

Highlights your final answer



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Try to avoid calculation mistake

ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-7 : Full Syllabus Test (Paper-I)

Name :

Roll No :

Test Centres

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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	37
Q.2	
Q.3	
Q.4	27
Section-B	
Q.5	26
Q.6	43
Q.7	45
Q.8	
Total Marks Obtained	178

Signature of Evaluator

Cross Checked by

Sourabh Kumar

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section-A

2.1 (a) Find the complete solution of differential equation $(D^2 - 4D + 3)y = \sin 3x \cos 2x$.

[12 marks]

$$(D^2 - 4D + 3)y = \sin 3x \cos 2x$$

for
CF

$$D^2 - 4D + 3 = 0$$

$$(D-3)(D-1) = 0$$

$$D = 1, 3$$

$$CF = c_1 e^x + c_2 e^{3x}$$

①

for PI

$$PI = \frac{\sin 3x \cos 2x}{D^2 - 4D + 3}$$

$$= \frac{1}{2} \times \frac{2 \sin 3x \cos 2x}{D^2 - 4D + 3}$$

$$= \frac{1}{2} \left[\frac{\sin 5x + \sin x}{D^2 - 4D + 3} \right]$$

$$= \frac{1}{2} \left[\frac{\sin 5x}{D^2 - 4D + 3} + \frac{\sin x}{D^2 - 4D + 3} \right]$$

$$= \frac{1}{2} \left[\frac{\sin 5x}{-25 - 4D + 3} + \frac{\sin x}{-1 - 4D + 3} \right]$$

$$\Rightarrow \frac{1}{2} \left[\frac{\sin 5x}{-22 - 4D} + \frac{\sin x}{-2 - 4D} \right]$$

$$\Rightarrow \frac{1}{2} \left[\frac{-\sin 5x}{(22+4D)(22-4D)} + \frac{2+4D}{(2+4D)(2-4D)} \sin x \right]$$

$$\Rightarrow \frac{1}{2} \left[-\frac{(22-4D)}{22^2 - 16D^2} \sin 5x - \frac{(2-4D)}{4 - 16D^2} \sin x \right]$$

$$= \frac{1}{2} \left[\frac{-(22-4D)}{884} \sin 5x - \frac{(2-4D)}{20} \sin x \right]$$

$$= \frac{1}{2} \left[-\frac{22 \sin 5x}{884} + \frac{4 \times 5 \cos 5x}{884} - \frac{2 \sin x}{20} + \frac{4 \cos x}{20} \right]$$

$$= \frac{1}{2} \left[-0.024 \sin 5x + 0.0226 \cos 5x - 0.1 \sin x + 0.2 \cos x \right]$$

$$\underline{PI} = -0.012 \sin 5x + 0.0113 \cos 5x - 0.05 \sin x + 0.1 \cos x$$

complete solⁿ

$$= CF + PI$$

$$\underline{CI} = C_1 e^x + C_2 e^{3x} - 0.012 \sin 5x + 0.0113 \cos 5x - 0.05 \sin x + 0.1 \cos x$$

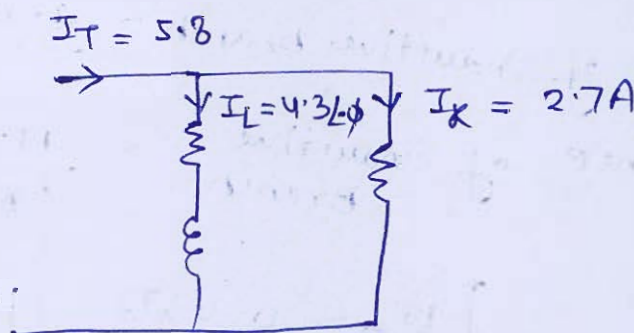
5

- 2.1 (b) An inductive circuit in parallel with a resistive circuit of $20\ \Omega$ is connected across 50-Hz supply. The inductive current is 4.3 A and the resistive current is 2.7 A. The total current is 5.8 A.

Find:

- Power absorbed by the inductive branch.
- Inductance of inductive branch.
- Power factor of the combined circuit. Also draw the phasor diagram.

[12 marks]



$$I_T = I_L + I_R$$

$$5.8 = 4.3 \angle -\phi + 2.7$$

$$(5.8)^2 = (4.3)^2 + (2.7)^2 + 2 \times 4.3 \times 2.7 \cos \phi$$

$$\phi = 70.21^\circ$$

- (i) Power absorbed in inductive branch

$$P = VI \cos \phi = \underline{\underline{I^2 R}}$$

Given

$$\therefore V_R = V_L = I_R \times 20 = 2.7 \times 20 = \underline{\underline{54 \text{ volt}}}$$

$$P = 54 \times 4.3 \cos (70.21)$$

$$\boxed{P = 78.61 \text{ watt}}$$

(ii) Impedance of Inductive branch

$$= \frac{V}{I} = \frac{54 \angle 0}{4.3 \angle -70.21}$$

$$Z = (4.251 + 11.81 j)$$

Resistance of Inductive branch = $\frac{4.251}{1}$

Inductance of Inductive Branch = $\frac{11.81}{2\pi \times 50}$

$$L = 0.0375 \text{ Henry}$$

(iii) Pf of combined ckt

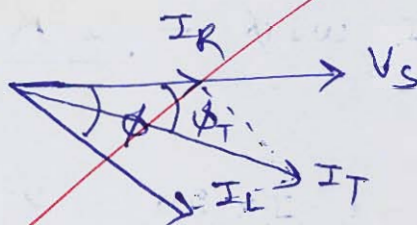
$$= \cos \phi$$

$$= \cos(70.21^\circ) = \underline{\underline{0.333}}$$

$$I_T = I_L + I_R = 4.3 \angle -70.21 + 2.7$$

$$= 5.80 \angle -44.23^\circ$$

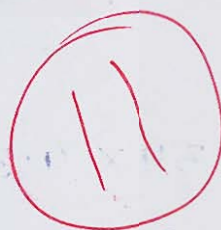
$$\text{Power factor} = \cos(44.23) = \underline{\underline{0.716}}$$



$$\phi_T = 44.23^\circ$$

$$\phi = 70.21^\circ$$

Good
Approach



- Q.1 (c) Determine the percentage of ionic polarizability in sodium chloride crystal which has the optical index of refraction and the static dielectric constant are 1.5 and 5.6 respectively. [12 marks]

- Q.1 (d) An energy meter is designed to have 80 revolutions of the disc per unit of energy consumed. Calculate the number of revolutions made by the disc when measuring the energy consumed by the load carrying 30 A at 230 V and 0.6 power factor. Find the percentage error if the meter actually makes 330 revolutions. Also specify whether the meter runs slower or faster.

[12 marks]

$$K = 80$$

$$P = \frac{VI \cos \phi}{1000} = \frac{230 \times 30 \times 0.6}{1000} \times 1 = 4.14 \text{ kWh}$$

$$\frac{\text{Revolution}}{\text{kWhr}} = K$$

$$\text{Revolution} = 80 \times 4.14$$

$$\boxed{\text{Revolutions} = 331.2}$$

$$\text{Actual Revolution } \boxed{R_a = 330} \text{ given}$$

$$\text{error} = \frac{330 - 331.2}{331.2} \times 100$$

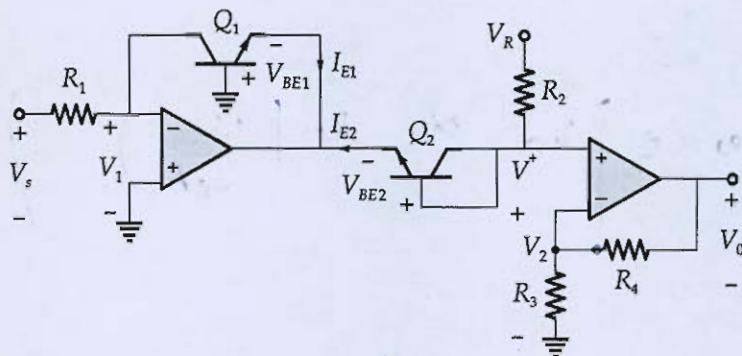
$$= -0.362\%$$

Meter should have made 331.2 revolution but it makes 330 revolution hence it runs slow.

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Good
Approach

- Q.1 (e) The figure shows a modified logarithmic amplifier to overcome the undesirable effects of temperature-dependent V_T and I_S (reverse saturation current). Show that if the two transistors Q_1 and Q_2 are matched transistors, then the output V_0 is truly proportional to $\ln(V_s)$.



$$\frac{V_s}{R_1} = I_{E1} = I_S e^{\left(\frac{V_{BE1}}{nV_T}\right)} \quad \text{--- (1)}$$

[12 marks]

~~$$V_2 = V_0 \times \left(\frac{R_3}{R_3 + R_4}\right) = V^+$$~~

~~$$I_{E2} = I_S e^{\left(\frac{V^+ - V_{BE2}}{nV_T}\right)}$$~~

~~if $I_{E1} = I_{E2}$ since both transistors are matched~~

~~$$\frac{V_s}{R_1} =$$~~

Applying KVL

~~$$+V_{BE1} - V_{BE2} + V^+ = 0$$~~

~~$$V_{BE1} - V_{BE2} + V_0 \frac{R_3}{R_3 + R_4} = 0$$~~

~~$$-nV_T \ln\left(\frac{V_s}{I_S R_1}\right) - V_{BE2} + \frac{V_0 R_3}{R_3 + R_4} = 0$$~~

$$V_o \left(\frac{R_3}{R_4 + R_3} \right) = nV_T \ln \left(\frac{V_s}{I_s R_1} \right) + V_{BE2}$$

Hence $V_o \propto \ln \left(\frac{V_s}{I_s R_1} \right)$

10

- Q.2 (a) (i) The read access times and the hit ratios for different caches in a memory hierarchy are as given below:

Code	Read access time (in nanoseconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

The read access time of main memory is 90 nanoseconds. Assume that the caches use the referred word-first read policy and the write back policy. Assume that all the caches are direct mapped caches. Assume that the dirty bit is always 0 for all the blocks in the caches. In execution of a program, 60% of memory reads are for instruction fetch and 40% are for memory operand fetch. Find the average read access time in nanoseconds.

[10 marks]

- Q.2 (a) (ii) A certain processor uses a fully associative cache of size 16 kB. The cache block size is 16 bytes. Assume that the main memory is byte addressable and uses a 32-bit address. How many bits are required for the Tag and the Index fields respectively in the addresses generated by the processor?

[10 marks]

- Q.2 (b) (i) Find the value of $\int_C \frac{\cos \pi z^2}{(z-2)(z-1)} dz$, where 'C' is $|z| = 3$.

[8 marks]

Q.2 (b) (ii) Solve $(x^2 - yz)\frac{\partial p}{\partial x} + (y^2 - zx)\frac{\partial p}{\partial y} = z^2 - xy$.

[12 marks]

- Q.2 (c) Draw the circuit arrangement for power measurement in a 3-phase, 3-wire balanced supply and load using two-wattmeter method, and show that the power factor of the load is given by

$$\cos \phi = \frac{1}{\sqrt{1 + 3 \left(\frac{P_1 - P_2}{P_1 + P_2} \right)^2}}$$

where P_1 and P_2 are powers indicated by Wattmeter 1 and Wattmeter 2, respectively.

[20 marks]

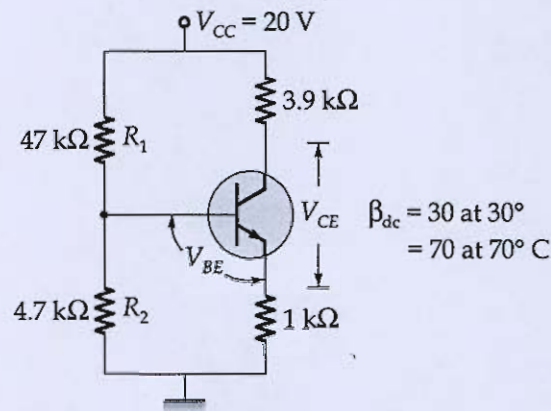
Q.3 (a)

Find the matrix P which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form.

Hence calculate A^4 by using matrix P .

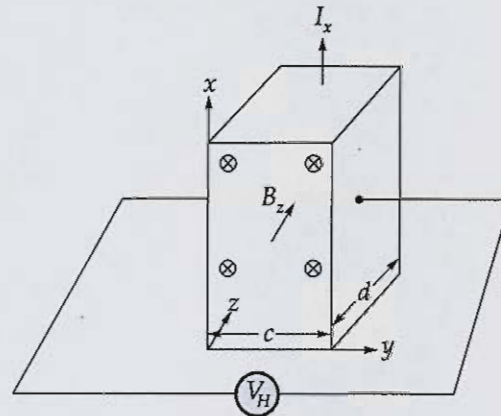
[20 marks]

- Q.3 (b) The transistor shown in figure is a silicon transistor. The junction temperature increases from 30° to 70° . If $\beta = 30$ at 30° and $\beta = 70$ at 70° , determine the percent change in D.C. bias point over the temperature range 30° to 70° neglecting change in base to emitter voltage.



[20 marks]

- Q.3 (c) (i) What is Hall effect? For a parallelepiped specimen having one corner situated at origin and externally applied electric field causing current in positive x -direction as shown below:



State what happens when magnetic field B_z is applied in positive z -direction in reference to Hall voltage. Determine electron mobility relation using Hall coefficient and conductivity (σ).

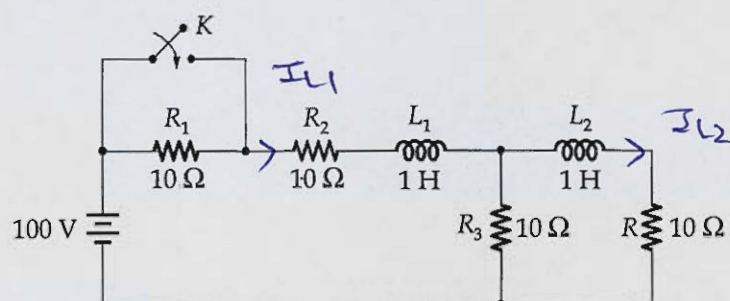
[10 marks]

- Q.3 (c) (ii) What is Meissner effect and how it can be used to justify negative susceptibility of superconductors? How critical field, H_C for a superconductor material varies with temperature? Explain briefly factors that affect transition temperature of superconductor.

[10 marks]

Q.4 (a)

In the network of below figure, the switch K is closed at time $t = 0$, a steady state having previously existed. Obtain the expression of current in the resistor R using Thevenin's theorem.



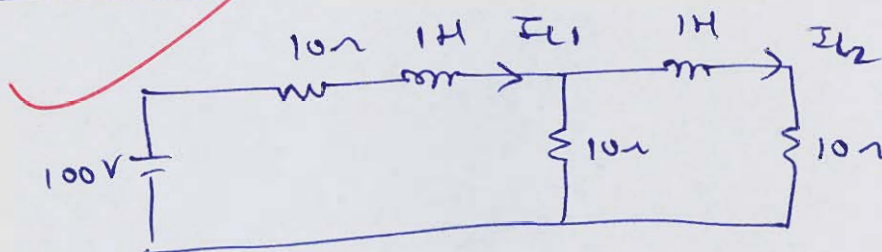
[20 marks]

$$t < 0$$

$$I_1(0^-) = \frac{100}{25} = 4A$$

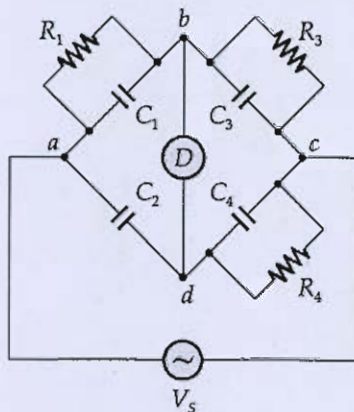
$$I_2(0^-) = 2A$$

$$\text{for } t > 0$$



Not complete
Solution

- Q.4 (b) (i) In a low-voltage bridge designed for the measurement of permittivity, the branch ab consists of two electrodes between which the specimen under test may be inserted; arm bc is a non-reactive resistor R_3 in parallel with a standard capacitor C_3 , arm cd is non-reactive resistor R_4 in parallel with a standard capacitor C_4 ; arm da is a standard air capacitor of capacitance C_2 without the specimen between the electrodes, balance is obtained with the following values :
- $C_3 = C_4 = 120 \text{ pF}$, $C_2 = 150 \text{ pF}$, $R_3 = R_4 = 5000 \Omega$ with the specimen inserted these values become $C_3 = 200 \text{ pF}$, $C_4 = 1000 \text{ pF}$, $C_2 = 900 \text{ pF}$ and $R_3 = R_4 = 5000 \Omega$. In each test $\omega = 5000 \text{ rad/sec}$. Find the relative permittivity of the specimen.



[12 marks]

$$Z_1 = \frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{R_1 j\omega C_1 + 1}$$

$$Z_2 = \frac{1}{j\omega C_2} \quad Z_3 = \frac{R_3}{R_3 j\omega C_3 + 1}$$

$$Z_4 = \frac{R_4}{j\omega C_4 R_4 + 1}$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(\frac{R_1}{R_1 j\omega C_1 + 1} \right) \times \left(\frac{R_4}{j\omega C_4 R_4 + 1} \right) = \frac{1}{j\omega C_2} \times \left(\frac{R_3}{j\omega C_3 R_3 + 1} \right)$$

$$R_1 R_4 (j\omega C_2) [j\omega C_3 R_3 + 1] = R_3 [R_1 j\omega C_1 + 1] [j\omega C_4 R_4 + 1]$$

$$R_1 R_4 [-\omega^2 C_2 C_3 R_3 + j\omega C_2] = R_3 [-\omega^2 R_1 C_1 C_4 R_4 + j\omega (R_1 C_1 + R_4 C_4 + 1)]$$

$$R_1 R_4 W C_2 = R_3 (R_1 C_1 + R_4 C_4)$$

$$\frac{R_1 R_4 W C_2}{R_3} - R_4 C_4 = R_1 C_1 \quad \text{--- ①}$$

3

Incomplete
Solution

- Q.4(b) (ii) A CRT has an anode voltage of 2000 V and parallel deflecting plates 2 cm long and 5 mm apart. The screen is 30 cm from the centre of the plates. Find the input voltage required to deflect the beam through 3 cm. The input voltage is applied to the deflecting plates through amplifiers having an overall gain of 100.

[8 marks]

$$V_a = 2000 \text{ V}$$

$$D = \frac{l \cdot l_d \cdot E_d}{2d \cdot E_a}$$

$l \rightarrow$ length of deflected plate
 $E_a \rightarrow$ anode voltage

$$3 \times 10^{-2} = \frac{2 \times 10^{-2} \times 30 \times 10^{-2} \times E_d}{2 \times 5 \times 10^{-3} \times 2000}$$

$$E_d = \frac{3 \times 10^{-2} \times 2 \times 5 \times 10^{-3} \times 2000}{2 \times 10^{-2} \times 30 \times 10^{-2}}$$

$$E_d = 100 \text{ volt}$$

$$\frac{V_o}{V_{in}} = 100$$

$$V_o = E_d$$

$$\frac{E_d}{V_{in}} = 100$$

$$V_{in} = \frac{E_d}{100}$$

$$V_{in} = \frac{100}{100} = 1 \text{ volt}$$

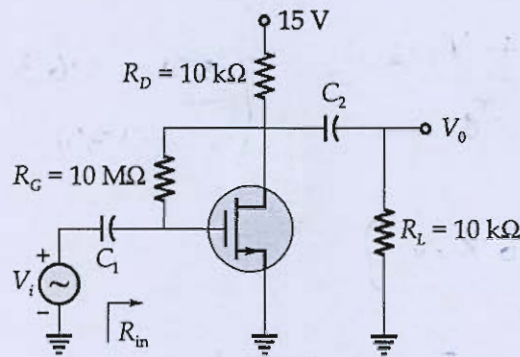
$$V_{in} = 1 \text{ volt}$$

Applied input voltage = 1 volt

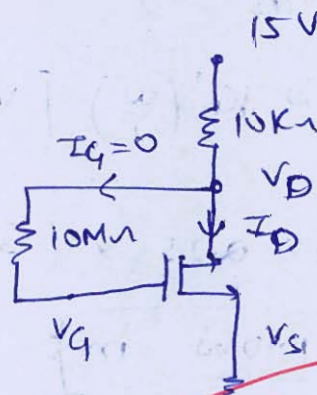
Good
Approach

8

- Q.4 (c) Determine the small-signal voltage gain, its input resistance and the largest allowable input signal. The transistor has $V_t = 1.5$ V, $K_n' \left(\frac{W}{L} \right) = 0.25$ mA/V² and $V_A = 50$ V. Assume the coupling capacitors to be sufficiently large so as they act as short circuits at the signal frequencies of interest.



[20 marks]

DC Analysis

$$V_{GS} = V_D - 0 = V_D$$

$$I_D = \frac{1}{2} \cdot K_n' \left(\frac{W}{L} \right) [V_{GS} - V_t]^2$$

$$I_D = 0.25 [V_{GS} - 1.5]^2$$

$$\frac{15 - V_D}{10} = 0.25 [V_{GS} - 1.5]^2$$

$$\frac{15 - V_{GS}}{10} = 0.25 (V_{GS} - 1.5)^2$$

$$15 - V_{GS} = 2.5 (V_{GS}^2 + 2.25 - 3 V_{GS})$$

$$2.5 V_{GS}^2 + 5.625 - 15 + V_{GS} - 7.5 V_{GS} = 0$$

$$2.5 V_{GS}^2 - 6.5 V_{GS} - 9.375 = 0$$

$$V_{GS} = 3.63 = V_{DS} = 3.63$$

$$I_D = \frac{15 - 3.63}{10} = 1.137 \text{ mA}$$

$$r_{ds} = \frac{V_A + V_{DS}}{I_D} = \left(\frac{50 + 3.63}{1.137} \right) \text{ k}\Omega$$

$$r_{ds} = 47.16 \text{ k}\Omega$$

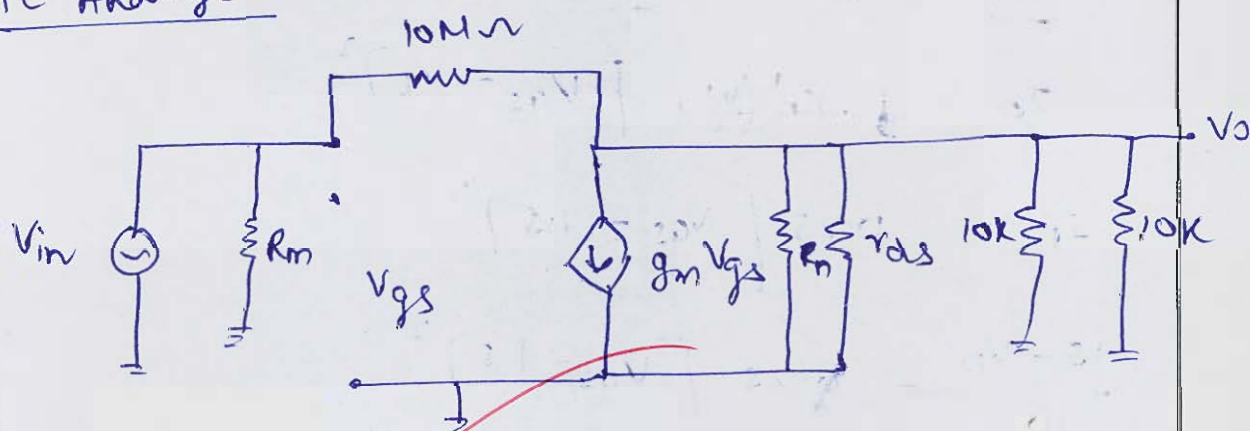
$$I_D = K_n' \left(\frac{W}{L} \right) (V_{GS} - V_T)^2$$

$$\frac{\partial I_D}{\partial V_{GS}} = g = 2 K_n' \left(\frac{W}{L} \right) (V_{GS} - V_T)$$

$$= 2 \times 0.25 \times (3.63 - 1.5)$$

$$g_m = 1.065 \text{ mS}$$

AC Analysis



$$A_v = -g_m (r_{ds} \parallel 10\text{k} \parallel 10\text{k})$$

$$= -1.065 \times (47.16 \parallel 5\text{k})$$

$$A_v = -4.81$$

$$\text{small signal voltage gain} = \underline{\underline{-4.81}}$$

$$R_m = \frac{R_4}{1 - A_v} = \frac{10 \text{ M}\Omega}{1 + 4.81} = \underline{\underline{1.719 \text{ M}\Omega}}$$

$$\text{Input Resistance } R_m = \underline{\underline{1.719 \text{ M}\Omega}}$$

$$V_{DS} = 3.63$$

$$\text{largest output signal} = 3.63 - 1.5 = \underline{\underline{2.13 \text{ volt}}}$$

largest allowable input signal

$$V_{in} = \frac{V_o}{4.81}$$

$$V_{in} = \underline{\underline{0.442 \text{ volt}}}$$

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Section-B

- Q.5 (a) Consider a 2-way set associative cache memory with 4 sets and total 8 cache blocks (0-7) and a main memory with 128 blocks (0-127). What memory blocks will be present in the cache after the following sequence of memory block references if LRU policy is used for cache block replacement? Assuming that initially the cache did not have any memory block from the current job.

0 5 3 9 7 0 16 55

[12 marks]

Q.5 (b) Obtain the partial differential equation from function $f(xy + z, x^2 + y^2 - z^2) = 0$.

[12 marks]

$$xy + z = \phi(x^2 + y^2 - z^2)$$

$$z = \phi(x^2 + y^2 - z^2) - xy \quad \text{--- (1)}$$

$$\frac{\partial z}{\partial x} = (2x - 2z \frac{\partial z}{\partial x}) \phi'(x^2 + y^2 - z^2) - y$$

$$\text{let } \frac{\partial z}{\partial x} = p$$

$$p = (2x - 2zp) \phi'(x^2 + y^2 - z^2) - y$$

$$\frac{p+y}{(2x-2zp)} = \phi'(x^2 + y^2 - z^2) \quad \text{--- (2)}$$

differentiating eqⁿ (1) w.r.t y

$$\frac{\partial z}{\partial y} = (2y - 2z \frac{\partial z}{\partial y}) \phi'(x^2 + y^2 - z^2) - x$$

$$\text{let } \frac{\partial z}{\partial y} = q$$

$$q + x = (2y - 2zq) \phi'(x^2 + y^2 - z^2) \quad \text{---}$$

$$\frac{q+x}{(2y-2zq)} = \phi'(x^2 + y^2 - z^2) \quad \text{--- (3)}$$

$$\text{eqⁿ (2) } \div \text{eqⁿ (3)}$$

$$\frac{p+y}{2x-2zp} = \frac{0+x}{2y-2z0}$$

$$(p+y)(y-z0) = (0+x)(x-zp)$$

$$py - \cancel{z0p} + y^2 - \cancel{z0y} = 0x - \cancel{zp0} + x^2 - zp x$$

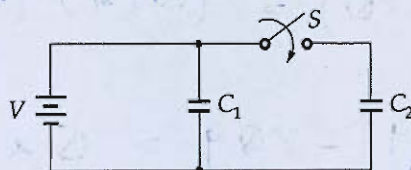
$$p(y+zx) + y^2 = x^2 + 0(x+zy)$$

$$\left[\frac{\partial z}{\partial x} \mid y+zx \right] + y^2 = x^2 + \frac{\partial z}{\partial y} (x+zy)$$

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Good
Approach

- Q.5 (c) Figure below shows the two identical parallel-plate capacitors connected to a battery with switch S closed. The switch S is opened and the free space between the plates of the capacitor is filled with a dielectric of dielectric coefficient $K = 2$. Find the ratio of the electrostatic energy stored in capacitor C_2 to electrostatic energy in capacitor C_1 after the introduction of the dielectric.



[12 marks]

suppose initial capacitance
 $C_1 = C_2 = C$

After capacitor is filled with dielectric

$$C_1' = C_2' = KC$$

Initially voltage across C_1 & $C_2 = V$

$$\text{net charge stored } Q_1 = Q_2 = CV$$

After switch is open

$$V_1' = V$$

$$Q_2' = Q_2 \rightarrow (\text{since charge should be same})$$

$$C_2' V_2' = CV$$

$$V_2' = \frac{CV}{KC}$$

$$V_2' = \frac{V}{K}$$

$$E_1 = \frac{1}{2} C_1' V_1'^2$$

$$= \frac{1}{2} (KC) \times V^2$$

$$E_1 \Rightarrow \frac{KCV^2}{2} \quad \text{--- (1)}$$

$$E_2 = \frac{1}{2} C_2' V_2'^2$$

$$= \frac{1}{2} (KC) \times \left(\frac{V}{K}\right)^2$$

$$= \frac{1}{2} \frac{CV^2}{K} \quad \text{--- (2)}$$

$$\frac{E_2}{E_1} = \frac{\frac{1}{2} \frac{CV^2}{K}}{\frac{K \frac{CV^2}{2}}{2}}$$

$$\boxed{\frac{E_2}{E_1} \Rightarrow \frac{1}{K^2}}$$

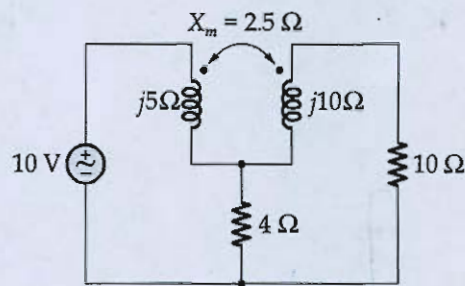
Hence ratio of electrostatic energy

$$\boxed{\frac{E_2}{E_1} = \frac{1}{K^2}}$$

④

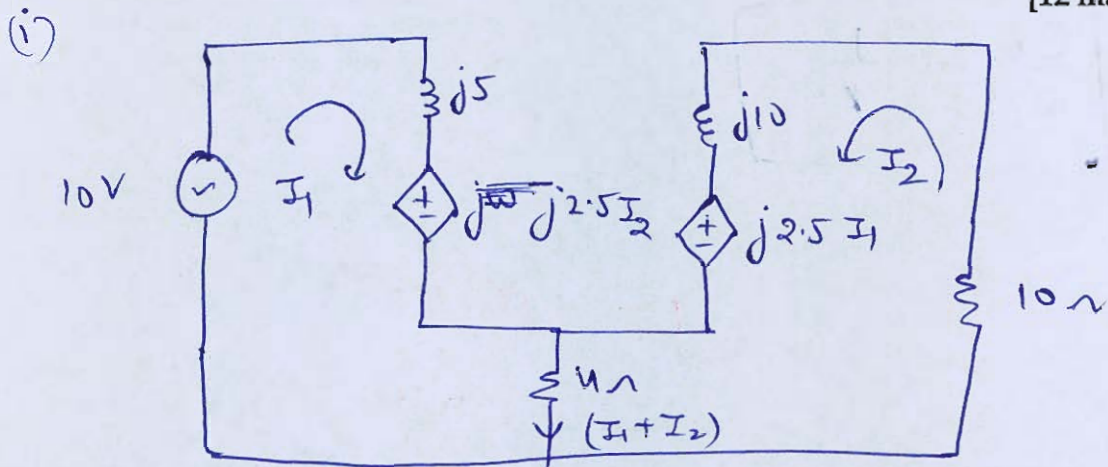
Incomplete
Solution

Q.5 (d) Figure below shows a network with mutual coupling.



- (i) Find the current in the $10\ \Omega$ resistance. Assume that inductor have negligible resistance.
 (ii) If the direction of winding of one of the coils is reversed, find the current in the $10\ \Omega$ resistance.

[12 marks]



In loop -1

$$-10 + j5 I_1 + j2.5 I_2 + 4(I_1 + I_2) = 0$$

$$(4 + j5) I_1 + (4 + j2.5) I_2 = 10 \quad \text{--- (1)}$$

In loop -2

$$+10 I_2 + j10 I_2 + j2.5 I_1 + 4(I_1 + I_2) = 0$$

$$(14 + j10) I_2 + (4 + j2.5) I_1 = 0 \quad \text{--- (2)}$$

$$I_1 = \frac{-(14 + j10) I_2}{(4 + j2.5)} \quad \text{--- (3)}$$

Putting in eq (1)

$$(4+j5) \left[\frac{-(14+j10)}{(4+j2.5)} I_2 \right] + (4+j2.5) I_2 = 10$$

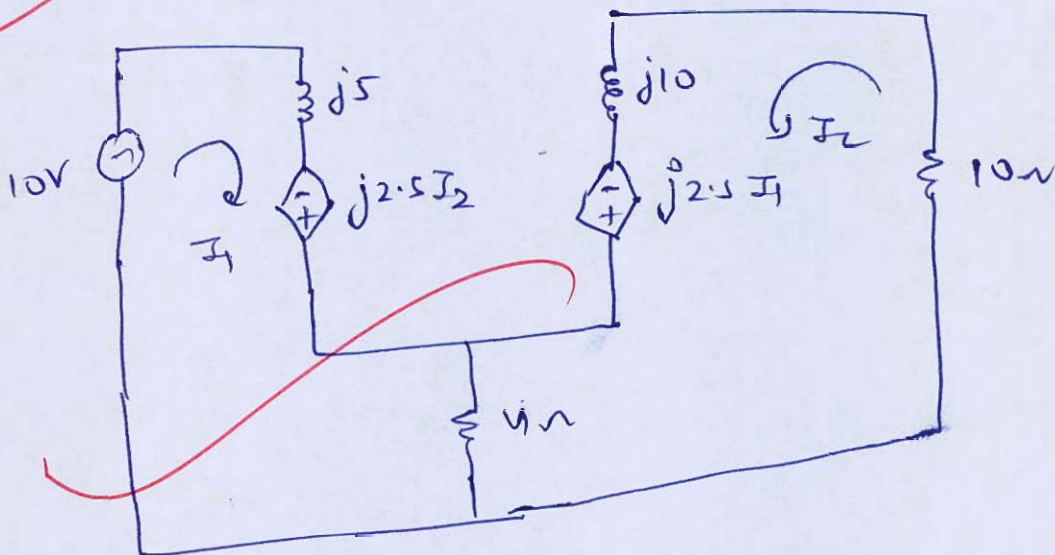
$$I_2 \left[(4+j2.5) - \frac{(4+j5)(14+j10)}{(4+j2.5)} \right] = 10$$

$$I_2 \left[(4+j2.5) - (13.438 + 19.101i) \right] = 10$$

$$I_2 = 0.523 \angle 119.61^\circ$$

current in 10Ω resistor.

(ii)



$$-10 + j5I_1 - j2.5I_2 + 4(I_1 + I_2) = 0$$

$$(4+j5)I_1 + (4-j2.5)I_2 = 10 \quad \text{--- (1)}$$

$$10I_2 + j10I_2 - j2.5I_1 + 4(I_1 + I_2) = 0$$

$$(14+j10)I_2 + (4-j2.5)I_1 = 0$$

$$I_1 = \frac{-(14+j10)I_2}{(4-j2.5)}$$

$$\left[- \frac{(4+j5)(14+j10)}{(4-j2.5)} + (4-j2.5) \right] \bar{I}_2 = 10$$

$$\bar{I}_2 = 0.3626 \angle 56.34^\circ$$

11

Good
Approach

- 2.5 (e) Consider a common-emitter circuit using a BJT having $I_s = 10^{-15}$ A, a collector resistance $R_C = 6.8 \text{ k}\Omega$ and a power supply $V_{CC} = 10 \text{ V}$ and $V_{CE} = 3.2 \text{ V}$.
- (i) Find the positive increment in V_{BE} (above V_{BE}) that drives the transistor to the edge of saturation, where $V_{CE(\text{sat})} = 0.3 \text{ V}$.
- (ii) Find the negative increment in V_{BE} that drives the transistor to within 1% of cut-off (i.e. to $V_0 = 0.99 V_{CC}$).
- [Take $V_T = 25 \text{ mV}$]
- [12 marks]

- 2.6 (a) (i) Compare RISC and CISC architecture.
 (ii) State and explain the instruction and data stream types based on Flynn's classification.
 (iii) Consider the process table with time quantum '4'.

P_{id}	Arrival time	Burst time
1	2	5
2	4	3
3	1	6
4	2	2
5	3	7

What is the average TAT and average WT using Round Robin scheduling?

[4 + 8 + 8 marks]

(i)

RISC

CISC

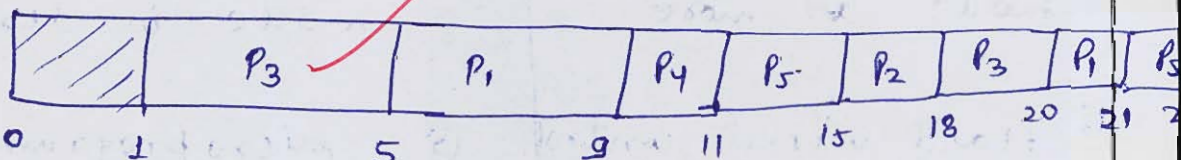
- | | |
|---|--|
| <p>① Number of instruction set is less</p> <p>② No. of program addressing mode is more</p> <p>③ Hard wired control unit is used.</p> <p>④ Used in super computers.</p> <p>⑤ They are costly</p> <p>⑦ Design is complex</p> | <p>① No. of instruction set is more.</p> <p>② No. of addressing mode is less.</p> <p>③ microprogrammed control unit is used.</p> <p>⑤ Used in general purpose computer.</p> <p>⑥ comparatively cheap.</p> <p>⑦ Design is comparatively less complex.</p> |
|---|--|

Pid	Arrival Time	Burst Time	Completion Time	TAT	WT
1	2	5	21	19	14
2	4	3	18	14	11
3	1	6	20	19	13
4	2	2	11	9	7
5	3	7	24	21	14

$$TAT = \text{Completion Time} - \text{Arrival time}$$

$$WT = TAT - BT$$

Gantt chart : time quantum : 4



$P_3: 6$ $P_1: 5$ $P_4: 2$ $P_5: 7$ $P_2: 3$ $P_3: 2$ $P_1: 1$
 $P_4: 2$ $P_5: 7$ $P_2: 3$ $P_3: 2$ $P_1: 1$ $P_5: 3$
 $P_5: 7$ $P_2: 3$ $P_3: 2$ $P_1: 1$ $P_5: 3$
 $P_2: 3$ $P_3: 2$ $P_1: 1$ $P_5: 3$
 $P_3: 2$ $P_1: 1$

$$\text{Avg. TAT} = \frac{19 + 14 + 19 + 9 + 21}{5}$$

$$\boxed{\text{Avg. TAT} = 16.4 \text{ sec}}$$

Avg. waiting time =
$$\frac{14 + 11 + 13 + 7 + 14}{5}$$

Avg. waiting time =
$$\underline{11.8 \text{ sec}}$$

✓ (18)

Good Approach

Q.6 (b) Find the Fourier series of $f(x)$,

$$f(x) = \begin{cases} \pi x & 0 \leq x \leq 1 \\ \pi(1-x) & 1 \leq x \leq 2 \end{cases}$$

Also find the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

[20 marks]

$$a_0 = \frac{1}{T} \int_0^T f(x) dx$$

$$= \frac{1}{2} \int_0^2 f(x) dx = \frac{1}{2} \left[\int_0^1 \pi x dx + \int_1^2 \pi(1-x) dx \right]$$

$$= \frac{1}{2} \left[\pi \frac{x^2}{2} \Big|_0^1 + \left(\pi x - \pi \frac{x^2}{2} \right) \Big|_1^2 \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{2} + \pi(2-1) - \pi \left(\frac{4-1}{2} \right) \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{2} + \pi - \pi \frac{3}{2} \right]$$

$$= \frac{1}{2} \left[\frac{3\pi}{2} - \frac{3\pi}{2} \right] = 0$$

$$\boxed{a_0 = 0}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2}$$

$$\boxed{\omega = \pi}$$

$$a_n = \frac{2}{T} \int_0^T f(x) \cos n\omega x dx$$

$$= \frac{2}{2} \left[\int_0^1 \pi x \cos n\pi x dx + \int_1^2 \pi(1-x) \cos n\pi x dx \right]$$

$$= \left[\pi \left(x \frac{\sin n\pi x}{n\pi} - \int \frac{\sin n\pi x}{n\pi} dx \right) + \int_1^2 \pi \cos n\pi x dx \right]$$

$$= \left[\pi \left(x \frac{\sin n\pi x}{n\pi} - \int \frac{\sin n\pi x}{n\pi} dx \right) + \int_1^2 \pi \cos n\pi x dx \right]$$

$$= \left[\pi \left(x \frac{\sin n\pi x}{n\pi} - \int \frac{\sin n\pi x}{n\pi} dx \right) + \int_1^2 \pi \cos n\pi x dx \right]$$

$$\pi \left[\frac{x \sin n\pi x}{n\pi} + \frac{\cos n\pi x}{n^2 \pi^2} \right] \Big|_0^1 + \left[-\frac{\pi \sin n\pi x}{n\pi} \right]_1^2$$

$$a_n \Rightarrow \pi \left[\frac{\sin n\pi}{n\pi} + \frac{\cos n\pi}{n^2 \pi^2} - \frac{1}{n^2 \pi^2} \right] + \pi \frac{\sin 2n\pi - \sin n\pi}{n\pi}$$

$$- \pi \left[\left(\frac{2 \sin 2n\pi}{n\pi} - \frac{\sin n\pi}{n\pi} \right) + \frac{\cos 2n\pi - \cos n\pi}{n^2 \pi^2} \right]$$

$$\Rightarrow \pi \left[\frac{(-1)^n - 1}{n^2 \pi^2} \right] - \pi \left[\frac{1 - (-1)^n}{n^2 \pi^2} \right]$$

$$a_n \Rightarrow 2\pi \left[\frac{(-1)^n - 1}{n^2 \pi^2} \right] \Rightarrow a_n = \frac{2[(-1)^n - 1]}{n^2 \pi}$$

for $n = 1, 3, 5$

$$a_n = \frac{-4}{n^2 \pi}$$

$$\begin{aligned} b_n &= \frac{2}{T} \int_0^T f(x) \sin n\pi x \, dx = \frac{2}{2} \int_0^2 f(x) \sin n\pi x \, dx \\ &= \int_0^1 \pi x \sin n\pi x \, dx + \int_1^2 \pi \sin n\pi x \, dx - \int_1^2 \pi x \sin n\pi x \, dx \\ &= \pi \left[-\frac{x \cos n\pi x}{n\pi} + \frac{\sin n\pi x}{n^2 \pi^2} \right]_0^1 + \left[-\frac{\pi \cos n\pi x}{n\pi} \right]_1^2 - \pi \left[-\frac{x \cos n\pi x}{n\pi} + \frac{\sin n\pi x}{n^2 \pi^2} \right]_1^2 \end{aligned}$$

$$-\frac{\cos n\pi}{n} - \left[\frac{\cos 2n\pi - \cos n\pi}{n\pi} \right] + \frac{2\cos 2n\pi}{n\pi}$$

$$\Rightarrow -\frac{\cos n\pi}{n} - \frac{\cos 2n\pi}{n} + \frac{\cos n\pi}{n} + \frac{2\cos 2n\pi}{n}$$

$$b_n = \left(\frac{\cos 2n\pi}{n} - \frac{\cos n\pi}{n} \right)$$

$$\left(\frac{1}{n} - \frac{(-1)^n}{n} \right)$$

for $n=1, 3, 5$

$$b_n = \frac{2}{n}$$

$$f(x) = a_0 + \sum_{n=1,2,3}^{\infty} a_n \cos n\pi x + \sum_{n=1,3,5}^{\infty} b_n \sin n\pi x$$

$$= 0 + \sum_{n=1,2,3}^{\infty} \frac{-4}{n^2\pi} \cos n\pi x + \sum_{n=1,3,5}^{\infty} \frac{2}{n} \sin n\pi x$$

for $n=1, 3, 5$ for $x=1$

$$\pi = \frac{-4}{\pi} \cos \pi x - \frac{4}{3^2\pi} \cos 3\pi x + \frac{4}{5^2\pi} \cos 5\pi x + \dots$$

$$\pi = -\frac{4}{\pi} (-1) - \frac{4}{3^2\pi} (-1) - \frac{4}{5^2\pi} (-1) + \dots$$

$$\pi = \frac{4}{\pi} + \frac{4}{3^2\pi} + \frac{4}{5^2\pi} + \dots$$

$$\boxed{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{4}}$$

Good
Approach

18

- 6 (c) (i) The number of atoms in a volume of one cubic metre of hydrogen gas is 9.8×10^{26} . The radius of hydrogen atom is 0.53 \AA . Calculate the polarizability and relative permittivity of the hydrogen gas.
- (ii) The magnetic field in a piece of copper and another piece of Fe_2O_3 is 10^6 Am^{-1} . Their magnetic susceptibilities are -0.5×10^{-5} and 1.4×10^{-3} respectively. Compare the flux density and magnetization in the two pieces.

[10 + 10 marks]

(ii) given $H = 10^6 \text{ A/m}$

$$\chi_1 = -0.5 \times 10^{-5} \quad \chi_2 = 1.4 \times 10^{-3}$$

$$B_1 = \mu_0 \mu_r H$$

$$= \mu_0 (1 + \chi_1) H$$

$$= 4\pi \times 10^{-7} (1 - 0.5 \times 10^{-5}) \times 10^6$$

$$= 4\pi \times 10^{-1} (1 - 0.5 \times 10^{-5})$$

$$= 4\pi \times 10^{-1}$$

$$B_2 = \mu_0 \mu_r H$$

$$B_2 = \mu_0 (1 + \chi_2) H$$

$$\frac{B_1}{B_2} = \frac{1 + \chi_1}{1 + \chi_2} = \frac{1 - 0.5 \times 10^{-5}}{1 + 1.4 \times 10^{-3}}$$

$$= \frac{(10^5 - 0.5) \times 10^3}{10^3 + 1.4} \times 10^3$$

$$\boxed{\frac{B_1}{B_2} \approx \frac{100}{100} = 1}$$

$$M = \chi H$$

$$\frac{M_1}{M_2} = \frac{\chi_1}{\chi_2} = \frac{-0.5 \times 10^{-5}}{1.4 \times 10^{-3}} = -0.35 \times 10^{-2}$$

$$\frac{M_1}{M_2} = -0.35 \times 10^{-2}$$

~~$$\frac{M_2}{M_1} = -285.71$$~~

flux density will be same but
magnetisation will vary.

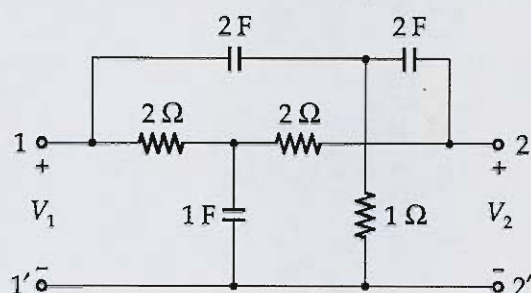


$$\begin{aligned} \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) &= \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) \\ \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) &= \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) \end{aligned}$$

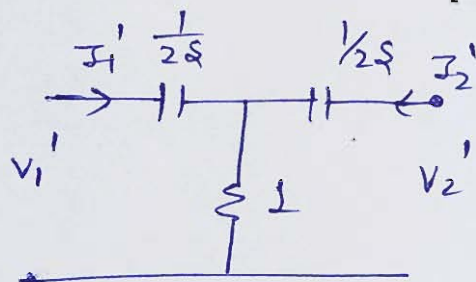
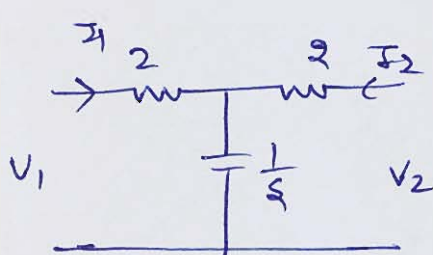
$$\begin{aligned} \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) &= \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) \\ \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) &= \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) &= \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) \\ \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) &= \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) \end{aligned}$$

Q.7 (a) For the notch-filter network, determine the y -parameters.



[20 marks]



$$I_1 =$$

$$V_1 = 2I_1 + \frac{1}{s}(I_1 + I_2)$$

$$V_1 = \left(2 + \frac{1}{s}\right)I_1 + \frac{1}{s}I_2$$

$$V_2 = 2I_2 + \frac{1}{s}(I_1 + I_2)$$

$$V_2 = \frac{1}{s}I_1 + \left(2 + \frac{1}{s}\right)I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 + \frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & 2 + \frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2s} + 1\right) & 1 \\ 1 & \left(\frac{1}{2s} + 1\right) \end{bmatrix} \begin{bmatrix} I_1' \\ I_2' \end{bmatrix}$$

$$Y_1 = \frac{\begin{bmatrix} 2 + \frac{1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & 2 + \frac{1}{s} \end{bmatrix}}{\left(2 + \frac{1}{s}\right)^2 - \frac{1}{s^2}}$$

$$Y_2 = \frac{\begin{bmatrix} \frac{1}{2s} + 1 & -1 \\ -1 & \left(\frac{1}{2s} + 1\right) \end{bmatrix}}{\left(\frac{1}{2s} + 1\right)^2 - 1}$$

$$Y_1 = \begin{bmatrix} \frac{2 + \frac{1}{s}}{4(1 + \frac{1}{s})} & \frac{-\frac{1}{s}}{4(1 + \frac{1}{s})} \\ \frac{-\frac{1}{s}}{4(1 + \frac{1}{s})} & \frac{2 + \frac{1}{s}}{4(1 + \frac{1}{s})} \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} \frac{(\frac{1}{2s} + 1)}{\frac{1}{s}(\frac{1}{4s} + 1)} & \frac{-1}{\frac{1}{s}(\frac{1}{4s} + 1)} \\ \frac{-1}{\frac{1}{s}(\frac{1}{4s} + 1)} & \frac{(\frac{1}{2s} + 1)}{\frac{1}{s}(\frac{1}{4s} + 1)} \end{bmatrix}$$

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$$Y_T = Y_1 + Y_2$$

Elaborate it more

$$Y_T = \begin{bmatrix} \frac{2 + \frac{1}{s}}{4(1 + \frac{1}{s})} + \frac{(\frac{1}{2s} + 1)}{\frac{1}{s}(\frac{1}{4s} + 1)} & \frac{-\frac{1}{s}}{4(1 + \frac{1}{s})} - \frac{1}{\frac{1}{s}(\frac{1}{4s} + 1)} \\ \frac{-\frac{1}{s}}{4(1 + \frac{1}{s})} - \frac{1}{\frac{1}{s}(\frac{1}{4s} + 1)} & \frac{2 + \frac{1}{s}}{4(1 + \frac{1}{s})} + \frac{(\frac{1}{2s} + 1)}{\frac{1}{s}(\frac{1}{4s} + 1)} \end{bmatrix}$$

$$= \frac{(2s+1)(8s^2+12s+1)}{4(4s+1)(s+1)}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- 7 (b) (i) Find the resistivity of
1. intrinsic Silicon and
 2. p -type silicon with $N_A = 10^{16}/\text{cm}^3$.
- Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$ and assume that intrinsic Silicon $\mu_n = 1350 \text{ cm}^2/\text{V-s}$ and $\mu_p = 480 \text{ cm}^2/\text{V-s}$ and for doped Silicon $\mu_n = 1110 \text{ cm}^2/\text{V-s}$ and $\mu_p = 400 \text{ cm}^2/\text{V-s}$ and comment on result.
- (Note that doping results in reduced carrier mobilities).

[10 marks]

(i) for intrinsic silicon
conductivity

$$\sigma = (n e \mu_n + p e \mu_p)$$

$$np = n_i^2$$

for intrinsic $n = p = n_i$

$$\sigma = n_i (\mu_n + \mu_p) \times e$$

$$= 1.5 \times 10^{10} (1350 + 480) \times 1.6 \times 10^{-19} \frac{1}{\text{cm}}$$

$$\sigma = 4392 \times 10^{-9} \frac{1}{\text{cm}}$$

$$\text{Resistivity} = \frac{1}{\sigma}$$

$$= \frac{1}{4392 \times 10^{-9}} \text{ cm}$$

$$\rho = 2.276 \times 10^5 \times 10^{-2} \text{ m}$$

$$\boxed{\rho = 2.276 \times 10^3 \text{ m}}$$

(11) for p type silicon

$$p + N_D = n + N_A$$

$$p = N_A$$

$$p = \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{10^{16}}$$

$$n = \frac{2.25 \times 10^{20}}{10^{16}} = \frac{2.25 \times 10^4}{1}$$

$$\sigma = ne\mu_n + pe\mu_p$$

$$= pe\mu_p$$

(conduction due to holes)

$$\sigma = 2.25 \times 10^4 \times 1.6 \times 10^{19} \times 480 \frac{1}{\text{cm}}$$

$$= 1728 \times 10^{-15} \frac{1}{\text{cm}}$$

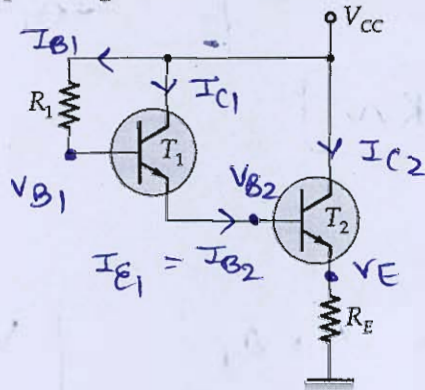
$$\text{Resistivity } \rho = \frac{1}{\sigma} = 5.7870 \times 10^{15} \text{ cm}$$

$$\rho = 5.7870 \times 10^{15} \times 10^{-2} \text{ m}$$

$$\boxed{\rho = 5.78 \times 10^8 \text{ m}}$$

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- 7 (b) (ii) For the transistor pair circuit is shown in below figure. Both the transistors have dc current gain β of 30. In the circuit $V_{CC} = +12V$, $R_E = 1.5 k\Omega$.
1. Find the same value of R_1 needed to bias the circuit such that $V_{CEQ2} = 5 V$ for transistor T_2 .
 2. With the value of R_1 as obtained above, determine the value of V_{CEQ1} .
(Assume both T_1 and T_2 are Si transistors)



[10 marks]

(i) $\beta = 30$

$V_{CEQ2} = 5 V$

$V_E = V_{CC} - V_{CEQ2} = 12 - 5$

$V_E = 7 \text{ volt}$

$I_{E2} = \frac{V_E}{R_E} = \frac{7}{1.5 K} = 4.66 \text{ mA}$

$I_{E2} = 4.66 \text{ mA}$

$V_{B1} = V_E + 0.7 + 0.7 = 8.4 \text{ volt}$

$I_{B2} + I_{C2} = I_{E2}$

$I_{B2} + \beta I_{B2} = I_{E2}$

$I_{B2} = \frac{I_{E2}}{(1+\beta)} = \frac{4.66 \text{ mA}}{31} = 0.150 \text{ mA}$

$I_{B1} + I_{C1} = (I_{E1} = I_{B2}) \Rightarrow I_{B1} = \frac{I_{E1}}{(1+\beta)}$

$$I_{B1} = \frac{0.150 \text{ mA}}{31} = \underline{4.849 \text{ }\mu\text{A}}$$

$$I_{B1} = \frac{V_{CC} - V_{B1}}{R_1} = \frac{12 - 8.4}{I_{B1}} = R_1$$

$$R_1 = 742.40 \text{ K}\Omega$$

(ii)

$$V_{CC} - V_{B2} = V_{CEQ1}$$

$$12 - V_{B2} = V_{CEQ1}$$

$$12 - 7.7 = V_{CEQ1}$$

$$V_{CEQ1} = 4.3 \text{ Volt}$$

$$\begin{aligned} V_{B2} &= V_E + 0.7 \\ V_{B2} &= 7.7 \text{ Volt} \end{aligned}$$

9

Good
Approach

- 7 (c) (i) Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$. Also calculate the magnitude of the maximum directional derivative.

(ii) What is phantom loading? What is the advantage of it?

[14 + 6 marks]

$$(i) \nabla f = \left(\frac{\partial f}{\partial x}\right)\hat{i} + \left(\frac{\partial f}{\partial y}\right)\hat{j} + \left(\frac{\partial f}{\partial z}\right)\hat{k}$$

$$= (2x\hat{i} - 2y\hat{j} + 4z\hat{k})$$

at point $(1, 2, 3)$

$$\Delta f = (2\hat{i} - 4\hat{j} + 12\hat{k})$$

$$\vec{PQ} = \vec{Q} - \vec{P}$$

$$= (5, 0, 4) - (1, 2, 3)$$

$$\vec{PQ} = (4\hat{i} - 2\hat{j} + \hat{k})$$

$$\hat{PQ} = \frac{\vec{PQ}}{|\vec{PQ}|} = \frac{4\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{16 + 4 + 1}}$$

$$\hat{PQ} = \left(\frac{4\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{21}} \right)$$

Directional Derivative in dirⁿ of PQ

$$\Rightarrow [\nabla f \cdot (\hat{PQ})] \hat{PQ}$$

$$\Rightarrow (8 + 8 + 12) \left(\frac{4\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{21}} \right)$$

$$\Rightarrow \frac{28}{\sqrt{21}} (4\hat{i} - 2\hat{j} + \hat{k}) \Rightarrow (24.44\hat{i} - 12.22\hat{j} + 6.11\hat{k})$$

Directional Derivative in the dirⁿ of \vec{PQ}

$$= (24.44\hat{i} - 12.22\hat{j} + 6.11\hat{k})$$

Max Directional Derivative

$$= \left| \sqrt{(24.44)^2 + (12.22)^2 + (6.11)^2} \right|$$

$$= \left| \sqrt{783.97} \right|$$

Max	= 27.99
D.D	

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(ii) In ~~watt~~ energy meter during testing we do not provide rated voltage & current instead we provide reduced voltage & current in order to overcome losses.

This arrangement is known as phantom loading.

Advantage:

- ① Reduced wastage of power
- ② Reduce I^2R loss in current coil.

Q.8 (a) (i) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. Also

express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A .

(ii) Find the area of the tangent cut-off from the parabola $x^2 = 8y$ by the line $x - 2y - 8 = 0$.

[10 + 10 marks]

- Q.8 (b) $\vec{F}(x, y, z) = yz\hat{i} - xz\hat{j} + \hat{k}$. Let 's' be the portion of surface of the paraboloid $z = 4 - x^2 - y^2$ which lies above the first octant, and let 'C' be closed curve $C = C_1 + C_2 + C_3$, where curves C_1 , C_2 and C_3 are the three curves formed by intersecting 's' with the xy , yz and xz planes respectively so that C is boundary of 's'. Orient C so that it is traversed CCW when seen from above the first octant.

(i) Set up and evaluate the loop integral $\oint_C \vec{F} \cdot d\vec{r}$ by parameterizing each piece of curve C.

(ii) Verify using Stoke's theorem that loop integral $\oint_C \vec{F} \cdot d\vec{r}$ is equal to surface integral

$$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{s}.$$

[20 marks]

- (c) (i) The electrical conductivity and electron mobility of aluminium are $3.8 \times 10^7 (\Omega\text{-m})^{-1}$ and $0.0012 \text{ m}^2/\text{V-s}$, respectively. Calculate the Hall voltage for an aluminium specimen that is 15 mm thick for a current of 25 A and a magnetic field of 0.6 T. Given that field is perpendicular to current.
- (ii) Briefly explain why the ferroelectric behavior of BaTiO_3 ceases above its ferroelectric curie temperature.
- (iii) Name the types of polarization and briefly explain the type of materials and about mechanism by which dipolar are induced or oriented by the action of an applied electric field. For gaseous-argon, solid LiF, liquid H_2O , what kind(s) of polarization is/are possible?

[6 + 4 + 10 marks]

Space for Rough Work

[Faint handwritten notes and calculations are visible throughout the page, including various mathematical expressions and numbers.]

Space for Rough Work

$$\frac{2+\frac{1}{s}}{4(1+\frac{1}{s})} + \frac{\frac{1}{2s}}{\frac{1}{s}(\frac{1}{4s}+1)}$$

$$\frac{-1}{4(s+1)} \left[\frac{1}{4s^2} + \frac{1}{s} \right]$$

$$\frac{2s+1}{4(s+1)} \quad \frac{1}{4s^2} + \frac{1}{s} + \frac{1}{s} - \frac{1}{s}$$

$$\frac{1}{s} \left(\frac{1}{4s} + 1 \right)$$

$$\frac{-1}{4(s+1)} - \frac{4s^2}{(1+4s)} = \frac{-1-4s-(4s^2+4s)}{4(s+1)(4s+1)}$$

$$+ \frac{1+2s}{\frac{1}{s}(2s+\frac{1}{2})}$$

$$\frac{(1+2s)2s}{(4s+1)} + \frac{2s+1}{4(s+1)} \int \pi x \sin n\pi x dx$$

$$(2s+1) \left[\frac{2s}{4s+1} + \frac{1}{4(s+1)} \right] \int \frac{x(-\cos n\pi x)}{n\pi} - \int \frac{-\cos n\pi x}{n\pi} dx + \frac{\sin n\pi x}{n^2 \pi^2}$$

$$\frac{2s \times 4(s+1) + 1 \times (s+1)}{4(4s+1)(s+1)} \left[\frac{x \cos n\pi x}{n\pi} \right] - \left[\frac{-\cos n\pi x}{n\pi} + \frac{\sin n\pi x}{n^2 \pi^2} \right]$$

$$\pi \left[\frac{-2 \cos 2n\pi}{n\pi} + \frac{\sin 2n\pi}{n^2 \pi^2} + \frac{(-1)^n}{n} \right]$$

$$4 + \frac{1}{s} + \frac{1}{s}$$

$$\frac{(2s+1)(8s^2+8s+4s+1)}{4(4s+1)(s+1)} \left(\frac{-\pi \cos n\pi}{n\pi} \right)$$

for $n=1, 3, 5$

$$\pi \left[\frac{-2}{n\pi} + \frac{(-1)^n}{n} \right]$$