. Highlights your final answer

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ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-7 : Full Syllabus Test (Paper-I)

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	46
Q.2	33
Q.3	48
Q.4	
Section-B	
Q.5	29
Q.6	
Q.7	40
Q.8	
Total Marks Obtained	196

Signature of Evaluator: Sourabh Kumar

Cross Checked by: ....

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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Section-A

Q.1 (a) Find the complete solution of differential equation $(D^2 - 4D + 3)y = \sin 3x \cos 2x$.

[12 marks]

Given differential equation –

$(D^2 - 4D + 3)y = \sin 3x \cos 2x$

For CF :- $m^2 - 4m + 3 = 0$

$(m-3)(m-1) = 0$

$m = 1, 3$

$\therefore$ CF $= c_1 e^{x} + c_2 e^{3x}$

For PI

$\sin 3x \cos 2x = \frac{1}{2}[\sin(3x+2x) + \sin(3x-2x)]$

$= \frac{1}{2}[\sin 5x + \sin x]$

PI $= \frac{1}{D^2-4D+3}\left[\frac{1}{2}(\sin 5x + \sin x)\right]$

$= \frac{1}{D^2-4D+3} \cdot \frac{1}{2}(\sin 5x) + \frac{1}{D^2-4D+3} \cdot \frac{1}{2}(\sin x)$

$= \frac{1}{-25-4D+3} \cdot \frac{1}{2}\sin 5x + \frac{1}{-1-4D+3} \cdot \frac{1}{2}\sin x$

$= \frac{1}{2} \cdot \frac{1}{-4D-21}(\sin 5x) + \frac{1}{2(-4D-1)}(\sin x)$

$$= -\frac{1}{2}\left(\frac{4D-21}{16D^2-(21)^2}\right)(\sin 5x) - \frac{1}{2}\left(\frac{4D-1}{16D^2-1}\right)\sin x$$

$$= -\frac{1}{2}\left(\frac{4D-21}{16(-25)-(21)^2}\right)(\sin 5x) - \frac{1}{2}\left(\frac{4D-1}{16(-1)-1}\right)\sin x$$

$$= \frac{1}{1682}(4D-21)\sin 5x + \frac{1}{34}(4D-1)\sin x$$

$$= \frac{1}{1682}\left[4\frac{d}{dx}(\sin 5x) - 21\sin 5x)\right] + \frac{1}{34}\left\{4\frac{d}{dx}(\sin x) - \sin x\right]$$

$$PI = \frac{1}{1682}\left[20\cos 5x - 21\sin 5x\right) + \frac{1}{34}\left[4\cos x - \sin x\right)$$

Hence, complete solution -

$$y = C.F + P.I$$

$$y = c_1 e^{x} + c_2 e^{3x} + \frac{1}{1682}(20\cos 5x - 21\sin 5x) + \frac{1}{34}(4\cos x - \sin x)$$

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Q.1 (b) An inductive circuit in parallel with a resistive circuit of 20 Ω is connected across 50-Hz supply. The inductive current is 4.3 A and the resistive current is 2.7 A. The total current is 5.8 A.

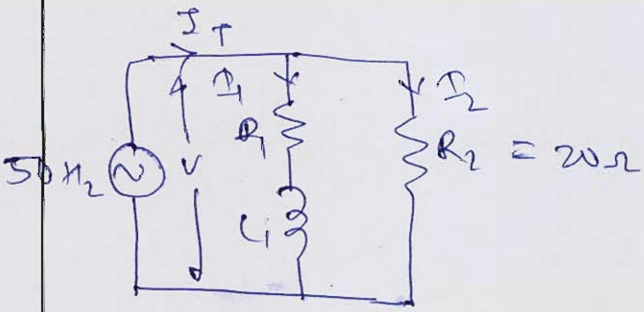
Find:

(i) Power absorbed by the inductive branch.

(ii) Inductance of inductive branch.

(iii) Power factor of the combined circuit. Also draw the phasor diagram.

[12 marks]

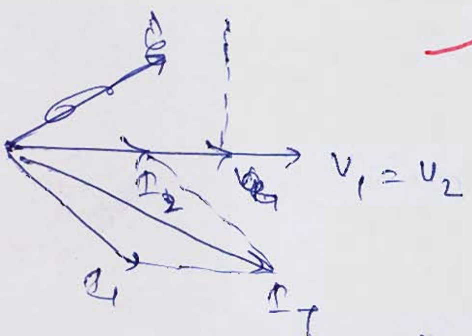


$I_1 = 4.3A$

$I_2 = 2.7 A$

$I_T = 5.8 A$

phasor diagram



① $V_L = I_2 R_2$

$= 2.7 \times 20$

$= 54 V$

$\bar{I}_T = \bar{I}_1 + \bar{I}_2$

$I_T^2 = I_1^2 + I_2^2 + 2 I_1 I_2 \cos\phi$

$\Rightarrow (5.8)^2 = 4.3^2 + 2.7^2 + 2 \times 4.3 \times 2.7 \times \cos\phi$

$\phi = 70.21°$

Power consumed by inductive branch

$P = V I_1 \cos\phi$

$= 54 \times 4.3 \times \cos 70.21$

$P = 78.61 W$

(ii) $I_1 = 4.3\,A$ $\quad \cos\phi = 70.21$

$V = 54\,V$

$$Z = \frac{V}{I_1} = \frac{54}{4.3} = 12.55\,\Omega$$

$X_L = Z \sin\phi$

$= 12.55 \sin 70.21$

$X_L = 11.81\,\Omega$

$$L = \frac{11.81}{2\pi \times 50} = 0.0376\,H$$

$$\boxed{L = 37.61\,mH}$$

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(iii) Total power consumed

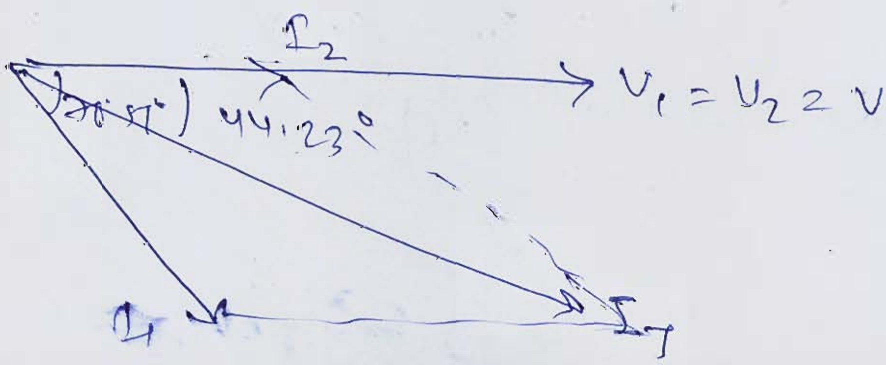
$P_T = P_1 + P_2$

$= 78.61 + (2.7)^2 \times 20 = 224.41$

Good Approach

overall power factor

$$\cos\phi = \frac{224.41}{54 \times 5.8} = 0.7165 \text{ lag}$$



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Q.1 (c) Determine the percentage of ionic polarizability in sodium chloride crystal which has the optical index of refraction and the static dielectric constant are 1.5 and 5.6 respectively.
[12 marks]

Clausius equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

$\therefore \eta = 1.5 \quad \epsilon_r = 5.6$

$$\frac{5.6-1}{5.6+2} = \frac{N\alpha_T}{3\epsilon_0}$$

$$\alpha_T = \frac{1.815\,\epsilon_0}{N} \quad — (1)$$

and
$$\frac{(1.5)^2-1}{(1.5)^2+2} = \frac{N\alpha_e}{3\epsilon_0}$$

$$\alpha_e = \frac{0.882\,\epsilon_0}{N}$$

$\therefore$ ionic polarizability

$$\alpha_i = \alpha_T - \alpha_e = \frac{1.815\,\epsilon_0}{N} - \frac{0.882\,\epsilon_0}{N}$$

$$= \frac{0.9326\,\epsilon_0}{N}$$

% ionic polarizability

$$= \frac{\alpha_i}{\alpha_T} \times 100 = \frac{0.9326}{1.815} \times 100$$

$$= 51.38\,\%$$

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Good Approach

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Q.1 (d) An energy meter is designed to have 80 revolutions of the disc per unit of energy consumed. Calculate the number of revolutions made by the disc when measuring the energy consumed by the load carrying 30 A at 230 V and 0.6 power factor. Find the percentage error if the meter actually makes 330 revolutions. Also specify whether the meter runs slower or faster.

[12 marks]

Given :- $K = 80$ rev/ kWh

Load current $I = 30\,A$

$V = 230\,V$ and $\cos\phi = 0.6$

Total energy consumed in one hour -

$$E = \frac{VI\cos\phi}{1000}$$

$$= \frac{230 \times 30 \times 0.6}{1000}$$

$E = 4.14$ kWh

∴ Number of revolution made by the disc -

$N_T = E \times K$

$= 4.14 \times 80$

$N_T = 331.2$ rev

∴ measured no. of revolution

$N_m = 330$ rev

% error $= \dfrac{N_m - N_T}{N_T} \times 100$

$= \dfrac{330 - 331.2}{331.2} \times 100$

% error = − 0.36 %

-ve sign shown meter runs slower

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Good Approach

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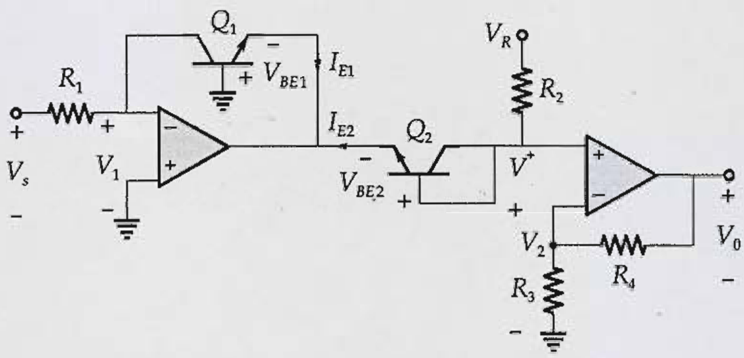
Q.1 (e) The figure shows a modified logarithmic amplifier to overcome the undesirable effects of temperature-dependent V_T and I_S (reverse saturation current). Show that if the two transistors Q_1 and Q_2 are matched transistors, then the output V_0 is truly proportional to $\ln(V_s)$.



[12 marks]

KCL at input node –

$$\frac{V_s}{R_1} = I_{E_1}$$

$$V_s = I_{E_1} \cdot R_1$$

and $$V_2 = \frac{R_3}{R_3 + R_4} \cdot V_0$$

$$V_+ = V_- = V_2 = \frac{R_3}{R_3 + R_4} \cdot V_0$$

$$I_{E_2} = \frac{V_R - V_+}{R_2}$$

$$= \frac{V_R - \left(\frac{R_3}{R_3 + R_4}\right) V_0}{R_2}$$

$$I_{E_2} = \frac{V_R (R_3 + R_4) - R_3 V_0}{R_2 (R_3 + R_4)}$$

If transistor are matched

$I_{E_1} = I_{E_2}$

$$\frac{V_S}{R_1} = \frac{V_R (R_3 + R_4) - R_3 V_O}{R_2 (R_3 + R_4)}$$

~~$R_1 = R_2 = R_3$~~

$$I_{E_1} = I_S e^{\frac{V_{BE_1}}{V_T}}$$

$$I_{E_2} = I_S \cdot e^{\frac{V_{BE_2}}{V_T}}$$

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$$V_{BE_1} = -\frac{R_f}{R_1} V_S$$

$$V_{BE_2} = \frac{R_3 V_O}{R_2 + R_4}$$

$$\frac{V_S}{R_1} = I_S e^{\frac{R_3 V_O}{(R_2 + R_4) V_T}}$$

$$\frac{R_3}{R_2 + R_4} \cdot \frac{V_O}{V_T} = \ln \left(\frac{V_S}{R_1 I_S} \right)$$

$$V_O = \frac{V_T (R_3 + R_4)}{R_4} \ln \left(\frac{V_S}{R_1 I_S} \right)$$

$$\boxed{V_O \propto \ln V_S}$$

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Q.2 (a) (i) The read access times and the hit ratios for different caches in a memory hierarchy are as given below:

Code	Read access time (in nanoseconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

The read access time of main memory is 90 nanoseconds. Assume that the caches use the referred word-first read policy and the write back policy. Assume that all the caches are direct mapped caches. Assume that the dirty bit is always 0 for all the blocks in the caches. In execution of a program, 60% of memory reads are for instruction fetch and 40% are for memory operand fetch. Find the average read access time in nanoseconds.

[10 marks]

∴ Read access time of

$t_{mm} = 90\ ns$

For memory operand fetch

$t_{avg_1} = 0.9 \times 8 + 0.1 \times 90$

$= 16.2\ ns$

(2)

For memory reads –

$t_{avg_2} = 2 \times 0.8 + 0.1 \times 90$

$= 10.6\ ns$

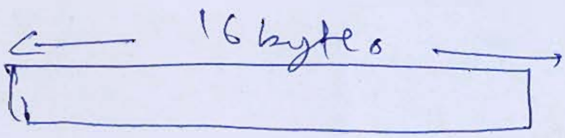
average read access time

$t_{avg} = 0.6 \times 10.6 + 0.4 \times 16.2$

$= 12.84\ ns$

Q.2 (a) (ii) A certain processor uses a fully associative cache of size 16 kB. The cache block size is 16 bytes. Assume that the main memory is byte addressable and uses a 32-bit address. How many bits are required for the Tag and the Index fields respectively in the addresses generated by the processor?

[10 marks]



cache size $= 16\,KB$

$= 2^{10} \cdot 2^{11}$

$= 2^{16}$

for 32 bit address

index field $= \frac{16\,KB}{32}$

$= \frac{2^{16}}{2^{5}} = 2^{13}$

$= 13$ bit

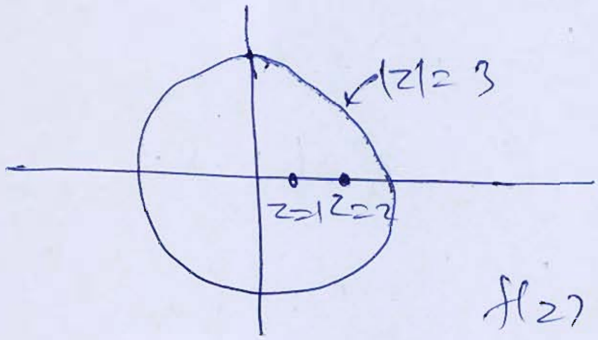
tag field $= 16 - 13$

$= 3$ bit

(2)

Q.2 (b) (i) Find the value of $\int_C \frac{\cos \pi z^2}{(z-2)(z-1)} dz$, where 'C' is $|z| = 3$.

[8 marks]



$f(z) = \cos \pi z^2$

For pole $z = 1$

Residue $R_1 = \left[\frac{[\cos \pi z^2]}{(z-1)(z-2)} \times (z-1) \right]_{z=1}$

$= \frac{\cos \pi (1)^2}{(1-2)}$

$R_1 = \frac{\cos \pi}{-1} = 1$

For pole $z = 2$

Residue $R_2 = \left[(z-2) \times \frac{\cos \pi z^2}{(z-1)(z-2)} \right]_{z=2}$

$= \frac{\cos \pi (2)^2}{(2-1)}$

$= \frac{\cos 4\pi}{1}$

$R_2 = 1$

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$\therefore \int \frac{\cos \pi z^2}{(z-2)(z-1)} dz = 2\pi i \{\text{sum of Residues}\}$

$= 2\pi i \times (1+1)$

$= 4\pi i$ (Ans)

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Good Approach

Q.2 (b) (ii) Solve $(x^2 - yz)\frac{\partial p}{\partial x} + (y^2 - zx)\frac{\partial p}{\partial y} = z^2 - xy$.

[12 marks]

for the given linear partial differential equation —

$$P = x^2 - yz, \quad Q = y^2 - zx$$
$$R = z^2 - xy$$

$$\therefore \quad \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$$

$$\Rightarrow x\,dx = y\,dy = z\,dz$$
$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 + y^3 + z^3 - 3xyz}$$

$$x\,dx + y\,dy + z\,dz = 0$$

$$\boxed{x^2 + y^2 + z^2 = c_1} \quad \text{— (1)}$$

$$\Rightarrow \frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx}$$

$$y^2\,dx - xz\,dx = x^2\,dy - yz\,dy$$

$$\Rightarrow x^2\,dy - y^2\,dx = yz\,dy - xz\,dx$$

$$x + yz = c_2 \quad \text{— (2)}$$

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Hence, the solution is

$f(x+yz, x^2+y^2+z^2) = 0$

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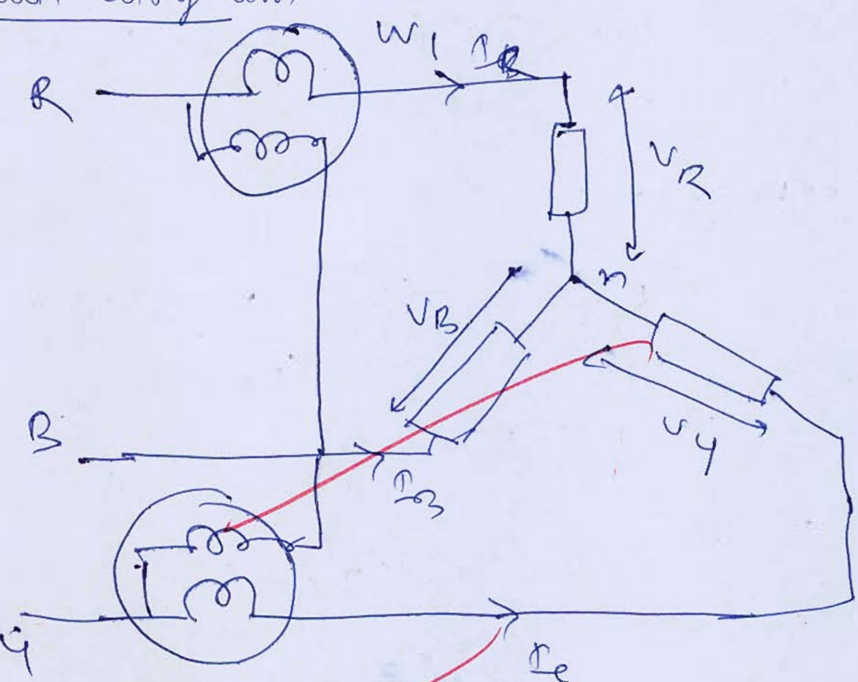
Q.2 (c) Draw the circuit arrangement for power measurement in a 3-phase, 3-wire balanced supply and load using two-wattmeter method, and show that the power factor of the load is given by

$$\cos\phi = \frac{1}{\sqrt{1+3\left(\frac{P_1-P_2}{P_1+P_2}\right)^2}}$$

where P_1 and P_2 are powers indicated by Wattmeter 1 and Wattmeter 2, respectively.

[20 marks]

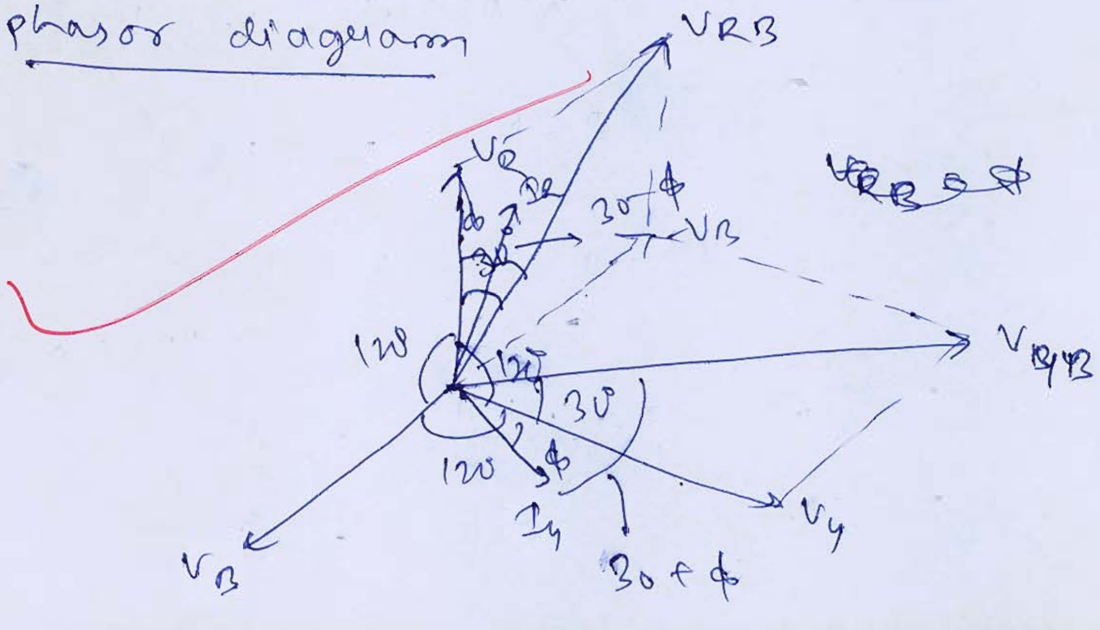
circuit diagram



$\therefore$ $V_R = V_Y = V_B = V$ (per phase)

$I_R = I_Y = I_B = I$ (per phase)

phasor diagram



For wattmeter 1

$P_1 = V_{RB} I_R \cos\phi$ (angle b/w V_{RB} & I_R)

$= \sqrt{3} V \times I \times \cos(30-\phi)$

$P_1 = \sqrt{3} VI \cos(30-\phi)$ — ①

For wattmeter 2

$P_2 = V_{YB} I_Y \cos$ (angle b/w V_{YB} & I_Y)

$= \sqrt{3} V \times I \times \cos(30+\phi)$

$P_2 = \sqrt{3} VI \cos(30+\phi)$

$P_1 + P_2 = \sqrt{3} VI \cos(30-\phi) + \sqrt{3} VI \cos(30+\phi)$

$= \sqrt{3} VI \times 2\cos 30^\circ \times \cos\phi$

$P_1 + P_2 = 3 VI \cos\phi$ — ①

$P_1 - P_2 = \sqrt{3} VI \cos(30-\phi) - \sqrt{3} VI \cos(30+\phi)$

$= \sqrt{3} VI \times 2 \times \sin 30^\circ \sin\phi$

$P_1 - P_2 = \sqrt{3} VI \sin\phi$ — ②

Dividing (2) by (1) -

$$\frac{1}{\sqrt{3}} \tan\phi = \left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

∴ power of the load

$$\cos\phi = \frac{1}{\sqrt{1 + \left[\sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)\right]^2}}$$

$$\cos\phi = \frac{1}{\left[1 + 3\left(\frac{P_1 - P_2}{P_1 + P_2}\right)^2\right]^{1/2}}$$

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Q.3 (a) Find the matrix P which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. Hence calculate A^4 by using matrix P.

[20 marks]

Given matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

eigen values of matrix A –

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow (1-\lambda)[(5-\lambda)(1-\lambda)-1] - 1[1-\lambda-3] + 3(1-3(5-\lambda)] = 0$

$\Rightarrow (1-\lambda)[5-6\lambda+\lambda^2-1] - 1(-\lambda-2) + 3(3\lambda-14) = 0$

$\Rightarrow 4 - 6\lambda + \lambda^2 - 4\lambda + 6\lambda^2 - \lambda^3 + \lambda + 2 + 9\lambda - 42 = 0$

$\lambda^3 - 7\lambda^2 + 36 = 0$

$\lambda = -2, 6, 3$

For eigen vector $[A - \lambda I][X] = 0$

for $\lambda = -2$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

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$$3x + y + 3z = 0 \quad --- (1)$$
$$x + 7y + z = 0 \quad --- (2)$$
$$3x + y + 3z = 0 \quad --- (3)$$

solving (1) and (2) –

$$\frac{x}{-20} = \frac{-y}{0} = \frac{z}{20} = k$$

$$x = -k \quad y = 0 \quad z = k$$

∴ For $\lambda = -2$ eigen vector $X_1 = k\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

For $\lambda = 6$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-5x + y + 3z = 0 \quad -- (4)$$
$$x - y + z = 0 \quad -- (5)$$
$$3x + y - 5z = 0 \quad -- (6)$$

solving (4) and (5) we get –

$$\frac{x}{4} = \frac{-y}{-8} = \frac{z}{4} = k$$

$$x = k \quad y = 2k \quad z = k$$

For $\lambda = 6$ eigen vector $X_2 = k\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

for $\lambda = 3$
$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2x + y + 3z = 0$ --- (7)
$x + 2y + z = 0$ --- (8)
$3x + y - 2z = 0$ --- (9)

solving (7) and (8) we get -

$$\frac{x}{-5} = \frac{-y}{-5} = \frac{z}{-5} = k$$

$x = -k \quad y = k \quad z = -k$

for $\lambda = 3$, eigen vector of $X_3 = k\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

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$\therefore$ matrix $P = \left[\frac{X_1}{|X_1|} \quad \frac{X_2}{|X_2|} \quad \frac{X_3}{|X_3|} \right]$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & -1/\sqrt{3} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 1/\sqrt{3} \end{bmatrix}$$

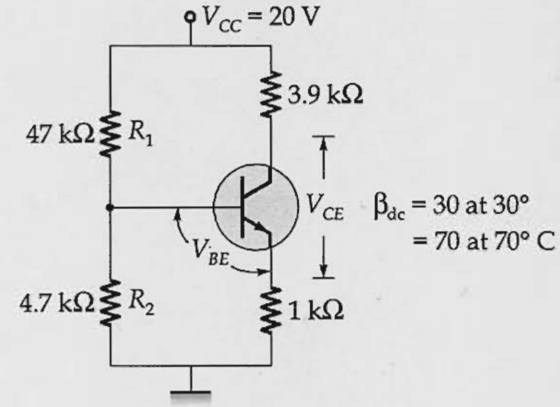
Try to avoid

and $A^4 = P D^4 P^{-1}$

$$= \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & -1/\sqrt{3} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 16 & 0 & 0 \\ 0 & 1296 & 0 \\ 0 & 0 & 81 \end{bmatrix} \begin{bmatrix} -0.70 & \\ 0.40 & \\ 0.5 & \end{bmatrix}$$

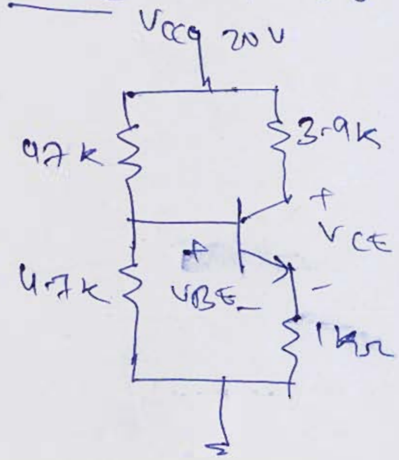
$$A^4 = \begin{bmatrix} 250.97 & 404.91 & 234.97 \\ 404.95 & 890.58 & 404.95 \\ 224.97 & 404.96 & 250.97 \end{bmatrix}$$ (Ans)

Q.3 (b) The transistor shown in figure is a silicon transistor. The junction temperature increases from 30° to 70°. If β = 30 at 30° and β = 70 at 70°, determine the percent change in D.C. bias point over the temperature range 30° to 70° neglecting change in base to emitter voltage.



[20 marks]

Case I at 30° $\beta_1 = 30$ $V_{BE} = 0.7\,V$

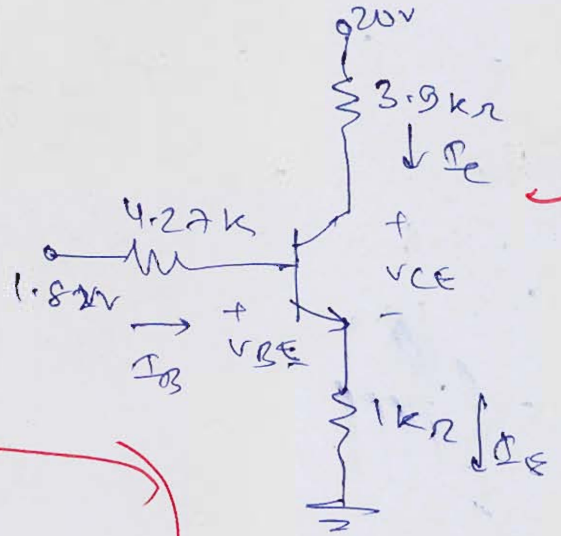


$V_{th} = 20 \times \left(\frac{4.7k}{47k + 4.7k}\right)$

$V_{th} = 1.82\,V$

$R_{th} = 4.7k \,||\, 47k$

$= 4.27k$



KVL in B-E loop –

$1.82 - 4.27 I_B - 0.7 - (30+1) \times 1\, I_B = 0$

$I_{B_1} = 0.0317\,mA$

$I_{C_1} = \beta I_B$

$I_{C_1} = 30 \times 0.0317$

$I_{C_1} = 0.9526\,mA$

$I_{E_1} = I_{B_1} + I_{C_1} = 0.984\,mA$

KVL in CE loop

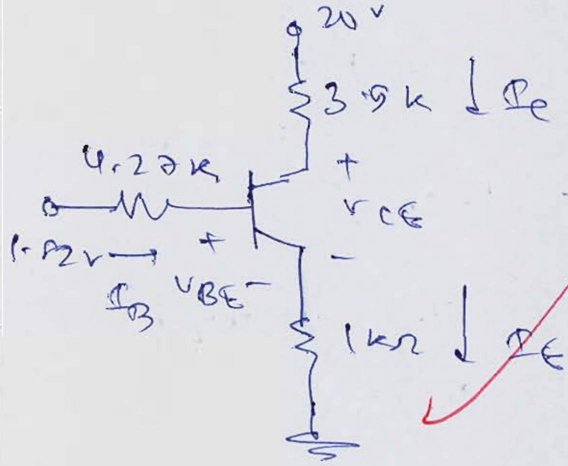
$$20 - 3.9 \times 0.9526 - V_{CE_1} - 0.954 \times 1 = 0$$

$$V_{CE_1} = 15.3\ V$$

Hence, at 30°C $\beta_1 = 30$ DC bias point

$$I_{C_1} = 0.9526\ mA$$
$$V_{CE_1} = 15.3\ V$$

Case II at 70°C $\beta = 70$



KVL in B-E loop

$$1.82 - 4.27 I_B - 0.7 - (70+1) \times 1 \times I_B = 0$$

$$I_{B_2} = 0.0148\ mA$$

$$I_{C_2} = \beta_2 I_{B_2}$$
$$= 70 \times 0.0148$$
$$I_{C_2} = 1.041\ mA$$

$$I_{E_2} = I_{B_2} + I_{C_2} = 1.056\ mA$$

KVL in CE loop

$$20 - 3.9 \times 1.041 - V_{CE_2} - 1 \times 1.056 = 0$$

$$V_{CE_2} = 14.88\ V$$

Hence at 70°C $\beta_2 = 70$ DC bias point –

$I_{C_2} = 1.041 mA$

$V_{CE_2} = 14.58 V$

∴ % change in $I_C = \frac{I_{C_2} - I_{C_1}}{I_{C_1}} \times 100$

$= \frac{1.041 - 0.9526}{0.9526} \times 100$

$= + 9.279 \%$

% change in $V_{CE} = \frac{V_{CE_2} - V_{CE_1}}{V_{CE_1}} \times 100$

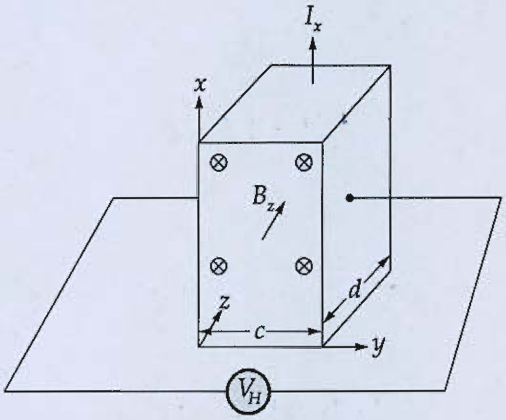
$= \frac{14.58 - 15.3}{15.3} \times 100$

$= - 2.745 \%$

18

Good Approach

Q.3 (c) (i) What is Hall effect? For a parallelepiped specimen having one corner situated at origin and externally applied electric field causing current in positive x-direction as shown below:



State what happens when magnetic field B_z is applied in positive z-direction in reference to Hall voltage. Determine electron mobility relation using Hall coefficient and conductivity (σ).

[10 marks]

Hall effect – when ~~semiconductor~~ current following semiconductor is placed in magnetic field then emf is induced. this effect is called Hall effect.

$\therefore \quad E_H = \frac{V_H}{c}(-\hat{a}_y)$

$F_m = q \; I \; \hat{a}_x \times B(-\hat{a}_z)$

$F_m = B I (\hat{a}_y)$

$F_e = q E_H$

$= (-e) \cdot \frac{V_H}{c}(-\hat{a}_y)$

$\therefore \quad F_m + F_e = 0$

$$BI = (ne)\frac{V_H}{c}$$

$R_H = \frac{1}{ne}$ = Hall coefficient

$$V_H = \frac{BI \cdot c}{ne}$$

$$V_H = \frac{R_H BI}{c}$$

Conductivity

$$\sigma = ne\mu$$

$$= \frac{\mu}{R_H}$$

∴ mobility, $\mu = \sigma \times R_H$

$$\mu = \sigma R_H$$

8

Application of Hall voltage –

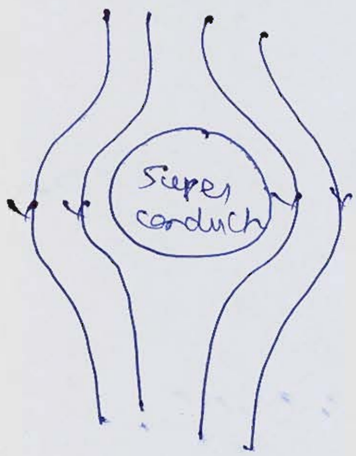
Good Approach

- To determine type of semiconductor
- To determine mobility of electron and hole
- Determine the conductivity

Q.3 (c) **(ii)** What is Meissner effect and how it can be used to justify negative susceptibility of superconductors? How critical field, H_C for a superconductor material varies with temperature? Explain briefly factors that affect transition temperature of superconductor.

[10 marks]

Meissner effect - when super conductor is plac in a magnetic field then the ~~magnetic~~ super conductor perfectly repel the ~~field repels~~ the magnetic field.



$\mu_r = 0$ inside the super conductor

$\mu_r = \chi_m + 1 = 0$

$\boxed{\chi_m = -1}$ — condition from

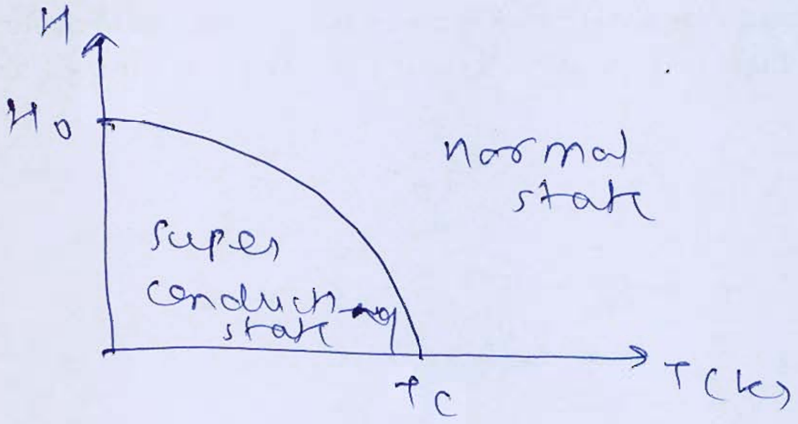
per diamagnetic super conductor behaviour in a external magnetic field

$\therefore$

$$H_C = H_0\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

H_C = critical magnetic field at temp T

H_0 = critical magnetic field at 0 K



The transition temperature of superconductor is affect by -

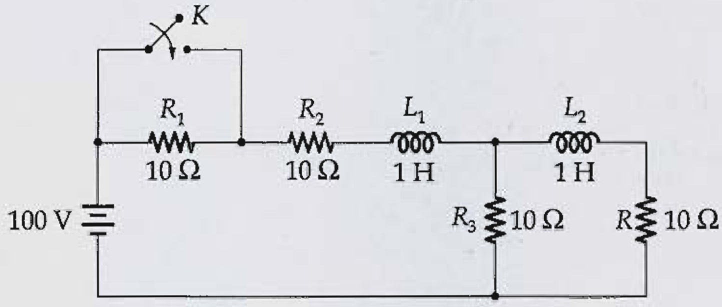
① current following the conductor if current is more than critical current then superconducting state is destroy below the transition temperature

② External magnetic H_c is varies with temperature. If the external magnetic is more then the transition temperature is reduced.

8

Good Approach

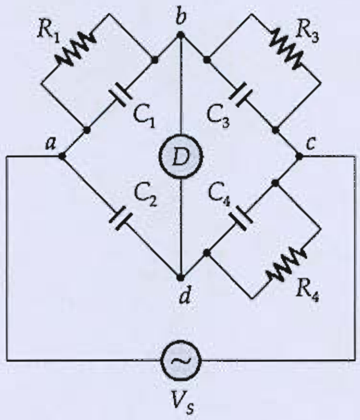
Q.4 (a) In the network of below figure, the switch K is closed at time $t = 0$, a steady state having previously existed. Obtain the expression of current in the resistor R using Thevenin's theorem.



[20 marks]

Q.4 (b) (i) In a low-voltage bridge designed for the measurement of permittivity, the branch *ab* consists of two electrodes between which the specimen under test may by inserted; arm *bc* is a non-reactive resistor R_3 in parallel with a standard capacitor C_3, arm *cd* is non-reactive resistor R_4 in parallel with a standard capacitor C_4; arm *da* is a standard air capacitor of capacitance C_2 without the specimen between the electrodes, balance is obtained with the following values :

$C_3 = C_4 = 120$ pF, $C_2 = 150$ pF, $R_3 = R_4 = 5000\ \Omega$ with the specimen inserted these values become $C_3 = 200$ pF, $C_4 = 1000$ pF, $C_2 = 900$ pF and $R_3 = R_4 = 5000\ \Omega$. In each test $\omega = 5000$ rad/sec. Find the relative permittivity of the specimen.



[12 marks]

Q.4 (b) (ii) A CRT has an anode voltage of 2000 V and parallel deflecting plates 2 cm long and 5 mm apart. The screen is 30 cm from the centre of the plates. Find the input voltage required to deflect the beam through 3 cm. The input voltage is applied to the deflecting plates through amplifiers having an overall gain of 100.

[8 marks]

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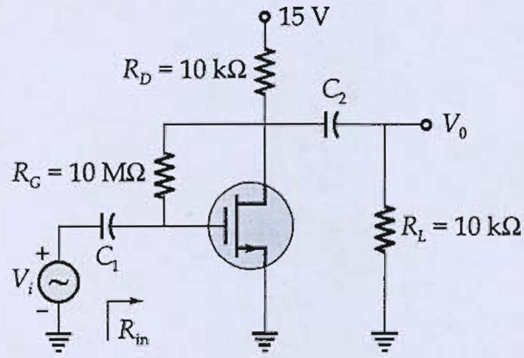
Q.4 (c) Determine the small-signal voltage gain, its input resistance and the largest allowable input signal. The transistor has $V_t = 1.5$ V, $K_n'\left(\frac{W}{L}\right) = 0.25$ mA/V^2 and $V_A = 50$ V. Assume the coupling capacitors to be sufficiently large so as they act as short circuits at the signal frequencies of interest.



[20 marks]

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Section-B

Q.5 (a) Consider a 2-way set associative cache memory with 4 sets and total 8 cache blocks (0-7) and a main memory with 128 blocks (0-127). What memory blocks will be present in the cache after the following sequence of memory block references if LRU policy is used for cache block replacement? Assuming that initially the cache did not have any memory block from the current job.

0 5 3 9 7 0 16 55

[12 marks]

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Q.5 (b) Obtain the partial differential equation from function $f(xy + z, x^2 + y^2 - z^2) = 0$.

[12 marks]

$u = xy + z$

$v = x^2 + y^2 - z^2$

$u_x = y + \frac{\partial z}{\partial x} = y + zp \quad -(1)$

$u_y = x + \frac{\partial z}{\partial y} = x + zq \quad -(2)$

$v_x = 2x - 2z\frac{\partial z}{\partial x} = 2x - 2zp \quad -(3)$

$v_y = 2y - 2z\frac{\partial z}{\partial y} = 2y - 2zq \quad -(4)$

$f = u + v$

$f_x = u_x + v_x$

$f_x = y + zp + 2x - 2zp$

$f_x = 2x + y - zp \quad -(5)$

$f_y = u_y + v_y$

$f_y = x + zq + 2y - 2zq$

$f_y = x + 2y - zq \quad -(6)$

Solving eq - (5) and (6) we get

$(2x - 2zp)(x + q) = (2y - 2zq)(y + p)$

(3)

Im

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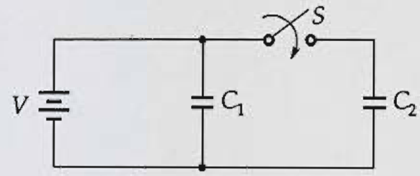
Q.5 (c) Figure below shows the two identical parallel-plate capacitors connected to a battery with switch S closed. The switch S is opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric coefficient $K = 2$. Find the ratio of the electrostatic energy stored in capacitor C_2 to electrostatic energy in capacitor C_1 after the introduction of the dielectric.



[12 marks]

Given initially, $C_1 = C_2 = C$

$V_1 = V_2 = V$

Now capacitor filled with dielectric of coefficient $K = 2$

$\therefore C_1 = 2C$

Electrostatic energy in C_1

$\therefore$ ~~$E_0 = \epsilon E$~~ $E_1 = E_2 = E$ (same electric field)

$W_{E_1} = \frac{1}{2} \epsilon E_1^2$

$= \frac{1}{2} \times \epsilon 2 \times \epsilon_0 \times E_1^2$

$W_{E_1} = \epsilon_0 E^2$

electrostatic energy in C_2

$W_{E_2} = \frac{1}{2} \epsilon E_2^2$

$= \frac{1}{2} \times \epsilon_0 E^2$

$$\frac{W_{E_2}}{W_{E_1}} = \frac{\frac{1}{2}\epsilon_0 E^2}{\epsilon_0 E^2}$$

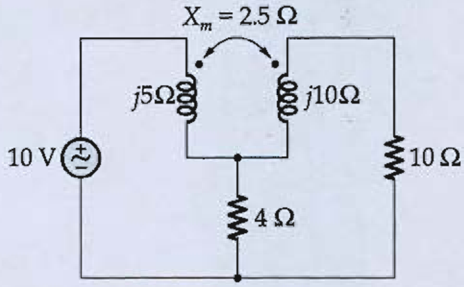
$$\boxed{\frac{W_{E_2}}{W_{E_1}} = \frac{1}{2}}$$

Hence, electrostatic energy stored in C_2 to C_2 is 1 : 2

11

Good Approach

Q.5 (d) Figure below shows a network with mutual coupling.

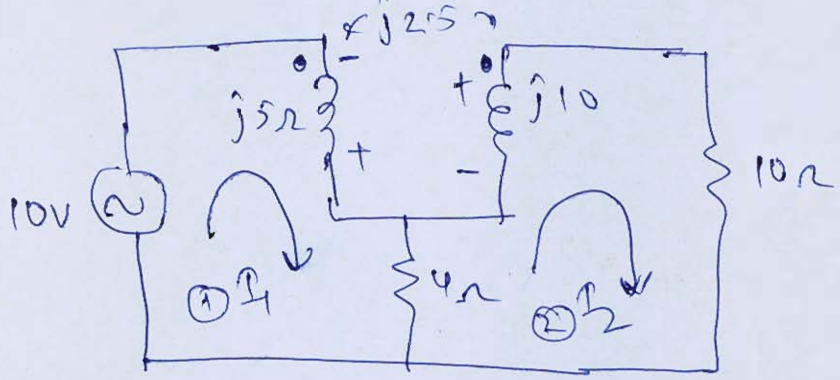


(i) Find the current in the 10 Ω resistance. Assume that inductor have negligible resistance.

(ii) If the direction of winding of one of the coils is reversed, find the current in the 10 Ω resistance.

[12 marks]

(1)



KVL in loop 1

$10 - j5 I_1 + j2.5 I_2 - 4(I_1 - I_2) = 0$

$(4+j5) I_1 - (4+j2.5) I_2 = 10 \quad -(1)$

KVL in loop 2

$-4(I_2 - I_1) - j10 I_2 + j2.5 I_1 - 10 = 0$

$(4+j2.5) I_1 - (14+j10) I_2 = 0 \quad -(2)$

Solving equation (1) and (2) by Cramer's rule —

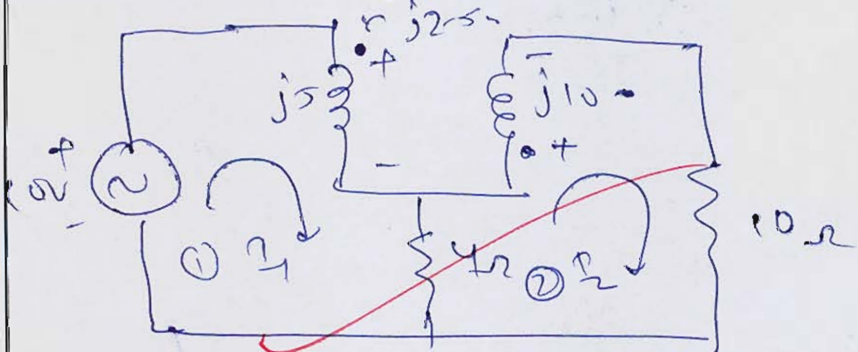
$$I_2 = \frac{\begin{vmatrix} 4+j5 & 10 \\ 4+j2.5 & 0 \end{vmatrix}}{\begin{vmatrix} 4+j5 & -(4+j2.5) \\ (4+j2.5) & -(14+j10) \end{vmatrix}}$$

$$I_2 = \frac{-10(4+j2.5)}{-(4+j5)(14+j10) + (4+j2.5)^2}$$

$$I_2 = \frac{47.17\angle -148}{90\angle -87.61}$$

$$\boxed{I_2 = 0.523\angle -60.38^\circ \text{ A}}$$

(ii) Direction of winding is reverse of.



KVL in loop ① -

$$10 - j5 I_1 - j2.5 I_2 - 4(I_1 - I_2) = 0$$

$$(4+j5) I_1 - (4-j2.5) I_2 = 10 \quad \text{—③}$$

KVL in loop ② -

$$-4(I_2 - I_1) - j10 I_2 - j2.5 I_1 - 10 I_2 = 0$$

$$(4-j2.5) I_1 - (14+j10) I_2 = 0 \quad \text{—④}$$

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solving eq- (3) and (4) we get -

$$I_2 = \frac{\begin{vmatrix} 4+j5 & 10 \\ 4-j2.5 & 0 \end{vmatrix}}{\begin{vmatrix} 4+j5 & -(4-j2.5) \\ (4-j2.5) & -(14+j10) \end{vmatrix}}$$

$$= \frac{-10(4-j2.5)}{-(4+j5)(14+j10) + (4-j2.5)^2}$$

$$I_2 = \frac{47.17 \angle 148^\circ}{130 \angle -88.34}$$

$$\boxed{I_2 = 0.362 \angle -123.65^\circ \text{ A}}$$

11

Good Approach

Q.5 (e) Consider a common-emitter circuit using a BJT having $I_s = 10^{-15}$ A, a collector resistance $R_C = 6.8$ kΩ and a power supply $V_{CC} = 10$ V and $V_{CE} = 3.2$ V.

(i) Find the positive increment in V_{BE} (above V_{BE}) that drives the transistor to the edge of saturation, where $V_{CE(sat)} = 0.3$ V.

(ii) Find the negative increment in V_{BE} that drives the transistor to within 1% of cut-off (i.e. to $V_0 = 0.99\ V_{CC}$).

[Take $V_T = 25$ mV] **[12 marks]**

Given:- $I_s = 10^{-15}$ A

$R_C = 6.8$ kΩ $V_{CC} = 10$ V $V_{CE} = 3.2$ V

① ~~At saturation~~ $I_C = \frac{V_{CC} - V_{CE}}{R_C}$

$I_C = \frac{10 - 3.2}{6.8} = 1$ mA

$\therefore I_C = I_0 e^{\frac{V_{BE}}{V_T}}$

$\Rightarrow 1 \times 10^{-3} = 10^{-15} e^{\frac{V_{BE}}{25 mV}}$

$\frac{V_{BE}}{25 mV} = \ln\left(\frac{1 \times 10^{-3}}{10^{-15}}\right)$

$V_{BE} = 25 mV \times 27.63$

$V_{BE_1} = 0.690$ V

4

saturation $I_C = \frac{10 - 0.3}{6.8} = 1.42$ mA

$1.42 \times 10^{-3} = 10^{-15} e^{\frac{V_{BE_2}}{25 mV}}$

$\frac{V_{BE_2}}{25 mV} = \ln\left(\frac{1.42 \times 10^{-3}}{10^{-15}}\right)$

$$V_{BE_2} = 0.6995\,V$$

% increase in V_{BE} = $\frac{V_{BE_2} - V_{BE_1}}{V_{BE_1}} \times 100$

$= \frac{0.6995 - 0.690}{0.6995} \times 100$

$= 1.376\,\%$

(ii) $I_C = \frac{0.01\,V_{CE}}{R_C} = \frac{0.01 \times 10}{6.8}$

$= 0.0147\,mA$

$0.0147 \times 10^{-3} = 10^{-15}\, e^{\frac{V_{BE_3}}{25\,mV}}$

$\frac{V_{BE_3}}{25\,mV} = \ln\left(\frac{0.0147 \times 10^{-3}}{10^{-15}}\right)$

$$V_{BE_3} = 0.585\,V$$

% decrease in V_{BE} = $\frac{V_{BE_3} - V_{BE_1}}{V_{BE_1}} \times 100$

$= \frac{0.585 - 0.690}{0.690} \times 100$

$= -15.21\,\%$

Q.6 (a) (i) Compare RISC and CISC architecture.

(ii) State and explain the instruction and data stream types based on Flynn's classification.

(iii) Consider the process table with time quantum '4'.

P_{id}	Arrival time	Burst time
1	2	5
2	4	3
3	1	6
4	2	2
5	3	7

What is the average TAT and average WT using Round Robin scheduling?

[4 + 8 + 8 marks]

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Q.6 (b) Find the Fourier series of $f(x)$,

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(1-x) & 1 \le x \le 2 \end{cases}$$

Also find the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$

[20 marks]

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.6 (c) (i) The number of atoms in a volume of one cubic metre of hydrogen gas is 9.8×10^{26}. The radius of hydrogen atom is 0.53 Å. Calculate the polarizability and relative permittivity of the hydrogen gas.

(ii) The magnetic field in a piece of copper and another piece of Fe_2O_3 is 10^6 Am^{-1}. Their magnetic susceptibilities are -0.5×10^{-5} and 1.4×10^{-3} respectively. Compare the flux density and magnetization in the two pieces.

[10 + 10 marks]

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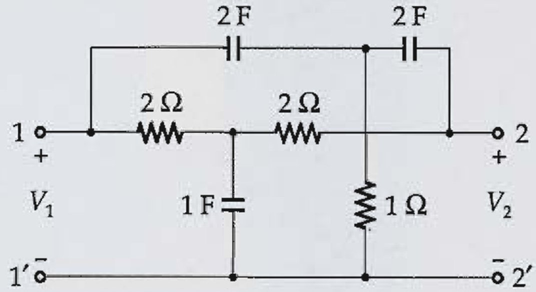
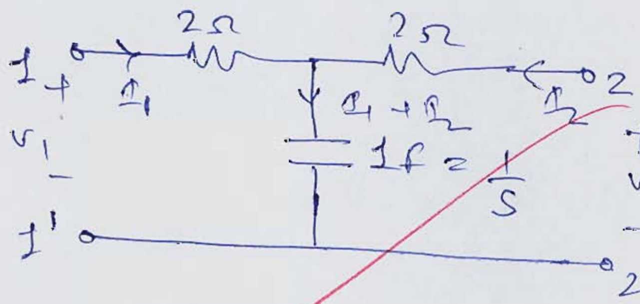
Q.7 (a) For the notch-filter network, determine the y-parameters.



[20 marks]

Given notch filter is parallel combination of 2 circuit –

1st circuit



KVL

$$v_1 = \left(2 + \frac{1}{s}\right) I_1 + \frac{1}{s} I_2$$

$$V_2 = \frac{1}{s} I_1 + \left(2 + \frac{1}{s}\right) I_2$$

$$\therefore \quad Z_1 = \begin{bmatrix} 2 + \frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & 2 + \frac{1}{s} \end{bmatrix}$$

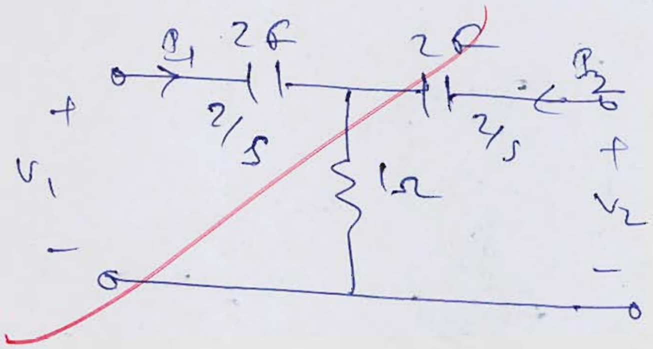
$$Y_1 = [Z_1]^{-1} = \begin{bmatrix} 2 + \frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & 2 + \frac{1}{s} \end{bmatrix}^{-1}$$

$$Y_1 = \frac{\begin{bmatrix} 2 + \frac{1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & 2 + \frac{1}{s} \end{bmatrix}}{\left(2 + \frac{1}{s}\right)^2 - 1/s^2}$$

$$= \frac{\begin{bmatrix} 2+\frac{1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & 2+\frac{1}{s} \end{bmatrix}}{4 + \frac{1}{s^2} + \frac{4}{s} - \frac{1}{s^2}}$$

$$Y_1 = \frac{s}{4s+4}\begin{bmatrix} 2+\frac{1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & 2+\frac{1}{s} \end{bmatrix}$$

2nd circuit



KVL

$$V_1 = \left(\frac{2}{s}+1\right) I_1 + I_2$$

$$V_2 = I_1 + \left(\frac{2}{s}+1\right) I_2$$

$$Z_2 = \begin{bmatrix} \frac{2}{s}+1 & 1 \\ 1 & \frac{2}{s}+1 \end{bmatrix}$$

$$Y_2 = [Z_2]^{-1} = \begin{bmatrix} \frac{2}{s}+1 & 1 \\ 1 & \frac{2}{s}+1 \end{bmatrix}^{-1}$$

$$Y_2 = \frac{\begin{bmatrix} 2/s+1 & -1 \\ -1 & 2/s+1 \end{bmatrix}}{\left(\frac{2}{s}+1\right)^2 - 1}$$

$$Y_2 = \frac{\begin{bmatrix} \frac{2}{s}+1 & -1 \\ -1 & \frac{2}{s}+1 \end{bmatrix}}{\frac{4}{s^2} + 1 + \frac{4}{s} - 1}$$

$$Y_2 = \frac{s^2}{4s+4} \begin{bmatrix} \frac{2}{s}+1 & -1 \\ -1 & \frac{2}{s}+1 \end{bmatrix}$$

∴ For the notch filter Y parameter

$$Y = Y_1 + Y_2$$

$$= \frac{s}{4s+4} \begin{bmatrix} 2+\frac{1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & 2+\frac{1}{s} \end{bmatrix} + \frac{s^2}{4s+4} \begin{bmatrix} \frac{2}{s}+1 & -1 \\ -1 & \frac{2}{s}+1 \end{bmatrix}$$

6

$$Y = \frac{1}{4s+4} \left[\begin{bmatrix} 2s+1 & -1 \\ -1 & 2s+1 \end{bmatrix} + \begin{bmatrix} 2s+s^2 & -s^2 \\ -s^2 & 2s+s^2 \end{bmatrix} \right]$$

$$Y = \frac{1}{4s+4} \begin{bmatrix} s^2+4s+1 & -(s^2+1) \\ -(s^2+1) & s^2+4s+1 \end{bmatrix}$$

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7 (b) (i) Find the resistivity of

1. intrinsic Silicon and
2. p-type silicon with $N_A = 10^{16}/cm^3$.

Use $n_i = 1.5 \times 10^{10}/cm^3$ and assume that intrinsic Silicon $\mu_n = 1350\ cm^2/V\text{-}s$ and $\mu_p = 480\ cm^3/V\text{-}s$ and for doped Silicon $\mu_n = 1110\ cm^2/V\text{-}s$ and $\mu_p = 400\ cm^2/V\text{-}s$ and comment on result.

(Note that doping results in reduced carrier mobilities).

[10 marks]

① Intrinsic silicon

$n_i = 1.5 \times 10^{10}/cm^3$

$\mu_n = 1350\ cm^2/Vs \quad \mu_p = 480\ cm^3/Vs$

resistivity of intrinsic silicon –

$\frac{1}{\rho_i} = n_i e(\mu_n + \mu_p)$

$= 1.5\times10^{10} \times 1.6\times10^{-19}(1350+480)$

$\boxed{\rho_i = 2.27\times10^{5}\ \Omega\text{-cm}}$

② p-type silicon

$N_A = 10^{16}/cm^3 \quad \mu_n = 1110\ cm^2/V\text{-}s$

$\mu_p = 400\ cm^2/V\text{-}s$

∴ concentration of hole

$n_p \simeq N_A = 10^{16}/cm^3$

concentration of electron

$n_e = \frac{n_i^2}{n_p} = \frac{(1.5\times10^{10})^2}{10^{16}}$

$n_e = 2.25\times10^{4}/cm^3$

resistivity of doped p-type silicon –

$$\frac{1}{\rho_p} = e(n_n \mu_n + n_p \mu_p)$$

$$= 1.6\times10^{-19}\left[2.25\times10^{4}\times1110 + 10^{16}\times400\right]$$

$$\boxed{\rho_p = 1.5625\ \Omega\text{-cm}}$$

Hence, when intrinsic silicon is doped with the p-type silicon has resistivity reduced.

9

Good Approach

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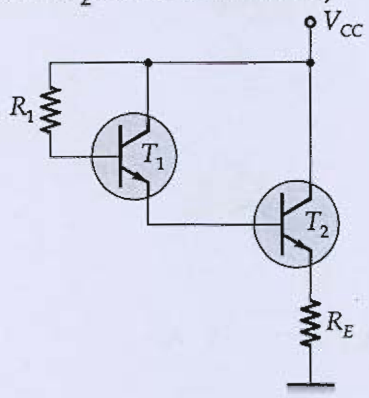
7 (b) (ii) For the transistor pair circuit is shown in below figure. Both the transistors have dc current gain β of 30. In the circuit V_{CC} = +12V, R_E = 1.5 kΩ.

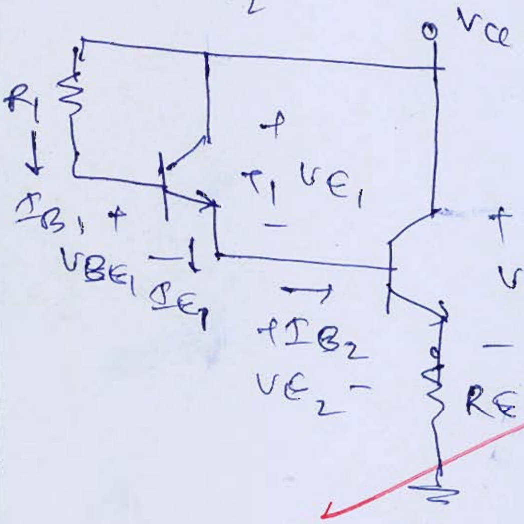
1. Find the same value of R_1 needed to bias the circuit such that V_{CEQ2} = 5 V for transistor T_2.
2. With the value of R_1 as obtained above, determine the value of V_{CEQ1}.

(Assume both T_1 and T_2 are Si transistors)



[10 marks]

(1) $V_{CEQ_2} = 5V$



$V_E = 12 - 5 = 7V$

$I_{E_2} = \frac{7V}{1.5k\Omega}$

$I_{E_2} =$ ~~3.33 mA~~ $4.667 mA$

$I_{E_1} = I_{B_2} = \frac{I_{E_2}}{\beta+1} = \frac{4.667}{30+1}$ = ~~0.1075 mA~~ $0.1505 mA$

$I_{B1} = \frac{I_{E1}}{\beta+1} = \frac{0.1505}{30+1}$ = ~~3.468 μA~~ $4.856 μA$

∴ ~~Here R1 resistance R1 = Vcc~~

KVL

$12 - I_{B_1} \times R_1 - 0.7 - 0.7 = 7$

$R_1 = \frac{12 - 7 - 1.4}{4.856 \mu A}$

~~$R_2 = 1.614 M\Omega$~~

$R_1 = 741.3\ k\Omega$

(2)

Now $R_1 = 741.3\ k\Omega$

$I_{B_1} = 4.856\ \mu A$

$V_{E_1} = 12 - R_1 I_{B_1} - V_{BE_1}$

$= 12 - 0.741 \times 4.856 - 0.7$

$V_{E_1} = 7.702V$

$V_{C_1} = 12\ V$

$\therefore V_{CE_1} = V_{C_1} - V_{E_1} = 12 - 7.702$

$V_{CE_1} = 4.298\ V$

9

Good approach

7 (c) (i) Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point (5, 0, 4). Also calculate the magnitude of the maximum directional derivative.

(ii) What is phantom loading? What is the advantage of it?

[14 + 6 marks]

Given :- $f = x^2 - y^2 + 2z^2$

directional derivative of function f

$$= \text{grad } f = \nabla f$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{a}_x + \frac{\partial f}{\partial y}\hat{a}_y + \frac{\partial f}{\partial z}\hat{a}_z$$

$$= \frac{\partial}{\partial x}(x^2 - y^2 + 2z^2)\hat{a}_x + \frac{\partial}{\partial y}(x^2 - y^2 + 2z^2)\hat{a}_y + \frac{\partial}{\partial z}(x^2 - y^2 + 2z^2)\hat{a}_z$$

$$\nabla f = 2x\,\hat{a}_x - 2y\,\hat{a}_y + 4z\,\hat{a}_z$$

At point $P(1,2,3)$

$$(\nabla f)_{(1,2,3)} = 2\times 1\,\hat{a}_x - 2\times 2\,\hat{a}_y + 4\times 3\,\hat{a}_z$$

$$\nabla f = 2\hat{a}_x - 4\hat{a}_y + 12\hat{a}_z$$

direction of line $\overline{PQ} = \dfrac{(5-1)\hat{a}_x + (0-2)\hat{a}_y + (4-3)\hat{a}_z}{\sqrt{4^2 + 2^2 + 1^2}}$

$$\overline{PQ} = \frac{4\hat{a}_x - 2\hat{a}_y + \hat{a}_z}{\sqrt{21}}$$

Directional derivation in the direction of $\overline{PQ}$

$$= \nabla f \cdot \overline{PQ}$$

$$= (2\hat{a}_x - 4\hat{a}_y + 12\hat{a}_z) \cdot \frac{(4\hat{a}_x - 2\hat{a}_y + \hat{a}_z)}{\sqrt{21}}$$

$$= \frac{4\times 2 + 4\times 2 + 12\times 1}{\sqrt{21}}$$

$$= \frac{28}{\sqrt{21}} = \underline{6.11} \quad \text{(Ans)}$$

Magnitude of maximum directional derivative at P (1, 4, 3)

$$= \frac{\nabla f}{|\nabla f|} = \frac{2\hat{a}_x - 4\hat{a}_y + 12\hat{a}_z}{\ldots}$$

$$= |\nabla f|$$

$$= \sqrt{(2)^2 + (4)^2 + (12)^2}$$

$$= 12.8 \quad \text{(Ans)}$$

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(1f) Phantom loading – (also called as false loading) refers to a method used in testing and calibrating wattmeter, especially electrodynamometer-type wattmeters, without drawing large power from the supply.

- when testing high voltage, high current energy meter directly applying full rated current and voltage require a lot of power and could be costly or unsafe. So we phantom loading

Advantages –

- saves power during calibration
- safer to test meters rated for high power
- ~~(its)~~ useful in labs and utility testing

16

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Q.8 (a) **(i)** Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

(ii) Find the area of the tangent cut-off from the parabola $x^2 = 8y$ by the line $x - 2y - 8 = 0$.

[10 + 10 marks]

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Q.8 (b) $\vec{F}(x,y,z) = yz\hat{i} - xz\hat{j} + \hat{k}$. Let 's' be the portion of surface of the paraboloid $z = 4 - x^2 - y^2$ which lies above the first octant, and let 'C' be closed curve $C = C_1 + C_2 + C_3$, where curves C_1, C_2 and C_3 are the three curves formed by intersecting 's' with the xy, yz and xz planes respectively so that C is boundary of 's'. Orient C so that it is traversed CCW when seen from above the first octant.

(i) Set up and evaluate the loop integral $\oint_C \vec{F} \cdot \overrightarrow{dr}$ by parameterizing each piece of curve C.

(ii) Verify using Stoke's theorem that loop integral $\oint_C \vec{F} \cdot \overrightarrow{dr}$ is equal to surface integral

$$\iint_s \vec{\nabla} \times \vec{F} \cdot \overrightarrow{ds}.$$

[20 marks]

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(c) (i) The electrical conductivity and electron mobility of aluminium are 3.8×10^7 $(\Omega\text{-m})^{-1}$ and 0.0012 $m^2/V\text{-s}$, respectively. Calculate the Hall voltage for an aluminium specimen that is 15 mm thick for a current of 25 A and a magnetic field of 0.6 T. Given that field is perpendicular to current.

(ii) Briefly explain why the ferroelectric behavior of $BaTiO_3$ ceases above its ferroelectric curie temperature.

(iii) Name the types of polarization and briefly explain the type of materials and about mechanism by which dipolar are induced or oriented by the action of an applied electric field. For gaseous-argon, solid LiF, liquid H_2O, what kind(s) of polarization is/are possible?

[6 + 4 + 10 marks]

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Space for Rough Work

Space for Rough Work