MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Civil Engineering Test No : 11

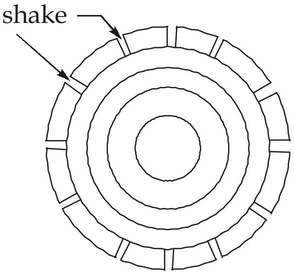
Section - A

Q.1 (a) Solution:

Shakes are the cracks and splits developed in the trees due to rupture of tissues as a result of extreme climatic disturbances such as extreme heat or severe frost during the growth of the trees. Shakes can also develop in filled logs due to rapid drying and shrinkage during seasoning which results in the rupture of wood fibres. Thus, a shake is a crack along which the fibres of wood have separated partly or completely.

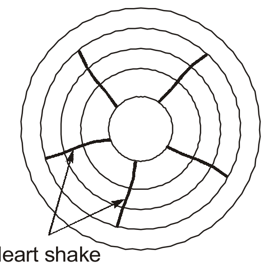
Shakes reduce the durability of timber and make it unsuitable for structural and decorative purposes as they admit moisture and air, causing decay and fungal attacks. They also reduce the allowable shear strength without much effect on compressive and tensile values. But when the shakes run diagonally across the tension side of timber, they reduce the tensile strength.

1. **Star Shakes:** These are radial splits or cracks wide at circumference and diminishing towards the centre of the tree. They are usually confined to the sapwood. This defect may arise due to severe frost and fierce heat of sun. When a log containing star shakes is saven, it separates into a number of non-usable pieces.

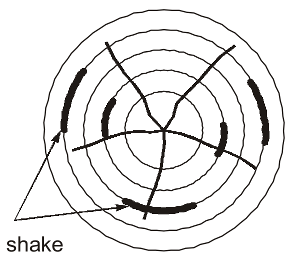


Star shake

2. **Hear shakes:** These splits occur in the central part of the stem, extending from pith to sap wood in the direction of medullary rays dividing the cross-section into several parts. They occur due to shrinkage of heartwood, when tree is over matured.



3. **Cup shakes:** They appear as curved split which partly separates annual rings from one another. It is caused due to excessive frost action on the sap present in the tree, especially when the true is young.



Q.1 (b) Solution:

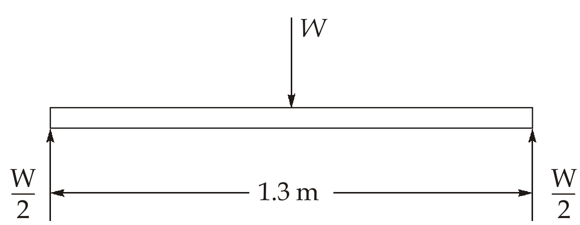
Span, $L = 1.3$ m

Width, $b = 150$ mm

Depth, $d = 250$ mm

Bending stress, $\sigma = 7 \text{N/mm}^2$

Shearing stress, $\tau = 1 \text{ N/mm}^2$



Maximum B.M., $$M = \frac{W \times L}{4} = \frac{W}{4} \times 1.3 \text{ Nm}$$

$$= \frac{W}{4} \times 1.3 \times 1000 \text{ Nmm} = 325\,W \text{ Nmm}$$

Maximum S.F. $$= \frac{W}{2} \text{ N.}$$

Value of W from bending stress consideration

Using bending equation

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{...(i)}$$

Where $M = 325\,W\,\text{Nmm}$

$$I = \frac{bd^3}{12} = \frac{150 \times 250^3}{12} = 195312500 \text{ mm}^4$$

$$\sigma = 7 \text{ N/mm}^2$$

and $$y = \frac{d}{2} = \frac{250}{2} = 125 \text{ mm}$$

Substituting these values in the above equation (i), we get

$$\frac{325W}{195312500} = \frac{7}{125}$$

$$W = \frac{7 \times 195312500}{325 \times 125} = 33653.8 \text{ N} = 33.65 \text{ kN}$$

Value of *W* from shear stress consideration

Average shear stress,

$$\tau_{avg} = \frac{\text{Shear force}}{\text{Area}} = \frac{\left(\frac{W}{2}\right)}{b \times d} = \frac{W}{2 \times 150 \times 250}$$

Maximum shear stress is given by equation

$\therefore$ $$\tau_{max} = \frac{3}{2} \times \tau_{avg}$$

But $$\tau_{max} = 1 \text{ N/mm}^2$$

$\therefore$ $$1 = \frac{3}{2} \times \frac{W}{2 \times 150 \times 250}$$

or $$W = \frac{2 \times 2 \times 150 \times 250}{3} = 50000 \text{ N} = 50 \text{ kN}$$

Hence, $$P_{safe} = \min \begin{cases} 33.64 \text{ kN} \\ 50 \text{ kN} \end{cases}$$

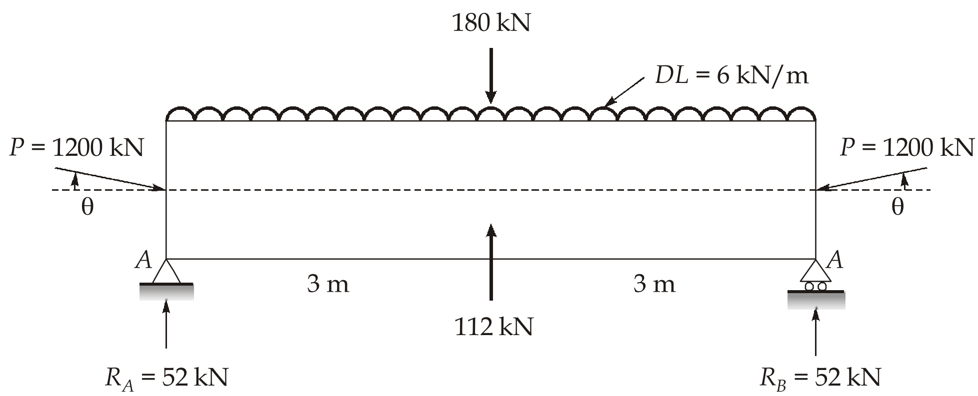
$$P_{safe} = 33.65 \text{ kN}$$ **Ans.**

Q.1 (c) Solution:

The inclined tendon will provide an upward force at the midspan

$$\text{Upward force} = 2\,P \sin\theta \simeq 2\,P \tan\theta \text{ (Since } \theta \text{ is small)}$$

$$= 2 \times 1200 \times \frac{140}{3000} = 112 \text{ kN}$$



$$\text{Net vertical load at mid span} = 180 - 112 = 68 \text{ kN}$$

$$\text{Dead load of the beam} = 0.4 \times 0.6 \times 25 = 6 \text{ kN/m}$$

Vertical reaction at A and B

$$R_A = R_B = \frac{6 \times 6}{2} + \frac{68}{2} = 52 \text{ kN}$$

Net bending moment at mid span (i.e. at C) is,

$$BM_C = 52 \times 3 - 6 \times 3 \times \frac{3}{2}$$

$$\Rightarrow \quad BM_C = 129 \text{ kNm}$$

Resultant extreme stress in the top most fibre is,

$$f_{top} = \frac{P}{A} + \frac{M_C}{I} y_{top}$$

$$\Rightarrow \quad f_{top} = \frac{1200 \times 10^3}{400 \times 600} + \frac{\left(129 \times 10^6\right)}{\frac{400 \times 600^3}{12}} \times 300$$

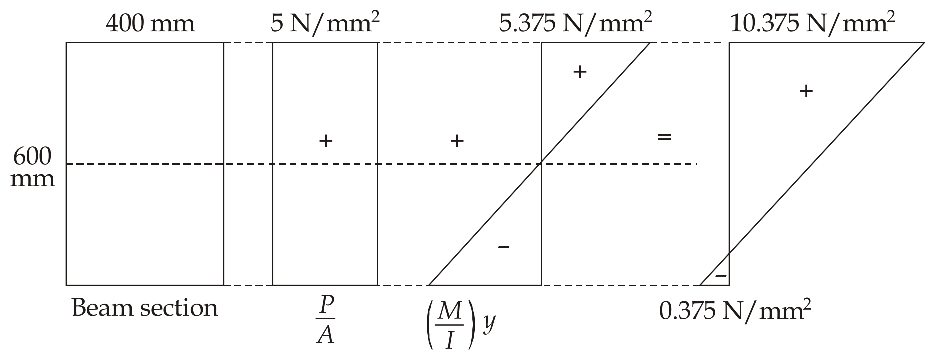
$$\Rightarrow \quad f_{top} = 5 + 5.375 = 10.375 \text{ N/mm}^2$$

Resultant extreme stress in the bottom most fibre is,

$$f_{bot} = \frac{P}{Q} - \frac{M_c}{I} y$$

$$\Rightarrow \quad f_{bot} = \frac{1200 \times 10^3}{400 \times 600} - \frac{\left(129 \times 10^6\right)}{\frac{400 \times 600^3}{12}} \times 300$$

$$\Rightarrow \quad f = 5 - 5.375 = -0.375 \text{ N/mm}^2$$

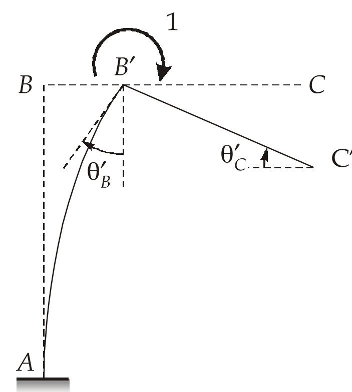


Q.1 (d) Solution:

Given:

Number of coordinates = 2

∴ Size of flexibility matrix will be (2 × 2)

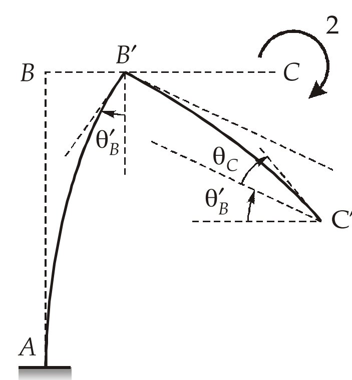


For I[st] column: Apply unit moment along coordinate 1 only

$$f_{11} = \theta'_B = \frac{M_B L_{AB}}{(EI)_{AB}} = \frac{1\times 4}{2EI} = \frac{2}{EI}$$

$$f_{21} = \theta'_C = \theta'_B = \frac{2}{EI}$$

For 2[nd] Column: Apply unit moment along coordinate 2 only



$$\theta_C = \frac{M_C L_{BC}}{(EI)_{BC}} = \frac{1\times 3}{EI}$$

$$f_{12} = \theta'_B = \frac{M_B L_{AB}}{(EI)_{AB}} = \frac{1\times 4}{2EI} = \frac{2}{EI}$$

$$f_{22} = \theta'_B + \theta_C = \frac{M_B L_{AB}}{(EI)_{AB}} + \frac{M_C L_{BC}}{(EI)_{BC}}$$

$$\Rightarrow \quad f_{22} = \frac{1\times 4}{2EI} + \frac{1\times 3}{EI} = \frac{5}{EI}$$

Hence the flexibility matrix is:

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$\Rightarrow \quad [f] = \frac{1}{EI}\begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$$

Q.1 (e) Solution:

Degree of static indeterminacy, $D_S = 6 - 3 = 3$

Number of independent mechanisms = Possible location of plastic hinges – D_S

$= 5 - 3 = 2$

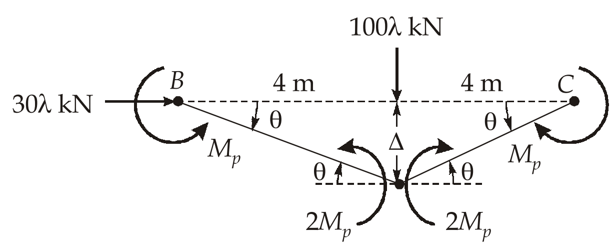
2 mechanisms → 1 Beam mechanism, 1 Sway mechanism

Let load factor is 'λ'

$$\therefore \quad \lambda = \frac{\text{Ultimate collapse load}}{\text{Working load}}$$

$$\Rightarrow \quad W_u = \gamma W_{\text{working load}}$$

1. Beam mechanism :



Beam mechanism

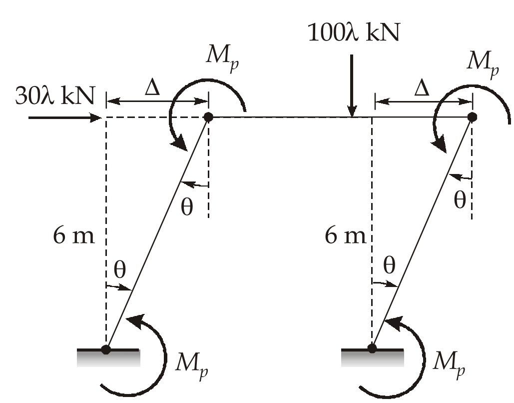
$$\text{External workdone} = \text{Internal work done}$$

$$\Rightarrow \quad 100\,\lambda \times \Delta = M_p\theta + 2M_p(\theta + \theta) + M_p\theta$$

$$\Rightarrow \quad 100\,\lambda \times 4\theta = 6\,M_p\theta \qquad (\because \quad \Delta = 4\theta)$$

$$\Rightarrow \quad \lambda = 0.015\,M_p \qquad \ldots(i)$$

2. Sway mechanism :



Sway mechanism

$$\text{External workdone} = \text{Internal work done}$$

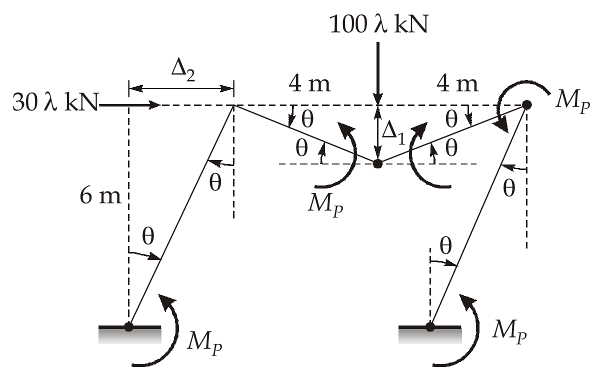
$$\Rightarrow \quad 30\,\lambda \times \Delta = M_p\theta + M_p\theta + M_p\theta + M_p\theta$$

$$\Rightarrow \quad 30\,\lambda \times 6\theta = 4\,M_p\theta$$

$$\Rightarrow \quad \lambda = 0.022\,M_p \qquad \text{...(ii)}$$

Note: Combined mechanism is a dependent mechanism

3. **Combined mechanism :**



(Combined mechanism)

$$\text{External workdone} = \text{Internal workdone}$$

$$\Rightarrow \quad 30\,\lambda \times \Delta_2 + 100\,\lambda \times \Delta_1 = M_p\theta + 2M_p(\theta + \theta) + 2M_p\theta + M_p\theta$$

$$\Rightarrow \quad 30\,\lambda \times 6\theta + 100\,\lambda \times 4\theta = 8\,M_p\theta$$

$$\Rightarrow \quad 580\,\lambda\theta = 8\,M_p\theta$$

$$\Rightarrow \quad \lambda = 0.0138\,M_p \qquad \text{...(iii)}$$

$$\text{Collapse load factor} = \min\{(i), (ii), (iii)\}$$

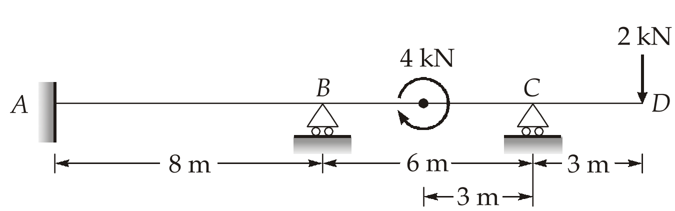
i.e. $\lambda = 0.0138\,M_p$ **Ans.**

Q.2 (a) Solution:

Rotation θ_A at fixed end A will be zero.

Let θ_B and θ_C be the rotations at 'B' and 'C' respectively.

The loading in the span BC will produce a clockwise moment of 4 kN-m at the centre of span. The fixed end moments due to this moment are computed below.



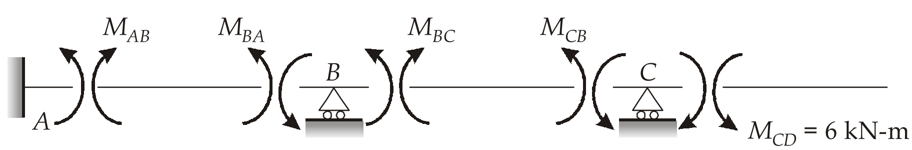
Fixed end moment:

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = M_{FCB} = +\frac{4}{4} = 1 \text{ kN-m}$$

$$M_{FCD} = -2 \times 3 = -6 \text{ kN-m}$$

FBD of all spans and joints:



The slope deflection equations in terms of unknowns are shown below.

For span AB

$$M_{AB} = 0 + \frac{2EI}{8}(0 + \theta_B) = \frac{EI\theta_B}{4}$$

$$M_{BA} = 0 + \frac{2EI}{8}(2\theta_B + 0) = \frac{EI\theta_B}{2}$$

For span BC

$$M_{BC} = +1 + \frac{2EI}{6}(2\theta_B + \theta_C) = 1 + \frac{EI}{3}(2\theta_B + \theta_C)$$

$$M_{CB} = +1 + \frac{2EI}{6}(2\theta_C + \theta_B) = 1 + \frac{EI}{3}(2\theta_C + \theta_B)$$

For equilibrium, the sum of the moments at the joint B is zero.

$$\therefore \quad M_{BA} + M_{BC} = 0$$

$$\Rightarrow \quad \frac{EI\theta_B}{2} + \frac{EI}{3}(2\theta_B + \theta_C) + 1 = 0$$

$$\Rightarrow \quad 3\theta_B + 4\theta_B + 2\theta_C + \frac{6}{EI} = 0$$

$$\Rightarrow \quad 7\theta_B + 2\theta_C = -\frac{6}{EI} \qquad \text{...(i)}$$

For equilibrium the sum of the moments at the joint C is zero.

$$\therefore \quad M_{CB} + M_{CD} = 0$$

$$\Rightarrow \quad 1 + \frac{EI}{3}(\theta_B + 2\theta_C) - 6 = 0$$

$$\Rightarrow \qquad \theta_B + 2\theta_C = \frac{15}{EI} \qquad \text{...(ii)}$$

Subtracting (ii) from (i),

$$6\theta_B = -\frac{21}{EI}$$

$$\Rightarrow \qquad \theta_B = -\frac{3.5}{EI}$$

Substituting the value of θ_B in equation (ii)

$$2\theta_C = \frac{15}{EI} + \frac{3.5}{EI}$$

$$\therefore \qquad \theta_C = \frac{9.25}{EI}$$

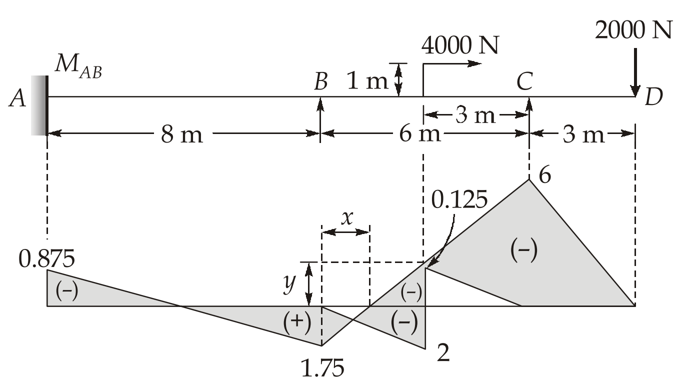
Final moments :

$$M_{AB} = \frac{EI\theta_B}{4} = \frac{EI}{4} \times \left(-\frac{3.5}{EI}\right) = -\,0.875 \text{ kN-m}$$

$$M_{BA} = \frac{EI\theta_B}{2} = \frac{EI}{2} \times \left(-\frac{3.5}{EI}\right) = -\,1.75 \text{ kN-m}$$

$$M_{BC} = 1 + \frac{EI}{3}\left[2\left(-\frac{3.5}{EI}\right) + \frac{9.25}{EI}\right] = +\,1.75 \text{ kN-m}$$

$$M_{CB} = 1 + \frac{EI}{3}\left[2\left(\frac{9.25}{EI}\right) + \left(-\frac{3.5}{EI}\right)\right] = +\,6 \text{ kN-m}$$



Calculation for net *BM* at mid span of *BC*.

$$\frac{1.75}{x} = \frac{6}{6-x}$$

$$\Rightarrow \qquad 6 \times 1.75 - 1.75x = 6x$$

$$\Rightarrow \qquad x = 1.3548 \text{ m}$$

Now, $$\frac{y}{3-x} = \frac{1.75}{x}$$

$$\Rightarrow \qquad y = 2.125 \text{ kN-m} \qquad \text{(Hogging)}$$

Net value of BM at mid span of BC = 2.125 – 2

$$= 0.125 \text{ kN-m} \qquad \text{(Hogging)}$$

Q.2 (b) Solution:

(i)

The proportions of various ingredients are as follows:

Silica	50 – 60%
Alumina	20 – 30%
Lime	10%
Magnesia	$\ngtr$ 1%
Iron oxides	5% to 6% $\ngtr$ 7%
Alkalies	$\ngtr$ 10%

1. **Silica:** It enables the brick to retain its shape and imparts durability, prevents shrinkage and warping. It is present either as free sand or in combination as silicate of alumina. Excess of silica makes the brick brittle and weak on burning. A large percentage of it is undesirable but is added to decrease shrinkage and increase refractoriness of low alumina clays.
2. **Alumina:** It absorbs water and renders the clay plastic. If it is present in excess, it produces cracks in brick upon drying. Clays having exceedingly high alumina content are likely to be very refractory.
3. **Lime:** It reduces shrinkage on drying and acts as a flux during burning process enabling silica to fuse and bind the brick particles together. In carbonated form, it lowers the fusion point. Excess of lime causes the brick to melt and the brick loses its shape.
4. **Magnesia:** It affects the colour and makes the brick yellow. It reduces warping while burning. If it is present in excess, it causes the brick to decay.
5. **Iron oxides:** It acts as a flux helping silica to fuse during burning process and bind the brick particles together, thus providing the brick its strength and hardness. It gives red colour on burning when excess of oxygen is available and dark brown or even black colour when oxygen available is insufficient (excess of ferric oxide makes the brick dark blue). They also increase the impermeability and durability of brick.

(ii)

1. **Rapid Hardening Portland Cement:** It has high lime content and can be obtained by increasing the C_3S content but is normally obtained from OPC clinker by finer grinding (450 m^2/kg). The basis of application of rapid hardening cement (RHC) is hardening properties and heat emission rather than setting rate. This permits addition of a little more gypsum during manufacture to control the rate of setting. RHC attains same strength in one day which an ordinary cement may attain in 3 days. However, it is subjected to large shrinkage and water requirement for workability is more. The cost of rapid hardening cement is about 10 percent more than the ordinary cement. Concrete made with RHC can be safely exposed to frost, since it matures more quickly.

Properties:	Initial setting time	30 minutes (minimum)
	Final setting time	10 hours (maximum)
	Compressive strength	
	1 day	16.0 N/mm^2
	3 day	27.5 N/mm^2

Uses: It is suitable for repair of roads and bridges and when load is required to be applied in a short period of time.

2. **Sulphate Resisting Portland Cement:** In this cement the amount of tricalcium aluminate is restricted to on acceptably low value(< 5). It should not be mistaken for super-sulphated cement. It is manufactured by grinding and intimately mixing together calcareous and argillaceous and/or other silica, alumina and iron oxide bearing materials. The Materials are burnt to clinkering temperature. The resultant clinker is ground to produce the cement. No material is added after burning except gypsum and not more than one per cent of air-entraining agents are added.

Properties: The specific surface of the cement should not be less than 225m^2/kg. The expansion of cement is limited to 10 mm and 0.8 per cent, when tested by Le-chatelier method and autoclave test, respectively. The setting times are same as that for ordinary Portland cement. The compressive strength of the cubes should be as follows.

72 ± 1 hour	$\nless$ 10 N/mm^2
168 ± 2 hours	$\nless$ 16N/mm2
672 ± 4 hours	$\nless$ 33 N/mm^2

It should have a fineness of 400 m^2/kg. The expansion of cement is limited to 5 mm. The initial setting time of the cement should not be less than 30 mm and the final setting time should not be more than 600 mm.

This cement can be used as an alternative to ordinary Portland cement or Portland pozzolana cement or Portland slag cement under normal conditions. Its use however is restricted where the prevailing temperature is below 40°C. Use of sulphate resisting cement is particularly beneficial in conditions where the concrete is exposed to the risk of deterioration due to sulphate attack; concrete in contact with soils or ground waters containing excessive sulphate as well as concrete in sea water or exposed directly to sea coast.

Q.2 (c) Solution:

(i)

Tensile design strength of each channel (based on gross-section yielding) is,

$$T_{dg} = \left(\frac{A_g f_y}{\gamma_{m0}}\right) = \left(\frac{3867 \times 250}{1.10}\right) N = 878.864 \text{ kN}$$

Weld thickness:

As per IS 800:2007

Minimum weld thickness = 3 mm

Maximum weld thickness = 7.1 – 1.5 = 5.6 mm

Provide 5 mm weld size.

∴ Throat thickness, $t = 0.7s = 0.7 \times 5 = 3.5$ mm

$$\text{Strength of weld} = L_w t \times \frac{f_u}{\sqrt{3}} \times \frac{1}{\gamma_{mw}} \qquad \text{(Fillet weld)}$$

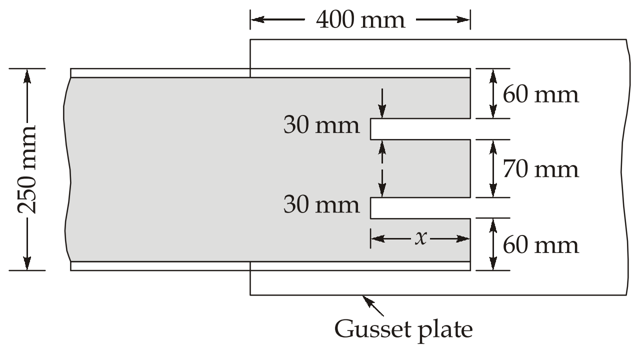
$$= L_w \times 3.5 \times \frac{410}{\sqrt{3} \times 1.25}$$

Equating strength of weld to tensile strength of the channel, we get

$$L_w \times 3.5 \times \frac{410}{\sqrt{3}} \times \frac{1}{1.25} = 878.864 \times 10^3$$

$$\Rightarrow \quad L_w = 1326 \text{ mm}$$

Since, allowable length is limited to (400 mm + 400 mm), it needs to be slot welded. The arrangement can be as shown in figure with two slots of length 'x' each.



∴ $400 + 400 + (250 - 2 \times 30) + 4x = 1326$

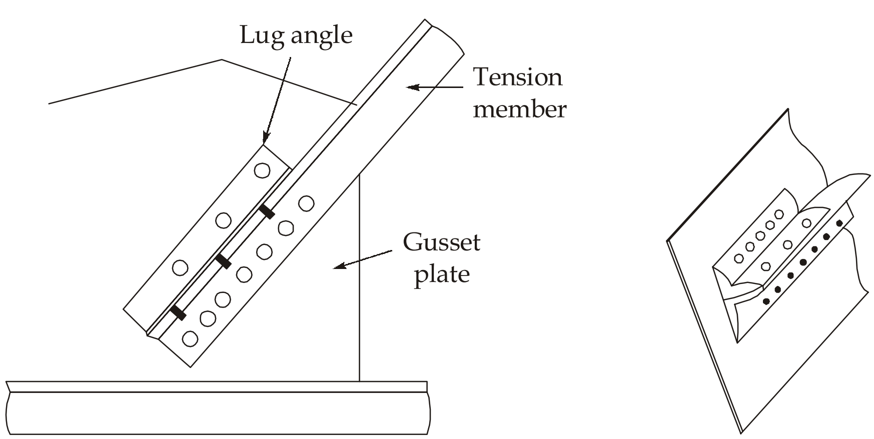
⇒ $x = 84 \text{ mm} \simeq 90 \text{ mm (say)}$

∴ Provide x = 90 mm slots as shown in above figure.

(ii)

Lug angle is designed as follows:

1. In the case of angle members, lug angles and their connection to the gussets or other supporting members should be capable of developing strength not less than 20% in excess of the force in the outstanding leg of the angle. The attachment of the lug angle to the angle member should be capable of developing 40% in excess of that force.
2. Where lug angles are used to connect an angle member, the whole area of the member should be taken as effective. The net area is calculated by deducting the area of the bolt holes from the gross-sectional area.



Connections of Bars and Rods

3. In the case of channel member and the like, the lug angle and their connections to the gusset or other supporting member should be capable of developing strength of not less than 10% of excess of the force not accounted for by the direct connection of the member. The attachment of lug angles to the member should be capable of developing 20% in excess of that force.
4. Lug angles connecting a channel shaped member should as far as possible be placed/disposed symmetrically with respect to the section of the member.

Q.3 (a) Solution:

(i)

When a bar is stretched only upto the elastic limit, the deformation will disappear completely on the removal of the load and there will be no residual deformation. The strain energy stored in the bar is then known as elastic strain energy as shown shaded in figure (a). If, however, the bar is loaded upto point B, beyond the elastic limit (A), the deformation will not disappear completely on the removal of the load as shown in figure (b). During loading, the work done is equal to the area under the curve, i.e. the area *OABCDO*. However when the load is removed, the load deflection follows the line *BD*, and a permanent elongation *OD* remains, *OD* being known as the permanent set. Hence the recovered strain energy is only the one represented by the triangle *BCD*. This recoverable energy is known as elastic strain energy while the area *OABDO* represents the inelastic strain energy which is lost in the process of permanently deforming the bar.

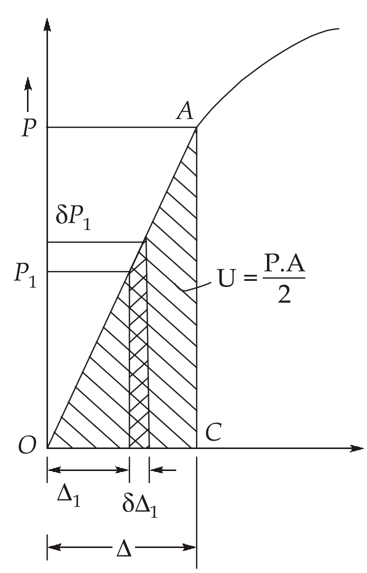


Figure (a)

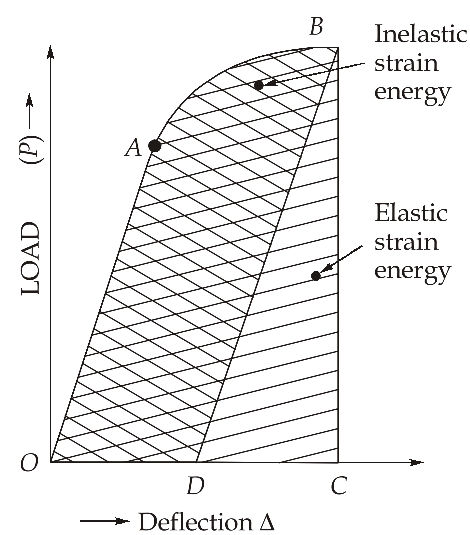


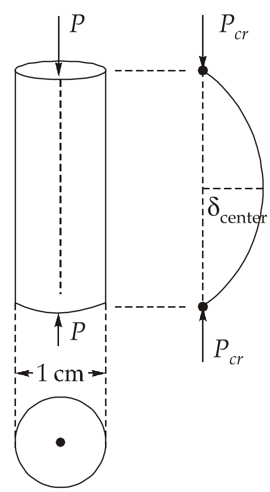
Figure (b)

(ii)

Given: Diameter $d = 1 \text{ cm} = 10 \text{ mm}$

Length $l = 120 \text{ cm} = 1200 \text{ mm}$

$\sigma_{max} = 350 \text{ N/mm}^2$



Area moment of inertia

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\times 10^4 = 490.8738 \text{ mm}^4$$

Area of cross-section, $$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(10)^2 = 78.5398 \text{ mm}^2$$

Euler's buckling load

$$P_{cr} = \frac{\pi^2 EI}{{l_{eff}}^2}$$

$$\Rightarrow \quad P_{cr} = \frac{\pi^2\times 2.1\times 10^5\times 490.8738}{(1200)^2} = 706.523 \text{ N}$$

Direct stress, $$\sigma_d = \frac{P_{cr}}{A} = \frac{706.523}{78.5398} = 9 \text{ N/mm}^2$$

Maximum stress, $$\sigma_{max} = \sigma_d + (\sigma_b)_{max}$$

$$\Rightarrow \quad 350 = 9 + (\sigma_b)_{max}$$

$\therefore$ Maximum bending stress, $\sigma_b = 341 \text{ N/mm}^2$

Now, $$\frac{M}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$\Rightarrow \quad \frac{P_{cr} \times \delta_c}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$\Rightarrow \quad \frac{706.523}{490.8738}\delta_c = \frac{341}{(10/2)}$$

$$\Rightarrow \quad \delta_c = 47.38 \text{ mm}$$

Q.3 (b) Solution:

Given: Working load, P_w = 100 kN

Factored load, P = 1.5 × 100 = 150 kN

Depth, h = 280 mm

Eccentricity, e = 150 mm

Let throat thickness of weld = t

Direct shear stress due to direct load

$$q = \frac{P}{2(ht)} = \frac{150 \times 10^3}{2 \times 280 \times t}$$

$$\Rightarrow \quad q = \frac{267.857}{t} \text{ (N/mm}^2\text{)}$$

Bending stress at the extreme edge of the weld.

$$f = \frac{M}{Z} = \frac{P.e}{\frac{1}{6}(2th^2)} = \frac{6Pe}{2th^2}$$

$$\Rightarrow \quad f = \frac{6 \times 150 \times 10^3 \times 150}{2 \times (280)^2 t}$$

$$\Rightarrow \quad f = \frac{860.969}{t} \text{ (N/mm}^2\text{)}$$

Now equivalent stress

$$f_e = \sqrt{f^2 + 3q^2} \le \frac{f_u}{\sqrt{3}\gamma_{mw}}$$

$$\Rightarrow \quad \sqrt{\left(\frac{860.969}{t}\right)^2 + 3\left(\frac{267.857}{t}\right)^2} \le \frac{410}{\sqrt{3} \times 1.25}$$

$$\Rightarrow \quad \frac{956509.7363}{t^2} \le 35861.33$$

$\Rightarrow \quad t^2 \geq 26.67245$

$\Rightarrow \quad t = 0.7\,S \geq 5.16 \text{ mm}$

$\Rightarrow \quad S \geq 7.37 \text{ mm}$

Use 8 mm weld size.

Q.3 (c) Solution:

Size of footing:

Given: $P = 200$ kN/m, $q_a = 150$ kN/m^2 at a depth of 1 m.

Let wall length = 1 m

$\therefore$ Required width of footing $= \dfrac{200 \times 1.1}{150 \times 1} = 1.47$ m

Therefore provide 1.5 m wide footing.

Thickness of footing based on shear consideration.

Factored net soil pressure, $q_u = \dfrac{200 \times 1.5}{1.5 \times 1} = 200 \text{ kN/m}^2 = 0.2 \text{ N/mm}^2$

(Assuming load factor = 1.5)

The critical section for one way shear is located at a distance 'd' away from the face of the wall.

$$\therefore \quad V_u = 0.2 \times 1000 \times \left[\frac{(1500 - 230)}{2} - d\right]$$

$$= (127000 - 200d)$$

Assuming 0.25% as percentage of tensile reinforcement.

$$\tau_c = 0.36 \text{ MPa for M20 concrete.}$$

$$V_{us} = 0.36 \times 1000 \times d = 360\,d \text{ N}$$

$$\therefore \quad V_u \leq V_{uc}$$

$$\Rightarrow \quad (127000 - 200d) \leq 360\,d$$

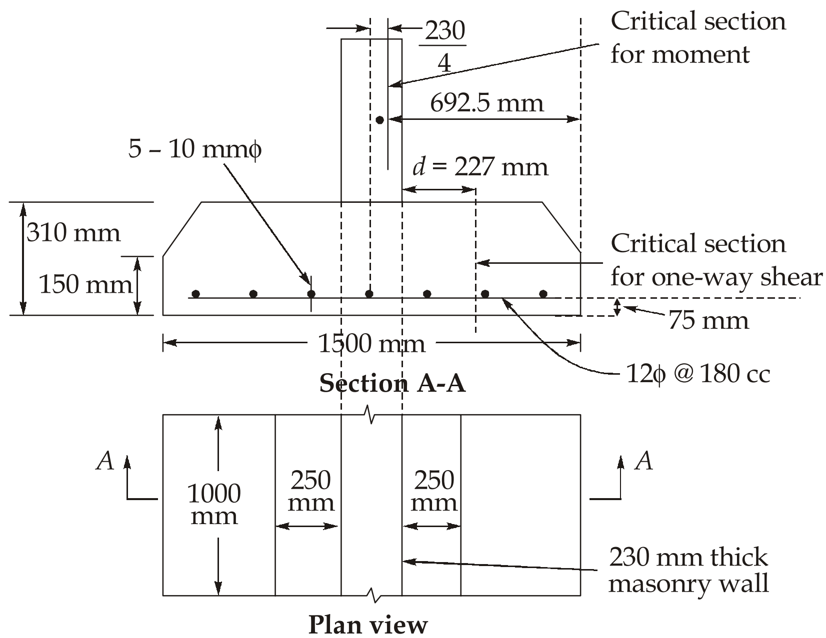
$$\Rightarrow \quad d \geq 227 \text{ mm}$$

Assuming a clear cover of 75 mm and 16ϕ bars

$$\text{Overall thickness, } D \geq \left(227 + 75 + \frac{16}{2}\right) = 310 \text{ mm}$$

Provide $D = 310$ mm upto a distance of 250 mm from the face of wall.

At the edge of the footing, a minimum thickness of 150 mm may be provided linearly tapered upto 310 mm as shown below.



Design of flexural reinforcement.

The critical section for maximum moment is located halfway between the centerline and edge of the wall i.e., $\left(\frac{1500}{2}-\frac{230}{4}\right)$

$$= 692.5 \text{ mm from the edge of footing.}$$

Considering a 1 m long strip of footing with

$$d = 227 \text{ mm}$$

$$M_u = 0.2\times1000\times\frac{692.5^2}{2}\text{ N.mm} = 48 \text{ kNm}$$

$$R_u = \frac{M_u}{bd^2} = \frac{48\times10^6}{1000\times227^2} = 0.932 \text{ MPa}$$

$$\frac{p_t}{100} = \frac{20}{2\times415}\left[1-\sqrt{1-\frac{4.598\times0.932}{20}}\right] = 0.274\times10^{-2}$$

$$A_{st\ req.} = 0.27\times10^{-2}\times1000\times227 = 622 \text{ mm}^2/\text{m length}$$

$$\text{Spacing of 16 mm } \phi \text{ bars} = \frac{1000\times\frac{\pi}{4}(16)^2}{622} = 323 \text{ mm c/c}$$

$$\text{Spacing of 12 mm } \phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 12^2}{622} = 182 \text{ mm} \quad (<3d \text{ or } 300 \text{ mm})$$

Provide 12 ϕ @ 180 mm c/c

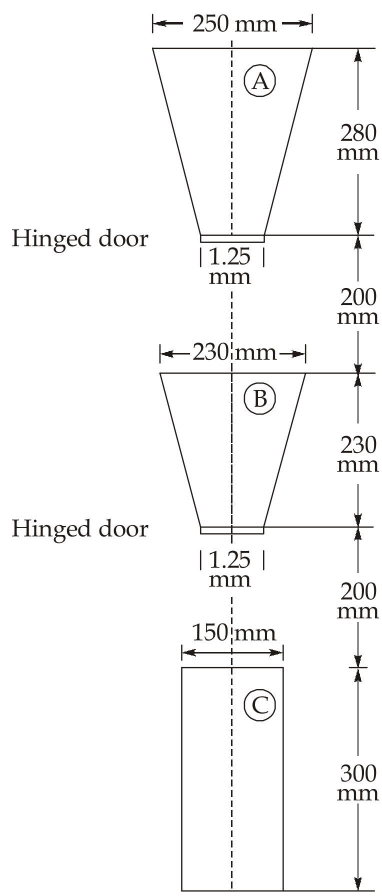
$$\text{Development length required} = \frac{0.87 f_y \cdot \phi}{4\tau_{bd}} = \frac{0.87 \times 415\phi}{4 \times 1.6 \times 1.2} = 47\phi = 47 \times 12 = 564 \text{ mm}$$

$$\text{Length available} = 692.5 - 75$$
$$= 617.5 \text{ mm} > 564 \text{ mm (OK)}$$

Q.4 (a) Solution:

(i)

Compacting Factor Test: It is particularly useful for concrete mixes of medium and low workabilities as are normally used when concrete is to be compacted by vibration. For concrete of very low workability of the order of 0.7 or below, the test is not suitable, because this concrete cannot be fully compacted for comparison.



Compacting factor apparatus

Procedure:

The sample of concrete to be tested is placed gently in the upper hopper. The hopper is filled level with its brim and the trap-door is opened to allow the concrete to fall into the lower hopper. Certain mixes have a tendency to stick in one or both of the hoppers. If this occurs, the concrete may be helped through by pushing the rod gently into the concrete from the top. During this process, the cylinder should be covered by the trowels. Immediately after the concrete has come to rest, the cylinder is uncovered, the trap-door of the lower hopper is opened, and the concrete is allowed to fall into the cylinder. The excess of concrete remaining above the level of the top of the cylinder is then cut-off. The weight of the concrete in the cylinder is then determined to the nearest 10 gram as the weight of partially compacted concrete. The cylinder is refilled with concrete from the same sample in layers of approximately 50 mm, the layers being heavily rammed or preferably vibrated so as to obtain full compaction. The top surface of the fully compacted concrete is carefully struck off level with the top of the cylinder. Compacting factor is defined as the ratio of the weight of partially compacted concrete to the weight of fully compacted concrete. It is normally stated to the nearest second decimal place.

Degree of Workability	Consistency	Slump (mm)	Compacting factor
Extremely low	Moist earth	0	0.65 - 0.7
Very low	Very dry	0 - 25	0.7 - 0.8
Low	Dry	25 - 50	0.8 - 0.85
Medium	Plastic	50 - 100	0.85 - 0.95
High	Semi-fluid	100 - 175	0.95 - 1

(ii)

The use of natural stones in construction involves a systematic sequence of operations starting from quarrying to dressing. Proper processing ensures that stones are extracted with minimum wastage and shaped suitably for masonry or decorative works.

1. **Quarrying of Stone:** The first stage in stone processing is quarrying, which is the extraction of blocks of stone from natural rock beds. Quarrying is done at exposed rock sites where suitable stone is available near the surface.

   The main methods used for quarrying are:

   Hand Tools: This is the simplest method for small-scale extraction using tools like chisels, wedges, hammers, and crowbars. It is suitable for soft stones or where precise cuts are required.

Channelling Machine: This method uses machines to cut continuous channels around a block of stone, helping separate it from the rock bed. It ensures smooth cuts and reduces wastage.

Blasting: This is the most widely used method for medium to large-scale extraction of hard stones such as granite. Blasting involves four main steps:

1. **Boring:** Holes of appropriate diameter and depth are drilled into the rock using jumpers or drilling machines.
2. **Charging:** The drilled holes are filled with explosive materials like gunpowder or dynamite.
3. **Tamping:** The holes are packed with damp clay or similar material and rammed tightly to confine the explosive charge.
4. **Firing:** The charge is ignited using a fuse or electric detonator to blast the rock apart into manageable blocks.

2. **Seasoning of Stone:** After quarrying, the stone contains quarry sap or interstitial moisture that must be removed. Seasoning is done by exposing the stones to the atmosphere in a shaded area to allow gradual drying and to prevent frost damage. Seasoning improves the hardness and durability of the stone by allowing the cementing material to bind the particles more firmly.
3. **Dressing of Stone:** Dressing is the final stage, where quarried and seasoned stones are cut, shaped, and finished to required sizes and textures for masonry work. Stones may be dressed into blocks, slabs, or tiles depending on use. Surface finishes can be natural, honed, or polished. Proper dressing reduces wastage and improves the appearance and workability of stones in construction.

Q.4 (b) Solution:

Given : 14 steps; T = 300 mm; R = 180 mm; 2 Landings = 1.25 m

Design constants: M20, Fe415; $Q = 0.138\, f_{ck}$

Loading on flight:

Let us assume bearing of landing slab in wall to be 160 mm,

$$\text{Effective span} = \left(1.25 + \frac{13 \times 300}{1000} + 0.16 + 1.25\right) \text{m} = 6.56 \text{ m}$$

Let us assume waist slab thickness = 280 mm {Assuming thickness @ 40 mm to 50 mm per m span}

Weight of slab on slope, $w' = 0.28 \times 1 \times 1 \times 25 = 7 \text{ kN/m}^2$

Dead weight of slab on horizontal projection, $w_1 = \dfrac{w'\sqrt{R^2 + T^2}}{T}$

$$w_1 = \frac{7\sqrt{180^2+300^2}}{300} = 8.16 \text{ kN/m}^2$$

$$\text{Dead weight of steps} = \frac{180}{2\times1000}\times 25 = 2.25 \text{ kN/m}^2$$

$$\text{Live load} = 5 \text{ kN/m}^2$$

$$\text{Assuming load due to finishing} = 0.1 \text{ kN/m}^2$$

$$\text{Total load, } w = (8.16 + 2.25 + 5 + 0.1) \text{ kN/m}^2 = 15.51 \text{ kN/m}^2$$

$$\text{Load on landing} = (15.51 - 2.25) \text{ kN/m}^2 = 13.26 \text{ kN/m}^2$$

But assuming uniform weight for design.

Design of waist slab:

$$\text{Factored bending moment} = 1.5\frac{wl^2}{8} = \frac{1.5\times15.51\times6.56^2}{8} = 125.15 \text{ kN-m}$$

$$\text{Depth required, } d = \sqrt{\frac{M_u}{Q.B}} = \sqrt{\frac{125.15\times10^6}{0.138\times20\times1000}} = 212.94 \text{ mm}$$

Adopt effective depth, d = 230 mm and effective cover of 30 mm and thus total depth = 260 mm

Reinforcement:

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6\,M_u}{f_{ck}\cdot b\cdot d^2}}\right]bd$$

$$= \frac{0.5\times20}{415}\left[1-\sqrt{1-\frac{4.6\times125.15\times10^6}{20\times1000\times230^2}}\right]1000\times230$$

$$= 1800.2 \text{ mm}^2$$

Using 16 mm ϕ bars, no. of bars required = $\dfrac{1800.2}{\frac{\pi}{4}\times(16)^2} = 8.9$

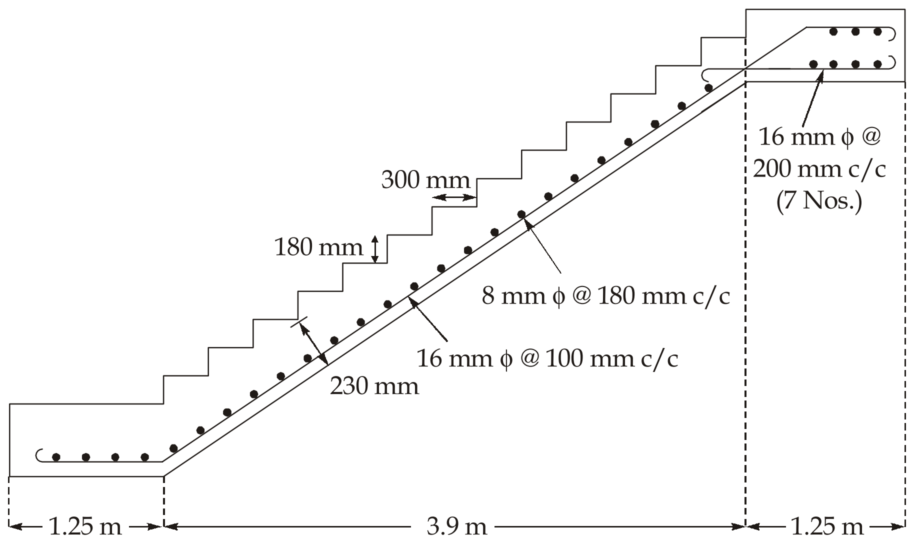
$\therefore$ No. of bars required in 1.4 m width = $1.4\times\dfrac{1800.2}{\frac{\pi}{4}\times16^2} = 12.53 \simeq 13$

$\therefore$ Spacing of bars = $\dfrac{1400}{13}$ = 107.7 mm $\simeq$ 100 mm (say)

Distribution reinforcement: $A_{sd} = \dfrac{0.12}{100}\times1000\times230$ = 276 mm^2

Using 8 mm ϕ bars, spacing = $\frac{1000}{276}\times\frac{\pi}{4}\times 8^2 = 182.12 \text{ mm} \simeq 180 \text{ mm (say)}$

$\therefore$ **Provide 8 mm ϕ at 180 mm c/c spacing.**



Reinforcement detailing of stair case

Q.4 (c) Solution:

Fixed end moments:

$$\overline{M}_{AB} = -\left[\frac{wl^2}{12}+\frac{6EI\delta}{l_{AB}^2}\right] = -\left[\frac{40\times 3^2}{12}+\frac{6\times 7000\times 2.5\times 10^{-3}}{3^2}\right]$$

$$= -[30 + 11.67] = -41.67 \text{ kNm}$$

$$\overline{M}_{BA} = +\frac{wl^2}{12}-\frac{6EI\delta}{l_{BA}^2} = +30 - 11.67 = +18.33 \text{ kNm}$$

$$\overline{M}_{BC} = -\frac{Wl}{8}+\frac{6EI\delta}{l_{BC}^2} = -\frac{100\times 2}{8}+\frac{6\times 7000\times 2.5\times 10^{-3}}{2^2}$$

$$= -25 + 26.25 = +1.25 \text{ kNm}$$

$$\overline{M}_{CB} = +\frac{Wl}{8}+\frac{6EI\delta}{l_{CB}^2} = +25 + 26.25 = +51.25 \text{ kNm}$$

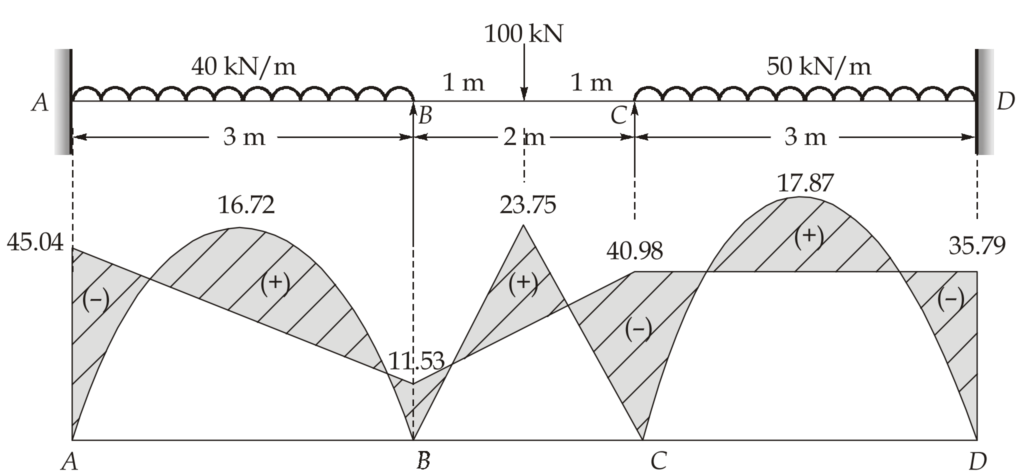
$$\overline{M}_{CD} = -\frac{wl^2}{12} = -\frac{50\times 3^2}{12} = -37.5 \text{ kNm}$$

$$\overline{M}_{DC} = +\frac{wl^2}{12} = +37.5 \text{ kNm}$$

Distribution Factors:

Joint	Member	Stiffness	Total stiffness	Distribution factors
B	BA	$\frac{4\ EI}{3}$	$\frac{10\ EI}{3}$	$\frac{2}{5}$
	BC	$\frac{4\ EI}{2}$		$\frac{3}{5}$
C	CB	$\frac{4\ EI}{2}$	$\frac{10\ EI}{3}$	$\frac{3}{5}$
	CD	$\frac{4\ EI}{3}$		$\frac{2}{5}$

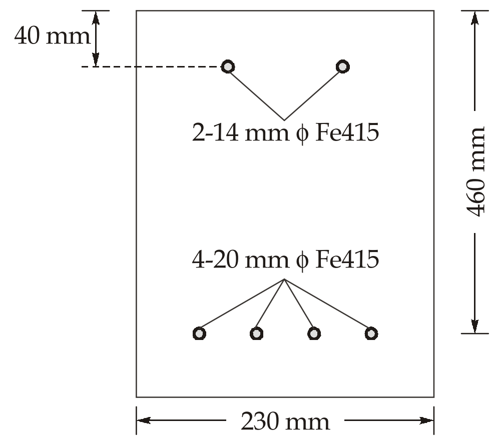
A	B ($\frac{2}{5}$)	B ($\frac{3}{5}$)	C ($\frac{3}{5}$)	C ($\frac{2}{5}$)	D
–41.67	+18.33	+1.25	+51.25	–37.50	+37.50
	–7.83	–11.75	–8.25	–5.50	
–3.92		–4.13	–5.88		–2.75
	+1.65	+2.48	+3.53	+2.35	
+0.83		+1.77	+1.24		+1.18
	–0.71	–1.06	–0.74	–0.50	
–0.36		–0.37	–0.53		–0.25
	+0.15	+0.22	+0.32	+0.21	
+0.08		+0.16	+0.11		+0.11
	–0.06	–0.10	–0.07	–0.04	
–45.04	+11.53	–11.53	+40.98	–40.98	+35.79



BMD

Section - B

Q.5 (a) Solution:



$$\text{Area of compressions steel, } A_{sc} = 2\times\frac{\pi}{4}\times(14)^2 = 307.8761 \text{ mm}^2$$

$$\text{Area of tension steel, } A_{st} = 4\times\frac{\pi}{4}\times(20)^2 = 1256.6371 \text{ mm}^2$$

$$\text{For Fe415, } (x_u)_{\lim} = 0.48d = 0.48 \times 460 = 220.8 \text{ mm}$$

Actual depth of neutral axis is determined as;

$$C = T$$

$$\Rightarrow \quad 0.36 f_{ck} B x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$\Rightarrow \quad 0.36 \times 20 \times 230 x_u + (353 - 0.45 \times 20) \times 307.8761 = 0.87 \times 415 \times 1256.6371$$

$$\Rightarrow \quad x_u = 210.02 \text{ mm} < x_{u,\lim} \ (= 220.8 \text{ mm})$$

Hence, the section is under-reinforced

$$\text{Moment of resistance, MOR} = 0.36 f_{ck} B x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) \times A_{sc} \times (d - d')$$

$$= [0.36 \times 20 \times 230 \times 210 \times (460 - 0.42 \times 210)$$

$$+ (353 - 0.45 \times 20) \times 307.876 \times (460 - 40)] \text{ N-mm}$$

$$= 173.78 \text{ kN-m}$$

Q.5 (b) Solution:

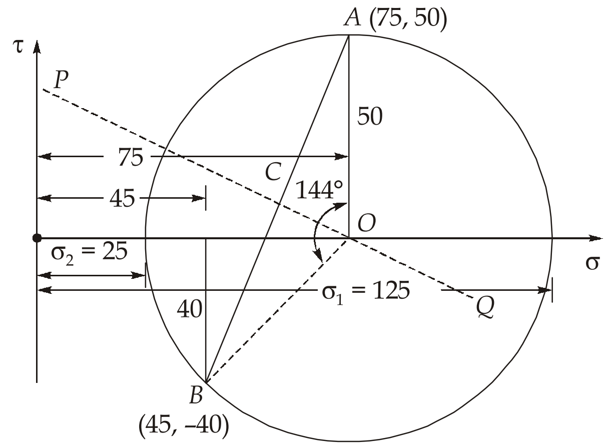
Normal and shear stress on planes A and B:

$$\sigma_A = 75 \text{ MPa}$$

$$\tau_A = 50 \text{ MPa}$$

$$\sigma_B = 45 \text{ MPa}$$

$$\tau_B = -40 \text{ MPa}$$



(Stress values in MPa)

$$\text{Slope of line } AB,\ m_{AB} = \frac{(50+40)}{(75-45)} = 3$$

$$\text{Slope of line } PQ,\ m_{PQ} = \frac{-1}{3} \qquad (\because AB \perp PQ)$$

$$\text{Mid point of } AB,\ C \equiv (60, 5)$$

Now line PQ passes through C (60, 5) and is having slope of $-\frac{1}{3}$ and thus equation of line PQ is,

$$y - 5 = \frac{-1}{3}(x-60)$$

For centre of Mohr's circle, $y = 0$

$$0 - 5 = \frac{-1}{3}(x-60)$$

$\Rightarrow$ $x = 75$

Centre of Mohr's circle = (75, 0)

Radius of Mohr's circle, $r = \sqrt{(75-75)^2 + (50-0)^2}$

$\Rightarrow$ $r = 50$

Principal stress, $\sigma_{1,2} = 75 \pm r$

$\therefore$ $\sigma_1 = 125$ MPa and $\sigma_2 = 25$ MPa

Q.5 (c) Solution:

Constant percentage method of depreciation:

This method of computing depreciation is also called as Declining Balance Method. Here the asset is assumed to lose its value annually at a constant percentage of its book value.

Let FDB = Fixed percentage i.e., factor for declining balance (FDB) method

C_i = Initial cost of asset

∴ Depreciation in first year, $D_1 = C_1 \times FDB$

∴ Book value at the end of first year, $B_1 = C_i - D_1 = C_i(1 - FDB)$

Depreciation in second year, D_2 = Book value at the beginning of 2nd year × FDB

Book value at the end 2nd year

$$B_2 = C_i(1 - FDB) - C_i(1 - FDB) \times FDB$$

$$= C_i(1 - FDB)^2$$

∴ Book value at the end of n^{th} year,

$$B_n = C_i(1 - FBD)^n$$

But book value at the end of n^{th} year,

$$B_n = \text{Salvage value} = C_s$$

∴ $$B_n = C_s = C_i(1 - FDB)^n$$

Given: Initial cost, C_i = Rs. 80000

Life, n = 8 years

Salvage value, C_s = Rs. 8000

∴ $$FDB = 1 - \left(\frac{C_s}{C_i}\right)^{1/n} = 1 - \left(\frac{8000}{80000}\right)^{1/8} = 0.25$$

∴ Book value after 5 years = Book value at the end of 5th year

$$= C_i(1 - \text{FDB})^5$$

$$= 80000(1 - 0.25)^5$$

$$= \text{Rs. } 18984.375$$

$$\simeq \text{Rs. } 18984.4$$

Q.5 (d) Solution:

Equivalent stiffness is given by,

$$\frac{1}{K_{eq}} = \frac{1}{K_{\text{spring}}} + \frac{1}{K_{\text{cantilever}}}$$

$$K_{\text{cantilever}} = \frac{3EI}{L^3} = \frac{3 \times 2 \times 10^5 \times \left(\frac{25 \times 5^3}{12}\right)}{(60)^3}$$

$$= \frac{12.5 \times 10^5}{12^3} = 723.4 \text{ N/mm} = 7234 \text{ N/cm}$$

$$\frac{1}{K_{eq}} = \frac{1}{20} + \frac{1}{7234}$$

$$\therefore \quad K_{\text{eq}} = 19.94 \text{ N/cm}$$

Natural frequency is given by

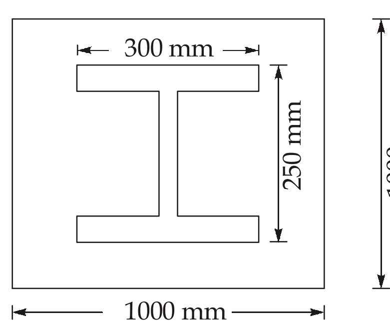
$$\omega = \sqrt{\frac{K_{eq} \times g}{W}} = \sqrt{\frac{19.94 \times 981}{250}} \qquad g = 981 \text{ cm/sec}^2$$

$$\therefore \quad \omega = 8.85 \text{ radian/sec}$$

$$f = \frac{\omega}{2\pi} = \frac{8.85}{2\pi} = 1.41 \text{ Cycle/sec}$$

Q.5 (e) Solution:

Let Area of steel plate = 1 m^2 (1 m × 1 m)



$$b_f = 300 \text{ mm}$$

$$\therefore \quad \text{Short outstand, } b = \frac{1000 - 300}{2} = 350 \text{ mm}$$

$$\therefore \quad \text{Long outstand, } a = \frac{1000 - 250}{2} = 375 \text{ mm}$$

As per IS : 800-2007

$$\text{Thickness of slab base, } t = \sqrt{\frac{2.5w\left(a^2 - 0.30b^2\right)}{\sigma_{bc}}} \quad \text{(OK)}$$

$$\text{Factored load, } P = 1.5 \times 750 = 1125 \text{ kN}$$

$$\therefore \quad w = \frac{P}{A} = \frac{1125 \times 10^3}{10^6} = 1.125 \text{ MPa} < 4 \text{ MPa}$$

$$\therefore \quad w < \sigma_{bc} \quad \text{(OK)}$$

$$\therefore \quad t = \sqrt{\frac{2.5 \times 1.125 \times \left(375^2 - 0.30 \times 350^2\right)}{185}} = 39.74 \text{ mm}$$

∴ Provide, 40 mm thickness of slab base plate.

Q.6 (a) Solution:

(i)

Given: $f_y = 250$ MPa, $f_u = 410$ MPa

Gross tensile strength of plate

$$T_{dg} = \frac{f_g}{\gamma_{mo}} A_g$$

$$\Rightarrow \quad T_{dg} = \frac{250 \times 160 \times 8}{1.1 \times 1000} \text{ kN} = 290.91 \text{ kN}$$

Diameter of hole for 16 mm diameter bolts, $d_o = 18$ mm

Design strength of plate in rupture,

$$T_{dn} = \frac{0.9 f_u A_n}{\gamma_{m1}}$$

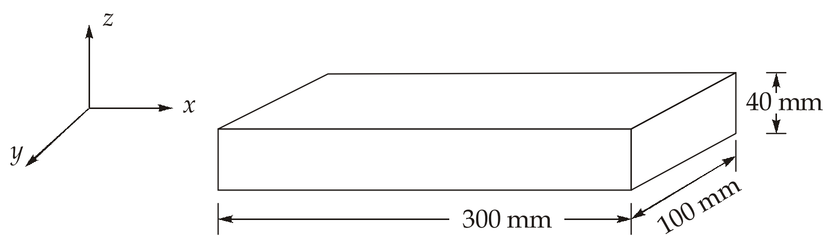
$$A_n = (B - nd_0) \times t$$

$$\therefore \quad T_{dn} = \frac{0.9 \times 410 \times (160 - 3 \times 18) \times 8}{1.25 \times 10^3} \text{ kN}$$

$$= 250.33 \text{ kN}$$

$$\text{Design strength of plate} = \min \begin{cases} 290.91 \text{ kN} \\ 250.33 \text{ kN} \end{cases}$$

$$= 250.331 \text{ kN}$$



(ii)

Given:

$$\text{Dimensions of bar} = 300 \text{ mm} \times 100 \text{ mm} \times 40 \text{ mm}$$

$$\therefore \quad x = 300 \text{ mm}, y = 100 \text{ mm and } z = 40 \text{ mm}$$

$$\therefore \quad \text{Volume, } V = xyz = 300 \times 100 \times 40$$

$$= 1200000 \text{ mm}^3$$

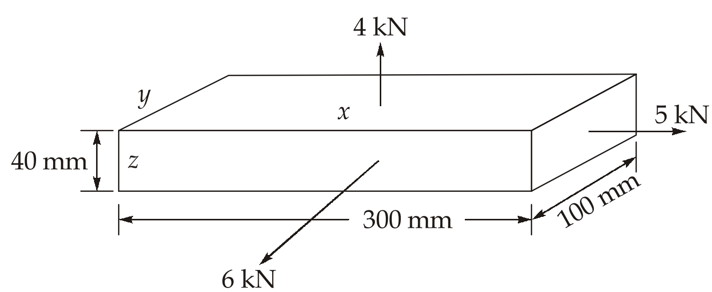
$$\text{Load in the direction of } x = 5 \text{ kN} = 5000 \text{ N}$$

$$\text{Load in the direction of } y = 6 \text{ kN} = 6000 \text{ N}$$

$$\text{Load in the direction of } z = 4 \text{ kN} = 4000 \text{ N}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\text{Poisson's ratio, } \mu = 0.25$$



$$\therefore \quad \text{Stress in the } x\text{-direction, } \sigma_x = \frac{\text{Load of } x\text{-direction}}{yz}$$

$$= \frac{5000}{100 \times 40} = 1.25 \text{ N/mm}^2$$

Similarly the stress in y-direction is given by,

$$\sigma_y = \frac{\text{Load in } y\text{-direction}}{xz}$$

$$= \frac{6000}{300 \times 40} = 0.5 \text{ N/mm}^2$$

$$\text{Stress in } z\text{-direction} = \frac{\text{Load in } x\text{-direction}}{xy}$$

$$= \frac{4000}{300 \times 100} = 0.133 \text{ N/mm}^2$$

Now,
$$\in_V = \frac{\Delta V}{V} = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}(1 - 2\mu)$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{1}{2 \times 10^5}(1.25 + 0.5 + 0.133)(1 - 2 \times 0.25)$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{1.883}{2 \times 10^5 \times 2}$$

$$\Rightarrow \quad \Delta V = \frac{1.883}{4 \times 10^5} \times V$$

$$\Rightarrow \quad \Delta V = \frac{1.883}{4 \times 10^5} \times 1200000$$

$$\Rightarrow \quad \Delta V = 5.649 \text{ mm}^3$$ **Ans.**

Q.6 (b) Solution:

(i)

S.no.	Nominal mix	Design mix
1	A mix where the cement concrete is specified by properties of different ingredients is called nominal mix concrete.	A mix where properties are determined and the properties of concrete are specified is called design mix concrete.
2	It is presumed that by these properties satisfactory performance can be achieved	Target performance is achieved by mix design process based on calculations and trials using properties of materials.
3	Variations in materials is not considered.	Mix is tailored considering actual properties of materials.
4	Generally used for grades upto M20.	Used for M25 and above grades.
5	Used in small scale, non-structural or general construction works.	Used in large-scale structural and critical infrastructure works.

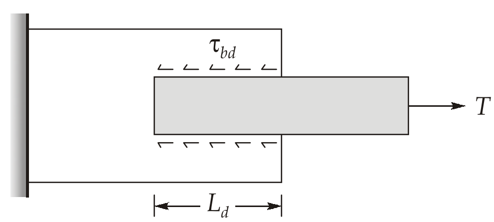
(ii)

Carbonation: Concrete is alkaline in nature and has an initial pH value of 12-13. As long as, the steel reinforcement is in alkaline environment, it corrosion cannot start. However, carbondioxide present in the atmosphere reacts with concrete in presence of water (the moist conditions). In fact in the presence of moisture, carbon dioxide changes to carbonic acid and attacks the concrete; $Ca(OH)_2$ present in concrete is converted to $CaCO_3$ causing reduction in pH of concrete. The concrete thus turns acidic. Once pH value in the concrete cover drops below 10, corrosion of steel reinforcement begins due to reduction in the alkaline environment. Because of corrosion of steel reinforcement, its volume increases and consequently the concrete cracks and spalling of concrete takes place.

(iii)

Development length: Development length is an embedded length of the bar required to develop the design strength of reinforcement at the critical section.

Let us consider a steel bar embedded in concrete. The bar is subjected to a tensile force *T*. Due to this tensile force, the steel bar will tend to come out and slip out of the concrete. This tendency of slipping is resisted by the bond stress developed over the surface of the bar.



Bond strength (τ_{bd}) is the shear stress developed along the contact surface between the reinforcing steel and the surrounding concrete, which prevents the bar from slipping out of concrete.

To avoid slipping $T \le \tau_{bd}\,(\pi\phi L_d)$ [Surface area = $\pi\phi L_d$]

$$\text{Shear strength, } T = 0.87 f_y A_s = 0.87 f_y \times (\pi/4) \times \phi^2$$

$$\therefore \quad 0.87 f_y \times \left(\frac{\pi}{4}\right) \times \phi^2 \le \tau_{bd} \times (\pi\phi) \times L_d$$

$$\Rightarrow \quad L_d \ge \frac{0.87 f_y \phi}{4\tau_{bd}}$$

where, L_d = Development length of steel bar.

f_y = Characteristic strength of the bar material

τ_{bd} = Design bond stress in plain bars in tension

ϕ = Diameter of bar

L_d is called the development length. It is the minimum length of bar which must be embedded in concrete beyond any section to prevent slipping out of bar from concrete.

Factors affecting the development length: Various factors affecting the development length are as follows:

1. **Grade of Concrete**: Since bond resistance is essentially an interfacial shear, it is a function of shear strength of concrete, and hence, of grade of concrete. Higher the grade, greater is the strength.
2. **Diameter of Bar**: Greater is the bar diameter lesser is the bond resistance for the same surface area because larger diameter of bar leads to greater cracking.
3. **Nature of Stress**: Since the transverse compression from concrete increases the grip and frictional resistance, bond strength is higher for bars in compression than in tension.
4. **Bends and Hooks**: The increase in bond resistance at bends is due to increase in frictional resistance on account of confinement of concrete inside the bend by radial component of the bar tension. The increase in bond strength is measured in terms of additional anchorage length provided by the bar bend.
5. **Cover**: If the cover is inadequate or when the horizontal distance between two parallel main reinforcing bars is less, the splitting occurs resulting in ultimate cracking and in reduction in bond strength to a large extent.
6. **Curtailment of Bars in Tension Zone**: The curtailment of bars in tension zone creates a condition of differential strains in adjacent bars effecting loss of shear and bond.
7. **Grouping of Bars:** Bond strength gets reduced for bundled bars due to reduction in surface area.

Q.6 (c) Solution:

(i)

Ist alternative:

Transportation by truck Stone aggregate required = 1.5 million tons

Temporary road cost = Rs. 60 lacs

Hauling charges = Rs. 10 per tonne of truck

Total cost of hauling = Rs. $10 \times 1.5 \times 10^6$ = Rs. 150 lacs

∴ Total expenditure incurred = (150 + 60) = 210 lacs

IInd alternative: Transportation by belt conveyor

Original initial cost of belt conveyor = Rs. 70 lacs

Salvage value = 20% of Original cost = 0.20×70 = Rs. 14 lacs

Depreciated value = Original value – Salvage value

= Rs. (70 – 14)lacs = Rs. 56 lacs

Maintenance cost = 30% of depreciated cost

$$= 0.30 \times 56 = 16.8 \text{ lacs}$$

$$\text{Electric power cost} = 0.60 \times \text{Original cost}$$

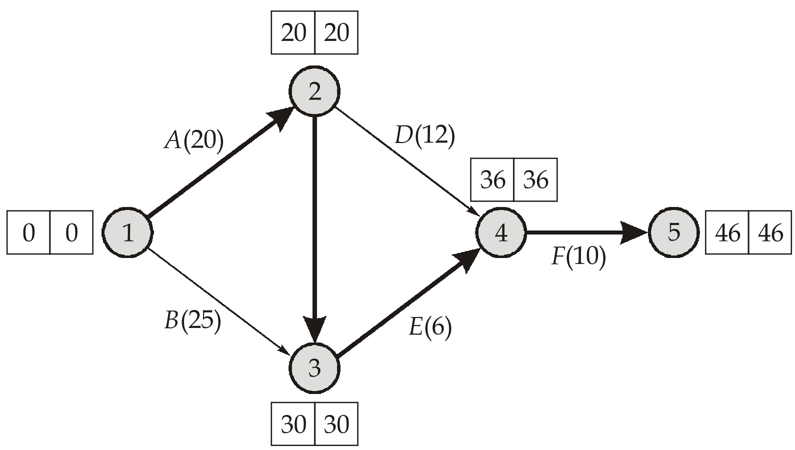
$$= 0.60 \times 70 = 42 \text{ lacs}$$

Total net expenditure incurred

$$= (70 + 16.8 + 42) - 14 = \text{Rs. } 114.8 \text{ lacs} < \text{Rs. } 210 \text{ lacs}$$

∴ The best alternative is transportation with belt conveyor.

(ii)



$$\text{Total float } (F_T) = LST - EST = LFT - EFT$$

$$\text{Free float } (F_F) = F_T - S_j = \text{Total float} - \text{Head event slack}$$

$$\text{Independent float } (F_{ID}) = F_F - S_i = \text{Free float} - \text{Tail event slack}$$

Activity ($i - j$)	Duration (t_{ij}) (days)	Earliest		Latest		Float		
		Start time (EST)	Finish time (EFT)	Start time (LST)	Finish time (LFT)	Total float (F_T)	Free float (F_F)	Independent float (F_{ID})
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1 - 2	20	0	20	0	20	0	0	0
1 - 3	25	0	25	5	30	5	5	5
2 - 3	10	20	30	20	30	0	0	0
2 - 4	12	20	32	24	36	4	4	4
3 - 4	6	30	36	30	36	0	0	0
4 - 5	10	36	46	36	46	0	0	0

Activities 1-2, 2-3, 3-4 and 4-5 having total float equal to zero and thus are critical. Hence, path 1-2-3-4-5 is the critical path as shown with thick lines.

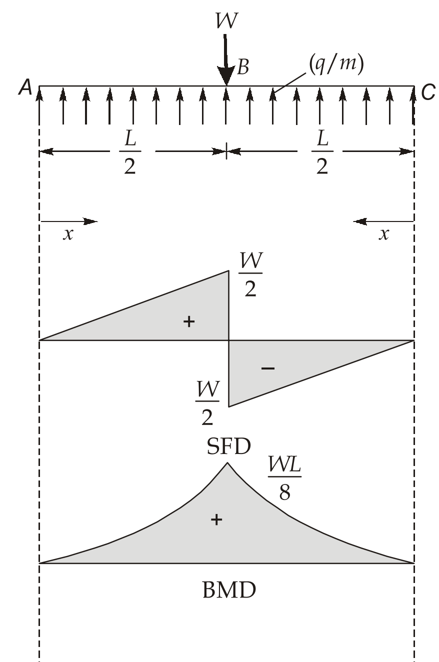
Q.7 (a) Solution:

(i)

Given: Span of wooden log = L

Concentrated load at the mid-span = W

The forces on the wooden log floating on water is shown below:



By balancing the forces in vertical direction,

$$q \times L = W$$

$$\Rightarrow \quad q = \frac{W}{L}$$

Now, taking x from A, shear force in any section between A and B,

$$S_x = q \cdot x = \frac{W \cdot x}{L}$$

$$\therefore \quad \text{At } x = 0,\ S_A = q \cdot 0 = 0$$

$$\text{and} \quad \text{at } x = \frac{L}{2}$$

$$\Rightarrow \quad S_B = q \cdot \frac{L}{2} = \frac{W}{L} \times \frac{L}{2} = \frac{W}{2}$$

Now, bending moment in any section between A and B,

$$M_x = q\, x \cdot \frac{x}{2} = \frac{W}{2L} x^2$$

At $x = 0$, $M_A = 0$

$$\text{and at } x = \frac{L}{2}, \quad M_B = \frac{W}{2L} \times \left(\frac{L}{2}\right)^2 = \frac{WL}{8}$$

Similarly, taking x from C, shear force in any section between C and B,

$$S_x = -q\, x = \frac{-Wx}{L}$$

$$\therefore \quad \text{At } x = 0, \quad S_C = 0$$

$$\text{At } x = \frac{L}{2}, \quad S_B = -\frac{W}{2}$$

Bending moment in any section between C and B, $M_x = q\, x \frac{x}{2} = \frac{Wx^2}{2L}$

$$M_C = 0$$

$$\text{and} \quad M_B = \frac{WL}{8}$$

(ii)

$$AB = BD = DE = l/3$$

Let R_A and R_E be the reaction force at A and E respectively.

$$R_A = R_E$$

$\Sigma M_A = 0$: $$R_E \times l = +2M$$

$\Rightarrow$ $$R_E = +\frac{2M}{l}(\downarrow) \quad \text{(in downward direction)}$$

$\therefore$ $$R_A = \frac{2M}{l}(\uparrow) \quad \text{(in upward direction)}$$

SFD $$V = \text{constant} = \frac{2M}{l}$$

BMD In portion AB $$M_x = R_A x = \frac{2Mx}{l}$$

$\therefore$ $$M_B = \frac{2M}{l} \times \frac{l}{3} = \frac{2M}{3}$$

In portion BD, $$M_x + M = R_A x$$

$\Rightarrow$ $$M_x = R_A x - M = \frac{2M}{l}x - M = M\left(\frac{2x}{l} - 1\right)$$

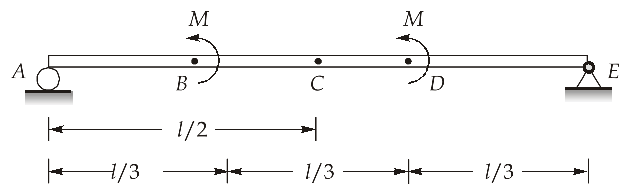
$\therefore$ $$M_B = -\frac{M}{3}$$

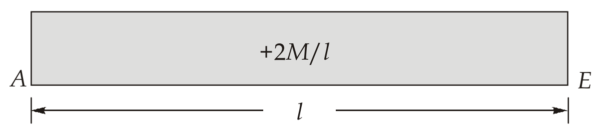
$$M_C = M\left(\frac{2(l/2)}{l} - 1\right) = 0$$

and $$M_D = M\left(\frac{4}{3} - 1\right) = \frac{M}{3}$$

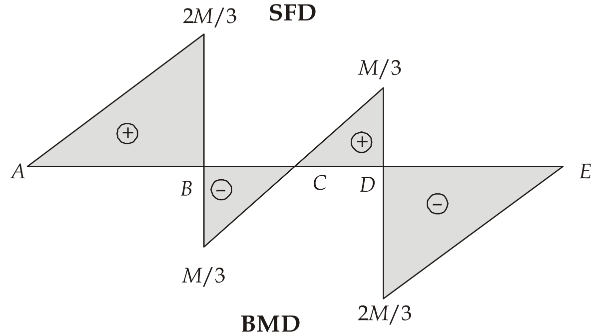
In portion DE $$M_x = \frac{2Mx}{l} - M - M = 2M\left(\frac{x}{l} - 1\right)$$

$\therefore$ $$M_D = 2M\left(\frac{2l/3}{l} - 1\right) = -\frac{2M}{3}$$





SFD



BMD

Q.7 (b) Solution:

(i)

Resource levelling: The analysis aiming at stabilization of rate of resource utilization (relatively constant) by various activities at different times without changing the project duration is called resource levelling.

In order to stabilize the use of existing level of resources, the total float of non-critical activities is used. By shifting a non-critical activity between its earliest start time and latest allowable time, project manager may be able to lower the maximum resource requirement.

The following two general rules are generally used in scheduling non-critical activities:

- If the total float of a non-critical activity is equal to its free float, then it can be scheduled anywhere between its earliest start and latest completion or finish times.
- If the total float of a non-critical activity is more than its free float, then its starting time can be delayed relative to its earliest start time by no more than the amount of its float without affecting the scheduling of its immediately succeeding activities.

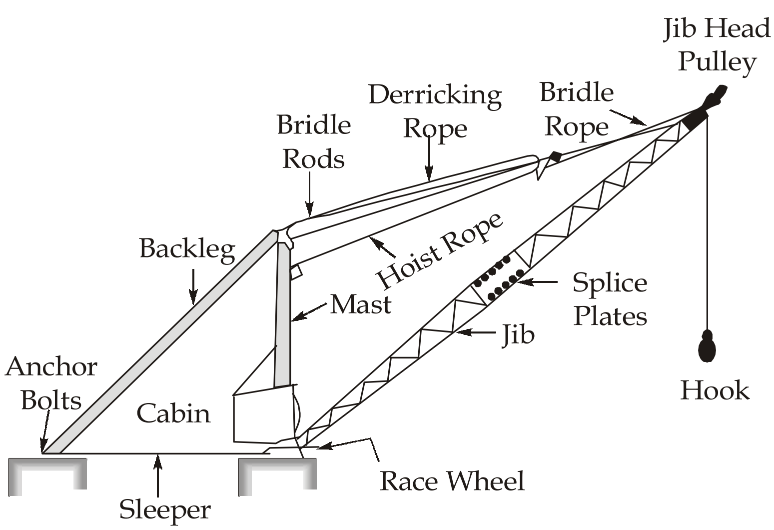
(ii)

Resource smoothening: The analysis aiming to reduce peak demand for resource and reallocating among activities of a project in a manner so that the total project duration remains shortest is known as resource smoothening (or loading).

The procedure of carrying out resource smoothening can be summarized in the following steps:

- **Step 1**: Calculate the earliest start and latest finish times of each activity and then draw a time scaled version (or squared) of the network. In this network, the critical path is drawn along a straight line and non-critical activities are shown on both sides of this line. Resource requirement of each activity is marked along the arrows.
- **Step 2**: Draw the resource histogram by taking earliest start times or latest start times of the activities on the x-axis and cumulative resource required on y-axis.
- **Step 3**: Shift start time of non-critical activities first having largest float in order to smoothen the demand for resources.

(iii)



Power driven Scotch Derrick Crane

Q.7 (c) Solution:

Let V_A and V_F be the vertical reactions at A and F respectively.

Taking moments about A,

$$V_F \times 4 = 200 \times 4(1+\sqrt{3})$$

$$\therefore \quad V_F = 200(1+\sqrt{3})\,\text{kN}\uparrow$$

$$\therefore \quad V_A = 200\sqrt{3}\ \text{kN}\downarrow$$

Considering the joint F we conclude $P_{FB} = 0$

$\therefore \quad P_{FE} = 200(1+\sqrt{3})\text{kN (Compressive)}$

Joint *A*:

$$P_{AB} = 200\sqrt{3} \text{ kN (Tensile)}$$

Joint *B*:

$$P_{BC}\sin 45° = 200\sqrt{3} \text{ kN (Tensile)}$$

$\Rightarrow \quad P_{BC} = 200\sqrt{6} \text{ kN (Tensile)}$

$$P_{BE} = 200\sqrt{6} \cos 45° = 200\sqrt{3} \text{ kN (Compressive)}$$

Joint *C*: $\quad P_{CD} = 200\sqrt{6} \cos 45° = 200\sqrt{3} \text{ kN (Tensile)}$

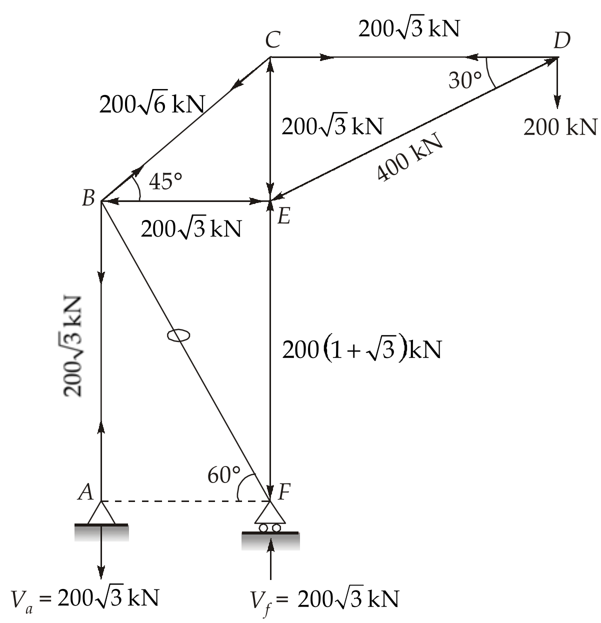
Resolving vertically, $P_{CE} = 200\sqrt{6} \sin 45° = 200\sqrt{3} \text{ kN (Compressive)}$

Joint *D*:

$\Rightarrow \quad P_{DE}\sin 30° = 200$

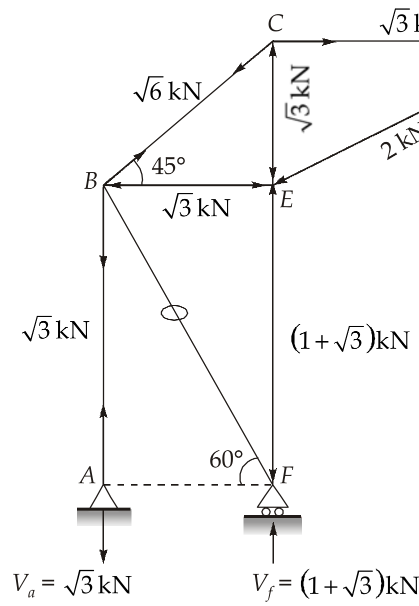
$$P_{DE} = 400 \text{ kN (Compressive)}$$

The forces in the members of the truss due to the given loading are shown in figure below.



To find the vertical deflection of the joint *D*, remove the given loading and apply a 1 kN load vertically at *D*. The forces in the various members due to the application of a vertical load of 1 kN at D are shown in figure.

Vertical deflection of $D = \delta = \sum\left(\frac{PKl}{AE}\right)$



Member	P	K	l(mm)	PKl
AB	$-200\sqrt{3}$	$-\sqrt{3}$	$4\sqrt{3}$	$2400\sqrt{3}$
BC	$-200\sqrt{6}$	$\sqrt{6}$	$4\sqrt{2}$	$4800\sqrt{3}$
CD	$-200\sqrt{3}$	$-\sqrt{3}$	$4\sqrt{3}$	$2400\sqrt{3}$
DE	$+400$	$+2$	8	6400
EF	$200(1+\sqrt{3})$	$(1+\sqrt{3})$	$4\sqrt{3}$	$(3200\sqrt{3}+4800)$
BF	0	0	8	0
BE	$200\sqrt{3}$	$\sqrt{3}$	4	2400
EC	$200\sqrt{3}$	$\sqrt{3}$	4	2400
			Total	$12800\sqrt{3}+16000$ kN

$$\delta = \sum\frac{PKl}{AE} = \frac{(12800\sqrt{3}+16000)1000}{10000\times 200}\text{ mm}$$

$$= 19.085\text{ mm}$$

Q.8 (a) Solution:

Given: Cantilever projection,

$$L = 2.4 \text{ m}$$

Materials used: M20, Fe415

$$f_{ck} = 20 \text{ N/mm}^2, \quad f_y = 415 \text{ N/mm}^2$$

Depth of slab:

$$\frac{\text{Span}}{\text{Overall depth}} = 10 \qquad \text{(For cantilever slab)}$$

$$\Rightarrow \quad \text{Overall depth} = \frac{2.4 \times 1000}{10} = 240 \text{ mm}$$

$$\text{Nominal cover} = 20 \text{ mm}$$

$$\text{Diameter of bar used} = 10 \text{ mm}$$

$$\therefore \quad \text{Effective depth,} \quad d = 240 - 20 - \frac{10}{2} = 215 \text{ mm}$$

Let us provide maximum depth of slab as 240 mm at support and gradually reduce the depth to 120 mm at free end.

Load calculation:

$$\text{Self weight of slab} = 0.5\,(0.24 + 0.12) \times 25 = 4.5 \text{ kN/m}^2$$

$$\text{Live load} = 2 \text{ kN/m}^2$$

$$\text{Load due to finishes} = 1.5 \text{ kN/m}^2$$

$$\text{Total working load} = 8 \text{ kN/m}^2$$

$$\therefore \quad \text{Factored load } (w_u) = 1.5 \times 8 = 12 \text{ kN/m}^2$$

Check for depth:

$$BM_u = \frac{w_u L^2}{2} = \frac{12 \times (2.4)^2}{2} = 34.56 \text{ kN-m/m}$$

For Fe 415 steel,

$$BM_{u,\,\lim} = 0.138\, f_{ck}\, bd^2$$

$$\therefore \quad BM_u = BM_{u,\,\lim}$$

$$\Rightarrow \quad 34.56 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$$

$$\Rightarrow \quad d = 111.9 \text{ mm} < 215 \text{ mm.} \qquad \text{(OK)}$$

Hence, the effective depth provided is sufficient to resist the design moment.

Also, provided depth is more than that required for balanced section and hence section is under-reinforced.

Reinforcement calculation:

$$A_{st} = \frac{0.5\, f_{ck}\, bd}{f_y}\left(1 - \sqrt{1 - \frac{4.6\, BM_u}{f_{ck}\, bd^2}}\right)$$

$$= \frac{0.5 \times 20 \times 1000 \times 215}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 34.56 \times 10^6}{20 \times 1000 \times 215^2}}\right)$$

$$= 466.43 \text{ mm}^2$$

$$\therefore \quad \text{Spacing of 10 mm } \phi \text{ bars} = \frac{1000 \times \frac{\pi}{4}(10)^2}{466.43} = 168.385 \text{ mm c/c}$$

$\therefore$ Provide 10 mm ϕ bars @ 160 mm c/c

$$(A_{st})_{\text{provided}} = \frac{1000 \times \frac{\pi}{4}(10)^2}{160} = 490.87 \text{ mm}^2 > (A_{st})_{\text{req.}}$$

Distribution reinforcement: $A_{st} = 0.12\%$ of A_g (For Fe 415 steel)

$$= \frac{0.12}{100} \times 1000 \times 240 = 288 \text{ mm}^2$$

Let us provide 10 mm ϕ bars as distribution bars

$$\therefore \quad \text{Spacing} = \frac{1000 \times \frac{\pi}{4}(10)^2}{288} = 272.7 \text{ mm c/c}$$

$\therefore$ Provide 10 mm ϕ bars @ 270 mm c/c.

$$(A_{st})_{\text{provided}} = \frac{1000 \times \frac{\pi}{4}(10)^2}{270} = 290.89 \text{ mm}^2$$

Anchorage length: $L_d = \dfrac{(0.87 \cdot f_y)\phi}{4\,\tau_{bd}} = \dfrac{0.87 \times 415 \times 10}{4 \times 1.2 \times 1.6} = 470 \text{ mm}$

Main tension bars are extended into the support to a minimum length of 470 mm including anchorage value of hooks and 90° bends.

Check for deflection control:

$$\left(\frac{L}{d}\right)_{\max} = \left(\frac{L}{d}\right)_{\text{Basic}} \times k_t \times k_c \times k_f$$

$$P_t(\%) = \frac{A_{st}}{bd} \times 100 = \frac{490.87}{1000 \times 215} \times 100 = 0.23\%$$

$$\therefore \quad f_s = 0.58 \times 415 \times \frac{466.43}{490.87} = 228.72 \text{ N/mm}^2$$

From graph given in question, modification factor for tension reinforcement is,

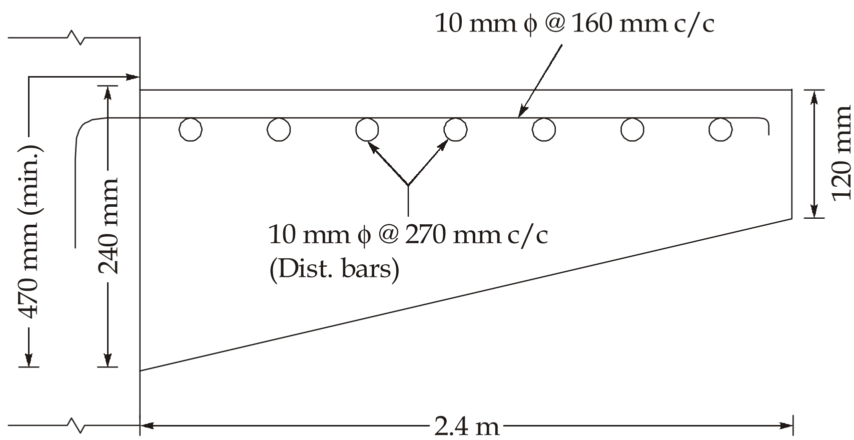
$$k_t = 1.8$$

$$\therefore \quad \left(\frac{L}{d}\right)_{\max} = 7 \times 1.8 \times 1 \times 1 = 12.6$$

$$\left(\frac{L}{d}\right)_{\text{provided}} = \frac{2400}{215} = 11.16 < 12.6 \quad \text{(OK)}$$

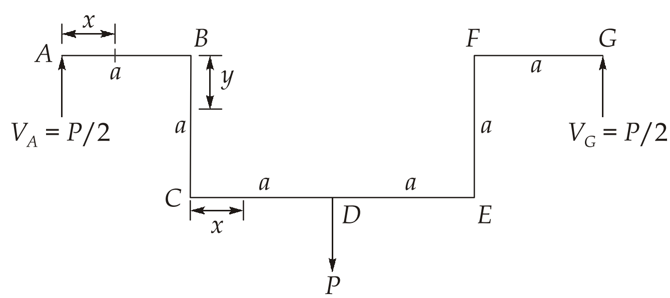
Hence, the cantilever slab satisfies the deflection limit.

Reinforcement details:



Q.8 (b) Solution:

(i)



Support reaction, $V_A = V_G = \dfrac{P}{2}$

Strain energy stored by the whole structure is:

$$U = 2(U_{AB} + U_{BC} + U_{CD})$$

$$\Rightarrow \quad U = 2\left[\int_0^a \frac{\left(\frac{P}{2}x\right)^2 dx}{2EI} + \int_0^a \frac{\left(\frac{P}{2}a\right)^2 dy}{2EI} + \int_0^a \left\{\frac{P}{2}(x+a)\right\}^2 \frac{dx}{2EI}\right]$$

$$\Rightarrow \quad U = 2\left[\frac{P^2}{4}\cdot\frac{a^3}{3}\cdot\frac{1}{2EI} + \frac{P^2}{4}\cdot\frac{a^2 a}{2EI} + \frac{P^2}{4}\cdot\frac{1}{2EI}\cdot\frac{1}{3}\left\{(x+a)^3\right\}_0^a\right]$$

$$\Rightarrow \quad U = 2\left[\frac{P^2a^3}{24EI} + \frac{P^2a^3}{8EI} + \frac{P^2}{24EI}(8a^3 - a^3)\right]$$

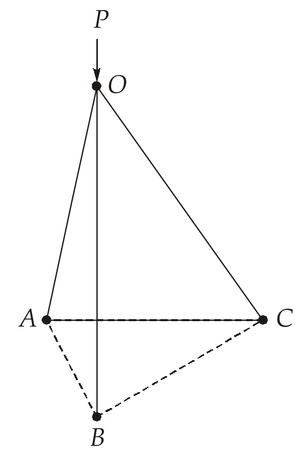
$$\Rightarrow \quad U = 2\times\frac{11}{24}\frac{P^2a^3}{EI} = \frac{11}{12}\frac{P^2a^3}{EI}$$

$\therefore$ Vertical deflection of D $= (\Delta_D)_{\text{Vertical}} = \dfrac{\partial U_i}{\partial P} = \dfrac{11}{12}\times\dfrac{(2P)a^3}{EI} = \dfrac{11Pa^3}{6EI}$

(ii)

By symmetry the force in each member is the same.

Let: Force in each rod is F



$$3F\cos\theta = P$$

$$\Rightarrow \quad F = \frac{P}{3\cos\theta}$$

Strain energy stored, $$W_i = \sum\frac{F^2 l}{2AE} = 3\times\frac{P^2}{9\cos^2\theta}\left(\frac{l}{2AE}\right)$$

$$\Rightarrow \quad W_i = \frac{P^2 l}{6\cos^2\theta AE}$$

$\therefore$ Vertical deflection, $$\delta = \frac{\partial W_i}{\partial P} = \frac{(2P)l}{6\cos^2\theta AE}$$

$$\Rightarrow \quad \delta = \frac{Pl}{3\,AE\cos^2\theta}$$

Q.8 (c) Solution:

(i)

Modular ratio, $$m = \frac{E_S}{E_C} = \frac{2\times10^5}{3\times10^4} = \frac{20}{3}$$

Area of the beam section, $A = 120 \times 300 = 36000\ \text{mm}^2$

Moment of inertia of the beam section

$$I = \frac{120\times300^3}{12} = 2.70\times10^8\ \text{mm}^4$$

Prestressing force, $$P = 6\times\frac{\pi}{4}\times6^2\times1150 = 195093\ \text{N}$$

Stress in concrete at the level of steel

$$f_c = \frac{P}{A}+\frac{Pe^2}{I}$$

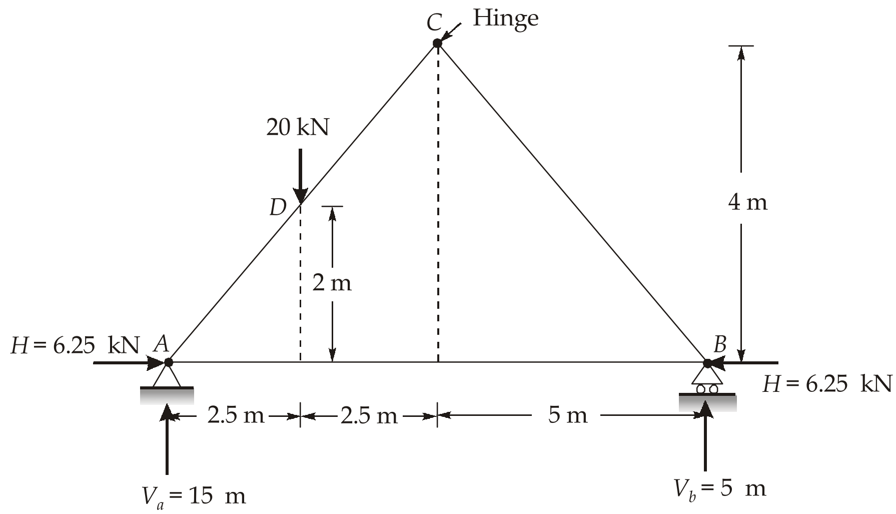
$$= \frac{195093}{36000}+\frac{195093\times55^2}{2.70\times10^8} = 7.61\ \text{N/mm}^2$$

Loss of stress in steel due to creep of concrete

$$= \phi m f_C$$

$$= 1.5\times\frac{20}{3}\times7.61\ \text{N/mm}^2 = 76.1 \simeq 76\ \text{N/mm}^2$$

(ii)



Let V_a and V_b be the vertical reactions at A and B respectively. Let H be the horizontal thrust at each support.

Taking moments about A,

$$V_b \times 10 = 20 \times 2.5$$

$$\therefore \quad V_b = 5 \text{ kN}$$

$$\therefore \quad V_a = 20 - 5 = 15 \text{ kN}$$

Taking moments about C from the right side,

$$H \times 4 = V_b \times 5$$

$$\Rightarrow \quad H \times 4 = 5 \times 5$$

$$\Rightarrow \quad H = 6.25 \text{ kN}$$

Bending moment calculations

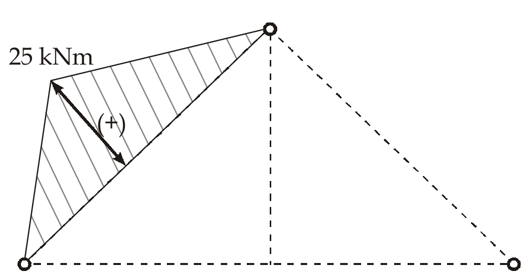
$$\text{B.M at } A = 0$$

$$\text{B.M at } D = 15 \times 2.5 - 6.25 \times 2$$

$$= +25 \text{ kNm}$$

$$\text{B.M at } C = 0$$

$$\text{B.M at } B = 0$$



BMD

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