MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

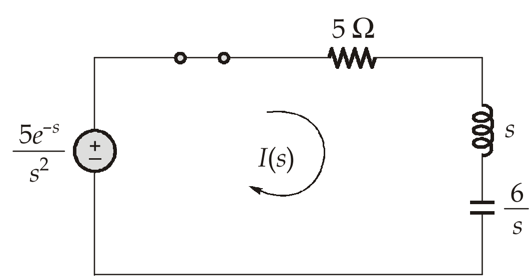
ESE-2025 Mains Test Series

E & T Engineering Test No : 9

Section A

Q.1 (a) Solution:

The s-domain equivalent of the given circuit is as shown below:



Applying KVL around the mesh for $t > 0$,

$$-\frac{5e^{-s}}{s^2} + \left(5 + s + \frac{6}{s}\right) I(s) = 0$$

$$\Rightarrow \quad I(s) = \frac{\left(\frac{5e^{-s}}{s^2}\right)}{\left(\frac{s^2 + 5s + 6}{s}\right)}$$

$$\Rightarrow \quad I(s) = \frac{5e^{-s}}{s(s^2 + 5s + 6)} = \frac{5e^{-s}}{s(s+2)(s+3)}$$

By partial-fraction expansion,

$$\frac{1}{s(s+2)(s+3)} = \frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+3}$$

$$A = \left.\frac{1}{(s+3)(s+2)}\right|_{s=0} = \frac{1}{6}$$

$$B = \left.\frac{1}{s(s+3)}\right|_{s=-2} = \frac{-1}{2}$$

$$C = \left.\frac{1}{s(s+2)}\right|_{s=-3} = \frac{1}{3}$$

$$\Rightarrow \quad I(s) = 5e^{-s}\left[\frac{1/6}{s}+\frac{-1/2}{s+2}+\frac{1/3}{s+3}\right]$$

$$\Rightarrow \quad I(s) = \frac{5}{6}\frac{e^{-s}}{s}-\frac{\frac{5}{2}e^{-s}}{s+2}+\frac{\frac{5}{3}e^{-s}}{s+3}$$

Taking inverse Laplace transform on both sides,

$$i(t) = \frac{5}{6}u(t-1)-\frac{5}{2}e^{-2(t-1)}u(t-1)+\frac{5}{3}e^{-3(t-1)}u(t-1); \quad t>0$$

Q.1 (b) Solution:

Given ,

$$V_{TN} = 1 \text{ V}$$

$$K = 0.9 \text{ mA/V}^2$$

(i) Given that n-channel MOSFET is working in saturation region. Thus,

$$\text{Drain current, } I_D = K(V_{GS} - V_{TN})^2$$

$$\Rightarrow \quad I_D = 0.9 \times 10^{-3}(2.1 - 1)^2$$

$$\Rightarrow \quad I_D = 1.089 \text{ mA}$$

(ii) Transconductance, $g_m = \dfrac{\partial I_D}{\partial V_{GS}}$

$$\Rightarrow \quad g_m = \frac{\partial\left[K(V_{GS}-V_{TN})^2\right]}{\partial V_{GS}}$$

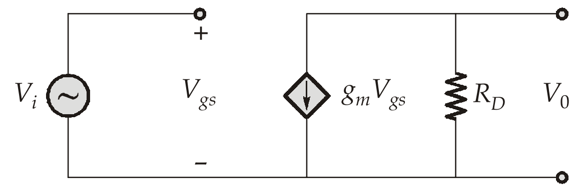
$$\Rightarrow \quad g_m = 2K(V_{GS} - V_{TN})$$

$$= 2 \times 0.9 \times 10^{-3}(2.1 - 1)$$

$$= 1.98 \text{ mA/V}$$

(iii) Since $V_i << V_{GS}$, thus it can be considered as small signal and hence it will not affect the operating region of MOSFET. Thus, we can assume that the transistor to still be in saturation region.

$\therefore$ Total current, $i_D = I_D$ + Current due to small signal (i_d)



where $i_d = g_mV_{gs} = g_mV_i$

$= 1.98 \times 10^{-3} \times 10 \times 10^{-3}$

$= 1.98 \times 10^{-5}$

$= 0.0198$ mA

$\Rightarrow$ $i_D = I_D + g_mV_i$

$= (1.098 + 0.0198 \sin \omega t)$ mA

DC Drain voltage, $V_D = V_{DD} - i_DR_D$

$= 8 - 1.098 \times 10^{-3} \times 2 \times 10^3$

$= 5.804$ V

Drain voltage due to small signal input,

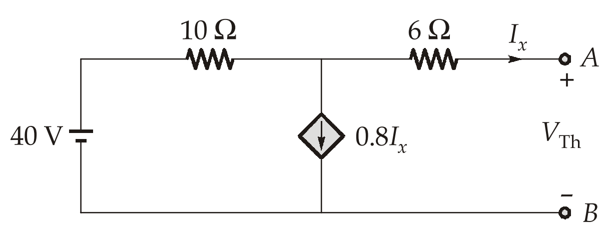
$$v_d = i_dR_D = 0.0198 \sin \omega t \times 10^{-3} \times 2 \times 10^3 = 0.0396 \sin \omega t \text{ V}$$

$\therefore$ Drain voltage, $v_D = V_D + v_d = (5.804 + 0.0396 \sin\omega t)$ V

Q.1 (c) Solution:

Step-I: Calculation of V_{Th}

'V_{Th}' is the open circuit voltage across the 16 Ω-resistor.



From the figure, $I_x = 0$

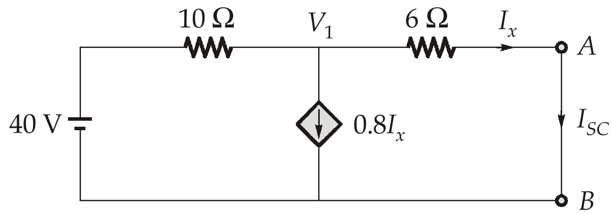
The dependent source $0.8I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes,

i.e., $0.8I_x = 0$

$\Rightarrow$ $V_{Th} = 40$ V

Step-II: Calculation of I_{SC}

'I_{SC}' is the short circuit current through the 16 Ω resistor.



From the figure, $$I_x = \frac{V_1}{6}$$

Applying KCL at node 1,

$$\frac{V_1 - 40}{10} + 0.8I_x + I_x = 0$$

$$\frac{V_1}{10} - 4 + 1.8I_x = 0$$

$$\frac{V_1}{10} + 1.8\frac{V_1}{6} = 4$$

$$0.1V_1 + 0.3V_1 = 4$$

$$0.4V_1 = 4$$

⇒ $$V_1 = \frac{4}{0.4} = 10 \text{ V}$$

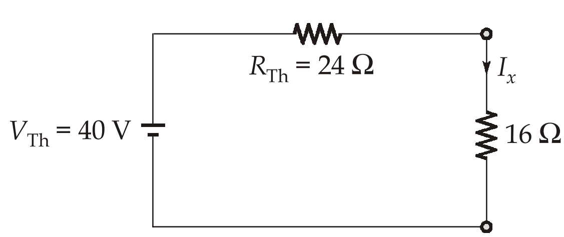
$$I_{SC} = I_x = \frac{V_1}{6} = \frac{10}{6} \text{ A}$$

Step III: Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_{SC}} = \frac{40}{\left(\frac{10}{6}\right)} = 24\ \Omega$$

Step IV: Calculation of I_x

The thevenin's equivalent of the given circuit is as shown below:



$$I_x = \frac{40}{24 + 16} = \frac{40}{40} = 1 \text{ A}$$

Q.1 (d) Solution:

(i) The magnetic susceptibility within a bar of some metal alloy when $M = 3.2 \times 10^5$ A/m and $H = 50$ A/m is,

$$\chi_m = \frac{M}{H} = \frac{3.2\times10^5 \text{ A/m}}{50 \text{ A/m}} = 6400$$

(ii) the permeability,

$$\mu = \mu_r\mu_0 = (\chi_m + 1)\mu_0$$
$$= (6400 + 1)(4\pi \times 10^{-7})$$
$$= 8.044 \times 10^{-3} \text{ H/m}$$

(iii) The magnetic flux density within the material is,

$$B = \mu H = (8.044 \times 10^{-3}) \times (50) = 0.4022 \text{ tesla}$$

(iv) This metal alloy would exhibit ferromagnetic behaviour due to very high value of magnetic susceptibility χ_m(6400), which is considerably larger than the χ_m values for diamagnetic ($-1 \le \chi_m < 0$) and paramagnetic materials (χ_m is small and positive).

Q.1 (e) Solution:

Slots per pole per phase, $q = \dfrac{144/3}{10} = 4.8$

$$\text{Angular slot pitch, } \gamma = \frac{180P}{\text{Total number of slots}} = \frac{10\times180}{144} = \frac{25}{2} = 12.5$$

$$\text{Coil span} = 12\gamma = 12\times\frac{25}{2} = 150°$$

$$\text{Chording angle, } \alpha = [\text{One pole pitch}] - [\text{Coil span}]$$
$$= 180° - 150° = 30°$$

$$\text{Distribution factor, } k_d = \frac{\sin\left(\frac{q\gamma}{2}\right)}{q\sin\left(\frac{\gamma}{2}\right)} = \frac{\sin\left(\frac{4.8\times12.5}{2}\right)}{4.8\sin\left(\frac{12.5}{2}\right)} = 0.95683$$

$$\text{Chording factor, } k_p = \cos\frac{\alpha}{2} = \cos 15° = 0.966$$

Number of turns per phase, $N_{ph} = \dfrac{144\times5}{3} = 240$

$$\text{Phase emf, } E_p = \sqrt{2}\times\pi f\, k_d k_p N_{ph}\phi$$
$$= 4.44 \times 50 \times 0.95683 \times 0.966 \times 240 \times 0.2$$
$$= 9855.71 \text{ V}$$

Q.2 (a) Solution:

(i) Given, $R_1 = 250\ \Omega$, $R_2 = 500\ \Omega$, $R_3 = 375\ \Omega$

The total resistance of resistors connected in parallel and neglecting their errors is:

$$R = \frac{1}{\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)+\left(\frac{1}{R_3}\right)} = \frac{1}{\left(\frac{1}{250}\right)+\left(\frac{1}{500}\right)+\left(\frac{1}{375}\right)}$$

$\Rightarrow$ $$R = 115.385\ \Omega$$

$$\delta R_1 = (0.025 \times 250) = +6.25\ \Omega$$

$\Rightarrow$ $$R_1' = 250 - 6.25 = 243.75\ \Omega$$

$$\delta R_2 = (-0.036 \times 500) = -18\ \Omega$$

$\Rightarrow$ $$R_2' = 500 + 18 = 518\ \Omega$$

$$\delta R_3 = (+0.014 \times 375) = +5.25\ \Omega$$

$\Rightarrow$ $$R_3' = 375 - 5.25 = 369.75\ \Omega$$

The resultant resistance of three resistors in parallel, considering the error of each resistor is,

$$R' = \frac{1}{\left(\frac{1}{R_1'}\right)+\left(\frac{1}{R_2'}\right)+\left(\frac{1}{R_3'}\right)}$$

$\Rightarrow$ $$R' = \frac{1}{\left(\frac{1}{243.75}\right)+\left(\frac{1}{518}\right)+\left(\frac{1}{369.75}\right)} = 114.45\ \Omega$$

The fractional error of the total resistance based on the rated values is:

$$\in = \frac{R - R'}{R}$$

$$= \frac{115.385 - 114.45}{115.385} = 0.0081 = +0.81\%$$

(ii)

Zener Breakdown	Avalanche Breakdown
1. Occurs due to large electric field intensity.	1. Occurs due to electron multiplication, which leads to multiple collisions between electrons and ions in the depletion layer.
2. The large electric field intensity leads to rupturing of covalent bonds between atoms.	2. Due to impact ionisation.
3. Occurs for breakdown voltage below 6 V.	3. Occurs for breakdown voltage typically greater than 6 V.
4. Zener breakdown voltage decreases with temperature (NTC).	4. Avalanche breakdown voltage increases with temperature (PTC).
5. The VI characteristics of Zener breakdown has a sharp curve.	5. The VI characteristic curve of the avalanche breakdown is not as sharp as the Zener breakdown.
6. Occurs in junctions with relatively high doping levels.	6. Occurs in junctions with relatively low doping levels.

Q.2 (b) Solution:

(i) For plane *A*, since the plane passes through the origin of the coordinate system as shown, we will move the origin of coordinate system one unit cell distance to the right along the *y*-axis; thus, this is a $(3\ \bar{2}\ 4)$ plane, as summarized below:

	x	y	z
Intercepts	$\frac{2a}{3}$	$-b$	$\frac{c}{2}$
Intercepts in terms of a, b and c	$\frac{2}{3}$	-1	$\frac{1}{2}$
Reciprocals of intercepts	$\frac{3}{2}$	-1	2
Reduction	3	-2	4
Enclosure		$(3\ \bar{2}\ 4)$	

For plane *B*, we will leave the origin at the unit cell as shown; this is a (2 2 1) plane, as summarized below:

	x	y	z
Intercepts	$\frac{a}{2}$	$\frac{b}{2}$	c
Intercepts in terms of a, b and c	$\frac{1}{2}$	$\frac{1}{2}$	1
Reciprocals of intercepts	2	2	1
Reduction		(not necessary)	
Enclosure		(2 2 1)	

(ii) Carbon Nanotube structure consists of a single sheet of graphite, rolled into a tube, both ends of which are capped with C_{60} fullerene hemispheres. These nanotubes are extremely strong and stiff, and relatively ductile. Carbon nanotubes also have unique and structure-sensitive electrical characteristics. Depending on the orientation of the hexagonal units in the graphene plane (i.e., tube wall) with the tube axis, the nanotube may behave electrically as either a metal or a semiconductor.

They have excellent thermal conductivity, meaning they can efficiently transfer heat. Their small size and cylindrical shape give them a very large surface area per unit volume, which is useful in applications like drug delivery.

Q.2 (c) Solution:

For $t < 0$, given circuit is source-free RLC circuit.

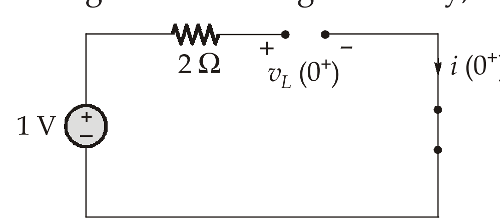
$$\Rightarrow \quad i(0^-) = 0 \text{ A}$$

$$v(0^-) = 0 \text{ V}$$

At $t = 0^+$: Since the inductor current and capacitor voltage cannot change instantly,

$$i(0^+) = i(0^-) = 0 \text{ A}$$

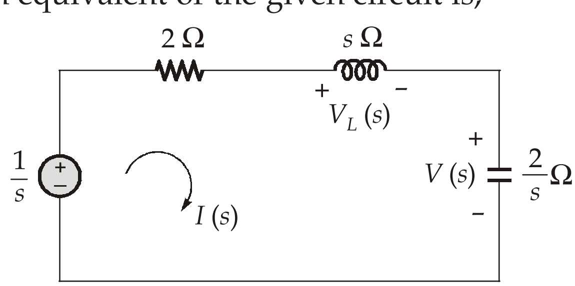
$$v(0^+) = v(0^-) = 0 \text{ V}$$



We have, $$v_L(0^+) = 1 \text{ V}$$

$$\Rightarrow \quad \frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{1}{1} = 1 \text{ A/s}$$

For $t \geq 0$: The s-domain equivalent of the given circuit is,



Apply KVL around the loop,

$$-\frac{1}{s}+\left(2+s+\frac{2}{s}\right)I(s) = 0$$

$$I(s) = \frac{\left(\frac{1}{s}\right)}{\left(2+s+\frac{2}{s}\right)}=\frac{\left(\frac{1}{s}\right)}{\left(\frac{s^2+2s+2}{s}\right)}$$

$\Rightarrow$ $$I(s) = \frac{1}{s^2+2s+2}$$

$\therefore$ $$V(s) = \frac{2}{s}\times I(s)=\frac{2}{s}\times\frac{1}{s^2+2s+2}=\frac{A}{s}+\frac{Bs+C}{s^2+2s+2}$$

$\Rightarrow$ $$2 = A(s^2+2s+2)+(Bs+C)s$$

On comparing the coefficients of 's^2'

$$0 = A + B \quad \text{...(i)}$$

On comparing the coefficients of 's^1'

$$0 = 2A + C \quad \text{...(ii)}$$

On comparing the coefficients of 's^0'

$$2 = 2A$$

$\Rightarrow$ $$A = 1$$

From equation (i), $B = -1$

From equation (ii), $C = -2$

$\therefore$ $$V(s) = \frac{1}{s}-\frac{(s+2)}{s^2+2s+2}=\frac{1}{s}-\frac{(s+1)}{(s+1)^2+1}-\frac{1}{(s+1)^2+1}$$

Take inverse Laplace transform on both sides,

$$v(t) = u(t) - e^{-t}\cos t\, u(t) - e^{-t}\sin t\, u\, lt)$$

$\Rightarrow$ $$v(t) = 1 - e^{-t}\cos t - e^{-t}\sin t;\ t \geq 0$$

$$\frac{dv(t)}{dt} = -[e^{-t}(-\sin t) - e^{-t}\cos t] - [e^{-t}(\cos t) - e^{-t}\sin t]$$

$$= e^{-t}\sin t + e^{-t}\cos t - e^{-t}\cos t + e^{-t}\sin t$$

$$= 2\, e^{-t}\sin t$$

$$\frac{d^2v(t)}{dt^2} = 2[e^{-t}\cos t - e^{-t}\sin t]$$

At $t = 0^+$:

$$\frac{d^2v(0^+)}{dt^2} = 2[1 - 0] = 2 \text{ V/s}^2$$

For the given RLC-circuit, $R = 2\ \Omega$, $L = 1$ H, $C = \frac{1}{2}$F. The roots of the characteristic equation are given by

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where $$\alpha = \frac{R}{2L} = \frac{2}{2\times 1} = 1$$

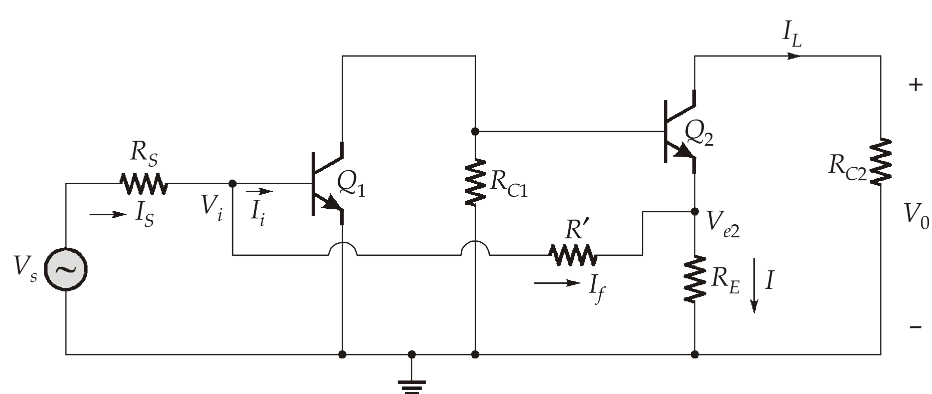
and $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times\frac{1}{2}}} = \sqrt{2} = 1.414$$

As $\alpha < \omega_0$, poles are complex conjugates with negative real parts. Thus, the circuit is underdamped.

Q.3 (a) Solution:

Given, $R_{C1} = 3\ \text{k}\Omega$, $R_{C2} = 500\ \Omega$, $R_E = 50\ \Omega$, $R' = R_S = 1.2\ \text{k}\Omega$, $h_{fe} = 50$, $h_{ie} = 1.1\ \text{k}\Omega$, and $h_{re} = h_{ce} = 0$.

Small signal equivalent circuit:



Step 1: Identify the type of feedback.

R' is not connected directly to output node $\Rightarrow$ Current sampling

R' is connected directly to input node $\Rightarrow$ shunt mixing

$\therefore$ The given feedback amplifier has current shunt feedback (current amplifier)

Step 2: Calculate the feedback factor (β):

KCL : $I_f = I_L + I$

$\Rightarrow \quad I = I_f - I_L$

We have, feedback current,

$$I_f = \frac{V_i - V_{e2}}{R'}$$

Put $V_i = 0 \quad \Rightarrow$

$$I_f = \frac{-V_{e2}}{R'} = \frac{-IR_E}{R'} = \frac{-(I_f - I_L)R_E}{R'}$$

$$\Rightarrow \quad I_f\left[1 + \frac{R_E}{R'}\right] = \frac{I_L R_E}{R'}$$

$$\Rightarrow \quad I_f(R' + R_E) = I_L R_E$$

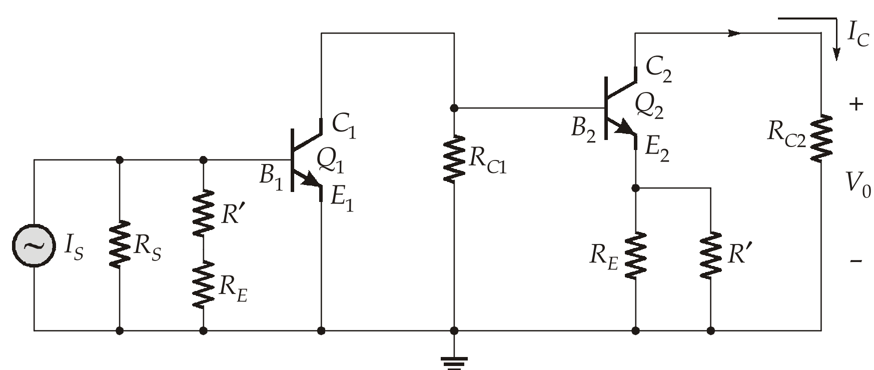
$$\Rightarrow \quad \frac{I_f}{I_L} = \beta = \frac{R_E}{R' + R_E} = \frac{50}{1200 + 50} = \frac{5}{125} = \frac{1}{25}$$

$$\Rightarrow \quad \beta = \frac{1}{25}$$

Step 3: Draw circuit without feedback:

Break output loop $\Rightarrow$ R' and R_E appear in series between B_1 and ground.

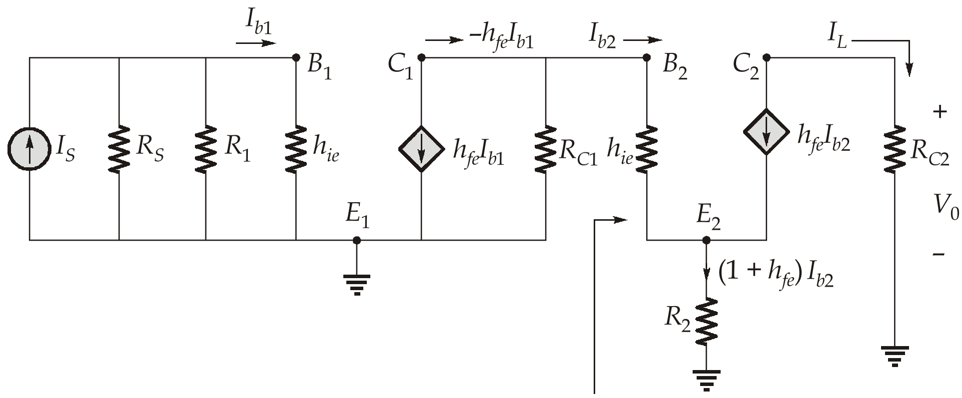
Ground input node $\Rightarrow$ R' and R_E appear in parallel between E_2 and ground.



$$R_1 = R' + R_E = 1200 + 50 = 1250\ \Omega = 1.25\ k\Omega$$

$$R_2 = R' \parallel R_E = 1200 \parallel 50 = \frac{1200 \times 50}{(1200 + 50)} = 48\ \Omega$$

Step 4: Replace Q_1 and Q_2 with h-parameter model:



$$R_{i2} = h_{ie} + (1 + h_{fe}) R_2$$
$$= 1100 + (51)(48)$$
$$= 3548\ \Omega$$
$$= 3.548\ \text{k}\Omega$$

$$I_{b1} = \frac{I_s(R_S \| R_1)}{(R_S \| R_1) + h_{ie}}$$

$$I_{b2} = \frac{-h_{fe} I_{b1} R_{C1}}{R_{C1} + R_{i2}}$$

$\therefore$ Current gain,
$$A_I = \frac{I_L}{I_S} = \left(\frac{I_L}{I_{b2}}\right)\cdot\left(\frac{I_{b2}}{I_{b1}}\right)\cdot\left(\frac{I_{b1}}{I_S}\right)$$
$$= (-h_{fe})\cdot\left(\frac{-h_{fe}R_{C1}}{R_{C1}+R_{i2}}\right)\cdot\left(\frac{R_S \| R_1}{R_S \| R_1 + h_{ie}}\right)$$
$$= (-50)\cdot\left(\frac{-50\times 3}{3+3.548}\right)\cdot\left(\frac{0.612}{0.612+1.1}\right) = 409.45$$

Resistance seen by current source 'I_s' is

$$R_{in} = R_S \| R_1 \| h_{ie}$$
$$= 1.2 \| 1.25 \| 1.1$$
$$= 0.3933\ \text{k}\Omega$$

Output resistance, $R_{C2} = 500\ \Omega$

Desensitivity, $D = 1 + \beta A_I$
$$= 1 + \left(\frac{1}{25}\times 409.45\right) = 17.378$$

With feedback:

$$A_{If} = \frac{A_I}{1+\beta A_I} = \frac{409.45}{17.378} = 23.56$$

For a current shunt amplifier, the input resistance decreases by a factor of $(1 + \beta A_I)$ and output impedance increases by a factor of $(1 + \beta A_I)$. Thus,

$$R_{in\,f} = \frac{R_{in}}{1+\beta A_I} = \frac{393.3}{17.378} = 22.63\ \Omega$$

$$R_{of} = R_0(1 + \beta A_I)$$

$$= 500 \times 17.378 = 8.689\ \text{k}\Omega$$

$$A_{vf} = \frac{V_0}{V_s} = \frac{I_L R_{c2}}{I_s R_s} = \frac{A_{If} R_{c2}}{R_s} = \frac{23.56\times 0.5}{1.2}$$

$$\Rightarrow \quad A_{vf} = 9.8167$$

Q.3 (b) Solution:

Given, $S = 100$ kVA $= 100 \times 10^3$ VA; $P_c = 1200$ W; $P_i = 960$ W

$$\text{Efficiency, } \eta = \frac{mS\cos\phi}{mS\cos\phi + P_i + m^2 P_{cu}}$$

where, $m = \dfrac{\text{Given load}}{\text{full load}}$ is the fraction of the full load.

(i) At full load ($m = 1$), $\cos\phi = 1$

$$\eta = \frac{1\times 100\times 10^3\times 1}{(1\times 100\times 10^3\times 1)+960+(1)^2\times 1200}$$

$$\Rightarrow \quad \eta = 0.97886 \text{ pu}$$

$$\eta = 97.886\%$$

(ii) At half load $\left(m = \dfrac{1}{2}\right)$, $\cos\phi = 0.8$

$$\Rightarrow \quad \eta = \frac{\frac{1}{2}\times 100\times 10^3\times 0.8}{\left(\frac{1}{2}\times 100\times 10^3\times 0.8\right)+960+\left(\frac{1}{2}\right)^2\times 1200}$$

$$\Rightarrow \quad \eta = 0.9695 \text{ pu}$$

$$\Rightarrow \quad \eta = 96.95\%$$

(iii) Power factor $\cos\phi = 0.7$,

At 75% full load, $m = \dfrac{75}{100} = 0.75$

$$\Rightarrow \quad \eta = \frac{0.75\times 100\times 10^3\times 0.7}{(0.75\times 100\times 10^3\times 0.7)+960+(0.75)^2\times 1200}$$

$$\Rightarrow \quad \eta = 0.9698 \text{ pu}$$

$$\Rightarrow \quad \eta = 96.98\%$$

(iv) At maximum efficiency, iron losses are equal to the copper losses. Thus, we have

$$P_i = m^2 P_{cu} \Rightarrow m = \sqrt{\frac{P_i}{P_{cu}}}$$

Thus,
$$S_M = S_{\text{Full-load}}\sqrt{\frac{P_i}{P_{cu}}} = 100\sqrt{\frac{960}{1200}} = 89.443 \text{ kVA}$$

(v) Maximum efficiency at $\cos\phi = 0.85$,

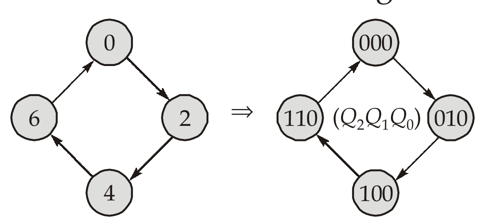
$$\eta_M = \frac{S_M \cos\phi}{S_M \cos\phi + 2P_i}$$

$$\Rightarrow \quad \eta_M = \frac{89.443\times 10^3 \times 0.85}{(89.443\times 10^3 \times 0.85) + (2\times 960)}$$

$$\Rightarrow \quad \eta_M = 0.9754 \text{ pu} = 97.54\%$$

3. (c) Solution

The sequence diagram of the 3-bit counter to be designed is,

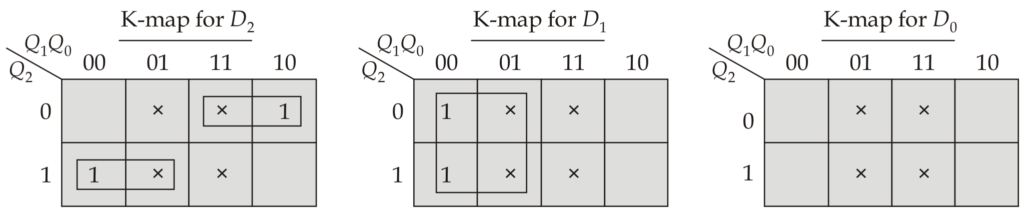


Excitation table:

	Present state			Next state			Excitations		
	Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	D_2	D_1	D_0
	0	0	0	0	1	0	0	1	0
*	0	0	1	×	×	×	×	×	×
	0	1	0	1	0	0	1	0	0
*	0	1	1	×	×	×	×	×	×
	1	0	0	1	1	0	1	1	0
*	1	0	1	×	×	×	×	×	×
	1	1	0	0	0	0	0	0	0
*	1	1	1	×	×	×	×	×	×

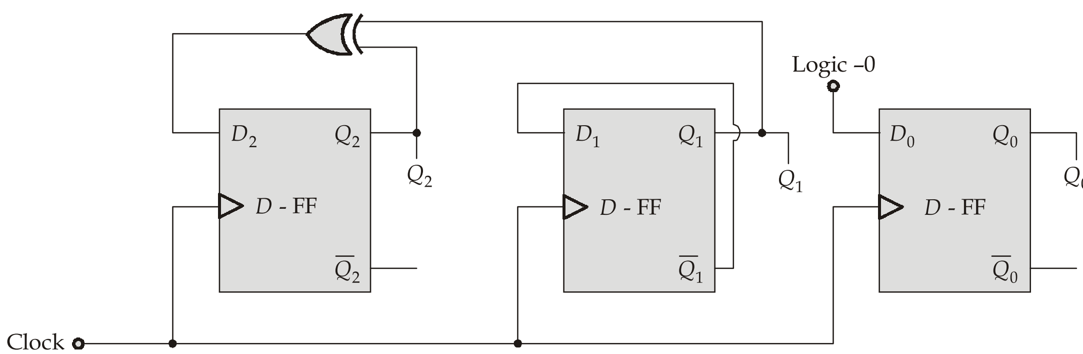
"*" indicates unused states of the counter

Minimization:



$D_2 = Q_2\overline{Q}_1 + \overline{Q}_2Q_1 = Q_2 \oplus Q_1$ $\quad$ $D_1 = \overline{Q}_1$ $\quad$ $D_0 = 0$

Logic circuit:



Checking for self starting:

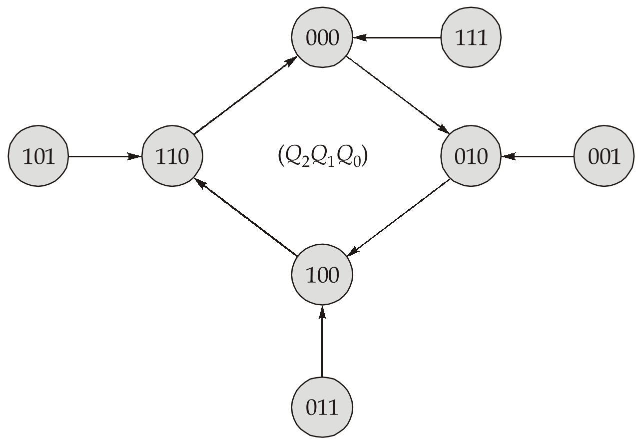
- A counter is said to be self starting when it enters into a used or valid state from an unused state within finite number of clock cycles.
- In the above designed counter, there are four unused states (1, 3, 5, 7). In order to determine the self starting capability of the counter, the next states of the unused states are to be examined, which can be done as shown below.

$$D_2 = Q_2 \oplus Q_1$$
$$D_1 = \overline{Q}_1$$
$$D_0 = 0$$

Present state			Excitations			Next state		
Q_2	Q_1	Q_0	D_2	D_1	D_0	Q_2^+	Q_1^+	Q_0^+
0	0	1	0	1	0	0	1	0
0	1	1	1	0	0	1	0	0
1	0	1	1	1	0	1	1	0
1	1	1	0	0	0	0	0	0

- From the above sequence table, it is clear that, from all the unused states, the counter will enter into a valid state within finite number of clock cycles. So, the designed counter is said to be self starting.

Complete sequence diagram:



Q.4 (a) Solution:

Given that, 3-phase, Y-connected 10 kVA and 230 V synchronous generator, with

$$X_s = 1.2 \text{ ohms/phase}, \; R_a = 0.5 \text{ ohms/phase}$$

(i) % voltage regulation at full load with 0.8 lagging power factor:

$$I_a \text{ per phase} = \frac{10\times10^3}{\sqrt{3}\times230} = 25.1 \text{ A} \qquad \text{(at full load)}$$

$$\cos\phi = 0.8;$$

$$\sin\phi = 0.6$$

$$\text{Terminal voltage, } V_t \text{ per phase} = \frac{230}{\sqrt{3}} = 132.8 \text{ V} \qquad [\because \text{Star Connection}]$$

Since the power factor is lagging, the generated emf per phase,

$$E_f = \sqrt{(V_t\cos\phi + I_aR_a)^2 + (V_t\sin\phi + I_aX_s)^2}$$

$$= \sqrt{[132.8\times0.8)+(25.1\times0.5)]^2 + [(132.8\times0.6)+(25.1\times1.2)]^2}$$

$$= \sqrt{(14111.0641)+(12056.04)}$$

$$\Rightarrow \qquad E_f = 161.76 \text{ volts}$$

$$\%\text{ voltage regulation} = \frac{E_f - V_t}{V_t} = \left[\frac{161.76-132.8}{132.8}\right] = 21.80\%$$

(ii) Let cos ϕ be the power factor at which voltage regulation becomes zero. The power factor must be leading.

$$E_f^2 = (V_t \cos\phi + I_a R_a)^2 + (V_t \sin\phi - I_a X_s)^2$$

$$= V_t^2 \cos^2\phi + I_a^2 R_a^2 + 2V_t \cos\phi I_a R_a + V_t^2 \sin^2\phi + I_a^2 X_s^2 - 2V_t \sin\phi I_a X_s$$

$$= V_t^2 + 2V_t I_a (R_a \cos\phi - X_s \sin\phi) + I_a^2 (R_a^2 + X_s^2)$$

For voltage regulation = 0,

$$E_f = V_t = 132.8 \text{ V}$$

$$\therefore \quad (132.8)^2 = (132.8)^2 + 2 \times 132.8 \times 25.1\,(0.5 \times \cos\phi - 1.2 \sin\phi) + (25.1)^2\,(0.5^2 + 1.2^2)$$

$$\Rightarrow \quad 1.2 \sin\phi - 0.5 \cos\phi = 0.15971$$

$$\Rightarrow \quad 1.2 \sin\phi - 0.5 \cos\phi \simeq 0.16$$

$$\Rightarrow \quad \phi = 30°$$

$$\therefore \quad \text{Power factor, } \cos\phi = \cos 30° = 0.866 \text{ (leading)}$$

Q.4 (b) Solution:

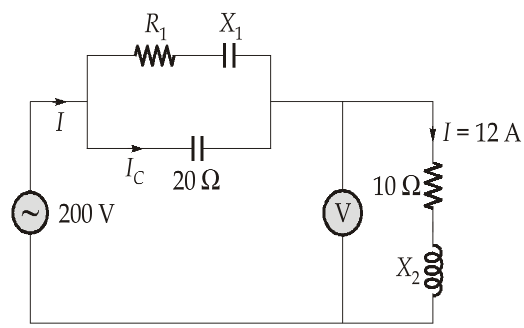
Carbon Dots	Quantum Dots
1. Carbon dots are small carbon nanoparticles having some form of surface passivation.	1. Quantum dots are small semiconductor particles on a nanoscale, having optical and electronic properties that differ from large particles according to quantum mechanics.
2. Top-down and bottom-up methods are used for production of carbon dots.	2. Colloidal synthesis, plasma synthesis, fabrication, electrochemical assembly can be used for production of Quantum dots.
3. The properties of carbon dots solely depends on their structures and compositions.	3. The properties of quantum dots are intermediate to those of bulk semiconductors and discrete atoms.
4. Carbon dots are used in bioimaging, sensing, drug delivery, catalysis, optronics etc.	4. Quantum dots are used in LEDs, single photon sources, quantum computing etc.

Q.4 (c) Solution:

(i) Given,

$$I = 12\angle 0°\text{A}$$

$$P = 1800 \text{ W}$$



Since the voltmeter reads 200 V. Thus,

$$|Z_2| = \frac{200}{12} = 16.667 \ \Omega$$

where $$|Z_2| = \sqrt{R_2^2 + X_2^2}$$

$\Rightarrow$ $$16.667 = \sqrt{(10)^2 + X_2^2}$$

$\Rightarrow$ $$(16.667)^2 = 100 + X_2^2 \quad \Rightarrow \quad X_2 = 13.33375 \ \Omega$$

We have, $$\overline{V_2} = IZ_2 = (12\angle 0°) \times (10 + j13.33375)$$

$$= (12\angle 0°) \times (16.667\angle 53.131°)$$

$$= 200\angle 53.131° \text{ V}$$

Also, $$P = VI \cos \phi$$

$$1800 = 200 \times 12 \times \cos \phi$$

$\Rightarrow$ $$\cos \phi = 0.75$$

$\Rightarrow$ $$\phi = 41.41°$$

$\therefore$ Applied voltage, $$\overline{V_s} = 200\angle 41.41°\text{V}$$

Voltage across parallel branches

$$= (200\angle 41.41°) - (200\angle 53.131°)$$

$$= 40.843\angle -42.73° \text{ V}$$

Current through capacitor,

$$I_C = \frac{40.843\angle -42.73°}{20\angle -90°}$$

$$= 2.042 \angle 47.27° \text{ A}$$

Current through R_1 and X_1,

$$= (12\angle 0° - 2.042\ \angle 47.27°)$$

$$= 10.72\ \angle{-8.043°}\ \text{A}$$

$$\therefore \qquad Z_1 = \frac{40.843\angle -42.73°}{10.72\angle -8.043°}$$

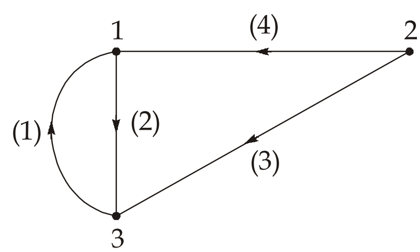
$$= 3.81\ \angle{-34.687°}\ \Omega = (3.133 - j2.1682)\ \Omega$$

$$\Rightarrow \qquad R_1 = 3.133\ \Omega$$

$$X_1 = 2.1682\ \Omega$$

(ii) For drawing the oriented graph,

1. replace all resistors, inductors and capacitors by line segments.
2. replace voltage source by short circuit and current source by an open circuit,
3. assume directions of branch currents arbitrarily, and
4. number the nodes and branches.



Complete Incidence Matrix (A_a):

Nodes ↓ \ Branches →	1	2	3	4
1	–1	1	0	–1
2	0	0	1	1
3	1	–1	–1	0

$$A_a = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & -1 & 0 \end{bmatrix}$$

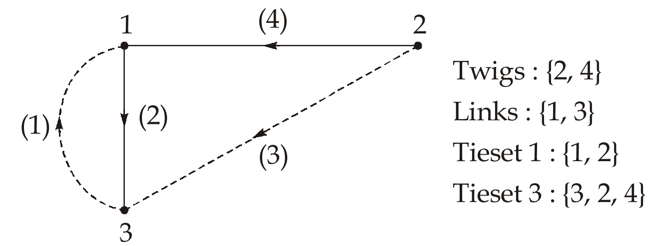
Eliminating the third row from the matrix A_a, we get the reduced incidence matrix A.

$$A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

To write the Tieset and f-cutset matrix, assume the tree with branches 2 and 4 as twigs and branches 1 and 3 as links.

Tieset Matrix (B):

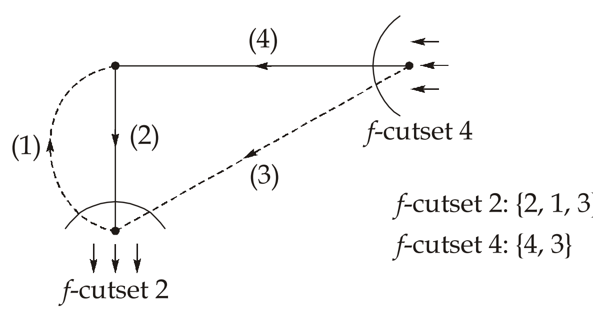
A tie-set is a closed path in a graph containing one link and remaining branches are twigs. The number of tie-sets is equal to the number of links in the graph.



$$B = \begin{array}{c} \\ 1 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

***f*-cutset matrix (Q):**

A cut-set is the smallest set of branches in a connected graph that, when removed, separates the graph into two sub-graphs. A fundamental cut-set of a graph with respect to a tree is a cut-set formed by one and only one twig and a set of links.

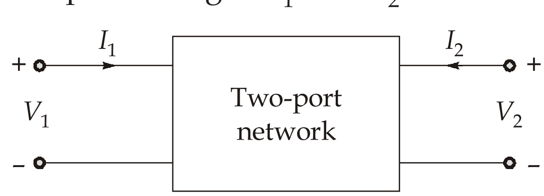


$$Q = \begin{array}{c} \\ 2 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Section B

Q.5 (a) Solution:

The Y-parameters of a two-port network are defined by expressing the two-port currents I_1 and I_2 in terms of the two-port voltages V_1 and V_2.



$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Condition for Reciprocity:

A two-port network is said to be reciprocal if the ratio of the response at port 2 to an excitation at port 1 is the same as the ratio of the response at port 1 to the same excitation at port 2, with the other port terminated identically in both cases.

(i) As shown in figure (a), Voltage V_s is applied at input port with the output port short circuited.

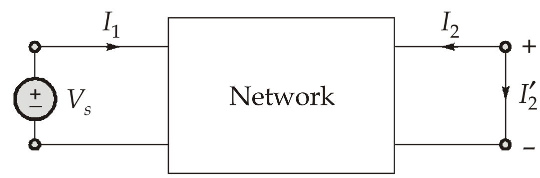


Fig. (a)

i.e.,
$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2 = -I_2'$$

From the Y-parameter equations,

$$-I_2' = Y_{21}V_s$$
$$\frac{I_2'}{V_s} = -Y_{21}$$

(ii) Now, when the voltage V_s is applied at output port with the input port short circuited.

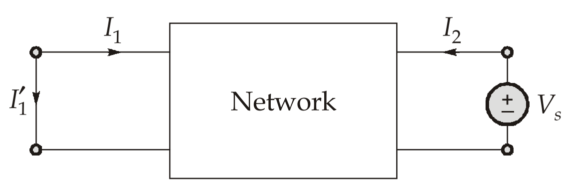


Fig. (b)

$$V_2 = V_s$$
$$V_1 = 0$$
$$I_1 = -I_1'$$

From the Y-parameter equations,

$$-I_1' = Y_{12}V_s$$
$$\frac{I_1'}{V_s} = -Y_{12}$$

Hence, for the network to be reciprocal,

$$\frac{I_2'}{V_s} = \frac{I_1'}{V_s}$$

$\Rightarrow$
$$Y_{12} = Y_{21}$$

Q.5 (b) Solution:

$$\text{Absolute error, } \delta A = A_m - A_t = 1.46 - 1.50 = -0.04 \text{ V}$$

$$\text{Absolute correction, } \delta C = -\delta A = +0.04 \text{ V}$$

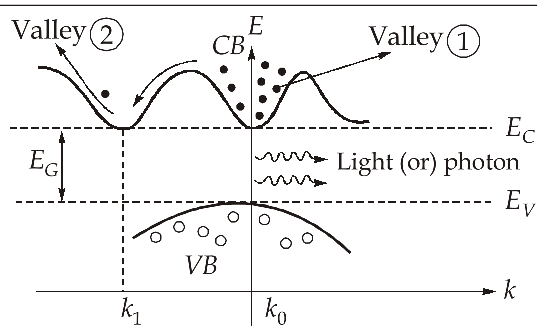
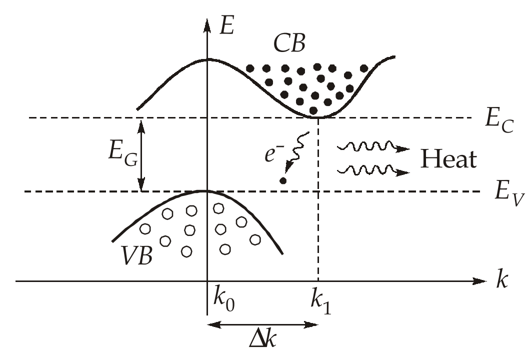
Relative error, ε_r (expressed as a fraction of the true value) $= \dfrac{\delta A}{A_t}$

$$= \frac{-0.04}{1.50} \times 100 = -2.67\%$$

Relative error (expressed as a percentage of f.s.d)

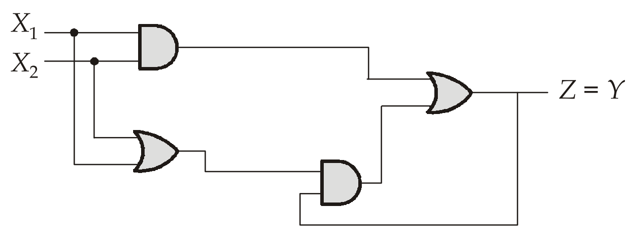
$$= \frac{-0.04}{2.5} \times 100 = -1.6\%$$

Q.5 (c) Solution:

Direct Band Gap Semiconductor	Indirect Band Gap Semiconductor
 	
• When minimum energy level of *CB* and maximum energy level of *VB* occur for same value of '*k*', then such semiconductors are known as 'direct band gap semiconductors'. Eg: GaAs • When an electron makes a transition from *CB* to *VB*, no change in momentum takes place and energy is released mainly in the form of light. So, these materials are suitable in fabrication of LED, LASER etc.	• When minimum energy level of *CB* and maximum energy level of *VB* occur for different value of '*k*', then the semiconductor is known as 'Indirect band gap semiconductors'. Eg : Si, Ge • When an electron makes a transition from *CB* to *VB*, change in momentum takes place and energy is released mainly in the form of heat. So, these materials are not suitable for fabrication of LED but are useful in making other semiconductor devices such as diode, BJT, MOSFET.

Q.5 (d) Solution:

The logic diagram of the circuit can be constructed by assuming X_1 and X_2 as the input and Y to be feedback input. The feedback input is equivalent to the output value i.e., $Y \equiv Z$.



The state table can be constructed as

Present state	Input		Next state	Output
Y	X_1	X_2	Y	Z
0	0	0	0	0
0	0	1	0	0
0	1	1	1	1
0	1	0	0	0
1	0	0	0	0
1	0	1	1	1
1	1	1	1	1
1	1	0	1	1

Q.5 (e) Solution:

Given,

$$\varepsilon_{r_{si}} = 11.7$$

$$N_d = 10^{16} \text{ cm}^{-3}$$

$$\mu_n = 1200 \text{ cm}^2/\text{V-s}$$

The conductivity is,

$$\sigma \approx q\mu_n N_d$$
$$= (1.6 \times 10^{-19}) \times (1200) \times (10^{16})$$
$$= 1.92\ (\Omega\text{-cm})^{-1}$$

The permittivity of silicon is,

$$\in = \in_r \in_0$$
$$= (11.7) \times (8.854 \times 10^{-14}) \text{ F/cm}$$
$$= 1.036 \times 10^{-12} \text{ F/cm}$$

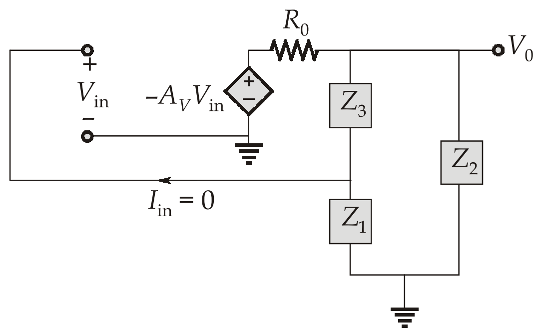
The dielectric relaxation time constant is then,

$$\tau_d = \frac{\in}{\sigma} = \frac{1.036 \times 10^{-12}}{1.92} = 0.5396 \times 10^{-12}$$

$$\Rightarrow \quad \tau_d = 0.5396 \text{ ps}$$

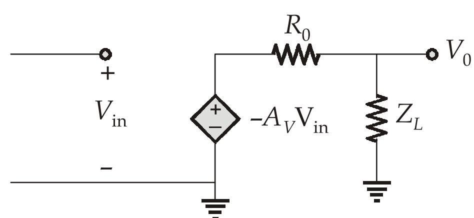
Q.6 (a) Solution:

Drawing the small signal model of the amplifier we have,



$\because \qquad I_{in} = 0;$

The above circuit can be reduced as

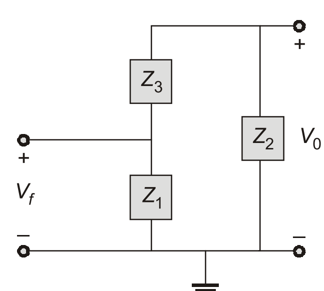


Thus, the overall gain of the amplifier,

$$A = \frac{V_0}{V_{in}} = \frac{-A_V Z_L}{Z_L + R_0}$$

where, $$Z_L = \frac{(Z_1 + Z_3)Z_2}{(Z_1 + Z_2 + Z_3)}$$

For the feedback circuit,



The feedback gain, $$\beta = \frac{V_f}{V_0} = \frac{Z_1}{Z_1 + Z_3}$$

According to the Barkhausen criterion, for sustained oscillations, the loop gain must be equal to or greater than unity (1), and the phase shift around the feedback loop must be a multiple of 360 degrees.

We have,
$$A\beta = \frac{-A_V Z_1 Z_L}{(R_0+Z_L)(Z_1+Z_3)} = \frac{-A_V Z_1\left[\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right]}{\left[R_0+\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right](Z_1+Z_3)}$$
$$= \frac{-A_V Z_1 Z_2}{R_0(Z_1+Z_2+Z_3)+Z_2(Z_1+Z_3)}$$

Now, $Z_1 = jX_1$, $Z_2 = jX_2$ and $Z_3 = jX_3$

$$\Rightarrow \quad A\beta = \frac{A_V(X_1X_2)}{jR_0(X_1+X_2+X_3)-X_2(X_1+X_3)}$$

To produce sustained oscillations, the phase shift of the loop gain $A\beta$ should be 0°.

Thus, $R_0(X_1 + X_2 + X_3) = 0$
$$\Rightarrow \quad X_1 + X_2 + X_3 = 0$$
$$(X_1 + X_3) = -X_2$$
$$\therefore \quad A\beta = \frac{-A_V X_1}{(X_1+X_3)}$$
$$\Rightarrow \quad A\beta = \frac{A_V X_1}{X_2}$$

Hence, X_1 and X_2 should be of the same type i.e., both must be either capacitive or inductive.

Q.6 (b) Solution:

The moving coil instrument reads average value of current while hot wire reads rms value of current. The electrostatic voltmeters do not take any current for their operation and they read the rms value of voltage.

Let 'i' be the instantaneous value of current,
$$i = I_0 + I_{1m}\sin\omega t + I_{2m}\sin 2\omega t$$
$$= 0.5 + 0.3\sin\omega t - 0.2\sin 2\omega t$$

Average value of i, $I_{av} = 0.5$ A

Hence, reading of moving-coil instrument = 0.5 A

RMS value of current i,
$$I_{rms} = \sqrt{I_0^2 + \left(\frac{I_{1m}}{\sqrt{2}}\right)^2 + \left(\frac{I_{2m}}{\sqrt{2}}\right)^2}$$

$$= \sqrt{(0.5)^2 + \left(\frac{0.3}{\sqrt{2}}\right)^2 + \left(\frac{-0.2}{\sqrt{2}}\right)^2} = 0.56125 \text{ A}$$

Hence, reading of hot-wire instrument = 0.56125 A

Reading of electrostatic voltmeter across 1000 Ω resistance,

$$V_R = 0.56125 \times 1000$$
$$= 561.25 \text{ V}$$

Instantaneous value of voltage across 1 mH inductor,

$$V_L = L\frac{di}{dt}$$
$$= (1\times 10^{-3})\frac{d}{dt}[0.5 + 0.3\sin\omega t - 0.2\sin 2\omega t]$$

For $\omega = 10^6$ rad/sec,
$$= (1\times 10^{-3})\left[(0.3)\times\omega\times\cos\omega t - 0.2\times 2\omega\times\cos 2\omega t\right]$$
$$V_L = (300\cos\omega t - 400\cos 2\omega t) \text{ V}$$

Hence, reading of electrostatic voltmeter across 1 mH inductor,

$$V_L = \sqrt{\left(\frac{300}{\sqrt{2}}\right)^2 + \left(\frac{-400}{\sqrt{2}}\right)^2} = 353.55 \text{ V}$$

Q.6 (c) Solution:

(i) The electromagnetic torque (armature torque) in a DC motor is directly proportional to the product of the flux per pole (ϕ) and the armature current (I_a) i.e.

$$T_a \propto \phi I_a$$

Given: Output power of the motor = 40 kW

Losses (Friction, windage and core-losses) = 8500 W = 8.5 kW

Thus,

Electromagnetic Power, P_{em} = 40 + 8.5 = 48.5 kW

The electromagnetic torque developed by the motor,

$$T_{em} = \frac{P_{em}}{\omega} = P_{em}\left(\frac{60}{2\pi N}\right)$$

$$T_{em} = 48.5\times 10^3 \times\left(\frac{60}{2\pi\times 1150}\right) = 402.73 \text{ N-m}$$

If the flux in each pole of the motor is reduced to 70% of its rated value i.e. $\phi = 0.7\phi$ and the armature current remains same $(I'_a = I_a)$, we have

$$\frac{T'_{em}}{T_{em}} = \frac{\phi' I'_a}{\phi I_a}$$

$$T'_{em} = 0.7T_{em} = 0.7 \times 402.73 = 281.91 \text{ N-m}$$

(ii) 1. Given, $2\theta = 27°$. Using Bragg's Law to obtain the interplanar spacing for the (3 2 1) set of planes for rubidium (Rb) (n = 1 for first order reflection)

$$d_{321} = \frac{n\lambda}{2\sin\theta} = \frac{(1)\times 0.0711\times 10^{-9}}{2\times\left(\sin\frac{27°}{2}\right)} = 0.1523 \text{ nm}$$

2. The interplanar spacing for a set of planes with Miller indices (hkl) in a cubic crystal system is given by

$$d = \frac{a}{\sqrt{h^2+l^2+k^2}}$$

Lattice parameter, $a = d_{321}\sqrt{(h)^2+(k)^2+(l)^2}$

$$\Rightarrow \quad a = (0.1523\times 10^{-9})\sqrt{(3)^2+(2)^2+(1)^2} \quad \Rightarrow \quad a = 0.57 \text{ nm}$$

∴ Atomic radius for BCC crystal structure is

$$R = \frac{a\sqrt{3}}{4} = \frac{0.57\times\sqrt{3}}{4} \text{nm} = 0.2468 \text{ nm}$$

Q.7 (a) Solution:

The built in voltage can be calculated as

$$V_j = V_T \ln\left[\frac{N_A N_D}{n_i^2}\right]$$

$$= 0.0259\ln\left[\frac{10^{18}\times 5\times 10^{15}}{(1.5\times 10^{10})^2}\right] = 0.796 \text{ V}$$

Area of the junction, $A = \pi(5 \times 10^{-4})^2$

$= 7.85 \times 10^{-7}$ cm^2

and Depletion layer width, $W = \left[\frac{2\varepsilon_{Si} V_j}{q}\left[\frac{1}{N_A}+\frac{1}{N_D}\right]\right]^{1/2}$

$$= \left[\frac{2(11.8)(8.85\times 10^{-14})(0.796)}{1.6\times 10^{-19}}(10^{-18}+10^{-16}\times 2)\right]^{1/2}$$

$= 0.457$ μm

Now,

(i) $$x_{no} = W\left(\frac{N_A}{N_A+N_D}\right) = \frac{W}{1+\frac{N_D}{N_A}} = \frac{0.457}{1+5\times10^{-3}} = 0.455\ \mu\text{m}$$

(ii) $$x_{po} = W\left(\frac{N_D}{N_A+N_D}\right) = \frac{0.457}{1+\frac{N_A}{N_D}} = \frac{0.457}{1+200} = 2.27\times10^{-3}\ \mu\text{m}$$

(iii) Accumulated space charge on either side of the junction,

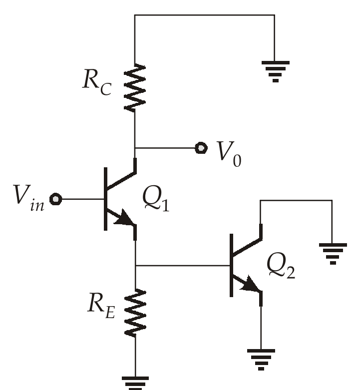
$$|Q_n| = |Q_p| = |qAx_{no}N_D| = |qAx_{po}N_A|$$

$$= |1.6\times10^{-19}\times(7.85\times10^{-7})(2.27\times10^{11})|$$

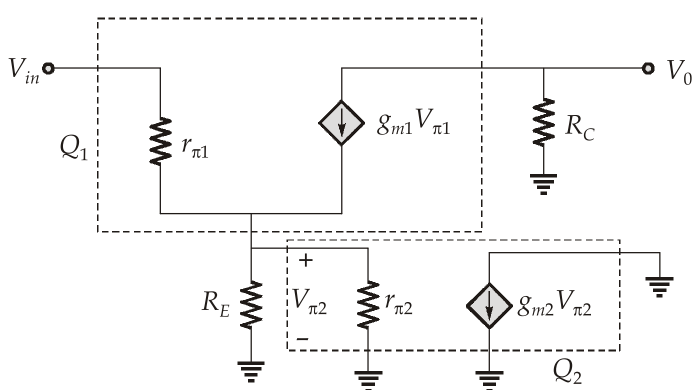
$$= 2.85\times10^{-14}\ \text{C}$$

Q.7 (b) Solution:

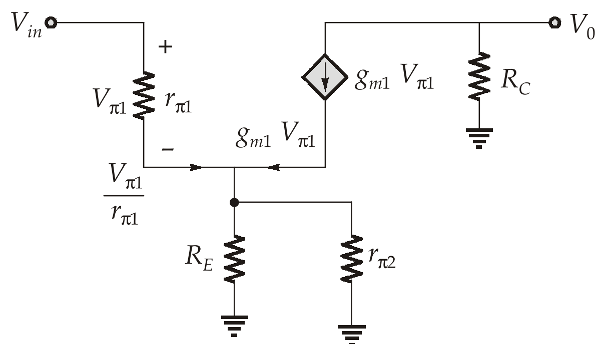
The ac equivalent of the circuit can be drawn as below,



Using the π-model of BJT, we have



The circuit can be redrawn as below,



$$V_{in} = V_{\pi 1} + \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right)(R_E \parallel r_{\pi 2})$$

$$= \left[1 + \left(\frac{1}{r_{\pi 1}} + g_{m1}\right)(R_E \parallel r_{\pi 2})\right]V_{\pi 1}$$

Now, $$V_0 = -g_{m1} R_C V_{\pi 1}$$

$\therefore$ $$\frac{V_0}{V_{in}} = \frac{-g_{m1}R_C}{1 + \left(\frac{1}{r_{\pi 1}} + g_{m1}\right)(R_E \parallel r_{\pi 2})}$$

Q.7 (c) Solution:

We need to consider the three time intervals $t \leq 0$, $0 \leq t \leq 4$, and $t \geq 4$ separately. For $t < 0$, switches S_1 and S_2 are open, thus $i = 0$. Since the inductor current cannot change instantly,

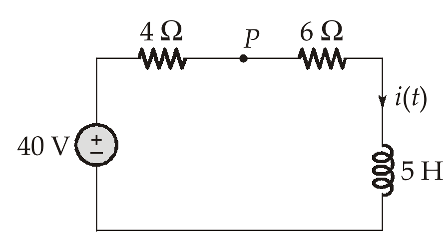
$$i(0^-) = i(0^+) = 0$$

For $0 \leq t \leq 4$, S_1 is closed, so the 4 Ω and 6 Ω resistors are in series. (Remember, at this time, S_2 is still open). Hence, assuming for now that S_1 is closed and S_2 is open forever,

$$i(\infty) = \frac{40}{4+6} = 4 \text{ A},$$

$$R_{Th} = 4 + 6 = 10\ \Omega$$

$$\tau = \frac{L}{R_{Th}} = \frac{5}{10} = \frac{1}{2} \text{ sec}$$



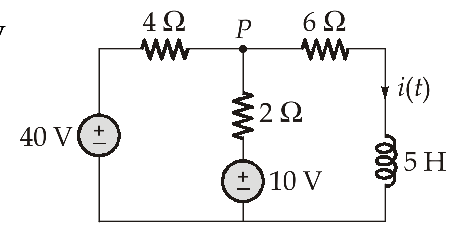
$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

$$= 4 + (0 - 4)e^{-2t} = 4(1 - e^{-2t}) \text{ A}, \quad 0 \leq t \leq 4$$

For $t \geq 4$, S_2 is closed; thus the 10 V voltage source is connected, and the circuit changes. This sudden change does not affect the inductor current because the current through the inductor cannot change abruptly. Thus, the inductor current is

$$i(4^+) = i(4^-) = 4(1 - e^{-8}) \simeq 4 \text{ A}$$

To find $i(\infty)$, let v be the voltage at node P in figure. Using KCL,



$$\frac{40 - v}{4} + \frac{10 - v}{2} = \frac{v}{6} \quad \Rightarrow \quad v = \frac{180}{11} \text{ V}$$

$$i(\infty) = \frac{v}{6} = \frac{30}{11} = 2.727 \text{ A}$$

The Thevenin resistance at the inductor terminals is

$$R_{Th} = (4||2) + 6 = \frac{4 \times 2}{6} + 6 = \frac{22}{3} \ \Omega$$

and
$$\tau = \frac{L}{R_{Th}} = \frac{5}{22/3} = \frac{15}{22} \text{ sec}$$

Hence,
$$i(t) = i(\infty) + [i(4^+) - i(\infty)]\, e^{-(t-4)/\tau}, \quad t \geq 4$$

We need $(t - 4)$ in the exponential because of the time delay. Thus,

$$i(t) = 2.727 + (4 - 2.727)\, e^{-(t-4)/\tau}, \quad \tau = \frac{15}{22}$$

$$= 2.727 + 1.273\, e^{-1.4667(t-4)}, \quad t \geq 4$$

Putting all this together,

$$i(t) = \begin{cases} 0, & t \leq 0 \\ 4(1 - e^{-2t}), & 0 \leq t \leq 4 \\ 2.727 + 1.273\, e^{-1.4667(t-4)}, & t \geq 4 \end{cases}$$

At $t = 2$,
$$i(2) = 4(1 - e^{-4}) = 3.93 \text{ A}$$

At $t = 5$,
$$i(5) = 2.727 + 1.273\, e^{-1.4667} = 3.02 \text{ A}$$

Q.8 (a) Solution:

(i) Given:

$$\mu_n = 1000 \text{ cm}^2/\text{V-s}$$

$$\mu_p = 600 \text{ cm}^2/\text{V-s}$$

$$N_C = N_V = 10^{19} \text{ cm}^{-3}$$

$$\sigma_i = 10^{-6} \ (\Omega\text{-cm})^{-1} \text{ at } T = 300 \text{ K}$$

The conductivity of the intrinsic semiconductor is given by,

$$\sigma_i = q n_i (\mu_n + \mu_p)$$

Then,
$$10^{-6} = (1.6 \times 10^{-19}) \times n_i \times (1000 + 600)$$

$\Rightarrow$
$$n_i(300 \text{ K}) = 3.9063 \times 10^9 \text{ cm}^{-3}$$

We know,
$$n_i^2 = N_C N_V e^{-\left(\frac{E_g}{kT}\right)}$$

(or) $$E_g = kT\ln\left(\frac{N_C N_V}{n_i^2}\right) = (0.0259)\ln\left(\frac{(10^{19})^2}{(3.9063\times10^9)^2}\right)$$

$\Rightarrow$ $$E_g = 1.1222 \text{ eV}$$

Now, at 500 K $$n_i^2(500\text{ K}) = (10^{19})^2 e^{-\left[\frac{1.1222}{(0.0259)\left(\frac{500}{300}\right)}\right]}$$

$\Rightarrow$ $$n_i^2(500\text{ K}) = 5.125 \times 10^{26}$$

$\Rightarrow$ $$n_i(500\text{ K}) = 2.264 \times 10^{13}\text{ cm}^{-3}$$

The conductivity at T = 500 K,

$$\sigma_i = qn_i(\mu_n + \mu_p)$$
$$= 1.6 \times 10^{-19} \times 2.264 \times 10^{13} \times (1000 + 600)$$
$$= 5.8 \times 10^{-3}\ (\Omega\text{-cm})^{-1}$$

(ii) The kinetic energy gained by an electron accelerated through a potential difference E_a is given by

$$qE_a = \frac{1}{2}mv^2$$

Velocity of electron beam,

$$v_{ox} = \sqrt{\frac{2qE_a}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2000}{9.1\times10^{-31}}}$$
$$= 26.52 \times 10^6 \text{ m/s}$$

The maximum transit time is given as 1/4 of a cycle i.e.

$$t_{\max} = \frac{T}{4} = \frac{l}{v_{ox}}$$

Where v_{ox} is the velocity of electron beam and l is the length of horizontal plates. Substituting $T = 1/f$, we get

Cutoff frequency, $$f_c = \frac{v_{ox}}{4l} = \frac{26.52\times10^6}{4\times50\times10^{-3}}$$
$$= 132.6 \times 10^6$$
$$= 132.6 \text{ MHz}$$

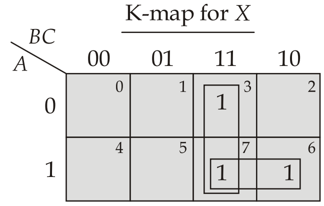
Q.8 (b) Solution:

(i) Truth table:

	A	B	C	X	Y	Z
m_0	0	0	0	0	0	1
m_1	0	0	1	0	1	0
m_2	0	1	0	0	1	1
m_3	0	1	1	1	0	0
m_4	1	0	0	0	1	0
m_5	1	0	1	0	1	1
m_6	1	1	0	1	0	0
m_7	1	1	1	1	0	1

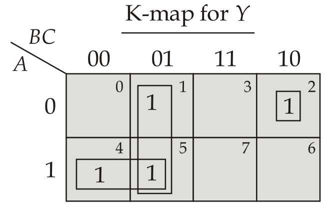
Minimization:

K-map for X



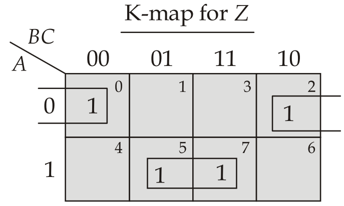
$$X = AB + BC$$

K-map for Y



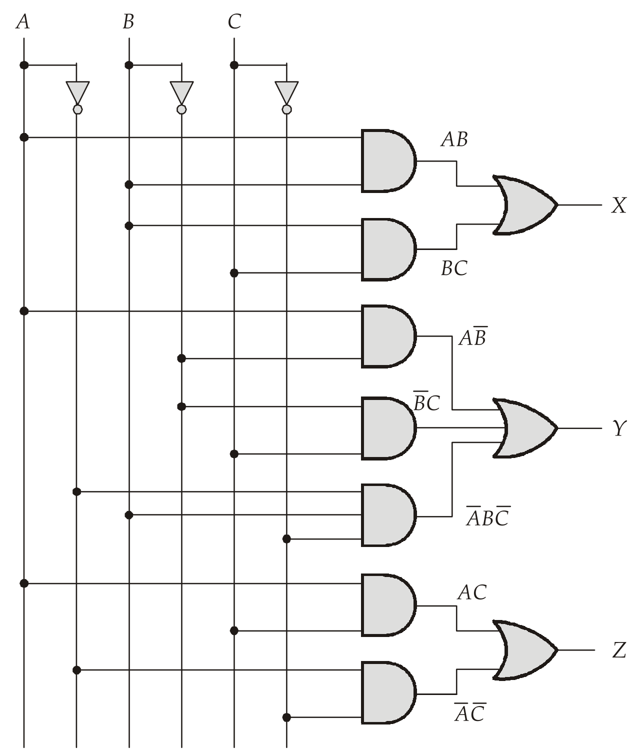
$$Y = A\bar{B} + \bar{B}C + \bar{A}B\bar{C}$$

K-map for Z



$$Z = AC + \bar{A}\bar{C}$$

Logic circuit:



(ii) The truth table of the given circuit can be constructed as shown below.

A	B	C	D	$X = \overline{A}BC$	$Y = A + D$	$Z = \overline{Y}$	$F = XZ$
0	0	0	0	0	0	1	0
0	0	0	1	0	1	0	0
0	0	1	0	0	0	1	0
0	0	1	1	0	1	0	0
0	1	0	0	0	0	1	0
0	1	0	1	0	1	0	0
0	1	1	0	1	0	1	1
0	1	1	1	1	1	0	0
1	0	0	0	0	1	0	0
1	0	0	1	0	1	0	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	0	1	0	0
1	1	0	1	0	1	0	0
1	1	1	0	0	1	0	0
1	1	1	1	0	1	0	0

The given circuit can be used to detect the binary combination $(ABCD) = (0110)_2$.

Q.8 (c) Solution:

Given: $R = 10\ \Omega$; $L = 0.2$ H; $C = 40\ \mu$F; $V = 100$ V

(i) Resonant frequency, $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{0.2\times40\times10^{-6}}} = 56.27 \text{ Hz}$$

(ii) As resonance, $X_L = X_C$. Thus, $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$.

Thus, $$I_0 = \frac{V}{R} = \frac{100}{10} = 10 \text{ A}$$

(iii) Power, $$P_0 = I_0^2 R = (10)^2 \times (10) = 1000 \text{ W}$$

(iv) Power factor, $$\cos\phi = \frac{R}{|Z|} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

But at resonance, $$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \quad \cos\phi = 1$$

(i.e. the current is in phase with the applied voltage)

(v) Voltage across $R = RI_0 = 10 \times 10 = 100$ V

Voltage across L, $$|V_L| = X_L I_0 = 2\pi f_0 L I_0 = 2\pi \times 56.27 \times 0.2 \times 10 = 707.11 \text{ V}$$

Voltage across C, $$|V_C| = X_C I_0 = \frac{I_0}{2\pi f_0 C} = \frac{10}{2\pi\times56.27\times40\times10^{-6}} = 707.11 \text{ V}$$

(vi) Quality factor, $$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.2}{40\times10^{-6}}}$$

$$\Rightarrow \quad Q = 7.071$$

(vii) Half-power frequencies:

$$f_1 = f_0 - \frac{R}{4\pi L} = 56.27 - \frac{10}{4\pi\times 0.2} = 52.3 \text{ Hz}$$

$$f_2 = f_0 + \frac{R}{4\pi L} = 56.27 + \frac{10}{4\pi\times 0.2} = 60.25 \text{ Hz}$$

Alternate Solution:

$$f_1 = \frac{1}{2\pi}\left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-10}{2\times 0.2} + \sqrt{\left(\frac{10}{2\times 0.2}\right)^2 + \frac{1}{0.2\times 40\times 10^{-6}}}\right]$$

$$= 52.3 \text{ Hz}$$

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$= \frac{1}{2\pi}\left[\frac{10}{2\times 0.2} + \sqrt{\left(\frac{10}{2\times 0.2}\right)^2 + \frac{1}{0.2\times 40\times 10^{-6}}}\right]$$

$$= 60.25 \text{ Hz}$$

(viii) The phasor diagram and variation of current with frequency can be drawn as below,

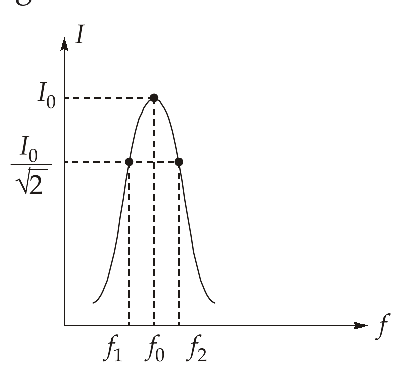


Fig. (a)

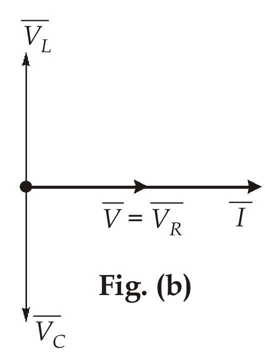


Fig. (b)

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