MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Electrical Engineering Test No : 10

Section-A

Q.1 (a) Solution:

(i)

$$V = 60 \text{ kV (rms)},$$

$$g_{max} = 4 \text{ kV/mm (rms)}$$

$$V_1 = \frac{V}{e} = \frac{60}{2.718} = 22.1 \text{ kV}$$

$$r = \frac{V}{e\,g_{max}} = \frac{60}{2.718 \times 4} = 5.5 \text{ mm}$$

$$\text{Diameter of core} = 2r = 2 \times 5.5 = 11 \text{ mm}$$

Radius of intersheath, $r_1 = \frac{V}{g_{max}} = \frac{60}{4} = 15$ mm

Diameter of intersheath, $d_1 = 2r_1 = 2 \times 15 = 30$ mm

$$V_2 = V - V_1 = 60 - 22.1 = 37.9 \text{ kV}$$

$$R = 1.881\frac{V}{g_{max}} = 1.881 \times \frac{60}{4} = 28.2 \text{ mm}$$

Minimum overall diameter of the cable,

$$D = 2R = 2 \times 28.2 = 56.4 \text{ mm}$$

(ii) Cable without intersheath

For economic cable size, $\frac{R}{r} = e = 2.718;$

$$\ln\frac{R}{r} = 1$$

$$V = g_{max} r \ln\frac{R}{r} = g_{max} r$$

$$r = \frac{V}{g_{max}} = \frac{60}{4} = 15 \text{ mm}$$

Diameter of conductor $= 2r = 2 \times 15 = 30$ mm

$$R = e\,r = 2.718 \times 15 = 40.77 \text{ mm}$$

$$D = 2R = 2 \times 40.77 = 81.54 \text{ mm}$$

Q.1 (b) Solution:

In per unit system,

$$V_t I_a \cos\theta = \text{power}$$

$$1 \times I_a \times 0.8 = 0.9$$

$$I_a = 1.125 \text{ p.u.}$$

$$\tan(\delta + \theta) = \frac{I_a X_q + V_t \sin\theta}{V_t \cos\theta} = \frac{1.125 \times 0.6 + 1 \times 0.6}{1 \times 0.8}$$

$$\delta + \theta = 57.89°$$

$$\delta = 57.89 - \cos^{-1}(0.8)$$

$$= 57.89° - 36.86° = 21.02°$$

$$I_d = I_a \sin(\delta + \theta)$$

$$= 1.125 \sin(57.89°) = 0.953 \text{ p.u.}$$

$$E_f = V_t \cos\delta + I_d X_d$$

$$= 1 \times \cos 21.02° + 0.953 \times 1 = 1.886 \text{ p.u.}$$

When loss of excitation takes place, $E_f = 0$ and the power is then given by

i.e.,
$$P_{max} = \frac{1}{2} V_t^2 \left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta$$

$$= \frac{1}{2}\left(\frac{1}{0.6} - 1\right) \sin 90° = 0.333 \text{ p.u.}$$

Q.1 (c) Solution:

It is seen from figure that for any current i_a, the voltage drop across thyristor is

$$v_T = 0.8 + \frac{2.0 - 0.8}{100} \times i_a = 0.8 + 0.012\, i_a$$

(i) Constant current of 80 A for one-half cycle. For i_a = 80 A, the voltage drop across thyristor is v_T = 0.8 + 0.012 × 80 = 1.76 V. The average on-state power loss in thyristor is

$$P_{av} = \frac{1}{T}\int_0^{T/2} v_T \cdot i_a \cdot dt = \frac{1}{T}\int_0^{T/2} 1.76 \times 80\, dt$$

$$= \frac{1.76 \times 80 \times T}{2T} = 70.4 \text{ W}$$

Waveform of i_a gives the rms current rating of thyristor as

$$\sqrt{\frac{80^2 \times T}{2T}} = 56.568 \text{ A}$$

(ii) Here, v_T = 0.8 + 0.012 × 30 = 1.16 V

$$\therefore \quad P_{av} = \frac{1.16 \times 30 \times T}{3T} = 11.6 \text{ W}$$

$$\text{Rms current rating} = 30 \times \frac{1}{\sqrt{3}} = 17.321 \text{ A}$$

(iii) Half-sine wave of peak value of 80 A,

$$i_a = 80 \sin \omega t$$

$$\therefore \quad v_T = 0.8 + 0.012 \times 80 \sin \omega t = 0.8 + 0.96 \sin \omega t$$

From the waveforms for i_a and v_T shown in figure, the average on-state power loss is given by

$$P_{av} = \frac{1}{2\pi}\int_0^{\pi} (0.8 + 0.96 \sin \omega t)\,(80 \sin \omega t)\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{\pi} 64 \sin \omega t \cdot d(\omega t) + \frac{1}{2\pi}\int_0^{\pi} 76.8 \sin^2 \omega t \cdot d(\omega t)$$

$$= \frac{1}{2\pi} \times 64 \left| -\cos \omega t \right|_0^{\pi} + \frac{76.8}{4\pi}\left| \omega t - \frac{\sin 2\omega t}{2} \right|_0^{\pi}$$

$$= 20.372 + 19.2 = 39.572 \text{ W}$$

$$\text{Rms current rating} = \frac{I_{max}}{2} = \frac{80}{2} = 40 \text{ A}$$

Q.1 (d) Solution:

Given, open loop transfer function of the given control system is,

$$G(s) = \frac{K(s+2)}{s^2}$$

put $s = j\omega$,

$$G(j\omega) = \frac{K(2+j\omega)}{(j\omega)^2}$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{2}\right) - 180°$$

Phase margin at gain crossover frequency, ω_{gc} is

$$PM = 180° + \angle G(j\omega)\Big|_{\omega = \omega_{gc}}$$

$$50° = 180° + \left[\tan^{-1}\frac{\omega_{gc}}{2} - 180°\right]$$

$$\therefore \quad \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) = 50°$$

$$\frac{\omega_{gc}}{2} = \tan 50°$$

$$\therefore \quad \omega_{gc} = 2.384 \text{ rad/sec}$$

At gain crossover frequency,

$$|G(j\omega)H(j\omega)|_{\omega = \omega_{gc}} = 1$$

$$\left|\frac{K(2+j\omega)}{(j\omega)^2}\right|_{\omega = \omega_{gc}} = 1$$

$$\frac{K\sqrt{2^2 + \omega_{gc}^2}}{\omega_{gc}^2} = 1$$

$$\frac{K\sqrt{4 + 2.384^2}}{(2.384)^2} = 1$$

$$\therefore \quad K = \frac{(2.384)^2}{\sqrt{4 + 2.384^2}}$$

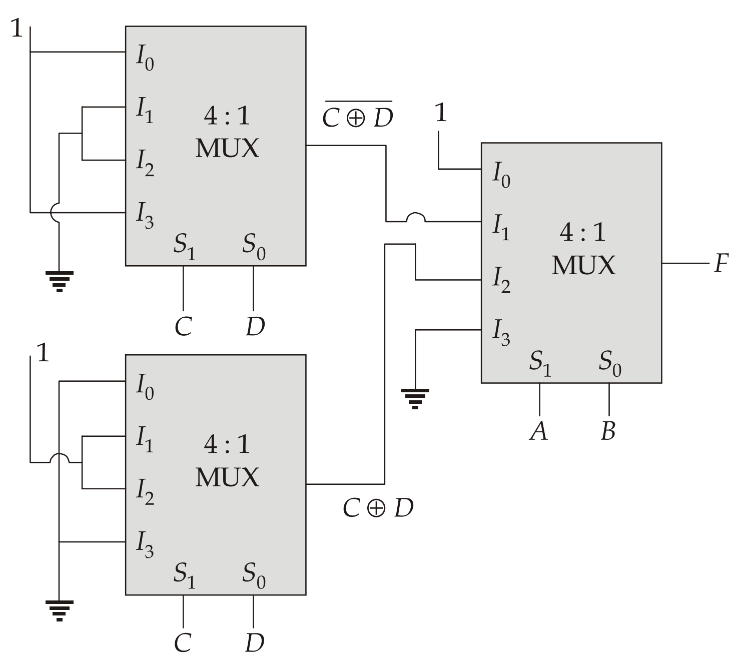
$\therefore$ $K \simeq 1.83$ to achieve the given phase margin of 50°.

Q.1 (e) Solution:

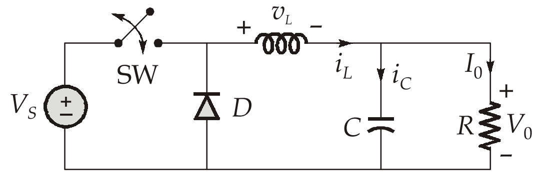
$$F(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 7, 9, 10)$$

Consider a 4 : 1 MUX with select lines as AB.

A	B	C	D	F	Input to 4 : 1 MUX
0	0	0	0	1	$I_0 = 1$
0	0	0	1	1	
0	0	1	0	1	
0	0	1	1	1	
0	1	0	0	1	$I_1 = \overline{C \oplus D}$
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	1	
1	0	0	0	0	$I_2 = C \oplus D$
1	0	0	1	1	
1	0	1	0	1	
1	0	1	1	0	
1	1	0	0	0	$I_3 = 0$
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	0	



Q.2 (a) Solution:



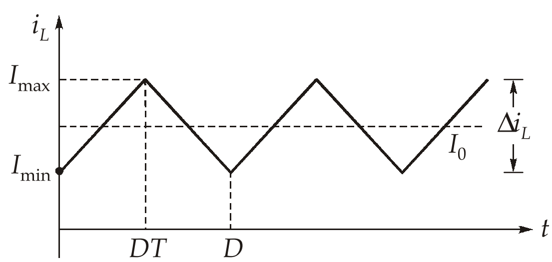
Assuming inductor current is in continuous conduction mode when switch is ON, SW → ON,

⇒ Diode D → OFF

(i)
$$v_L = V_S - V_0 = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_S - V_0}{L}$$

$$(\Delta i_L)_{\text{closed}} = \left(\frac{V_S - V_0}{L}\right)DT \quad \text{...(i)}$$



The average inductor current must be the same as the average current in the load resistor

$$I_L = I_0 = \frac{V_0}{R}$$

So minimum value of current,

$$I_{\text{min}} = I_L - \frac{\Delta i_L}{2} \quad \text{...(ii)}$$

$$I_{\text{min}} = \frac{V_0}{R} - \frac{1}{2}\left[\frac{V_0}{L}(1-D)T\right]$$

When switch is off,

Diode → ON

$$v_L = -V_0 = \frac{L\,di_L}{dt}$$

So,
$$(\Delta i_L)_{\text{open}} = -\left(\frac{V_0}{L}\right)(1-D)T \quad \text{...(iii)}$$

For inductor

$$(\Delta i_L)_{closed} + (\Delta i_L)_{open} = 0$$

$$\left(\frac{V_S - V_0}{L}\right)(DT) - \left(\frac{V_0}{L}\right)(1-D)T = 0$$

Solving this, $V_0 = V_S D$

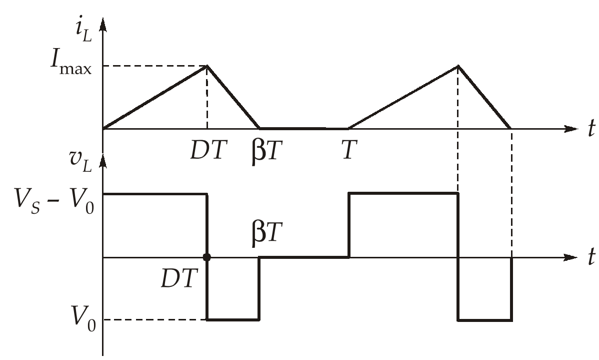
So minimum current in inductor

$$I_{min} = \frac{0.4 \times 24}{20} - \frac{1}{2}\left[\frac{0.4\times 24}{200\times 10^{-6}}(1-0.4)\times\frac{1}{10\times 10^{3}}\right]$$

$$I_{min} = -0.96 \text{ Amp} < 0$$

Since negative current is not possible, inductor current must be discontinuous

(ii)



The average inductor voltage is always zero. So

$$(V_S - V_0)DT = (\beta - D)T\, V_0$$

$$DV_S - DV_0 = \beta V_0 - DV_0$$

$$V_0 = \left(\frac{D}{\beta}\right)V_S$$

and $$I_L = \frac{1}{2} I_{max}(\beta) = \frac{V_0}{R}$$ ...(i)

When switch is closed, $$\frac{di_L}{dt} = \frac{V_S - V_0}{L}$$

$$I_{max} = \Delta i_L = \left(\frac{V_S - V_0}{L}\right)DT$$

$$I_{max} = \Delta i_L = \frac{V_S(1 - D/\beta)DT}{L}$$ ...(ii)

From equation (i) and (ii),

$$\frac{1}{2}\times\frac{V_S(1-D/\beta)DT\times\beta}{L} = \frac{D}{\beta}\frac{V_S}{R}$$

$$\frac{1}{2}\frac{(\beta-D)T\times\beta}{L} = \frac{1}{R}$$

$$\beta(\beta - D) = \frac{2L}{RT}$$

$$\beta(\beta - 0.4) = \frac{2\times 200\times 10^{-6}}{20\times\frac{1}{10\times 10^3}} = 0.2$$

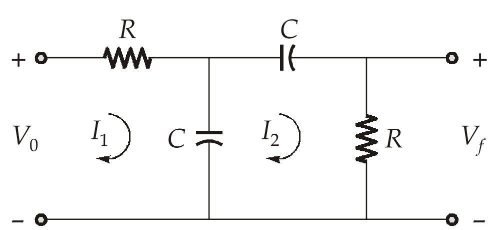
$$\beta^2 - 0.4\beta - 0.2 = 0$$

Solving this, $\beta = 0.6898 \approx 0.69$

So, Output voltage, $V_0 = \left(\frac{0.4}{0.69}\right)\times 24 = 13.91$ Volts

Q.2 (b) (i) Solution:

According to Barkhausen's criteria, the phase shift provided around the loop should be zero or 360°. Here in the given op-amp circuit, the non-inverting amplifier gives the zero phase shift therefore the phase shift provided by the feedback network should also be zero degree. Now the feedback network from output side,



$$-V_0 + RI_1 + (I_1 - I_2)\frac{1}{j\omega C} = 0$$

Let, $\frac{1}{j\omega C} = X_C$

$$(R + X_C)I_1 - X_C I_2 = V_0 \quad \text{...(i)}$$

Also, $(2X_C + R)I_2 - I_1 X_C = 0$

$$I_1 = \frac{(2X_C + R)}{X_C}I_2 \quad \text{...(ii)}$$

Putting equation (ii) in equation (i),

$$(R + X_C)\frac{[2X_C + R]}{X_C}I_2 - I_2X_C = V_0$$

$$[R+X_C][2X_C+R]I_2 - I_2X_C^2 = V_0X_C$$

$$I_2[2RX_C + R^2 + 2X_C^2 + X_CR - X_C^2] = V_0X_C$$

$$I_2[X_C^2 + 3X_CR + R^2] = V_0X_C$$

$$I_2 = \frac{V_0X_C}{X_C^2 + 3X_CR + R^2}$$

Now, $$V_f = RI_2$$

$$V_f = \frac{V_0X_CR}{X_C^2 + 3X_CR + R^2}$$

Therefore, feedback factor, $$\frac{V_f}{V_0} = \beta = \frac{X_CR}{X_C^2 + 3X_CR + R^2}$$

$$\beta = \frac{1}{3 + \frac{X_C}{R} + \frac{R}{X_C}}$$

On putting, $$X_C = \frac{1}{j\omega C}$$

$$\beta = \frac{1}{3 + j\left[\omega RC - \frac{1}{\omega RC}\right]} \quad \text{...(iii)}$$

Gain of amplifier, $$A = 1 + \frac{R_2}{R_1}$$

Now the loop gain the circuit,

$$A\beta = \frac{\left(1 + \frac{R_2}{R_1}\right)\times 1}{3 + j\left[\omega RC - \frac{1}{\omega RC}\right]}$$

Circuit at the oscillations, say $\omega = \omega_0$, the phase shift of loop gain should be zero, therefore

$$\omega_0 RC - \frac{1}{\omega_0 RC} = 0$$

$$\omega_0^2 = \frac{1}{(RC)^2}$$

$$\omega_0 = \frac{1}{RC} \quad \text{(Frequency of oscillations)}$$

and $$|A\beta| \geq 1$$

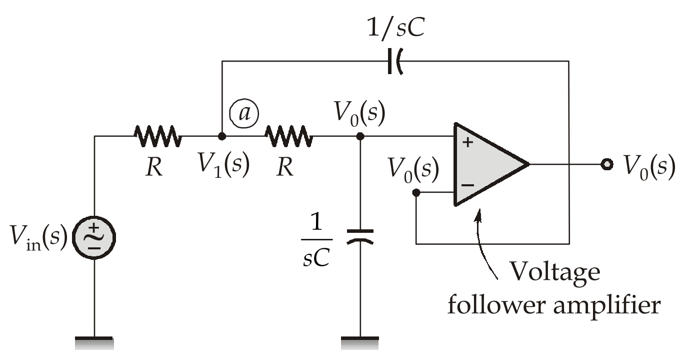
$$\frac{\left[1+\frac{R_2}{R_1}\right]}{3} \geq 1$$

$$\frac{R_2}{R_1} \geq 2$$

Therefore $R_2 \geq 2R_1$ (Condition of sustained oscillations)

Q.2 (b) (ii) Solution:

Given circuit is s-domain,



Apply KCL at node (a)

$$\frac{V_1(s)-V_{in}(s)}{R}+sC[V_1(s)-V_0(s)]+\left[\frac{V_1(s)-V_0(s)}{R}\right] = 0$$

$$[2+sRC]V_1(s) - V_{in}(s) - V_0(s)[1+sRC] = 0 \quad \text{...(i)}$$

Also,
$$V_0(s) = \frac{1}{1+sRC}V_i(s)$$

$$V_1(s) = (1+sRC)V_0(s) \quad \text{...(ii)}$$

On putting equation (ii) in equation (i),

$$(2+sRC)(1+sRC)V_0(s) - V_0(s)[1+sRC] = V_{in}(s)$$

$$V_0(s)[1+sRC][2+sRC-1] = V_{in}(s)$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{1}{(1+sRC)^2} \quad \text{(Transfer function of filter circuit)}$$

Now, frequency response,

$$\frac{V_0(j\omega)}{V_{in}(j\omega)} = \frac{1}{(1+j\omega RC)^2}$$

at $\omega = 0$, $\left|\dfrac{V_0(j\omega)}{V_{in}(j\omega)}\right| = 1$

at $\omega = \infty$, $\left|\dfrac{V_0(j\omega)}{V_{in}(j\omega)}\right| = 0$

at $\omega = \dfrac{1}{RC}$, $\left|\dfrac{V_0(j\omega)}{V_{in}(j\omega)}\right| = \dfrac{1}{\sqrt{2}}$

Therefore it is a low pass filter.

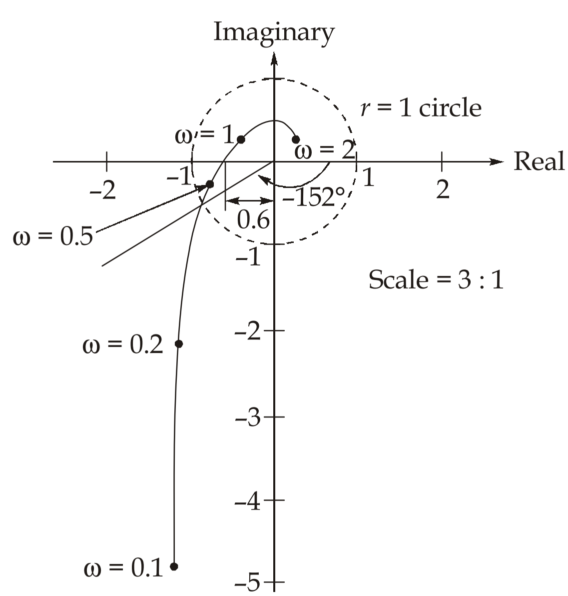
Q.2 (c) Solution:

$$L(s) = \frac{Ke^{-s}}{s(s+1)(s+2)}$$

For unity gain, $K = 1$

$$L(\omega) = \frac{e^{-j\omega}}{j\omega(j\omega+1)(j\omega+2)}$$

ω	$\lvert L(\omega)\rvert$	$\arg(L(\omega))$
0.1	4.969	–104.30°
0.2	2.439	–118.48°
0.5	0.868	–159.25°
1	0.316	–218.86°
2	0.079	–313.03°



$$\arg(L(\omega)) \text{ at } |L(\omega)| = 1 \text{ is } -152°$$

$$\therefore \quad \text{Phase margin} = 180° - 152° = 28°$$

$$|L(\omega)| \text{ at } \arg(L(\omega)) = -180° \text{ is } 0.6$$

$$\therefore \quad \text{Gain margin} = \frac{1}{0.6} = 1.67 \text{ or } 4.437 \text{ dB}$$

Q.3 (a) Solution:

(i) Magnetic bubble memory is a solid state device. It is highly reliable, small in size, light in weight and its power consumption is low. It access time is high, i.e. it is a sow device, typicaly 100 kbps. In this type of memory, data is stored in magnetic bubbles. This is done in a thin film of magnetic material. A '1' is represented by the presence of a bubble while the absence signifies a logical 'o'. It is a nonvolatile semi random access type memory. Its readout is on-destructive in nature. A bubble memory contains several loops, each loop of which contains a large number of bits.

For read or write operation to be done, each loop possesses a 1-bit viewing window.

E.g. Intel 7110

(ii) Total memory capacity: 2kB RAM and 2kB ROM = 4 kB

$$\text{Number of address lines} = \log_2 4 \times 2^{10} = 12$$

Out of 12, 11 address lines (A_o to A_{10}) of microprocessor are connected to 11 memory address lines. Address line A_{11} is used as chip select. The RAM chip is selected when A_{11} is low and ROM gets selected for $A_{11} = 1$

The two address lines are connected to A_{14} and A_{15}. Thus, the 16-bits address line would be:

A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0
I_1	I_2	X	X	CS	M	M	M	M	M	M	M	M	M	M	M

In which

I represents input/output devices

CS reresents chip select

M reresents memory

X reresents don't care

States of A_{15}, A_{14}, A_{11} selects either memory or Input/Output devices as follows:

A_{15}	A_{14}	A_{11}	Selected device
0	0	0	RAM
0	0	1	ROM
1	0	X	Output device
0	1	X	Input device

(iii) There are three ports in 8255 viz. Port A, Port B and Port C each having 8 pins. Again Port C can be divided into Port C_{upper} and Port C_{lower} each having four pins i.e. a nibble. So, 8255 can be viewed to have effectively four ports - Port A, Port B, Port C_{upper}, Port C_{lower}.

Q.3 (b) Solution:

$$P_{fan} = P_{dev} = P\text{; no rotational loss}$$

$$P = E_a I_a$$

For series motor,
$$T_{dev} = K_d I_a^2 \quad \text{...(i)}$$

$$T_{fan} = K_F n^2$$

But,
$$T_{dev} = T_{fan} = T$$

$\therefore$
$$I_a \propto n$$

(i) Operation at 400 rpm ($R_{ext} = 0$)

$$E_a = 220 - (0.6 + 0.4) \times 30 = 190 \text{ V} \quad \text{...(ii)}$$

$$I_a = 30 \text{ A}$$

$$P = 190 \times 30 = 5.7 \text{ kW}$$

$$T\omega = E_a I_a$$

or,
$$T = \frac{5700}{\frac{2\pi \times 400}{60}} = 136 \text{ Nm or } 136.08 \text{ N-m}$$

(ii) Operation at 200 rpm ($R_{ext} = ?$)

$$\frac{T_1}{T_2} = \frac{n_1^2}{n_2^2}$$

$\therefore$
$$T = 136 \times \left(\frac{200}{400}\right)^2 = 34 \text{ Nm}$$

$$\frac{I_{a_1}}{I_{a_2}} = \frac{n_1}{n_2}$$

$$\therefore \qquad I_a = 30 \times \left(\frac{200}{400}\right) = 15 \text{ A}$$

$$T\omega = E_a I_a$$

$$34 \times \left(\frac{2\pi \times 200}{60}\right) = [220 - (0.6 + 0.4 + R_{ext}) \times 15] \times 15$$

On solving we get, $R_{ext} = 10.5\ \Omega$

$$\therefore \qquad P = T\omega = 34 \times \left(\frac{2\pi \times 200}{60}\right) = 712.094 \text{ W}$$

Q.3 (c) Solution:

Let the base MVA be 10 and base kV be 13.8

Base current, $$I_B = \frac{\text{MVA} \times 1000}{\sqrt{3} \times 13.8} = \frac{10000}{\sqrt{3} \times 13.8} = 418.37 \text{ A}$$

Now, $$Z_n = j\,0.7\ \Omega$$

$$Z_{n\ (\text{p.u.})} = j0.7 \times \frac{10}{(13.8)^2} = j\,0.03675 \text{ p.u.}$$

$$Z_0 = Z_{g0} + 3Z_n$$

$$= j\,0.05 + 3 \times j\,0.03675 = j\,0.16 \text{ p.u.}$$

Also, $$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}}$$

$$E_a = \frac{13.2}{13.8} \text{p.u.} = 0.9565 \text{ p.u.}$$

$$I_{a1} = \frac{0.9565}{j0.15 + \dfrac{j0.15 \times j0.16}{j0.15 + j0.16}} = -j\,4.206 \text{ p.u.}$$

Now, $$V_{a0} = V_{a2} = V_{a1} = E_a - I_{a1} Z_1$$

$$= 0.9565 - (-j\,4.206) \times j\,0.15 = 0.3256 \text{ p.u.}$$

$$I_{a2} = \frac{-V_{a2}}{Z_2} = \frac{-0.3256}{j0.15} = j\,2.171 \text{ p.u.}$$

$$I_{a0} = \frac{-V_{a0}}{Z_0} = \frac{-0.3256}{j0.16} = j\,2.035 \text{ p.u.}$$

Initial symmetrical rms current in line *b*,

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$$

$$= -j4.206(-0.5 - j0.866) + j2.171(-0.5 + j0.866) + j2.035$$
$$= -5.52 + j3.05250 = 6.3\angle 151.06 \text{ p.u.}$$
$$= 2635.73\angle 151.06° \text{ A}$$

Initial symmetrical rms current in ground wire

$$= 3I_{a0} = 3 \times j2.035 \times 418.37$$
$$= 2554.15\angle 90° \text{ A}$$

Q.4 (a) Solution:

For a single phase full converter,

Average output voltage, $V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\times 230\sqrt{2}}{\pi}\cos(60°) = 103.54 \text{ V}$

Current, $I_0 = \frac{V_0}{R} = \frac{103.54}{10} = 10.354 \text{ A}$

RMS output voltage, $V_{0r} = \frac{V_m}{\sqrt{2}} = 230 \text{ V}$

As load current is ripple free, rms value of load current

$$I_{0r} = I_0 = 10.354 \text{ A}$$
$$P_{dc} = V_0 I_0 = (10.354) \times (103.54) = 1072.05 \text{ W}$$

RMS value of source, $I_S = I_0 = I_{0r} = 10.354 \text{ A}$

Output ac power, $P_{ac} = V_{0r} \times I_{0r} = 230 \times 10.354 = 2381.42 \text{ W}$

Rectification efficiency, $\eta = \frac{P_{dc}}{P_{ac}} = \frac{1072.05}{2381.42} = 0.45 \approx 45\%$

$$\text{Form factor} = \frac{V_{0r}}{V_0} = \frac{230}{103.54} = 2.22$$

Voltage ripple factor, $\text{VRF} = \sqrt{(FF)^2 - 1} = \sqrt{(2.22)^2 - 1} = 1.982$

As load current ripple free,

$$\text{CRF} = 0$$
$$I_{S1} = \frac{2\sqrt{2}}{\pi}\times I_S = \frac{2\sqrt{2}}{\pi}\times 10.354 = 9.32 \text{ A}$$
$$\theta_1 = -\alpha = -60°$$
$$\text{Distortion factor (DF)} = \cos\alpha = +0.5$$

RMS value of total input current $= I_S = I_0 = 10.354 \text{ A}$

$$\text{CDF} = \frac{I_{S1}}{I_S} = \frac{9.32}{10.354} = 0.9$$

Input power factor $= (\text{DF}) \times \text{DF} = 0.9 \times 0.5 = 0.45$ (lag)

$$\text{THD} = \left[\frac{1}{(CDF)^2} - 1\right]^{1/2} = \left[\frac{1}{(0.9)^2} - 1\right]^{1/2} = 0.4843$$

Active power $= V_0 I_0 = (10.354) \times (103.54) = 1072.05$ W

$$\text{Reactive power} = \frac{2V_m}{\pi} \cdot I_0 \sin\alpha$$

$$= \frac{2\sqrt{2}}{\pi} \times 230 \times 10.354 \times \sin(60^\circ) = 1856.79 \text{ VAr}$$

Q.4 (b) Solution:

Given, state matrices,

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

(i) State transition matrix,

$$\phi(t) = e^{At} = L^{-1}\left[(sI - A)^{-1}\right]$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} s+2 & 0 \\ 0 & s+3 \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} s+2 & 0 \\ 0 & s+3 \end{bmatrix}^{-1} = \frac{1}{(s+2)(s+3)}\begin{bmatrix} s+3 & 0 \\ 0 & s+2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\phi(t) = e^{At} = L^{-1}\begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$$

(ii) Zero input response, $X_{ZIR}(t)$

$$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$X_{ZIR}(t) = \phi(t)\, x(0)$$

$$= \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-3t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$X_{ZIR}(t) = \begin{bmatrix} e^{-2t} \\ e^{-3t} \end{bmatrix}$$

(iii) Zero state response, $X_{ZSR}(t)$

$$X_{ZSR}(s) = [sI - A]^{-1} \cdot B \cdot R(s)$$

given, $\quad r(t) = e^{-t}u(t)$

by taking laplace transform,

$$R(s) = \frac{1}{s+1}$$

$$X_{ZSR}(s) = \begin{bmatrix} s+2 & 0 \\ 0 & s+3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \left(\frac{1}{s+1} \right)$$

$$= \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+3 & 0 \\ 0 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \left(\frac{1}{s+1} \right)$$

$$X_{ZSR}(s) = \begin{bmatrix} \frac{1}{(s+1)(s+2)} & 0 \\ 0 & \frac{1}{(s+1)(s+3)} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Taking inverse Laplace transform,

$$X_{ZSR}(s) = L^{-1} \left\{ \begin{bmatrix} \frac{1}{(s+1)(s+2)} & 0 \\ 0 & \frac{1}{(s+1)(s+3)} \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= L^{-1} \left\{ \begin{bmatrix} \frac{1}{s+1} - \frac{1}{s+2} & 0 \\ 0 & \frac{1}{2}\left[\frac{1}{s+1} - \frac{1}{s+3} \right] \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$X_{ZSR}(t) = \begin{bmatrix} e^{-t} - e^{-2t} & 0 \\ 0 & \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} e^{-t} - e^{-2t} \\ \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t} \end{bmatrix} u(t)$$

(iv) Total response
$$y(t) = C\{X_{ZIR}(t) + X_{ZSR}(t)\}$$
$$= \begin{bmatrix} 2 & 1 \end{bmatrix}\left\{\begin{bmatrix} e^{-2t} \\ e^{-3t} \end{bmatrix} + \begin{bmatrix} e^{-t} - e^{-2t} \\ \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t} \end{bmatrix}\right\}$$
$$= \begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} e^{-t} \\ \frac{1}{2}e^{-t} + \frac{1}{2}e^{-3t} \end{bmatrix}$$
$$y(t) = 2e^{-t} + \frac{1}{2}\left(e^{-t} + e^{-3t}\right)$$
$$y(t) = \frac{5}{2}e^{-t} + \frac{1}{2}e^{-3t}$$

Q.4 (c) Solution:

(i) From the given bode magnitude plot, we can write the open loop transfer function as,

$$G(s)H(s) = \frac{K\left(\frac{s}{\omega_2} + 1\right)}{s\left(\frac{s}{\omega_1} + 1\right)\left(\frac{s}{\omega_3} + 1\right)}$$

The slope between the corner frequency ω_1 rad/s and 10 rad/s is –40 dB/decade, therefore, we can write

$$-40 = \frac{0 - 24.1}{\log 10 - \log \omega_1}$$
$$\log\left(\frac{10}{\omega_1}\right) = \frac{24.1}{40}$$

Therefore, corner frequency,

$$\omega_1 = 2.5 \text{ rad/sec}$$

The slope between 10 rad/s and the corner frequency ω_2 rad/s is –40 dB/decade, therefore, we can write,

$$-40 = \frac{-15.92 - 0}{\log \omega_2 - \log 10}$$

$$\log\left(\frac{\omega_2}{10}\right) = \frac{-15.92}{-40}$$

Therefore, corner frequency,

$$\omega_2 = 25 \text{ rad/sec}$$

The slope between the corner frequencies ω_2 rad/sec and ω_3 rad/sec is –20 dB/decade, therefore, we can write,

$$-20 = \frac{-23.52 + 15.92}{\log \omega_3 - \log 25}$$

$$\log\left(\frac{\omega_3}{25}\right) = \frac{-7.6}{-20}$$

$\therefore$ Therefore, corner frequency,

$$\omega_3 \simeq 60 \text{ rad/sec}$$

For the initial slope line, the magnitude,

$$M = -20 \times \log \omega + 20 \log K$$

At $\omega_1 = 2.5$ rad/sec, $M = 24.1$ dB

$$24.1 = -20 \times \log(2.5) + 20 \log K$$

$$\therefore \quad K = 40.08 \simeq 40$$

$$\therefore \quad G(s)H(s) = \frac{40\left(\frac{s}{25}+1\right)}{s\left(\frac{s}{2.5}+1\right)\left(\frac{s}{60}+1\right)}$$

$$\therefore \quad G(s)H(s) = \frac{240(s+25)}{s(s+2.5)(s+60)}$$

(ii) Given: open loop transfer function,

$$G(s) = \frac{K}{2s(1+0.1s)(1+s)}$$

Put $s = j\omega$,

$$G(j\omega) = \frac{K}{2j\omega(1+j0.1\omega)(1+j\omega)}$$

We know that,

The gain margin, $\text{G.M} = \left.\dfrac{1}{|G(j\omega)|}\right|_{\omega = \omega_{pc}}$

where, ω_{pc} is the frequency at which phase of the system is –180°.

$$\angle G(j\omega) = -90° - \tan^{-1}(0.1\,\omega) - \tan^{-1}(\omega)$$

At $\omega = \omega_{pc}$; $\angle G(j\omega_{pc}) = -180°$

$$-180° = -90° - \tan^{-1}(0.1\,\omega_{pc}) - \tan^{-1}(\omega_{pc})$$

$$-90° = -\tan^{-1}\left(\frac{0.1\omega_{pc} + \omega_{pc}}{1 - 0.1\omega_{pc}^2}\right)$$

$$1 - 0.1\omega_{pc}^2 = 0$$

$\therefore$ $\omega_{pc} = \sqrt{10} = 3.162 \text{ rad/sec}$

Magnitude of the system,

$$|G(j\omega)| = \frac{K}{2\omega\sqrt{\omega^2+1}\sqrt{1+(0.1\omega)^2}}$$

At phase crossover frequency ω_{pc},

$$|G(j\omega_{pc})| = \frac{K}{2\omega_{pc}\sqrt{1+\omega_{pc}^2}\sqrt{1+(0.1\omega_{pc})^2}}$$

$$|G(j\omega_{pc})| = \frac{K}{2\times 3.162\sqrt{(3.162)^2+1}\sqrt{1+(0.1\times 3.162)^2}}$$

$$= \frac{K}{22}$$

Gain margin, $\text{G.M} = \left.\dfrac{1}{|G(j\omega)|}\right|_{\omega = \omega_{pc}} = \dfrac{22}{K}$

Given, Gain margin = 14 dB

i.e., 20 log (G.M) = 14

$\therefore$ $\text{G.M} = 10^{14/20} = 5.012$

$$5.012 = \frac{22}{K}$$

$$K = \frac{22}{5.012}$$

$\therefore$ $K = 4.389$

Section-B

Q.5 (a) Solution:

Given data,

At supply side, $V_L = 400$ V, $f = 50$ Hz,

$\alpha = 60°$, source inductance, L_s

At DC side/load side, $I_0 = 25$ A,

$V_0' = 250$ V

Load resistance, $R_L = \frac{V_0}{I_0} = \frac{250}{25} = 10\,\Omega$

As we know, due to source inductance, there is reduction in average output voltage of converter. The reduced output voltage due to source inductance is given as

$$V_0 = \frac{3V_{ml}}{\pi}\cos\alpha - 6fL_sI_0$$

$$250 = \frac{3 \times 400 \times \sqrt{2}}{\pi} \times 0.5 - 6 \times 50 \times L_s \times 25$$

$$7500\,L_s = 270.10 - 250$$

$$L_s = \frac{20.10}{7500} = 2.67 \text{ mH}$$

Therefore the source inductance is 2.67 mH.

As we know, the performance of a 3-phase full converter as influenced by source inductance is given by relation

$$\cos(\alpha + \mu) = \cos\alpha - \frac{2\omega L_s I_0}{V_{ml}}$$

$$\cos(60° + \mu) = \cos(60°) - \frac{2 \times 2\pi \times 50 \times 2.67 \times 10^{-3} \times 25}{400\sqrt{2}}$$

$$= 0.5 - 0.0741 = 0.4258$$

Angle of overlap, $\mu = \cos^{-1}(0.4258) - 60°$

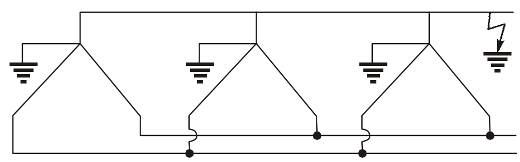
$$\mu = 4.8°$$

Q.5 (b) Solution:

Let base MVA be 10 MVA and base kV be 6.6 kV

$$\therefore \qquad \text{Base current} = \frac{10 \times 10^3}{\sqrt{3} \times 6.6} = 874.77 \text{ Amp}$$

When all the 3 alternators are solidly grounded,



Since all the alternators are operating in parallel, the resultant reactance will be one-third, i.e.,

$$Z_1 = \frac{j0.15}{3} = j0.05 \text{ p.u.}$$

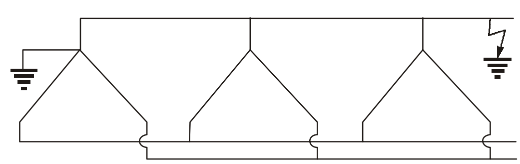
$$Z_2 = \frac{j0.1}{3} = j0.0333 \text{ p.u.}$$

$$Z_0 = \frac{j0.05}{3} = j0.0166 \text{ p.u.}$$

Fault current,
$$I_f = \frac{3}{Z_1 + Z_2 + Z_0} = \frac{3}{j0.05 + j0.0333 + j0.0166} = -j30 \text{ p.u.}$$

$$I_f = -j30 \times 874.77 = -j26243.1 \text{ Amp}$$

when only one alternator neutral is solidly grounded and the others are isolated.



$$Z_1 = \frac{j0.15}{3} \text{ p.u.}$$

$$Z_2 = \frac{j0.1}{3} \text{ p.u.}$$

$$Z_0 = j0.05 \text{ p.u.}$$

∴ Fault current,
$$I_f' = \frac{3}{Z_1 + Z_2 + Z_0}$$

$$= \frac{3}{j0.05 + j0.033 + j0.05} = -j22.55 \text{ p.u.}$$

$$= -j22.55 \times 874.77$$

$$I_f' = -j\,19731.65 \text{ Amp}$$

$$\therefore \quad \left|I_f - I_f'\right| = \left|-j26.243 + j19.731\right| = 6.512 \text{ kA}$$

Q.5 (c) Solution:

(i) We know that

$$e = N\frac{d\phi}{dt}$$

$$N.d\phi = edt$$

$$N.\Delta\phi = E.\Delta t$$

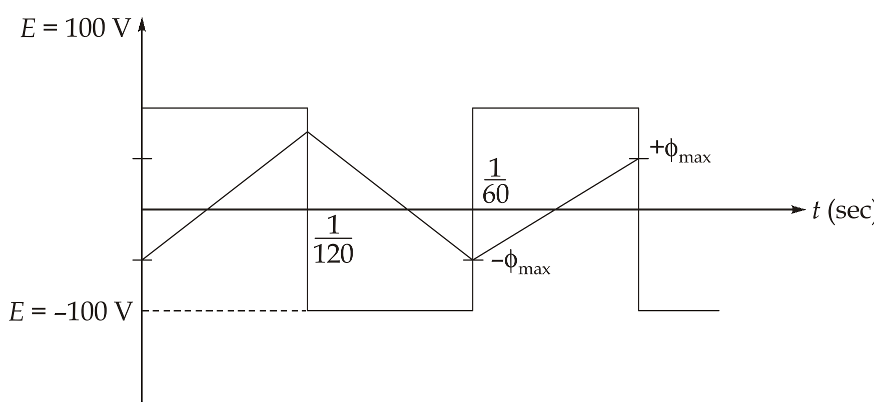
$$\text{Change in flux linkage} = \text{Volt-time product}$$

In the steady state, the positive volt-time area during positive half-cycle will change the flux from negative maximum flux ($-\phi_{max}$) to positive maximum flux ($+\phi_{max}$). Hence, the total change in flux is $2\phi_{max}$ during a half cycle of voltage. Since E is constant in each half cycle, therefore, flux will vary linearly with time. Hence,

$$500 \times (2\phi_{max}) = E \times \frac{1}{120}$$

$$\phi_{max} = \frac{100}{1000 \times 120} \text{ Wb} = 0.833 \times 10^{-3} \text{ Wb}$$

The waveforms of voltage and flux is given as



(ii) Given :

$$B_{max} = 1.2 \text{ T}$$

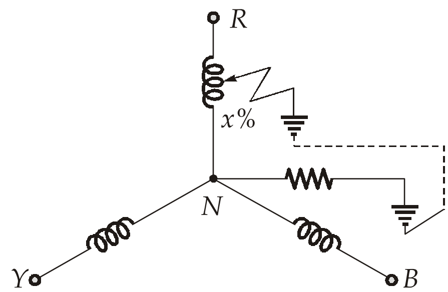
$$\phi_{max} = B_{max} \times A$$

$$= 1.2 \times 0.001 = 1.2 \times 10^{-3} \text{ Wb}$$

$$N \times 2\phi_{max} = E \times \frac{1}{120}$$

$$E = 120 \times 500 \times 2 \times 1.2 \times 10^{-3} = 144 \text{ V}$$

Q.5 (d) Solution:



Let $x\%$ of the winding be unprotected.

Voltage per phase is, $V_{ph} = \dfrac{13.8\times10^3}{\sqrt{3}} = 7.967 \text{ kV}$

Full load current, $I = \dfrac{\text{MVA}}{\sqrt{3}\,V_L} = \dfrac{125\times10^6}{\sqrt{3}\times13800} = 5.23 \text{ kA}$

Reactance per phase, $X = \left(\dfrac{V_{ph}}{I}\right)\times(\text{reactance pu})$

$$= \frac{7.969\times10^3}{5230}\times1.4 = 2.133\,\Omega$$

Emf induced in unprotected winding

$$= 7967\times\frac{x}{100} = 79.67x \text{ volts}$$

Earth fault current caused by unprotected winding $= I\times\dfrac{10}{100} = 5230\times\dfrac{10}{100} = 523\text{ A}$

$$\text{Earth impedance} = \frac{\text{emf induced}}{\text{Earth fault current}} = \frac{79.67x}{523}\,\Omega = 0.1523x\,\Omega$$

Reactance of unprotected winding

$$= \frac{x\times X_{(\text{reactance})}}{100} = \frac{x}{100}\times2.133 = 0.02133x\,\Omega$$

$\therefore$ Earth resistance, $R = 2\,\Omega$

Since (Earth impedance) $= \sqrt{(\text{Earth resistance})^2 + (\text{Earth reactance})^2}$

$\therefore$ $0.1523x = \sqrt{(2)^2 + (0.02133x)^2}$ or $x = 13.26$

$\therefore$ Percentage of protected winding

$$= (100 - 13.26)\% = 86.74\%$$

Q.5 (e) Solution:

$$m(t) = \cos(1000t)\cos(3000t)$$

$$= \underbrace{0.5\cos(4000t)}_{m_1(t)} + \underbrace{0.5\cos(2000t)}_{m_2(t)}$$

(i) Modulation index, $\mu = |K_a m(t)|_{\max}$, K_a : Amplitude sensitivity

$\Rightarrow$ $\mu_1 = |m(t)|_{\max} = 0.5 = \mu_2$

$\Rightarrow$ $\mu = \sqrt{\mu_1^2 + \mu_2^2} = 0.707$ ($\because$ $m_1(t)$ and $m_2(t)$ are orthogonal)

(ii) Power efficiency, $\eta = \dfrac{\mu^2}{2+\mu^2} = \dfrac{2\times 0.5^2}{2+2\times 0.5^2} = 0.2$

(iii) $m(t) = 0.5\cos(4000t) + 0.5(2000t)$

Frequency components in message signal

$= 2000$ rad/s, 4000 rad/s

Amplitude modulated signal:

$$s_{AM}(t) = (1 + m(t))c(t)$$

$$= (1 + 0.5\cos(4000t) + 0.5\cos(2000t))\cos(10000t)$$

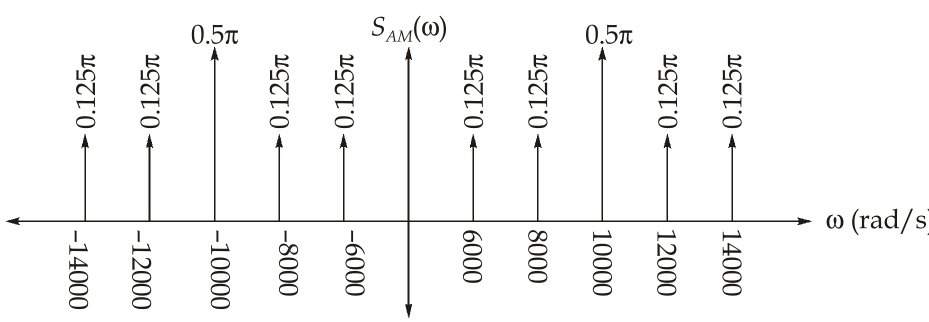
$$= \cos(10000t) + 0.25\cos(14000t) + 0.25\cos(6000t)$$

$$+ 0.25\cos(12000t) + 0.25(8000t)$$

Frequency components in AM signal:

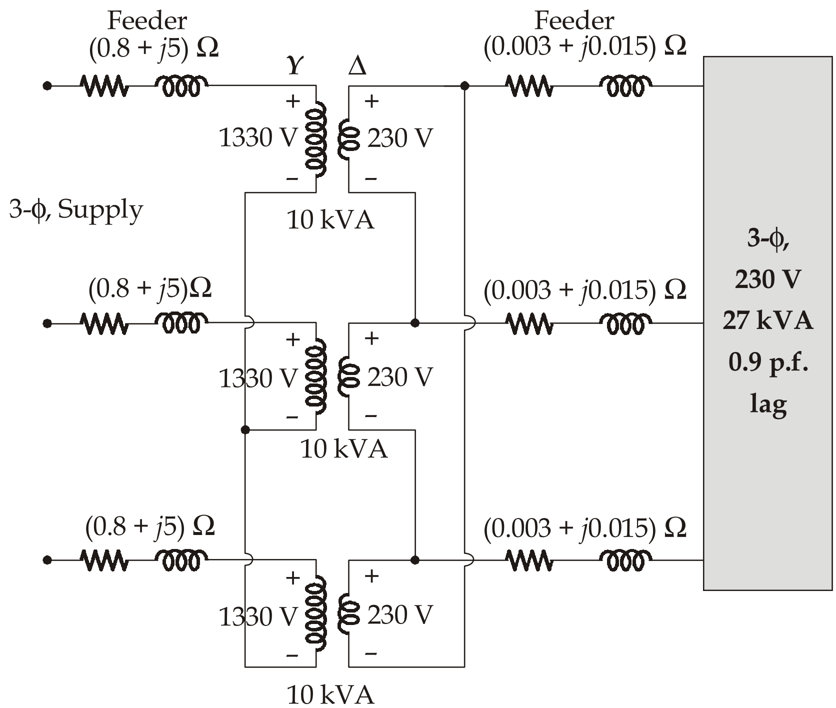
6000 rad/s, 8000 rad/s, 10000 rad/s, 12000 rad/s, 14000 rad/s

(iv)



Q.6 (a) Solution:

The circuit is shown below :



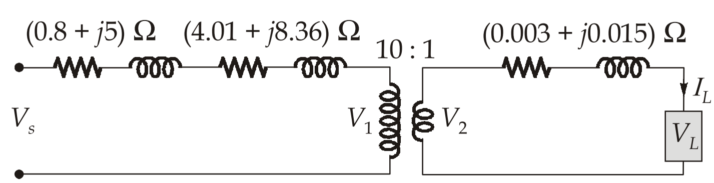
The equivalent circuit of the individual transformer reffered to high voltage side is,

$$R_{eqH} + jX_{eqH} = (0.12 + j0.25) \times \left[\frac{1330}{230}\right]^2 = (4.01 + j8.36)\Omega$$

The turns ratio of the equivalent Y-Y bank is,

$$a' = \frac{1330}{\left(\frac{230}{\sqrt{3}}\right)} = 10$$

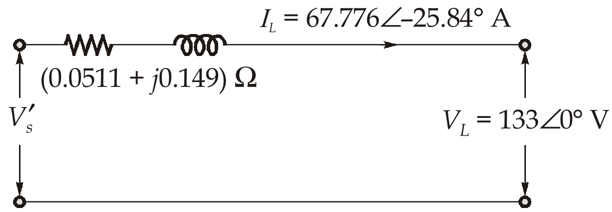
The single phase equivalent circuit of the system is shown below,



All the impedances from the primary side can be transferred to the secondary side and combined with the feeder impedance on the secondary side

$$Z = \left[(4.81 + j13.36) \times \left(\frac{1}{10}\right)^2\right] + (0.003 + j0.015)$$

$$Z = (0.0511 + j\,0.149)\Omega$$



$$V_L = \frac{230}{\sqrt{3}}\angle 0° = 133\angle 0° \text{ V}$$

$$I_L = \frac{27\times 10^3}{\sqrt{3}\times 230} = 67.776 \text{ A}$$

$$\phi_L = -\cos^{-1}(0.9) = -25.84°$$

$$V_s' = [(0.0511 + j0.149)(67.776\angle -25.84°)] + 133\angle 0°$$

$$V_s' = 140.7\angle 3.1° \text{ V}$$

Supply phase voltage, $V_s = 140.7 \times 10 = 1407$ V

Line to line supply voltage,

$$1407\sqrt{3} = 2437 \text{ V}$$

Q.6 (b) Solution:

(i) Given that,

$$x(n) = \left(\frac{1}{3}\right)^n u(n) + (2)^n u(-n-1)$$

Taking the z-transform of the above equation, we obtain

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}}\ ; \qquad \frac{1}{3} < |z| < 2$$

$$= \frac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)(1-2z^{-1})}$$

Now, consider the given output,

$$y(t) = 5\left(\frac{1}{3}\right)^n u(n) - 5\left(\frac{2}{3}\right)^n u(n)$$

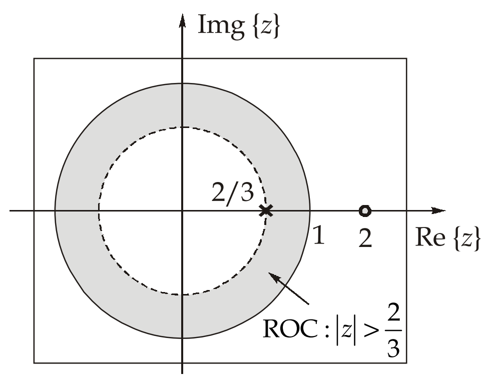
Taking the z-transform of the above equation, we obtain,

$$Y(z) = \frac{5}{1-\frac{1}{3}z^{-1}} - \frac{5}{1-\frac{2}{3}z^{-1}}; \qquad |z| > \frac{2}{3}$$

$$= \frac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{2}{3}z^{-1}\right)}$$

Now,
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}}; \qquad |z| > \frac{2}{3}$$

The pole-zero plot of $H(z)$ is shown in figure



(ii) Consider,
$$H(z) = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}}; \qquad |z| > \frac{2}{3}$$

$$= \frac{1}{1-\frac{2}{3}z^{-1}} - \frac{2z^{-1}}{1-\frac{2}{3}z^{-1}}$$

The inverse z-transform of the above equation yields

$$h(n) = \left(\frac{2}{3}\right)^n u(n) - 2\left(\frac{2}{3}\right)^{n-1} u(n-1)$$

$$= \left(\frac{2}{3}\right)^n [u(n) - 3u(n-1)]$$

(iii) Since,
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}}$$

$$Y(z)\left[1-\frac{2}{3}z^{-1}\right] = X(z)\left[1-2z^{-1}\right]$$

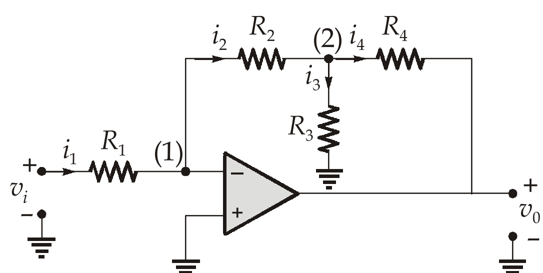
$$Y(z)-\frac{2}{3}z^{-1}Y(z) = X(z) - 2z^{-1}X(z)$$

The inverse z-transform of the above equation leads to

$$y(n)-\frac{2}{3}y(n-1) = x(n) - 2x(n-1)$$

(iv) The system is stable because the ROC includes the unit circle. It is also causal since the impulse response $h(n) = 0$ for $n < 0$.

Q.6 (c) Solution:



Assume opamp to be ideal: $R_i \to \infty$, $R_o \to 0$, $A_{OL} \to \infty$

Virtual Ground Theory:

$$v_+ = v_- = 0$$

$\Rightarrow$
$$i_1 = \frac{v_i - v_-}{R_1} = \frac{v_i}{R_1}$$

At node (1)
$$i_- = 0 \ (\because R_i \to \infty)$$

$\Rightarrow$
$$i_2 = i_1 = \frac{v_i}{R_1}$$

$$v_2 = v_- - i_2R_2 = -v_i\frac{R_2}{R_1}$$

KCL at node (ii)

$$i_2 = i_3 + i_4$$

$$\frac{v_i}{R_1} = \frac{v_2}{R_3}+\frac{v_2-v_o}{R_4}$$

$$\frac{v_i}{R_1} = -v_i\frac{R_2}{R_1R_3}-v_i\frac{R_2}{R_1R_4}-\frac{v_o}{R_4}$$

$$\Rightarrow \quad \frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right)$$

Now, Input resistance = 1 MΩ

$$\frac{v_i}{i_1} = R_1 = 1\text{ M}\Omega \text{ (Maximum limit)}$$

Voltage gain = 100

∵ R_1 is at maximum possible value choose R_2 also at maximum value,

$$R_2 = 1\text{ M}\Omega$$

Now, $$1+\frac{R_4}{R_2}+\frac{R_4}{R_3} = 100$$

$$R_4\left(\frac{1}{R_2}+\frac{1}{R_3}\right) = 99$$

If we choose R_4 at maximum value = 1 MΩ

Then, $$R_3 = 10.2\text{ k}\Omega < 1\text{ M}\Omega$$

Q.7 (a) Solution:

(i) Given, G = 100 MVA, H = 10 MJ/MVA

Kinetic energy stored in rotor= $G \cdot H$ × 100 × 10 MJ = 1000 MJ

(ii) $P_a = P_i - P_e$ = (60 – 50) MW = 10 MW

We know, $$M = \frac{GH}{180f} = \frac{100\times 10}{180\times 60} = \frac{5}{54}\text{ MJ-sec/ele-deg}$$

Now, $$M\cdot\frac{d^2\delta}{dt^2} = P_a$$

$$\Rightarrow \quad \frac{5}{54}\cdot\frac{d^2\delta}{dt^2} = 10$$

$$\therefore \quad \frac{d^2\delta}{dt^2} = \alpha = \frac{10\times 54}{5} = 108\text{ elec-deg/sec}^2$$

$$\alpha = 108 \text{ elec-deg/sec}^2 = 108 \times \frac{2}{P} \text{ mech-deg/sec}^2$$

$$= 108 \times \frac{2}{4} \times \left(\frac{60°}{360°}\right) = 9 \text{ rpm / sec}$$

(iii) 12 cycles is equivalent to $\frac{12}{60} = 0.2$ sec

Change in load angle, $\Delta\delta = \frac{1}{2}\alpha(\Delta t)^2 = \frac{1}{2} \times 108 \times (0.2)^2$

$$\Delta\delta = 2.16 \text{ elec-degree}$$

Now, $\alpha = 108$ elec-degree/sec^2

$$\alpha = 60 \times \frac{108}{360} \times \frac{2}{4} = 9 \text{ rpm/sec}$$

Assuming no accelerating torque, before beginning of 12 cycle period.

$$360° = 1 \text{ rev}$$

$$\Rightarrow \quad 1° = \frac{1}{360} \text{ rev}$$

$$\Rightarrow \quad 108° = \frac{108}{360} \text{ rev}$$

Thus, rotor speed at the end of 12 cycles

$$= \frac{120f}{P} + \alpha\Delta t = \left(\frac{120 \times 60}{4} + 9 \times 0.2\right) \text{rpm} = 1801.8 \text{ rpm}$$

Q.7 (b) (i) Solution:

If $x(t) \leftrightarrow X_n$

Then, $$\frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |X_n|^2 = X_0^2 + \sum_{n=1}^{\infty} 2|X_n|^2 \quad \text{...(i)}$$

Parseval's relation states that the total average power in a periodic signal equals the sum of the average powers in all of its harmonic components,

Consider the LHS of equation (i),

$$\frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{T}\int_0^T x(t)x^*(t)\,dt$$

$$= \frac{1}{T}\int_0^T x(t)\left(\sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}\right)^* dt$$

$$= \frac{1}{T}\int_0^T x(t)\left(\sum_{n=-\infty}^{\infty} X_n^* e^{-jn\omega_0 t}\right)dt$$

$$= \sum_{n=-\infty}^{\infty} X_n^*\left(\frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt\right) = \sum_{n=-\infty}^{\infty} X_n^* X_n$$

$$\frac{1}{T}\int_0^T |x(t)|^2\,dt = \sum_{n=-\infty}^{\infty} |X_n|^2 = X_0^2 + \sum_{n=1}^{\infty} 2|X_n|^2$$

The result indicates that the total average power of $x(t)$ is the sum of the average power in each harmonic component.

Q.7 (b) (ii) Solution:

1. Consider the equation,

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

$$\frac{dy(t)}{dt} + 10y(t) = [x(t) * z(t)] - x(t)$$

Taking the Fourier transform of the above equation, we have

$$j\omega Y(\omega) + 10Y(\omega) = X(\omega)Z(\omega) - X(\omega)$$

$$Y(\omega)[10 + j\omega] = X(\omega)[Z(\omega) - 1]$$

Given that, $z(t) = e^{-t}\,u(t) + 3\delta(t),$

This implies that,

$$Z(\omega) = \frac{1}{1+j\omega} + 3$$

Substituting $Z(\omega)$ in the above equation, we get

$$Y(\omega)[10 + j\omega] = X(\omega)\left[\frac{1}{1+j\omega} + 3 - 1\right]$$

$$\frac{Y(\omega)}{X(\omega)} = H(\omega) = \frac{3 + 2j\omega}{(1+j\omega)(10+j\omega)}$$

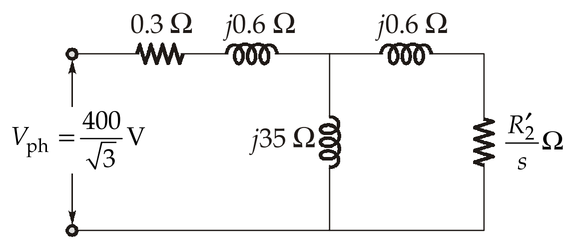
2. Finding the partial fraction expansion of $H(\omega)$, we obtain

$$H(\omega) = \frac{1}{9}\left(\frac{1}{1+j\omega}\right) + \frac{17}{9}\left(\frac{1}{10+j\omega}\right)$$

Taking the inverse Fourier transform, we obtain

$$h(t) = \frac{1}{9}e^{-t}u(t) + \frac{17}{9}e^{-10t}u(t)$$

Q.7 (c) Solution:



(i) At $s = 1$,

$$V_{ph} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$Z_f = j35 \,||\, (0.25 + j0.6) = 0.639\angle 67.78\ \Omega$$

$$= 0.241 + j0.591\ \Omega = R_f + jX_f$$

$$I_{st} = \frac{V_{ph}}{Z_1 + Z_f} = \frac{231}{0.3 + j0.6 + 0.639\angle 67.78°}$$

$$= 176.48\angle -65.56° \text{ A}$$

$$T_{st} = \frac{3I_{st}^2 R_f}{\omega_{sm}} = \frac{3 \times 176.48^2 \times 0.241}{2\pi \times 1500} \times 60 = 143.35 \text{ Nm}$$

(ii)

$$s_{fl} = \frac{N_S - N}{N_S} = \frac{1500 - 1450}{1500} = \frac{1}{30} = 0.033$$

$$\frac{R_2'}{s} = \frac{0.25}{\frac{1}{30}} = 7.5\ \Omega$$

$$Z_f = j35 \,||\, (7.5 + j0.6) = 7.23\angle 16.47\ \Omega$$

$$Z_f = 6.94 + j2.052\ \Omega$$

$$Z_{in} = Z_f + 0.3 + j0.6 = 7.71\angle 20.12\ \Omega$$

$$Z_{in} = 7.24 + j2.65\ \Omega$$

$$I_1 = \frac{V_{ph}}{Z_{in}} = \frac{231}{7.24 + j2.65} = 29.96\angle -20.12° \text{ A}$$

$$\text{Power factor} = \cos 20.12° = 0.938 \text{ lagging}$$

$$\text{Output, } P_{gross} = 3I_1^2 R_f (1 - s)$$

$$P_{gross} = 3 \times 29.96^2 \times 6.94(1 - 0.033) = 18.07 \text{ kW}$$

$$\text{Rotational loss} = 1.5 \text{ kW}$$

$$\text{Net, } P_{out} = 18.07 - 1.5 = 16.57 \text{ kW}$$

$$T_{net} = \frac{P_{out}}{\omega_n} = \frac{16.57 \times 10^3}{2\pi \times 1450} \times 60 = 109.12 \text{ Nm}$$

$$P_{in} = \sqrt{3} V_L I_1 \cos\phi_1$$

$$= \sqrt{3} \times 400 \times 29.96 \times \cos 20.12° = 19.49 \text{ kW}$$

Overall efficiency, $$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{16.57}{19.49} \times 100 = 85.02\%$$

Internal effeciency, $$\eta_{int} = \frac{P_{gross}}{P_{in}} \times 100 = \frac{18.07}{19.49} \times 100 = 92.71\%$$

(iii) $$\frac{R_2'}{s} = \sqrt{R_{th}^2 + (X_{th} + X_2)^2}$$

$$Z_{th} = j35 \parallel (0.3 + j0.6) = 0.29 + j0.592\ \Omega$$

$$\frac{0.25}{s_{max}} = \sqrt{0.29^2 + (0.592 + 0.6)^2}$$

$$s_{max} = 0.204$$

$$\frac{R_2'}{s_{max}} = \frac{0.25}{0.204} = 1.225\ \Omega$$

$$Z_f = (j35) \parallel (j0.6 + 1.225)$$

$$Z_f = 1.183 + j0.631\ \Omega$$

$$I_1 = \frac{V_{ph}}{Z_1 + Z_f} = \frac{231}{1.34\angle 28.1 + (0.3 + j0.6)}$$

$$= 119.89\angle -39.69° \text{ A}$$

$$P_G = 3 I_1^2 R_f$$

$$= 3 \times 119.89^2 \times 1.183 = 51.01 \text{ kW}$$

$$\text{Gross } P_{out} = P_G(1 - s) = 51.01(1 - 0.204) = 40.61 \text{ kW}$$

$$\text{Net } P_{out} = P_G(1 - s) - P_{rot}$$

$$= 40.61 - 1.5 = 39.11 \text{ kW}$$

$$T_{max} = \frac{P_{out}}{\omega_m} = \frac{39.11 \times 10^3}{2\pi \times \frac{1500}{60}(1 - 0.204)} = 312.76 \text{ Nm}$$

Q.8 (a) Solution:

- **Starting and terminating points**

Poles, $s = 0, \; -30 \pm j95.5$ and zero $s = -40$

Since, there are four poles, four root locus paths will originate from these poles and one of the four will terminate at the zero; $s = -40$ and the other three will terminate at infinity. Use rule no. 5 to find out which part of the real axis is part of the root locus.

- **Asymptotes directions and angles**

$$\text{Number} = P - Z = 4 - 1 = 3$$

Angle, $$\alpha_0 = \pm\frac{180^\circ}{4-1} = \pm 60^\circ$$

$$\alpha_1 = \pm\frac{3\times 180^\circ}{4-1} = \pm 180^\circ$$

$$\alpha_2 = \pm\frac{5\times 180^\circ}{4-1} = \pm 300^\circ$$

Intersection of asympotes,

$$s = \frac{0 - 20 - 30 + j95.5 - 30 - j95.5 - (-40)}{4-1} = -13.3$$

- **Branches of root locus on real axis**

The root locus exists on the real axis between poles at 0 and –20 and zero (–40) to infinity (rule 5).

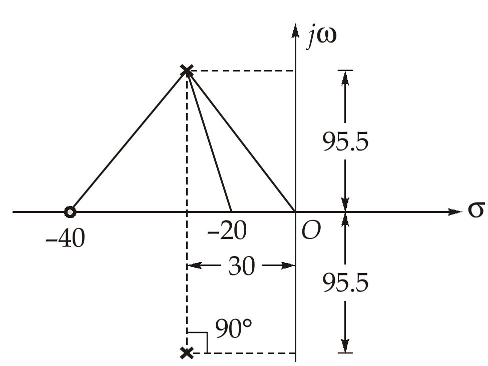
Breakaway point

Let, B = breakaway point

$$\frac{1}{B+40} - \left(\frac{1}{B+20} + \frac{1}{B+0} + \frac{2}{B+30}\right) = 0$$

$$B = -11.72 \text{ or } -68.28$$

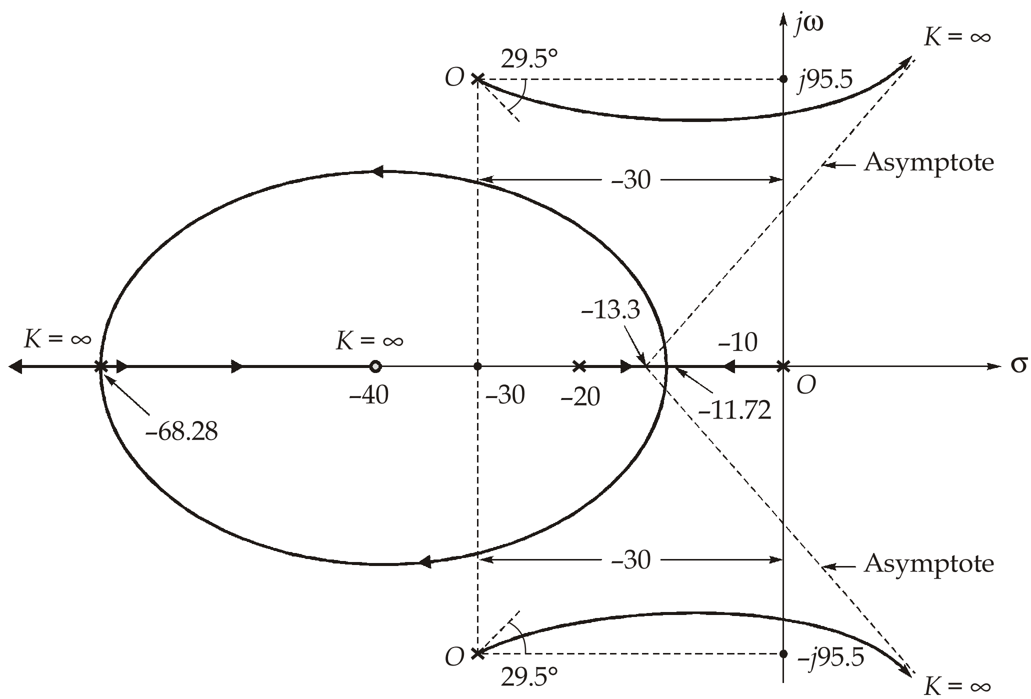
- **Angle of departure from complex poles**



$$= 180° - \left(180° - \tan^{-1}\frac{95.5}{30}\right) - \left(180° - \tan^{-1}\frac{95.5}{10}\right) - \left(90° + \tan^{-1}\frac{95.5}{10}\right)$$

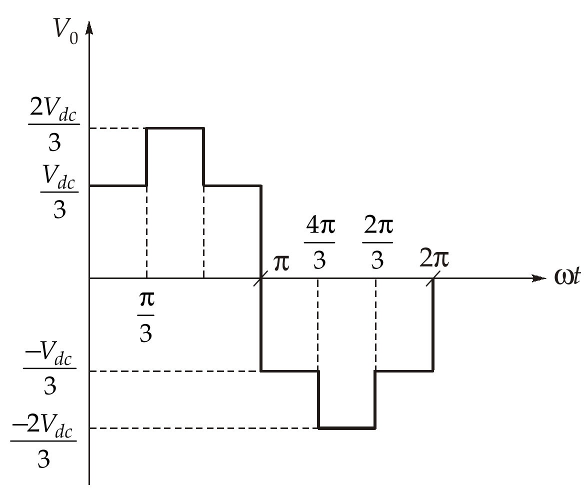
$$= 180° - 107.4° - 96° - 90° + 84° = -29.5°$$

The complete locus is shown below,



Q.8 (b) Solution:

(i) The output phase voltage waveform of 6-step, 3-ϕ, inverter is given as



Since given output voltage is having quarter wave symmetry, i.e., it is symmetric about 90°, therefore, Fourier series expression of output phase voltage can be given as

$$b_n = \frac{8}{T_o}\left[\int_0^{\pi/2} V_o(t)\sin(n\omega t)d(\omega t)\right]$$

$$= \frac{4}{\pi}\left[\int_0^{\pi/3} \frac{V_{dc}}{3}\sin(n\omega t)d(\omega t) + \int_{\pi/3}^{\pi/2} \frac{2V_{dc}}{3}\sin(n\omega t)d(\omega t)\right]$$

$$= \frac{4V_{dc}}{3\pi}\left[-\frac{\cos n\omega t}{n}\Big|_0^{\pi/3} + 2\left[-\frac{\cos n\omega t}{n}\right]_{\pi/3}^{\pi/2}\right]$$

$$= \frac{4V_{dc}}{3\pi}\left[-\frac{\cos n\frac{\pi}{3}}{n} + \frac{1}{n} - \frac{2}{n}\cos\frac{n\pi}{2} + \frac{2\cos n\frac{\pi}{3}}{n}\right]$$

For odd harmonics, $n = 1, 3, 5, 7, ...$

$$\cos\frac{n\pi}{2} = 0$$

Therefore, $b_n = \frac{4V_{dc}}{3n\pi}\left[1 + \cos\frac{n\pi}{3}\right]$

Therefore, output voltage expression,

$$V_o(t) = \sum_{n=6k\pm1}^{\infty} b_n \sin n\omega t$$

$$= \sum_{n=6k\pm1}^{\infty} \frac{4V_{dc}}{3n\pi}\left(1 + \cos\frac{n\pi}{3}\right)\sin n\omega t$$

Fundamental voltage, $V_{o1} = \frac{4V_{dc}}{3\pi}\left(1 + \cos\frac{\pi}{3}\right)\sin\omega t$

$$V_{o1} = \frac{2V_{dc}}{\pi}\sin\omega t$$

$$V_{o1(rms)} = \frac{\sqrt{2}}{\pi}V_{dc} \text{ (rms per phase)}$$

(ii) For 60 Hz output, $V_o = 400 \times \frac{60}{50} = 480$ V (line)

Therefore, Inverter output voltage = Motor input voltage

$$\frac{\sqrt{2}}{\pi}V_{dc} = \frac{480}{\sqrt{3}}$$

$$V_{dc} = 615.62 \text{ Volt}$$

For 50 Hz output,
$$\frac{\sqrt{2}}{\pi}V_{dc} = \frac{400}{\sqrt{3}}$$

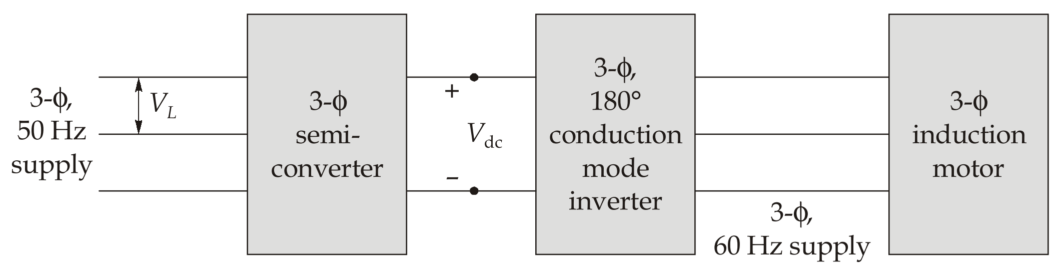
$$V_{dc} = 513 \text{ Volt}$$

For 40 Hz output,
$$V = 400\times\frac{40}{50} = 320 \text{ V}$$

Therefore,
$$\frac{\sqrt{2}}{\pi}V_{dc} = \frac{320}{\sqrt{3}}$$

$$V_{dc} = \frac{320\pi}{\sqrt{6}} = 410.41 \text{ Volt}$$

(iii) Now, inverter is fed through 3-ϕ, semi-converter



Given :
$$V_{dc} = 615.62 \text{ V}$$

Corresponds to 60 Hz and constant $\left(\frac{V}{f}\right)$ ratio.

$$V_L = 500 \text{ V [L – L]}$$

The average output voltage of 3-ϕ semi-converter is given as

$$V_{dc} = \frac{3V_{ml}}{2\pi}(1+\cos\alpha)$$

$$615.62 = \frac{3\sqrt{2}\times 500}{2\pi}(1+\cos\alpha)$$

$$1 + \cos\alpha = \frac{513\times 2\pi}{1500\sqrt{2}} = 1.8234$$

$$\cos\alpha = 0.8234$$

Firing angle,
$$\alpha = \cos^{-1}(0.8234) = 34.57°$$

Q.8 (c) (i) Solution:

Given that, $x(t) = \frac{1}{\sqrt{t}} = t^{-1/2}$

We know that,

$$x(t) \xleftrightarrow{\text{ULT}} X(s)$$

and $X(s) = \int_0^\infty x(t)e^{-st}dt = \int_0^\infty t^{-1/2}e^{-st}dt$

$$= \int_0^\infty t^{-1/2}e^{-st}dt$$

Let, $st = p$

$$dt = \frac{dp}{s}$$

When $t = 0$, $p = 0$

$t = \infty$, $p = \infty$

Then, $X(s) = \frac{1}{\sqrt{s}}\int_0^\infty p^{-1/2}e^{-p}dp$

$$= \frac{1}{\sqrt{s}}\underbrace{\int_0^\infty p^{\frac{1}{2}-1}e^{-p}dp}_{\text{Gamma function}}$$

$$= \frac{1}{\sqrt{s}}\Gamma\left(\frac{1}{2}\right) = \sqrt{\frac{\pi}{s}} \qquad \left[\because \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}\right]$$

Therefore,

$$\frac{1}{\sqrt{t}}u(t) \xleftrightarrow{\text{U.L.T.}} \sqrt{\frac{\pi}{s}}$$

Q.8 (c) (ii) Solution:

Output voltage, $V_0 = -V_s\left(\frac{D}{1-D}\right) = -24\left(\frac{0.4}{1-0.4}\right) = -16 \text{ V}$

$$I_L = \frac{V_s D}{R(1-D)^2} = \frac{24\times 0.4}{5(1-0.4)^2} = 5.33 \text{ A}$$

$$\Delta i_L = \frac{V_s DT}{L} = \frac{24 \times 0.4}{20 \times 10^{-6} \times 10^5} = 4.8 \text{ A}$$

$$I_{L(\max)} = I_L + \frac{\Delta i_L}{2} = 5.33 + \frac{4.8}{2} = 7.73 \text{ A}$$

$$I_{L(\min)} = I_L - \frac{\Delta i_L}{2} = 5.33 - \frac{4.8}{2} = 2.93 \text{ A}$$

$$\frac{\Delta V_0}{V_0} = \frac{D}{RCf} = \frac{0.4}{5 \times 80 \times 10^{-6} \times 100 \times 10^3} = 0.01 = 1\%$$

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