MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Mechanical Engineering Test No : 11

Section : A

1. (a) Solution:

(i)

The initial height of the space occupied by air is

$$x_1 = \frac{V_1}{A}$$

The initial height of the sand-filled space of the cylinder is

$$y_1 = L - l - x_1$$

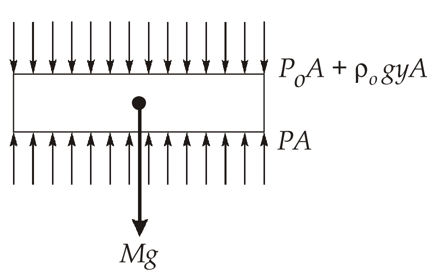
Consider the situation when the inner surface of the piston has moved to a height x above the bottom of the cylinder. Then

$$x = \frac{V}{A}$$

The height of the sand filled volume is

$$y = L - l - x$$

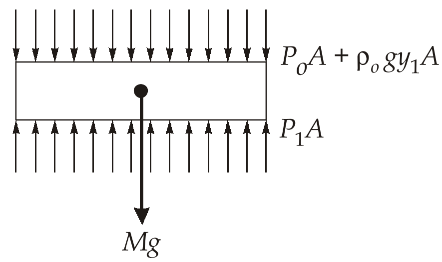
The force balance on the piston gives



$$PA = Mg + P_oA + yA\rho_o g$$

$$PA = Mg + P_oA + A\rho_o g\left(L - l - \frac{V}{A}\right) \quad \text{...(i)}$$

The force balance on the piston in the initial state gives



$$P_1A = Mg + P_oA + yA\rho_o g$$

$\therefore$
$$P_1A = Mg + P_oA + A\rho_o g\left(L - l - \frac{V_1}{A}\right) \quad \text{...(ii)}$$

Subtracting equation (ii) from equation (i), we obtain the linear P-V relation as

$$(P - P_1) = \frac{-\rho_o g(V - V_1)}{A} \quad \text{...(iii)}$$

(ii)

Work done by the gas, $W = \int_{V_1}^{V_2} PdV$

$$W = \int_{V_1}^{V_2} P_1 dV - \left(\frac{\rho_o g}{A}\right)\int_{V_1}^{V_2}(V - V_1)dV \quad \text{[from equation (iii)]}$$

$$W = P_1(V_2 - V_1) - \frac{\rho_o g(V_2 - V_1)^2}{2A}$$

Q.1 (b) Solution:

Lumped system analysis assumes a uniform temperature distribution throughout the body, which is the case only when the thermal resistance of the body to heat conduction is zero.

Thus, lumped system analysis is exact when

$$\text{Bi} = 0$$

and approximate when Bi > 0, smaller the Bi number, the more accurate the lumped system analysis.

It is generally accepted that lumped system analysis is applicable if

$$\text{Bi} \leq 0.1$$

When this criterion is satisfied, the temperature within the body relative to surroundings ($T - T_\infty$) remain within 5% of each other even for well rounded geometries such as a spherical ball.

When Biot number < 0.1, the variation of temperature with location within the body is slight and can reasonably be approximated as being uniform.

$$\text{Biot number (Bi)} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

$$= \frac{L_c/(KA)}{1/(hA)} = \frac{hL_c}{K}$$

$$L_c \text{ (characteristic length)} = \frac{V}{A_s} \frac{\text{(Volume of system)}}{\text{(Surface area)}}$$

Significance of Biot Number indicated that it is a measure of the relative magnitudes of the two heat transfer mechanisms : convection at the surface and conduction through the solid. A small value of Bi indicates that the inner resistance of the body to heat conduction is small relative to the resistance to convection between the surface and the fluid. As a result, the temperature distribution within solid becomes fairly uniform, and lumped system analysis becomes applicable.

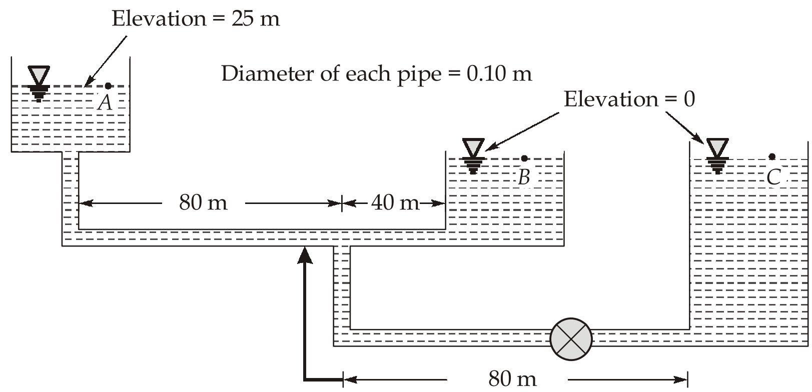
Physical Significance of the Fourier Number :

$$\text{Fourier Number, } (\tau) = \frac{K.L^2\left(\frac{1}{L}\right)\Delta T}{\frac{\rho C_P L^3}{t}\Delta T}$$

$$= \frac{\text{The rate of which heat is conducted across a body of thickness } L \text{ and normal area } L^2 \text{ (and the volume } L^3\text{)}}{\text{The rate at which heat is stored in a body of volume } L^3}$$

Therefore, the fourier number is a measure of heat conducted through a body relative to heat stored. Thus, a large value of the Fourier number indicates propagation of heat through a body.

1. (c) Solution:



$$f_1 = f_2 = f_3 = 4 \times 0.005 = 0.02 = f$$

From continuity equation, $Q_1 = Q_2 + Q_3$

Since, $d_1 = d_2 = d_3 = d$

$$V_1 = V_2 + V_3 \quad \text{...(i)}$$

for fluid flowing from A to B

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \frac{f_1 L_1}{d_1} \cdot \frac{V_1^2}{2g} + \frac{f_2 L_2}{d_2} \cdot \frac{V_2^2}{2g}$$

where, $P_A = P_B = 0$, $z_A = 25$ m, $z_B = 0$, $V_A = V_B = 0$

$$25 = \frac{f}{d(2g)}\left(l_1 V_1^2 + l_2 V_2^2\right)$$

$$= \frac{0.02}{0.1 \times 2 \times 10}\left(80V_1^2 + 40V_2^2\right)$$

$$2V_1^2 + V_2^2 = 62.5 \text{ m}^2/\text{s}^2 \quad \text{...(ii)}$$

Similarly, for flowing from A to C,

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + z_C + \frac{f_1 L_1}{d_1} \cdot \frac{V_1^2}{2g} + + \frac{f_3 L_3}{d_3} \cdot \frac{V_3^2}{2g}$$

where, $P_A = P_C = 0$, $V_A = V_C = 0$, $z_A = 25$ m, $z_C = 0$

$$25 = \frac{f}{d(2g)}\left(L_1 V_1^2 + L_3 V_3^2\right)$$

$$= \frac{0.02}{0.1 \times 2 \times 10}\left(80V_1^2 + 80V_3^2\right)$$

$$31.25 = V_1^2 + V_3^2 \quad \text{...(iii)}$$

Now, equation (ii) –2 × equation (iii)

$$V_2^2 - 2V_3^2 = 0$$

$$\Rightarrow \quad V_3 = \frac{V_2}{\sqrt{2}} \quad \text{...(iv)}$$

Putting value of V_1 in equation (ii)

$$2\left[V_2\left(1+\frac{1}{\sqrt{2}}\right)\right]^2 + V_2^2 = 62.5$$

$$\therefore \quad V_2 = 3.0254 \text{ m/s}$$

$$\text{Discharge to tank } B, Q_B = \frac{\pi d^2}{4}V_2 = \frac{\pi}{4}\times 0.1^2 \times 3.0254$$

$$= 0.02376 \text{ m}^3\text{/s or } 23.76 \text{ lit/s}$$

1. (d) Solution:

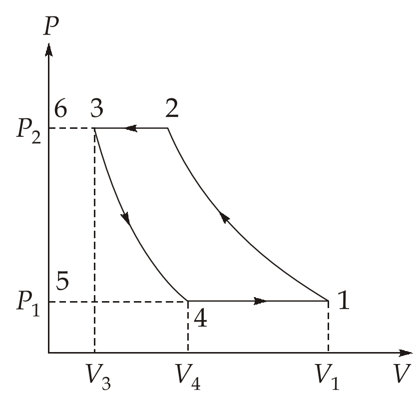
The volumetric efficiency of a compressor is the ratio of refrigerant delivered to the displacement of the compressor. It is also the ratio of effective swept volume to the swept volume.

Effective swept volume = $V_1 - V_4$

Swept volume = $V_1 - V_3 = V_s$

Clearance volume = $V_3 = V_c$

Total volume = V_1



$$\text{Volumetric efficiency} = \frac{\text{Effective swept volume}}{\text{Swept volume}} = \frac{V_1 - V_4}{V_1 - V_3}$$

$$\text{Clearance ratio, } C = \frac{\text{Clearance volume}}{\text{Swept volume}} = \frac{V_3}{V_1 - V_3} = \frac{V_c}{V_s}$$

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} = \frac{(V_1 - V_3) + (V_3 - V_4)}{(V_1 - V_3)}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3} \times \frac{V_3}{V_3}$$

$$= 1 + \frac{V_3}{V_1 - V_3}\left[1 - \frac{V_4}{V_3}\right] = 1 + C - C\frac{V_4}{V_3}$$

$$P_3 V_3^{\,n} = P_4 V_4^{\,n} \text{ or } \frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{1/n} = \left(\frac{P_2}{P_1}\right)^{1/n}$$

Hence, $$\eta_{vol} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{1/n} \qquad [P_3 = P_2 \text{ and } P_4 = P_1]$$

1. (e) Solution:

The declination, δ in degrees for any day of the year (N) can be calculated as:

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + N)\right]$$

Where, N = Jan + Feb + March + April + May + 15 day of June

N = 31 + 28 + 31 + 30 + 31 + 15

N = 166

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + 166)\right] = 23.45 \times 0.9942$$

δ = 23.314°

The hour angle 2h after local solar noon is

h = +0.25(120) = 30°

The mathematical expression for the solar altitude angle is

$\sin(\alpha) = \cos(\phi) = \sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(h)$

where, L = Local altitude

$\sin\alpha = \sin 40° \sin 23.314° + \cos 40° \cos 23.314° \cos 30°$

$= 0.864$

Therefore, α = 59.75°

Solar azimuth angle is, $$\sin(z) = \cos(23.35°)\frac{\sin(30°)}{\cos(59.75°)} = 0.911$$

Therefore, $z = 65.67°$

Length of day (in hours) is

$$\text{Day length} = \frac{2}{15}\cos^{-1}[-\tan(L)\tan(\delta)]$$

$$= \frac{2}{15}\cos^{-1}[-\tan(40°)\tan(23.314°)] = 14.83 \text{ hours}$$

This means that the sun rises at 12 – 7.4 = 4.6 = 4 : 36 am solar time and sets at 7.4 i.e. at 7 : 24 pm solar time.

2. (a)(i) Solution:

1. Since the expansion process is quasi-static, we have

$$W = \int_{v_1}^{v_2} PdV$$

Rearranging the van der Waals equation we obtain P as

$$P = \frac{\bar{R}T}{(v-b)} - \frac{a}{v^2}; \quad \text{where } v \text{ is the volume per kmol}$$

Therefore, the work done in the constant temperature expansion is

$$W = \bar{R}T\ln\left[\frac{(v_2-b)}{(v_1-b)} + a\left(\frac{1}{v_2} - \frac{1}{v_1}\right)\right]$$

Substituting numerical values in the above equation,

$$W = 8.3145\times10^3\times473\times\ln\left(\frac{5.2-0.0304}{3.2-0.0304}\right) + 5.52\times10^5\times\left(\frac{1}{5.2} - \frac{1}{3.2}\right)$$

$$W = 1923.86 - 66.346 = 1857.5 \text{ kJ}$$

2. The work done by an ideal gas during an isothermal expansion is

$$W_i = \bar{R}T\ln\left(\frac{V_2}{V_1}\right)$$

$$W_i = 8.3145\times10^3\times473\times\ln\left(\frac{5.2}{3.2}\right) \text{ kJ}$$

$$W_i = 1909.38 \text{ kJ}$$

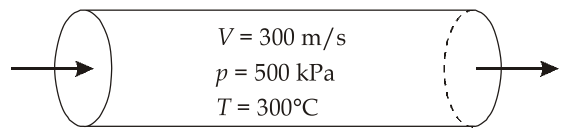
3. $$\text{Percentage difference} = \frac{W_i - W}{W}\times100 = \frac{1909.38-1857.5}{1857.5}\times100$$

$$= 2.793 \simeq 2.8\%$$

The prediction of the work done using the ideal gas equation is about 2.8% higher.

2. (a)(ii) Solution:

Given data: $V = 300$ m/s; $p = 500$ kPa; $T = 300°C$; $= (300 + 273)K = 573$ K



$$p_o = 100 \text{ kPa}$$

$$T_o = 20°C = (20 + 273)K = 293K$$

$$\text{Gas constant: } R = \frac{\bar{R}}{M} = \frac{8.314}{28} = 0.2969 \text{ kJ/kgK}$$

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 0.2969}{1.4 - 1} = 1.039 \text{ kJ/kgK}$$

$$\text{Availability: } a = (h - h_0) - T_o(s - s_0) + \frac{V^2}{2000}$$

$$= 1.039(573 - 293) - 293\left(1.039 \log_e \frac{573}{293} - 0.2969 \log_e \frac{500}{100}\right) + \frac{(300)^2}{2000}$$

$$= 290.92 - 293(0.6968 - 0.4778) + 45$$

$$= 271.75 \text{ kJ/kg}$$

2. (b) (i) Solution:

Average height of surface protrusions,

$$k = 0.12 \text{ mm} = 0.12 \times 10^{-3} \text{ m}$$

$$\text{Shear stress developed, } \tau_0 = 8.6 \text{ N/m}^2$$

$$\text{Density of water, } \rho = 1000 \text{ kg/m}^3$$

$$\text{kinematic viscosity, } \nu = 0.0093 \text{ stokes} = 0.0093 \times 10^{-4} \text{ m}^2/\text{s}$$

$$\text{Shear velocity, } u_f = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{8.6}{1000}} = 0.09274 \text{ m/s}$$

$$\text{Roughness Reynolds number} = \frac{u_f k}{\nu} = \frac{0.09274 \times 0.12 \times 10^{-3}}{0.0093 \times 10^{-4}} = 11.966$$

Since $\frac{u_f k}{\nu}$ lies between 4 and 100, the pipe surface behaves as in 'Transition'.

Note:

- For smooth boundary .... $\frac{u_f k}{\nu} < 4$

- For rough boundary .... $\frac{u_f k}{\nu} > 100$ and
- For boundary in transition stage .... $\frac{u_f k}{\nu}$ lies between 4 and 100.

(ii)

The velocity distribution near the rough boundaries is given by:

$$\frac{u}{u_f} = 5.75\log_{10}\left(\frac{y}{k}\right)+8.5 \quad \text{...(i)}$$

where, k = Average height of roughness elements, and

u_f = shear friction velocity

Equation (i) is known as Karman-Prandtl equation for the velocity distribution near hydrodynamically rough boundaries.

Let u = velocity at a distance (y) of 1 cm from the pipe wall

$1.2u$ = velocity at a distance (y) of 2 cm from the pipe wall

$$\frac{u}{u_f} = 5.75\log_{10}\left(\frac{1}{k}\right)+8.5 \quad \text{...(ii)}$$

$$\frac{1.2u}{u_f} = 5.75\log_{10}\left(\frac{2}{k}\right)+8.5 \quad \text{...(iii)}$$

Dividing (ii) by (iii), we get:

$$\frac{1}{1.2} = \frac{5.75\log_{10}\left(\frac{1}{k}\right)+8.5}{5.75\log_{10}\left(\frac{2}{k}\right)+8.5}$$

$$5.75\log_{10}\left(\frac{2}{k}\right)+8.5 = 1.2\times5.75\log_{10}\left(\frac{1}{k}\right)+1.2\times8.5$$

$$5.75\log_{10}\left(\frac{2}{k}\right)+8.5 = 6.9\log_{10}\left(\frac{1}{k}\right)+10.2$$

$$5.75\log_{10}(2) - 5.75\log_{10}(k) + 8.5 = 6.9\log_{10}(1) - 6.9\log_{10}(k) + 10.2$$

$$1.73 - 5.75\log_{10}(k) + 8.5 = 0 - 6.9\log_{10}(k) + 10.2$$

$$1.15\log_{10}(k) = 10.2 - 8.5 - 1.73 = -0.03$$

$$\log_{10}(k) = -\frac{0.03}{1.15} = -0.0261 \quad \text{or} \quad k = 0.942 \text{ cm}$$

Answer

2. (c) Solution:

(i) The hot and cold fluids have the same flow rate with

$$\dot{m}_h = \dot{m}_c = \frac{1200}{60} = 20 \text{ kg/s}$$

and
$$C_h = C_c = 20 \times 1005 = 20100 \text{ W/°C};$$

$$\frac{C_{min}}{C_{max}} = 1.0$$

The energy balance gives, $q = 210000 = C_h \Delta T_h = C_c \Delta T_c$

and
$$\Delta T_h = \Delta T_c = \frac{210000}{20100} = 10.45°\text{C}$$

The heat-exchanger effectiveness is

$$\in = \frac{\Delta T_{\text{min fluid}}}{\Delta T_{\text{max } HX}} = \frac{10.45}{(25-0)} = 0.4179$$

As we have, for a cross flow heat exchanger with both fluids unmixed, and $C = 1.0$.

$$\varepsilon = 1 - \exp\left[N^{0.22}\left(\exp\left(-N^{0.78}\right) - 1\right)\right] \quad \text{...(i)}$$

On putting $\varepsilon = 0.4179$, equation (i), we have

$$N = 0.8$$

and
$$N = \frac{UA}{C_{min}} = \frac{30(A)}{20100} = 0.8$$

$$A = 536 \text{ m}^2$$

This is the required area of the heat exchanger.

(ii)

We now examine the effect of reducing the flow rate by half, while keeping the inlet temperatures and value of U the same. Note that the flow rate of both fluids is reduced because they are physically the same fluid.

This means that the value of C_{min}/C_{max} will remain the same at a value of 1.0.

Also,
$$C_{min}\left(\frac{1}{2}\right)(20100) = 10050 \text{ W/°C}$$

so that
$$\text{NTU} = N = \frac{UA}{C_{min}} = \frac{(30)(536)}{10050} = 1.6$$

On putting the value of N in Equation (i), we have

$$\varepsilon = 1 - \exp\left[N^{0.22}\left(\exp\left(-N^{0.78}\right) - 1\right)\right]$$

$$\varepsilon = 0.5713$$

The temperature difference for each fluid is then

$$\Delta T = \in \Delta T_{\text{max HX}} = (0.5713)(25 - 0) = 14.28°\text{C}$$

The resulting heat transfer is then

$$\dot{q} = \dot{m}c\Delta T = (10050)(14.28) = 143.5 \text{ kW}$$

$$\text{Change in heat transfer rate} = \frac{210 - 143.5}{200} \times 100 = 32\% \text{ reduction}$$

(iii)

$$\text{NTU} = N = \frac{UA}{C_{\min}} \propto C^{0.8} \times C^{-1} = kC^{-0.2}$$

Our new value of N under these conditions would be

$$N = (0.8)\left(\frac{10050}{20100}\right)^{-0.2} = 0.919$$

On putting the value of N in Equation (i), we have

$$\in = 0.4494$$

$$\dot{q} = \dot{m}c\Delta T = (10050)(14.28) = 143.5 \text{ kW}$$

The corresponding temperature difference in each fluid is

$$\Delta T = T_{\text{min HX}} = (0.4494)(25 - 0) = 11.23°\text{C}$$

The heat transfer is calculated as

$$q = \text{m} \times \Delta T = (10050)(11.23) = 112.9 \text{ kW}$$

$$\% \text{ reduction in heat transfer rate} = \frac{210 - 112.9}{210} \times 100 = 46.24\%$$

3. (a)(i) Solution:

- The rate of heat interchange between the two plates

$$Q_{12} = \left(F_g\right)_{12} A_1 \sigma_b \left[T_1^4 - T_2^4\right]$$

$$(F_g)_{12} = \frac{1}{\left(\frac{1}{\epsilon_1}+\frac{1}{\epsilon_1}-1\right)}$$ For infinite long parallel plates.

$$(F_g)_{12} = \frac{1}{\frac{1}{0.6}+\frac{1}{0.4}-1} = 0.316$$

$$Q_{12} = 0.315 \times 1 \times (5.67 \times 10^{-8}) \times (800^4 - 500^4) = 6219 \text{ W/m}^2$$

- The emissivity of a polished aluminium shield if heat flow reduced to 40%.

$$\frac{\sigma \times \left(T_1^4 - T_2^4\right) \times A}{\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}+\frac{2}{\varepsilon_s}-2\right]} = 0.4 \times 6219$$

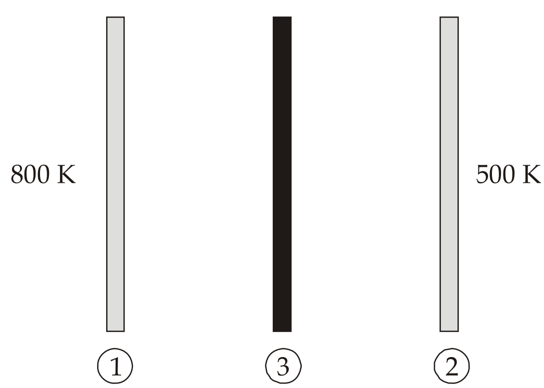
$$\frac{5.67 \times 10^{-8} \times \left[800^4 - 500^4\right] \times 1}{\left(\frac{1}{0.6}+\frac{1}{0.4}+\frac{2}{\varepsilon_s}-2\right)} = 2433$$

$$\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}+\frac{2}{\varepsilon_s}-2 = 7.91$$

$$\frac{1}{0.6}+\frac{1}{0.4}+\frac{2}{\varepsilon_s}-2 = 7.91$$

$$\varepsilon_s = 0.343$$

- Steady state temperature calculation:



Under steady state condition

$$Q_{12} = Q_{13} = Q_{32}$$

$$\frac{40}{100} \times 6219 = \frac{A\sigma_b\left[T_1^4 - T_s^4\right]}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_s}-1}$$

$$2483 = \frac{1 \times 5.67 \times 10^{-8}\left[800^4 - T_s^4\right]}{\frac{1}{0.6}+\frac{1}{0.343}-1}$$

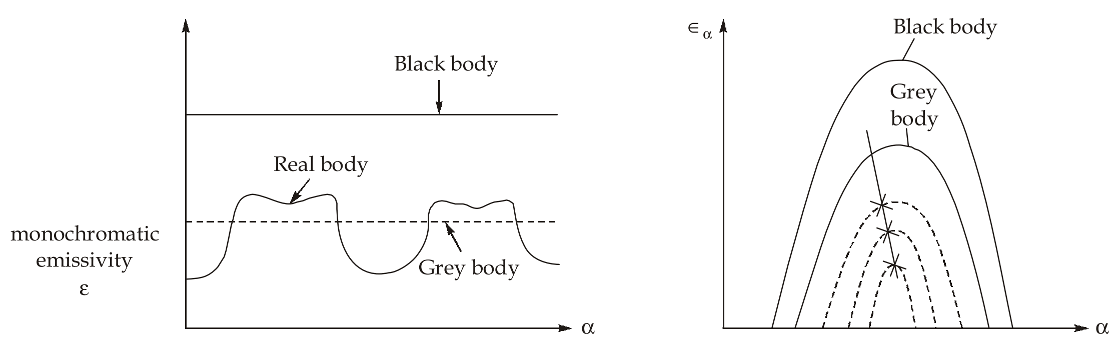
$$T_s = 710\text{ K}$$

3. (a)(ii) Solution:

Black body radiation: A body is said to be a black body if it is perfect absorber and perfect emitter of radiations. A black body absorbs all the radiations incident upon it and reflects the radiation absorbed by it. Also it emits radiation equally in all direction.

Grey body radiations: A body is said to be grey with respect to radiation if all its radiation properties are independent of wavelength.

Hence monochromatic emissivity of a grey body stay constant and does not change with wavelength of emission.



Weins displacement law can applied for grey bodies as well as shown in figure with increase in temp. wavelength at which maximum emission takes places as shown above for grey bodies as well, is like blackbodies.

3. (b) Solution:

$$\text{Shaft power, } P = 1\text{ MW} = 1000\text{ kW}$$

$$\text{Speed, } N = 120\text{ rpm}$$

$$\text{Overall efficiency, } \eta_0 = 92\%$$

$$\text{Head, } H = 12\text{ m}$$

$$\text{Overall efficiency, } \eta_0 = \frac{\text{Shaft power}}{\text{Water power}} = \frac{P}{\rho g Q H}$$

$$\text{Flow rate, } Q = \frac{P}{(\rho g H)\eta_0} = \frac{1000 \times 10^3}{10^3 \times 9.81 \times 12 \times 0.92} = 9.2334\text{ m}^3/\text{s}$$ **Ans**

$$\text{Specific speed, } N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{120\sqrt{1000}}{(12)^{5/4}} = 169.9 \text{ (S.I. unit)}$$ **Ans**

Model scale = 1 : 10

Head under which model is tested, $H_m = 7.2$ m

for similar turbine, each of the following parameters must be same for both model and prototype:

(i) Head coefficient, $C_H = \dfrac{H}{N^2D^2}$

(ii) Flow coefficient, $C_Q = \dfrac{Q}{ND^3}$

(iii) Power coefficient, $C_P = \dfrac{P}{N^3D^5}$

(i) $$\left(\frac{H}{N^2D^2}\right)_{\text{model}} = \left(\frac{H}{N^2D^2}\right)_{\text{prototype}}$$

$$\frac{H_m}{N_m^2D_m^2} = \frac{H_p}{N_p^2D_p^2}$$

$$N_m{}^2 = N_p^2\left(\frac{D_p}{D_m}\right)^2\frac{H_m}{H_p}$$

$$N_m = N_p \times \frac{D_p}{D_m} \times \sqrt{\frac{H_m}{H_p}} = 120\times 10\sqrt{\frac{7.2}{12}} = 929.5 \text{ rpm}$$

$$\frac{Q_m}{N_mD_m^3} = \frac{Q_p}{N_pD_p^3}$$

$$Q_m = Q_p\frac{N_m}{N_p}\left(\frac{D_m}{D_p}\right)^3$$

Discharge in the model:

$$Q_m = 9.2334\times\frac{929.5}{120}\times\left(\frac{1}{10}\right)^3$$

$$= 0.07152 \text{ m}^3/\text{s}$$ Answer

$$\text{Power coefficient, } \frac{P_m}{N_m^3D_m^5} = \frac{P_p}{N_p^3D_p^5}$$

Power produced by the model:

$$P_m = P_P\left(\frac{N_m}{N_p}\right)^3\left(\frac{D_m}{D_p}\right)^5$$

$$= 1000\left(\frac{929.5}{120}\right)^3 \times \left(\frac{1}{10}\right)^5 = 4.65 \text{ kW}$$ Answer

3. (c) Solution:

Given : Diameter of circular orifice, d = 3.2 cm = 0.032 m

Co-efficient of discharge, C_d = 0.62

Pressure across orifice, h_w = 150 mm of water

Temperature of air in the room = 20°C

Piston displacement = 0.00178 m^3

Compression ratio, r = 6.5

Fuel consumption = 0.135 kg/min

Calorific value of fuel, C = 43900 kJ/kg

Brake power, B.P. = 28 kW

Speed = 2500 rpm

$k = \frac{1}{2}$ ..... for 4-stroke cycle, engine

(i) Volumetric efficiency on the basis of air alone:

Characteristics gas equation is written as,

$$pV = mRT$$

or $$\frac{m}{V} = \frac{p}{RT} = \frac{1.0132\times 10^5}{287\times(20+273)} = 1.2 \text{ kg/m}^3$$

Also, 150 mm of H_2O $= \frac{150}{1000}\times 1000 = 150 \text{ kg/m}^2$

Thus head of air column causing flow,

$$= \text{Air consumption} = C_d \times A \times \sqrt{2gH}$$

$$= 0.62\times\frac{\pi}{4}\times(0.032)^2\times\sqrt{2\times 9.81\times 125} = 0.0247 \text{ m}^3/\text{s}$$

Therefore, air consumption per stroke = $\dfrac{0.0247\times 60}{\left(\dfrac{2500}{2}\right)} = 0.001185 \text{ m}^3$

$\therefore$ Volume efficiency, $\eta_{vol.} = \dfrac{\text{Air consumption of stroke}}{\text{Piston displacement}}$

$$= \frac{0.001185}{0.00178} = 0.665 \text{ or } 66.5\%$$ **Ans.**

(ii) Air-fuel ratio:

Mass of air drawn into the cylinder per min. = 0.0247 × 60 × 1.2 = 1.778 kg/min

$\therefore$ Air-fuel ratio $= \dfrac{1.778}{0.135} = 13.17 : 1$ **Ans.**

(iii) Brake mean effective pressure, p_{mb}:

$$\text{B.P.} = \frac{n \times p_{mb} LANk \times 10}{6}$$

$$28 = \frac{1 \times p_{mb} \times 0.00178 \times 2500 \times \frac{1}{2} \times 10}{6} \quad [\because LA = 0.00178 \text{ m}^3]$$

$\therefore$ $p_{mb} = \dfrac{28 \times 6 \times 2}{0.00178 \times 2500 \times 10} = 7.55 \text{ bar}$ **Ans.**

(iv) Relative efficiency:

$$\eta_{\text{air-standard}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(6.5)^{1.4-1}} = 0.527 \text{ or } 52.7\%$$

Brake thermal efficiency, $\eta_{th(B)} = \dfrac{B.P.}{\dot{m}_f \times C} = \dfrac{28}{\frac{0.135}{60} \times 43900} = 0.2835 \text{ or } 28.35\%$

$\therefore$ $\eta_{relative} = \dfrac{\eta_{thermal(B)}}{\eta_{\text{air-standard}}} = \dfrac{0.2835}{0.527}$

$= 0.5379 \text{ or } 53.79\%$ **Ans.**

4. (a) Solution:

$$\text{Re} = \frac{\rho l v}{\mu}$$

$$\text{Re} = \frac{1.076 \times \sqrt{1.2} \times 2.0}{19.8 \times 10^{-6}} = 119060.5$$

Re < 5 × 10^5 therefore flow is laminar

$$\text{Pr} = \frac{\mu c_p}{k} = \frac{19.8 \times 10^{-6} \times 1008}{0.0286} = 0.698$$

$$\text{Nu} = \frac{hl}{k} = 0.664(\text{Re})^{0.5}(\text{Pr})^{0.33}$$

$$= 0.664(119060.5)^{0.5}(0.698)^{0.33} = 203.48$$

$$h = \frac{203.48\,k}{l} = \frac{203.48 \times 0.0286}{\sqrt{1.2}}$$

$$= 5.3125 \text{ W/m}^2\text{K}$$

Heat flow from both sides of the plate is given by

$$Q = h(2A)\Delta t$$

$$Q = 5.3125 \times 2 \times 1.2\,(90 - 20)$$

$$Q = 892.5 \text{ W}$$ Answer

The instantaneous heat loss from the plate is also given by

$$= 2500 \times 1.2 \times 0.003 \times 650 \times \frac{dT}{d\tau} = 5850\frac{dT}{d\tau}$$

Also, $$-mc_p\frac{dT}{d\tau} = hA_s(T - T_\infty)$$

when $$\tau = 0,\ T = T_i$$

∴ $$\left(-\frac{dT}{d\tau}\right)_{\tau=0} = \text{Rate of cooling}$$

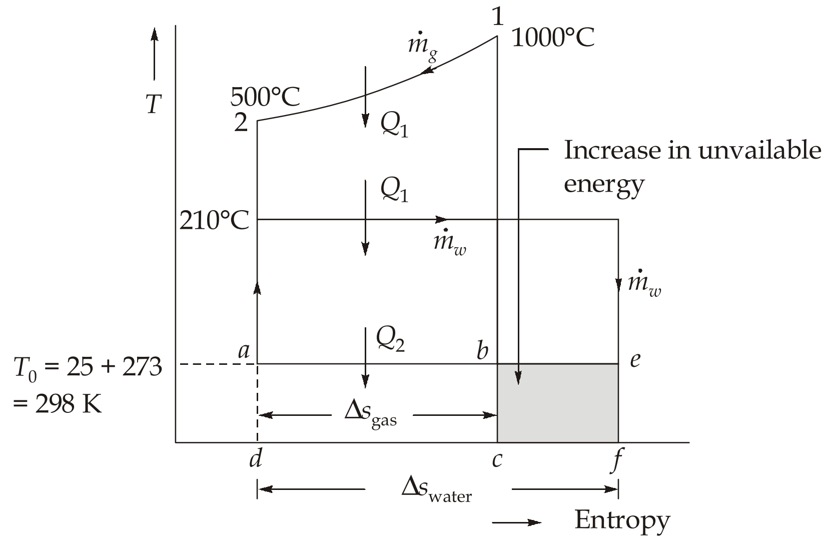
$$= \frac{892.5}{5850} = 0.15256\ ^\circ\text{C/s}$$ Answer

$$\frac{(\Delta t)_2}{(\Delta t)_1} = \frac{h_2}{h_1} = \left(\frac{\text{Re}_2}{\text{Re}_1}\right)^{0.5}$$

$$= \left(\frac{v_2}{v_1}\right)^{0.5} = \left(\frac{2.5}{2}\right)^{0.5} = 1.118$$

∴ Percentage increase in cooling rate = 11.8% Answer

4. (b) (i) Solution:



When water evaporates at 210°C as the gas gets cooled from 1000°C to 500°C, the resulting power cycle has an unavailable energy represented by the area *aefd*. The increase in unavailable energy due to irreversible heat transfer is thus given by area *befc*.

Entropy increase of 1 kg water

$$(\Delta S)_{water} = \frac{\text{Latent heat absorbed}}{T} = \frac{1 \times 1900.2}{(273+210)}$$

$$= 3.934 \text{ kJ/K}$$

$$Q_1 = \text{Heat transferred from the gas}$$

$$= \text{Latent heat transferred from water}$$

$$= m_g c_{pg}(1000 - 500) = 1 \times 1900.2 \text{ (for one kg of water)}$$

$$m_g c_{pg} = \frac{1900.2}{500} = 3.8004 \text{ kJ/K}$$

$$(\Delta S)_{Gas} = 3.8004\, ln\left(\frac{773}{1273}\right) = -1.896 \text{ kJ/K}$$

$$(\Delta S)_{Total} = (\Delta S)_{water} + (\Delta S)_{gas}$$

$$= 3.934 - 1.896 = 2.038 \text{ kJ/K}$$ Answer

Increase in unavailable energy = $T_0(\Delta S)_{total} = 298 \times 2.038 = 607.2966$ kJ Answer

4. (b) (ii) Solution:

The compressibility factor (Z) is a useful thermodynamic property for modifying the ideal gas law to account for behavior of real gases. It is measure of how much the thermodynamic properties of a real gas deviate from those expected of an ideal gas. Compressibility factor is defined as the ratio of actual volume at a given pressure and temperature to the ideal volume under same conditions of pressure and temperature. It is also known as the compression factor or the gas deviation factor.

Significance:

(i) For $Z \simeq 1$, the gas pressure becomes very low and all gases tend toward ideal behavior.

(ii) For $Z < 1$, the gas is at intermediate pressure because the intermolecular forces of attraction cause the actual volumes to be less than the ideal values.

(iii) For $Z > 1$, the gas is at high pressure because the intermolecular repulsive forces cause the actual volumes to be greater than the ideal values.

In case of a real gas, forces between the molecules are a function of the distance between them. When the molecules are far apart, there are attractive forces between them but when they are forced close together their electronic fields overlap due to which repulsive forces come into play. Pressure and molecular attraction tend to confine the molecules while temperature and molecular repulsion tend to separate them.

When the temperature of gas is high, kinetic energy of molecules is increased due to which the molecules tend to move apart. So, at high temperatures, the actual volume of gas will be more than the ideal volume. Hence the compressibility factor will be greater than one at high temperatures.

4. (c) Solution:

Given : $T_E = 233$ K; $T_A = 303$ K; $T_C = 313$ K; $T_G = 373$ K

(i) Mass flow rate of solution in evaporator for 1 tonne refrigeration capacity, m

$$m = \frac{\frac{(14000 \times 1)}{3600}}{(h_1 - h_4)} = \frac{\left(\frac{14000 \times 1}{3600}\right)}{(1388 - 470)} = 0.00424 \text{ kg/s}$$ **Ans.**

(ii) Mass flow rates of strong and weak solution, m_1; m_2:

The two equations are for overall mass balance for absorber, we have

$$m_1 + m_{2'} = m_{1'} \quad \text{(overall mass balance for absorber)}$$

$$m_1 x_1 + m_{2'} x_{2'} = m_{1'} x_{1'} = (m_1 + m_{2'}) x_{1'}$$

(Ammonia balance for the absorber)

$$0.00424 \times 0.945 + 0.375\,m_{2'} = (0.00424 + m_{2'})0.421$$

or $$0.004 + 0.375m_{2'} = 0.00178 + 0.421\,m_{2'}$$

$\therefore$ $$m_{2'} = \frac{(0.004 - 0.00178)}{(0.421 - 0.375)} = 0.0483 \text{ kg/s}$$ **Ans.**

and $$m_{1'} = m_1 + m_{2'} = 0.00424 + 0.0483 = 0.0525 \text{ kg/s}$$ **Ans.**

(iii) Absorber heat rejection (Q_A); Generator heat transfer (Q_G) ; Condenser heat rejection (Q_C)

- Energy balance for absorber gives,

$$Q_A + m_{1'}h_{1'} = m_1h_1 + m_{2'}h_{2'}$$

$\therefore$ $$Q_A = m_1h_1 + m_{2'}h_{2'} - m_{1'}h_{1'}$$
$$= 0.00424 \times 1388 + 0.0483 \times 340 - 0.0525 \times 30$$
$$= 20.73 \text{ kJ/s}$$ **Ans.**

- Energy balance for generator gives,

$$Q_G + m_{1'}h_{1'} = m_{2'}h_{2'} + m_2h_2 = m_{2'}h_{2'} + m_1h_2 \qquad (\because \; m_2 = m_1 = m)$$

$\therefore$ $$Q_G = m_{2'}h_{2'} + m_1h_2 - m_{1'}h_{1'}$$
$$= 0.0483 \times 340 + 0.00424 \times 1870 - 0.0525 \times 30$$
$$= 22.77 \text{ kJ/s}$$ **Ans.**

- Condenser heat rejection, $Q_C = m_2(h_2 - h_3) = 0.00424(1870 - 470)$
$$= 5.94 \text{ kJ/s}$$ **Ans.**

(iv) Overall energy balance, COP

Overall energy balance $= Q_G + Q_E - Q_A - Q_C$

$$= 22.77 + \frac{\left(\frac{14000 \times 1}{1}\right)}{3600} - 20.73 - 5.94 = 0$$

This checks the overall energy balance,

$$\text{COP} = \frac{Q_{ref}}{Q_A} = \frac{\left(\frac{14000 \times 1}{3600}\right)}{20.73} = 0.187$$ **Ans.**

Section : B

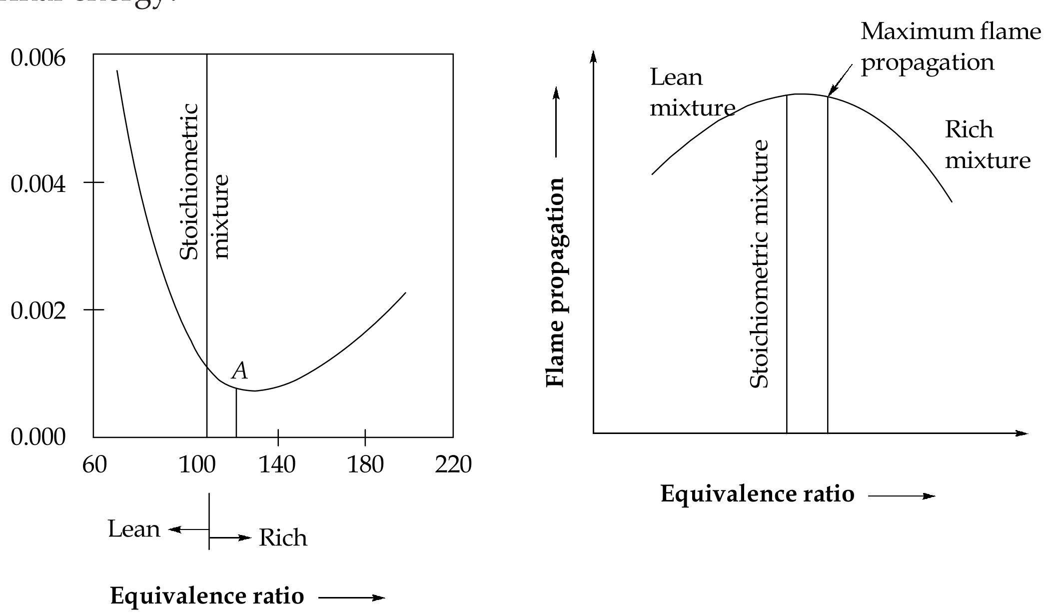
5. (d)

Effect of the following on the flame speed during combustion in an SI engine is:

(i) Compression ratio: A high compression ratio increases the pressure and temperature of the working mixture which reduces the initial preparation phase of combustion and hence less ignition advance is needed. High pressure and temperature of the compressed mixture also speed up the second phase of combustion. Thus engine with higher compression ratio have higher flame speeds.

(ii) Intake pressure: Flame speed increases with the increase in the intake pressure and temperature. A higher initial pressure and temperature may help to form a better homogenous air-vapour mixture which helps in increasing the flame speed.

(iii) Air-fuel ratio: The air-fuel ratio has a very significant influence on the flame speed. The highest flame velocities are obtained with somewhat richer mixture. When the mixture is made leaner or richer the flame speed decreases. Less thermal energy is released in the case of lean mixtures resulting in lower flame temperature. Very rich mixtures lead to incomplete combustion which results again in the release of less thermal energy.



(iv) Engine load: The cycle pressure increases when the engine output is increased or the load on engine is increased. With the increased throttle opening the cylinder gets filled to a higher density. This results in increased flame speed. When the output is decreased by throttling, the initial and final compression pressures decrease and dilution of the working mixture increases. The smooth development of self-propagation nucleus of flame becomes unsteady and difficult.

5. (b) Solution:

Given: $U_o = 10$ m/s, $P = 1$ atm $= 1.01325 \times 10^5$ Pa, $T = 273 + 27 = 300$ K

$$\text{Radius of rotor, } r = \frac{D}{2} = \frac{120}{2} = 60 \text{ m}$$

$$\text{Speed of rotor, } \omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.1888 \text{ rad/s}$$

$$\text{Air density, } \rho = \frac{P}{RT} = \frac{1.01325 \times 10^5}{287 \times 300} = 1.1768 \text{ kg/m}^3$$

$$\text{Area of rotor, } A = \pi r^2 = \pi \times 60^2 = 11309.73 \text{ m}^2$$

For maximum output,
$$a = \frac{1}{3}$$

$$\text{Power coefficient, } c_p = 4a(1-a)^2$$

$$C_{p,\max} = 4 \times \frac{1}{3}\left(1 - \frac{1}{3}\right)^2 = \frac{4}{3} \times \frac{4}{9} = 0.593$$

$$\text{Tip speed ratio, } \lambda = \frac{\omega r}{U_o} = \frac{60 \times 4.1888}{10} = 25.1328$$

$$\text{Theoretical power, } P_o = \frac{1}{2}(\rho A)U_o^3 = \frac{1}{2} \times 11309.73 \times 1.1768 \times 1000 = 6.65 \text{ MW}$$

$$T_m = \frac{P_o}{U_o} r = \frac{6.65}{10} \times 60 = 39.9 \times 10^6 \text{ Nm}$$

$$C_{T,\max} = \frac{C_{p,\max}}{\lambda} = \frac{0.593}{25.1328} = 0.0236$$

Torque produced at the shaft for maximum output,

$$T_{\text{shaft, max}} = 39.9 \times 10^6 \times 0.0236 = 941640 \text{ Nm} = 9.4164 \times 10^5 \text{ Nm}$$

5. (c) Solution:

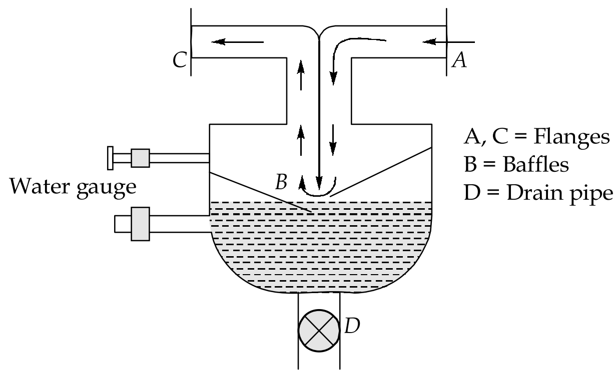
Steam separator: The steam available from a boiler may be either wet, dry; or superheated, but in many cases there will be loss of heat from it during its passage through steam pipe from the boiler to engine or turbine tending to produce wetness.

The use of wet steam in the turbine is uneconomical besides involving some risk. Hence it is usual to endeavour to separate any water particles that may be present from the steam before the steam enters the turbine. This is done by steam separator.

Thus the function of a steam separator is to remove the entrained water particles from the steam conveyed to the steam turbine. Types of steam separator according to principle of operation are:

1. Baffle type
2. Reverse current type
3. Centrifugal type

Baffle type steam separator: Figure below shows the Baffle type steam separator. The steam enters the flange A and flows down. In its passage it strikes the baffle B; as a result it gets deflected but water particle having greater density and greater inertia fall to the bottom of separator. The drier steam discharges through the flange C. To see the level of water collected a water gauge is provided. The water collected in the vessel is removed at intervals through the drain pipe.



5. **(d) Solution:**

Given: t_{db} = 22°C, t_{dp} = 15°C, p_v = 0.017 bar, p_{vs} = 0.0264 bar

$$p_t = 1.01325 \times \frac{730}{760} = 0.9732 \text{ bar}$$

∴ Relative humidity, $\phi = \frac{p_v}{p_{vs}} = \frac{0.017}{0.0264} = 0.644$

(i) Specific humidity, $\omega = \frac{0.622 p_v}{p_t - p_v} = \frac{0.622 \times 0.017}{0.9732 - 0.017}$

= 0.011 kg/kg of dry air

(ii) Enthalpy of air, $h = 1.005\ t_{db} + \omega(2500 + 1.82 t_{db})$

= 1.005 × 22 + 0.011 (2500 + 1.88 × 22)

= 22.11 + 27.95

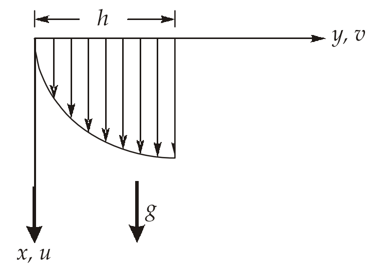
= 50.06 kJ/kg of dry air

(iii) Specific volume of air is equal to the volume of 1 kg of dry air or 0.011 kg of water vapour. Based on dry air part.

$$v = v_a = \frac{R_a \times T_a}{p_a} = \frac{R_a \times T_a}{(p_t - p_v)}$$

$$= \frac{287 \times (22 + 273)}{(0.9732 - 0.017) \times 10^5} = 0.885 \text{ m}^3/\text{kg of dry air}$$

5. (e) Solution:



Given coordinate system,

$$v = 0 \text{ and } u = u(y) \text{ only}$$

- Acceleration of fluid in vertical direction (x-coordinate)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \quad \text{...(i)}$$

Since, $u = u(y)$ only, i.e. $\frac{\partial u}{\partial x} = 0$ and $v = 0$

- $$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 + \frac{\partial^2 u}{\partial y^2}$$

$\therefore$ $$\nabla^2 u = \frac{\partial^2 u}{\partial y^2} \quad (\because u = u(y) \text{ only}) \quad \text{...(ii)}$$

- Now applying Navier-Stokes equation in x-direction

$$\rho a_x^{\nearrow 0} = -\frac{\partial P}{\partial x}^{\nearrow 0} + \mu\nabla^2 u + \rho g \quad \text{(from equation (i) and (ii))}$$

$\frac{\partial P}{\partial x} = 0$ (because according to question flow is driven by gravity)

$\Rightarrow$ $$\mu\frac{d^2u}{dy^2} + \rho g = 0$$

$\Rightarrow$ $$\frac{d^2u}{dy^2} = -\frac{\rho}{\mu}g$$

$\Rightarrow$ $$\frac{du}{dy} = -\frac{\rho}{\mu}gy + C_1$$

According to question shear stress at $y = h$ is zero, i.e., $\left.\frac{du}{dy}\right|_{y=h} = 0$

$\Rightarrow$ $$0 = -\frac{\rho g h}{\mu} + C_1$$

$\therefore$ $$C_1 = \frac{\rho g h}{\mu}$$

$$\frac{du}{dy} = -\frac{\rho g}{\mu}y + \frac{\rho g h}{\mu}$$

$$\tau = \tau_{max}, \text{ where } \frac{du}{dy} \text{ is maximum}$$

$$\frac{du}{dy} = -\frac{\rho g}{\mu}[y - h]$$

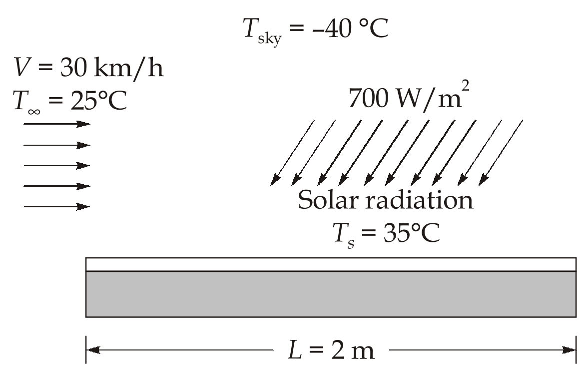
$$\left(\frac{du}{dy}\right)_{max} = \left(\frac{du}{dy}\right)_{y=0} = \frac{\rho g h}{\mu}$$

$$\tau_{max} = \mu\left(\frac{du}{dy}\right)_{max} = \rho g h$$

6. (a)

Assumption:

- Steady state conditions
- Critical Reynold's number = 5×10^5
- Negligible heat exchange on the back surface of the absorber plate
- Air is an ideal gas with constant properties
- The local atmospheric pressure is 1 atm.



Assuming wind flows across 2 m surface, the Reynold's number is determined as

$$Re_L = \frac{VL}{\nu} = \frac{30 \times 1000 \times 2}{3600 \times 1.608 \times 10^{-5}} = 1.036 \times 10^6$$

As $Re_L > R_{critical}$, we have relation for combined laminar and turbulent flow, the average heat transfer coefficient can be calculated as

$$Nu = \frac{hL}{k} = \left(0.037\, Re^{0.8} - 871\right) Pr^{1/3}$$

$$= [0.037 \times (1.036 \times 10^6)^{0.8} - 871]\,(0.7282)^{1/3}$$

$$= 1378$$

$$h = \frac{k}{L} Nu = \frac{0.02588 \times 1378}{2} = 17.83\ W/m^2K$$

Rate of heat loss from the collector by convection is

$$Q_{conv} = hA_s(T_s - T_\infty)$$

$$= 17.83 \times 2 \times 1.2\,(35 - 25) = 427.9\ Watt$$

The rate of heat loss from the collector by radiation is

$$Q_{\text{radiation}} = \in A_s \sigma \left(T_s^4 - T_{sky}^4\right)$$
$$= 0.90 \times 2 \times 1.2 \times 5.67 \times 10^{-8} [(35 + 273)^4 - (-40 + 273)^4]$$
$$= 741.2 \text{ Watt}$$
$$Q_{\text{total}} = Q_{\text{convection}} + Q_{\text{radiation}} = 427.9 + 741.2$$
$$= 1169.1 \approx 1169 \text{ Watt}$$

The net rate of heat transferred to the water is

$$Q_{\text{net}} = Q_{\text{in}} - Q_{\text{out}} = \alpha AI - Q_{\text{out}} = 0.88 \times 2 \times 1.2 \times 700 - 1169$$
$$= 1478 - 1169 = 309 \text{ Watt}$$

Efficiency of collector is given as,

$$\eta_{\text{collector}} = \frac{Q_{net}}{Q_{in}} = \frac{309}{1478} = 0.209 \text{ or } 20.9\%$$

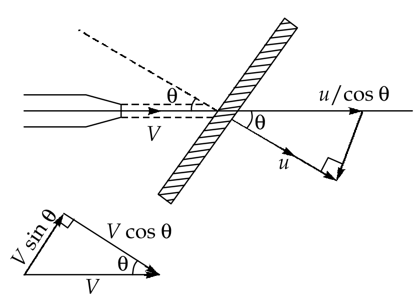
Temperature rise of water as it flows through the collector is

$$Q_{\text{net}} = \dot{m} c_p \Delta T$$

$$\Delta T = \frac{309}{\left(\frac{1}{60}\right)(4180)} \approx 4.4354°\text{C}$$

6. (b) Solution:

Given :
$$A = 13 \text{ cm}^2 = 13 \times 10^{-4} \text{ m}^2$$
$$V = 25 \text{ m/s}$$
$$u = 10 \text{ m/s}$$



From velocity triangle,

Relative velocity of jet w.r.t. plate in normal direction:

$$= V\cos\theta - u$$

Also, relative velocity along axis of jet :

$$= V - \frac{u}{\cos\theta}$$

So, mass flow rate, $\dot{m} = \rho A\left(V - \frac{u}{\cos\theta}\right)$

1. (i) F_n, Normal Force :

$$F_n = \dot{m}(V_i - V_f) = \rho A\left(V - \frac{u}{\cos\theta}\right) \times (V\cos\theta - u)$$

$$= \frac{\rho A(V\cos\theta - u)^2}{\cos\theta}$$

$$= 10^3 \times (13 \times 10^{-4}) \times (25\cos 40 - 10)^2 \times \frac{1}{\cos 40}$$

$$= 142.11 \text{ N}$$

(ii) Power Produced, P :

$$P = F_n \times u = 142.11 \times 10 = 1421.1 \text{ W}$$

(iii) Efficiency, η :

$$\eta = \frac{P}{\text{K.E.}} = \frac{P}{\frac{1}{2}(\dot{m}V^2)} = \frac{P}{\frac{1}{2}(\rho AV)V^2} = \frac{2P}{\rho AV^3}$$

$$= \frac{2 \times 1421.1}{10^3 \times 13 \times 10^{-4} \times (25)^3} = 0.1399 \approx 0.14 \text{ or } 14\%$$

2. Let Q_1 and Q_2 be volume flow rate along the tangential to plate in upward and downward direction respectively and Q is rate of flow striking on plate.

$\therefore$ $$Q_1 + Q_2 = Q \quad ...(i)$$

Since, the resultant force in tangential direction on plate is zero.

$$\therefore \left(\frac{\rho Q_1(V\cos\theta - u)}{\cos\theta} - \frac{\rho Q_2(V\cos\theta - u)}{\cos\theta}\right) - \frac{\rho Q(V\cos\theta - u)}{\cos\theta} \times \sin\theta = 0$$

$$\Rightarrow \quad Q_1 - Q_2 - Q\sin\theta = 0$$

$$\Rightarrow \quad Q_1 - Q_2 = Q\sin\theta \quad ...(ii)$$

From equation (i) and (ii)

$$Q_1 = \frac{Q}{2}(1 + \sin\theta)$$

$$Q_2 = \frac{Q}{2}(1 - \sin\theta)$$

So, $$\frac{Q_1}{Q_2} = \frac{1+\sin\theta}{1-\sin\theta} = \frac{1+\sin 40^\circ}{1-\sin 40^\circ} = 4.598 \approx 4.6$$

OR

$$\frac{Q_2}{Q_1} = \frac{1-\sin\theta}{1+\sin\theta} = 0.217$$

6. (c) Solution:

Given: $m_1 = 60$ kg/min, $T_1 = 5°\text{C} = T_2'$, $\phi_1 = 100\% = 1$

From steam table given, at $T_1 = 5°\text{C}$

$$P_{vs1} = 0.00872 \text{ bar}$$

$$\phi_1 = \frac{P_{v1}}{P_{vs1}}$$

$\Rightarrow$ $P_{v1} = 1 \times P_{vs1} = 0.00872$ bar

$$\omega_1 = \frac{0.622 P_{v1}}{P - P_{v1}} = \frac{0.622 \times 0.00872}{1 - 0.00872}$$

$$= 5.47 \times 10^{-3} \text{ kg/kg d.a}$$

$$h_1 = 1.005 T_1 + \omega_1 (2500 + 1.88 T_1)$$

$$= 1.005 \times 5 + 5.47 \times 10^{-3} (2500 + 1.88 \times 5)$$

$$= 18.75 \text{ kJ/kg}$$

For sensible heating,

$\therefore$ $\omega_1 = \omega_2 \Rightarrow P_{v1} = P_{v2}$

So, $P_{v2} = 0.00872$ bar

Now, $P_{v2} = \phi_2 P_{vs2}$

$\Rightarrow$ $P_{vs2} = \dfrac{0.00872}{0.2} = 0.0436$ bar

Now, at $P_{vs2} = 0.0436$ bar

By interpolation

$$\frac{T_2 - 30}{31 - 30} = \frac{0.0436 - 0.04246}{0.04496 - 0.04246}$$

$$T_2 = 30.456°\text{C}$$

$$h_2 = 1.005 \times 30.456 + 5.47 \times 10^{-3} (2500 + 1.88 \times 30.456)$$

$$= 44.60 \text{ kJ/kg}$$

$$T_3 = 24°\text{C (given)}$$

(i) Let m_2 is mass flow rate through the heating coil and m_2' is mass flow rate by passed. Now after mixing of heated air and bypassed air,

$$m_3 T_3 = m_2 T_2 + m_2' T_2'$$

where, $m_3 = m_1 = 60$ kg/min

$$m_2 = m_1 - m_2' = 60 - m_2'$$

So, $60 \times 24 = (60 - m_2') \times 30.456 + m_2' \times 5$

$$1440 = 1827.36 - 30.456 m_2' + 5 m_2'$$

$$\Rightarrow \qquad m_2' = \frac{1440 - 1827.36}{-25.456} = 15.22 \text{ kg/min} \qquad \textbf{Answer}$$

(i) Heat added by coil, $\dot{Q} = m_2(h_2 - h_1) = (m_1 - m_2')(h_2 - h_1)$

$$= (60 - 15.22)\ (44.60 - 18.75)$$

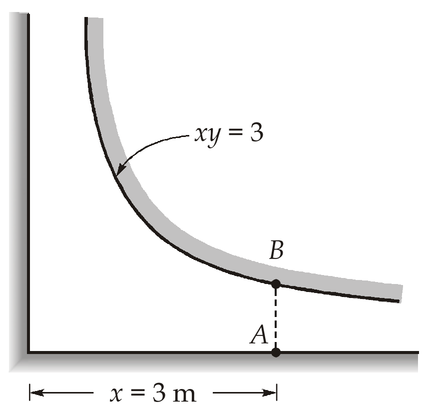
$$= 1157.56 \text{ kJ/min}$$

$$\dot{Q} = 19.29 \text{ kW} \qquad \textbf{Answer}$$

7. (a) Solution:

The stream function is given as

$$\psi = 4xy$$



We have, $\frac{\partial \psi}{\partial x} = v = \frac{4y}{100} \text{cm/s} \qquad [y \text{ in m}]$

and $\frac{\partial \psi}{\partial y} = -u = \frac{4x}{100} \text{cm/s} \qquad [x \text{ in m}]$

(i) For point B, $x = 3$ m and $y = 1$ m, and hence at this point

$$u = \frac{4 \times 3}{100} = -0.12 \text{ cm/s}$$

and $v = \frac{4 \times 1}{100} = 0.04 \text{ cm/s}$

$\therefore$ Velocity at point B is

$$V = \sqrt{u^2 + v^2} = \sqrt{(-0.12)^2 + (0.04)^2} = 0.126 \text{ cm/s}$$

(ii) For two dimensional flow the components of convective acceleration are

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

and $$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial x} = -4;\ \frac{\partial u}{\partial y} = 0;\ \frac{\partial v}{\partial x} = 0 \text{ and } \frac{\partial v}{\partial y} = 4$$

Thus at point B, we have

$$a_x = -0.12 \times (-4) \times 10^{-4} \text{ cm/s}^2$$
$$= 48 \times 10^{-6} \text{ cm/s}^2$$

and $$a_y = 0.04 \times 4 \times 10^{-4} \text{ cm/s}^2$$
$$= 16 \times 10^{-6} \text{ cm/s}^2$$

∴ Convective acceleration at B is

$$a_x = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(48\times10^{-6}\right)^2 + \left(16\times10^{-6}\right)^2}$$
$$= 50.6 \times 10^{-6} \text{ cm/s}^2$$

(iii) The flow rate per unit width across AB is given by

$$Q = u \times AB \times 1$$
$$u = -0.12 \text{ cm/s} = -0.12 \times 10^{-2} \text{ m/s; and } AB = 1 \text{ m}$$
∴ $$Q = -0.12 \times 10^{-2} \times 1 \times 1 \text{ m}^3\text{/s}$$
$$= -1200 \text{ cm}^3\text{/s}$$

Note : The negative sign simply indicates the flow in the negative x-direction. However, the flow rate will be the same if it is in the positive x-direction.

Alternatively the flow rate per unit width across AB is also given by

$$Q = (\psi_2 - \psi_1) \times 1$$

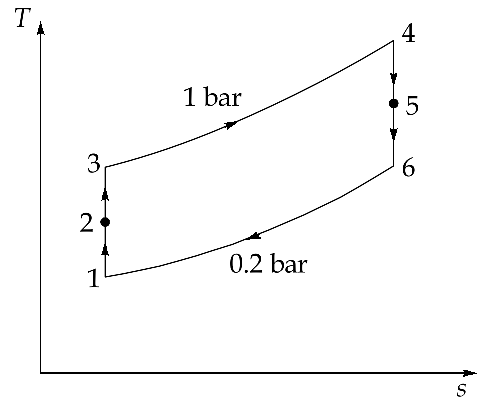
where ψ_2 is the value of the stream function for the curved boundary through point B; and ψ_1 is the value of the stream function for the straight horizontal boundary through point A.

$$\psi_2 = 12 \text{ cm}^2\text{/s; and } \psi_1 = 0$$
∴ $$Q = 12 \times 1 \times 100 \text{ cm}^3\text{/s}$$
$$= 1200 \text{ cm}^3\text{/s}$$

7. (b) Solution:

Given : $V_1 = 268$ m/s; $\eta_{comb} = \eta_c = \eta_T = 100\%$; $T_1 = 220$ K; $C_{P_a} = 1.005$ kJ/kgK; $\gamma = 1.4$;

$T_4 = 1350$ K; $C_{P_g} = 1.102$ kJ/kgK; $n = 1.33$



Process 1 - 2 (diffuser)

$$T_2 = T_1 + \frac{V_1^2}{2C_P} \qquad [h_{01} = h_{02} \text{ and } V_2 = 0]$$

$$T_2 = 220 + \frac{(268)^2}{2(1.005)\times 10^3}$$

$$T_2 = 255.733 \text{ K}$$

and

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$$

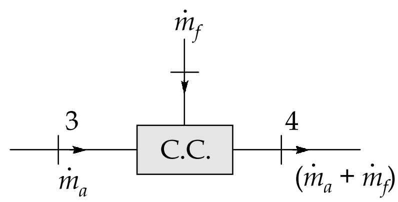
$$P_2 = 0.2\left(\frac{255.733}{220}\right)^{\frac{1.4}{0.4}} = 0.3387 \text{ bar}$$

Process 2 - 3 (compression):

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_3 = 255.733\left(\frac{1}{0.3387}\right)^{\frac{0.4}{1.4}} = 348.44 \text{ K}$$

Process 3 - 4 (combustion) :



$$\dot{m}_a C_P T_3 + \dot{m}_f CV \times \eta_{comb} = \left(\dot{m}_a + \dot{m}_f\right) C_{P_g} T_4$$

$$\dot{m}_f (43000 - 1.102 \times 1350) = \dot{m}_a (1.102 \times 1350 - 1.005 \times 348.44)$$

$$\dot{m}_f = \dot{m}_a \left(\frac{1137.518}{41512.3}\right)$$

$$\text{AFR} = \frac{\dot{m}_a}{\dot{m}_f} = \frac{1}{0.0274} = 36.4964$$

We know, $W_C = W_T$

$$\dot{m}_a C_P \left(T_3 - T_2\right) = \left(\dot{m}_a + \dot{m}_f\right) C_{P_g} \left(T_4 - T_5\right)$$

$$1.005(348.44 - 255.733) = (1 + 0.0274) \times 1.102 \times (1350 - T_5)$$

$$T_5 = 1267.708 \text{ K}$$

and
$$\frac{P_5}{P_4} = \left(\frac{T_5}{T_4}\right)^{\frac{n}{n-1}}$$

$$P_5 = 1 \times \left(\frac{1267.708}{1350}\right)^{\frac{1.33}{0.33}} = 0.7761 \text{ bar}$$

Now,
$$\frac{T_6}{T_5} = \left(\frac{P_6}{P_5}\right)^{\frac{n-1}{n}}$$

$$T_6 = 1267.708 \left(\frac{0.2}{0.7761}\right)^{\frac{0.33}{1.33}}$$

$$= 905.533 \text{ K}$$

Exit velocity,
$$T_5 = T_6 + \frac{V_e^2}{2C_P}$$

$$V_e = \sqrt{2 \times 1102(1267.708 - 905.533)}$$

$$V_e = 893.44 \text{ m/s}$$

and, thrust power $(TP) = \left\{\left(\dot{m}_a + \dot{m}_f\right) \times V_e - \dot{m}_a V_1\right\} V_1$

$$\frac{TP}{\dot{m}_a} = \{(1 + 0.0274) \times 893.44 - 268\} \times 268$$

$$\frac{TP}{\dot{m}_a} = 174178.63 \text{ J/kg}$$

$\therefore$ Specific thrust power $= 174.178$ kJ/kg

Now, Propulsive power $(PP) = \frac{1}{2}\dot{m}_a\left\{\left(1 + \frac{1}{AFR}\right)V_e^2 - V_1^2\right\}$

$$\frac{PP}{\dot{m}_a} = \frac{1}{2}\left\{(1 + 0.0274)(893.44)^2 - 268^2\right\}$$

$\therefore$ Specific propulsive power $= 374.1413$ kJ/kg

hence, $\eta_{prop} = \frac{TP/\dot{m}_a}{PP/\dot{m}_a}$

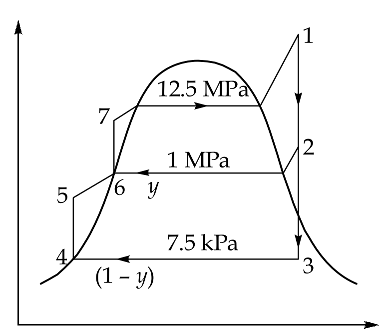
$$= \frac{174.178}{374.1413} = 0.46554 = 46.554\%$$

7. **(c) Solution:**

At State 1

$P_1 = 12.5$ MPa, $T_1 = 550°C$, $h_1 = 3476.5$ kJ/kg, $s_1 = 6.6317$ kJ/kgK

At State 2



$P_2 = 1$ MPa (10 bar), $s_2 = s_1 = 6.6317$ kJ/kgK $> s_g$

So it is in superheated state

By linear interpolation

$$\frac{2777.1 - 2828.3}{6.5850 - 6.6956} = \frac{h_2 - 2777.1}{6.6317 - 6.5850}$$

$$h_2 = 2798.718 \text{ kJ/kg}$$

At State 3

$P_3 = 7.5$ kPa, $h_f = h_4 = 168.75$ kJ/kg, $h_{fg} = 2405.3$ kJ/kg, $s_f = s_4 = 0.5763$ kJ/kg.K, s_{fg}=7.6738 kJ/kg.K

$$s_1 = s_3 = s_f + xs_{fg}$$
$$6.6317 = 0.5763 + x(7.6738)$$
$$x = 0.7891$$
$$h_3 = h_f + xh_{fg} = 168.75 + 0.7891 \times 2405.3$$
$$h_3 = 2066.773 \text{ kJ/kg}$$

At State 4

$h_4 = 168.75$ kJ/kg, $s_4 = 0.5763$ kJ/kgK, $v_4 = 0.001008$ m^3/kg

At State 5

$s_4 = s_5 = 0.5763$ kJ/kg, $h_5 = h_4 + v_4\, dp = 168.75 + 0.001008\,(1000 - 7.5)$

$$h_5 = 169.75 \text{ kJ/kg}$$

At State 6

$P_6 = 1$ MPa (10 bar), $s_6 = s_f = 2.1381$ kJ/kgK,

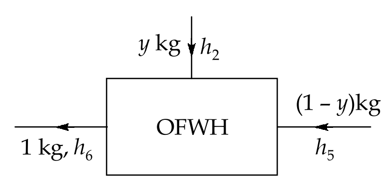
$$h_6 = h_f = 762.51 \text{ kJ/kg}, \; v_6 = 0.001127 \text{ m}^3\text{/kg}$$

At State 7

$P_7 = 12.5$ MPa, $s_7 = s_6 = 2.1381$ kJ/kgK,

$$h_7 = h_6 + v_6 dp = 762.51 + 0.001127\,(12500 - 1000)$$
$$h_7 = 775.4705 \text{ kJ/kg}$$

Energy balance in open feed water heater,



$$\dot{E}_{in} = \dot{E}_{out}$$
$$yh_2 + (1 - y)h_5 = h_6$$
$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.51 - 169.75}{2798.718 - 169.75} = 0.2254$$

(i)
$$\frac{W_T}{\dot{m}_1} = (h_1 - h_2) + (1 - y)\,(h_2 - h_3)$$
$$= (3476.5 - 2798.718) + (1 - 0.2254)\,(2798.718 - 2066.773)$$
$$\frac{W_T}{\dot{m}_1} = 1244.746 \text{ kJ/kg}$$
$$\frac{W_P}{\dot{m}_1} = (1 - y)\,(h_5 - h_4) + (h_7 - h_6)$$
$$= (1 - 0.2254)\,(169.75 - 168.75) + (775.4705 - 762.51)$$
$$\frac{W_P}{\dot{m}_1} = 13.7351 \text{ kJ/kg}$$
$$\frac{q_{in}}{\dot{m}_1} = h_1 - h_7 = 3476.5 - 775.4705 = 2701.0295 \text{ kJ/kg}$$

$$\eta_T = \frac{\frac{W_T}{\dot{m}_1} - \frac{W_P}{\dot{m}_1}}{\frac{q_{in}}{\dot{m}_1}} = \frac{1244.746 - 13.7351}{2701.0295} = 0.4557$$

$$\eta_T = 45.57\%$$

(ii)
$$\frac{W_{cycle}}{\dot{m}_1} = \frac{W_T}{\dot{m}_1} - \frac{W_P}{\dot{m}_1} = 1244.746 - 13.7351$$
$$= 1231.011 \text{ kJ/kg}$$

$$\dot{m}_1 = \frac{P}{W_{cycle}} = \frac{330 \times 10^3 \text{ kW}}{1231.011 \text{ kJ/kg}} = 268.0723 \text{ kg/s}$$

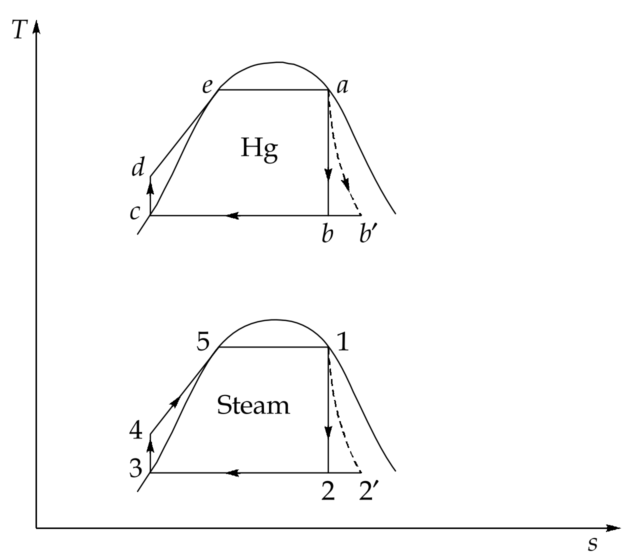
(iii) Entropy balance, $\dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{system}$ $\quad\left[\frac{dS}{dt} = 0 \quad \because \text{ it is steady flow}\right]$

$$\dot{S}_{gen} = \dot{S}_{out} - \dot{S}_{in}$$
$$= \dot{m}_1\left[s_6 - (1-y)s_5 - ys_2\right]$$
$$= 268.0723\ [2.1381 - (1 - 0.2254) \times 0.5763 - 0.2254 \times 6.6317]$$
$$= 52.7868 \text{ kW/K}$$

8. (a) Solution:

$P_a = 10$ bar, $P_b = P_{b'} = P_c = 0.1$ bar, $P_1 = 40$ bar, $P_2 = P_{2'} = P_3 = 0.06$ bar

$\mu_m = 0.75$, $\eta_s = 0.80$



Cycle $a - b' - c - d - e$ = Mercury cycle

Cycle $1 - 2' - 3 - 4 - 5$ = Steam cycle

For mercury cycle,

$$s_a = s_b$$

$$0.5158 = 0.089 + x(0.6604 - 0.089)$$

$$x = 0.747$$

$$(h_b)_{0.1\text{ bar}} = h_f + x(h_g + h_f)$$

$$h_b = 34.485 + 0.747\,(332.975 - 34.485)$$

$$h_b = 257.457 \text{ kJ/kg}$$

Isentropic work by mercury turbine = $h_a - h_b$ = 362.406 – 257.457

$$= 104.95 \text{ kJ/kg}$$

Actual work done by mercury turbine = $h_a - h_{b'}$

$$W_T = 104.95 \times 0.75$$

$$= 78.7125 \text{ kJ/kg}$$

$$h_{b'} = h_a - 78.7125$$

$$= 362.406 - 78.7125$$

$$= 283.6935 \text{ kJ/kg}$$

Neglecting the pump work,

Heat supplied in the mercury power cycle,

$$Q_s = h_a - h_d$$

$$= h_a - h_d \quad [\because \; h_d = h_c]$$

$$= 362.406 - 34.485$$

$$= 327.921 \text{ kJ/kg}$$

Heat rejected in the mercury power cycle,

$$Q_R = Q_s - W_T$$

$$= 327.921 - 78.7125$$

$$= 249.2085 \text{ kJ/kg}$$

For steam power cycle,

$$s_2 = s_1$$

$$0.521 + x(8.331 - 0.521) = 6.070$$

$$x = 0.71$$

$$(h_2)_{0.06\text{ bar}} = h_f + x(h_g - h_f)$$

$= 151.5 + 0.71(2567 - 151.5)$

$= 1866.505$ kJ/kg

Isentropic turbine work $= h_1 - h_2$

$= 2801 - 1866.505 = 934.495$ kJ/kg

Actual turbine work of steam power cycle,

$$W_T = 934.495 \times 0.80$$

$= 747.596$ kJ/kg

Neglecting pump work,

Heat supplied in the steam power cycle,

$$Q_s = h_1 - h_4$$

$= h_1 - h_3$ $[\because\ h_3 = h_4]$

$= 2801 - 151.5 = 2649.5$ kJ/kg

(1) By energy conservation,

Heat rejection by mercury power cycle in condenser = Heat received by feed water in steam power cycle

$$\therefore \quad \dot{m}_m\left(h_{b'} - h_c\right) = 1 \times \left(h_1 - h_3\right)$$

$$\dot{m}_m = \frac{2801 - 151.5}{249.2085}$$

$= 10.6332$ kg of mercury per kg of steam

(2) Thermal efficiency of cycle $= \dfrac{\text{Net work done}}{\text{Total heat supplied in mercury power cycle}}$

$$= \frac{78.7125 \times 10.6332 + 747.596}{10.6316 \times 327.921}$$

$= 0.4544 = 45.44\%$

(3) Work done by the mercury turbine

$= 10.6332 \times 78.7125$

$= 836.97$ kJ/kg of steam

Work done by steam turbine $= 747.596$ kJ/kg of steam

8. (b) Solution:

(i) The discharge Q delivered by the pump is given by

$$Q = k\pi B_1 D_1 V_{f_1}$$

Given:

$$B_1 = 15 \text{ mm} = 0.015 \text{ m}$$

$$D_1 = 400 \text{ mm} = 0.400 \text{ m}$$

$$V_{f_1} = 3 \text{ m/s}$$

Assuming $k = 1$, we get

$$Q = 1 \times \pi \times 0.015 \times 0.400 \times 3$$

or $$Q = 0.05654 \text{ m}^3/\text{s} = 3393 \text{ litres/minute}$$

(ii) Manometric efficiency is given by

$$\eta_{\text{mano}} = \frac{gH_m}{V_{w_1} u_1}$$

or $$\frac{V_{w_1} u_1}{g} = \frac{H_m}{\eta_{mano}}$$

$\therefore$ $$\frac{V_{w_1} u_1}{g} = \frac{35}{0.8} = 43.75 \text{ m}$$

i.e. total head developed by the pump = 43.75 m

The pressure rise through the impeller $\left(\frac{p_1 - p}{w}\right)$ is 65% of the total head developed by the pump.

Thus, $$\left(\frac{p_1 - p}{w}\right) = (0.65 \times 43.75) = 28.437 \text{ m}$$

Applying Bernoulli's equation between the inlet and outlet tips of the impeller and neglecting the head loss in the impeller, we have

$$\frac{p}{w} + \frac{V^2}{2g} = \frac{p_1}{w} + \frac{V_1^2}{2g} - \frac{V_{w_1} u_1}{g}$$

or $$\left(\frac{p_1 - p}{w}\right) = \frac{V^2}{2g} + \frac{V_{w_1} u_1}{g} - \frac{V_1^2}{2g}$$

As $$V = V_f = 3 \text{ m/s}$$

$$28.437 = \frac{(3)^2}{2\times 9.81} + 43.75 - \frac{V_1^2}{2g}$$

or $$V_1 = 17.59 \text{ m/s}$$

But $$V_1 = \sqrt{V_{w_1}^2 + V_{f_1}^2}$$

∴ $$V_{w_1} = \sqrt{V_1^2 - V_f^2}$$

$$V_{f_1} = 3 \text{ m/s}$$

Thus by substitution, we get

$$V_{w_1} = \sqrt{(17.59)^2 - (3)^2} = 17.33 \text{ m/s}$$

By substitution in equation, we get

$$\frac{17.33\times u_1}{9.81} = 43.75$$

or $$u_1 = \frac{43.75\times 9.81}{17.33} = 24.76 \text{ m/s}$$

But $$u_1 = \frac{\pi D_1 N}{60}$$

or $$24.76 = \frac{\pi\times 0.400\times N}{60}$$

or $$N = \frac{24.76\times 60}{\pi\times 0.400} = 1182.20 \text{ rpm}$$

i.e., speed of the pump = 1182.20 rpm

(iii) From outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_1}}{u_1 - V_{w_1}}$$

or $$\tan\phi = \frac{3}{24.76 - 17.33} = 0.4037$$

∴ $$\phi = 21.98°$$

i.e., blade angle at output = 21.98°

(iv) From inlet velocity triangle

$$\tan\theta = \frac{V_f}{u}$$

or $$u = V_f \cot\theta$$

$$\theta = 60°$$

$\therefore \quad u = 3\cot 60° = 1.732$ m/s

Also, $\quad u = \dfrac{\pi DN}{60}$

or $\quad 1.732 = \dfrac{\pi \times D \times 1182.20}{60}$

or $\quad D = \dfrac{1.732 \times 60}{\pi \times 1182.20} = 0.02798$ m = 27.98 mm

i.e. the diameter of the impeller at inlet = 27.98 mm

8. (c) Solution:

Given : $K = 1$, four stroke; $d = 18$ cm; $l = 36$ cm; $N = 285$ rpm; $T_b = 393$ Nm; imep = 7.2 bar; $\dot{m}_f = 3.5$ kg/hr; $\dot{m}_w = 4.5$ kg/min; $(\Delta T)_w = 36°C$; $T_g = 415°C$; $P_a = 1.013$ bar; $T_a = 21°C$; CV = 45200 kJ/kg; $H_2 = 15\%$ by mass

We know, $\quad ip = (\text{imep}) \times LAN$

$$= (7.2 \times 10^2)\left\{\frac{\pi}{4}(0.18)^2\right\}(0.36) \times \frac{285}{60 \times 2}$$

$ip = 15.7$ kW

Now, $\quad \eta_{ith} = \dfrac{ip \times 3600}{\dot{m}_f \times CV}$

$$= \frac{15.7 \times 3600}{3.5 \times 45200} = 0.357 = 35.7\%$$

and $\quad$ air inhaled, $\dot{m}_a = \dot{m}_f \times \dfrac{A}{F}$

$$= \frac{3.5 \times 25}{60} = 1.46 \text{ kg/min}$$

and $\quad PV = mRT \qquad (\because$ air as an ideal gas)

So,

$\therefore \quad$ Volume of air inhaled $= \dfrac{\dot{m}_a R T_a}{P_a} = \dfrac{1.46 \times 0.287 \times 294}{1.013 \times 10^2}$

$= 1.216$ m^3/min

Swept volume $= \dfrac{\pi}{4} d^2 l \times$ Number of cycles/min

$$= \frac{\pi}{4}(0.18)^2(0.36) \times \frac{285}{2}$$

$= 1.305\ m^3/min$

So, volumetric efficiency $= \frac{\text{Air inhaled}}{\text{Swept volume}} = \frac{1.216}{1.305} = 0.932$

$= 93.2\%$

Heat balance sheet on minute basis

Heat input $= \dot{m}_f \times CV$

$$= \frac{3.5}{60} \times 45200 = 2636.7\ \text{kJ/min} \qquad (100\%)$$

(i) Brake power, $bp = 2\pi N T_b$

$$= 2\pi \times \frac{285}{60} \times (393 \times 10^{-3})$$

$= 11.73$ kW

So, heat equivalent to bp = 11.73 × 60

= 703.8 kJ/min (26.7%)

(ii) Heat in cooling water $= \dot{m}_w \times c_{pw} \times (\Delta T)_w$

= 4.5 × 4.186 × 36

= 678.13 kJ/min (25.7%)

Now, mass of the exhaust gases $= \dot{m}_f + \dot{m}_a$

$$= \frac{3.5}{60} + 1.46 = 1.518\ \text{kg/min}$$

Mass of steam in exhaust gases $= \frac{9 \times 0.15 \times 3.5}{60} = 0.07875$ kg/min

So, mass of dry exhaust gases = 1.518 – 0.07875

= 1.439 kg/min

∴ Heat in dry exhaust gases $= (m_g)_{dry} \times C_p \times \Delta T$

= 1.439 × 1.005 × (415 – 21)

= 569.9 kJ/min (21.6%)

Heat in steam in exhaust gases (assuming at atmospheric pressure)

$$= (9 \times 0.15) \times \frac{3.5}{60}\left[4.186 \times (100-21) + 2256.9 + 2.05(415-100)\right]$$

$$= 254.62 \text{ kJ/min} \qquad (9.6\%)$$

Heat in radiation (*efc*) = Heat input – Heat losses

= 2636.7 – 703.8 – 678.13 – 569.9 – 254.62

= 430.25 kJ/min (16.3%)

Heat balance sheet

	Heat losses (kJ/min)			
1.	Brake power	=	703.8	(26.7%)
2.	Cooling water	=	678.3	(25.7%)
3.	Dry exhaust	=	569.9	(21.6%)
4.	Steam in exhaust	=	254.62	(9.6%)
5.	Radiation & other	=	430.25	(16.3%)
	Heat Input	=	2636.7	(100%)

