• Try to attempt all five question

• Try to avoid calculation mistake

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Leading Institute for ESE, GATE & PSUs

ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-7 : Full Syllabus Test (Paper-I)

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	36
Q.2	40
Q.3	50
Q.4	
Section-B	
Q.5	28
Q.6	
Q.7	37
Q.8	
Total Marks Obtained	191

Signature of Evaluator: Sourabh kumar

Cross Checked by: ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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Section-A

.1 (a) Find the complete solution of differential equation $(D^2 - 4D + 3)y = \sin 3x \cos 2x$.

[12 marks]

$(D^2 - 4D + 3)\, y = \sin 3x \cos 2x$

Complementary function

$m^2 - 4m + 3 = 0$

$\boxed{m = 1, 3}$

$y_{cf} = c_1 e^{x} + c_2 e^{3x} \rightarrow (1)$

Particular Integral

$$PI = \frac{1}{D^2 - 4D + 3} \sin 3x \cos 2x$$

$$= \frac{1}{2(D^2 - 4D + 3)} (\sin 5x + \sin x)$$

$(D^2 = -a^2)$

$$= \frac{1}{2(D^2 - 4D + 3)} \sin 5x + \frac{1}{2(D^2 - 4D + 3)} \sin x$$

$$= \frac{1}{2(-25 - 4D + 3)} \sin 5x + \frac{1}{2(-1 - 4D + 3)} \sin x$$

$$= \frac{1}{-4(2D + 11)} \sin 5x - \frac{1}{4(2D - 1)} \sin x$$

$$= -\frac{1}{4}\left[\frac{1}{2D + 11} \sin 5x + \frac{1}{2D - 1} \sin x\right]$$

$$= -\frac{1}{4}\left[\frac{2D - 11}{4D^2 - 121} \sin 5x + \frac{2D + 1}{4D^2 - 1} \sin x\right]$$

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$$= -\frac{1}{4}\left[\frac{2D-11}{-221}\sin 5x + \frac{2D+1}{-5}\sin x\right]$$

$$= -\frac{1}{4}\left[\frac{-1}{221}\times(2\times 5\cos 5x - 11\sin 5x) - \frac{1}{5}(2\cos x+\sin x)\right]$$

$$PI = \frac{1}{4\times 221}(10\cos 5x - 11\sin 5x) + \frac{1}{20}(2\cos x+\sin x)$$

$$y = y_{cf} + y_{PI}$$

$$y = C_1e^{x} + C_2e^{3x} + \frac{10\cos 5x - 11\sin 5x}{884} + \frac{2\cos x+\sin x}{20}$$

$$+ C$$

11

Good Approach

Q.1 (b) An inductive circuit in parallel with a resistive circuit of 20 Ω is connected across 50-Hz supply. The inductive current is 4.3 A and the resistive current is 2.7 A. The total current is 5.8 A.

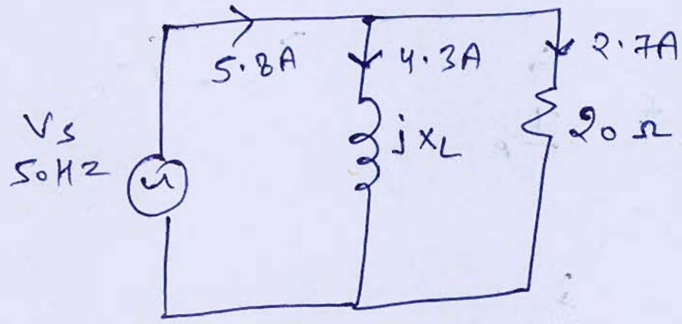
Find:

(i) Power absorbed by the inductive branch.

(ii) Inductance of inductive branch.

(iii) Power factor of the combined circuit. Also draw the phasor diagram.

[12 marks]



~~5.8 = 4. ...~~

~~$I = I_L + I_R$~~

$$V = 20 \times 2.7 = 54\,V$$

$$X_L = \frac{V}{I_L} = \frac{54}{4.3\angle -90^\circ} = j12.55$$

$$\text{(i)}\ P = P_{abs.} = I^2 X_L = (4.3)^2 \times 12.55 = 232.04\,W$$

(ii)

$$X_L = 12.55\,\Omega$$

$$L = \frac{12.55}{2\pi \times 50} \Rightarrow \boxed{L = 39.96\,mH}$$

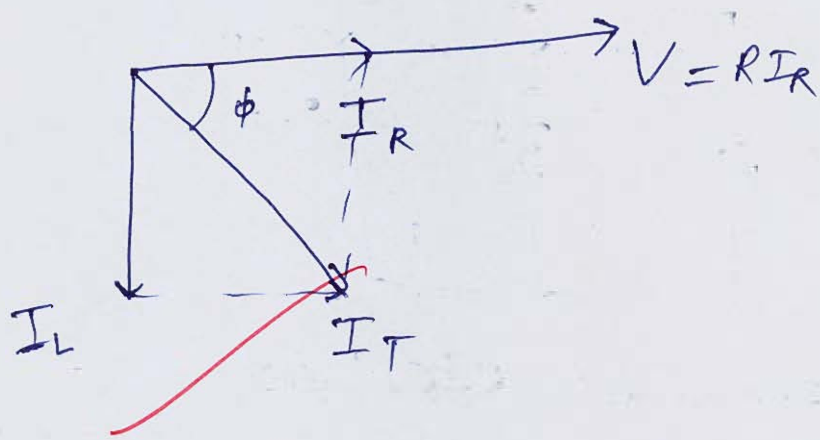
(iii)

$$Z = R + jX = 20 + j12.55$$

$$pf = \cos\phi = \cos 32.1$$

$$pf = 0.847\ \text{lagging}$$

Phasor diag.



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Q.1 (c) Determine the percentage of ionic polarizability in sodium chloride crystal which has the optical index of refraction and the static dielectric constant are 1.5 and 5.6 respectively.
[12 marks]

NaCl $\rightarrow \alpha_i = ?$

By using Clausis Maussati equation

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{\alpha_i + \alpha_e}{3\varepsilon_0} \rightarrow (1)$$

Given $\varepsilon_r = 5.6$
$n = 1.5$

from maxwell relation

$$\boxed{\varepsilon_r = n^2} \rightarrow (2)$$

$$\frac{n^2 - 1}{n^2 + 2} = \frac{\alpha_e}{3\varepsilon_0} \rightarrow (3)$$

from (1) & (3)

$$\frac{5.6 - 1}{5.6 + 2} = \frac{\alpha_i + \alpha_e}{3\varepsilon_0}$$

$$\alpha_i + \alpha_e = 3\varepsilon_0 \times \frac{4.6}{7.6}$$

$$\boxed{\alpha_i + \alpha_e = 1.815\,\varepsilon_0} \rightarrow (4)$$

and

$$\frac{1.5 - 1}{1.5 + 2} = \frac{\alpha_e}{3\varepsilon_0}$$

$$\boxed{\alpha_e = 3\varepsilon_0 \times \frac{0.5}{3.5}} \rightarrow (5)$$

from 4 & 8

$$\alpha_i = 1.815\,\varepsilon_0 - 0.428\,\varepsilon_0$$

$$\boxed{\alpha_i = 1.386\,\varepsilon_0}$$

$$\alpha_i = 1.386 \times 8.854 \times 10^{-12}$$

$$\alpha_i = 1.227 \times 10^{-11}$$

$$\% \alpha_i = 1.227 \times 10^{-9}$$

4

Q.1 (d) An energy meter is designed to have 80 revolutions of the disc per unit of energy consumed. Calculate the number of revolutions made by the disc when measuring the energy consumed by the load carrying 30 A at 230 V and 0.6 power factor. Find the percentage error if the meter actually makes 330 revolutions. Also specify whether the meter runs slower or faster.

[12 marks]

Given,

$K = 80 \frac{rev}{kWh}$ → meter constant

$E = VI\cos\phi \times t$

$E = \frac{230 \times 30 \times 0.6}{1000} \times 1hr = 4.14\, kWhr$

No. of revolution = E × meter const.

$= 4.14 \times 80$

$= 331.2$

Meter actually takes = 330 revolution

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So

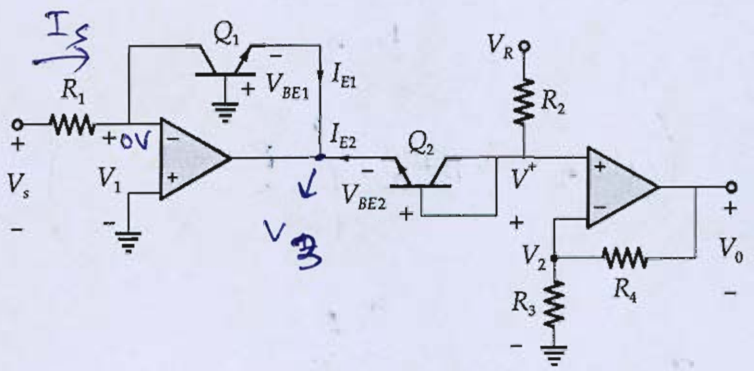
$\%\ error = \frac{330 - 331.2}{331.2} \times 100$

Good Approach

$= -0.36\,\%$

Meter runs slower as no. of revolutions make are less

Q.1 (e) The figure shows a modified logarithmic amplifier to overcome the undesirable effects of temperature-dependent V_T and I_S (reverse saturation current). Show that if the two transistors Q_1 and Q_2 are matched transistors, then the output V_0 is truly proportional to $\ln(V_s)$.



[12 marks]

From the given figure

$$V_2 = V_0 \times \frac{R_3}{R_3 + R_4} \rightarrow (4)$$

From virtual ground

$$V_2 = V^+$$

From figure

Applying KVL in Q_1 side

$$V_3 = -V_{BE1} \rightarrow (1)$$

and

$$I_s = \frac{V_s}{R_1} \rightarrow (2)$$

Again KVL in Q_2 side

$$-V_3 - V_{BE2} + V^+ = 0$$

$$V^+ = V_2 = V_3 + V_{BE2}$$

$$\boxed{V_2 = V_{BE2} - V_{BE1}} \rightarrow (3)$$

from (3) & (4)

$$V_o = V_2 \times \left(\frac{R_3+R_4}{R_3}\right)$$

$$V_o = \left(V_{BE_2} - V_{BE_1}\right)\left(\frac{R_3+R_4}{R_3}\right) \rightarrow (5)$$

Now,

$$I_s = I_o e^{\frac{V_{BE_1}}{\eta V_T}} = \frac{V_s}{R_1}$$

$$\boxed{\frac{V_{BE_1}}{\eta V_T} I_o R_1 = \ln V_s} \rightarrow (6)$$

from (5) & (6)

$$\boxed{V_o = \left(V_{BE_2} - \frac{\eta V_T}{I_o R_1} \ln V_s\right)\left(\frac{R_3+R_4}{R_2}\right)}$$

$$V_o \propto \ln(V_s)$$

5

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.2 (a) (i) The read access times and the hit ratios for different caches in a memory hierarchy are as given below:

Code	Read access time (in nanoseconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

The read access time of main memory is 90 nanoseconds. Assume that the caches use the referred word-first read policy and the write back policy. Assume that all the caches are direct mapped caches. Assume that the dirty bit is always 0 for all the blocks in the caches. In execution of a program, 60% of memory reads are for instruction fetch and 40% are for memory operand fetch. Find the average read access time in nanoseconds.

[10 marks]

$t_{mm} = 90ns$

$(t_{access})_{avg\,read} = t_{cm} \times H + (1-H)\,t_{mm}$

$\Rightarrow t_{cm} = 0.8 \times 2 + 0.9 \times 2 + 0.9 \times 8$

$= 1.6 + 1.8 + 7.2$

$= 10.6\,ns$

$(t_{access})_{avg} = 10.6 \times 0.6 + 0.4 \times 90$

$= 42.36\,ns$

2

.2 (a) **(ii)** A certain processor uses a fully associative cache of size 16 kB. The cache block size is 16 bytes. Assume that the main memory is byte addressable and uses a 32-bit address. How many bits are required for the Tag and the Index fields respectively in the addresses generated by the processor?

[10 marks]

Given, <u>Fully Associative Cache</u>

$$Size = 16\,KB = 2^{14}\,B$$

Block size = 16 B

Main memory address uses = 32 bit address

$$\text{No of Cache lines} = \frac{16\,KB}{16\,B} = 2^{10}$$

So 10 bits are used in cache

Associative cache is represented by

Tag	Index field

Bits required for Index field = 10

Bits required for tag = 32 - 10
= 22

5

Tag	Index field
22 bits	10 bits

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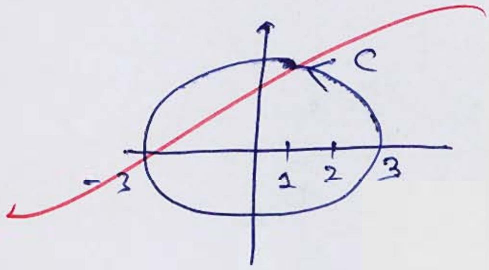
Q.2 (b) (i) Find the value of $\int_C \frac{\cos \pi z^2}{(z-2)(z-1)} dz$, where 'C' is $|z| = 3$.

[8 marks]

$$\int \frac{\cos \pi z^2}{(z-2)(z-1)} dz$$

Poles $\Rightarrow$ $z = 1, 2$

Both poles lie in given contour $\Rightarrow |z| = 3$



Residue for $z = 1$

$$R_1 = \lim_{z \to 1} \frac{\cos \pi z^2}{z-2} = \frac{\cos \pi}{-1} = 1$$

Residue for $z = 2$

$$R_2 = \lim_{z \to 2} \frac{\cos \pi z^2}{z-1} = \frac{\cos 4\pi}{1} = 1$$

$$\int_C \frac{\cos \pi z^2}{(z-2)(z-1)} = 2\pi i \times (R_1 + R_2)$$

$$= 2\pi i \times 2$$

$$= 4\pi i$$

8

Good Approach

Q.2 (b) (ii) Solve $(x^2 - yz)\frac{\partial p}{\partial x} + (y^2 - zx)\frac{\partial p}{\partial y} = z^2 - xy$.

[12 marks]

The given form of partial differential equation Can be written as :-

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy} \rightarrow (1)$$

Multiplying by y, z and x respectively

$$\frac{y\,dx}{x^2 y - y^2 z} = \frac{z\,dy}{zy^2 - z^2 x} = \frac{x\,dz}{z^2 x - x^2 y}$$

Adding all together

$$\frac{y\,dx + z\,dy + x\,dz}{0} = 0$$

$$y\,dx + z\,dy + x\,dz = 0$$

$$\boxed{xy + yz + zx = 0} \rightarrow (2)$$

Now, equation (1) Can also be written as

$$\frac{dx - dy}{x^2 - yz - y^2 + zx} = \frac{dy - dz}{y^2 - zx - z^2 + xy} = \frac{dz - dx}{z^2 - xy - x^2 + yz}$$

$$\frac{dx - dy}{(x+y+z)(x-y)} = \frac{dy - dz}{(x+y+z)(y-z)} = \frac{dz - dx}{(x+y+z)(z-x)}$$

$$\frac{dx - dy}{x - y} = \frac{dy - dz}{y - z} = \frac{dz - dx}{z - x}$$

on solving $\boxed{x = y = z} \rightarrow (3)$

$$f\left(xy+yz+zx,\ \phi(x=y=z)\right)=0$$

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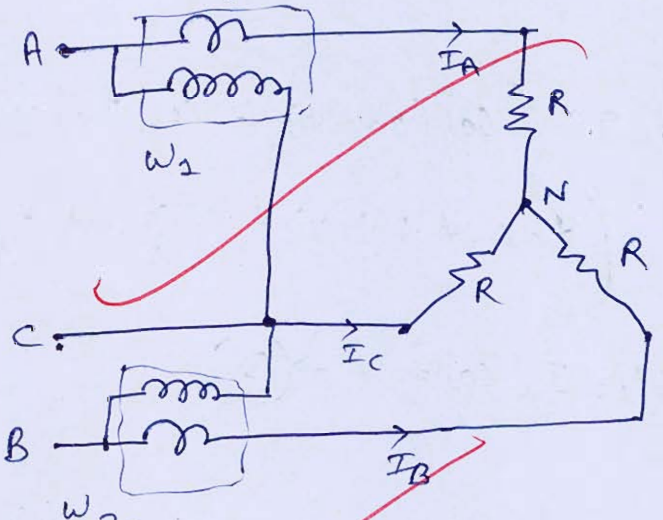
Q.2 (c) Draw the circuit arrangement for power measurement in a 3-phase, 3-wire balanced supply and load using two-wattmeter method, and show that the power factor of the load is given by

$$\cos\phi = \frac{1}{\sqrt{1+3\left(\frac{P_1-P_2}{P_1+P_2}\right)^2}}$$

where P_1 and P_2 are powers indicated by Wattmeter 1 and Wattmeter 2, respectively.

[20 marks]

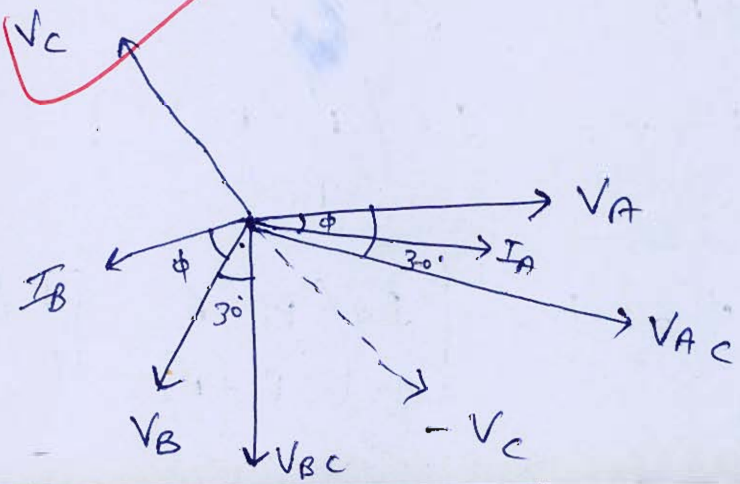
In 3ϕ 3 wire balanced supply and load, the circuit arrangement for 2-Wattmeter method is



The power measured by two wattmeters is given by

$$P_1 = I_A V_{AC} \cos\phi_1 \rightarrow (1)$$

$$P_2 = I_B V_{BC} \cos\phi_2 \rightarrow (2)$$



From the phasor diagram, its clear that

$$\phi_1 = 30 - \phi$$

$$\phi_2 = 30 + \phi$$

$\phi \rightarrow$ angle b/w phase voltage & phase current

$$P_1 = V_L I_L \cos(30 - \phi)$$

$$P_2 = V_L I_L \cos(30 + \phi)$$

Now,

$$P_1 + P_2 = V_L I_L \left(\cos(30-\phi) + \cos(30+\phi)\right)$$

$$P_1 + P_2 = V_L I_L \left(\frac{\sqrt{3}}{2}\cos\phi + \frac{1}{2}\sin\phi + \frac{\sqrt{3}}{2}\cos\phi - \frac{1}{2}\sin\phi\right)$$

$$P_1 + P_2 = \sqrt{3} V_L I_L \cos\phi \rightarrow 3$$

And

$$P_1 - P_2 = V_L I_L \left(\cos(30-\phi) - \cos(30+\phi)\right)$$

$$P_1 - P_2 = V_L I_L (\sin\phi) \rightarrow 4$$

from equation 3 & 4

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{\tan\phi}{\sqrt{3}}$$

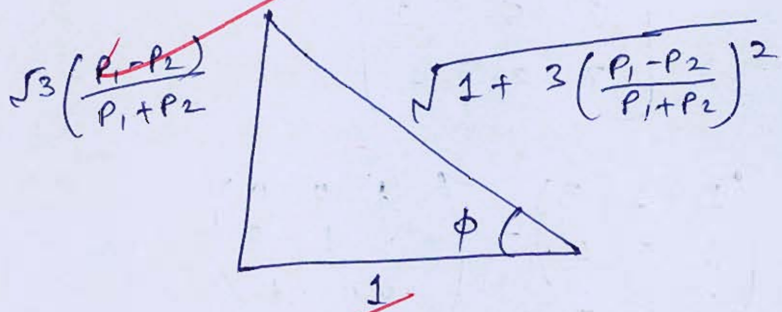
$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

$$\boxed{\phi = \tan^{-1}\left[\sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)\right]}$$

So power factor of load is given by

$$\cos\phi = pf = \cos\left(\tan^{-1}\left[\sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)\right]\right)$$

or it can also be expressed as



using phytagoras theorem

$$\cos\phi = pf = \frac{1}{\sqrt{1 + 3\left(\frac{P_1 - P_2}{P_1 + P_2}\right)^2}}$$

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Good Approach

Q.3 (a) Find the matrix P which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. Hence calculate A^4 by using matrix P.

[20 marks]

Given,

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

$$(A - \lambda I) = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$(A - \lambda I) = \begin{pmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{pmatrix}$$

$$|A - \lambda I| = 0$$

$$(1-\lambda)\left[(5-\lambda)(1-\lambda) - 1\right] - \left[1 - \lambda - 3\right] + 3\left[1 - 15 + 3\lambda\right] = 0$$

$$(1-\lambda)\left[5 - 5\lambda - \lambda + \lambda^2 - 1\right) + \lambda + 2 + 3 - 45 + 9\lambda = 0$$

$$\Rightarrow \lambda^2 - 6\cancel{\lambda} + 4 - \lambda^3 + 6\lambda^2 - \cancel{4\lambda} + \cancel{10\lambda} - 40 = 0$$

$$\Rightarrow \quad -\lambda^3 + 7\lambda^2 - 36 = 0$$

$$\Rightarrow \lambda^3 - 7\lambda^2 + 36 = 0$$

$$\boxed{\lambda = -2, 6; 3} \rightarrow \text{eigen values}$$

For eigen vectors

$\lambda = -2$

$$(A - \lambda I)X = \begin{pmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\Rightarrow \quad 3x_1 + x_2 + 3x_2 = 0 \rightarrow (1)$$

$$\& \quad x_1 + 7x_2 + x_3 = 0 \rightarrow (2)$$

$$3x_1 + x_2 + 3x_3 = 0$$
$$3x_1 + 21x_2 + 3x_3 = 0 \Rightarrow \text{solving ① \& ②}$$
$$-$$
$$-20x_2 = 0$$
$$\boxed{x_2 = 0}$$

$x_1 = -x_3$ Let $x_3 = k$ then $x_1 = -k$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \rightarrow ③$$

For $\lambda = 6$

$$(A - \lambda I)X = \begin{pmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-5x_1 + x_2 + 3x_3 = 0$$
$$x_1 - x_2 + x_3 = 0$$
$$3x_1 + x_2 - 5x_3 = 0$$

on solving

$$x_1 = x_3$$
$$x_2 = 2x_1 = 2x_3$$

Let $x_3 = k$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \rightarrow ④$$

For $\lambda = 3$

$$(A - \lambda I)X = \begin{pmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-2x_1 + x_2 + 3x_3 = 0$$
$$x_1 + 2x_2 + x_3 = 0$$
$$3x_1 + x_2 - 2x_3 = 0$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \rightarrow ⑤$$

on solving $x_1 = x_3 = -x_2$

from (3), (4) & (5) equations

$$\boxed{P = \begin{pmatrix} -1 & 1 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}} \rightarrow$$ This matrix will transform the A into diagonal form

Diagonal form

$$D = P^{-1} A P$$

$$P^{-1} = \frac{adj(P)}{|P|} = -\frac{1}{6}\begin{bmatrix} 3 & 0 & 3 \\ -1 & -2 & -1 \\ -2 & 2 & -2 \end{bmatrix}$$

$$D = -\frac{1}{6}\begin{bmatrix} 3 & 0 & 3 \\ -1 & -2 & -1 \\ -2 & 2 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$D = \begin{pmatrix} 0 & -6 & -3 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A^4 = P^{-1} D^4 P$$

$$A^4 = \begin{pmatrix} 251 & 405 & 235 \\ 405 & 891 & 405 \\ 235 & 405 & 251 \end{pmatrix}$$

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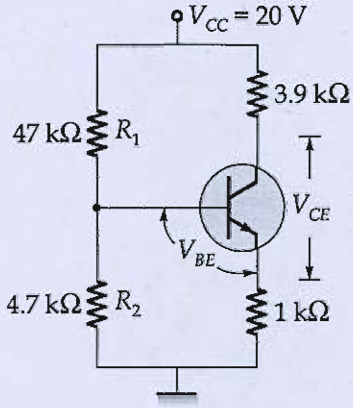
Q.3 (b) The transistor shown in figure is a silicon transistor. The junction temperature increases from 30° to 70°. If β = 30 at 30° and β = 70 at 70°, determine the percent change in D.C. bias point over the temperature range 30° to 70° neglecting change in base to emitter voltage.



β_{dc} = 30 at 30°
= 70 at 70° C

[20 marks]

from given transistor

$$V_{th} = 20 \times \frac{4.7}{47+4.7} = 1.818\ V$$

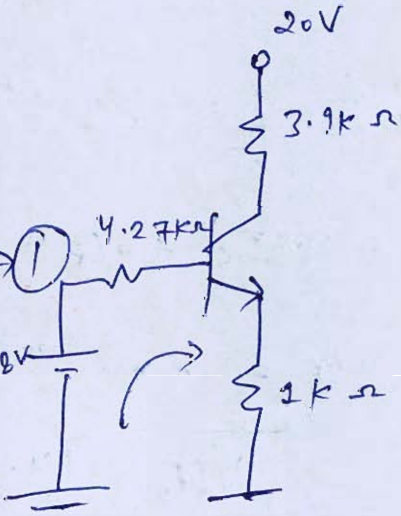
$$R_{th} = \frac{47 \times 4.7}{47+4.7} = 4.27\ k\Omega$$

at T = 30°C

β = 30

KVL in inner loop

$\Rightarrow -1.818 + 4.27 I_B + 0.7 + I_E = 0 \rightarrow (1)$



$\Rightarrow 4.27 I_B + 31 I_B = 1.118$

$$\boxed{I_B = 0.0317\ mA}$$

$$\boxed{I_C = \beta I_B = 0.951\ mA}$$

KVL in outer loop

$\Rightarrow -20 + 3.9 I_C + V_{CE} + I_E R_E = 0 \rightarrow (2)$

$V_{CE} = 20 - 3.9 I_C - I_E$

$V_{CE} = 20 - 3.9 \times 0.951 - 0.982$

$\boxed{V_{CE_1} = 15.31 \text{ V}} \rightarrow$ at $T = 30^\circ C$

at $T = 70^\circ C$

$\beta = 70$

from equation (1)

$4.27 I_B + 71 I_B = 1.118$

$\boxed{I_B = 0.0148 \text{ mA}}$

$I_C = \beta I_B = 70 \times 0.0148 = 1.039 \text{mA}$

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Again from equation (2)

$-20 + 3.9 I_C + V_{CE} + I_E R_E = 0$

Good Approach

$V_{CE} = 20 - 3.9 \times 1.029 - 1.0545$

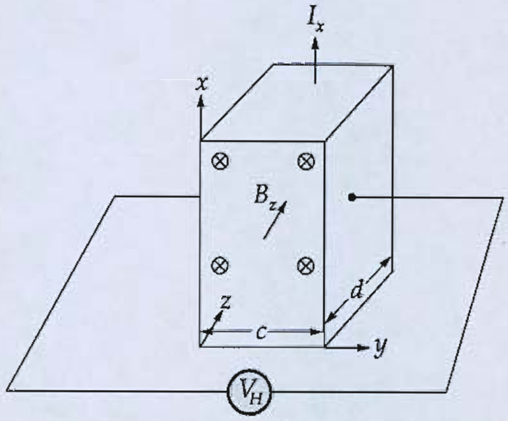
$\boxed{V_{CE_2} = 14.89 \text{ V}}$ at $T = 70^\circ C$

% Change in dc bias $= \dfrac{V_{CE_2} - V_{CE_1}}{V_{CE_1}} \times 100$

$= \dfrac{14.89 - 15.31}{15.31} \times 100$

$= -2.74\%$

Q.3 (c) (i) What is Hall effect? For a parallelepiped specimen having one corner situated at origin and externally applied electric field causing current in positive x-direction as shown below:



State what happens when magnetic field B_z is applied in positive z-direction in reference to Hall voltage. Determine electron mobility relation using Hall coefficient and conductivity (σ).

[10 marks]

Hall Effect → When a specimen (metal or semi conductor) is placed in a transverse magnetic field having current flowing in it perpendicular to magnetic field then a electric field is generated perpendicular to both magnetic field & current flow which further generates a Hall voltage in specimen.

From the given specimen, I flowing in +ve x direction and B_z is applied in +z direction then an electric field is generated in the specimen in +y direction which generates the Hall voltage which is given by $\boxed{V_H = \dfrac{BI}{new}}$ $\left(V_H = \dfrac{E_H}{d}\right)$

$$\vec{F}_H = (\vec{B} \times \vec{I})\, l = (a\hat{z} \times a\hat{n})\, l$$

$$\vec{F}_H = a\hat{y}\, l.$$

$$\vec{F_H} = q\vec{E}_H$$

for $q = +ve$

$$\vec{E}_H = +a\hat{y}$$

$q = -ve$

$$\vec{E}_H = -a\hat{y}$$

The Hall coefficient is given by

$$\boxed{R_H = \frac{1}{ne}} \rightarrow (1)$$

So, electron mobility can be expressed as

$$\sigma = ne\mu \rightarrow (2)$$

from (1) & (2)

$$\mu = \frac{\sigma}{ne}$$

$$\boxed{\mu = \sigma R_H}$$

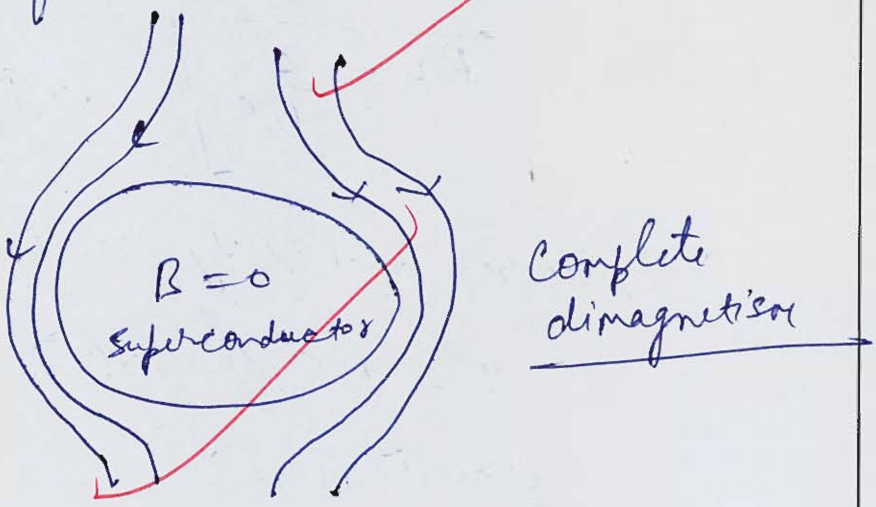
electron mobility

9

Q.3 (c) (ii) What is Meissner effect and how it can be used to justify negative susceptibility of superconductors? How critical field, H_C for a superconductor material varies with temperature? Explain briefly factors that affect transition temperature of superconductor.

[10 marks]

Meissner effect → This effect states that when magnetic field lines are passed through the superconductor then the material repels these lines completely to pass from it and magnetic field inside the superconductor is zero.



We know,

$$B = \mu_0 (H + M)$$

Since $B = 0$

$$\boxed{M = -H}$$

$M = \chi_M H$ it is clear that

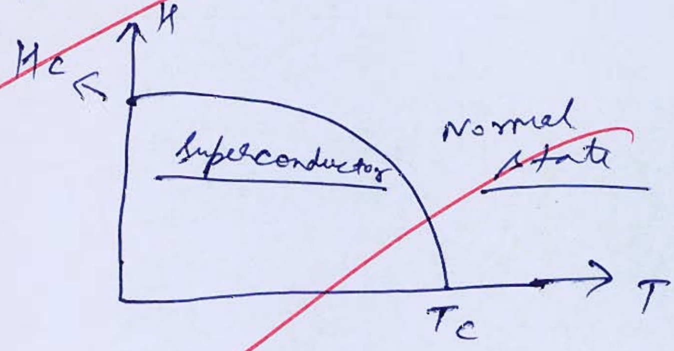
$\boxed{\chi_M = -1}$ or $\boxed{\mu_r = 0}$

negative susceptibility

Variation of H_c with temperature

$$\boxed{H = H_c\left(1 - \left(\frac{T}{T_c}\right)^2\right)}$$

→ H_c varies with T according to this relation



Factors affecting transition temperature are

1) T_c depends on atomic mass of material

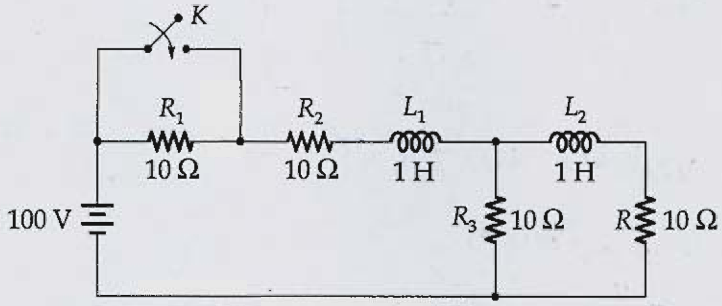
$$\boxed{T_c \propto \frac{1}{\sqrt{M}}}$$

2) for $T < T_c$, superconducting state
$T > T_c$, normal state

3) T_c also depends on critical magnetic field.

9

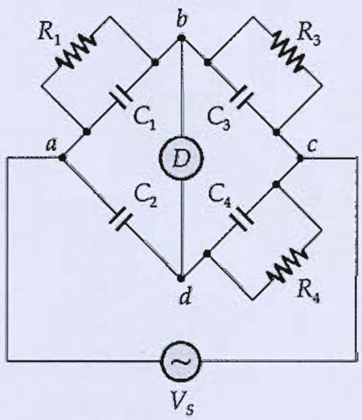
Q.4 (a) In the network of below figure, the switch K is closed at time $t = 0$, a steady state having previously existed. Obtain the expression of current in the resistor R using Thevenin's theorem.



[20 marks]

Q.4 (b) (i) In a low-voltage bridge designed for the measurement of permittivity, the branch *ab* consists of two electrodes between which the specimen under test may by inserted; arm *bc* is a non-reactive resistor R_3 in parallel with a standard capacitor C_3, arm *cd* is non-reactive resistor R_4 in parallel with a standard capacitor C_4; arm *da* is a standard air capacitor of capacitance C_2 without the specimen between the electrodes, balance is obtained with the following values :

$C_3 = C_4 = 120$ pF, $C_2 = 150$ pF, $R_3 = R_4 = 5000\ \Omega$ with the specimen inserted these values become $C_3 = 200$ pF, $C_4 = 1000$ pF, $C_2 = 900$ pF and $R_3 = R_4 = 5000\ \Omega$. In each test $\omega = 5000$ rad/sec. Find the relative permittivity of the specimen.



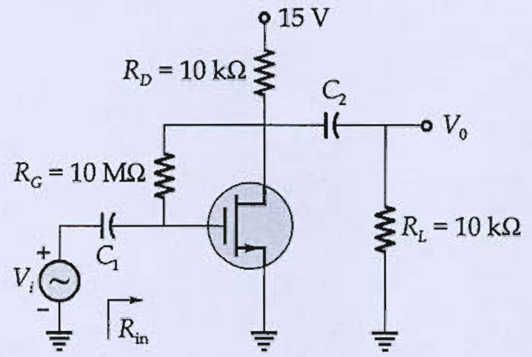
[12 marks]

Do not write in this marg

Q.4 (b) **(ii)** A CRT has an anode voltage of 2000 V and parallel deflecting plates 2 cm long and 5 mm apart. The screen is 30 cm from the centre of the plates. Find the input voltage required to deflect the beam through 3 cm. The input voltage is applied to the deflecting plates through amplifiers having an overall gain of 100.

[8 marks]

Q.4 (c) Determine the small-signal voltage gain, its input resistance and the largest allowable input signal. The transistor has $V_t = 1.5$ V, $K_n'\left(\frac{W}{L}\right) = 0.25$ mA/V^2 and $V_A = 50$ V. Assume the coupling capacitors to be sufficiently large so as they act as short circuits at the signal frequencies of interest.



[20 marks]

Section-B

Q.5 (a) Consider a 2-way set associative cache memory with 4 sets and total 8 cache blocks (0-7) and a main memory with 128 blocks (0-127). What memory blocks will be present in the cache after the following sequence of memory block references if LRU policy is used for cache block replacement? Assuming that initially the cache did not have any memory block from the current job.

0 5 3 9 7 0 16 55

[12 marks]

<u>2-way set associative cache</u>

~~N = 4 sets~~

~~MM = 128 blocks~~

~~CM = 8 blocks~~

<u>LRU</u>

0	0	0	0	0	0	0	0	0	0
	5	5	5	5	5	5	5	5	5
		3	9	7	7	1	6	5	5
M	M	M	M	M	H	M	M	H	H

Tag	SO	~~WO~~
	~~2 bit~~	4 bit

1

Incomplete Solution

Q.5 (b) Obtain the partial differential equation from function $f(xy + z, x^2 + y^2 - z^2) = 0$.

[12 marks]

$$f(xy+z,\ x^2+y^2-z^2)=0$$

$$P = xy+z \qquad Q = x^2+y^2-z^2$$

$$F = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$P\frac{dz}{dx} + Q\frac{dz}{dy} = \underbrace{f(x,y,z)}_{R}$$

$$P = \begin{vmatrix} \frac{\partial P}{\partial y} & \frac{\partial P}{\partial z} \\ \frac{\partial Q}{\partial y} & \frac{\partial Q}{\partial z} \end{vmatrix}$$

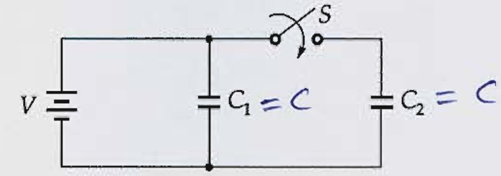
$$Q = \begin{vmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial z} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial z} \end{vmatrix}$$

$$R = \begin{vmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{vmatrix}$$

2

Incomplete Solution

Q.5 (c) Figure below shows the two identical parallel-plate capacitors connected to a battery with switch S closed. The switch S is opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric coefficient $K = 2$. Find the ratio of the electrostatic energy stored in capacitor C_2 to electrostatic energy in capacitor C_1 after the introduction of the dielectric.

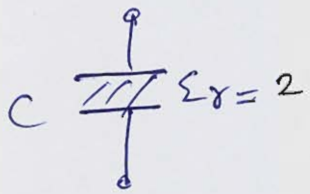


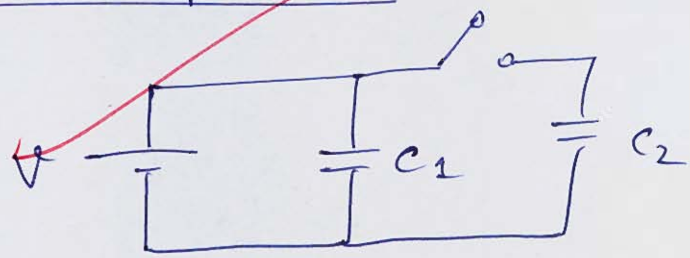
[12 marks]

<u>when switch S closed</u>

$$V_{C_1} = V_{C_2} = V \qquad C_1 = C_2 = C$$

<u>when switch S opened</u>





$C_1 = 2C$

Energy stored in $C_1 = \frac{1}{2} C_1 V^2$

$= \frac{1}{2} \times 2C \times V^2$

$= CV^2 = E_1$

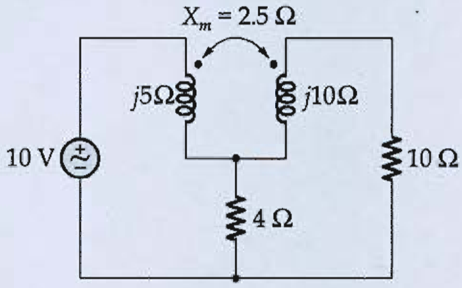
Energy stored in $C_2 = \frac{1}{2} CV^2 = E_2$

$$\frac{E_2}{E_1} = \frac{\frac{1}{2}CV^2}{CV^2} = \frac{1}{2} = 0.5$$

11

Good Approach

Q.5 (d) Figure below shows a network with mutual coupling.

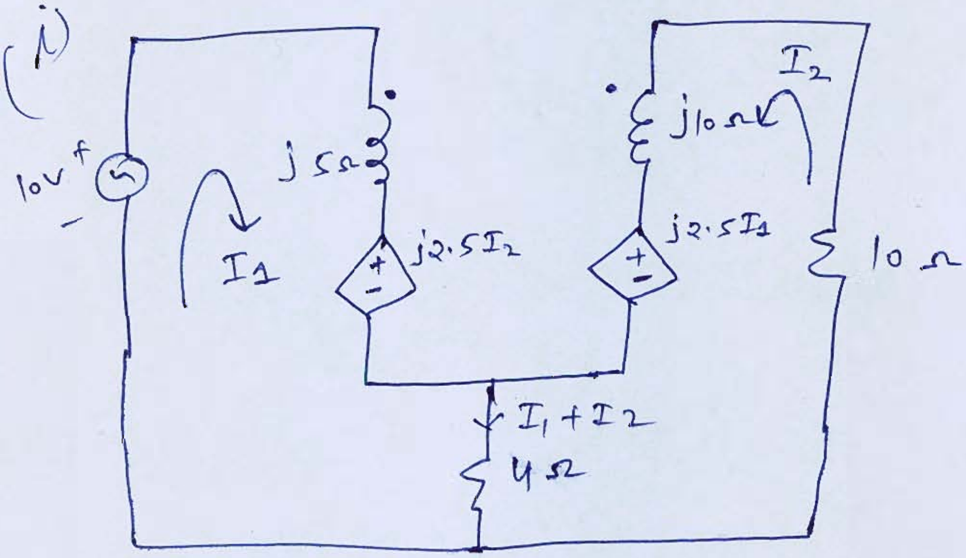


(i) Find the current in the 10 Ω resistance. Assume that inductor have negligible resistance.

(ii) If the direction of winding of one of the coils is reversed, find the current in the 10 Ω resistance.

[12 marks]

(i)



KVL

$$-10 + j5 I_1 + j2.5 I_2 + 4 I_1 + 4 I_2 = 0$$

$$I_1(4 + j5) + I_2(4 + j2.5) = 10 \rightarrow (1)$$

Again KVL

$$10 I_2 + j10 I_2 + j2.5 I_1 + 4 I_1 + 4 I_2 = 0$$

$$I_1(4 + j2.5) + I_2(14 + j10) = 0 \rightarrow (2)$$

$$\begin{pmatrix} 4+j5 & 4+j2.5 \\ 4+j2.5 & 14+j10 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

$$\Delta = (4+j5)(14+j10) - (4+j2.5)^2$$

$$\Delta = 90.07\angle 92.38$$

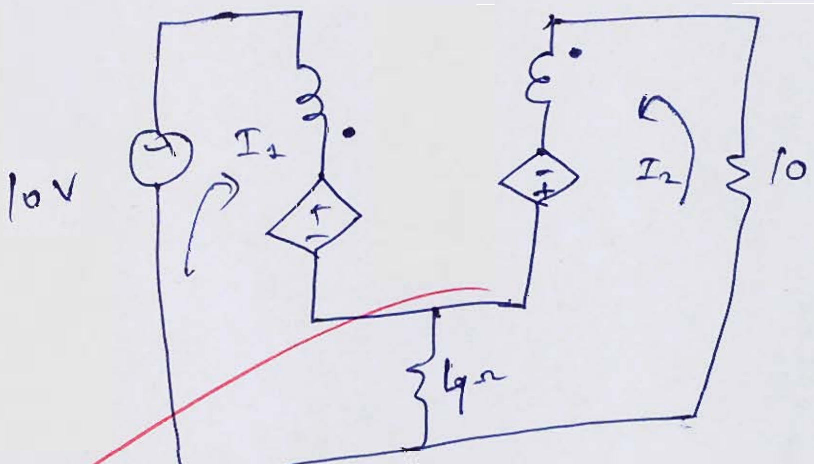
$$\Delta_2 = \begin{pmatrix} 4+j5 & 10 \\ 4+j2.5 & 0 \end{pmatrix}$$

$$\Delta_2 = -40 - j25 = 47.17\angle -148$$

$$\boxed{I_2 = \frac{\Delta_2}{\Delta} = 0.523\angle 119.62\ A}$$

→ Current in 10Ω resistance

(ii) when one of the coil is reversed.



KVL

$$-10 + j5I_1 + j2.5I_2 + 4(I_1+I_2) = 0$$

$$I_1(4+j5) + I_2(4+j2.5) = 10 \rightarrow (1)$$

KVL

$$10I_2 + j10I_2 - j2.5I_1 + 4I_1 + 4I_2 = 0$$

$$\Rightarrow I_1(4-j2.5) + I_2(14+j10) = 0 \rightarrow (2)$$

$$\Delta = (4+j5)(14+j10) - (4+j2.5)(4-j2.5)$$

$$\Delta = 111.2\angle 98.4\ A$$

$$\Delta_2 = (4+j2.5) \times -10 = -40 + j25$$

$$\Delta_2 = 47.17\angle 148$$

$$I_2 = \frac{\Delta_2}{\Delta} = 0.424\angle 49.6\ A$$

9

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Q.5 (e) Consider a common-emitter circuit using a BJT having $I_s = 10^{-15}$ A, a collector resistance $R_C = 6.8\ \text{k}\Omega$ and a power supply $V_{CC} = 10$ V and $V_{CE} = 3.2$ V.

(i) Find the positive increment in V_{BE} (above V_{BE}) that drives the transistor to the edge of saturation, where $V_{CE(sat)} = 0.3$ V.

(ii) Find the negative increment in V_{BE} that drives the transistor to within 1% of cut-off (i.e. to $V_0 = 0.99\ V_{CC}$).

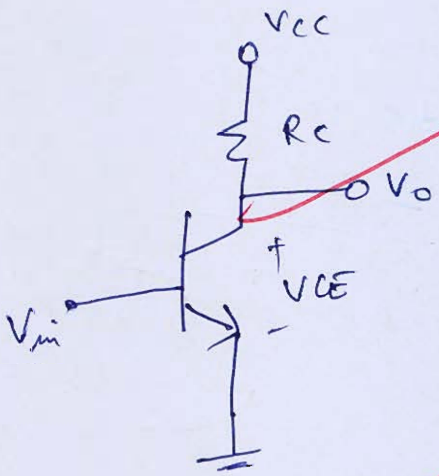
[Take $V_T = 25$ mV] **[12 marks]**

CE config

$I_S = 10^{-15}$ A $R_C = 6.8k\Omega$

$V_{CC} = 10V$ $V_{CE} = 3.2V$

$V_T = 25mV$



At edge of saturation

$-V_{CC} + I_C R_C + (V_{CE})_{sat} = 0$

$$I_C = \frac{10 - 0.3}{6.8}$$

$$\boxed{I_C = 1.426\ mA}$$

$$I_C = I_0\, e^{\frac{V_{BE}}{V_T}}$$

$$\frac{1.426\times10^{-3}}{10^{-15}} = e^{\frac{V_{BE}}{V_T}} = 27.98$$

$$V_{BE} = 27.98 \times 25\times10^{-3}$$

$$\boxed{V_{BE} = 0.699 \approx 0.7V}$$

(ii) $V_o = 0.99 \times 10 = 9.9 = V_{CE}$

$I_C = \frac{10 - 9.9}{6.8} = 0.0147 mA$

$I_C = I_0 e^{V_{BE}/V_T}$

$V_{BE} = -0.585 V$

5

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.6 (a) (i) Compare RISC and CISC architecture.

(ii) State and explain the instruction and data stream types based on Flynn's classification.

(iii) Consider the process table with time quantum '4'.

P_{id}	Arrival time	Burst time
1	2	5
2	4	3
3	1	6
4	2	2
5	3	7

What is the average TAT and average WT using Round Robin scheduling?

[4 + 8 + 8 marks]

Q.6 (b) Find the Fourier series of $f(x)$,

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(1-x) & 1 \le x \le 2 \end{cases}$$

Also find the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$.

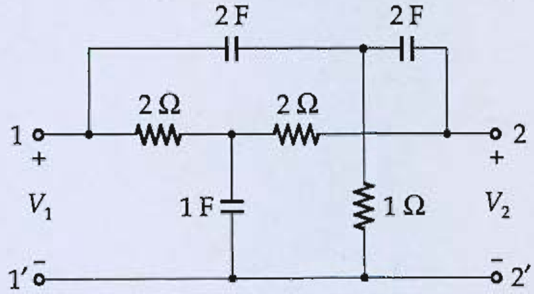
[20 marks]

6 (c) (i) The number of atoms in a volume of one cubic metre of hydrogen gas is 9.8×10^{26}. The radius of hydrogen atom is 0.53 Å. Calculate the polarizability and relative permittivity of the hydrogen gas.

(ii) The magnetic field in a piece of copper and another piece of Fe_2O_3 is 10^6 Am^{-1}. Their magnetic susceptibilities are -0.5×10^{-5} and 1.4×10^{-3} respectively. Compare the flux density and magnetization in the two pieces.

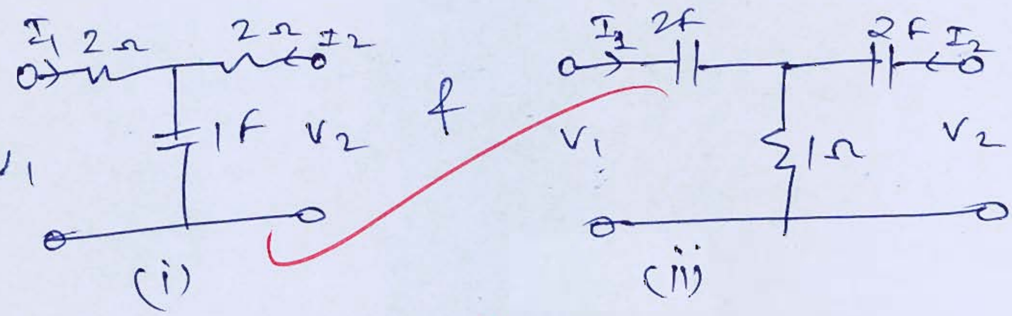
[10 + 10 marks]

Q.7 (a) For the notch-filter network, determine the y-parameters.

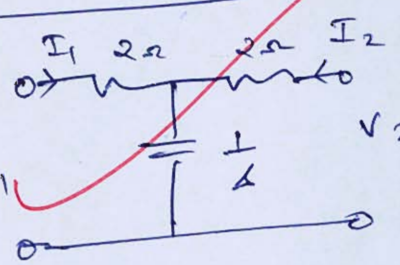


[20 marks]

The above network is cascade of two network



for Network(i)



$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 2+1/s & 1/s \\ 1/s & 2+1/s \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$V_1 = \left(2+\frac{1}{s}\right) I_1 + \frac{1}{s} I_2 \rightarrow (1)$$

$$V_2 = \frac{1}{s} I_1 + \left(2+\frac{1}{s}\right) I_2 \rightarrow (2)$$

On solving

$$V_1 = \left(2+\frac{1}{s}\right)\left[\frac{V_2 - \left(2+\frac{1}{s}\right) I_2}{1/s}\right] + \frac{1}{s} I_2$$

$$V_1 = \left(\frac{2s+1}{1}\right)\left[\frac{sV_2 - (2s+1) I_2}{s}\right] + \frac{1}{s} I_2$$

$$V_1 = (2s+1) V_2 - \frac{(2s+1)^2}{s} I_2 + \frac{1}{s}$$

$$\cancel{V_1 = (s+2)V_2 + \left(\frac{s^2+4s+3}{s}\right) I_2 \rightarrow (3)}$$

$$\cancel{I_1 = V_2 - 2I}$$

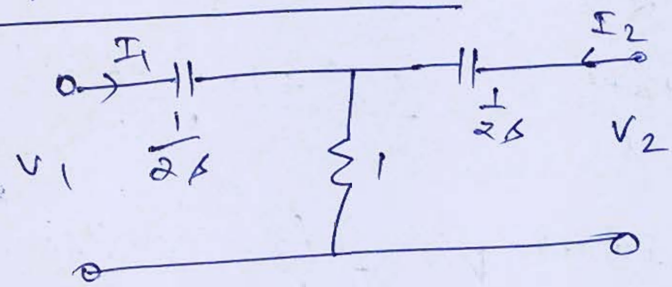
$$V_1 = (2s+1)V_2 - 4(s+1) I_2 \rightarrow (3)$$

$$I_2 = sV_2 - s\left(\frac{2s+1}{s}\right) I_2$$

$$I_1 = sV_2 - (2s+1) I_2 \rightarrow (4)$$

$$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 2s+1 & 4(s+1) \\ s & 2s+1 \end{pmatrix}$$

For Network (ii)



$$V_1 = \left(1 + \frac{1}{2s}\right) I_1 + I_2 \rightarrow (5)$$

$$V_2 = I_1 + \left(1 + \frac{1}{2s}\right) I_2 \rightarrow (6)$$

on solving

$$I_1 = V_2 - \left(\frac{2s+1}{2s}\right) I_2 \rightarrow (7)$$

$$V_1 = \left(\frac{2s+1}{2s}\right)\left(V_2 - \left(\frac{2s+1}{2s}\right) I_2\right) + I_2$$

$$V_1 = \left(\frac{2s+1}{2s}\right) V_2 - \left(\frac{4s+1}{4s^2}\right) I_2 \rightarrow (8)$$

$$\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} = \begin{pmatrix} 1 & \frac{2s+1}{2s} \\ \frac{2s+1}{2s} & \frac{4s+1}{4s^2} \end{pmatrix}$$

ABCD parameters of complete network

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 2s+1 & 4s+4 \\ s & 2s+1 \end{pmatrix} \begin{pmatrix} 1 & \frac{2s+1}{2s} \\ \frac{2s+1}{2s} & \frac{4s+1}{4s^2} \end{pmatrix}$$

$$= \begin{pmatrix} 2s+1 + \frac{8s^2+12s+4}{2s} & \frac{4s^2+1+4s}{2s} + \frac{16s^2+20s+4}{4s^2} \\ s + \frac{4s^2+1+4s}{2s} & \frac{2s^2+s}{2s} + \frac{8s^2+6s+1}{4s^2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{12s^2+14s+4}{2s} & \frac{8s^3+24s^2+22s+4}{4s^2} \\ \frac{6s^2+4s+1}{2s} & \frac{4s^3+10s^2+6s+1}{4s^2} \end{pmatrix}$$

6

$$V_1 = \left(\frac{12s^2+14s+4}{2s}\right) V_2 - \left(\frac{8s^3+24s^2+22s+4}{4s^2}\right) I_2 \rightarrow (9)$$

$$I_1 = \left(\frac{6s^2+4s+1}{2s}\right) V_2 - \left(\frac{4s^3+10s^2+6s+1}{4s^2}\right) I_2 \rightarrow (10)$$

Y-parameters $\Rightarrow$ $\boxed{\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}}$

solving (9) & (10) equations

$$I_2 = \frac{2s(12s^2+14s+4)}{8s^3+24s^2+22s+4} V_2 - \frac{4s^2}{8s^3+24s^2+22s+4} V_1$$

$$I_2 = \frac{-4s^2}{8s^3+24s^2+22s+4} V_1 + \frac{4s(6s^2+7s+2)}{8s^3+24s^2+22s+4} V_2$$

$$I_1 = y_{21} V_1 + y_{22} V_2$$

7 (b) (i) Find the resistivity of

1. intrinsic Silicon and
2. p-type silicon with $N_A = 10^{16}/cm^3$.

Use $n_i = 1.5 \times 10^{10}/cm^3$ and assume that intrinsic Silicon $\mu_n = 1350\ cm^2/V\text{-}s$ and $\mu_p = 480\ cm^3/V\text{-}s$ and for doped Silicon $\mu_n = 1110\ cm^2/V\text{-}s$ and $\mu_p = 400\ cm^2/V\text{-}s$ and comment on result.

(Note that doping results in reduced carrier mobilities).

[10 marks]

Given $n_i = 1.5 \times 10^{10}/cm^3$

Intrinsic Si $\Rightarrow \mu_n = 1350 \frac{cm^2}{V-s}$ $\mu_p = 480 \frac{cm^2}{V-s}$

Doped Si $\Rightarrow \mu_n = 1110 \frac{cm^2}{V-s}$ $\mu_p = 400 \frac{cm^2}{V-s}$

1) <u>for intrinsic Si</u>

$$\sigma = n_i e(\mu_n + \mu_p)$$

$$\sigma = 1.5 \times 10^{10} \times 1.6 \times 10^{-19} \times 1830 \text{ per } \Omega cm$$

$$\sigma = 4392 \times 10^{-9} / \Omega cm$$

<u>Resistivity</u>

$$\boxed{\rho = \frac{1}{\sigma} = 2276.86\ \Omega m}$$

$\hookrightarrow$ 227686 Ωcm

2) <u>for p-type Si</u>

$$\sigma = N_A q(\mu_n + \mu_p)$$

$$\sigma = 10^{16} \times 1.6 \times 10^{-19} \times 1510$$

$$\sigma = 2416 \times 10^{-3}$$

$$\boxed{\rho = \frac{1}{\sigma} = 4.14 \times 10^{-3}\ \Omega\text{-}m}$$

$\hookrightarrow$ 0.414 Ωcm

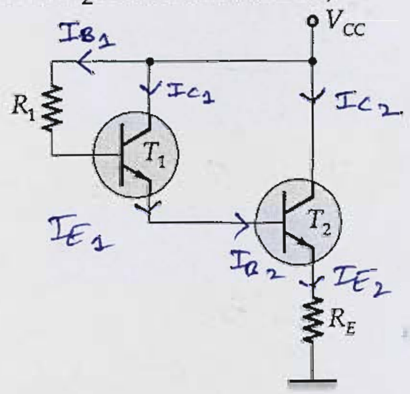
When Si is doped with p-type impurity since mobilities decreases then resistivity also decreases & conductivity increases

$$\boxed{\sigma(\text{p-type}) > \sigma(\text{intrinsic})}$$

6

7 (b) (ii) For the transistor pair circuit is shown in below figure. Both the transistors have dc current gain β of 30. In the circuit V_{CC} = +12V, R_E = 1.5 kΩ.

1. Find the same value of R_1 needed to bias the circuit such that V_{CEQ2} = 5 V for transistor T_2.
2. With the value of R_1 as obtained above, determine the value of V_{CEQ1}. (Assume both T_1 and T_2 are Si transistors)



[10 marks]

Given, $\boxed{\beta = 30}$

$V_{CC} = 12 \quad R_E = 1.5 k\Omega$

1) $R_1 = ? \quad V_{CE_2} = 5V$

Applying KVL in outer loop of T_2

$$-V_{CC} + V_{CE_2} + I_{E_2} R_E = 0$$

$$\Rightarrow I_{E_2} = \frac{12-5}{1.5} = 4.67 mA$$

$$I_{B_2} = \frac{I_{E_2}}{1+\beta} = 0.15 mA$$

since $I_{E_1} = I_{B_2} = 0.15 mA$

$$I_{B_1} = \frac{I_{E_1}}{1+\beta} = 4.856 \times 10^{-3} mA$$

Again KVL

$$-V_{CC} + I_{B_1} R_1 + 0.7 + 0.7 + I_{E_2} R_E = 0$$

$I_{B_1} R_1 = 12 - 1.4 - 4.67 \times 1.5$

$\boxed{R_1 = 740.32\,k\Omega}$

2) KVL in T_1

$-V_{CC} + V_{CE_1} + 0.7 + I_{E_2} R_E = 0$

$V_{CE_1} = \dfrac{12 - 0.7}{4.67 \times 1.5}$

$\boxed{V_{CE_1} = 1.61V}$

7

Do not write in this margin

7 (c) (i) Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point (5, 0, 4). Also calculate the magnitude of the maximum directional derivative.

(ii) What is phantom loading? What is the advantage of it?

[14 + 6 marks]

(i) $f = x^2 - y^2 + 2z^2$

$$\nabla f = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)(x^2 - y^2 + 2z^2)$$

$$\boxed{\nabla f = 2x\hat{i} - 2y\hat{j} + 4z\hat{k}}$$

at $P(1,2,3)$

$$\nabla f\Big|_{at\,P} = 2\hat{i} - 4\hat{j} + 12\hat{k}$$

PQ line $\Rightarrow$ $P(1,2,3)$ $Q(5,0,4)$ 13

$\Rightarrow$ $\hat{a_n} = 4\hat{i} - 2\hat{j} + \hat{k}$

Good Approach

Directional derivative of $f = \nabla f \cdot \hat{a_n}$

$$= (2\hat{i} - 4\hat{j} + 12\hat{k})\left(\frac{4\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{16+4+1}}\right)$$

$$= \frac{8 - 8 + 12}{\sqrt{21}}$$

$$= \frac{12}{\sqrt{21}} \text{ or } \frac{4\sqrt{21}}{7}$$

$$(\nabla f\Big|_{max.} = \sqrt{4 + 16 + 144} = \sqrt{164} = 2\sqrt{41}$$

(ii) Phantom loading is done when the ~~[illegible]~~ low value of current is required to measure so that it helps in saving of wastage of power if it was done on rated load.

In Phantom loading, the required value is not measured at rated load but it is measured at less than rated load which helps in giving accurate value of the required parameter.

Advantages

1) Saving in wastage of power i.e. low power consumption

2) gives accurate values.

5

Q.8 (a) **(i)** Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

(ii) Find the area of the tangent cut-off from the parabola $x^2 = 8y$ by the line $x - 2y - 8 = 0$.

[10 + 10 marks]

Q.8 (b) $\vec{F}(x,y,z) = yz\hat{i} - xz\hat{j} + \hat{k}$. Let '$s$' be the portion of surface of the paraboloid $z = 4 - x^2 - y^2$ which lies above the first octant, and let 'C' be closed curve $C = C_1 + C_2 + C_3$, where curves C_1, C_2 and C_3 are the three curves formed by intersecting 's' with the xy, yz and xz planes respectively so that C is boundary of 's'. Orient C so that it is traversed CCW when seen from above the first octant.

(i) Set up and evaluate the loop integral $\oint_C \vec{F} \cdot \overrightarrow{dr}$ by parameterizing each piece of curve C.

(ii) Verify using Stoke's theorem that loop integral $\oint_C \vec{F} \cdot \overrightarrow{dr}$ is equal to surface integral

$$\iint_s \vec{\nabla} \times \vec{F} \cdot \overrightarrow{ds}.$$

[20 marks]

(c) (i) The electrical conductivity and electron mobility of aluminium are 3.8×10^7 $(\Omega\text{-m})^{-1}$ and 0.0012 $m^2/V\text{-s}$, respectively. Calculate the Hall voltage for an aluminium specimen that is 15 mm thick for a current of 25 A and a magnetic field of 0.6 T. Given that field is perpendicular to current.

(ii) Briefly explain why the ferroelectric behavior of $BaTiO_3$ ceases above its ferroelectric curie temperature.

(iii) Name the types of polarization and briefly explain the type of materials and about mechanism by which dipolar are induced or oriented by the action of an applied electric field. For gaseous-argon, solid LiF, liquid H_2O, what kind(s) of polarization is/are possible?

[6 + 4 + 10 marks]

OOOO

Space for Rough Work

Space for Rough Work