \sin\phi = tV_LI_L \sin\phi$ Wh

Reactive kVAh $Q_{3\phi}$ $= \sqrt{3}V_LI_L \sin\phi \times t = \sqrt{3} \times$ Wh meter readings of W_1 or W_2

$$\text{Watt hour meter readings} = 1\,\text{hr} \times 400 \times \left(\frac{\sqrt{3} \times 400}{\sqrt{5^2 + 7^2}}\right) \times \sin(54.46°) = 26.22\ \text{kWh}$$

$$\text{Total reactive kVAh} = \sqrt{3} \times 26.22 = 45.4\ \text{kVAh}$$

For unbalanced load, $I_R \neq I_Y \neq I_B$

Phase load angles, $\phi_R \neq \phi_Y \neq \phi_B$

Therefore, this method won't work to calculate reactive VAh of the unbalanced load.

Q.6 (b) Solution:

(i) Nominal ratio, $K_n = \dfrac{6900}{115} = 60$

Primary winding turns, $N_p = 22500$

Secondary winding turns, $N_s = 375$

Turns ratio, $n = \dfrac{22500}{375} = 60$

No load current, $I_0 = 0.005\ \text{A}$

No load power factor $= \cos 73.7° = 0.28$

$\sin 73.7° = 0.96$

Primary current, $I_p = 0.0125\ \text{A}$

Primary power factor $= \cos 53.1° = 0.6$

and $\sin 53.1° = 0.8$

Now primary voltage V_p is taken as reference and therefore we can write,

$$V_p = 6900 + j0$$

$$I_p = 0.0125\,(0.6 - j0.8)$$

$$= 0.0075 - j0.01$$

$$I_0 = 0.005(0.28 - j0.96)$$

$$= 0.0014 - j0.0048$$

Phasor $\dfrac{I_s}{n}$ is the phasor difference of I_p and I_0

$$\therefore \quad \frac{I_s}{n} = (0.0075 - j0.01) - (0.0014 - j0.0048)$$

$$= 0.0061 - j0.0052$$

$$I_s\,(\text{reversed}) = n \times \frac{I_s}{n} = 60(0.0061 - j0.0025)$$

$= 0.366 - j0.312$

or $I_s = -(0.366 - j0.312)$

$= -0.366 + j0.31\ \Omega$

$\therefore$ Secondary current, $I_s = 0.48$ A

Primary induced voltage, $E_p = V_p - I_p Z_p$

$= 6900 - j0 - (0.0075 - j0.01)(1200 + j2000)$

$= 6900 - j0 - (9 + j3)$

$= 6891 - j3$ V

Secondary induced voltage E_s (reversed)

$$= \frac{E_s}{n} = \frac{6891 - j3}{60} = 114.85 - j0.05 \text{ V}$$

Secondary terminal voltage,

$V_s = E_s - I_s Z_s$

$= -114.85 + j0.05 - (-0.366 + j0.312)(0.4 + j0.7)$

$= -114.49 + j0.18$

$V_s = 114.49$ V

Secondary burden $= V_s I_s = 114.49 \times 0.48 = 55$ VA

(ii) Actual ratio $= \dfrac{V_p}{V_s} = \dfrac{6900}{114.48} = 60.27$

$V_s =$ (reversed) $= -(-114.49 + j0.18)$

$= 114.49 - j0.18$ V

Angle by which V_s (reversed) lags V_p

$$= \tan^{-1}\frac{0.18}{114.49} \approx \frac{0.18}{114.49} \text{rad}$$

(iii) The solution to the problem lies in reducing the turns ratio (decreasing number of primary turns) so that the actual ratio equals the nominal ratio,

$$\text{New turns ratio} = \frac{60}{60.27} \times 60$$

$\therefore$ New value of primary turns

$$= \frac{60}{60.27} \times 60 \times N_s = \frac{60}{60.27} \times 60 \times 375 = 22399$$

$\therefore$ Reduction in primary turns $= 22500 - 22399 = 101$

Here we are neglecting the change in voltage drops caused by change in turns ratio and hence the solution is only approximate.

Q.6 (c) Solution:

(i) Taylor series of a function $f(x)$ centered at $x = a$ is

$$T(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + ...$$

For $f(x) = \ln(1 + x)$ centered at $a = 0$ is

$$T(x) = \ln(1+0) + \frac{(1+0)^{-1}(x-0)}{1!} - \frac{(1+0)^{-2}}{2!}(x-0)^2 + \frac{2(1+0)^{-3}}{3!}(x-0)^3 - ....$$

$$= \ln(1) + \frac{x}{1!} - \frac{x^2}{2!} + \frac{2x^3}{3!} - ....$$

$$= 0 + \frac{x}{1} - \frac{x^2}{2} + \frac{x^3}{3} - ....$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

(ii) Using the ratio test,

$$|x| < \left|\frac{C_n}{C_{n+1}}\right| = \left|\frac{(-1)^{n+1} n}{(-1)^{n+2}(n+1)}\right| = \left|\frac{n}{n+1}\right|$$

Because n is the index of summation (an increasing integers), $n + 1$ is always greater than n and therefore,

$$|x| < \left|\frac{n}{n+1}\right| < 1$$

Thus radius of convergere is : $|x| < 1$

(iii) $\text{In}\left(\frac{3}{2}\right) = \text{In}\left(1 + \frac{1}{2}\right)$ can be approximated by the first two non-zero terms of the Taylor series as

$$\ln(1 + x) \approx \frac{x}{1} + \frac{-x^2}{2} = \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^2}{2} = \frac{3}{8}$$

(iv) The upper bound of error is found using Taylor's inequality for an approximation of n-terms

$$|R_n(x)| \leq M_n \frac{|x^{n+1}|}{(n+1)!}$$

Where, $x = \frac{1}{2}$ and $n = 2$

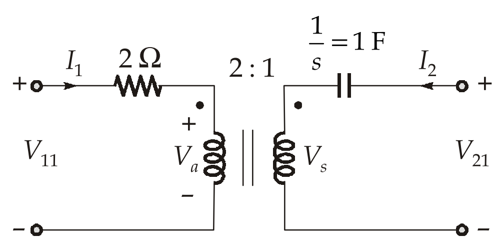
In addition, $M_n \geq |f^{(n+1)}(x)| \Rightarrow M_2 \geq \frac{2}{(1+x)^3}$

For all $|x| \leq \frac{1}{2}$, the maximum of M_2 in this range is for $x = -\frac{1}{2}$ which gives $M_2 = 16$.

Putting these numbers into the above,

$$|R_2(0.5)| \leq \frac{16(0.5)^3}{3!} = \frac{1}{3}$$

Q.7 (a) Solution:



$$\frac{I_1}{I_2} = -\frac{1}{2}$$

$$\frac{V_a}{V_b} = 2$$

$$V_{11} = 2I_1 + V_a$$

and $$V_{21} = \frac{I_2}{s} + V_b$$

$\Rightarrow$ $$V_{21} - \frac{I_2}{s} = \frac{V_a}{2}$$

$\Rightarrow$ $$2V_{21} - \frac{2I_2}{s} = V_a$$

$\Rightarrow$ $$V_{11} = 2\times\left(-\frac{I_2}{2}\right) + 2V_{2_1} - \frac{2I_2}{s}$$

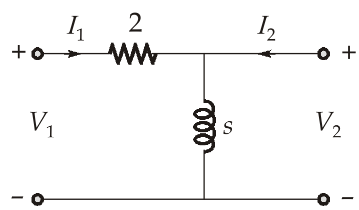
$\Rightarrow$ $$V_{11} = -I_2 - \frac{2}{s}I_2 + 2V_{21}$$

$$\Rightarrow \quad V_{11} - 2V_{21} = -\left[\frac{s+2}{s}\right]I_2$$

$$\Rightarrow \quad I_2 = -\frac{s}{s+2}V_{11} + \frac{2s}{s+2}V_{21}$$

$$I_1 = \frac{s}{2(s+2)}V_{11} - \frac{s}{s+2}V_{21}$$

So,
$$Y_1 = \begin{bmatrix} \frac{s}{2(s+2)} & -\frac{s}{s+2} \\ \frac{-s}{s+2} & \frac{2s}{s+2} \end{bmatrix}$$



$$V_1 = (s+2)I_1 + sI_2$$

$$V_2 = sI_1 + sI_2$$

So,
$$Z = \begin{bmatrix} s+2 & s \\ s & s \end{bmatrix}$$

$$[Y_2] = \frac{1}{-s^2 + s^2 + 2s}\begin{bmatrix} s & -s \\ -s & s+2 \end{bmatrix}^T$$

$$= \frac{1}{2s}\begin{bmatrix} s & -s \\ -s & s+2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{s+2}{2s} \end{bmatrix}$$

So,
$$Y_1 + Y_2 = Y_{net}$$

$$Y_{net} = \begin{bmatrix} \frac{(s+1)}{(s+2)} & -\frac{(3s+2)}{2(s+2)} \\ \frac{-(3s+2)}{2(s+2)} & \frac{-(3s^2+4s+4)}{2s(s+2)} \end{bmatrix}$$

Q.7 (b) Solution:

(i)

$$f(z) = \frac{2z^3+1}{z^2+2} = 2z-2+\frac{2z+1}{z(z+1)}$$

$$= (2i-2)+2(z-i)+\frac{1}{z}+\frac{1}{z+1} \quad \text{...(i)}$$

[By partial fractions]

To expand $\frac{1}{z}$ and $\frac{1}{(z+1)}$ about $z = i$, put $z - i = t$,

So that, $$\frac{1}{z} = \frac{1}{(t+i)} = \frac{1}{i}\left(1+\frac{t}{i}\right)^{-1}$$

Expanding by Binomial theorem

$$= \frac{1}{t}\left[1-\frac{t}{i}+\frac{t^2}{i^2}-\frac{t^3}{i^3}+\frac{t^4}{i^4}-\text{........}\infty\right]$$

$$= \frac{1}{i}+\frac{t}{1}+\frac{t^2}{i^3}-\frac{t^3}{i^4}+\frac{t^4}{i^5}-\text{.......}\infty$$

$$= -i+(z-i)+\sum_{n=2}^{\infty}(-1)^n\frac{(z-i)^n}{i^{n+1}}$$

[Expanding by Binomial theorem]

and $$\frac{1}{z+1} = \frac{1}{t+i+1} = \frac{1}{1+i}\left(1+\frac{t}{1+i}\right)^{-1}$$

$$= \frac{1}{1+i}\left[1-\frac{t}{1+i}+\frac{t^2}{(1+t)^2}-\frac{t^3}{(1+i)^3}+\frac{t^4}{(1+i)^4}-\text{......}\infty\right]$$

$$= \frac{1-i}{2}-\frac{t}{2i}+\left[\frac{t^2}{(1+i)^3}-\frac{t^3}{(1+i)^4}+\frac{t^4}{(1+i)^5}-\text{......}\infty\right]$$

$$= \frac{1}{2}-\frac{i}{2}-\frac{z-i}{2!}+\sum_{n=2}^{\infty}(-1)^n\frac{(z-i)^n}{(1+i)^{n+1}}$$

So, from eqn. (i), $$f(z) = (2z-2)+\left(-i+(z-i)+\sum_{n=2}^{\infty}(-1)^n\frac{(z-i)^n}{i^{n+1}}\right)$$

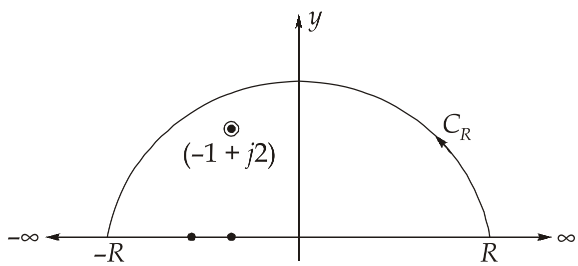
$$+\frac{1}{2}-\frac{i}{2}-\frac{z-i}{2!}+\sum_{n=2}^{\infty}(-1)^n\frac{(z-i)^n}{(1+i)^{n+1}}$$

(ii) We have $\displaystyle\int_{-\infty}^{\infty}\frac{x\sin\pi x}{x^2+2x+5}dx$

Let us consider $\displaystyle\int_C\frac{z\sin\pi z}{z^2+2z+5}dz$ (where $z = x + iy$)

Poles are at $z = -1 \pm j2$

Out of two poles, only $z = -1 + j2$ is inside the contour.



Residue at $z = -1 + j2$

$$= \lim_{z\to -1+j2}(z+1-j2)\frac{z\sin\pi z}{(z+1-j2)(z+1+j2)}$$

$$= \lim_{z\to -1+j2}\frac{z\sin\pi z}{(z+1+j2)}$$

$$= \frac{(-1+j2)\sin\pi(-1+j2)}{(-1+j2+1+j2)}$$

$$= \frac{(-1+j2)\sin\pi(-1+j2)}{4j}$$

$$\int_{-R}^{R}\frac{z\sin\pi z}{z^2+2z+5}dz = 2\pi j \times (\text{Residue})$$

$$= 2\pi j\times\frac{(-1+j2)\sin\pi[(-1+j2)]}{4j}$$

$$= \frac{\pi}{2}(-1+j2)(-\sin j2\pi)$$

$$= \frac{\pi}{2}(1-j2)(\sin 2\pi j)$$

$$= \frac{\pi}{2}(1 - j2)\, j \sinh 2\pi$$

$$= \frac{\pi}{2}(2 + j)\sinh 2\pi$$

Taking real part,

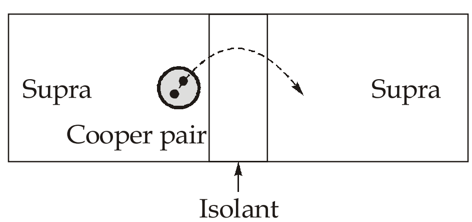
$$\int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} = \pi \sinh 2\pi$$

Q.7 (c) Solution:

(i) **Quantum dot** is semiconductor nanostructure that confines the motion of conduction band electrons, valence band holes or excitons in all three spatial directions. It has discrete quantized energy spectrum. Because the bandgap of the quantum dots can be adjusted, QDs are desirable for solar cells. It replaces bulky materials such as or silicon or copper indium gallium sellenide (*CIGS*) or cadmium telluride (*CdTe*).

QD can boost the solar power efficiency and saving in area occupied.

(ii) **Josephson effect:** When two superconductors are separated by a very thin insulating layer, a continuous electric current appears, the value of which is linked to characteristics of superconductors. This superconductor-insulator - superconductor sandwich has been called a 'Josephson junction'.



When a material becomes superconducting, the electrons form "Cooper pairs" and condensate in the shape of a unique collective quantum wave. If the electric insulator separating the two super conductor is very thin (only a few nanometers), then the wave can somehow spill out of the superconductor which enables the electron pairs to go through the insulator due to a quantum effect called 'tunneling effect'. When spontaneously going from one superconductor to the other, the pair creates an electric current.

This effect finds application in:

1. Fast switching devices 'cryotron switches'.
2. Accurate magnetic field measuring devices 'Squids'.
3. Magnetic leviation, etc.

(iii) **Seebeck effect** is the production of an electromotive force (emf) and consequently an electric current in a loop of material consisting of at least two dissimilar conductors when two junctions are maintained at different temperatures.

Thermo-emf, $V = a_0 + a_1 T + a_2 T^2 + ...$

Assuming linear interpolation,

$$\Rightarrow \quad V = a_0 + a_1 T$$

$$11.167 \times 10^{-3} = a_0 + a_1(207 + 273)$$

$$11.223 \times 10^{-3} = a_0 + a_1(208 + 273)$$

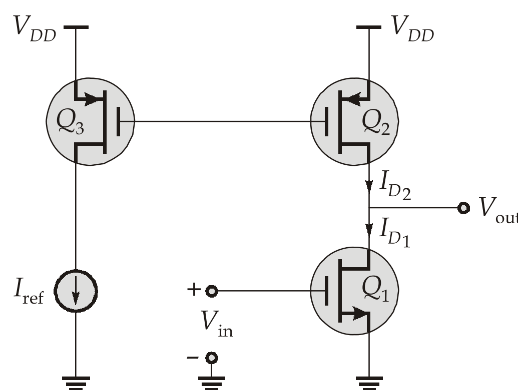
$$a_1 = 0.056 \times 10^{-3} \text{ V-K}^{-1}$$

$$\Rightarrow \quad a_0 = -15.713 \times 10^{-3} \text{ V}$$

$$11.191 \times 10^{-3} = -15.713 \times 10^{-3} + 0.056 \times 10^{-3} \text{ T}$$

$$\Rightarrow \quad T = 480.43 \text{ K} = 207.43 \text{ °C}$$

Q.8 (a) Solution:



Since, V_{GS} for Q_2 and Q_3 is same. Hence,

$$I_{D_2} = I_{D_1} = I_{ref}$$

$$\therefore \quad g_{m_2} = g_{m_3} = \sqrt{\frac{2\mu_p C_{ox} W I_{D_2}}{L}} = \sqrt{\frac{2 \times 10 \times 10^{-6} \times 100 \times 10^{-6} \times 100}{10}}$$

$$\therefore \quad g_{m_2} = g_{m_3} = 1.41 \times 10^{-4} \text{ mA/V}$$

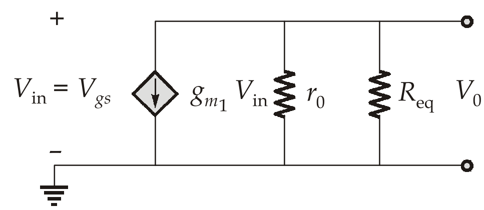
$$g_{m_1} = \sqrt{\frac{2\mu_n C_{ox} W I_{D_1}}{L}}$$

$$= \sqrt{\frac{2 \times 20 \times 10^{-6} \times 100 \times 10^{-6} \times 100}{10}} = 0.2 \text{ mA/V}$$

Since, the drain current is same for all the transistors. Hence,

$$r_o = \frac{V_A}{I_D} = \frac{100}{100 \times 10^{-6}} = 1 \text{ M}\Omega$$

Using the above small-signal parameters, the small-signal equivalent circuit can be drawn as below:



Here, R_{eq} is the resistance seen at the drain of Q_2. We get, $R_{eq} = r_0$

Hence, $$V_o = -g_{m_1} \cdot V_{in} \left[r_o \parallel R_{eq} \right]$$

$\Rightarrow$ $$A_v = \frac{V_o}{V_{in}} = -g_{m_1}[r_o \parallel r_o] = -0.2 \times 10^{-3} \times 500 \times 10^3 = -100$$

Q.8 (b) Solution:

(i) $$f(x) = x^4 - x - 10 = 0 \text{ and } f'(x) = 4x^3 - 1$$

$$f(2) = 16 - 2 - 10 = 4 \text{ and } f'(2) = 32 - 1 = 31$$

By Newton-Raphson's method

$$a_1 = a - \frac{f(a)}{f'(a)} = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{4}{31}$$

$$= 2 - 1.29 = 1.871$$

$$f(1.871) = (1.871)^4 - 1.871 - 10 = 12.25 - 1.871 - 10$$

$$= 0.379$$

$$f'(1.871) = 4(1.871)^3 - 1$$

$$= 4 \times 6.5497 - 1 = 25.1988$$

$$a_2 = 1.871 - \frac{f(1.871)}{f'(1.871)} = 1.871 - \frac{0.379}{25.1988} = 1.856$$

$$f(1.856) = (1.856)^4 - (1.856) - 10 = 0.0102$$

$$f'(1.856) = 4(1.856)^3 - 1 = 4 \times 6.3934 - 1$$

$$= 24.5736$$

$$a_3 = 1.856 - \frac{f(1.856)}{f'(1.856)} = 1.856 - \frac{0.0102}{24.5736} = 1.8556$$

$$f(1.8556) = (1.8556)^4 - 1.8556 - 10 = 0.00038$$

$$f'(1.856) = 4(1.8556)^3 - 1 = 24.5572$$

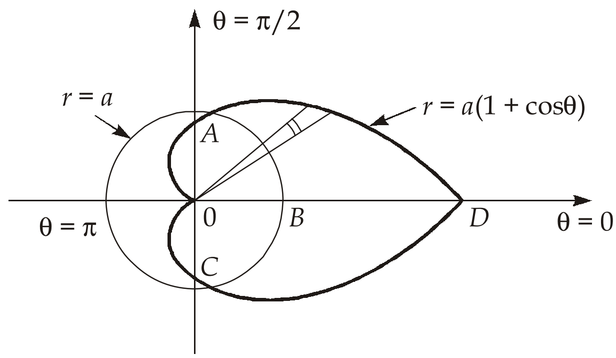
$$a_4 = 1.8556 - \frac{f(1.8556)}{f'(1.8556)} = 1.8556 - \frac{0.0038}{24.5572} = 1.85558$$

$$\text{Required root} = 1.85558$$

(ii)

$$r = a(1 + \cos\theta) \quad \text{...(i)}$$

$$r = a \quad \text{...(ii)}$$



Solving (i) and (ii), by eliminating r, we get

$$a(1 + \cos\theta) = a$$

$$1 + \cos\theta = 1$$

$$\cos\theta = 0$$

$$\theta = -\frac{\pi}{2} \text{ or } \frac{\pi}{2}$$

$$\text{Required area} = \text{Area } ABCDA$$

$$= \int_{-\pi/2}^{\pi/2} \int_{r \text{ for circle}}^{\text{for cardioid}} r\, d\theta\, dr = \int_{-\pi/2}^{\pi/2} \int_{a}^{a(1+\cos\theta)} r\, d\theta\, dr$$

$$= \frac{a^2}{2} \int_{-\pi/2}^{\pi/2} [(1+\cos\theta)^2 - 1]\, d\theta$$

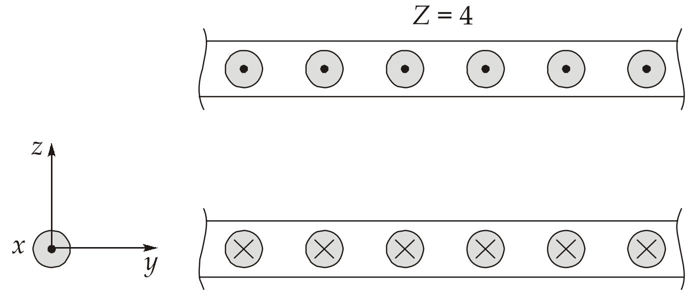
$$= \frac{2a^2}{2} \int_{0}^{\pi/2} (\cos^2\theta + 2\cos\theta)\, d\theta$$

$$= a^2 \left[\frac{\pi}{4} + 2(\sin\theta)_0^{\pi/2}\right]$$

$$= a^2 \left[\frac{\pi}{4} + 2\right] = \frac{a^2}{4}(\pi + 8)$$

Q.8 (c) Solution:

(i) The parallel current sheets are shown in figure,



Let, $H = H_0 + H_4$

where H_0 and H_4 are the contributions due to the current sheets $Z = 0$ and $Z = 4$, respectively.

In general, for an infinite sheet of current density K A/m,

$$H = \frac{1}{2}K \times \hat{a}_n$$

where $\hat{a}_n$ is a unit normal vector directed from the current sheet to the point of interest

$$H_0 = \frac{1}{2}K \times \hat{a}_n = \frac{1}{2}(-10\hat{a}_x) \times \hat{a}_z = 5\hat{a}_y \text{ A/m}$$

$$H_4 = \frac{1}{2}K \times \hat{a}_n = \frac{1}{2}(10\hat{a}_x) \times (-\hat{a}_z) = 5\hat{a}_y \text{ A/m}$$

Hence, $H = 10\hat{a}_y$ A/m

At (0, –3, 10) which is above the two sheets ($Z = 10 > 4 > 0$)

$$H_0 = \frac{1}{2}(-10\hat{a}_x) \times \hat{a}_z = 5\hat{a}_y \text{ A/m}$$

$$H_4 = \frac{1}{2}(10\hat{a}_x) \times \hat{a}_z = -5\hat{a}_y \text{ A/m}$$

Hence, $H = 0$ A/m

(ii) The magnetic flux density is given as

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & -\frac{I_0}{4\pi a^2}\mu(x^2+y^2) \end{vmatrix} = \frac{-I_0}{2\pi a^2}\mu_0\left(y\,\hat{a}_x - x\,\hat{a}_y\right)$$

So, the magnetic field intensity is given as

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{-I_0}{2\pi a^2}(y\,\hat{a}_x - x\,\hat{a}_y)$$

We calculate the closed line integral of this field as follows

$$\oint \vec{H}\cdot\vec{dl} = -\frac{I_0}{2\pi a^2}\oint_L (y\,\hat{a}_x - x\,\hat{a}_y)(a\,d\phi\,\hat{a}_\phi)$$

$$= -\frac{I_0}{2\pi a^2}\oint_L a\,d\phi(y\,\hat{a}_x - x\,\hat{a}_y)\cdot(\hat{a}_\phi)$$

$$= -\frac{I_0}{2\pi a^2}\oint_L a\,d\phi(y\,\hat{a}_x - x\,\hat{a}_y)(-\sin\phi\,\hat{a}_z + \cos\phi\,\hat{a}_z)$$

$$= -\frac{I_0}{2\pi a^2}\oint ad\phi(-y\sin\phi - x\cos\phi)$$

$$= \frac{I_0}{2\pi a^2}\oint_L ad\phi(a\sin^2\phi + a\cos^2\phi)$$

$\{\because x = r\cos\phi \text{ and } y = r\sin\phi\}$

$$= \frac{I_0}{2\pi}\oint_L d\phi(\sin^2\phi + \cos^2\phi)$$

$$= \frac{I_0}{2\pi}\oint_L d\phi = \frac{I_0}{2\pi}\times 2\pi = I_0$$

Since $\oint_L \vec{H}\cdot\vec{dl} = I_0$ Ampere's law is verified.

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