MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

E & T Engineering Test No : 10

Section A

Q.1 (a) Solution:

(i) Similarities between JUMP and CALL:

1. Both transfer program control to a certain memory location.
2. Both are 3 byte instructions.
3. They do not have any effect on flags, but they depends on status of flags in conditional type instructions.

Differences between JUMP and CALL:

1. The contents of program counter are stored on the stack by CALL instruction but not by JUMP instruction.
2. JUMP requires 10T states while CALL requires 18T states.
3. JUMP is used for branching the program control to a location in memory that is a part of the main program while CALL is used for calling a subroutine. Using a CALL instruction, the program control is transferred to a location in memory that is not a part of the main program.
4. A CALL instruction has to be followed by a RETURN instruction. There is no such compulsion for JUMP.

(ii) Similarities between STA and STAX:

1. Both STA and STAX are data transfer instructions in the 8085 that involve storing data from the accumulator to a memory location.
2. They do not affect the flags.

Differences between STA and STAX:

1. STA uses direct addressing mode while STAX uses register indirect addressing mode i.e. in STA instruction, the 16-bit address of memory location is specified in the instruction itself whereas in STAX instruction, 16-bit address is stored in a register pair (BC or DE).
2. STA is 3 bytes instruction while STAX is 1 byte instruction.
3. STA needs 13T-states while STAX needs 7T states.

Q.1 (b) Solution:

Given that,
$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Taking the DTFT, we obtain

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$e^{j\omega_0 n}$ is an eigen function of an LTI system i.e. for an LTI system with impulse response $H(e^{j\omega})$, if the input is $e^{j\omega_0 n}$, the output is given by

$$y[n] = e^{j\omega_0 n}H(j\omega_0)$$

We have, Input sequence,
$$x[n] = Ce^{j\left(\frac{\pi}{2}\right)n}$$

$$\Rightarrow \quad \omega_0 = \frac{\pi}{2}\text{ rad/s}$$

At $\omega_0 = \frac{\pi}{2}$,
$$H(e^{j\pi/2}) = \frac{1}{1-\frac{1}{2}e^{-j\pi/2}} = \frac{1}{1-\frac{1}{2}\left(\cos\frac{\pi}{2}-j\sin\frac{\pi}{2}\right)}$$

$$\Rightarrow \quad H(e^{j\pi/2}) = \frac{1}{1+j\left(\frac{1}{2}\right)} = \frac{2}{\sqrt{5}}e^{-j(\tan^{-1}0.5)}$$

The response of the system to the complex exponential is given by,

$$y[n] = C\times H(e^{j\pi/2})e^{j\omega_0 n}$$
$$= CH(e^{j\pi/2})e^{jn\pi/2}$$

$$\Rightarrow \quad y[n] = C\left[\frac{2}{\sqrt{5}} e^{-j(\tan^{-1} 0.5)} \cdot e^{jn\pi/2}\right]$$

$$= \frac{2C}{\sqrt{5}} e^{j\left(\frac{\pi n}{2} - \tan^{-1} 0.5\right)} = 0.894 C e^{j\left(\frac{\pi n}{2} - 26.57^\circ\right)}$$

Q.1 (c) Solution:

Given,

Refractive index of fiber core, $n_1 = 1.5$

The magnitude of partial reflection of the light transmitted through the interface can be estimated using the classical Fresnel formula for light of normal incidence and is given by,

$$r = \left(\frac{n_1 - n}{n_1 + n}\right)^2$$

where

r : Fraction of the light reflected at a single interface.

n_1 : Refractive index of the fiber core.

n : Refractive index of the medium between the two jointed fibers (i.e., for air $n = 1$)

The magnitude of the Fresnel reflection at the fiber-air interface is given by,

$$r = \left(\frac{n_1 - n}{n_1 + n}\right)^2 = \left(\frac{1.5 - 1}{1.5 + 1}\right)^2 = \left(\frac{0.5}{2.5}\right)^2 = 0.04$$

The value obtained for 'r' corresponds to a reflection of 4% of the transmitted light at the single interface.

The loss in decibels due to Fresnel reflection at a single interface is given by:

$$\text{Loss}_{\text{Fres}} = -10 \log_{10}(1 - r)$$

$$= -10 \log_{10}(1 - 0.04)$$

$$= -10 \log_{10} 0.96 \simeq 0.18 \text{ dB}$$

A similar calculation may be performed for the other interface (air-fiber). However, from considerations of symmetry, it is clear that the optical loss at the second interface is also 0.18 dB.

Hence, the total loss due to Fresnel reflection at the fiber joint is approximately 0.36 dB.

Q.1 (d) Solution:

Given, $P = 2Q$

The sum of probabilities of all possible input symbols must equal one i.e.

$$P(x_1) + P(x_2) + P(x_3) = 1$$
$$P + Q + Q = 1$$
$$P + 2Q = 1$$
$$2Q + 2Q = 1$$

$$\Rightarrow \quad Q = \frac{1}{4}, \quad P = \frac{1}{2}$$

The channel matrix $P(y_j/x_i)$ is

$$P(y/x) = \begin{bmatrix} P & 1-P & 0 \\ 1-P & P & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The joint probability matrix $P(x_i, y_j)$ is

$$P(x, y) = [P(x)]_d P(y/x) = \begin{bmatrix} P & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & Q \end{bmatrix} \begin{bmatrix} P & 1-P & 0 \\ 1-P & P & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P(x, y) = \begin{bmatrix} P^2 & P(1-P) & 0 \\ Q(1-P) & QP & 0 \\ 0 & 0 & Q \end{bmatrix}$$

From the joint probability matrix, we can write

$$P(y_1) = P^2 + Q(1-P) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

$$P(y_2) = QP + P(1-P) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$

$$P(y_3) = Q = \frac{1}{4}$$

Thus,

$$H(x) = \Sigma P(x_i) \log_2 \frac{1}{P(x_i)}$$
$$= -P\log_2 P - 2Q \log_2 Q$$
$$H(x) = -\left[P\log_2 P + P\log_2\left(\frac{P}{2}\right)\right]$$

$$= -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right)+\frac{1}{2}\log_2\left(\frac{1}{4}\right)\right]$$

$$= -\left[-\frac{1}{2}-1\right]=\frac{3}{2}\text{ bits/symbol}$$

$$H(y/x) = \sum_i\sum_j P(x_i,y_j)\log_2\frac{1}{P(y_j/x_i)}$$

$$= \begin{bmatrix} P^2\log_2 P+P(1-P)\log_2(1-P)+Q(1-P) \\ \log_2(1-P)+QP\log_2 P+Q\log_2 1 \end{bmatrix}$$

$$= -\left[\frac{1}{4}\log_2\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\log_2\frac{1}{2}+\frac{1}{8}\log_2\frac{1}{2}+\frac{1}{8}\log_2\frac{1}{2}\right]$$

$$= \left[\frac{1}{4}\times 1+\frac{1}{4}\times 1+\frac{1}{8}\times 1+\frac{1}{8}\times 1\right]$$

$$= \frac{2}{4}+\frac{2}{8}=\frac{6}{8}=\frac{3}{4}=0.75\text{ bits/symbol}$$

$$H(y) = \sum_i P(y_i)\log_2\frac{1}{P(y_i)}$$

$$= \frac{3}{8}\log_2\frac{8}{3}+\frac{3}{8}\log_2\frac{8}{3}+\frac{1}{4}\log_2 4$$

$$= 1.5612\text{ bits/symbol}$$

$$I(x:y) = H(x) + H(y) - H(x, y)$$

$$= H(y) - H(y/x)$$

$$= 1.5612 - 0.75$$

$$= 0.8112\text{ bits}$$

Q.1 (e) Solution:

In the network shown in figure, the current through the inductor i_L and the voltage across the capacitor v are considered as the state variables, and the current i_0 through L and the voltage v_0 across L as the output variable.

Writing the KCL at node 1

$$i = i_R+i_C+i_L=\frac{v}{R}+C\frac{dv}{dt}+i_L$$

i.e., $$\frac{dv}{dt} = \frac{-1}{C}i_L - \frac{v}{RC} + \frac{1}{C}i$$

The voltage across the inductor is

$$v = L\frac{di_L}{dt}$$

i.e., $$\frac{di_L}{dt} = \frac{v}{L}$$

Also, $$i_0 = i_L$$

and $$v_0 = v$$

Based on the above equations, the state model is obtained as

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ \frac{-1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} i$$

$$\begin{bmatrix} i_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v \end{bmatrix}$$

If the state variables are selected such as i_L is x_1 and v is x_2, and the output i_0 is y_1 and v_0 is y_2 and the input current i is u, then the state model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ \frac{-1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q.2 (a) Solution:

(i) 1. The phase difference between the radiated waves from two antenna elements is

$$\psi = \beta + kd\cos\theta$$

where β is the phase difference between the feed of the two array elements and k is the propagation constant. We have, $k = 2\pi/\lambda$.

Assuming E_0 as the electric field pattern of the infinitesimal dipole, the electric field pattern of the antenna array is given by

$$E_t = E_0 + E_0 e^{j\psi} = E_0(1 + \cos\psi + j\sin\psi)$$

$$|E_t| = E_0\sqrt{(1+\cos\psi)^2 + \sin^2\psi} = E_0\sqrt{2(1+\cos\psi)}$$

$$|E_t| = 2E_0\cos\left(\frac{\psi}{2}\right)$$

For infinitesimal dipole, $E_0(\theta, \phi) = K\cos\theta$ for $\theta = 90°$. Thus, the normalized field is given by

$$E_{tn} = \cos\theta\cos\left(\frac{\beta + kd\cos\theta}{2}\right)$$

For $d = \lambda/4$, we have

$$E_{tn} = \cos\theta\cos\left(\frac{\beta}{2} + \frac{\pi}{4}\cos\theta\right)$$

For $\beta = 0$, normalized field is given by

$$E_{tn} = \cos\theta\cos\left(\frac{\pi}{4}\cos\theta\right)$$

The nulls are obtained by setting the total field equal to zero, or

$$E_{tn} = \cos\theta\cos\left(\frac{\pi}{4}\cos\theta\right)\Big|_{\theta = \theta_n = 0}$$

Thus, $\cos\theta_n = 0 \Rightarrow \theta_n = 90°$

and $\cos\left(\frac{\pi}{4}\cos\theta_n\right) = 0$

$$\frac{\pi}{4}\cos\theta_n = \frac{\pi}{2}, \frac{-\pi}{2}$$

$$\theta_n = \text{does not exist}$$

The only null occurs at $\theta = 90°$ and is due to the pattern of the individual elements. The array factor does not contribute any additional nulls because there is not enough separation between the elements to introduce a phase difference of 180° between the elements, for any observation angle.

2. $\beta = +\frac{\pi}{2}$

The normalized field is given by

$$E_{tn} = \cos\theta\cos\left[\frac{\pi}{4}(\cos\theta+1)\right]$$

The nulls are found from

$$E_{tn} = \cos\theta\cos\left[\frac{\pi}{4}(\cos\theta+1)\right]\Bigg|_{\theta=\theta_n} = 0$$

Thus, $\cos\theta_n = 0$

$$\theta_n = 90°$$

and $\cos\left[\frac{\pi}{4}(\cos\theta+1)\right]\Bigg|_{\theta=\theta_n} = 0$

$$\frac{\pi}{4}(\cos\theta_n+1) = \frac{\pi}{2} \Rightarrow \theta_n = 0°$$

and $\frac{\pi}{4}(\cos\theta_n+1) = -\frac{\pi}{2} \Rightarrow$ No real value of θ_n exists

Thus, the nulls occurs at $\theta = 0°$ and $\theta = 90°$.

3. $\beta = -\frac{\pi}{2}$

The normalized field is given by

$$E_{tn} = \cos\theta\cos\left[\frac{\pi}{4}(\cos\theta-1)\right]$$

The nulls are given by

$$\cos\theta\cos\left[\frac{\pi}{4}(\cos\theta-1)\right]\Bigg|_{\theta=\theta_n} = 0$$

Thus, $\cos\theta_n = 0 \Rightarrow \theta_n = 90°$

and $\frac{\pi}{4}(\cos\theta_n-1) = \frac{\pi}{2}$

$\Rightarrow$ θ_n = does not exist and $$\frac{\pi}{4}(\cos\theta_n - 1) = -\frac{\pi}{2}$$

$$\Rightarrow \quad \theta_n = 180°$$

Therefore, nulls occur at $\theta = 90°$ and $\theta = 180°$.

(ii) The normalized electric field pattern for a half-wave dipole is given by,

$$f(\theta) = \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|$$

The nulls in the E-field pattern can be obtained as,

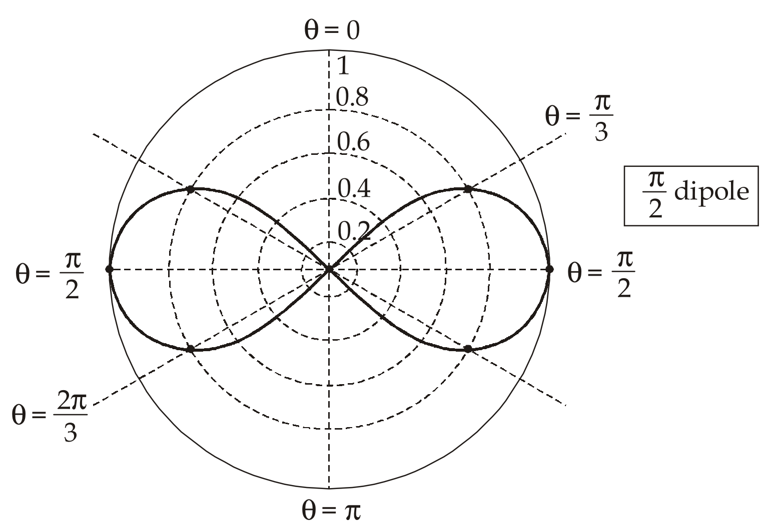
$$\cos\left(\frac{\pi}{2}\cos\theta_n\right) = 0 \quad \Rightarrow \quad \cos\theta_n = \pm 1$$

$$\Rightarrow \quad \theta_n = 0, \pi$$

For various values of θ, the values of $f(\theta)$ can be obtained as

θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
$f(\theta)$	0	0.8165	1	0.8165	0

The normalized E-field pattern is thus obtained as below,



Q.2 (b) Solution:

(i) RISC (Reduced Instruction Set Computing) and CISC (Complex Instruction Set Computing) are two approaches to processor architecture. The RISC processors use fewer and simpler instructions that execute quickly while the CISC processors have the complex instructions that can perform the multiple operations. The differences between CISC and RISC are as detailed below:

CISC (Complex Instruction Set Computer)	RISC (Reduced Instruction Set Computer)
• Large instruction set.	• Compact instruction set.
• Instruction formats are of different lengths.	• Instruction formats are all of the same length.
• Instructions perform both elementary and complex operations.	• Fixed length simple instruction format.
• Control unit is micro-programmed.	• Control unit is simple and hardwired.
• Not pipelined or less pipelined.	• Pipelined.
• Single register set.	• Multiple register set to enhance performance.
• Numerous memory addressing options for operands.	• Few addressing modes.
• Emphasis on hardware for optimizing performance.	• Emphasis on software for optimizing performance .
• Instructions may take multiple clock cycles.	• Instruction takes one clock cycle.
• Can perform register to register, register to memory or memory to memory operations.	• Can perform only Register to Register operations.
• Small code sizes, high cycles per second.	• Low cycles per second, large code sizes.
• Transistors used for storing complex instructions.	• Transistors are used for more registers.
Examples of CISC processors: ♦ VAX ♦ Motorola 68000 family ♦ Intel X86 architecture based processors.	Examples of RISC processors: ♦ Apple iPods (custom ARM 7TDMI SoC) ♦ Apple iphone (Samsung ARM1176JZF) ♦ Nintends Game Boy Advance (ARM7)

(ii) Given,

Time to perform a task is non-pipeline system,

$$(t_n) = 60 \text{ nsec}$$

$$\text{Pipeline cycle time } (t_p) = 10 \text{ nsec}$$

$$\text{speed up } (S) = 5.66$$

$$\text{Number of tasks } (n) = 100$$

For 100 tasks, speed up, $S = \dfrac{nt_n}{(k+n-1)t_p}$

$$\Rightarrow \quad 5.66 = \frac{(60\times10^{-9})\times100}{(k+100-1)(10\times10^{-9})}$$

$$\Rightarrow \quad k + 99 = \frac{600}{5.66}$$

$$\Rightarrow \quad k \simeq 7$$

∴ The number of segments (k) in the pipeline system is 7.

Q.2 (c) Solution:

(i) We have, frequency range: 4 to 10 MHz

IF range = 1.8 MHz

We know,

$$\text{image frequency}, f_{si} = f_s + 2\text{IF}$$

$$f_{si(\min)} = f_{s(\min)} + 2\text{IF}$$

$$= 4 + 2(1.8) = 7.6 \text{ MHz}$$

$$f_{si(\max)} = f_{s(\max)} + 2\text{IF}$$

$$= 10 + 2(1.8) = 13.6 \text{ MHz}$$

∴ Image frequency range = (7.6 – 13.6) MHz

Since the receiver allows the frequency from 4 to 10 MHz, thus the image frequencies in the range 7.6 MHz to 10 MHz fall in the receiver passband.

Image frequency rejection ratio:

(i) f_s = 4 MHz

IF = 1.8 MHz, Q = 50, f_s = 4 MHz, f_{si} = 7.6 MHz

The image frequency rejection ratio, a using a RF filter with Quality Factor Q is given by

$$\alpha = \sqrt{1+Q^2\rho^2}$$

where,

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{7.6}{4} - \frac{4}{7.6}$$

$$\rho = 1.37$$

$$\alpha = \sqrt{1+(50)^2(1.37)^2}$$

$$\alpha = 68.51$$

Similarly for (ii) $f_s = 10$ MHz

$$IF = 1.8\text{ MHz}, Q = 50, f_s = 10\text{ MHz}, f_{si} = 13.6\text{ MHz}$$

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{13.6}{10} - \frac{10}{13.6}$$

$$= 0.62$$

$$\alpha = \sqrt{1+Q^2\rho^2}$$

$$\alpha = \sqrt{1+(50)^2(0.62)^2} = 31.02$$

(ii) Input signal, $m(t) = 5\cos(2000\,\pi t) + 2\cos(4000\,\pi t)$ V

$$= m_1(t) + m_2(t)$$

To avoid slope slope overload distortion,

$$\Delta f_s \geq \left|\frac{d}{dt}m(t)\right|_{max}$$

$$\Delta \times 56000 \geq [5 \times 2000\pi + 2 \times 4000\pi]$$

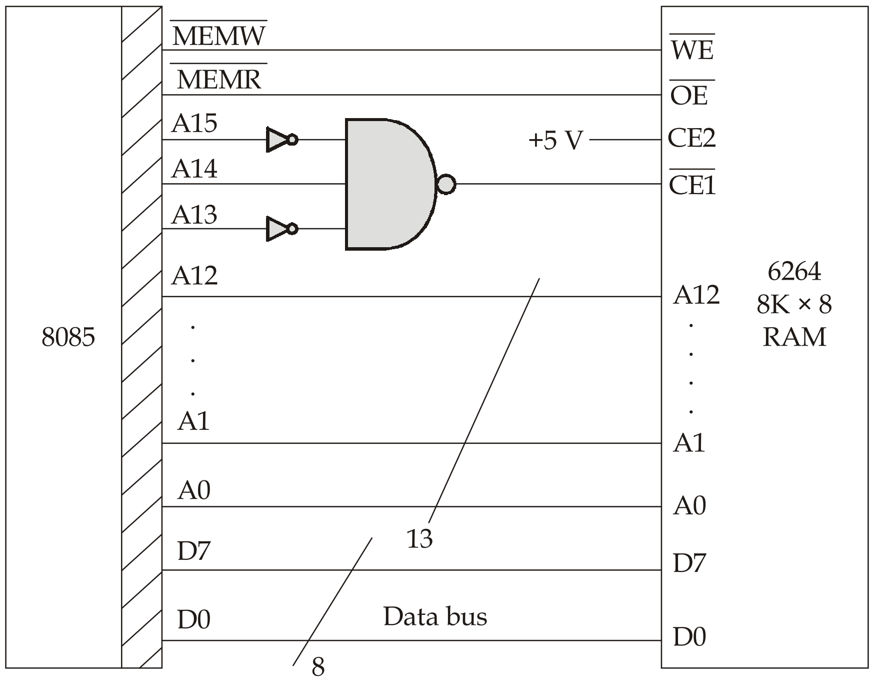
$$\Delta \times 56000 \geq [10000\pi + 8000\pi]$$

$$\Delta \geq \frac{18000\pi}{56000} \Rightarrow \Delta \geq 1\text{ V}$$

$$\Rightarrow \quad \Delta_{min} = 1\text{ V}$$

Q.3 (a) Solution:

(i) The 6264 IC has 13 address lines (A0-A12), since 8 KB = 2^{13} bytes. The ending address of the chip is 5FFFH (since 4000H + 1FFFH = 5FFFH). When the addresses 4000H to 5FFFH are written in binary form, the values in the lines A15, A14 and A13 are 0, 1, and 0, respectively. The NAND gate decoder is designed such that when the lines A15 and A13 carry 0 and A14 carries 1, the output of the NAND gate is 0. The NAND gate output is in turn connected to the $\overline{\text{CE1}}$ pin of the RAM chip. A NAND output of 0 selects the RAM chip for read or write operation and CE2 is connected to +5 V. Figure below show the interfacing of IC 6264 with the 8085.



(ii) In the 8085 microprocessor, the contents of the flag register and the accumulator together are called program status word (PSW). The PSW can be accessed only using PUSH and POP instructions. The PUSH instruction moves the flag register contents (as the lower-order byte) and the accumulator contents (as the higher-order byte) to the stack.

The contents of the flag register can be read and complemented using the program given in table below.

Program for reading and complementing the contents of the flag register

Mnemonics		Comments
PUSH PSW	;	Push the contents of the accumulator and the flag register onto the stack.
POP H	;	Bring the PSW to the HL pair.
MOV A, L	;	Move the contents of the flag register to the accumulator.
CMA	;	Complement the accumulator.
MOV L, A	;	Store the complemented flags in register L.
PUSH H	;	Push the contents of the HL register pair onto the stack.
POP PSW	;	Pop the contents of the accumulator and the flag register back to the PSW.
HLT	;	Terminate program execution

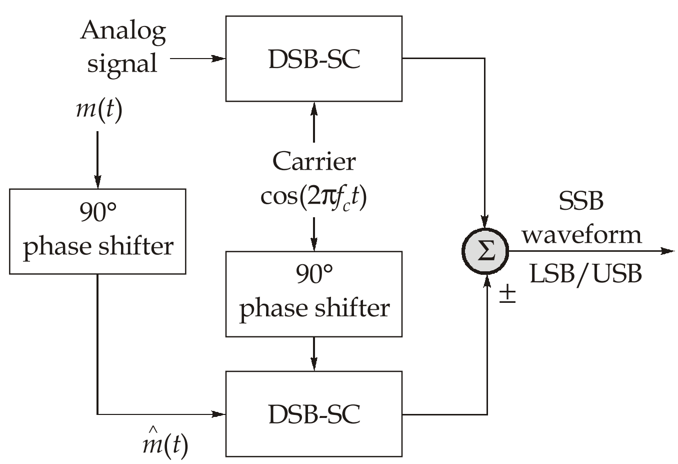
Q.3 (b) Solution:

Let the message signal be denoted by $m(t)$ and carrier signal be denoted by $\cos(\omega_c t)$, then the SSB signal can be written as,

$$s(t) = m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t$$

where $\hat{m}(t)$ is the hilbert transform of $m(t)$ and $\sin\omega_c t$ is the phase shifted version of the carrier signal.

The following figure shows the block diagram of SSBSC modulator using phase discrimination method.



Now, $m(t)\cos\omega_c t \xleftrightarrow{\text{F.T.}} \frac{1}{2}M(\omega-\omega_c) + \frac{1}{2}M(\omega+\omega_c)$ ...(i)

$\hat{m}(t)\sin\omega_c t \xleftrightarrow{\text{F.T.}} \frac{1}{2j}\hat{M}(\omega-\omega_c) - \frac{1}{2j}\hat{M}(\omega+\omega_c)$ ...(ii)

Taking the difference of equation (i) and (ii), we get

$$s(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t$$

Taking Fourier transfer,

$$S(\omega) = \frac{1}{2}M(\omega-\omega_c)+\frac{1}{2}M(\omega+\omega_c)-\left[\frac{1}{2j}\hat{M}(\omega-\omega_c)-\frac{1}{2j}\hat{M}(\omega+\omega_c)\right]$$

Now, $\hat{M}(\omega-\omega_c) = -j\,\text{sgn}(\omega-\omega_c)\,M(\omega-\omega_c)$ $[\because F[\hat{m}(t)] = j\,\text{sgn}(\omega)M(\omega)]$

and $\hat{M}(\omega+\omega_c) = -j\,\text{sgn}(\omega+\omega_c)\,M(\omega+\omega_c)$

$$\therefore \qquad S(\omega) = \frac{1}{2}M(\omega-\omega_c)+\frac{1}{2}M(\omega+\omega_c)-\left[-\frac{1}{2}\text{sgn}(\omega-\omega_c)M(\omega-\omega_c)+\frac{1}{2}\text{sgn}(\omega+\omega_c)M(\omega+\omega_c)\right]$$

$$S(\omega) = \frac{1}{2}M(\omega-\omega_c)[1+\text{sgn}(\omega-\omega_c)]+\frac{1}{2}M(\omega+\omega_c)[1-\text{sgn}(\omega+\omega_c)]$$

We have, $1 + \text{sgn}(\omega - \omega_c) = \begin{cases} 2; & \text{for } \omega > \omega_c \\ 0; & \text{for } \omega < \omega_c \end{cases}$

and $1 - \text{sgn}(\omega + \omega_c) = \begin{cases} 2; & \text{for } \omega < -\omega_c \\ 0; & \text{for } \omega > \omega_c \end{cases}$

$$S(\omega) = \begin{cases} M(\omega-\omega_c); & \text{for } \omega > \omega_c \\ 0; & \text{for } -\omega_c < \omega < \omega_c \\ M(\omega+\omega_c); & \text{for } \omega < -\omega_c \end{cases}$$

Thus, in phase discrimination method of SSB modulation, the difference of the signal at summing junction produces the upper sideband of the SSB signal.

Q.3 (c) Solution:

Comparing the given state equation with $\dot{q}(t) = Aq(t) + Bx(t)$, we get

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Taking Laplace Transform of the state equation, we get

$$sQ(s) - q(0) = AQ(s) + BX(s)$$

We get, $$Q(s) = \phi(s)[q(0) + BX(s)] \qquad \text{...(i)}$$

where $$\phi(s) = [sI - A]^{-1}$$

$$[sI - A] = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s+1 & -1 \\ 0 & s+2 \end{bmatrix}$$

and $$\phi(s) = \begin{bmatrix} \frac{1}{(s+1)} & \frac{1}{(s+1)(s+2)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}$$

Given that $x(t) = u(t)$, thus $X(s) = \frac{1}{s}$. Therefore,

We have,
$$BX(s) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s} \end{bmatrix} = \begin{bmatrix} 0 \\ 1/s \end{bmatrix} \text{ and } q[0] = \begin{bmatrix} q_1(0) \\ q_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Therefore,
$$q(0) + BX(s) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/s \end{bmatrix} = \begin{bmatrix} -1 \\ 1/s \end{bmatrix}$$

and
$$Q(s) = \phi(s)[q(0) + BX(s)]$$

$$= \begin{bmatrix} \frac{1}{s+1} & \frac{1}{(s+1)(s+2)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}\begin{bmatrix} -1 \\ 1/s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{s+1} + \frac{1}{s(s+1)(s+2)} \\ \frac{1}{s(s+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{(s+1)} + \frac{1/2}{s} - \frac{1}{(s+1)} + \frac{1/2}{(s+2)} \\ \frac{1/2}{s} - \frac{1/2}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1/2}{s} - \frac{2}{s+1} + \frac{1/2}{s+2} \\ \frac{1/2}{s} - \frac{1/2}{s+2} \end{bmatrix}$$

Taking the inverse Laplace transform on both sides,

$$q(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2} - 2e^{-t} + \frac{1}{2}e^{-2t}\right)u(t) \\ \left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)u(t) \end{bmatrix}$$

Q.4 (a) Solution:

The waveform is periodic with period $T = \pi$ and fundamental frequency,

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2 \text{ rad/s.}$$

Given, $$x(t) = M\sin(t); \quad 0 < t < \pi$$

The exponential Fourier series coefficient is,

$$C_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt$$

$$= \frac{1}{\pi}\int_0^{\pi} M\sin(t)e^{-jn2t}dt$$

$$= \frac{M}{\pi}\int_0^{\pi} e^{-jn2t}\cdot \sin t\, dt$$

$$= \frac{M}{\pi}\left[\frac{e^{-jn2t}}{(-jn2)^2+(1)^2}[(-jn2)\sin t-(1)\cos t]\right]_0^{\pi}$$

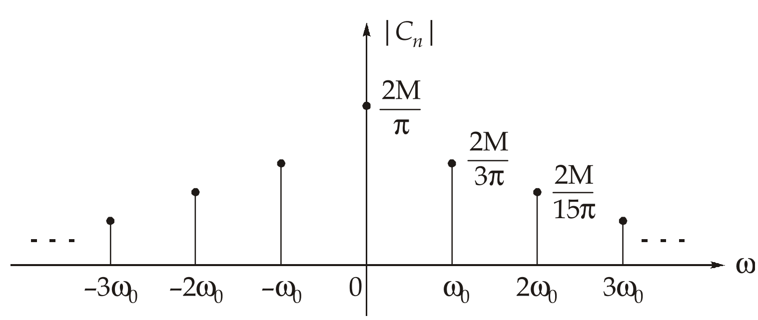
$$= \frac{M}{\pi}\left[\frac{e^{-jn2\pi}+1}{(1-4n^2)}\right]$$

$$= \frac{M}{\pi}\left[\frac{2}{1-4n^2}\right] \qquad \because (e^{-jn2\pi}=1)$$

$$\therefore \quad C_n = \frac{2M}{\pi(1-4n^2)}$$

The magnitude spectrum is as shown below:

n	$\lvert C_n \rvert$
0	$\frac{2M}{\pi}$
1	$\frac{2M}{3\pi}$
2	$\frac{2M}{15\pi}$
⋮	⋮



The exponential Fourier series for the given waveform is,

$$x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{+\infty} \left[\frac{2M}{\pi[1-4n^2]} e^{jn2t} \right]$$

$$\Rightarrow \quad x(t) = \frac{2M}{\pi} \sum_{n=-\infty}^{+\infty} \frac{1}{1-4n^2} e^{jn2t}$$

Q.4 (b) Solution:

(i) Given, $Q_1 = 4$ nC; $Q_2 = 6$ nC

The magnitude of electric field intensity at some arbitrary observation point $P(x, y)$ due to Q_1 is:

$$E_1 = \frac{Q_1}{4\pi\varepsilon_0 R_1^2}$$

Similarly, E_2 at P due to Q_2:

$$E_2 = \frac{Q_2}{4\pi\varepsilon_0 R_2^2}$$

Here,

$$R_1 = \sqrt{(x-1)^2 + (y-1)^2}$$

$$R_2 = \sqrt{(x-5)^2 + (y-7)^2}$$

For equal electric field intensities due to Q_1 and Q_2,

$$\frac{Q_1}{4\pi\varepsilon_0 R_1^2} = \frac{Q_2}{4\pi\varepsilon_0 R_2^2}$$

$$\frac{Q_1}{R_1^2} = \frac{Q_2}{R_2^2}$$

$$\Rightarrow \quad \frac{R_1^2}{R_2^2} = \frac{Q_1}{Q_2} = \frac{4}{6}$$

$$\Rightarrow \quad \frac{(x-1)^2 + (y-1)^2}{(x-5)^2 + (y-7)^2} = \frac{4}{6}$$

$$\Rightarrow \quad 6[x^2 + 1 - 2x + y^2 + 1 - 2y] = 4[x^2 + 25 - 10x + y^2 + 49 - 14y]$$

$$\Rightarrow \quad 6x^2 + 6y^2 - 12x - 12y + 12 = 4x^2 + 4y^2 - 40x - 56y + 296$$
$$\Rightarrow \quad 2x^2 + 2y^2 + 28x + 44y - 284 = 0$$
$$\Rightarrow \quad x^2 + y^2 + 14x + 22y - 142 = 0$$
$$\Rightarrow \quad (x^2 + 14x + 49) + (y^2 + 22y + 121) = 142 + 49 + 121$$
$$\Rightarrow \quad (x + 7)^2 + (y + 11)^2 = 312$$

This represents a circle whose center is at (–7, –11) and radius is $\sqrt{312} = 17.664$ units.

(ii) Given,
$$U(\theta, \phi) = \begin{cases} \sin\theta \ , & 0 \le \theta \le \dfrac{\pi}{2}, 0 \le \phi \le 2\pi \\ 0 \ , & \text{otherwise} \end{cases}$$

We have, $U_{max} = 1$

and Power radiated, $P_{rad} = \int\limits_{\theta}\int\limits_{\phi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi$

$$\Rightarrow \quad P_{rad} = \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \sin\theta \cdot \sin\theta \, d\theta \, d\phi$$

$$\Rightarrow \quad P_{rad} = \left[\int_{\theta=0}^{\pi/2} \sin^2\theta \, d\theta\right] \cdot \left[\int_{\phi=0}^{2\pi} d\phi\right]$$

$$\Rightarrow \quad P_{rad} = \left[\int_{\theta=0}^{\pi/2} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta\right] \cdot [\phi]_0^{2\pi}$$

$$= \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/2} \cdot [\phi]_0^{2\pi} = \left(\frac{\pi}{4}\right) \cdot (2\pi) = \frac{\pi^2}{2}$$

$$\therefore \quad \text{Directivity, } D = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \times 1}{\left(\dfrac{\pi^2}{2}\right)} = \frac{8}{\pi} = 2.5465$$

Q.4 (c) Solution:

(i) The network layer of the OSI model is responsible for the delivery of data packets from the source to the destination across multiple networks. It handles several critical functions to ensure effective and efficient communication. Below are the primary functions of the network layer:

1. Routing
2. Logical addressing
3. Path determination
4. Packet forwarding.

1. Routing:

Routing is the process by which data packets are forwarded from one network to another. The network layer is responsible for determining the best path through the network to ensure that data reaches its intended destination efficiently.

Routing Tables: Routers maintain routing tables that contain information about various paths through the network. These tables are used to make forwarding decisions.

Routing protocols: Protocols like OSPF (Open Shortest Path First), BGP (Border Gateway Protocol) and RIP (Routing Information Protocol) are used to exchange routing information between routers and update routing tables dynamically.

Route Selection: The network layer uses metrics such as hop count, bandwidth, delay, and cost to select the optimal path for data transmission.

2. Logical addressing:

Logical addressing is used to uniquely identify each device on a network. The network layer assigns a unique address, known as an IP address, to each device. This allows for the correct delivery of packets to the destination. Key components of logical addressing include:

- **IP addressing:** IP addresses are assigned to devices to facilitate identification and communication. IPv4 and IPv6 are the two versions of IP addressing currently in use.
- **Hierarchical addressing:** IP addresses are structured hierarchically, allowing for efficient routing and management. This hierarchy typically includes a network portion and a host portion.
- **Subnetting:** Subnetting divides a larger network into smaller, manageable sub- networks improving routing efficiency and network organization.

3. Path determination:

Path determination involves selecting the most appropriate path for data to travel from the source to the destination. This function ensures that data packets take the optimal route through the network. Components of path determination include:

- **Routing Algorithms:** Algorithms such as distance-Vector and link-state are used to calculate the best path for data transmission.

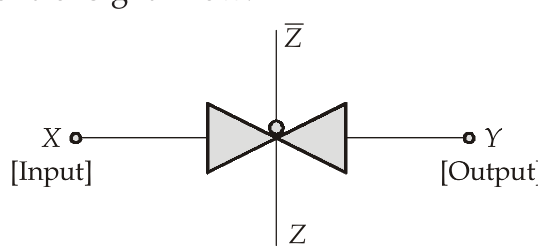
  Distance -vector algorithms use hop count as a metric, while link-state algorithms use a map of the network to determine the shortest path.
- **Metrics and Policies:** Various metrics (e.g. bandwidth, latency, reliability) and routing policies (e.g. administrative preferences) are considered to determine the best path for data packets.

4. Packet Forwarding:-

Packet forwarding is the actual process of moving packets from one network interface to another within a router or between routers based on the destination address. The key aspects include:

- **Forwarding Decisions:** Routers examine the destination IP address of incoming packets and look up their routing tables to determine the next hop for each packet.
- **Switching:** The network layer performs packet switching, directing packets to the appropriate output interface to continue their journey to the destination.
- **Encapsulation:** Data is encapsulated in packets with appropriate headers and trailers that contain information necessary for routing and delivery.
- **Fragmentation:** When packets are too large to pass through a network segment, the network layer fragments the packet into smaller units that can be reassembled at the destination.

(ii) Transmission gate is a bidirectional electronic switch controlled logically, used in digital circuits to control signal flow.



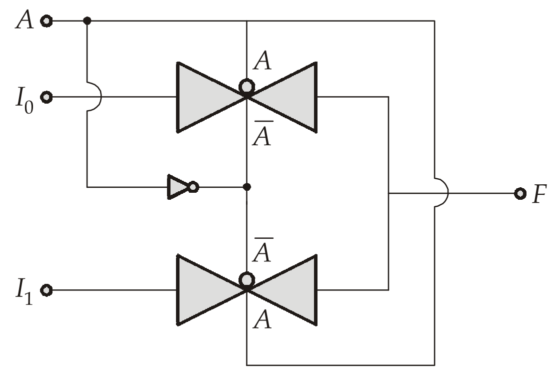
$$\left.\begin{array}{l}\text{If } Z = 1 \Rightarrow Y = X \\ \text{If } Z = 0 \Rightarrow Y = 0\end{array}\right\}$$ By applying complementary control signals to the gates of these transistors, the transmission gate can either allow a signal to pass through or block it.

Transmission gates are used to implement all the Boolean expressions.

Implementation of a 2 : 1 MUX using transmission gates:

For a 2 : 1 MUX, output expression is,

$$F = \bar{A}I_0 + AI_1$$



Section B

Q.5 (a) Solution:

Computer and network security address four requirements:

- **Confidentiality:** Requires that data can only be accessible by authorized parties. This type of access includes printing, displaying, and other forms of disclosure, including simply revealing the existence of an object.
- **Integrity:** Requires that only authorized parties can modify data. Modification includes writing, changing, changing status, deleting, and creating.
- **Availability:** Requires that data are available to authorized parties.
- **Authenticity:** Requires that a host or service be able to verify the identity of a user.

A useful means of classifying security attacks is in terms of passive attacks and active attacks. A passive attack attempts to learn or make use of information from the system but does not affect system resources. An active attack attempts to alter system resources or affect their operation.

Passive Attacks

Passive attacks are in the nature of eavesdropping on, or monitoring of transmissions. The goal of the opponent is to obtain information that is being transmitted. Two types of passive attacks are release of message contents and traffic analysis.

The **release of message contents** is easily understood. A telephone conversation, an electronic mail message, or a transferred file may contain sensitive or confidential information. We would like to prevent an opponent from learning the contents of these transmissions.

A second type of passive attack, **traffic analysis**, is subtler. Suppose that we had a way of masking the contents of messages or other information traffic so that opponents, even if they captured the message, could not extract the information from the message. The common technique for masking contents is encryption. Even with encryption protection in place, an opponent might still be able to observe the pattern of these messages. The opponent could determine the location and identity of communicating hosts and could observe the frequency and length of messages being exchanged. This information might be useful in guessing the nature of the communication that was taking place.

Passive attacks are very difficult to detect because they do not involve any alteration of the data. Typically, the message traffic is sent and received in an apparently normal fashion and neither the sender nor receiver is aware that a third party has read the messages or observed the traffic pattern. However, it is feasible to prevent the success of these attacks, usually by means of encryption. Thus, the emphasis in dealing with passive attacks is on prevention rather than detection.

Active Attacks

Active attacks involve some modification of the data stream or the creation of a false stream and can be subdivided into four categories: masquerade, replay, modification of messages, and denial of service.

A **masquerade** takes place when one entity pretends to be a different entity. A masquerade attack usually includes one of the other forms of active attack. For example, authentication sequences can be captured and replayed after a valid authentication sequence has taken place, thus enabling an authorized entity with few privileges to obtain extra privileges by impersonating an entity that has those privileges.

Replay involves the passive capture of a data unit and its subsequent retransmission to produce an unauthorized effect.

Modification of messages simply means that some portion of a legitimate message is altered, or that messages are delayed or reordered, to produce an unauthorized effect. For example, a message meaning “Allow John Smith to read confidential file accounts" is modified to mean “Allow Fred Brown to read confidential file accounts”.

The **denial of service** prevents or inhibits the normal use or management of communications facilities. This attack may have a specific target; for example, an entity may suppress all messages directed to a particular destination (e.g., the security audit service). Another form of service denial is the disruption of an entire network or a server, either by disabling the network server or by overloading it with messages so as to degrade performance.

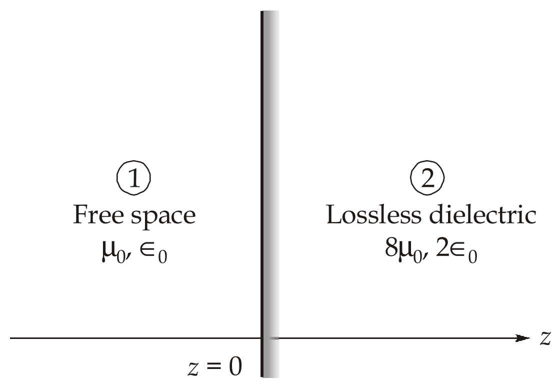
Active attacks present the opposite characteristics of passive attacks. Whereas passive attacks are difficult to detect, measures are available to prevent their success. On the other hand, it is quite difficult to prevent active attacks absolutely, because to do so would require physical protection of all communications facilities and paths at all times. Instead, the goal is to detect them and to recover from any disruption or delays caused by them. Because the detection has a deterrent effect, it may also contribute to prevention.

Q.5 (b) Solution:

Given that,

In medium-1, $\vec{H}_i = 10\cos(10^8 t - \beta z)\hat{a}_x \text{ mA/m}$

For medium-2, $\epsilon = 2\epsilon_0, \mu = 8\mu_0$



We have, $$\beta_1 = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \frac{1}{3}\text{ rad/m}$$

$$\eta_1 = \eta_0 = 120\pi\ \Omega$$

For the lossless dielectric medium,

$$\beta_2 = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r}$$

$$= \frac{\omega}{c}\times 4 = 4\beta_1 = \frac{4}{3}\text{ rad/m}$$

$$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_r}{\epsilon_r}} = 2\eta_0$$

Given that $\vec{H}_i = 10\cos(10^8 t - \beta_1 z)\hat{a}_x \text{ mA/m}$, we expect that

$$\vec{E}_i = E_{i0}\cos(10^8 t - \beta_1 z)\hat{a}_{Ei}$$

Where $\hat{a}_{Ei} = \hat{a}_{Hi}\times\hat{a}_{Ki} = \hat{a}_x\times\hat{a}_z = -\hat{a}_y$

and $E_{i0} = \eta_1 H_{i0} = 10\eta_0$

Hence, $\vec{E}_i = -10\eta_0 \cos(10^8 t - \beta_1 z)\,\hat{a}_y$ mV/m

Now, $$\frac{E_{r0}}{E_{i0}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_0 - \eta_0}{2\eta_0 + \eta_0} = \frac{1}{3}$$

$$E_{r0} = \frac{1}{3}E_{i0}$$

Thus, $$\vec{E}_r = E_{r0}\cos(\omega t + \beta_i z)(-\hat{a}_y)$$

$$\vec{E}_r = \frac{10}{3}\eta_0 \cos\left(10^8 t + \frac{1}{3}z\right)\hat{a}_y \text{ mV/m}$$

From $\vec{E}_r$, which we easily obtain $\vec{H}_r$ as

$$\vec{H}_r = \frac{-10}{3}\cos\left(10^8 t + \frac{1}{3}z\right)\hat{a}_x \text{ mA/m}$$

$$\left[H_{r0} = \frac{E_{r0}}{\eta_0} \text{ and } \hat{a}_{Er} \times \hat{a}_{Hr} = -\hat{a}_z\right]$$

Similarly, $$\frac{E_{t0}}{E_{i0}} = \tau = 1 + \Gamma = \frac{4}{3} \text{ or } E_{t0} = \frac{4}{3}E_{i0}$$

Thus, $$\vec{E}_t = E_{t0}\cos(10^8 t - \beta_2 z)\hat{a}_{Et}$$

Where, $\hat{a}_{Et} = \hat{a}_{Ei} = -\hat{a}_y$, hence,

$$\vec{E}_t = \frac{-40}{3}\eta_0 \cos\left(10^8 t - \frac{4}{3}z\right)\hat{a}_y \text{ mV/m}$$

From which, we obtain $$\vec{H}_t = \frac{20}{3}\cos\left(10^8 t - \frac{4}{3}z\right)\hat{a}_x \text{ mA/m}$$

$$\left[H_{t0} = \frac{E_{t0}}{\eta_2} \text{ and } \hat{a}_{Et} \times \hat{a}_{Ht} = \hat{a}_z\right]$$

Q.5 (c) Solution:

Fault equivalence:

Two faults f_1 and f_2 are equivalent if all tests that defect f_1 also detect f_2 and vice versa. If faults f_1 and f_2 are equivalent, then the corresponding faulty functions are identical.

Faulty function for the circuit,

$$i \text{ when } c(s\text{-}a\text{-}0) = \overline{(0 + b)\cdot \overline{ab}}$$

$$= \overline{b} + ab$$

$$= a + \overline{b}$$

$$i \text{ when } f(s\text{-}a\text{-}1) = \overline{(a+b)\cdot\overline{a\cdot b}}$$
$$= \overline{b\cdot\overline{a}}$$
$$= a+\overline{b}$$

Two faulty functions are indistinguishable, hence the two faults are equivalent.

Q.5 (d) Solution:

From the pole locations shown in figure, the system is underdamped. Thus, the poles are given by $s = -\xi\omega_n + j\omega_d$. The real part of the complex poles is

$$-\xi\omega_n = -4$$

i.e. $$\xi\omega_n = 4$$

or $$\omega_n = \frac{4}{\xi}$$

and $$\omega_d = 2$$

We know, $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

i.e., $$2 = \frac{4}{\xi}\sqrt{1-\xi^2}$$

∴ $$\xi = 2\sqrt{1-\xi^2}$$

i.e. $$\xi^2 = 4(1-\xi^2)$$

i.e., $$\xi^2 = \frac{4}{5} = 0.8$$

or $$\xi = 0.894$$

∴ $$\omega_n = \frac{4}{\xi} = \frac{4}{0.894} = 4.474 \text{ rad/s}$$

The unit-step response of the underdamped system is

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$
$$= 1 - \frac{e^{-4t}}{0.447}\sin\left(2t + \tan^{-1}\frac{0.447}{0.894}\right)$$
$$= 1 - \frac{e^{-4t}}{0.447}\sin(2t + 26.56°)$$

For 2% tolerance band, the settling time

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{4} = 1s$$

Q.5 (e) Solution:

Given data:

Bit rate, $R_b = 3.5 \times 10^6$ bits per sec

PSD, $\frac{N_0}{2} = 2.5 \times 10^{-20}$ W/Hz

$\Rightarrow$ $N_0 = 5 \times 10^{-20}$ W/Hz

$A_C = 1.2\ \mu$V

$\therefore$ Bit duration, $T_b = \frac{1}{R_b} = \frac{1}{3.5\times10^6} = 2.85 \times 10^{-7} = 0.285\ \mu$sec

The signal energy per bit is

$$E_b = \frac{A_c^2 T_b}{2} = \frac{(1.2\times10^{-6})^2}{2}\times0.285\times10^{-6}$$

$$= 2.0519 \times 10^{-19} \text{ Joules}$$

(i) Coherent binary FSK : The average probability of error is

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2.0519\times10^{-19}}{5\times10^{-20}}}\right) = Q\left(\sqrt{4.1038}\right)$$

$$\because \quad Q(x) = \frac{1}{2}erfc\left[\frac{x}{\sqrt{2}}\right]$$

$$\therefore \quad P_e = \frac{1}{2}erfc\left[\sqrt{\frac{4.1038}{2}}\right]$$

$$P_e = \frac{1}{2}erfc\left[\sqrt{2.0519}\right]$$

(ii) Coherent MSK:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = erfc\left[\sqrt{\frac{E_b}{N_0}}\right] = erfc\left[\sqrt{\frac{2.0519\times10^{-19}}{5\times10^{-20}}}\right]$$

$$= erfc\left[\sqrt{4.1038}\right]$$

(iii) Non-coherent binary FSK:

$$P_e = \frac{1}{2}\exp^{\left[\frac{-E_b}{2N_0}\right]} = \frac{1}{2}\exp^{\left[\frac{-2.0519\times10^{-19}}{2\times5\times10^{-20}}\right]}$$

$$= \frac{1}{2}\exp^{[-2.0519]}$$

$$P_e = 6.424 \times 10^{-2}$$

Q.6 (a) Solution:

(i) The IEEE 754 single-precision format uses 32 bits, with 1 sign bit, 8 bits for exponent and 23 bits for Mantissa, as shown below:

Sign(S)	Exponent(E)	Mantissa(M)
1	10000101	11000000000000000000000
	8 bits	23 bits

From above representation, the floating point number is given by

$$N = (-1)^S \times (1.M)_2 \times 2^{E-127}$$

We have,

$$\text{Biased exponent value} = 10000101 = 133$$

$$\text{Actual exponent} = 133 - 127 \qquad [\because 127 \text{ is bias}]$$

$$= 6$$

$$\text{Mantissa, M} = 11000000000000000000000$$

$$\text{Sign Bit, S} = 1$$

Thus, $$\text{Decimal number} = (-1) \times (1.11)_2$$

$$= -(1110000)_2 = -112$$

(ii) The main problems in instruction pipelining that must be solved to achieve high-speed performance include:

Timing Variations: Not all stages take the same amount of time. This means that the speed gain of a pipeline will be determined by its slowest stage. This problem is particularly acute in instruction processing, since different instructions have different operand requirements and sometimes vastly different processing time. Moreover, synchronization mechanisms are required to ensure that data is passed from stage to stage only when both stages are ready.

Data Hazards: When several instructions are in partial execution, a problem arises if they reference the same data. We must ensure that a later instruction does not attempt to access data sooner than a preceding instruction, if this will lead to incorrect results. For example, instruction N+1 must not be permitted to fetch an operand that is yet to be stored into by instruction N.

Branching: In order to fetch the "next" instruction, we must know which one is required. If the present instruction is a conditional branch, the next instruction may not be known until the current one is processed.

Interrupts: Interrupts insert unplanned "extra" instructions into the instruction stream. The interrupt must take effect between instruction that is, when one instruction has completed and the next has not yet begun. With pipelining, the next instruction has usually begun before the current one has completed.

Q.6 (b) Solution:

(i) Given:

$$f_{MUF} = 15 \text{ MHz}$$

$$h = 500 \text{ km}$$

$$n = 0.8$$

Skip distance, $$d_{skip} = 2h\sqrt{\left[\left(f_{MUF}/f_c\right)^2 - 1\right]}$$

where, $$f_c = 9\sqrt{N_{max}}$$

We know,

refractive index, $$n = \sqrt{1 - \frac{81N_{max}}{f_{MUF}^2}}$$

$$\Rightarrow \quad 0.8 = \sqrt{1 - \frac{81 \times N_{max}}{(15 \times 10^6)^2}}$$

$$\Rightarrow \quad N_{max} = 1 \times 10^{12} \text{ electrons/m}^3$$

$$f_c = 9\sqrt{N_{max}} = 9\sqrt{1 \times 10^{12}} = 9 \times 10^6 \text{ Hz}$$

$$\Rightarrow \quad f_c = 9 \text{ MHz}$$

$$\therefore \quad d_{skip} = 2 \times 500 \times 10^3 \sqrt{\left[\left(\frac{15}{9}\right)^2 - 1\right]}$$

$$\Rightarrow \quad d_{skip} = 1333.33 \text{ km}$$

(ii) The single-mode fibers are the dominant and the most widely used fiber type within telecommunications due to the following reasons:

1. They exhibit the greatest transmission bandwidths and the lowest losses of the fiber transmission media.
2. They have a superior transmission quality over other fiber types because of the absence of modal noise.
3. They offer a substantial upgrade capability (i.e., future proofing) for future wide-bandwidth services using either faster optical transmitters and receivers or advanced transmission techniques (e.g. coherent technology).
4. They are compatible with the developing integrated optics technology.
5. The installation of single-mode fiber will provide a transmission medium which will have adequate performance such that it will not require replacement over its anticipated lifetime of more than 20 years.

Q.6 (c) Solution:

(i) Step 1: Draw the message signal $m(t)$:

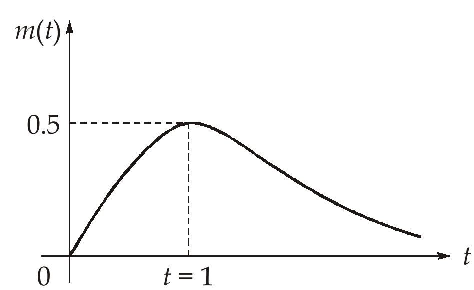
- To plot $m(t)$, the values of modulating signal $m(t) = t/(1 + t^2)$ for different values of "t" are calculated as below:

t	0	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0
$m(t)$	0	0.192	0.345	0.44	0.49	0.5	0.4	0.3	0.235	0.192

Peak or maximum value of $m(t)$ (at $t = 1.0$)

- From table, the maximum value of $m(t) = 0.5$.

The message signal $m(t)$ can be plotted as below:



The equation for AM wave is given by

$$s_{AM}(t) = [E_c + m(t)] \cos \omega_c t$$

Where E_c is the amplitude of carrier and ω_c is the frequency of carrier. Thus,

$$s_{AM}(t) = E_c \left[1 + \frac{m(t)}{E_c}\right] \cos \omega_c t$$

The modulation index of the above AM wave is given by

$$\mu = \frac{m(t)|_{\max}}{E_c}$$

Step 2: Calculate E_c for different values of μ:

$$\text{Modulation index } \mu = \frac{E_m}{E_c} \qquad \text{where } E_m = 0.5$$

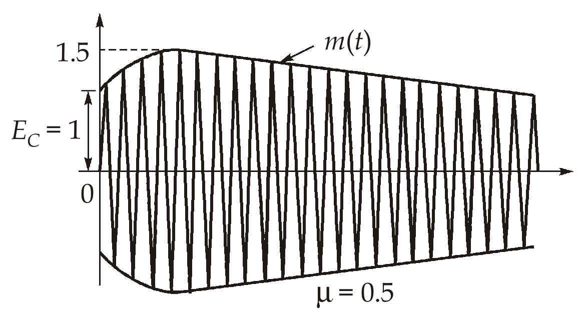
$$\therefore \quad E_c = \frac{E_m}{\mu} = \frac{0.5}{0.5} = 1 \qquad \text{...for } \mu = 0.5$$

$$E_c = \frac{0.5}{1} = 0.5 \qquad \text{...for } \mu = 1$$

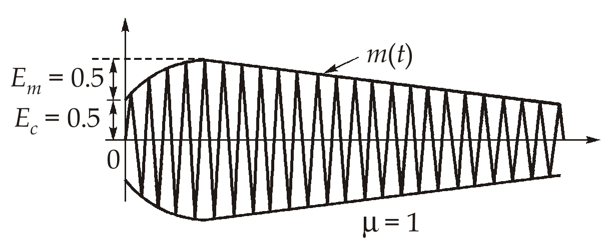
$$\text{and} \quad E_c = \frac{0.5}{1.25} = 0.4 \qquad \text{...for } \mu = 1.25$$

Step 3: Plot the waveforms:

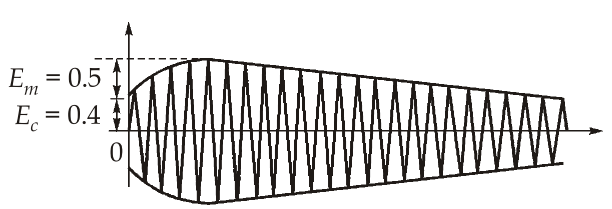
The waveforms of the AM wave for different values of μ are shown below in Fig. (a), (b) and (c).



(a) AM wave for μ = 0.5



(b) AM wave for μ = 1.0



(c) AM wave for μ = 1.25

(ii) It is given that:

$$f_c = 20 \text{ MHz}, f_m = 400 \text{ Hz}, A = 5 \text{ V}, \Delta f = 10 \text{ kHz}$$

1. The modulation index,

$$m_f = \frac{\Delta f}{f_m} = \frac{10\times10^3}{400}$$

$$= 25 \quad ...(1)$$

2. Equation of FM wave is:

$$e_{FM} = A\cos[\omega_c t + m_f \sin\omega_m t]$$

$$= 5\cos[(2\pi\times 20\times 10^6 t) + 25\sin(2\pi\times 400t)] \quad ...(2)$$

3. New carrier voltage, $A_c' = 10$ volts.

   New modulating frequency,

$$f_m' = 2 \text{ kHz}$$

   The frequency deviation $\Delta f = K_f A_m$ is independent of f_m and thus, remains unchanged.

$\therefore$ New value of $m_f = \dfrac{10 \text{ kHz}}{2 \text{ kHz}} = 5$

$\therefore$ Equation of FM wave is:

$$e_{FM} = 10\cos[(2\pi\times 20\times 10^6 t) + 5\sin(2\pi\times 2\times 10^3 t)] \quad ...(4)$$

   Since the value of modulation index m_f is greater than 1, hence both the FM wave represents WBFM. Thus, the power of the FM wave is equal to the unmodulated carrier power.

4. Power dissipated across 100 Ω resistor by the first wave

$$= \frac{A_c^2}{2R} = \frac{(5/\sqrt{2})^2}{R} = \frac{12.5}{100}$$

$$= 0.125 \text{ W} \quad ...(5)$$

5. Power dissipated across 100 Ω resistor by the second wave

$$= \frac{A_c'^2}{2R} = \frac{(10/\sqrt{2})^2}{100}$$

$$= 0.5 \text{ W} \quad ...(6)$$

Q.7 (a) Solution:

(i) Given that, $$X(s) = \frac{1}{e^{(s+11)}}\left[\frac{s^2}{(s+1)(s+2)}\right]$$

Assume $$X(s) = \frac{1}{e^{(s+11)}}F(s) \quad \text{...(i)}$$

where, $$F(s) = \frac{s^2}{(s+1)(s+2)} = \frac{s^2}{(s^2+3s+2)}$$

Since the order of the numerator and denominator polynomial is equal, the rational Laplace transform $F(s)$ is improper. We use long division to express $F(s)$ as the sum of a proper rational function and a polynomial in 's':

$$\begin{array}{r|l} & \quad 1 \\ s^2+3s+2 & \; s^2 \\ & \; s^2+3s+2 \\ & (-)\;(-)\;\;(-) \\ \hline & \;-3s-2 \end{array}$$

Thus, we may write

$$F(s) = 1-\frac{(3s+2)}{(s^2+3s+2)} = 1-\frac{(3s+2)}{(s+1)(s+2)}$$

Using the partial fraction expansion to expand the rational function, we obtain

$$F(s) = 1+\frac{1}{s+1}-\frac{4}{s+2}$$

Taking the inverse Laplace transform,

$$f(t) = \delta(t) + e^{-t}u(t) - 4e^{-2t}u(t)$$

From equation (i),

$$X(s) = \frac{1}{e^{(s+11)}}F(s) = \frac{1}{e^{11}}e^{-s}F(s)$$

Using time-shifting property of Laplace transform, we obtain

$$x(t) = \frac{1}{e^{11}}f(t-1) = \frac{1}{e^{11}}\left[\delta(t-1)+e^{-(t-1)}u(t-1)-4e^{-2(t-1)}u(t-1)\right]$$

(ii) Given, λ = 1.3 μm, L_{B1} = 0.7 mm, L_{B2} = 80 m

The modal birefringence is given by

$$B_F = \frac{\lambda}{L_B}$$

Hence, for a beat length of 0.7 mm:

$$B_F = \left(\frac{\lambda}{L_{B1}}\right) = \frac{1.3\times10^{-6}}{0.7\times10^{-3}} = 1.86\times10^{-3}$$

This typifies a high birefringence fiber.

For a beat length of 80 m:

$$B_F = \frac{1.3\times10^{-6}}{80} = 1.625\times10^{-8}$$

which indicates a low birefringence fiber.

Q.7 (b) Solution:

(i) By definition, the probability distribution function is given by

$$P_R(r) = \int_{-\infty}^{r} p_R(r)dr = \int_0^r \frac{r}{b} e^{-\frac{r^2}{2b}} dr \quad ...(i)$$

Assuming, $r^2 = t$

$dt = 2rdr$

Now, equation (i) becomes,

$$P_R(r) = \int_0^{r^2} \frac{1}{2b} e^{-\frac{t}{2b}} dt = \frac{1}{2b}\int_0^{r^2} e^{-\frac{t}{2b}} dt$$

$$P_R(r) = \frac{-1}{2b} 2b\, e^{-\frac{t}{2b}}\Big|_0^{r^2} = -\left[e^{-\frac{r^2}{2b}} - 1\right]$$

$$\therefore \quad P_R(r) = \left(1 - e^{-\frac{r^2}{2b}}\right); \quad r \geq 0$$

(ii) We know that the mean value of RV 'R' is given by,

$$E(R) = m_R = \int_{-\infty}^{\infty} r p_R(r) dr = \int_0^{\infty} r \frac{r}{b} e^{-\frac{r^2}{2b}} dr$$

$$= \int_0^{\infty} \frac{r^2}{b} e^{-\frac{r^2}{2b}} dr = \int_0^{\infty} r \cdot \frac{r}{b} e^{-\frac{r^2}{2b}} dr$$

$$= \int_0^\infty -r d\left[e^{-\frac{r^2}{2b}}\right]dr = -\left[re^{-\frac{r^2}{2b}}\Big|_0^\infty - \int_0^\infty e^{-\frac{r^2}{2b}}dr\right]$$

$$= -\left[0 - \int_0^\infty e^{-\frac{r^2}{2b}}dr\right] = \int_0^\infty e^{-\frac{r^2}{2b}}dr$$

$$m_R = \int_0^\infty e^{-\frac{\pi r^2}{2b\pi}}dr \quad \text{...(ii)}$$

Now, put $\frac{r^2}{2b\pi} = t^2$

$$\Rightarrow \quad \frac{r}{\sqrt{2b\pi}} = t$$

$$\frac{dr}{\sqrt{2b\pi}} = dt$$

Now, equation (ii) may be written as:

$$m_R = \int_0^\infty e^{-\pi t^2}dt\sqrt{2b\pi} = \sqrt{2b\pi}\int_0^\infty e^{-\pi t^2}dt = \sqrt{2b\pi}\frac{1}{2}$$

$$\therefore \quad m_R = \sqrt{\frac{b\pi}{2}}$$

(iii) The mean-square value of RV 'R' is given by,

$$E[R^2] = \int_{-\infty}^{\infty} r^2 p_R(r)dr = \int_0^\infty \frac{r^3}{b}e^{-\frac{r^2}{2b}}dr$$

$$= \int_0^\infty r^2 \cdot \frac{2r}{2b}e^{-\frac{r^2}{2b}}dr$$

$$= -\int_0^\infty r^2 d\left(e^{-\frac{r^2}{2b}}\right)dr = -\left[r^2 \cdot e^{\frac{-r^2}{2b}}\Big|_0^\infty - \int_0^\infty e^{-\frac{r^2}{2b}}2rdr\right]$$

$$E[R^2] = 2\int_0^\infty e^{-\frac{r^2}{2b}}rdr \quad \text{...(iii)}$$

Put $r^2 = t$, thus

$$2rdr = dt$$

Now, equation (iii) may be written as

$$E[R^2] = \int_0^\infty e^{-\frac{t}{2b}} dt = e^{-\frac{t}{2b}}(-2b)\Big|_0^\infty = -2b[0-1]$$

$$\therefore \quad E[R^2] = 2b$$

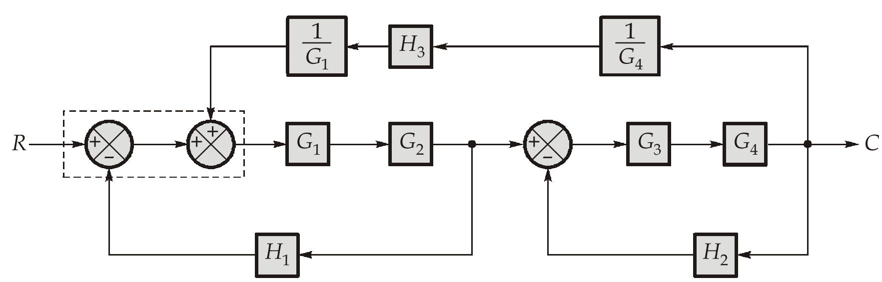
(iv) By the definition, the variance of RV 'R' is given by

$$\sigma_R^2 = E[R^2] - m_R^2 = 2b - \left(\sqrt{\frac{b\pi}{2}}\right)^2 = 2b - \frac{b\pi}{2} = \frac{4b - b\pi}{2}$$

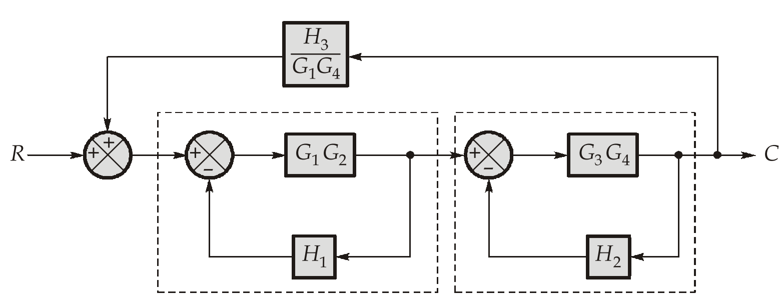
$$\therefore \quad \text{Variance } (\sigma_R^2) = \frac{b}{2}[4-\pi]$$

Q.7 (c) Solution:

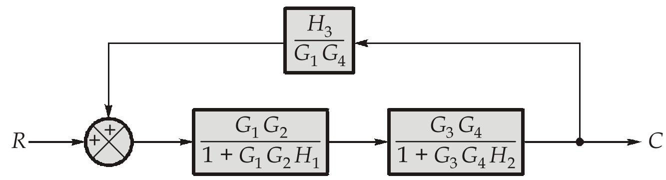
(i) • By moving the summing point between the blocks G_1 and G_2 to the left of G_1, and also shifting the signal take-off point between the block G_3 and G_4 to the right of G_4, the given block diagram can be modified as follows:



• By interchanging the two summing points indicated in the dotted box and simultaneously reducing the possible cascaded blocks into single block, the given block diagram can be minimized to the form as shown below:



- By reducing the loops indicated by dotted boxes, the block diagram will be further reduced to the following form:

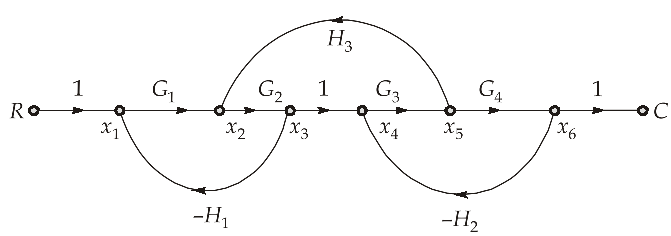


- By reducing the above block diagram, the overall transfer function $\frac{C}{R}$ can be given as,

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)-\left(\frac{G_1G_2G_3G_4H_3}{G_1G_4}\right)}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{1+G_1G_2H_1+G_3G_4H_2+G_1G_2G_3G_4H_1H_2-G_2G_3H_3}$$

(ii) The equivalent signal flow graph of the given block diagram can be drawn as shown below.



- There is only one forward path in the given signal flow graph. The forward path and the corresponding gain is as follows:

  $R - x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - C$,

  $$P_1 = G_1G_2G_3G_4$$

- The individual loops and the corresponding gains are as follows:

  $x_1 - x_2 - x_3 - x_1$, $\quad L_1 = -G_1G_2H_1$

  $x_2 - x_3 - x_4 - x_5 - x_2$, $\quad L_2 = G_2G_3H_3$

  $x_4 - x_5 - x_6 - x_4$, $\quad L_3 = -G_3G_4H_2$

- The product of gains of two non-touching loops is,

  $$L_{13} = G_1G_2G_3G_4H_1H_2$$

- The determinant of the signal flow graph can be given by,

$$\Delta = 1 - (L_1 + L_2 + L_3) + L_{13}$$
$$= 1 + G_1G_2H_1 + G_3G_4H_2 + G_1G_2G_3G_4H_1H_2 - G_2G_3H_3$$

- All the loops are touching the forward path. So, $\Delta_1 = 1$.
- So, using the Mason's Gain Formula, the overall transfer function $\frac{C}{R}$ will be,

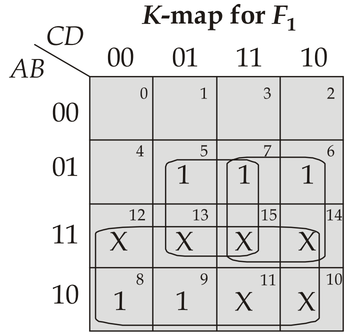
$$\frac{C}{R} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 + G_1G_2G_3G_4H_1H_2 - G_2G_3H_3}$$

Q.8 (a) Solution:

Programmable Array Logic (PAL) has programmable AND array and fixed OR array. The 4-wide PAL implies that each OR gate in the fixed OR array has 4 inputs coming from the programmable AND array. Thus, to restrict the product terms for each function to less than or equal to 4, we carry out the minimization using K-Map.

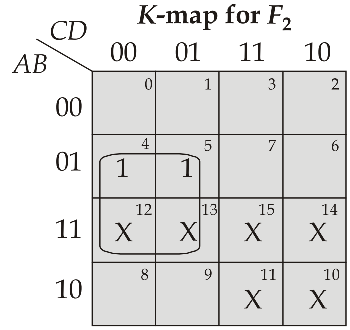
Minimization:

K-map for F_1



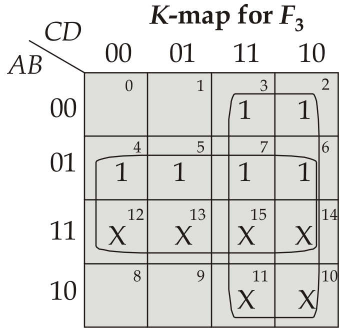
$F_1 = A + BD + BC$

K-map for F_2



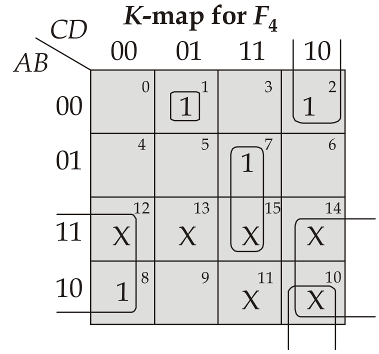
$F_2 = B\overline{C}$

K-map for F_3



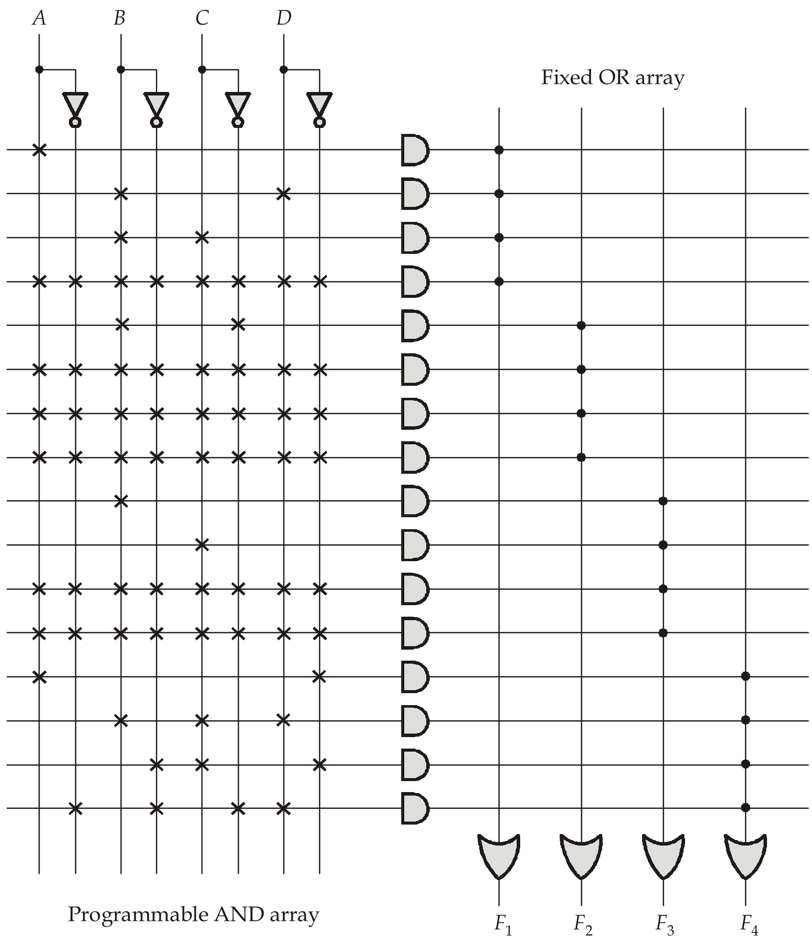
$F_3 = B + C$

K-map for F_4



$F_4 = A\overline{D} + BCD + \overline{B}C\overline{D} + \overline{A}\overline{B}\overline{C}D$

Implementation using PAL:

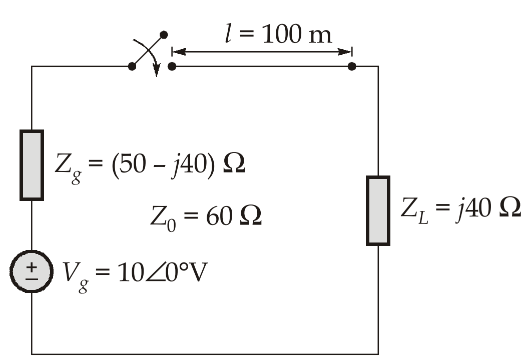


Q.8 (b) Solution:

Given that;

$Z_0 = 60\ \Omega$; $V_g = 10\angle 0°\ V_{rms}$; $Z_g = (50 - j40)\Omega$; $Z_L = j40\ \Omega$; $l = 100$ m; $\beta = 0.25$ rad/m

where Z_0 = Characteristic impedance; V_g = Source voltage; Z_g = Source Impedance; Z_L = Load Impedance; l = length of line; β = Propagation constant



(i) We have,

$$\beta l = \frac{1}{4}\times 100 = 25 \text{ rad} = 1432.4° = 352.4°$$

The input impedance of a transmission line of length 'l' and terminated with load impedance Z_L is given by

$$Z_{\text{in}} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$= 60\left[\frac{j40 + j60\tan(352.4°)}{60 - 40\tan(352.4°)}\right]$$

$$= j29.375\ \Omega$$

Using voltage division rule,

$$V(z=0) = V_0 = \frac{Z_{in}}{Z_{in}+Z_g}V_g = \frac{j29.375(10\angle 0°)}{j29.375 + 50 - j40}$$

$$= \frac{293.75\angle 90°}{51.116\angle -12°} = 5.75\angle 102°$$

(ii)

$$Z_{\text{in}} = Z_L = j40\ \Omega$$

The voltage equation of transmission line is given by

$$V(x) = V_0^+ e^{j\beta x} + V_0^+ e^{-j\beta x} = V_0^+(e^{j\beta x} + \Gamma_L e^{-j\beta x}) \quad \text{...(i)}$$

where x is the distance from the load towards the generator.

At $x = l$, $V = V_g$. Thus,

$$V_0^+ = \frac{V_g}{(e^{j\beta l} + \Gamma_L e^{-j\beta l})}$$

$$\left[\text{where } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{j40-60}{j40+60} = 1\angle 112.62°\right]$$

To calculate voltage at the receiving end, we put $x = 0$. We get,

$$V_L = V_0^+(1+\Gamma_L) = \frac{V_g(1+\Gamma_L)}{(e^{j\beta l} + \Gamma_L e^{-j\beta l})} = \frac{10(1+1\angle 112.62°)}{(e^{25j} + 1\angle 112.62° e^{-25j})}$$

$$= \frac{10(1+\angle 112.62°)}{\angle 1432.4° + \angle -1319.774°} = 12.615\angle 0°\ \text{V}$$

(iii) At 4 m from the load, $\beta x = \frac{1}{4}\times 4 = 1 \text{ rad} = 57.3°$

$$Z_{in} = 60\left[\frac{j40 + j60\tan 57.3°}{60 - 40\tan 57.3°}\right] = -j3471.88\ \Omega$$

Substituting $x = 4$ m in equation (i), we get,

$$V(x = 4\text{ m}) = \frac{V_g(e^{j\beta x} + \Gamma_L e^{-j\beta x})}{(e^{j25} + \Gamma_L e^{-j25})}$$

$$= \frac{10\left[(e^{j57.3°}) + (1\angle 112.62°)\left(e^{-j57.3°}\right)\right]}{e^{j1432.394°} + (1\angle 112.62°)e^{-j1432.394°}} = 22.74\angle 0°\text{ V}$$

(iv) 3 m from the source is the same as 97 m from the load, i.e.,

$$l' = 100 - 3 = 97\text{ m}$$

$$\beta l' = \frac{1}{4}\times 97 = 24.25\text{ rad} = 309.42°$$

We have,

$$Z_{in} = 60\left[\frac{j40 + j60\tan 309.42°}{60 - 40\tan 309.42°}\right] = -j18.2\ \Omega$$

$$V(l' = 97\text{m}) = \frac{V_g(e^{j\beta l'} + \Gamma_L e^{-j\beta l'})}{\left[e^{j\beta l} + \Gamma_L e^{-j\beta l}\right]}$$

$$= \frac{10\left[e^{j\frac{97}{4}} + (1\angle 112.62°)\left(e^{-j\frac{97}{4}}\right)\right]}{\left(e^{j25} + (1\angle 112.62°)\left(e^{-j25}\right)\right)}$$

$$= \frac{10[\angle 1389.422° + \angle(112.62° - 1389.422°)]}{\angle 1432.394° + \angle -1319.774°}$$

$$= \frac{10[\angle 1389.422° + \angle -1276.802°]}{\angle 1432.394° + \angle -1319.774°}$$

$$= \frac{10[\cos(1389.422°) + j\sin(1389.422°) + \cos(1276.802°) - j\sin(1276.802°)]}{[\cos(1432.394°) + j\sin(1432.394°) + \cos(1319.774°) - j\sin(1319.774°)]}$$

$$= \frac{5.8066\angle -123.68°}{0.8793\angle 56.31°} = 6.604\ \angle -180°\text{ V}$$

Q.8 (c) Solution:

Given, $$G(s) = \frac{K}{(s+1)(s+4)(s+a)}$$

Characteristic equation:

$$1 + G(s) = 0$$

$$\Rightarrow \quad (s+1)(s+4)(s+a) + K = 0$$

$$(s+1)(s^2 + (4+a)s + 4a) + K = 0$$

$$s^3 + (4+a)s^2 + 4as + s^2 + (4+a)s + (4a+K) = 0$$

$$s^3 + (5+a)s^2 + (4+5a)s + 4a + K = 0$$

From Routh array:

s^3	1	$(4+5a)$
s^2	$(5+a)$	$(4a+K)$
s^1	$\dfrac{(5+a)(4+5a)-(4a+K)}{(5+a)}$	0
s^0	$(4a+K)$	

For sustained oscillations, odd row of routh array must be zero i.e.

$$(5+a)(4+5a) - (4a+K) = 0$$

$$20 + 25a + 4a + 5a^2 - 4a - K = 0$$

$$5a^2 + 25a + (20 - K) = 0 \qquad \text{...(1)}$$

The frequency of sustained oscillation, can be obtained from the auxiliary equation as

$$(5+a)s^2 + (4a+K) = 0$$

$$s^2 = -\frac{(4a+K)}{(5+a)} \Rightarrow (j\omega)^2 = -\left(\frac{4a+K}{5+a}\right)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4a+K}{5+a}} \qquad \text{...(2)}$$

From the given figure, Time period of oscillation,

$$T = 2x = 2 \times 1.0471$$

$$= 2.0942$$

$$\therefore \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2.0942} = 3\ \text{rad/sec}$$

From equation (2),

$$3 = \sqrt{\frac{4a+K}{5+a}}$$

$$\Rightarrow \quad 45 + 9a = 4a + K$$

$$\Rightarrow \quad K = 5a + 45 \qquad ...(3)$$

From equations (1) and (3),

$$5a^2 + 25a + (20 - 5a - 45) = 0$$

$$\Rightarrow \quad 5a^2 + 20a - 25 = 0$$

$$a^2 + 4a - 5 = 0$$

On solving, $a = 1, -5$

From Routh Array, $a > -5$ for the system to be stable. Thus, $a = 1$.

From equation (3),

$$K = (5 \times 1) + 45$$

$$\Rightarrow \quad K = 5 + 45 = 50$$

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