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Leading Institute for ESE, GATE & PSUs

# ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

## Mechanical Engineering

Test-5 : Production Engineering & Material Science

+ Mechatronics and Robotics

Heat Transfer  
Renewable Sources of Energy

Name : .....

Roll No :

### Test Centres

### Student's Signature

Delhi ☒ Bhopal ☐ Jaipur ☐  
Pune ☐ Kolkata ☐ Hyderabad ☐

### Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

### FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	32
Q.2	—
Q.3	43
Q.4	47
Section-B	
Q.5	13
Q.6	44
Q.7	—
Q.8	—
<b>Total Marks Obtained</b>	<b>179</b>

Signature of Evaluator

Cross Checked by

*Cama Sharm*

Keep it up, well done!

## IMPORTANT INSTRUCTIONS

**CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.**

### DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

### DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.



**Section A : Heat Transfer + Renewable Sources of Energy**

- (a) A long conducting rod of diameter  $D$  and electrical resistance per unit length  $R_e$  is initially in thermal equilibrium with the ambient air and its surroundings. This equilibrium condition is disturbed when an electrical current  $I$  is passed through the rod. Derive an expression for the variation of the rod temperature with time during passage of the current.

[12 marks]



$R_e$  = Resistance per unit length

$$q_{gen} = I^2 R_e \quad \text{heat gen. per unit length.}$$

$$\text{at } t = 0, \quad T = T_{\infty}$$

assuming the rod to be a lump.

$$T = T(\text{time}) \text{ only.}$$

$$\text{vol. } \dot{q}_{gen} = \frac{I^2 R_e}{\left(\frac{\pi}{4} D^2 \times 1\right)} = \text{volumetric heat generation.}$$

let  $h$  be convective H.T. coeff.

$T_{\infty}$  ambient temp.

at steady state heat generated is  
convected at the boundary.

$$\dot{m} c_p (dT) = (\dot{q}_{gen} \cdot \text{vol.}) \cdot \cancel{h(\pi D \times 1)(T - T_{\infty})(dt)}$$

$$\dot{m} c_p dT = I^2 R_e dt$$

$$\left(\frac{dT}{dt}\right) = \frac{I^2 R_e}{\rho \left(\frac{\pi}{4} D^2 \times 1\right) c_p}$$

$$T = \frac{I^2 R_e}{\rho \left(\frac{\pi}{4} D^2\right) c_p} t + C_1$$

Initial condn  $T = T_{\infty}$  at  $t = 0$

$$C = T_{\infty}$$

$$T = \frac{I^2 R e}{\rho \frac{\pi}{4} D^2 C_p} (t) + T_{\infty}$$

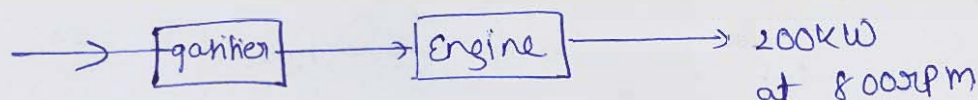
temp distribution as a function of time.

where  $\rho \rightarrow$  density of rod

$C_p \rightarrow$  sp. heat capacity of rod.

- Q.1 (b) A biomass gasifier is used to run a CI engine in a dual fuel model with 80% diesel replacement. The gasifier engine system produces 200 kW of power at 800 rpm. Calculate the biomass feeding rate of the gasifier if the efficiency of the engine is 35% and the calorific value of the producer gas is 17000 kJ/kg, assuming the efficiency of gasifier to be 75%.

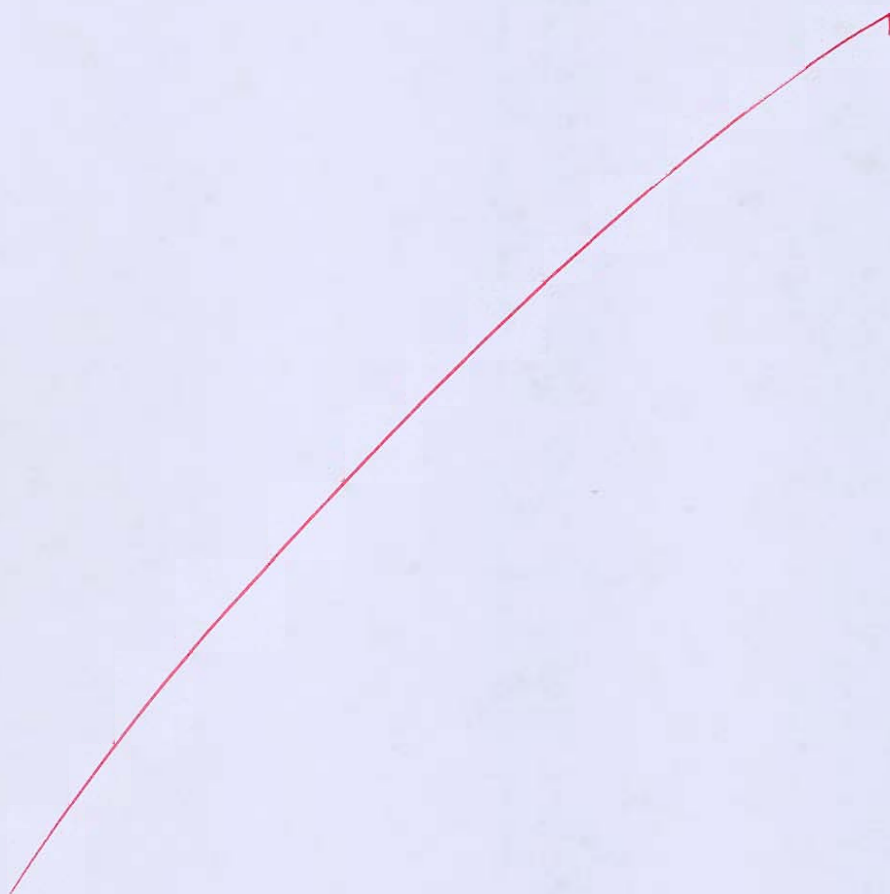
[12 marks]



$$(H.A.)_s \text{ to Engine} = \frac{200 \text{ kW}}{0.35}$$

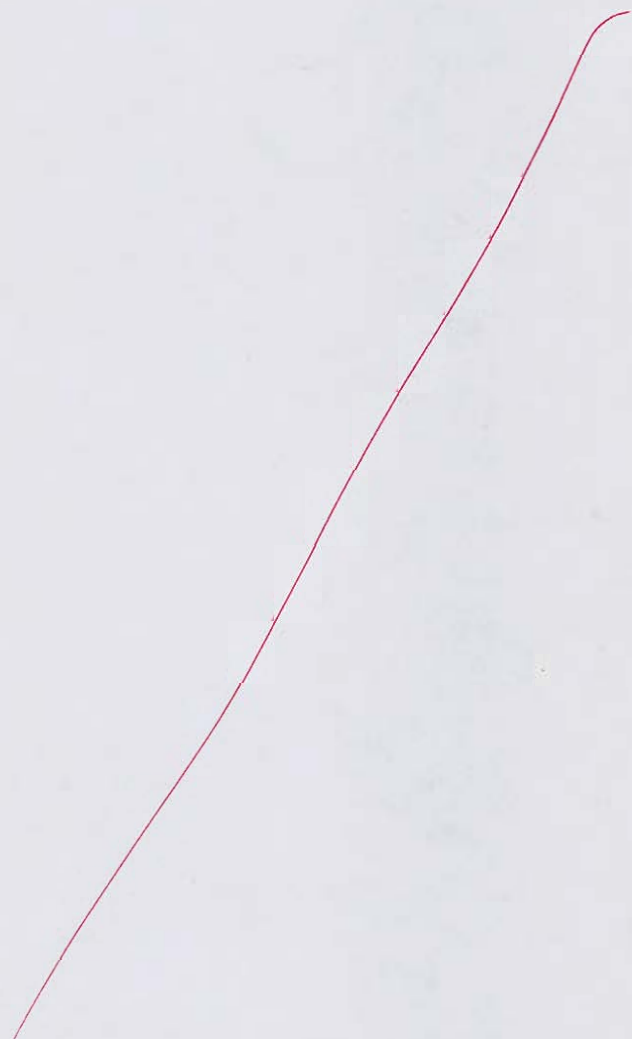
$$= 571.4286 \text{ kW}$$

m



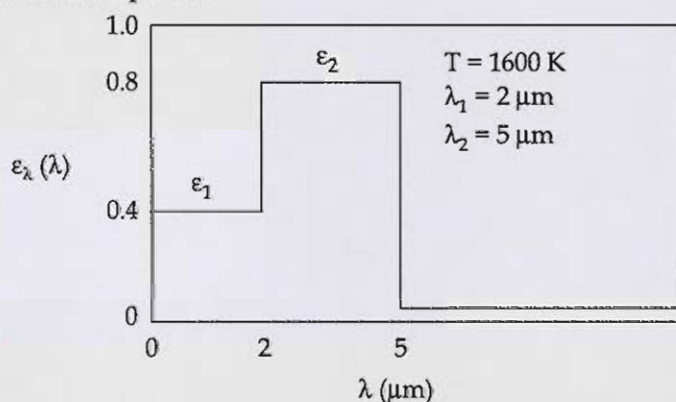
Q.1 (c) Discuss how energy is stored in reversible chemical reactions. What are their main advantages? What do you understand by turning temperature?

[12 marks]





- 1 (d) A diffuse surface at 1600 K has the spectral emissivity as shown in figure. Determine
- the total emissivity.
  - the total emissive power.



[Use blackbody radiation band emission fraction table attached at the end]

[12 marks]

$T = 1600 \text{ K}$

$\lambda_1 T = 2 \times 1600 \text{ } \mu\text{m K} = 3200 \text{ } \mu\text{m K} \Rightarrow f_{0-2} = \text{---}$

$\lambda_2 T = 5 \times 1600 \text{ } \mu\text{m K} = 8000 \text{ } \mu\text{m K} \Rightarrow f_{0-5} = \text{---}$

$= 8000 \text{ } \mu\text{m K}$

$$\epsilon = \frac{\int_0^{\lambda_1} \epsilon_1 E_b \lambda d\lambda + \int_{\lambda_1}^{\lambda_2} \epsilon_2 E_b \lambda d\lambda + \int_{\lambda_2}^{\infty} \epsilon_3 E_b \lambda d\lambda}{(\sigma T^4)}$$

2  
table is attached.

Table not given

$$\epsilon = \frac{(f_{0-2\mu m})(0.4)(\sigma T^4) + (f_{0-5} - f_{0-2})(0.8)\sigma T^4}{(\sigma T^4)}$$

$$\epsilon = 0.4(f_{0-2\mu m}) + 0.8(f_{0-5} - f_{0-2})$$

$f_{0-2}$  = Black body Radn function.

→ Total Emissivity

$E = \epsilon \sigma T^4$  = total emissive power.

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

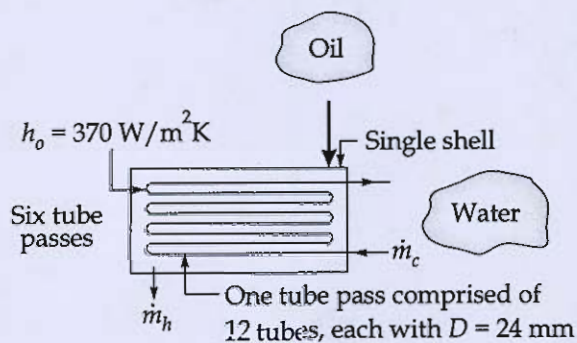
$$K = 1600 \text{ K}$$

?



- (e) A shell and tube exchanger must be designed to heat 3 kg/s of water from 20 to 85°C. The heating is to be accomplished by passing hot oil, which is available at 180°C, through the shell and tube side of the heat exchanger. The oil is known to provide an average convection coefficient  $h_o = 370 \text{ W/m}^2\text{K}$  on the outside of the tube. Twelve tubes pass the water through the shell. Each tube is thin walled, of diameter  $D = 24 \text{ mm}$ , and makes six passes through the shell. If the oil leaves the exchanger at 110°C. Determine

- The required oil flow rate in kg/s.
- The required tube length to accomplish the desired heating.



Use the equation  $Nu = 0.023 (Re)^{0.8} (Pr)^{0.4}$

Correction factor for the heat exchanger to be 0.87

Properties of oil,  $c_p = 2300 \text{ J/kgK}$ ; Properties of water,  $c_p = 4180 \text{ J/kgK}$ ;

$\mu = 560 \times 10^{-6} \text{ Ns/m}^2$ ;  $k = 0.65 \text{ W/mK}$

[12 marks]

$$\dot{m}_{\text{water}} = 3 \text{ kg/s} \quad \left. \begin{array}{l} T_{ci} = 20^\circ\text{C} \\ T_{co} = 85^\circ\text{C} \end{array} \right\} \text{(water tube side)}$$

$$T_{hi} = 180^\circ\text{C} \quad \dot{m}_{\text{oil}} = ?$$

$$T_{ho} = 110^\circ\text{C}$$

$$(\dot{m} c_p)_{\text{oil}} (T_{hi} - T_{ho}) = (\dot{m} c_p)_{\text{water}} (T_{co} - T_{ci})$$

$$\dot{m}_{\text{oil}} \times 2300 \times (180 - 110) = 3 \times 4180 \times (85 - 20)$$

$$\boxed{\dot{m}_{\text{oil}} = 5.627 \text{ kg/s}}$$

$$h_o = 370 \text{ W/m}^2\text{K}$$

$$3 \text{ kg/s} = 12 \left[ 1000 \times \frac{\pi}{4} (0.024)^2 \times V \right]$$

$$\boxed{V = 0.5526 \text{ m/s}}$$

velocity of water in tube  $= 0.5526 \text{ m/s}$ .

$$Re = \frac{1000 \times 0.5526 \times 0.024}{560 \times 10^{-6}}$$

$$Re = 23683.77 > 2000 \Rightarrow \text{Turbulent.}$$

$$Pr = \frac{\mu C_p}{k_f} = \frac{560 \times 10^{-6} \times 4180}{0.65}$$

$$Pr = 3.6012$$

$$Nu = \frac{h_i D}{k_f} = 0.023 \times Re^{0.8} Pr^{0.4}$$

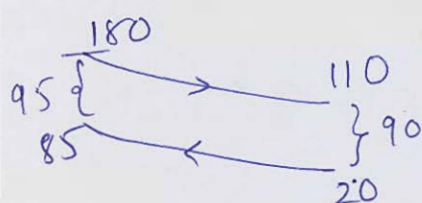
$$\frac{h_i D}{k_f} = 121.3022$$

$$h_i = 3285.27 \text{ W/m}^2\text{K}$$

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} \quad (\text{since tube is very thin})$$

$$U = 332.54 \text{ W/m}^2\text{K}$$

$$(LMTD) = 0.87 (LMTD)_{\text{counter}}$$



$$(LMTD)_{\text{count}} = \frac{95 - 90}{\ln\left(\frac{95}{90}\right)} = 92.4775$$

$$(LMTD) = 80.4554^\circ\text{C}$$

$$Q = U A_s (LMTD) \Rightarrow A_s = 30.4657 \text{ m}^2$$

Total length of tube  $\leftarrow$

$$\frac{A_s}{\text{tube}} = 2.5388 \text{ TTDL}$$

$$L = 33.672 \text{ m}$$

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(a) A submarine is to be designed to provide a comfortable temperature for the crew of not less than  $21^{\circ}\text{C}$ . The submarine can be idealized as a cylinder 10 m in diameter and 70 m in length. The combined heat transfer coefficient on the interior is  $15 \text{ W/m}^2\text{K}$ , while on the outside surface the heat transfer coefficient is vary from  $60 \text{ W/m}^2\text{K}$  (not moving) to  $880 \text{ W/m}^2\text{K}$  (top speed). For the following wall constructions, determine the minimum size in kilowatts of the heating unit required if the sea water temperatures vary from  $2.2^{\circ}\text{C}$  to  $14.5^{\circ}\text{C}$  during operation.

- (i) 1.2 cm aluminium
  - (ii) 1.8 cm stainless steel with a 2.5 cm thick layer fiberglass insulation on the inside
  - (iii) of sandwich construction with a 1.8 cm thickness of stainless steel, a 2.5 cm thick layer of fiberglass insulation, and a 0.6 cm thickness of aluminium on the inside.
- What conclusions can you draw?

Take, for aluminium ( $K_a$ ) =  $236 \text{ (W/mK)}$  at  $0^{\circ}\text{C}$

for stainless steel ( $K_s$ ) =  $0.035 \text{ (W/mK)}$  at  $20^{\circ}\text{C}$

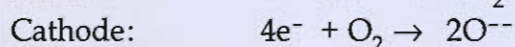
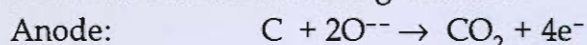
for fiberglass insulation ( $K_{fg}$ ) =  $0.035 \text{ (W/mK)}$  at  $20^{\circ}\text{C}$

[20 marks]





2 (b) A fuel cell has the following reactions:



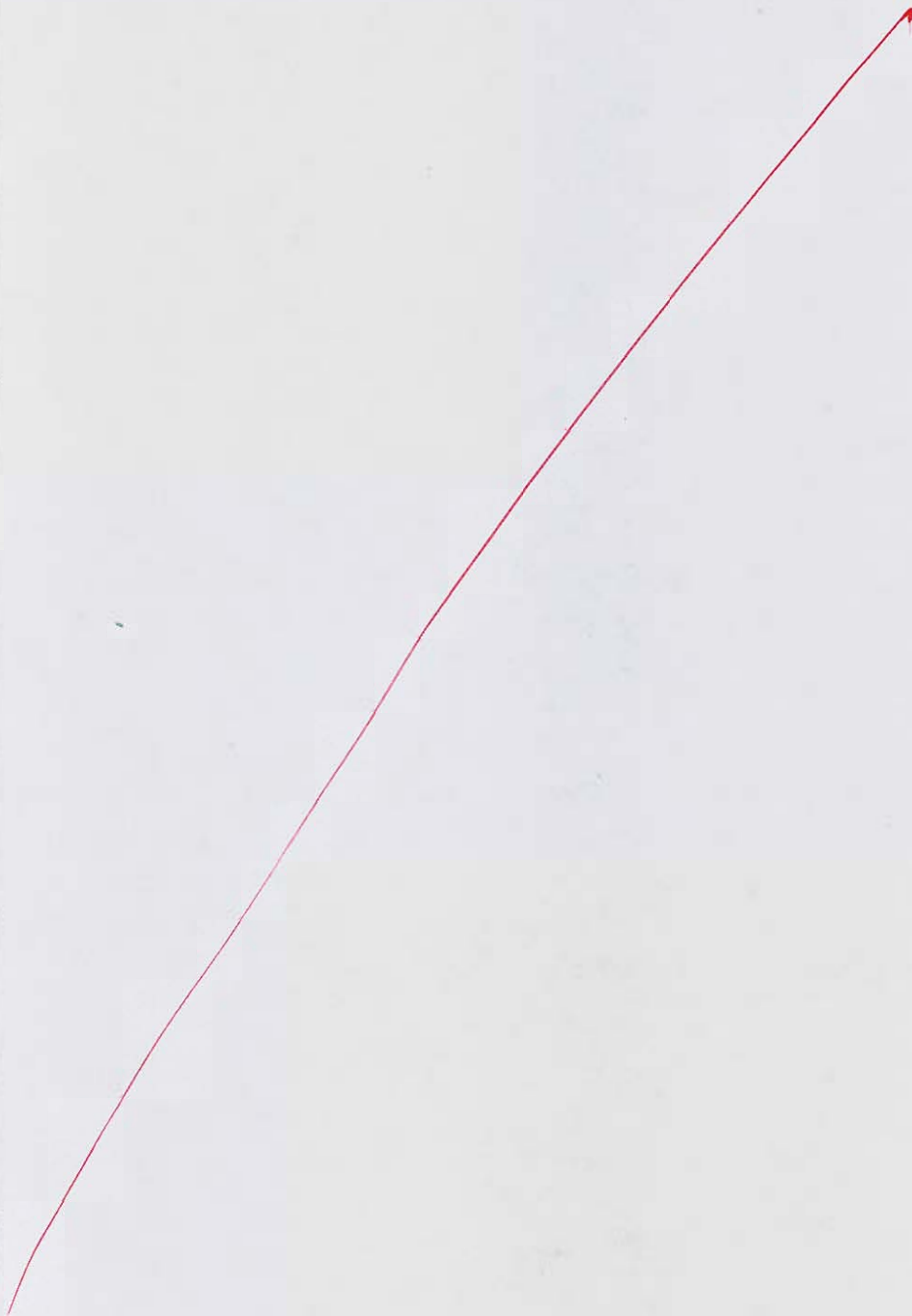
At reference temperature and pressure (298 K and 1 bar), the changes of enthalpy and of free energy, per kilomole of  $\text{CO}_2$ , are:

$$\Delta \bar{h}_f^\circ = -393.5 \text{ MJ}$$

$$\Delta \bar{g}_f^\circ = -394.5 \text{ MJ}$$

What is the overall reaction? What is the ideal emf? What is the difference in entropy between reactants and products? Assume that the internal resistance of the cell is 1 milli-Ohm. Otherwise, the cell behaves as an ideal voltage source. How much carbon is needed to deliver 1 MWh of electricity to the load in minimum possible time? What is the load resistance under such conditions?

[20 marks]





- Q.2 (c) A square plate maintained at  $95^{\circ}\text{C}$  experiences a force of  $10.5\text{ N}$  when air at  $25^{\circ}\text{C}$  flows over it at a velocity of  $30\text{ m/s}$ . Assuming the flow to be turbulent and using Colburn analogy, calculate (a) heat transfer coefficient and (b) heat loss from the plate surface.

Properties of air at the mean film temperature are:

$$\rho = 1.06\text{ kg/m}^3, c_p = 1.005\text{ kJ/kgK}, \nu = 18.97 \times 10^{-6}\text{ m}^2/\text{s}, \text{Pr} = 0.696$$

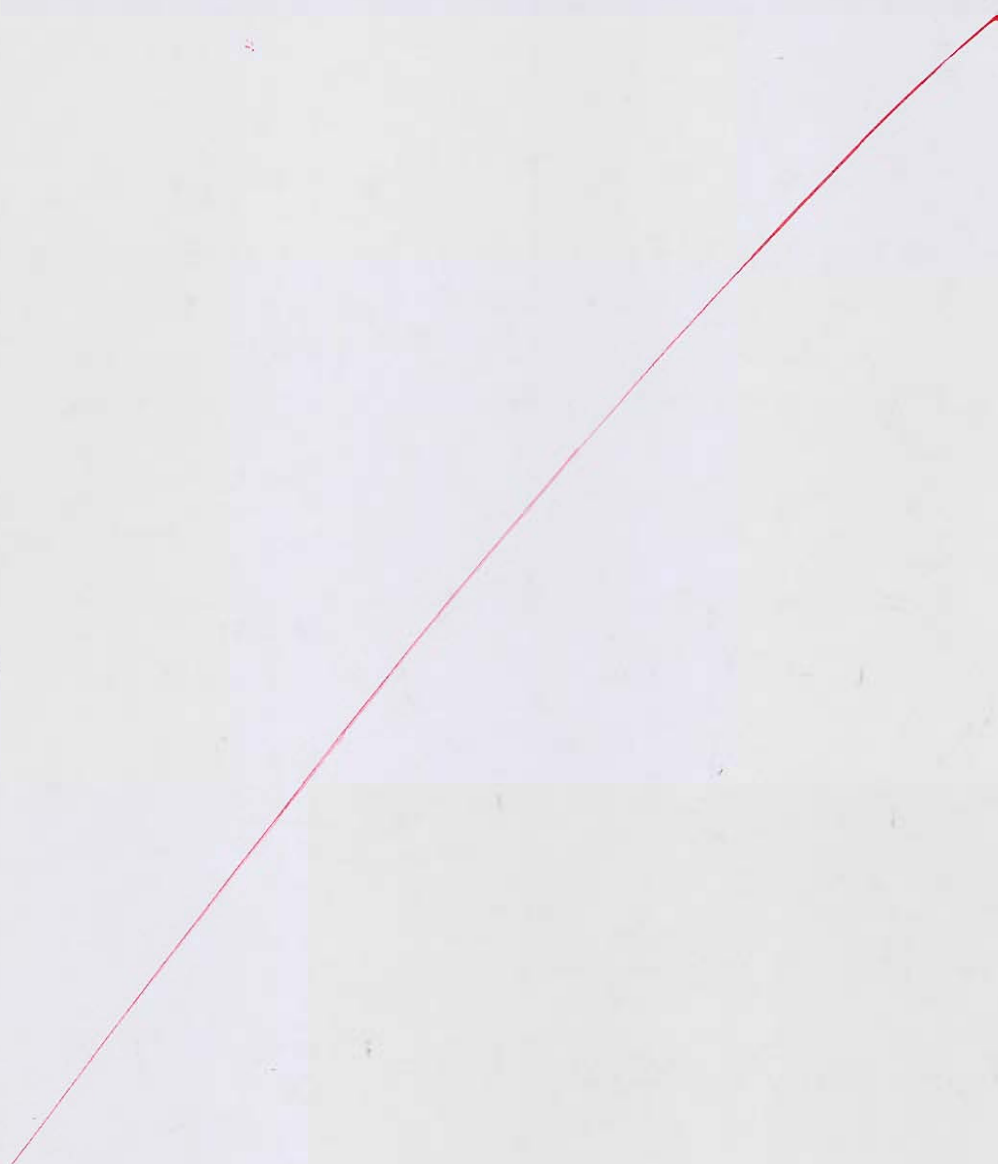
For turbulent flow, take drag coefficient

$$C_D = \frac{0.0742}{\text{Re}_L^{1/5}}$$

[20 marks]





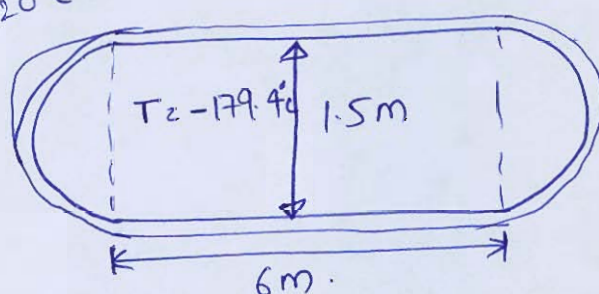


- Q.3 (a) A cylindrical liquid oxygen (LOX) tank has a diameter of 1.5 m, a length of 6 m, and hemispherical ends. The boiling point of LOX is  $-179.4^{\circ}\text{C}$ . An insulation is sought which will reduce the boil-off rate in the steady state 12 kg/hr. The heat of vapourization of LOX is 282 kJ/kg. If the thickness of the insulation 7 cm, then determine the value of its thermal conductivity?

[Assume surrounding temperature to be  $20^{\circ}\text{C}$ ]

[20 marks]

$T_{\text{surround}} = 20^{\circ}\text{C}$



thickness of  
insulation  
 $= 7\text{ cm}$

$$\text{Boil off rate} = \frac{12 \text{ kg}}{\text{hr}}$$

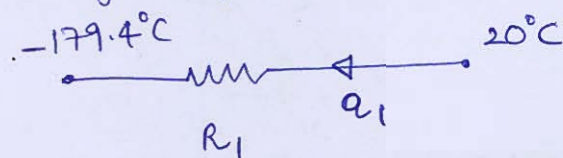
$$\text{Heat transfer required} = \frac{12 \text{ kg}}{\text{hr}} \times 282 \frac{\text{kJ}}{\text{kg}}$$

$$= 3384 \frac{\text{kJ}}{\text{hr}}$$

$$= 940 \text{ watts}$$

let  $k$  be the thermal conductivity of insulation

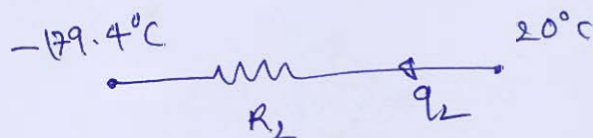
1. heat transfer through cyl. surface.



$$R_1 = \frac{\ln\left(\frac{R_2}{R_1}\right)}{2\pi k L} = \frac{\ln\left(\frac{0.75+0.07}{0.75}\right)}{2\pi \times k \times 6}$$

$$R_1 = \frac{2.3669 \times 10^{-3}}{k} \rightarrow \textcircled{1}$$

2. Heat transfer through hemispherical ends.



$$r_2 = 0.82 \text{ m}$$

$$r_1 = 0.75 \text{ m}$$

$$R_2 = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{0.82 - 0.75}{4\pi \times k \times (0.82) \times (0.75)}$$

$$R_2 = \frac{9.0576 \times 10^{-3}}{k} \rightarrow \textcircled{2}$$

$$q = 940 \text{ watts} = q_1 + q_2$$

$$940 = \frac{20 - (-179.4)}{\left(\frac{2.3669 \times 10^{-3}}{k}\right)} + \frac{20 - (-179.4)}{\left(\frac{9.0576 \times 10^{-3}}{k}\right)}$$

$$940 = 199.4 \left[ \frac{k}{2.3669 \times 10^{-3}} + \frac{k}{9.0576 \times 10^{-3}} \right]$$

$$k = 8.846 \times 10^{-3} \frac{\text{W}}{\text{mK}}$$

→ Thermal conductivity of insulation required.

Q.3 (b) A school in a remote place has the following energy requirements:

- Ten lamps each of 100 CP, that operate for 6 hours daily.
- Ten computers each of 250W, that operate for 6 hours daily by a dual fuel engine driven generator.
- 2 hp water pump driven by dual fuel engine for two hours daily.

Use the following data:

Gas required for lighting a 100 Candle power lamp =  $0.126 \text{ m}^3/\text{hour}$

Thermal efficiency of both the engines = 25%

Conversion efficiency of generator = 80%

Collectable cow dung per cow = 7 kg

Percentage of dry matter in cow dung = 18%

Biogas yield =  $0.34 \text{ m}^3/\text{kg}$  of dry matter

Retention period = 50 days

Density of slurry =  $1090 \text{ kg}/\text{m}^3$

Heating value of biogas =  $23 \text{ MJ}/\text{m}^3$

Volume occupied by gas = 10% of digester volume

Equal amount of water is added in cowdung for producing slurry.

Calculate the size of digester and the number of cows required to feed the plant.

[20 marks]



Total Biogas Requirement / Day

$$\begin{aligned}\underline{\text{Lamps}} &= 10 \times 6 \text{ hrs/day} \times 0.126 \text{ m}^3/\text{hr} \\ &= 7.56 \text{ m}^3/\text{day} \rightarrow \textcircled{1}\end{aligned}$$

$$\begin{aligned}\underline{\text{computers}} &= 10 \times 250 \frac{\text{J}}{\text{S}} \times 6 \times 3600 \text{ S} \\ &= 54 \times 10^6 \text{ J/day}\end{aligned}$$

$$\text{HV of biogas} = 23 \text{ MJ/m}^3$$

let  $V$  be volume of biogas used by engine.

$$\text{Power of (Engine)} = V \times 23 \times 10^6 \frac{\text{J}}{\text{m}^3} \times \underbrace{0.25}_{\eta_{\text{engine}}} \times \underbrace{0.81}_{\eta_{\text{gen}}}$$

$$V_{\text{engine}} = \frac{54 \times 10^6}{23 \times 10^6 \times 0.25 \times 0.8}$$

$$= \frac{9.3913 \text{ m}^3/\text{day}}{0.8}$$

$$= 11.7391 \text{ m}^3/\text{day} \rightarrow \textcircled{2}$$

2hp water pump

$$= 2 \times 746 \text{ Watts} \times 2 \times 3600 \text{ S/day}$$

$$= 10.5984 \times 10^6 \text{ J/day}$$

let  $V$  be the volume reqd by engine

$$= V \times 23 \times 10^6 \frac{\text{J}}{\text{m}^3} \times 0.25 \times 0.8$$

$$V = 2.304 \text{ m}^3/\text{day} \rightarrow \textcircled{3}$$

$$\text{Total Biogas Requirement} = 21.603 \frac{\text{m}^3}{\text{day}}$$

let ' $n$ ' be number of cows.

$$\text{Biogas produced by } n \text{ cows} = n \times 7 \times 0.18 \times 0.34 = 0.4284 n$$

$$0.4284 n = 21.6031$$

$$n = 50.4274$$

$\therefore$  No of cows required to feed the plant = **51**

$$\text{mass of slurry} = \left( \text{mass of collectible cow dung} \right) + \left( \text{Equal amount of water} \right)$$

$$= (51 \times 7) + (51 \times 7)$$

$$= 714 \text{ kg}$$

$$\text{volume of slurry added per day} = \frac{714 \text{ kg}}{1090 \frac{\text{kg}}{\text{m}^3}}$$

$$V_d = 0.655 \text{ m}^3$$

$$\frac{V_s}{V_d} = 50 = \text{Retention time}$$

$$V_s = 32.7523 \text{ m}^3 = \left\{ \begin{array}{l} 90\% \text{ of volume of} \\ \text{digester} \end{array} \right\}$$

**17**

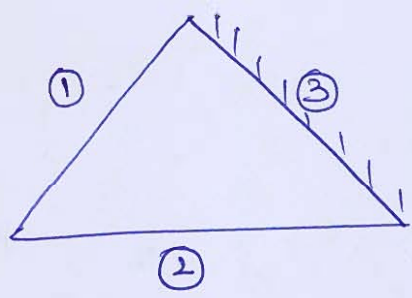
$$V = 36.3914 \text{ m}^3$$

→ volume of digester.



- 3 (c) A paint baking oven consists of a long, triangular (equilateral) duct in which a heated surface is maintained at 1200 K and another surface is insulated. Painted panels, which are maintained at 500 K, occupy the third surface. The triangle is of width  $W = 1$  m, and the heated and insulated surfaces have an emissivity of 0.8. The emissivity of the panels is 0.4. Determine:
- (i) During steady-state operation, the rate at which energy is supplied to the heated side per unit length of the duct to maintain its temperature at 1200 K?
  - (ii) The temperature of the insulated surface?

[20 marks]



① → heated surface

$$\epsilon_1 = 0.8 \quad T_1 = 1200 \text{ K}$$

② → Panel

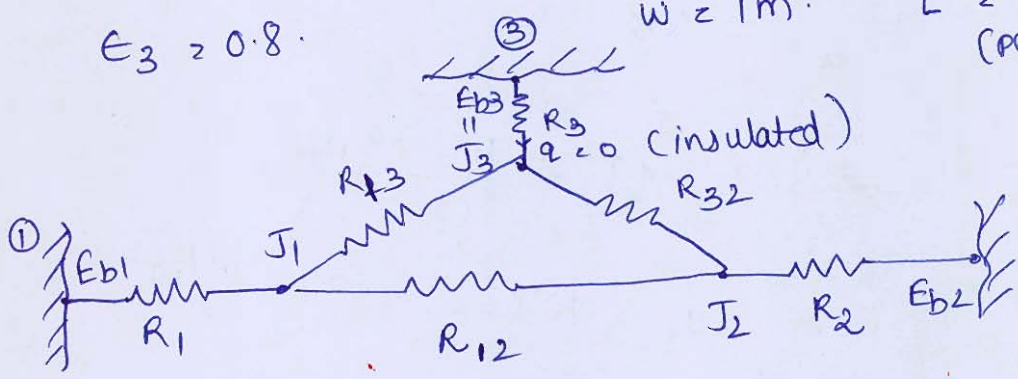
$$\epsilon_2 = 0.4 \quad T_2 = 500 \text{ K}$$

③ → insulated surface

$$\epsilon_3 = 0.8$$

$$W = 1 \text{ m}$$

$$L = 1 \text{ m (per unit length)}$$



$$R_1 = \frac{1 - \epsilon_1}{\epsilon_1 A_1} = \frac{1 - 0.8}{0.8 \times 1 \times 1} = 0.25$$

$$R_2 = \frac{1 - \epsilon_2}{\epsilon_2 A_2} = \frac{1 - 0.4}{0.4 \times 1 \times 1} = 1.5$$

$$R_3 = \frac{1 - \epsilon_3}{\epsilon_3 A_3} = 0.25$$

$$R_{12} = \frac{1}{A_1 f_{12}} = 2$$

$$f_{12} = 0.5 = f_{23} = f_{31}$$

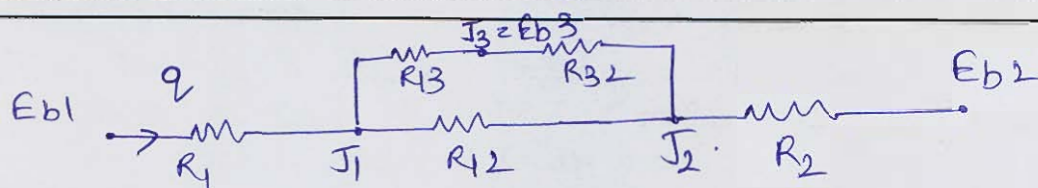
$$R_{13} = 2$$

$$R_{32} = 2$$

$$E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8} \times 1200^4$$

$$E_{b1} = 117573.12 \text{ W/m}^2$$

$$E_{b2} = \sigma T_2^4 = 3543.75 \text{ W/m}^2$$



$$q = \frac{Eb1 - Eb2}{(R_{total})} ; R_{total} = R1 + \frac{(R13 + R32)R12}{R12 + R13 + R32} + R2$$

$$= 0.25 + \frac{(2+2)2}{2+2+2} + 1.5$$

$$q = 36982.49 \text{ Watt}$$

$$= 3.083 \left[ \frac{1}{m^2} \right]$$

during steady state

(Ans)  $q = 36.9825 \text{ kW}$  of heat has to be supplied to heated surface to maintain  $1200 \text{ K}$ .

$$\frac{Eb1 - J1}{R1} = 36982.49 = \frac{117573.12 - J1}{0.25}$$

$$J1 = 108327.4975 \text{ W/m}^2$$

$$\frac{J2 - 3543.75}{1.5} = 36982.49 \Rightarrow J2 = 59017.485$$

$$\frac{J1 - J3}{R13} = \frac{J3 - J2}{R32}$$

$$J3 = \frac{J1 + J2}{2}$$

$$J3 = 83672.24 = Eb3$$

(since insulated  
reradiating)



$$83672.24 = (5.67 \times 10^{-8}) T_3^4$$

$$T_3 = 1102.17 \text{ K}$$

Temperature of insulated surface.

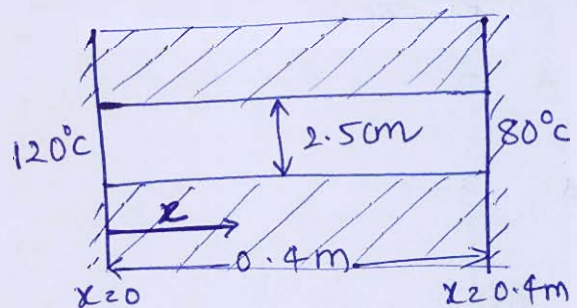
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4 (a) Two large steel plates at temperature of  $120^\circ\text{C}$  and  $80^\circ\text{C}$  are separated by a steel rod 0.4 m long and 2.5 cm in diameter. The rod is welded to each plate. The space between the plates is filled with insulation, which also insulates the circumference of the rod. Due to voltage difference between the two plates, an electric current flows through the rod, dissipating electrical energy at a rate of 15 W. Determine:

- (i) The maximum temperature in the rod.
- (ii) The heat flow rate at each end of the rod.

Take thermal conductivity of steel as  $43 \text{ W/mK}$  at  $20^\circ\text{C}$ . Also, compare the net heat flow rate at the two ends with the total rate of heat generation.

[20 marks]



Electrical energy dissipation in rod  $= 15 \text{ W}$ .

$$V_{\text{rod}} = \frac{\pi}{4} (0.025)^2 \times 0.4 = 1.9635 \times 10^{-4} \text{ m}^3$$

$$\dot{q}_{\text{gen}} = \text{Vol. heat generation} = \frac{15}{V_{\text{rod}}}$$

$$\dot{q}_{\text{gen}} = 76.3944 \frac{\text{W}}{\text{m}^3}$$

Energy Equation

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \dot{q}_{\text{gen}} = 0 \quad \left( \begin{array}{l} \text{1D} \\ \text{steady} \\ \text{heat gen} \end{array} \right)$$

$$k \frac{\partial^2 T}{\partial x^2} + \dot{q}_{\text{gen}} = 0$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_{\text{gen}}}{k} = 0$$

$$\frac{\partial^2 T}{\partial x^2} = - \frac{\dot{q}_{\text{gen}}}{k}$$

$$\frac{\partial T}{\partial x} = - \frac{\dot{q}_{\text{gen}}}{k} x + C_1$$

$$T = - \frac{\dot{q}_{\text{gen}} x^2}{2k} + C_1 x + C_2$$

B.C

$$\text{@ } x = 0 \quad T = 120$$

$$\boxed{C_2 = 120}$$

$$\text{@ } x = 0.4 \quad T = 80$$

$$80 = - \frac{\dot{q}_{\text{gen}} x^2}{2k} + C_1 (0.4) + 120$$

$$(0.4) C_1 = 80 + \left( \frac{\dot{q}_{\text{gen}} x^2}{2k} \right) - 120$$

$$\boxed{C_1 = 255.3226}$$

at max temperature point

$$\frac{\partial T}{\partial x} = 0$$

$$C_1 = \frac{\dot{q}_{\text{gen}}}{k} x$$

$$\frac{C_1 k}{\dot{q}_{\text{gen}}} = \boxed{x = 0.1437 \text{ m}}$$

$$T(x=0.1437\text{m}) = -\frac{76.3944 \times 10^3 \times 0.1437^2}{2 \times 43} + \left( \frac{255.3226}{0.1437} \right) + 120$$

$$T_{\text{max}} = 138.3466^\circ\text{C}$$

$$q = -kA \frac{\partial T}{\partial x}$$

$$q = -kA \left[ -\frac{\dot{q}_{\text{gen}}}{k} x + C_1 \right]$$

at  $x=0$  (left end).

$$q = -kA C_1 = -43 \times \frac{\pi}{4} (0.025)^2 \times 255.3226$$

$$q_{\text{left}} = -5.3892 \text{ Watt}$$

(-ve sign indicates -ve direction)

at  $x=0.4\text{m}$  (Right End).

$$q = -43 \times \frac{\pi}{4} \times 0.025^2 \times \left[ -\frac{\dot{q}_{\text{gen}}}{k} x + C_1 \right]$$

$$q_{\text{Right}} = 9.6107 \text{ watt}$$



Q.4 (b) (i) With the help of a neat sketch, explain the working of a Vertical Axis Wind Turbine (VAWT). Describe the function of its main components. Also, discuss the key advantages of VAWTs.

(ii) A propeller type wind turbine has following data

Speed of free wind at a height of 10 m = 15 m/s ; Air density = 1.23 kg/m<sup>3</sup>;  
 $\alpha = 0.15$ ; Height of tower = 120 m; Diameter of rotor = 85 m;

Wind velocity at the turbine reduces by 25%; Generator efficiency = 90%

Find :

- (i) Total power available in wind.
- (ii) Power extracted by the turbine.
- (iii) Electrical power generated.
- (iv) Axial thrust on the turbine.
- (v) Maximum axial thrust on the turbine.

[10 + 10 marks]

(iii)

$$\frac{V_i}{15} = \left( \frac{120}{10} \right)^\alpha$$

$$V_i = 21.7755 \text{ m/s} \quad \left. \vphantom{\begin{matrix} V_i \\ 21.7755 \end{matrix}} \right\} \begin{array}{l} \text{wind velocity at} \\ \text{tower height.} \end{array}$$

$$a = 0.25 \quad (\text{interference factor.})$$

$$D = 85 \text{ m.}$$

$$\begin{aligned} \text{(i) Total power available in wind} &= \frac{1}{2} \rho A V_i^3 \\ &= \frac{1}{2} \times 1.23 \times \frac{\pi}{4} \times 85^2 \times 21.7755^3 \\ &= 36.0336 \times 10^6 \text{ W} \end{aligned}$$

$$P_0 = 36.0336 \text{ MW} \quad \rightarrow \text{(i)}$$

(ii)

$$C_p = 4a(1-a)^2 = 0.5625$$

$$P_t = C_p P_0$$

$$P_t = 20.2689 \text{ MW} \quad \rightarrow \text{Power extracted by turbine.}$$



(iii) Electrical power generated  $= \eta_{\text{gen}} P_t$

$$P_{\text{electrical}} = 18.242 \text{ MW}$$

(iv)  $P_t = \dot{m} (V_i - V_e) V_t = F_a \times V_t$

$$V_t = V_i (1-a) = 16.3316$$

$$F_a = \frac{20.2689 \times 10^6}{16.3316} \text{ N}$$

axial thrust  $\rightarrow F_a = 1.2411 \times 10^6 \text{ N}$

(v)  $F_{\text{maue}} = \text{maue} \cdot \text{axial thrust}$

$$= \frac{1}{2} \rho a V_i^2$$

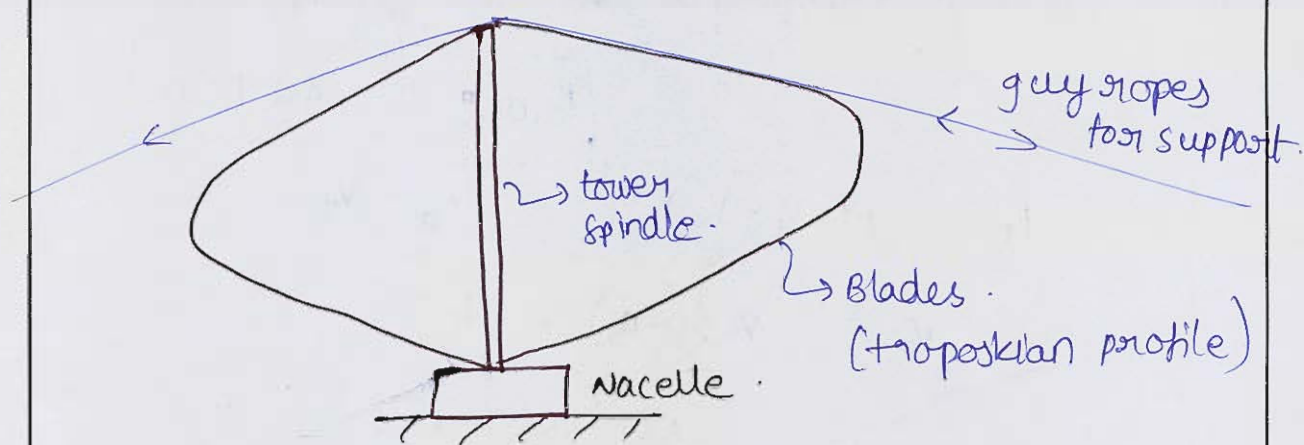
$$= 0.5 \times 1.23 \times \frac{\pi}{4} (85)^2 \times 21.7755^2$$

$$F_{\text{maue}} = 1.6547 \times 10^6 \text{ N}$$

(i) VAWT

It has a vertical axis, it can accept wind from any direction and it has its nacelle at the bottom.

The blades are of tropostriach profile to minimise centrifugal stresses.



All the gearbox, transformer, generator are setup at the ground.

### Key Advantages of VAWT

1. No need of yaw control mechanism.
2. Can accept wind from any direction.
3. maintenance and inspection is easy as nacelles is at the bottom.

main disadvantage is that it have severe vibration problem and is still in R&D stage.

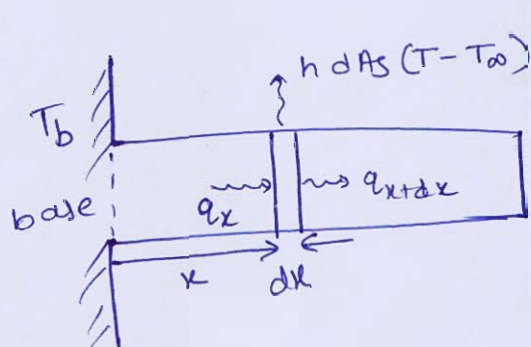
18



4 (c) Derive an expression for temperature distribution in case of infinite fin.

Two long slender rods A and B, made of different materials having same diameter of 12 mm and length 1 m, are attached to a surface maintained at a temperature of  $100^\circ\text{C}$ . The surfaces of the rods are exposed to ambient still air at  $20^\circ\text{C}$ . By traversing along the length of the rods with a temperature sensor, it is found that the surface temperatures of rods A and B are equal at positions 15 cm and 7.5 cm respectively away from the base surface. If material of A is carbon steel with thermal conductivity  $60 \text{ W/mK}$ , what is the thermal conductivity of rod B? List the assumptions made. Assume that the average convection coefficient of air is  $5 \text{ W/m}^2\text{K}$ . Find the ratio of the rate of heat transfer for rods A and B.

[20 marks]



consider a small element,

$$\dot{E}_{in} = \dot{E}_{out} \quad (\text{steady state})$$

$$q_x = q_{x+dx} + h dA_s (T - T_\infty)$$

$$q_x - q_{x+dx} = h dA_s (T - T_\infty)$$

$$q_x - \left( q_x + \frac{\partial q_x}{\partial x} dx \right) = h dA_s (T - T_\infty)$$

$$-\frac{\partial q_x}{\partial x} dx = h P dx (T - T_\infty)$$

$$-\frac{\partial}{\partial x} \left( -k A_c \frac{\partial T}{\partial x} \right) = h P (T - T_\infty)$$

$$\frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) - \frac{h P}{k A_c} (T - T_\infty) = 0$$

assuming crosssection area  $A_c$  constant through out the length.

$$\frac{\partial^2 T}{\partial x^2} - \frac{h P}{k A_c} (T - T_\infty) = 0$$

→ Governing Diff. Egn.

let  $\theta = T - T_{\infty}$

$$\frac{d^2 \theta}{dx^2} - \frac{hP}{kAc} \theta = 0.$$

auxiliary eqn  $\alpha^2 - \frac{hP}{kAc} = 0$

$$\alpha = \sqrt{\frac{hP}{kAc}} = m.$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx}.$$

Boundary condn

(a)  $x = 0 \quad \theta = \theta_b$

$$\theta_b = C_1 + C_2.$$

(a)  $x \rightarrow \infty \quad \theta = 0$

$$C_1 = 0 \Rightarrow C_2 = \theta_b.$$

$$\frac{\theta}{\theta_b} = e^{-mx} \Rightarrow$$

$$\frac{T - T_{\infty}}{T_b - T_{\infty}} = e^{-\sqrt{\frac{hP}{kAc}} x}$$

Temperature distribution  
in case of  $\infty$  fin.

$$D = 0.012 \text{ m} \quad L = 1 \text{ m} \quad T_b = 100^\circ\text{C} \quad T_{\infty} = 20^\circ\text{C}$$

for rod A,

$$\frac{T - T_{\infty}}{T_b - T_{\infty}} = e^{-m_A(0.15)} \rightarrow (1)$$

for rod B,

$$\frac{T - T_{\infty}}{T_b - T_{\infty}} = e^{-m_B(0.075)} \rightarrow (2)$$

$$(1) = (2)$$

since Temperatures are equal.



$$m_A (0.15) = m_B (0.075)$$

$$2 \sqrt{\frac{hP}{k_A A_c}} = \sqrt{\frac{hP}{k_B A_c}}$$

$$\frac{4}{k_A} = \frac{1}{k_B} \Rightarrow$$

$$k_B = \frac{k_A}{4}$$

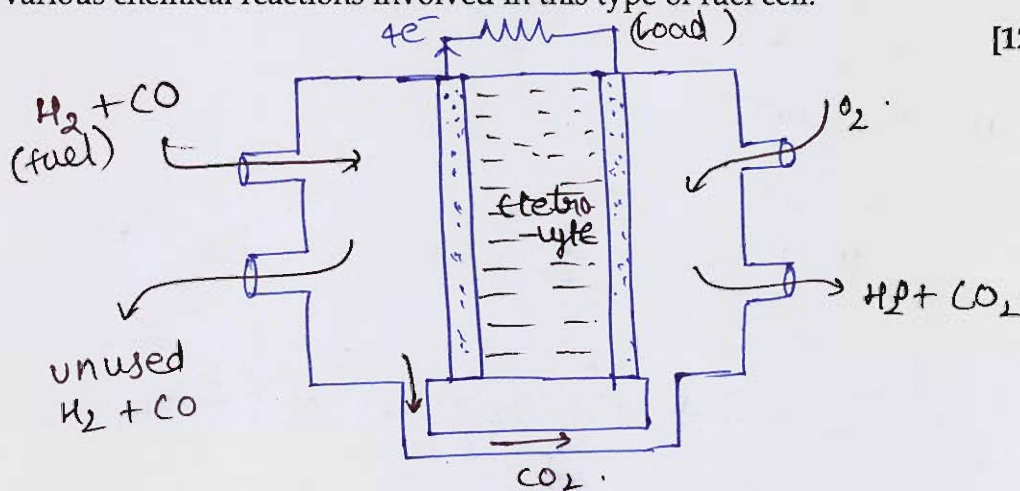
$$k_B = \frac{60}{4}$$

Thermal conductivity  
of B.

$$k_B = 15 \text{ W/mK}$$

### Section B : Heat Transfer + Renewable Sources of Energy

- 5 (a) Explain the working of molten carbonate fuel cell using appropriate diagram and write the various chemical reactions involved in this type of fuel cell. [12 marks]



MCFC

Electrolyte: molten carbonate salts of sodium and potassium.

Fuel : H<sub>2</sub> + CO (Producer gas).

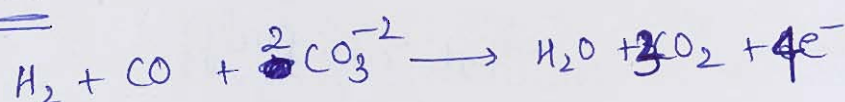
Oxidant : oxygen.

Byproduct :  $H_2O + CO_2$  .

It is high temperature fuel cell, hence catalyst use is not necessary .

operating temperature :  $600 - 800^\circ C$  .

Anode Reaction



Cathode Reaction



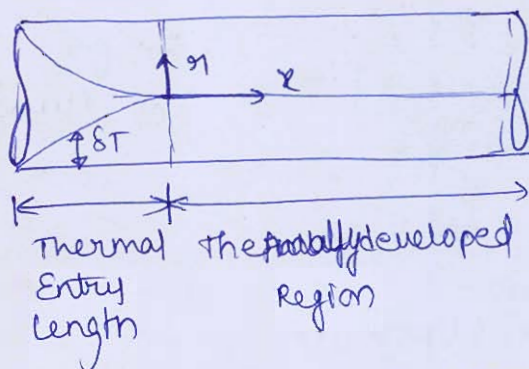
Overall Reaction



It involves transfer of  $4e^-$  .

- 5 (b) Explain clearly what is "thermally developed zone" in case of laminar flow through a tube and compare it to hydrodynamically developed zone. Draw the temperature distribution for (i) constant wall temperature (ii) constant heat flux.

[12 marks]

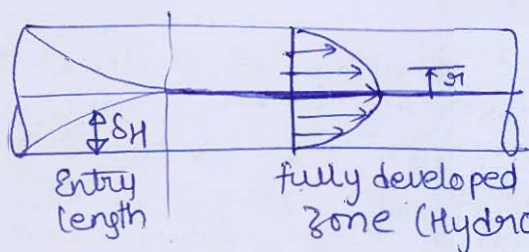


$\delta T =$  Thermal boundary layer.

$$\frac{\partial T}{\partial x} \neq 0$$

$$T = f(r, x)$$

It is the region where the thermal boundary layer from all around merge and the gradients along the flow direction vanish.



$$\frac{\partial U}{\partial x} = 0$$

$$U = f(r) \text{ only.}$$

It fully developed (Thermally) zone,

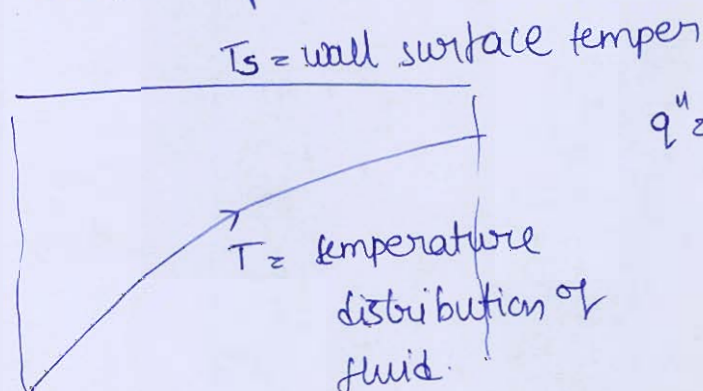
$\left(\frac{\partial T}{\partial r}\right)$  at boundary becomes constant

there by

$$h = \text{constant}$$

convective heat transfer coefficient.

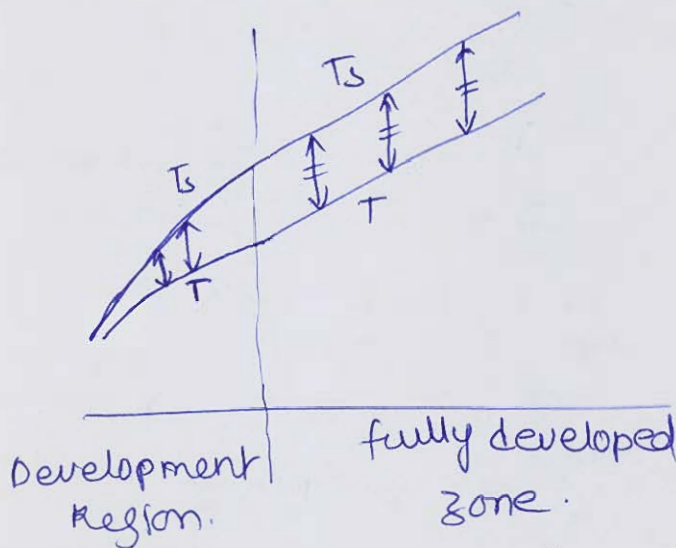
(i) Constant wall temperature



$$q'' = h(T_s - T)$$



(ii) Const wall heat flux.



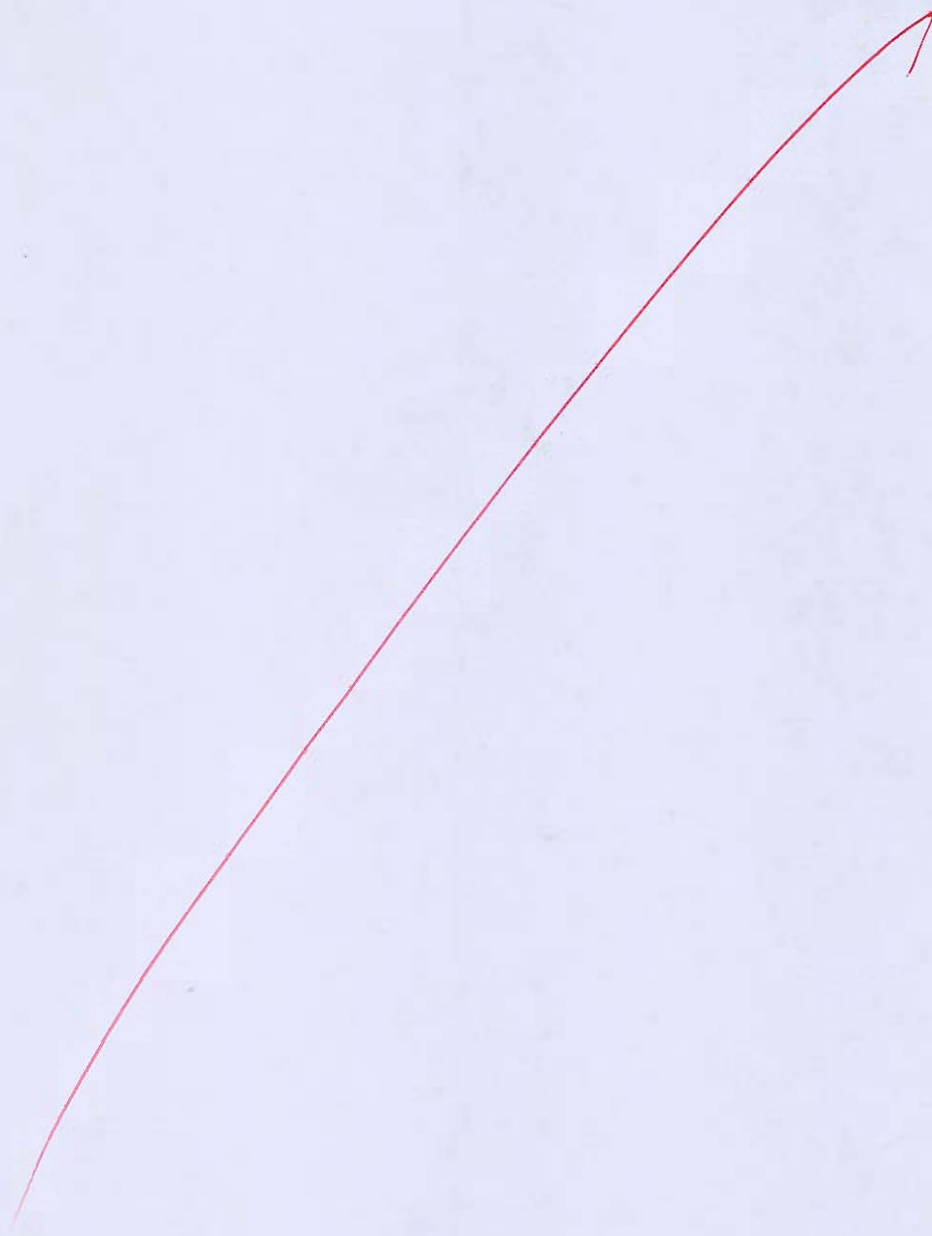
$T_s$  = wall surface temperature

$T$  = temperature of fluid.

- Q.5 (c) A 3 mm-thick panel of aluminum alloy ( $\rho = 2770 \text{ kg/m}^3$ ,  $c = 875 \text{ J/kgK}$  and  $k = 177 \text{ W/mK}$ ) is finished on both sides with an epoxy coating that must be cured at or above  $T_c = 150^\circ\text{C}$  for at least 5 min. The curing operation is performed in a large oven with air at  $175^\circ\text{C}$  and convection coefficient of  $h = 40 \text{ W/m}^2\text{K}$ . The coating has an emissivity of  $\epsilon = 0.8$ , and the temperature of the oven walls is  $175^\circ\text{C}$ , providing an effective radiation coefficient of  $h_{\text{rad}} = 12 \text{ W/m}^2\text{K}$ . If the panel is placed in the oven at an initial temperature of  $25^\circ\text{C}$ , at what total elapsed time,  $t$ , will the cure process be completed? Also show the variation of temperature with time during the curing process.

[12 marks]





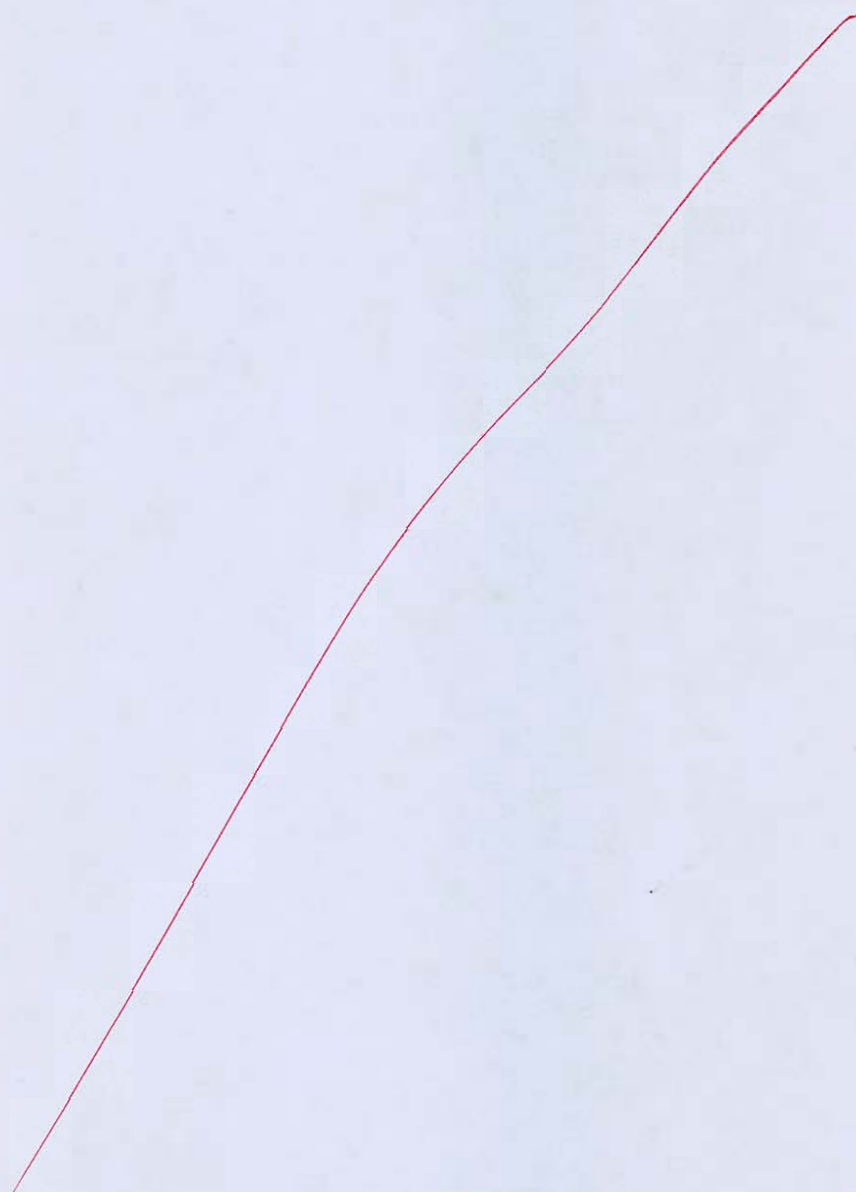
- Q.5 (d) Assuming the sun's surface as black and an equivalent black body temperature of 5779 K,
1. Estimate the rate at which the sun emits radiant energy
  2. What fraction of this energy is intercepted by the earth?
  3. What is the amount intercepted?

Given: Diameter of sun =  $1.39 \times 10^9$  m

Diameter of earth =  $1.27 \times 10^7$  m

Distance between the sun and the earth =  $1.5 \times 10^{11}$  m

[12 marks]





- 5 (e) A compound parabolic concentrator (CPC), 1.5 m long has an acceptance angle of  $20^\circ$ . The surface of the absorber is flat with a width of 15 cm. Evaluate the concentration ratio, the aperture height and the surface area of the concentrator.

[12 marks]

$$L = 1.5 \text{ m}$$

$$2\theta_a = 20^\circ \text{ acceptance angle}$$

$$\theta_a = 10^\circ$$

$$W = 0.15 \text{ m}$$

$$C = \frac{1}{\sin \theta_a}$$

$$C = 5.7587 \text{ conc. Ratio}$$

$$\frac{H}{W} = \frac{1}{2} \left( 1 + \frac{1}{\sin \theta_a} \right) \cos \theta_a$$

$$H = W \times 3.328$$

$$H = 0.4992 \text{ m} \rightarrow \text{Aperture height}$$

$$\frac{A_{\text{conc}}}{(WL)} = 1 + C \Rightarrow A_{\text{conc}} = 1.5207 \text{ m}^2$$

- Q.6 (a) A roof top collector is installed in a building at Agra ( $27.167^\circ\text{N}$ ,  $78.1^\circ\text{E}$ ). Determine the total extra-terrestrial radiation falling on the collector on 10<sup>th</sup> June. If the collector are installed horizontally and area covered by the collector is  $10\text{ m}^2$ . Calculate the change in the total extra-terrestrial radiation incident on the collector if it is inclined by  $15^\circ$ .

[20 marks]

$$\phi = 27.167^\circ$$

$\bar{I}_0$   $H_0$  = total extra terrestrial radiation

$$10^{\text{th}} \text{ June} \Rightarrow n = 31 + 28 + 31 + 30 + 31 + 10$$

$$n = 161$$

$$\delta = 23.45 \sin \left( \frac{360}{365} (n + 284) \right) \quad \left\{ \text{Coopers Relation} \right\}$$

$$\delta = 23.0116^\circ \quad (\text{Declination angle})$$

$$\omega_s = \cos^{-1} (-\tan \phi \tan \delta)$$

$$\omega_s = 102.589^\circ = 1.79 \text{ Rad} \Rightarrow \text{hour angle}$$



on horizontal surface

$$H_0 = 3600 \times \frac{24}{\pi} \times I_{sc} \left( 1 + 0.033 \cos \left( \frac{360n}{365} \right) \right) \times \left[ \cos \phi \cos \delta \sin \omega_s + \sin \phi \sin \delta \omega_s \right]$$

$$H_0 = 36438.56 \left[ \cos \phi \cos \delta \sin \omega_s + \sin \phi \sin \delta \omega_s \right] \frac{kJ}{m^2 \text{ day}}$$

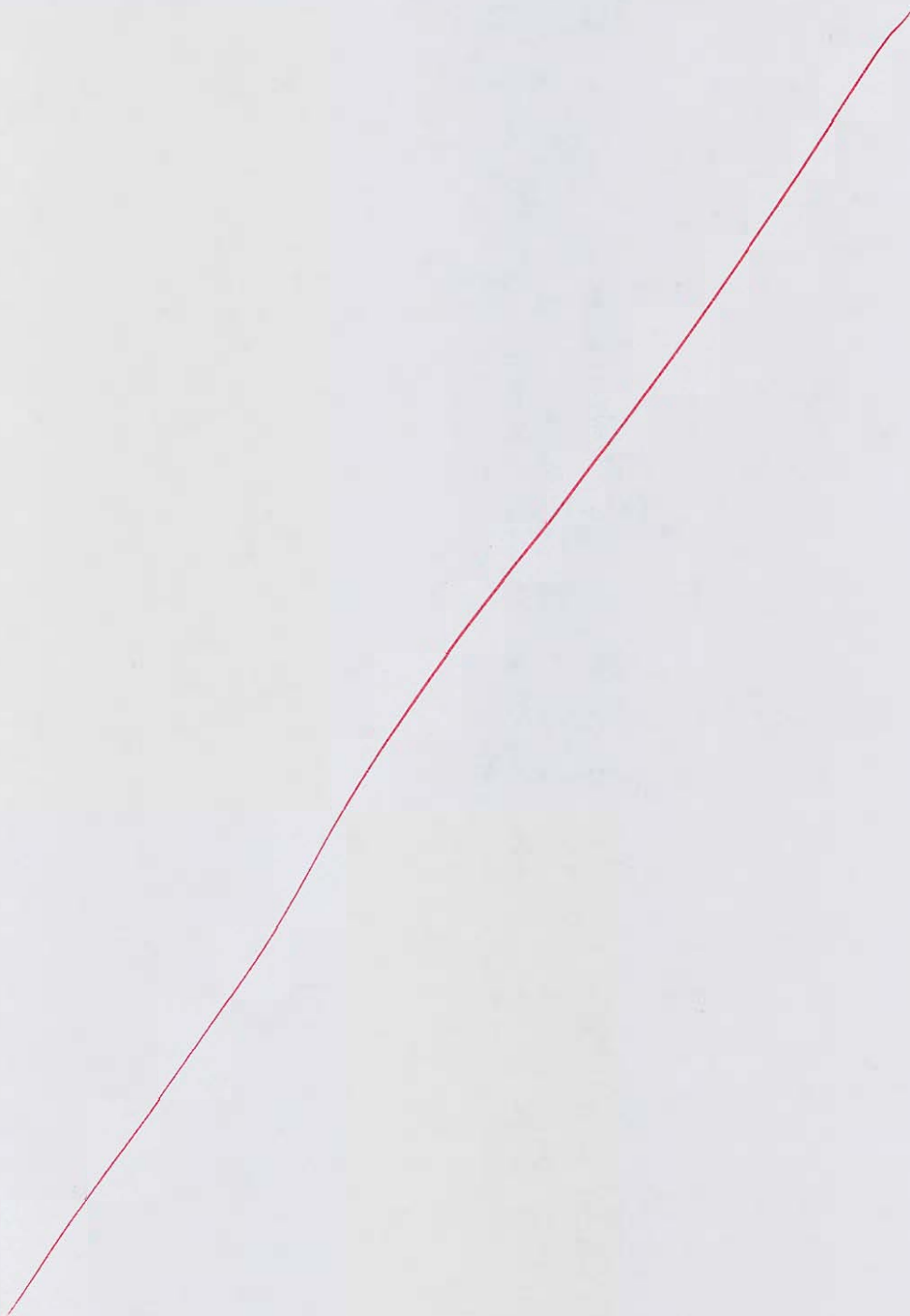
$$H_0 = 40763.4179 \frac{kJ}{m^2 \text{ day}}$$

$$\text{Area} = 10 m^2$$

$$H_0 A = 407634.18 \text{ kJ/day}$$

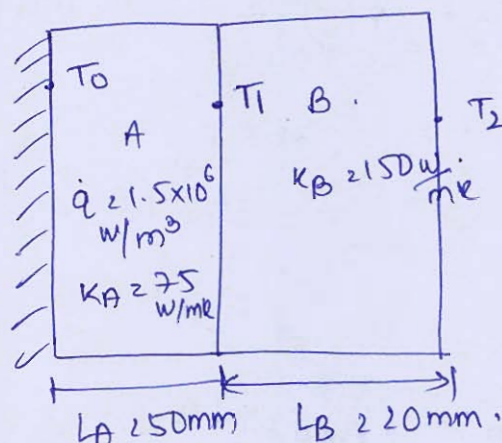
on inclined surface,

$$\beta = 15^\circ$$



- (b) A plane wall is composed of two materials, A and B. The wall of material A has uniform energy generation  $\dot{q} = 1.5 \times 10^6 \text{ W/m}^3$ ,  $K_A = 75 \text{ W/mK}$  and thickness  $L_A = 50 \text{ mm}$ . The wall of material B has no internal heat generation, with  $K_B = 150 \text{ W/mK}$  and thickness  $L_B = 20 \text{ mm}$ . The inner surface of material A is well insulated, while the outer surface of material B is cooled by a water stream  $T_\infty = 30^\circ\text{C}$  and  $h = 1000 \text{ W/m}^2\text{K}$ .
- (i) Determine the temperature  $T_0$  of the insulated surface and the temperature  $T_2$  of the cooled surface.
- (ii) Sketch the temperature distribution that exists in the composite under steady-state conditions.

[20 marks]



$$T_\infty = 30^\circ\text{C}$$

$$h = 1000 \frac{\text{W}}{\text{m}^2\text{K}}$$

$$\text{Total heat generated in A} = 1.5 \times 10^6 \frac{\text{W}}{\text{m}^3} \times 1 \text{ m}^2 \times 0.05$$

assuming area of heat cut  $= 1 \text{ m}^2$

$$q = 75000 \text{ Watt}$$

at cooled surface,

$$q = h(1 \text{ m}^2)(T_2 - 30)$$

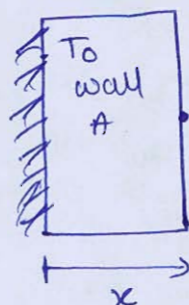
$$75000 = 1000 \times 1 (T_2 - 30)$$

Ans  $\leftarrow T_2 = 105^\circ\text{C} \rightarrow \text{Temperature of cooled surface.}$

$$\frac{T_1 - T_2}{\left(\frac{0.02}{150 \times 1}\right)} = 75000 \Rightarrow T_1 = 115^\circ\text{C}$$

Temp at junction of walls A, B.





$$T = \frac{\dot{q}}{2k} [L^2 - x^2] + T_w$$

$$T = \frac{1.5 \times 10^6}{2 \times 75} [0.05^2 - x^2] + T_w$$

$$T_w = 115^\circ\text{C}$$

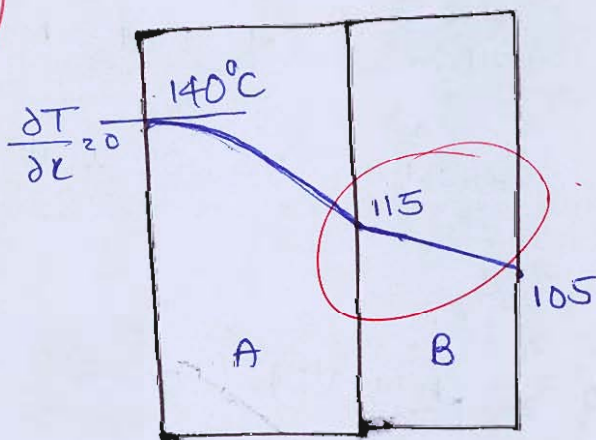
$$T = 10^4 (0.05^2 - x^2) + T_w \quad \left\{ \begin{array}{l} \text{Temp. distribution} \\ \text{in wall A.} \end{array} \right.$$

$$\text{at } x=0 \quad T = T_0 = 10^4 (0.05^2) + 115$$

$$T_0 = 25 + 115$$

$$\text{(Ans)} \quad T_0 = 140^\circ \quad \rightarrow \text{Temperature at insulated surface}$$

18



$$T_{\infty} = 30^\circ\text{C}$$

$$k_A < k_B$$

$$\left( \frac{\partial T}{\partial x} \right)_{\text{in A}} > \left( \frac{\partial T}{\partial x} \right)_{\text{in B}}$$

Temperature distribution in  
composite wall.

- (c) Discuss the relative merits and limitations of tidal power. What are the difficulties in tidal power developments? For a typical tidal power plant shown below, the basin area is  $25 \times 10^6 \text{ m}^2$ . The tide has a range of 10 m. However, turbine stops working when the head on it falls below 2 m. Assume that density of seawater is  $1025 \text{ kg/m}^3$ , acceleration due to gravity is  $9.81 \text{ m/s}^2$ , combined efficiency of turbine and generator is 75% and period of energy generation is 6h and 12.5 min.

Determine:

1. Work done in filling or emptying the basin
2. Average power
3. The energy generated in one filling process (in kWh)

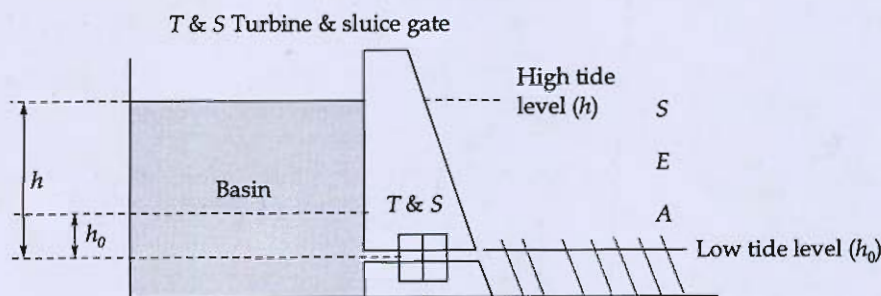


Figure: Single Basin tidal plant

[20 marks]

### Limitations

- \* Disturbs marine life.
- \* Site specific (feasible in regions where range is atleast 5m).
- \* Large number of turbine generator sets reqd.
- \* Geography dependent.

### merits

- \* Power generation through out the year.
- \* Unlike conventional power plant these doesn't harm the environment.

### Difficulties

- \* continuous change in range and head at which turbine operates.



\* large amount of volume of water has to be discharged or inducted in to basin in a short time (single emptying usually in or less than 6 hrs 12.5 minutes).

$$A = 25 \times 10^6 \text{ m}^2$$

$$R = 10 \text{ m}$$

$$r = 2 \text{ m}$$

$$\rho = 1025 \text{ kg/m}^3$$

$$g = 9.81$$

$$\eta_g = 0.75$$

$$t = 6 \text{ h } 12.5 \text{ min}$$

$$(W.D)_{\text{filling}} = \frac{1}{2} \rho A g R^2$$

$$= \frac{1}{2} \times 1025 \times 25 \times 10^6 \times 9.81$$

$$(W.D)_{\text{filling}} = 1.2569 \times 10^{13} \text{ J}$$

$$P_{\text{avg}} = 0.75 \times \frac{\frac{1}{2} \rho A g (R^2 - r^2)}{(22350 \text{ s})}$$

$$P_{\text{avg}} = 404.9094 \text{ MWatt} \rightarrow (ii)$$

$$\text{Energy generated in one filling} = 0.75 \times \frac{1}{2} \rho A g (R^2 - r^2)$$

$$= 9.0497 \times 10^{12} \text{ J}$$

$$= 2.5138 \times 10^6 \text{ kWh}$$

$$\text{Energy generated in one filling process} = 2.5138 \times 10^6 \text{ kWh}$$



- 7 (a) (i) Briefly explain how plastic solar cells with the help of nanotechnology can popularize the use of solar cells in the near future.
- (ii) Determine the solar array area and battery size for the average load of 67 W for 24 h. Solar cell efficiency is 10% and sum total of all array design and degrade array factor is 0.5. Battery charging efficiency is 60%. The load is to be supported for seven continuous days of cloudy weather (no sunshine) and the battery is to be fully recharged in 3 days. Average monthly insolation is  $181 \text{ kWh/m}^2$  and it is assumed that each winter day receives 9.7 hour of sunshine.

[10 + 10 marks]







Q.7 (b) The surface temperature of a thin, flat plate placed parallel to an air stream is  $80^{\circ}\text{C}$ . The free stream velocity is  $50\text{ m/s}$  and the temperature of the air is  $0^{\circ}\text{C}$ . The plate is  $60\text{ cm}$  wide and  $40\text{ cm}$  long in the direction of the air stream. Neglecting the end effect of the plate and assuming that the flow within the boundary layer changes abruptly from laminar to turbulent at a transition Reynolds number of  $Re_{tr} = 4 \times 10^5$ , find:

- (i) the average heat transfer coefficient in the laminar and turbulent regions.
- (ii) the rate of heat transfer for the entire plate, considering both sides.

Also plot the heat transfer coefficient and local friction coefficient as a function of the distance from the leading edge of the plate.

Take, kinematic viscosity ( $\nu$ ) =  $18.1 \times 10^{-6}\text{ m}^2/\text{s}$

Thermal conductivity ( $k$ ) =  $0.0269\text{ W/mK}$

Prandtl number ( $Pr$ ) =  $0.71$

Density ( $\rho$ ) =  $1.075\text{ kg/m}^3$

[20 marks]

