MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Mechanical Engineering Test No : 10

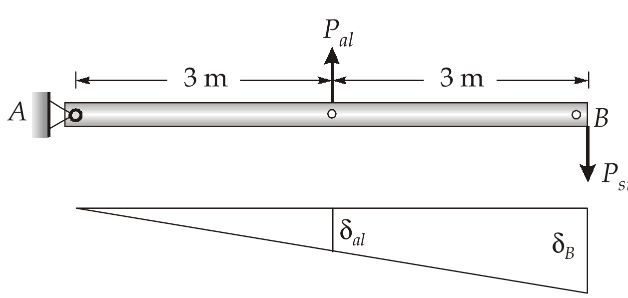
Full Syllabus Test (Paper-II)

Section : A

1. (a) Solution:

Given : maximum vertical moment of P = 5 mm

Member AB :



FBD* and movement diagram of bar *AB

$$\Sigma M_A = 0$$

$$3P_{al} = 6P_{st}$$

$$P_{al} = 2P_{st}$$

Also,
$$\frac{\delta_B}{6} = \frac{\delta_{al}}{3}$$

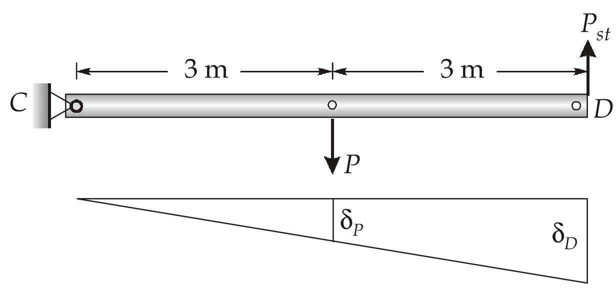
$$\delta_B = 2\delta_{al} = 2\left[\frac{PL}{AE}\right]_{al}$$

$$\delta_B = 2\left[\frac{P_{al}(2000)}{500(70000)}\right]$$

$$\delta_B = \frac{1}{8750}P_{al} = \frac{1}{8750}(2P_{st})$$

$$\delta_B = \frac{1}{4375}P_{st}$$

Member CD:



FBD* and movement diagram of bar *CD

$$\delta_D = \delta_{st} + \delta_B = \left[\frac{PL}{AE}\right]_{st} + \frac{1}{4375}P_{st}$$

$$\delta_D = \frac{P_{st}(2000)}{300(200000)} + \frac{1}{4375}P_{st}$$

$$\delta_D = \frac{11}{42000}P_{st}$$

$$\Sigma M_C = 0$$

$$6P_{st} = 3P$$

$$P_{st} = \frac{1}{2}P$$

Also,

$$\frac{\delta_P}{3} = \frac{\delta_D}{6}$$

$$\delta_P = \frac{1}{2}\delta_D = \frac{1}{2}\left(\frac{11}{42000}P_{st}\right)$$

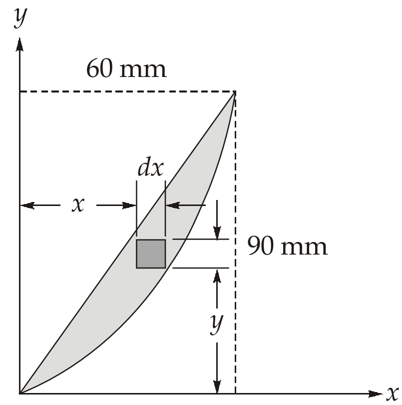
$$\delta_P = \frac{11}{84000}P_{st}$$

$$5 = \frac{11}{84000}\left(\frac{1}{2}P\right)$$

$$P = 76363.64 \text{ N} = 76.4 \text{ kN}$$

1. (b) Solution:

The double differential area element is shown in figure. Choosing to integrate over y first, the area A of the region A is



$$A = \int_A dA = \int_0^{60}\left(\int_{x^2/40}^{3x/2} dy\right)dx$$

$$= \int_0^{60}\left(\frac{3x}{2} - \frac{x^2}{40}\right)dx = \left[\frac{3x^2}{4} - \frac{x^3}{120}\right]_0^{60} = 900 \text{ mm}^2$$

We know that,

$$A\bar{x} = \int_A x\,dA = \int_0^{60}\left(\int_{x^2/40}^{3x/2} x\,dy\right)dx = \int_0^{60}\left(\frac{3x}{2} - \frac{x^2}{40}\right)x\,dx$$

$$A\bar{x} = \left[\frac{3x^3}{6} - \frac{x^4}{160}\right]_0^{60} = 27000 \text{ mm}^3$$

So, $$\bar{x} = \frac{27000}{900} = 30 \text{ mm}$$

Also, $$A\bar{y} = \int_A y\,dA = \int_0^{60}\left(\int_{x^2/40}^{3x/2} y\,dy\right)dx = \int_0^{60}\frac{1}{2}\left(\frac{9x^2}{4} - \frac{x^2}{1600}\right)dx$$

$$A\bar{y} = \frac{1}{2}\left[\frac{9x^3}{12} - \frac{x^5}{8000}\right]_0^{60} = 32400$$

$$\bar{y} = \frac{32400}{A} = \frac{32400}{900} = 36 \text{ mm}$$

Therefore, the coordinates of the centroid of the area are

$$\bar{x} = 30 \text{ mm and } \bar{y} = 36 \text{ mm}$$

1. (c) Solution:

Given : $a = 2.866 \times 10^{-8}$ cm; $\rho = 7.87$ g/cm^3; Atomic weight = 55.847 g/mol

The number of iron atoms would be present in each unit cell for the required density of 7.87 g/cm^3 in given by

$$\rho(\text{g/cm}^3) = \frac{x(\text{atom/cell}) \times 55.847(\text{g/mol})}{(2.866 \times 10^{-8}\text{cm})^3 \times 6.02 \times 10^{23}(\text{atoms/mol})}$$

$$7.87 = \frac{x \times 55.847}{(2.866 \times 10^{-8})^3 \times 6.02 \times 10^{23}}$$

$$x = 1.997 \text{ atoms/cell}$$

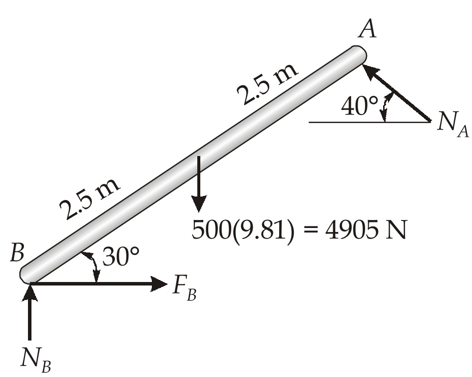
Since, the iron is BCC, two iron atoms are present in each unit cell.

Therefore, vacancies per unit cell that would be present for the required density of 7.87 g/cm^3 = 2 – 1.997 = 3×10^{-3}.

$$\text{The number of vacancies/cm}^3 = \frac{\text{Vacancies/cell}}{(a)^3} = \frac{3 \times 10^{-3}}{(2.866 \times 10^{-8})^3} = 1.274 \times 10^{20}$$

1. (d) Solution:

From FBD of *AB*,



$$\Sigma M_B = 0$$

$$N_A \sin 40°(5\cos 30°) + N_A \cos 40°(5\sin 30°) - 4905(2.5\cos 30°) = 0$$

$$N_A = 2260.24 \text{ N}$$

$$\Sigma F_x = 0$$

$$F_B - N_A \cos 40° = 0$$

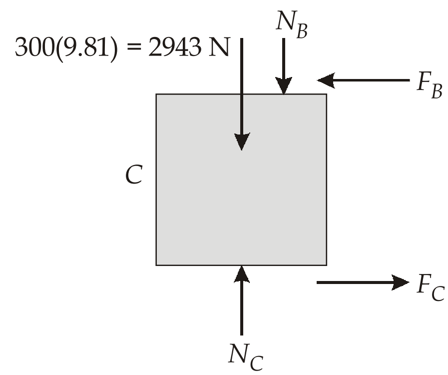
$$F_B = 2260.24\cos 40° = 1731.44 \text{ N}$$

$$\Sigma F_y = 0$$

$$N_B + N_A \sin 40° - 4905 = 0$$

$$N_B = -2260.24\sin 40° + 4905$$
$$= 3452.15 \text{ N}$$

From FBD of block C,



$$\Sigma F_y = 0$$
$$N_C - N_B - 2943 = 0$$
$$N_C = 3452.15 + 2943$$
$$= 6395.15 \text{ N}$$
$$\Sigma F_x = 0$$
$$F_C - F_B = 0$$
$$F_C = F_B = 1731.44 \text{ N}$$

We must compare each of the friction forces against its maximum static value.

$$(F_B)_{max} = 0.4N_B = 0.4(3452.15) = 1380.86 \text{ N} < F_B = 1731.44 \text{ N}$$
$$(F_C)_{max} = 0.4N_C = 0.4(6395.15) = 2558.06 \text{ N} > F_C = 1731.44 \text{ N}$$

We conclude that the system cannot be in equilibrium. Although there is sufficient friction beneath B, the friction force under C exceeds its limiting value.

1. (e) Solution:

Fibre-reinforced Plastic : Fibre-reinforced plastic is a composite material made of a polymer matrix reinforced with fibres. The fibers are usually glass, carbon, basalt or aramid. Other fibres such as paper or wood or asbestos is also sometimes used. It is a category of composite plastics which specifically use fibre materials to mechanically enhance the strength and elasticity of plastics. The extent that strength and elasticity are enhanced in a fibre reinforced plastic depends on the mechanical properties of both the fibre and matrix, their volume relative to one another and the fibre length and its orientation within the matrix.

Rule of mixture is a weighted mean used to predict various properties of a composite material made up of continuous and unidirectional fibres. It provides a theoretical upper and lower bound on properties such as the elastic modulus, mass density, UTS, thermal conductivity and electrical conductivity.

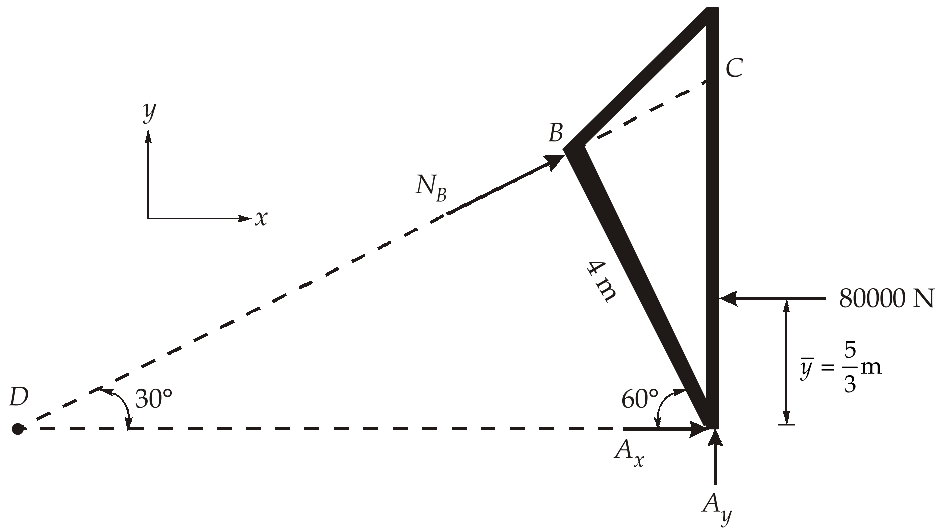
$$E_C = fE_f + (1-f)E_m$$
$$f = \frac{V_f}{V_f + V_m}$$

E_f = material property of fibres; E_m = material property of matrix

Two applications : Marine industries, construction industries.

2. (a) Solution:

The FBD of the barrier is shown in figure, where N_B is the reaction at B, acting perpendicular to the inclined surface, and A_x and A_y are the components of the pin reaction at A.



The resultant of a distributed load is equal to the area under the loading diagram, acting at the centroid of that area. Therefore, we obtain

$$R = \frac{1}{2}(5)(32000) = 80000 \text{ N}$$

and we know, $$\bar{y} = \frac{5}{3}\text{m}$$

Taking moment about A,

Because the unknown forces A_x and A_y intersect at A, a convenient starting point is

$$\Sigma M_A = 0$$

$$80000\left(\frac{5}{3}\right) - N_B(4) = 0$$

$$N_B = 33333.3 \text{ N}$$

For determining A_x, we use

$$\Sigma F_x = 0$$

$$N_B \cos 30° + A_x - 80000 = 0$$

$$A_x = 80000 - (33333.3)\cos 30° = 51132.5 \text{ N}$$

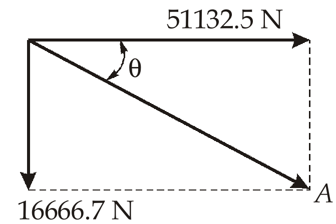
For determining A_y, we use

$$\Sigma F_y = 0$$

$$A_y + N_B \sin 30° = 0$$

$$A_y = -(33333.3)\sin 30° = -16666.7 \text{ N}$$

The signs indicate that N_B and A_x are directed as shown on the FBD, whereas the correct direction of A_y is opposite the direction shown on the FBD. Therefore, the force that acts on the barrier at A is

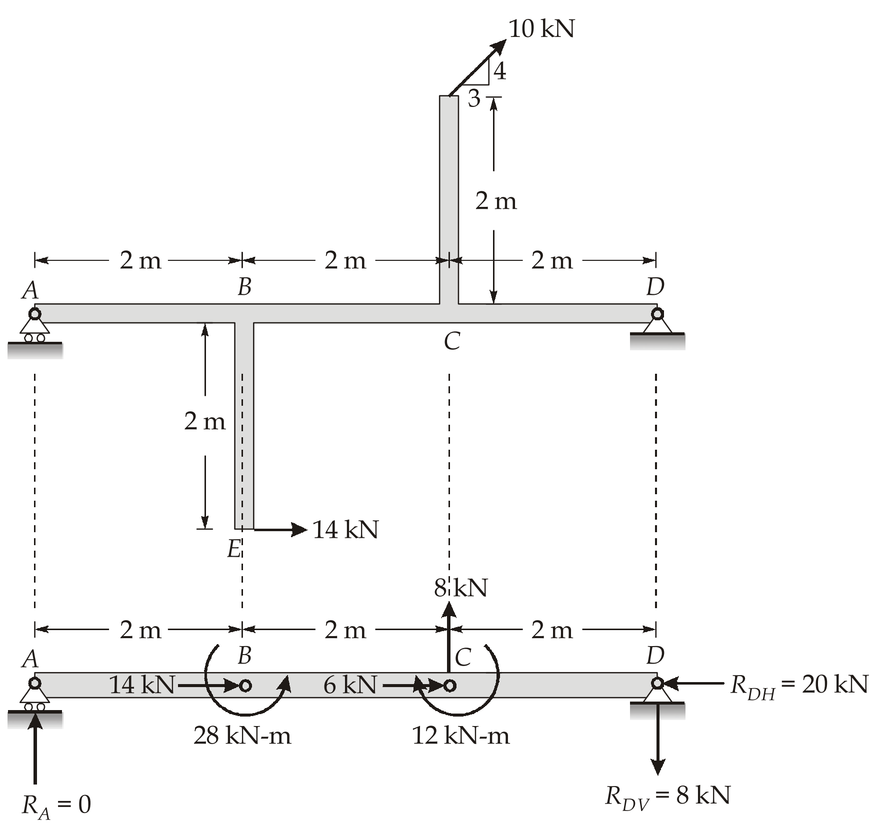


$$|A| = \sqrt{(51132.5)^2 + (16666.7)^2} = 53780 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{16666.7}{51132.5}\right) = 18.1°$$

and the force at B is 33333.3 N.

2. (b) Solution:



$$F_{BH} = 14 \text{ kN to the right}$$

Moment at B,

$$M_B = 14(2) = 28 \text{ kN.m counterclockwise}$$

$$F_{CH} = \frac{3}{8}(10) = 6 \text{ kN to the right}$$

$$F_{CV} = \frac{4}{5}(10) = 8 \text{ kN upward}$$

Moment at C,

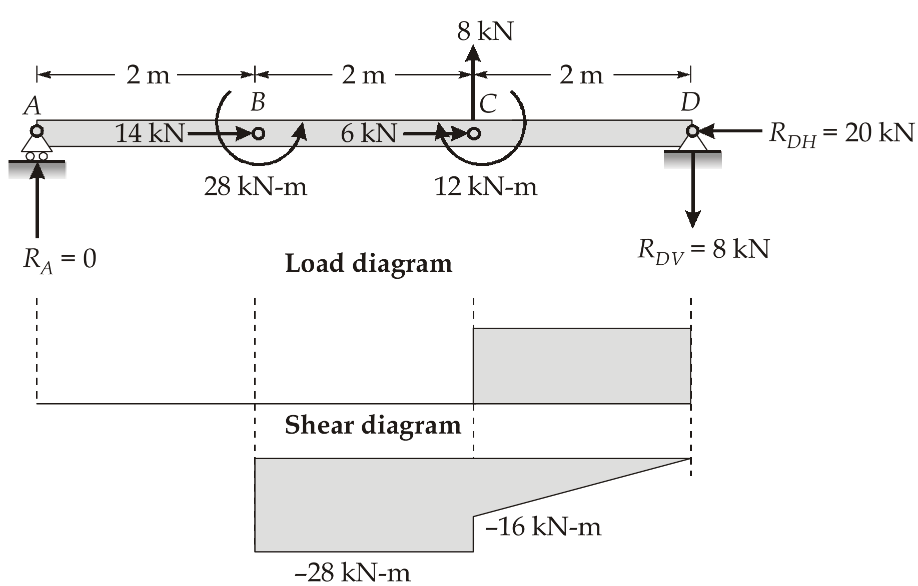
$$M_C = F_{CH}(2) = 6(2) = 12 \text{ kN.m clockwise}$$

Taking moment at D,

$$M_D = 0$$
$$6R_A + 12 + 8(2) = 28$$
$$R_A = 0$$
$$\Sigma M_A = 0$$
$$6R_{DV} + 12 = 28 + 8(4)$$
$$R_{DV} = 8 \text{ kN}$$
$$\Sigma F_H = 0$$
$$R_{DH} = 14 + 6 = 20 \text{ kN}$$

To draw the shear diagram

1. Shear in segments AB and BC is zero.
2. $V_C = 8$
3. $V_D = V_C$ + Area in load diagram
   $V_D = 8 + 0 = 8$ kN
   $V_{D2} = V_D - R_{DV} = 8 - 8 = 0$ kN



Moment diagram

To draw the Moment diagram

1. Shear in segments AB is zero.
2. $M_B = -28$ kNm

3. $M_C = M_B$ + Area in shear diagram

$M_C = -28 + 0 = -28$ kNm

$M_{C2} = M_C + 12 = -28 + 12$

$M_{C2} = -16$ kNm

4. $M_D = M_{C2}$ + Area in shear diagram

$M_D = -16 + 8(2)$

$M_D = 0$

2. (c) Solution :

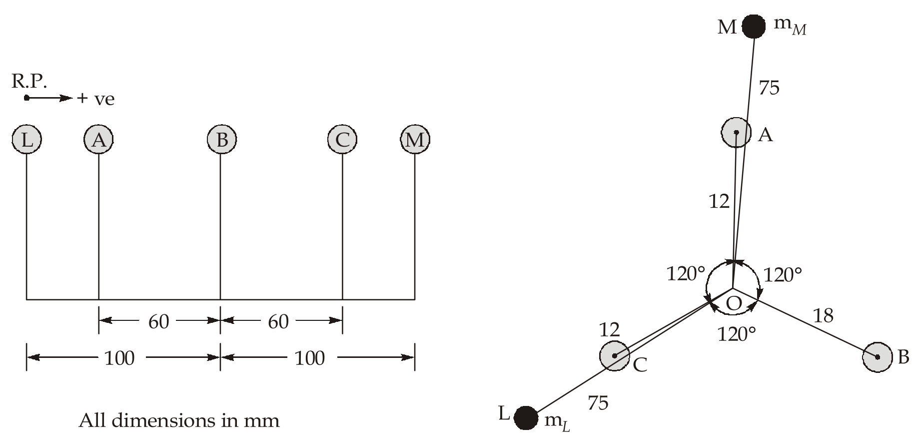
Given: D = 75 mm = 0.075 m; t = 25 mm = 0.025 m; r_A = 12 mm = 0.012 m; r_B = 18 mm = 0.018 m ;r_C = 12 mm = 0.012 mm; ρ = 7000 kg/m^3; N = 600 rpm

or

$$\omega = 2\pi \times \frac{600}{60} = 20\pi \text{ rad/s}$$

$$m = \pi r^2 t \times 7000$$

$$= \frac{\pi}{4} \times (0.075)^2 \times (0.025) \times 7000 = 0.773 \text{ kg}$$



All dimensions in mm

(a) Position of planes

(b) Angular position of masses

Plane	Mass (m) kg	Radius (r) m	(m.r.) kgm	(l)m	(m.r.l.)	θ
L (R.P.)	m_L	0.075	0.075 m_L	0	0	θ_L
A	0.773	0.012	9.276×10^{-3}	0.04	0.371×10^{-3}	0
B	0.773	0.018	13.91×10^{-3}	0.1	1.391×10^{-3}	120°
C	0.773	0.012	9.276×10^{-3}	0.16	1.484×10^{-3}	240°
M	m_M	0.075	0.075 m_M	0.2	15×10^{-3} m_M	θ_M

$$\Sigma(mrl)_x = \Sigma(mrl)\cos\theta$$

$$= -1.0665 \times 10^{-3}$$

$$\Sigma(mrl)_y = -0.0805 \times 10^{-3}$$

$$R_{mrl} = 1.0695 \times 10^{-3} = m_M \times 15 \times 10^{-3}$$

$\Rightarrow$ $$m_M = 0.0713 \text{ kg}$$

$$\theta_{mrl} = \tan^{-1}\left(\frac{0.0805}{-1.0665}\right) = 4.3° \quad \text{(3rd quadrant)}$$

$$\theta_M = 4.3° \text{ (1st quadrant) CCW from A}$$

$$\Sigma(mr)_x = \Sigma(mr)\cos\theta \qquad (m_m \times 0.075 = 5.348 \times 10^{-3})$$

$$= 3.016 \times 10^{-3}$$

$$\Sigma(mr)_y = \Sigma(mr)\sin\theta$$

$$= 4.414 \times 10^{-3}$$

$$R(mr) = 5.346 \times 10^{-3} = \text{mL} \times 0.075$$

$\Rightarrow$ $$m_L = 0.0713 \text{ kg}$$

$$\theta(mr) = \tan^{-1}\left(\frac{4.414}{3.016}\right) = 55.65°$$

$$\theta_{mL} = 180 + 55.65 = 235.65° \text{ CCW from A}$$

$$= 124.34° \text{ CW from A}$$

So, $$m_L = m_M = 0.0713 \text{ kg}$$

$$\theta_{mL} = 124.39° \text{ CW from A,}$$

$$\theta_{mM} = 4.3° \text{ CCW from A}$$

3. (a)(i) Solution:

1. Sensitiveness of a governor

A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of sensitivity.

As a governor is used to limit the change of speed of the engine between minimum to full-load conditions, the sensitiveness of a governor is also defined as the ratio of the difference between the maximum and the minimum speeds (range of speed) to the mean equilibrium speed. Thus,

$$\text{Sensitiveness} = \frac{\text{Range of speed}}{\text{Mean speed}} = \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

When N = Mean speed; N_1 = Minimum speed corresponding to full-load conditions;
N_2 = Maximum speed corresponding to no-load conditions

2. Hunting of a governor

Sensitiveness of a governor is a desirable quality. However, if a governor is too sensitive, it may fluctuate continuously, because when the load on the engine falls, the sleeve rises rapidly to a maximum position. This shuts off the fuel supply to the extent to affect a sudden fall in the speed. As the speed falls to below the mean value, the sleeve again moves rapidly and falls to a minimum position to increase the fuel supply. The speed subsequently rises and becomes more than the average with the result that the sleeve again rises to reduce the fuel supply. This process continues and is known as hunting.

3. (a) (ii) Solution:

For spring controlled governor.

We know that, $F = br + C$ ... (i)

When, $r = 0.075$ mm and $F = 22.5$ N

Then, $22.5 = 0.075 \times b + C$ ... (ii)

When, $r = 0.150$ mm and $F = 60$ N

$60 = 0.15 \times b + C$ ... (iii)

On solving equation (ii) and (iii), we get

$$b = 500 \text{ and } C = -15$$

∴ Controlling force equation is given by

$$F = 500r - 15$$

For $r = 0.1$ m,

Controlling force, $F = 500 \times 0.1 - 15 = 35$ N

Also, $F = mr\omega^2$

$$\therefore \quad 35 = \frac{8}{9.81} \times \omega^2 \times 0.1$$

$$\omega = 20.72 \text{ rad/s}$$

or, $$\omega = \frac{2\pi N}{60}$$

$$N = \frac{20.72 \times 60}{2\pi} = 197.83 \text{ rpm}$$

For isochronous governor, $F = b \times r$ when $C = 0$

$$\frac{F}{r} = b = 500$$

∴ To make the governor isochronous, the ratio of $\frac{F}{r}$ should be made constant.

$$F = mr\omega^2 = br + C$$

$$\frac{8}{9.81} \times \omega^2 \times 0.1 = 500 \times 0.1 + 0$$

$$\omega = 24.76 \text{ rad/s}$$

$$N = \frac{\omega \times 60}{2\pi} = \frac{24.76 \times 60}{2 \times \pi} = 236.44 \text{ rpm}$$

3. (b) Solution:

The normal stress due to bending in the top fiber at point A is due to the moment

$$M = 600 \times 2 = 1200 \text{ Nm}$$

There is also a shearing stress in the outer fiber at point A due to the torque

$$T = 600 \times 1 = 600 \text{ Nm}$$

In addition, the tensile force of 10 kN produces a normal stress at point A. The stresses are

$$\text{Shear stress, } \tau = \frac{TR}{J} = \frac{600 \times 0.03}{\pi \times \frac{0.06^4}{32}} = 14.15 \times 10^6 \text{ Pa} = 14.15 \text{ MPa}$$

$$\text{Bending stress, } \sigma_1 = \frac{My}{I} = \frac{1200 \times 0.03}{\pi \times \frac{0.06^4}{64}} = 56.6 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

$$\text{Normal stress, } \sigma_2 = \frac{P}{A} = \frac{10000}{\pi \times \frac{0.06^2}{4}} = 3.54 \times 10^6 \text{ Pa} = 3.54 \text{ MPa}$$

$$\sigma = (\sigma_1 + \sigma_2) = 60.14 \text{ MPa}$$

The vertical shearing stress $\frac{FA\bar{y}}{Ib}$ is zero in the top fiber. The maximum normal and shearing stresses at point A is given as

$$\sigma_{max} = \frac{60.14}{2} + \sqrt{(30.07)^2 + (14.15)^2} = 63.3 \text{ MPa}$$

$$\tau_{max} = \sqrt{(30.07)^2 + (14.15)^2} = 33.23 \text{ MPa}$$

At point B the normal stress due to bending is zero and σ_2 is the same as at point A. So, the normal stress of 3.54 MPa replaces 60.1 MPa on the element. The normal and shearing stresses at point B are then calculated to be (the $\frac{FA\bar{y}}{Ib}$ shear will add on one side of the shaft or the other)

$$\text{Shear stress due to torsion, } \tau_1 = \frac{TR}{J} = 14.15 \times 10^6 \text{ Pa or } 14.15 \text{ MPa}$$

Shear stress due to shear force,

$$\tau_2 = \frac{FA\bar{y}}{Ib} = \frac{600 \times \pi \times (0.03)^2 \times \left(4 \times \frac{0.03}{3\pi}\right)}{\left(\pi \times \frac{0.06^4}{64}\right) \times 0.06} = 0.566 \times 10^6 \text{ Pa}$$

$$\tau = \tau_1 + \tau_2 = 14.7 \text{ MPa}$$

$$\text{Normal stress, } \sigma_2 = \frac{P}{A} = 3.54 \times 10^6 \text{ Pa or } 3.54 \text{ MPa}$$

The maximum normal and shearing stresses at point *B* are found to be

$$\sigma_{max} = \frac{3.54}{2} + \sqrt{(1.77)^2 + (14.7)^2} = 16.6 \text{ MPa}$$

$$\tau_{max} = \sqrt{(1.77)^2 + (14.7)^2} = 14.8 \text{ MPa}$$

At point *C* is the normal stress due to bending is –56.6 MPa and t and σ_2 are the same as at point *A*. So, the normal stress is –56.6 + 3.54 = –53.06 MPa.

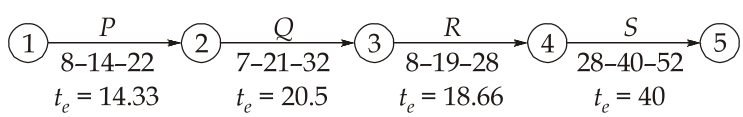
The normal stress with maximum magnitude and the maximum shear stresses at point *C* are

$$\sigma_{max} = \frac{-53.06}{2} \pm \sqrt{(26.53)^2 + (14.15)^2} = -56.6 \text{ MPa}$$

$$\tau_{max} = \sqrt{(26.53)^2 + (14.15)^2} = 30.0 \text{ MPa}$$

Obviously, the normal and shearing stresses are maximum in the shaft at point *A*.

3. (c) (i) Solution:



Using equations,

$$\text{Expected mean time } (t_e) = \frac{t_p + 4t_m + t_o}{6}$$

$$\text{Standard deviation, } (\sigma_o) = \left(\frac{t_p - t_o}{6}\right)$$

$$\text{Variance, } (\sigma_o)^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Activity	Optimum Time	Most Likely Time	Pessimistic Time	Expected Time	Standard Deviation	Variance	Remarks
P	8	14	22	14.33	2.33	5.44	Most Reliable
Q	7	21	32	20.5	4.16	17.36	–
R	8	19	28	18.66	3.33	11.11	–
S	28	40	52	40.00	4	16	–

Expected time of the project = T_E

14.33 + 20.5 + 18.66 + 40 = 93.49

Variance of the project, (σ_o^2) = 5.44 + 17.36 + 11.11 + 16 = 49.91

Standard deviation of the project (σ_o) = $\sqrt{49.91} = 7.065$

z(Normal deviate) or probability factor for 95% probability,

$$z = +1.647,$$

$$z = \frac{x - T_E}{\sigma_o}$$

$$\therefore \quad x = z\sigma_o + T_E = 1.647 \times 7.065 + 93.49$$

$$= 105.126 \text{ days}$$

z for 5% probability = –1.647

$$z = T_E - z\sigma$$

$$= 93.49 - 1.647 \times 7.065 = 81.86 \text{ days}$$

Variance of activity P is the least, therefore, P has the most reliable time estimates mentioned in the above table.

3. (c) (ii) Solution:

Optimal sequence,

4	1	3	2	5	6

Job	M-I			M-II		
	Time In		Time Out	Time In		Time Out
4	0	2	2	2	6	8
1	2	3	5	8	8	16
3	5	5	10	16	9	25
2	10	12	22	25	10	35
5	22	9	31	35	3	38
6	31	11	42	42	1	43
		42			37	

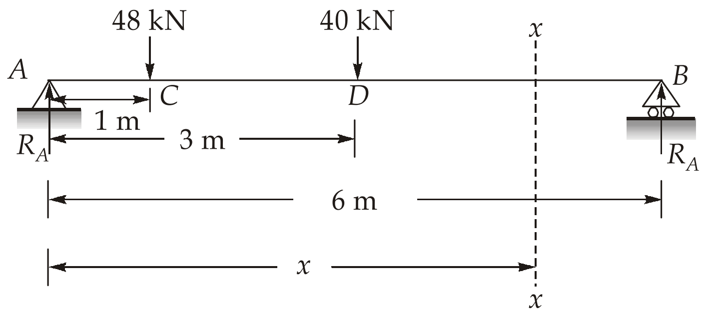
Total elapsed time = 43 hours

Idle time for M-I = 43 – 42 = 1 hour

Idle time for M-II = 43 – 37 = 6 hours

4. (a) Solution:

Given: $I = 90 \times 10^6$ mm^4, $E = 2 \times 10^5$ N/mm^2



$\Sigma F_V = 0$

$$R_A + R_B - 48 - 40 = 0$$
$$R_A + R_B = 88$$

Taking moment about A, we get

$$48 \times 1 + 40 \times 3 = R_B \times 6$$
$$R_B = \frac{48 + 120}{6} = \frac{168}{6} = 28 \text{ kN}$$

$\therefore$ $$R_A = 88 - 28 = 60 \text{ kN}$$

Consider a section x-x at a distance x from A.

BM at this section is given by.

$$EI\frac{d^2y}{dx^2} = R_A \times x - 48(x-1) - 40(x-3)$$
$$= 60x - 48(x-1) - 40(x-3)$$

Integrating the above equation, we get

$$EI\frac{dy}{dx} = \frac{60x^2}{2} - 48\frac{(x-1)^2}{2} - 40\frac{(x-3)^2}{2} + C_1$$
$$= 30x^2 - 24(x-1)^2 - 20(x-3)^2 + C_1 \quad \text{... (i)}$$

Again integrating the above equation.

$$EIy = \frac{30x^3}{3} - 24\frac{(x-1)^3}{3} - 20\frac{(x-3)^3}{3} + C_1x + C_2$$
$$EIy = 10x^3 - 8(x-1)^3 - \frac{20}{3}(x-3)^3 + C_1x + C_2 \quad \text{... (ii)}$$

The boundary conditions are:

1. at $x = 0$, $y = 0$
2. at $x = 6$, $y = 0$

On putting $x = 0$ and $y = 0$ in equation (ii)

$$0 = 0 + C_1 \times 0 + C_2$$

$\therefore$ $$C_2 = 0$$

Again putting $x = 6$, $y = 0$ and $C_2 = 0$ in equation (ii)

$$0 = 10 \times (6)^3 - 8(6-1)^3 - \frac{20}{3}(6-3)^3 + C_1 \times 6 + 0$$

$$C_1 = -\frac{980}{6} = -163.33$$

Hence, $$EIy = 10x^3 - 8(x-1)^3 - \frac{20}{3}(x-3)^3 - 163.33x \quad \text{... (iii)}$$

(i) Deflection under first load (i.e. at $x = 1$ m) y_c:

$$EIy_C = 10 \times (1)^3 - 163.33 \times 1 = -153.33 \text{ kNm}^3$$
$$= -153.33 \times 10^{12} \text{ Nmm}^3$$

$$y_C = -\frac{153.33 \times 10^{12}}{2 \times 10^5 \times 90 \times 10^6} = -8.52 \text{ (mm)}$$

Deflection under second load (i.e. at $x = 3$ m) y_D:

$$EIy_D = 10 \times (3)^3 - 8 \times (3-1)^3 - 163.33 \times 3$$
$$= -283.99 \text{ kNm}^3 = -283.99 \times 10^{12} \text{ Nmm}^3$$

$$y_D = -\frac{283.99 \times 10^{12}}{2 \times 10^5 \times 90 \times 10^6} = -15.78 \text{ mm}$$

(ii) For maximum deflection (y_{max}), slope is zero i.e., $\frac{dy}{dx} = 0$

Hence, from equation (i)

$$0 = 30x^2 - 24(x-1)^2 + C_1$$
$$0 = 30x^2 - 24(x^2 + 1 - 2x) - 163.33$$
$$0 = 30x^2 - 24x^2 - 24 + 48x - 163.33$$
$$0 = 6x^2 + 48x - 187.33$$
$$x = 2.87, -10.87 \text{ (Not possible)}$$

$\therefore$ $$x = 2.87 \text{ m}$$

(iii) On putting $x = 2.87$ m in equation (iii)

$$EIy_{max} = 10 \times (2.87)^3 - 8 \times (2.87 - 1)^3 - 163.33 \times 2.87$$
$$= -284.67 \text{ kNm}^3$$
$$EIy_{max} = -284.67 \times 10^{12} \text{ Nmm}^3$$

$$y_{max} = -\frac{284.67 \times 10^{12}}{2 \times 10^5 \times 90 \times 10^6} = -15.82 \text{ mm}$$

4. (b)Solution:

Given: $m = 5$ kg, $k = 1960$ N/m, $\dot{x} = 0.5$ m/s.

Viscous resistance, $F_r = 0.98$ N

(i)Natural frequency of the system,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1960}{5}} = 19.79 \text{ rad/s}$$

(ii) As we know,
$$F_r = C\dot{x}$$
$$0.98 = C \times 0.5$$
$$C = \frac{0.98}{0.5} = 1.96 \text{ N.s/m}$$
Also,
$$C_c = 2m\omega_n = 2 \times 5 \times (19.79)$$
$$= 197.9 \text{ Ns/m}$$
$$\text{Damping ratio } (\xi) = \frac{C}{C_c} = \frac{1.96}{197.9} = 9.9 \times 10^{-3}$$
(iii) The response of the system is given by.
$$x(t) = A \times e^{-\xi\omega_n t} \times \sin(\omega_d t + \phi) \quad \text{... (i)}$$
On differentiating equation (i)
$$\dot{x}(t) = A \times e^{-\xi\omega_n t}\omega_d \times \cos(\omega_d t + \phi) + A\sin(\omega_d t + \phi)(-\xi\omega_n)e^{-\xi\omega_n t} \quad \text{... (ii)}$$
Initial condition at
$$t = 0, \quad x = 20 \text{ mm}$$
$$t = 0, \quad \dot{x} = 0$$
On putting $t = 0$ and $x = 20$ in equation (i)
$$20 = Ae^{-\xi\omega_n \times 0}\sin(\omega_d \times 0 + \phi)$$
$$20 = A \sin\phi \quad \text{... (iii)}$$
Again putting $t = 0$ and $\dot{x} = 0$ in equation (ii)
$$0 = A \cos\phi\, \omega_d + A \sin\phi\, (-\xi\omega_n)$$
$$0 = A\cos\phi\, \omega_n\sqrt{1-\xi^2} + A\sin\phi(-\xi\omega_n)$$
$$0 = A\,\omega_n\left[\sqrt{1-\xi^2}\cos\phi - \xi\sin\phi\right]$$
$$\xi \sin\phi = \sqrt{1-\xi^2}\cos\phi$$
$$\tan\phi = \frac{\sqrt{1-\xi^2}}{\xi} = \frac{\sqrt{1-(9.9\times10^{-3})^2}}{9.9\times10^{-3}} = 101$$
$$\phi = 89.43°$$
On putting $\phi = 89.43°$ in equation (iii)
$$20 = A \sin\phi$$
$$20 = A \sin 89.43°$$
$$A = \frac{20}{\sin(89.43°)} = 20$$
Hence, the response of the system is,
$$x(t) =$$
$$20e^{-(9.9\times10^{-3}\times19.79)t} \times \sin\left(19.79 \times \sqrt{1-(9.9\times10^{-3})^2}\,t + 89.43°\right)$$

$$= 20e^{-0.1959t} \sin(19.789t + 89.43°)$$

Displacement of mass when $t = 0.5$ sec.

$$x(0.5) = 20 \times e^{-0.1959\times0.5} \sin(19.789 \times 0.5 + 89.43°)$$
$$= -16.25 \text{ mm}$$

4. (c) (i) Solution:

Brass: Brass is an alloy of copper and zinc. It generally contains 60 to 90% copper and 10 to 40% zinc. Brass is the most widely used copper alloy. It can be rolled into sheets, drawn into wires, cast into moulds and can be easily fabricated by processes like spinning and Y-chromium. According to the percentage of copper and zinc, brass is of following types:

1. **Low Brass:** It contains 80% copper and 20% zinc and is used in forming and musical equipments.
2. **Red Brass:** It contains 85% copper and 15% zinc and is used in plumbing pipe, electrical sockets, radiator core, etc.
3. **White Brass:** It contains 10% copper and 90% zinc and generally used in ornaments.
4. **Yellow Brass:** It contains 60% copper and 40% zinc and is used in rivets, tubes, valve stems, etc.
5. **Cartridge Brass:** It contains 70% copper and 30% zinc and is used in cartridges, tubes, springs, etc.

Bronze: Bronze is an alloy of copper and tin. It generally contains 75 to 95% copper and 5 to 25% tin. It has better mechanical and corrosion properties as compared to brass. It can be rolled into sheets, wire and rods. It is used in making utensils, pump linings, bushes, bearings and hydraulic fittings. Main types of Bronze are as follows:

1. **Phosphor Bronze:** It contains 9% to 10% tin, 0.1% to 0.3% phosphorus and remaining is copper. It has high strength and toughness, good bearing capacity, high elasticity, and resistance to corrosion. It is used in bearings, gears, pumps parts, springs, etc.
2. **Aluminium Bronze:** It does not contain tin and is an alloy of 90% copper and 10% aluminium. It has good strength and high corrosion resistance. It is used in condensers, heat exchangers, gears, cams and rollers, etc.
3. **Beryllium Bronze:** It contains 98% copper and 2% beryllium. It has high yield point and fatigue limit. It is used in springs, cams and bearings.
4. **Silicon Bronze:** It contains 96% copper, 3% silicon, 0.5% manganese, 0.5% iron. It has high strength and toughness. These are used in boilers, tubes, pumps, etc.

5. **Manganese Bronze:** It contains 55% copper, 38% zinc, 5% manganese and 2% tin. It is used in bushes, rods, ship propellers, etc.
6. **Gun Metal:** It contains 88% copper, 10% tin and 2% zinc. It is used in nut, bolts, bearings, casting, guns, etc.
7. **Bell Metal:** It contains 80% copper and 20% tin. It is used in casting of bells.

4. (c) (ii) Solution:

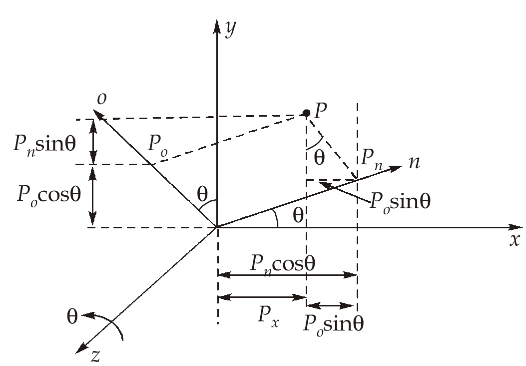
$$C_0 = 0.40 \qquad C_\alpha = 0.13$$
$$W_\alpha = 0.66 \qquad C_\beta = ?$$
$$W_\beta = 0.34$$
$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha}; \qquad 0.66 = \frac{C_\beta - 0.40}{C_\beta - C_\alpha}$$
$$W_\beta = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha}; \qquad 0.34 = \frac{0.40 - 0.13}{C_\beta - C_\alpha}$$
$$\frac{0.66}{0.34} = \frac{C_\beta - 0.40}{0.40 - 0.13}; \qquad 0.1782 = 0.34(C_\beta - 0.40)$$
$$C_\beta - 0.40 = 0.524$$

Solving for C_β;
$$C_\beta = 0.924$$

The composition of β-phase in 92 wt % *B* and 8 wt % *A*.

Section : B

5. (a) Solution:



So,
$$P_x = P_n\cos\theta - P_o\sin\theta$$

Similarly
$$P_y = P_n\sin\theta + P_o\cos\theta$$
$$P_z = P_a$$

In matrix form;
$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}$$

If $\theta = 60°$ and $P = [2, 3, 5]$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} -1.598 \\ 3.232 \\ 5 \end{bmatrix}$$

5. (b) Solution:

Given, $P = 12$ kW, $m_1 = 20$ kg, $R_1 = 80$ mm, $m_2 = 35$ kg, $R_2 = 120$ mm, $N_1 = 2000$ rpm

$$I_1 = m_1 R_1^2 = 20 \times 0.08^2 = 0.128 \text{ kg-m}^2$$

$$I_2 = m_2 R_2^2 = 35 \times 0.120^2 = 0.504 \text{ kg-m}^2$$

$$\omega_1 = \frac{2\pi N_1}{60} = 209.44 \text{ rad/s}$$

$$\text{Torque, } T = \frac{P \times 60}{2\pi N} = \frac{12 \times 10^3 \times 60}{2\pi \times 2000} = 57.295 \text{ Nm}$$

(i) Time required to bring the output shaft to the rated speed from rest

$$t = \frac{(\omega_1 - \omega_2) I_1 I_2}{(I_1 + I_2) T} \quad \text{(where, } \omega_2 = 0\text{)}$$

$$t = \frac{(209.44 - 0) \times 0.128 \times 0.504}{(0.128 + 0.504) \times 57.295}$$

$$t = 0.373 \text{ s}$$

(ii) Heat generated during clutching:

$$E = \frac{1}{2} \frac{(\omega_1 - \omega_2)^2 I_1 I_2}{(I_1 + I_2)}$$

$$= \frac{1}{2} \frac{(209.44 - 0)^2 \times 0.128 \times 0.504}{(0.128 + 0.504)}$$

$$E = 2238.786 \text{ J}$$

5. (c) Solution:

Sound Fields

The sound waves that emanate from a machinery source travel in all directions. However, depending on the interaction at the boundaries, they are absorbed, or reflected, or even transmitted further, or a combination of these phenomena occurs depending on the nature of the boundary. This is due to the impedance of the boundary. The sound waves are thus amplified or attenuated depending upon the boundary, and these conditions lead to different sound fields.

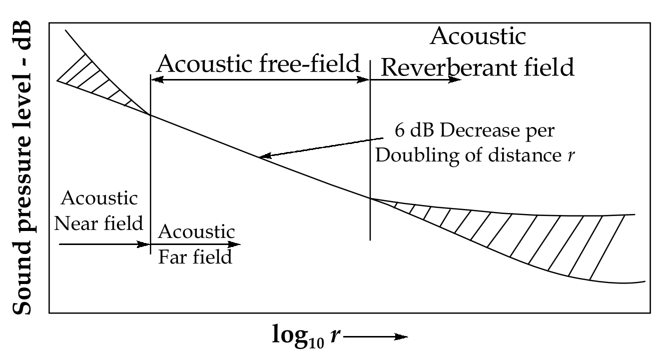
Near-Field Condition

The region very close to the source of sound has a sound level that is almost constant and equal to the maximum level of the generated sound. Thus, while monitoring the noise of a machine, measurements in this region should be avoided. The approximate distance from the machine where such a near-field condition exists is equal to one length of the largest dimension of the machine.

Far-Field Condition

Any region beyond the near field of the machinery is known as its far field. The far field can be further divided into two distinct regions-the free field region and the reverberant field region. In the free-field region it is assumed that there are no strong reflected waves, and for a point spherical source, the sound pressure level reduces by 6 dB for every doubling of the distance from the source. While performing noise monitoring of machines, it is preferred that the noise is measured in the free-field conditions where there are no reflections from the boundary.

The second region in the far field close to the boundary is known as the reverberant field where, because of strong reflections from hard walls, the sound level increases. On a shop floor where machinery noise is to be monitored, measurements away from hard reflecting walls should be done. The free condition of decrease of 6 dB for every doubling of the distance for a three-dimensional point source is violated. Figure below shows the variation of the noise field from a distance of r from the source.



Variation of sound pressure level at different field conditions.

5. (d) Solution:

- Cutting velocity, $V = \frac{\pi D_{avg} N}{1000 \times 60} = \frac{\pi \times 12.445 \times 400}{1000 \times 60} = 0.26$ m/s

  Depth of cut, $d = \frac{12.70 - 12.19}{2} = 0.255$ mm

  Feed, $f = \frac{203.20}{400} = 0.508$ mm/rev

- Metal removal rate,MRR= Area of cut × V

  Now area of cut = $b.t. = d.f.$

  $\therefore$ MRR = $d.f.\ \pi.D_{ave.}N$

  $D_{ave.}$ = 12.445 m

  $\therefore$ MRR = 0.255 × 0.508 × π × 12.445 × 400

  = 2025.86 mm^3/min

- Machining time, $T = \frac{L}{fN}$, L is tool travel

  $= \frac{150}{0.508 \times 400} = 0.738$ min

- Power = 56 dfV, Watts

  d and f are in mm, V is in m/min

  $\therefore$ Power = 0.255 × 0.508 × 0.26 × 60 × 56 = 113.17 watts

- Now, $P = T.\omega$

  $\therefore$ Torque, $T = \frac{113.17 \times 60}{2\pi \times 400} = 2.7$ Nm

  $\therefore$ Cutting force, $F_c = \frac{2T}{D_{ave.}} = \frac{2 \times 2.7 \times 1000}{12.445} = 434.17$ N

5. (e) Solution:

Given, Cost of failure, F = 1500 + 120x

Cost of control, $C = \frac{3000}{x}$

where, x = Percent defective

$\therefore$ Quality cost, $T = F + C = 1500 + 120x + \frac{3000}{x}$

For quality cost to be minimum

$$\frac{dT}{dx} = 0$$

$$\Rightarrow \quad 120 + \left(\frac{-3000}{x^2}\right) = 0$$

$$\Rightarrow \quad x = 5$$

$$\frac{d^2T}{dx^2} = \frac{6000}{x^3} \text{ And } \left.\frac{d^2T}{dx^2}\right|_{x=5} > 0$$

$\Rightarrow$ At $x = 5$; T has a minima.

$\therefore$ Percent defective for minimum quality cost,

$$x = 5$$

and Minimum quality cost, $T = \left(1500 + 120 \times 5 + \frac{3000}{5}\right) = 2700$

6. (a) Solution:

Hydraulic actuators: An actuator wherein hydraulic energy is used to impart motion is called hydraulic actuators. All systems involving high loads are operated by hydraulic actuators in which oil pressure is applied on mechanical actuator to give an output in terms of rotary or linear motion. Basic example of hydraulic actuator is steering gear of ship in which hydraulic pressure is used to move the rudder actuator. It is based on principle of Pascal's law.

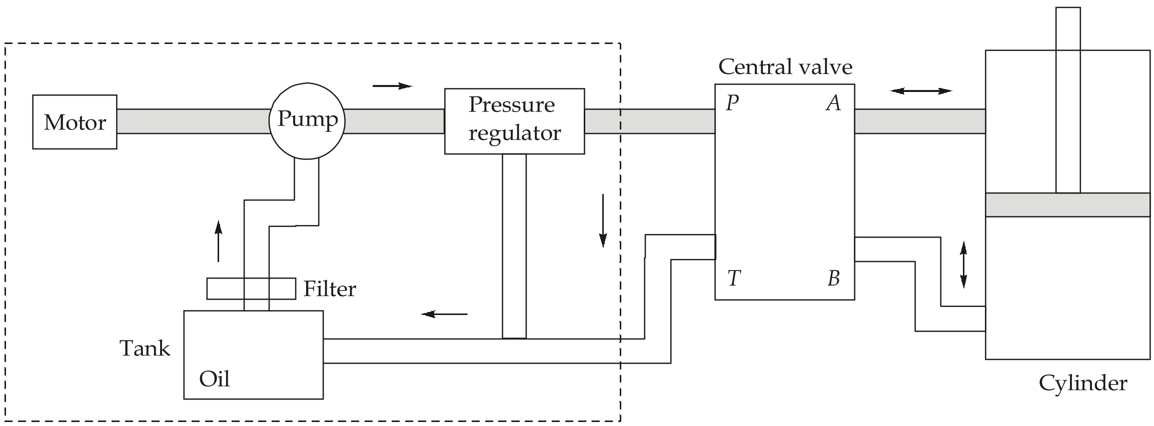
Advantages:

1. Easy to produce transmit, store, regulate and control, maintain and transform the hydraulic power.
2. Possible to generate high gain in force and power amplification.
3. Hydraulic systems are uniform and smooth, generate stepless motion and variable speed and force to a greater accuracy.
4. It is used where very high speed and large forces are required. It has higher load capacity.
5. Weight to power ratio of an hydraulic system is comparatively less than that for an electro mechanical system.

Disadvantages:

1. The manufacturing cost of the system is high since the hydraulic elements have to be machined to a high degree of precision.
2. Hydraulic elements have to be specially treated to protect them against rust, dirt, corrosion etc.

3. Petroleum based hydraulic oil may pose fire hazards thus limiting the upper level of working temperature.
4. Hydraulic power is not readily available compared to electric power.



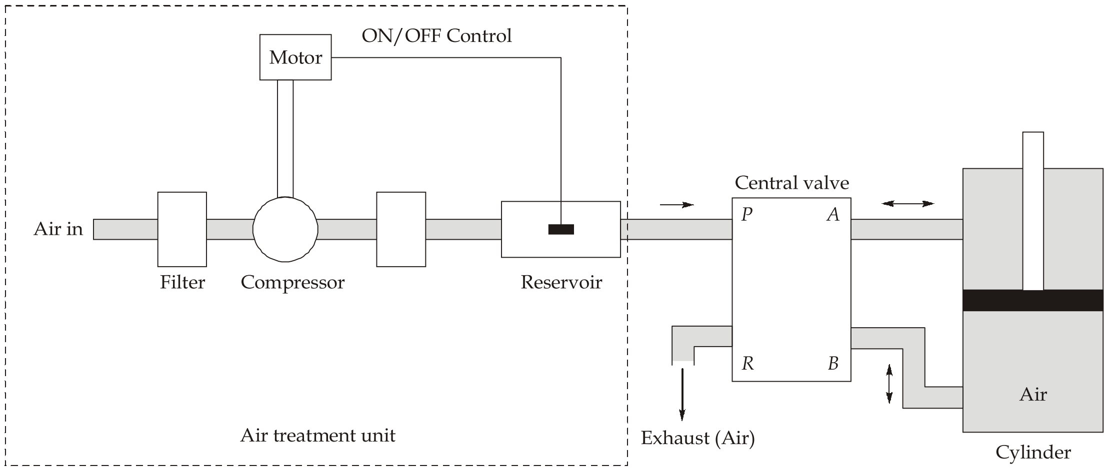
Block diagram of hydraulic system

Pneumatic actuators: It uses pressurized air to transmit and control power. In this type, compressed air at high pressure is used which converts this energy into either linear or rotary motion. Pneumatic actuators enable large forces to be produced from relatively small pressure changes. The most common example is "Main engine pneumatic Actuator" used for changing of roller position over cam shaft for reversing.

Advantages:
1. Pneumatic systems are fire and explosion proof whereas hydraulic system are not, unless non flammable liquid is used.
2. In this no return pipes are used when air is used.
3. Ecological purity.
4. Low cost
5. High speed operation
6. Pneumatic system are insensitive to temperature changes in contrast to hydraulic system in which fluid friction due to viscosity depends greatly on temperature.
7. Ease in reversal of movements.

Disadvantages:
1. The normal operating pressure of pneumatic system is lower than that of hydraulic system.
2. Accuracy of pneumatic actuators is poor at low velocities.
3. Output power is less compared to hydraulic system.
4. Difficulties in performance at slow speed.
5. In pneumatic systems, external leakage is permissible to a certain extent, but internal leakage must be avoided because the effective pressure difference is

rather small. In hydraulic system internal leakage is permissible to a certain extent, but external leakage must be avoided.



Block diagram of pneumatic system

6. (b) Solution:

Tensile load, $F_T = 180 \text{ kN} = 18 \times 10^3 \text{ N}$

Shear load, $F_s = 12 \text{ kN} = 12 \times 10^3 \text{ N}$

Yield stress, $\sigma_{ys} = 328.6 \text{ MPa}$, FOS = 2.5

$$\therefore \quad \text{Allowable stress, } \sigma_e = \frac{\sigma_{ys}}{FOS} = \frac{328.6}{2.5} = 131.44 \text{ MPa}$$

$$\text{Tensile stress, } \sigma_x = \frac{F_T}{A} = \frac{18 \times 10^3}{A}$$

$$\text{Shear stress, } \tau_{xy} = \frac{F_s}{A} = \frac{12 \times 10^3}{A} \qquad (\sigma_y = 0, \text{ not given})$$

(i) According to Rankine's theory of failure,

$$\sigma_e = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$131.44 = \frac{1}{2}\left[\frac{18 \times 10^3}{A} + \sqrt{\left(\frac{18 \times 10^3}{A}\right)^2 + 4\left(\frac{12 \times 10^3}{A}\right)^2}\right]$$

$$A = \quad 182.59 = \frac{\pi d_c^2}{4}$$

$$\therefore \quad \text{Core dia. } d_c = 15.25 \text{ mm}$$

(ii) According to maximum shear stress theory,

$$\sigma_e = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$131.44 = \sqrt{\left(\frac{18\times10^3}{A}\right)^2 + 4\left(\frac{12\times10^3}{A}\right)^2}$$

$\therefore$ $$A = 228.24 = \frac{\pi d_c^2}{4}$$

$\therefore$ Core diameter, $d_c = 17.05$ mm

(iii) According to Von-Mises theory of failure,

$$\sigma_e = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$131.44 = \sqrt{\left(\frac{18\times10^3}{A}\right)^2 + 3\left(\frac{12\times10^3}{A}\right)^2}$$

$\therefore$ $$A = 209.19 = \frac{\pi d_c^2}{4}$$

$\therefore$ Core diameter, $d_c = 16.32$ mm

(iv) According to Saint Venant's theory of failure,

$$\sigma_e = \frac{1}{2}\left[(1-\mu)(\sigma_x) + (1+\mu)\sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$131.44 = \frac{1}{2}\left[(1-0.298)\frac{18\times10^3}{A} + (1+0.298)\sqrt{\left(\frac{18\times10^3}{A}\right)^2 + 4\left(\frac{12\times10^3}{A}\right)^2}\right]$$

$$A = 196.196 = \frac{\pi d_c^2}{4}$$

$\therefore$ Core diameter, $d_c = 15.81$ mm

6. (c) Solution:

$\because$ $$\left(\frac{A}{V}\right)_{Casting} = \frac{2\times300\times200 + 25\times300\times2 + 25\times200\times2}{300\times200\times25}$$

$$= \frac{145000}{1500000} = 0.0967$$

Volume of riser, $V_r = 0.26 \times 1500000 = 390000 \text{ mm}^3$

For side placement of riser, $d = h$

$$V_r = \frac{\pi}{4}.d_r^3 = 390000$$

d_r = 79.2 mm which is far greater than the thickness of casting.

$$\left(\frac{A}{V}\right)_r = 0.07577 \ (< 0.0967)$$

Solidification time justify this placement for top placement of riser $h = d/2$

$$V_r = \frac{\pi}{4}.d^2.\frac{d}{2} = 390000$$

d_r = 99.77 mm (less than the X-section dimensions of casting)

$\left(\frac{A}{V_r}\right) = \frac{6}{d_r}$ = 0.06014 (< 0.0967) Time is also justifying this place. So put the riser in to this position.

Height of riser $$h = \frac{d}{2} = \frac{99.77}{2} = 49.885 \text{ mm}$$

Pipe height: We know that pipe cavity compensate the shrinkage allowance which is around 1/6th of riser volume, let the new modified height of riser is H.

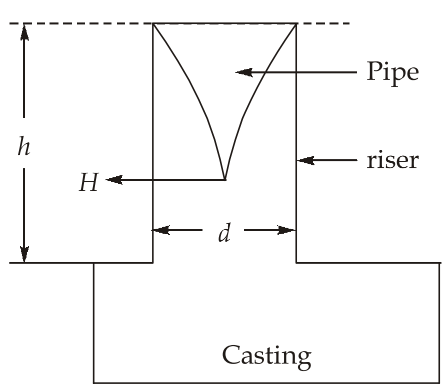
$$\therefore \quad V_s = \frac{7}{100} \times 300 \times 200 \times 25$$

$$V_s = 105000 \text{ mm}^3$$

$$\frac{1}{6}\frac{\pi}{4} d_r^2 \times H = 105000$$

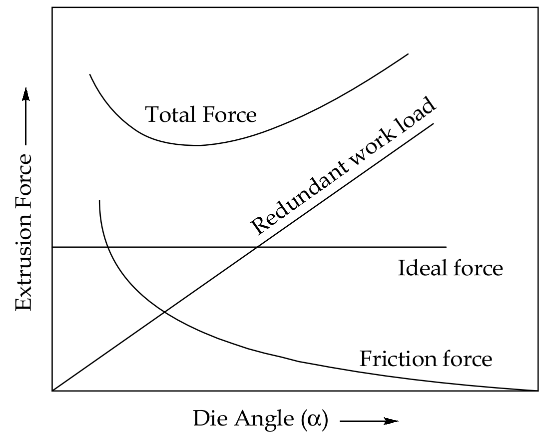
$$H = \frac{24 \times 105000}{\pi \times (99.77)^2} = 80.6 \text{ mm}$$

Hence the new height of riser should be more than 80.6 mm in order to avoid pipe extension in to the casting.



Q.7 (a) (i) Solution:

Extrusion Force Vs Die Angle (α) :-



Total extrusion force is the sum of ideal force, force due to friction and force due to redundant work. There is a specific angle at which this total extrusion force is minimum, referred as the optimum die angle.

7. (a) (ii) Solution:

Given : $d_o = 150$ mm, $d_f = 100$ mm

$K = 180$ MPa, $n = 0.2$ (strain hardening coefficient)

Average flow stress, $\overline{\sigma_f} = \dfrac{K\varepsilon^n}{1+n}$

where ε = True strain = $2\ln\left(\dfrac{d_o}{d_f}\right) = 2\ln\left(\dfrac{150}{100}\right) = 0.8109$

$$\therefore \quad \overline{\sigma_f} = \frac{180\times(0.8109)^{0.2}}{1+0.2} = 143.8427 \text{ MPa}$$

Ideal Extrusion Force, $F_{\text{Ideal}} = \overline{\sigma_f} A_o \ln\left(\dfrac{A_o}{A_f}\right)$

$$F_{\text{Ideal}} = 143.8427\times\frac{\pi}{4}\times(150)^2\times\ln\left(\frac{150^2}{100^2}\right)$$

$$= 2061311.9 \text{ N} = 2.0613 \text{ MN}$$

Total Force (F_{Total}) = $F_{\text{Ideal}} + F_{\text{Friction}} + F_{\text{Redundant deformation}}$

$$F_{\text{Total}} = F_{\text{Ideal}} + 0.25F_{\text{Total}} + 0.30F_{\text{Ideal}}$$

$$0.75F_{\text{Total}} = 1.30F_{\text{Ideal}}$$

$$F_{Total} = \frac{1.30}{0.75} F_{Ideal} = 1.7333 F_{Ideal} = 1.7333 \times 2.0613$$

$$F_{Total} = 3.573 \text{ MN}$$

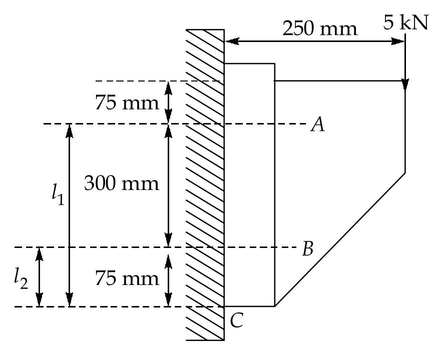
7. (b) Solution:

Given : Two bolt at point 'A' and two bolt at point 'B'

Maximum load, $P = 5$ kN

Yield strength in tension, $\sigma_{yt} = S_{yt} = 380 \text{ N/mm}^2$

FOS = 5



Direct shear stress in bolt. Assume two bolt at A denoted by 1 and two bolt at B by '2'.

$$P_{1'} = P_{2'} = \frac{P}{\text{No. of bolt}}$$

$$P_{1'} = P_{2'} = \frac{5000}{4} = 1250 \text{ N}$$

Direct shear stress, $$\tau_{direct} = \frac{P_{each}}{\text{Area}} = \frac{1250}{A} \text{ N/mm}^2 \quad ...(1)$$

Since the tendency of the bracket is to tilt about the edge 'C', the bolt at 'A' denoted by '1' are at the farthest distance from 'C'. Therefore, bolt at 'A' are subjected to maximum tensile force.

$$P_{1''} = \frac{P \times el_1}{2(l_1^2 + l_2^2)} = \frac{5000 \times 250 \times (300+75)}{2(375^2 + 75^2)}$$

$$P_{1''} = 1602.564 \text{ N}$$

Tensile stress in bolt at 'A' given by

$$\sigma_t = \frac{1602.564}{A}$$

Principal stress in bolt

$$\sigma_{1,2} = \left(\frac{\sigma_t}{2}\right) \pm \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + (\tau)^2}$$

$$\sigma_{1,2} = \left(\frac{1602.564}{2A}\right) \pm \sqrt{\left(\frac{1602.564}{2A}\right)^2 + \left(\frac{1250}{A}\right)^2}$$

$$\sigma_1 = \frac{2286.055}{A}, \sigma_2 = \frac{-683.491}{A}$$

For safe design, by maximum principal stress theory

$$\sigma_{\text{induced}} \le \sigma_{\text{permissible}}$$

$$\frac{2286.055}{A} \le \frac{380}{5}$$

$$A \ge 30.0796 \text{ mm}^2$$

$$\frac{\pi}{4} d_c^2 \ge 30.0796$$

$$d_c \ge \sqrt[2]{\frac{4 \times 30.0796}{\pi}}$$

$$d_c \ge 6.188 \text{ mm}$$

$$d \ge \left(\frac{6.188}{0.8}\right)$$

$$d \ge 7.735 \text{ mm}$$

Major diameter of the bolt, $d = 8$ mm

7. (c) Solution:

The given robot is simple 2-axis, planar robot. Corresponding a matrix for both the arm can be written as follows:

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & l_1C_1 \\ S_1 & C_1 & 0 & l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_H = A_1 5 A_2 = \begin{bmatrix} C_1C_2 - S_1S_2 & -C_1S_2 - S_1C_1 & 0 & l_2(C_1C_2 - S_1S_2) + l_1C_1 \\ S_1C_2 + C_1S_2 & -S_1S_2 + C_1C_2 & 0 & l_2(S_1C_2 + C_1S_2) + l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_1C_2 - S_1S_2 = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 = \cos(\theta_1 + \theta_2) = C(\theta_1 + \theta_2)$$
$$= C_{12}$$

$$S_1C_2 + C_1S_2 = \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2 = \sin(\theta_1 + \theta_2) = S_{12}$$

$$^0T_H = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_2C_{12}+l_1C_1 \\ S_{12} & C_{12} & 0 & l_2S_{12}+l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We can see that ^0T_H is also given as

$$^0T_H = \begin{bmatrix} -0.2924 & -0.9563 & 0 & 0.6978 \\ 0.9563 & -0.2924 & 0 & 0.8172 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & O_x & a_x & p_x \\ n_y & O_y & a_y & p_y \\ n_z & O_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equating the matrix terms, we get,

$$\theta_1 + \theta_2 = \text{ATAN2}(S_{12}, C_{12})$$
$$= \text{ATAN2}(0.9563, -0.2924) = 107°$$

$$l_2C_{12} + l_1C_1 = p_x$$

$$\begin{cases} C_1 = \dfrac{(p_x - l_2C_{12})}{l_1} \\ S_1 = \dfrac{(p_y - l_2S_{12})}{l_1} \end{cases}$$

$$l_2S_{12} + l_1S_1 = p_y$$

$$\theta_1 = \tan^{-1}\left(\frac{S_1}{C_1}\right) = \tan^{-1}\left[\frac{p_y - l_2S_{12}}{p_x - l_2C_{12}}\right]$$

$$= \tan^{-1}\left[\frac{0.8172 - 0.9563}{0.6978 + 0.2924}\right] = \tan^{-1}(-0.1405) = -8°$$

$$\theta_1 + \theta_2 = 107°,$$
$$\theta_2 = 107° - \theta_1 = 107° + 8° = 115°$$

8. (a) Solution:

$$p = 800 \text{ units/day}$$

$$d = \frac{60000}{300} = 200 \text{ units/day}$$

$$D = 60000 \text{ units/year}$$
$$C_u = \text{Rs. } 100/\text{unit}$$

$$C_h = \frac{4}{100} \times 100 = \text{Rs. } 4/\text{unit/year}$$

$$C_o = \text{Rs. } 400/\text{set-up}$$

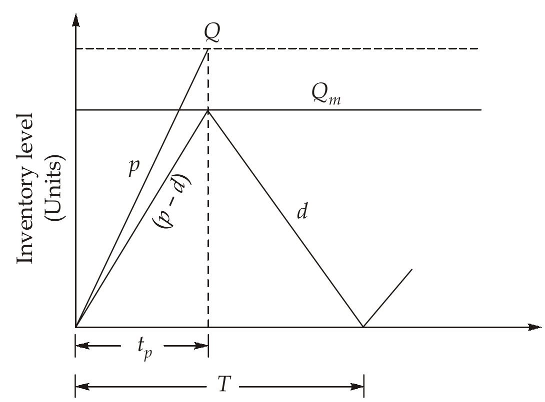
(i)
$$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{p}{p-d}\right)}$$

$$= \sqrt{\frac{2\times 60,000\times 400}{4}\times\left(\frac{800}{800-200}\right)}$$

$$= 4000 \text{ units/set-up}$$

(ii)
$$N^* = \frac{D}{Q^*} = \frac{60000}{4000} = 15 \text{ production runs/year}$$

(iii)



$$Q_m = Q\left(\frac{p-d}{p}\right) = 4000\left(\frac{800-200}{800}\right) = 3000 \text{ units}$$

(iv)
$$Q = p\cdot t_p$$
$$4000 = t_p \times 800$$
$$t_p = 5 \text{ working days}$$

Also,
$$Q_m = d(T - t_p)$$
$$3000 = 200(T - 5)$$
$$T = 20 \text{ working days}$$

Percentage of time facility will be producing sub components

$$= \frac{5}{20}\times 100 = 25\%$$

(v) Total annual cost,

$$TAC = DC_u + \frac{D}{Q}C_o + \frac{Q_m}{2}C_h$$

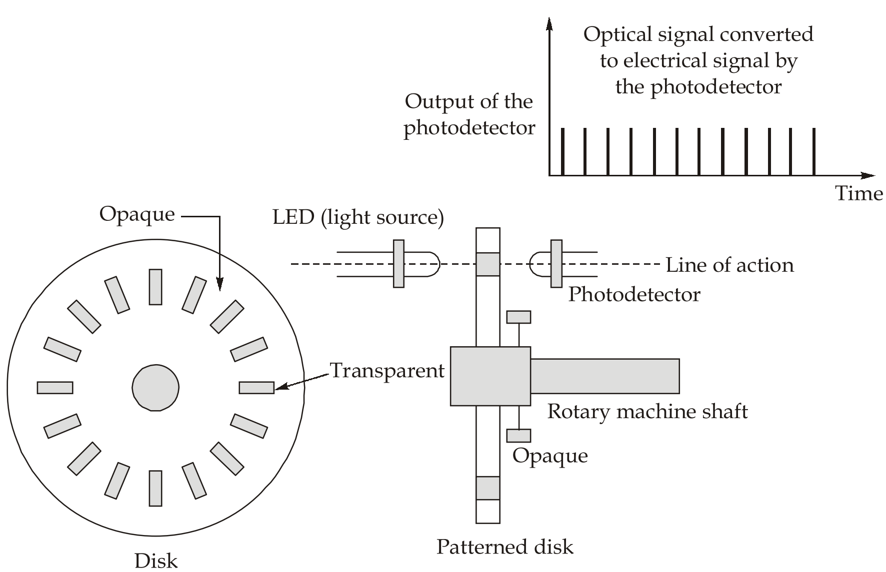
$$= (60,000\times 100) + \left(\frac{60,000}{4000}\times 400\right) + \frac{3000}{2}\times 4$$

$$= \text{Rs. } 60,12,000$$

8. (b) Solution:

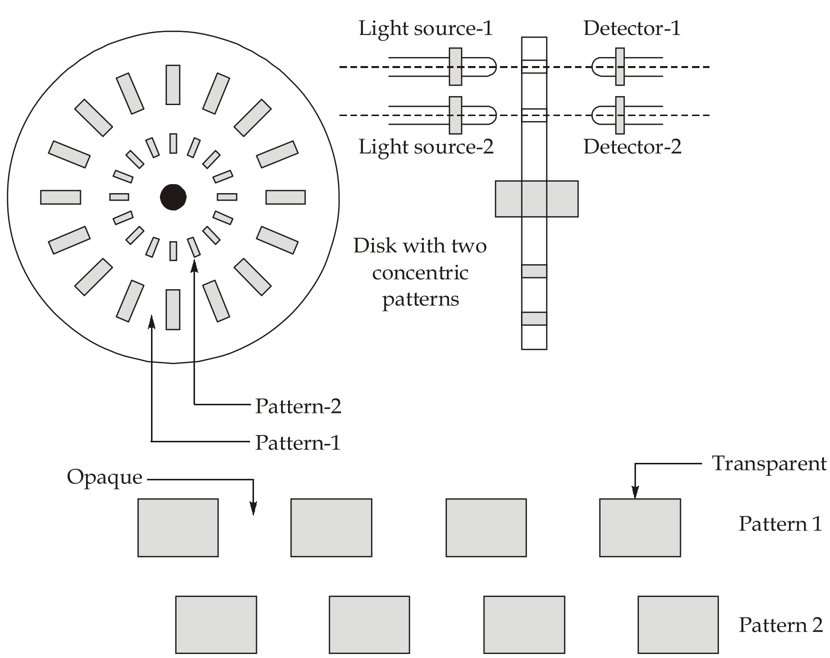
(i) Incremental Encoder : The important element in the encoder is the circular disk that contains alternative evenly spaced opaque and transparent segments over a circle. A light source is located on one side of the disk and a photo detecting device (photodetector) is placed on the other side of the disk as shown in the figure. Light Emitting Diode (LED) is used as the light source that provides continuous light signal. A photodiode or phototransistor is commonly used as the light detector, which is placed at the other side of the disk. The line of action of the light source, the circular pattern and the photo detector must match with each other. Light signals can be received if the transparent segment of the pattern is in between the light source and the photodetector. Conversely, the detector receives no light signal if opaque segment of the pattern is moved.

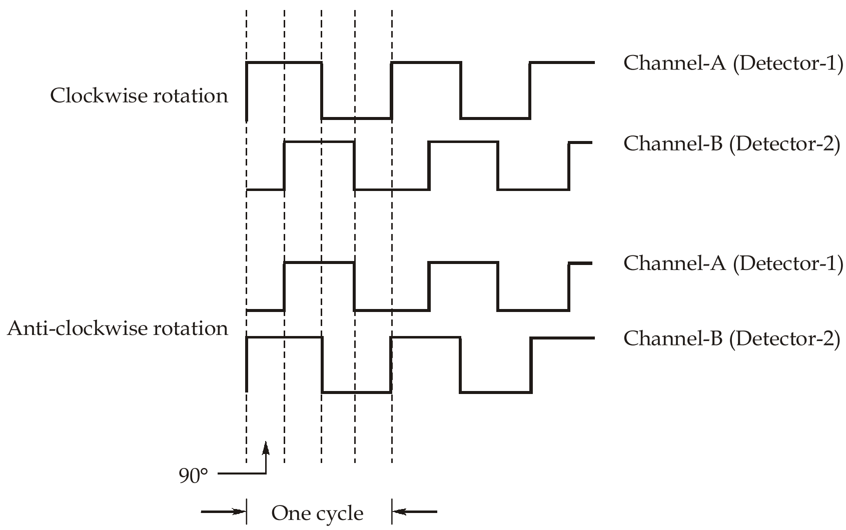
Under these design criteria, if the disc is made to rotate, the photodetector will receive pulsed light signal for every time it sees the light source, i.e. the LED. In the real application the disc is rigidly fixed with the rotor or shaft of the rotating element. The evenly spaced transparent radial lines on its surface, rotates past the light source. The output is taken from the photodetector. The number of pulses determines the position of the disk and the number of pulses per second measures the velocity of the disk. Thus, the basic principle of incremental rotary optical encoders is derived from the fact that the output signals are obtained by an electronic counter in which the measured value is derived by counting the 'increments'.



To know the direction of rotation or movement of a rotor, shaft or even a piston there must be a provision of another circular opaque and transparent pattern as shown in the figure. The pulsed light signal generated by utilizing the two patterns are usually called Channel-A signal and Channel-B signal. The relative angular placements of transparent segments of these two patterns are such that the outputs of the two photodetectors are like as shown in figure. In order to make it more clear an equivalent of the relative angular placements of two patterns have been shown in the figure as you can see figure the phase difference between the two signals is 90 degrees electrical. Such signals together are referred to as 'quadrature square-waves', which means that two square waves that are phase-shifted with respect to each other by 90 degrees electrical, or one-quarter of a cycle. For this reason some manufacturers assign the name of such type of encoder as quadrature incremental optical encoder. The phase relationship parameter, offset by 90 degrees electrical, determines the direction. If Channel-A leads Channel-B, then its direction of movement will be clockwise direction, and vice versa.

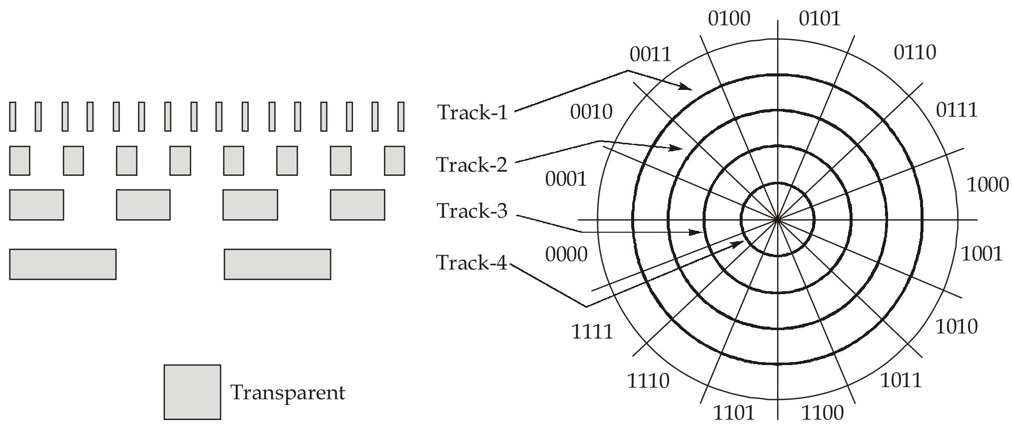
The number of transparent segments on the disk determines the resolution of the encoder. More the number of transparent segments more accurate position (angular) information can be obtained. Therefore, it is preferred to have higher number of transparent segments within the pattern for better resolution.

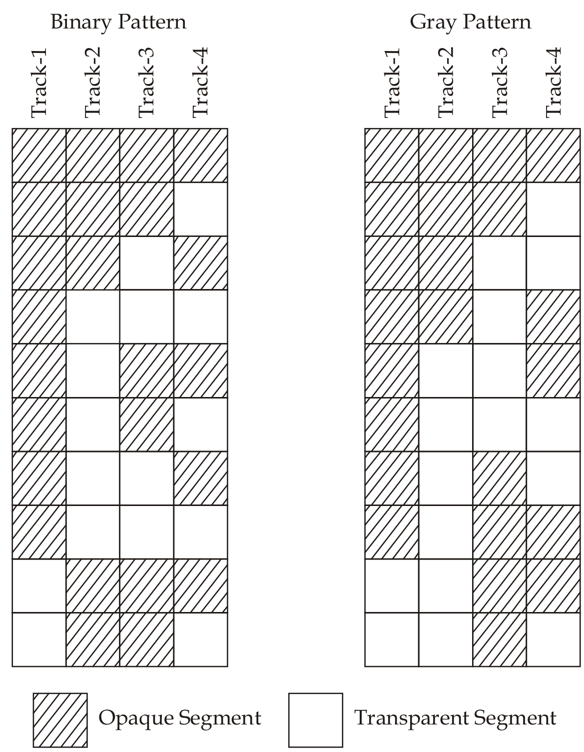




(ii) Absolute Encoder : Absolute rotary optical encoder also provides angular position and velocity value, which is derived from the pattern of the coded disc, but under a different scheme. These are a bit complex type but more capable than incremental ones. The principle of operation is that they provide a unique output for every position. The coded disk consists of a number of concentric patterns of opaque and transparent segments. The concentric patterns are called tracks. A typical disc is of an absolute encoder having only four numbers of tracks. However, typical encoders have usually 8 to 14 numbers of tracks. The number of track determines the resolution of the encoder. Each track has its own photodetector which notice the length of the transparent segment of each track. The length of the transparent segments of the track towards the center of the disk increases in a specific order to satisfy the binary coding technique. This encoder can detect 16 positions, obviously for a single rotation. If the number of tracks is 12 then the encoder will be able to detect $2^{12} = 4096$ positions for the same rotation.

In practice, binary patterned coding techniques are usually not employed. The most preferred coding technique used in absolute encoder is 'Gray' coding due to the reasons that Gray codes are reliable and are considered as versatile error detecting codes. The great merit of the absolute encoder is that if the power fails the exact position of the rotating disk can be known.





8. (c) (i) Solution:

Arc characteristics is given by, $I_a = 20(V - 15)$

Volt-ampere characteristic of power source,

$$I_t^2 = -500(V - 45)$$

As current voltage variation is parabolic in nature, hence power source is constant current source.

We know that, for stability of arc,

$$I_t = I_a$$

$$I_t^2 = I_a^2$$

$$-500(V-45) = [20(V-15)]^2$$
$$-500V + 500 \times 45 = 400[V^2 - 30V + 225]$$
$$400V^2 - 12000V + 90000 + 500V - 22500 = 0$$
$$400V^2 - 11500V + 67500 = 0$$
$$V = 20.53 \text{ Volt}, 8.22 \text{ Volt}$$

Now, $I_a = 20(V-15)$

If, $V = 20.53$ Volt,

$$I_a = 20(20.53 - 15) = 110.6 \text{ Ampere}$$

If, $V = 8.22$ Volt

$$I_a = 20(8.22 - 15) = -135.6 \text{ Ampere}$$

As current is negative for 8.22 Volt. Hence, voltage will be 20.53 Volt.

$$\text{Power of arc, } P = V \times I = 20.53 \times 110.6 = 2270.618 \text{ Watt}$$
$$P = 2.27 \text{ kW}$$

(ii)

Arc length voltage relationship is given as,

$$V_a = 25 + 5l$$

Voltage-Ampere characteristics of power source is given by,

$$I^2_t = -500(V-45)$$
$$I^2_t = -500(25 + 5l - 45)$$
$$I^2_t = -500 \times 5(l-4)$$
$$I_t = \{-2500(l-4)\}^{1/2}$$

We know that, Power is given by, $P = V \times I$

$$P = (25 + 5l) \times \{-2500(l-4)\}^{1/2}$$

Now, for optimum arc length, $\frac{dP}{dl} = 0$

$$(25+5l)\times\frac{1}{2}\{-2500(l-4)\}^{-1/2}\times(-2500)+5\{-2500(l-4)\}^{1/2} = 0$$

$$2500\times5(5+l)\times\frac{1}{2}\{-2500(l-4)\}^{-1/2} = 5\{-2500(l-4)\}^{1/2}$$

$$2500 \times \frac{1}{2}(5+l) = \{-2500(l-4)\}$$
$$1250 \times 5 + 1250l = -2500l + 10000$$
$$3750l = 10000 - 6250$$
$$\text{Optimum arc length, } l = 1 \text{ mm}$$