# Detailed Solutions

# ESE-2025 Mains Test Series

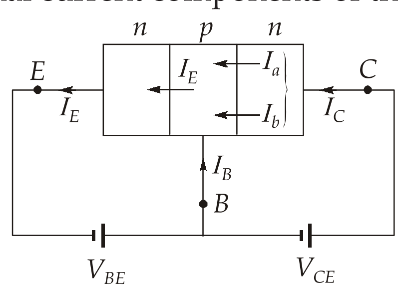
# Electrical Engineering Test No : 9

## Section-A

**Q.1 (a) Solution:**

**(i)** The solution for the given problem can be obtained from the basic transport mechanism of the BJT.

Let us consider the internal current components of the BJT as shown below.




$I_a$ = Current due to injected minority carriers crossing the CB junction

$I_b$ = Current due to thermally generated minority carriers crossing the CB junction

$I_a = \alpha I_E$

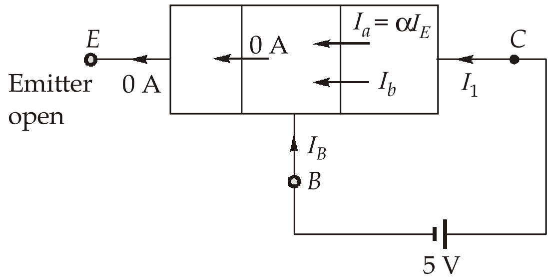
where,

$\alpha$ = large signal current gain of CB configuration

$\beta$ = large signal current gain of CE configuration

$I_b = I_{CO}$ = reverse saturation current of the CB junction

**For the circuit given in the figure (a):**




As the emitter is open circuited, $I_E = 0$ A

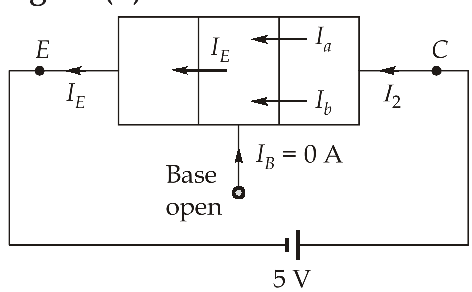
So, $I_a = \alpha I_E = 0$ A

$I_b = I_{CO}$ $\quad \because$ *CB* junction is reverse biased

and $I_1 = I_C = I_a + I_b = 0\text{ A} + I_{CO}$

$I_1 = I_{CO}$ ...(i)

**For the circuit given in the figure (b) :**




$$I_a = \alpha I_E$$

$$I_b = I_{CO}$$

$$I_2 = I_C = I_E \qquad \because I_B = 0 \text{ A}$$

So, $I_2 = I_E = \alpha I_E + I_{CO}$

$$I_E(1 - \alpha) = I_{CO}$$

$$I_E = \frac{1}{(1-\alpha)} I_{CO} = \left[1 + \frac{\alpha}{(1-\alpha)}\right] I_{CO}$$

$$I_E = (1 + \beta) I_{CO} \qquad \because \beta = \frac{\alpha}{(1-\alpha)}$$

$$I_2 = I_E = (1 + \beta) I_{CO} \qquad \text{...(ii)}$$

From the equations (i) and (ii), it is clear that, the current $I_2$ is $(1 + \beta)$ times of current $I_1$.

**(ii)** Given that, when the transistor is in CB configuration,

$$I_C = 2.98 \text{ mA}$$

$$I_E = 3 \text{ mA}$$

$$I_{CO} = 0.01 \text{ mA}$$

For CB configuration,

$$I_C = \alpha I_E + I_{CO}$$

$$\therefore \quad \alpha = \frac{I_C - I_{CO}}{I_E} = \frac{(2.98 - 0.01)\times 10^{-3}}{3\times 10^{-3}} = 0.99$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.99}{1-0.99} = 99$$

Also given that, when the same transistor is rebiased to get CE configuration,

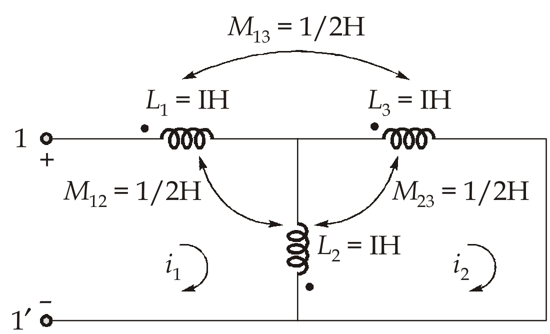
$$I_B = 30\ \mu A$$

$$I_C = \beta I_\beta + (1+\beta)I_{CO}$$

$$= 99 \times 30 \times 10^{-6} + (1+99) \times 0.01 \times 10^{-3}$$

$$I_C = 3.97\ \text{mA}$$

**Q.1 (b) Solution:**




Taking loop equation in mesh (1), we get

$$L_1\frac{di_1}{dt} + L_2\frac{d(i_1-i_2)}{dt} + M_{12}\frac{d(i_2-i_1)}{dt} - M_{21}\frac{di_i}{dt} + M_{13}\frac{di_2}{dt} - M_{23}\frac{di_2}{dt} = v_1$$

$$\text{or,} \quad \frac{di_1}{dt} + \frac{d(i_1-i_2)}{dt} + \frac{1}{2}\frac{d(i_2-i_1)}{dt} - \frac{1}{2}\frac{di_i}{dt} + \frac{1}{2}\frac{di_2}{dt} - \frac{1}{2}\frac{di_2}{dt} = v_1$$

$$\text{or,} \quad \frac{di_1}{dt} - \frac{1}{2}\frac{di_2}{dt} = v_i \quad \text{...(i)}$$

Taking loop equation from mesh (2), we get

$$L_2\frac{d(i_2-i_1)}{dt} + L_3\frac{di_2}{dt} + M_{32}\frac{d(i_2-i_1)}{dt} + M_{31}\frac{di_1}{dt} + M_{23}\frac{di_2}{dt} - M_{21}\frac{di}{dt} = 0$$

$$\text{or} \quad \frac{d(i_2-i_1)}{dt} + \frac{di_2}{dt} + \frac{1}{2}\frac{d(i_2-i_1)}{dt} + \frac{1}{2}\frac{di_1}{dt} + \frac{1}{2}\frac{di_2}{dt} - \frac{1}{2}\frac{di_2}{dt} = 0$$

$$\text{or} \quad \frac{-3}{2}\frac{di_1}{dt} + 3\frac{di_2}{dt} = 0 \quad \text{...(ii)}$$

or $$\frac{1}{2}\frac{di_1}{dt} = \frac{di_2}{dt}$$

Putting the value of $\frac{di_2}{dt}$ in equation (i), we get

$$\frac{di_1}{dt} - \frac{1}{2}\left(\frac{1}{2}\frac{di_1}{dt}\right) = v_1$$

or $$\frac{3}{4}\frac{di_1}{dt} = v_1$$

or $$v_1 = \frac{3}{4}\frac{di_1}{dt}$$

$\therefore$ $$L_{eq} = \frac{3}{4}\text{H}$$

or $$L_{eq} = 0.75 \text{ H}$$

**Q.1 (c) Solution:**

$$(D^2 - 1)y = x \sin x + (1 + x^2)e^x$$

Auxiliary equation is $D^2 - 1 = 0$

Roots are $D = 1, -1$

$$\text{C.F.} = C_1e^x + C_2e^{-x}$$

$$\text{P.I.} = \frac{1}{D^2-1}x\sin x + \frac{1}{D^2-1}(1+x^2)e^x$$

$$\text{P.I.} = PI_1 + PI_2 \quad \text{...(i)}$$

$$\text{P.I.}_1 = \frac{1}{D^2-1}x\sin x$$

$$= x\left\{\frac{1}{D^2-1}\sin x\right\} - \left\{\frac{2D}{(D^2-1)^2}\sin x\right\}$$

$$= x\left\{\frac{1}{(-1-1)}\sin x\right\} - \left\{\frac{2\cos x}{(-1-1)^2}\right\}_{D^2=-1^2}$$

$$= \frac{-x\sin x}{2} - \frac{\cos x}{2} = \frac{-1}{2}[x\sin x + \cos x]$$

$$\text{PI}_2 = \frac{1}{D^2-1}(1+x^2)e^x$$

$$= e^x\left\{\frac{1}{\left[(D+1)^2-1\right]}(1+x^2)\right\}$$

$$= e^x\left\{\frac{1}{(D^2+2D)}(1+x^2)\right\}$$

$$= e^x\left\{\frac{1}{2D\left(1+\frac{D}{2}\right)}(1+x^2)\right\}$$

$$= \frac{e^x}{2}\left\{\frac{1}{D}\left(1+\frac{D}{2}\right)^{-1}(1+x^2)\right\}$$

$$= \frac{e^x}{2}\left\{\frac{1}{D}\left(1-\frac{D}{2}+\frac{D^2}{4}-\frac{D^3}{8}\right)(1+x^2)\right\}$$

$$= \frac{e^x}{2}\left\{\left(\frac{1}{D}-\frac{1}{2}+\frac{D}{4}-\frac{D^2}{8}\right)(1+x^2)\right\}$$

$$= \frac{e^x}{2}\left\{\left(\frac{1}{D}-\frac{1}{2}+\frac{D}{4}-\frac{D^2}{8}\right)(1+x^2)\right\}$$

$$= \frac{e^x}{2}\left\{\left(x+\frac{x^3}{3}\right)-\frac{1}{2}(1+x^2)+\frac{1}{4}(2x)-\frac{1}{8}(2)\right\}$$

$$= \frac{e^x}{2}\left\{x+\frac{x^3}{3}-\frac{1}{2}-\frac{x^2}{2}+\frac{x}{2}-\frac{1}{4}\right\}$$

$$= \frac{e^x}{2}\left\{\frac{12x+4x^3-6-6x^2+6x-3}{12}\right\}$$

$$= \frac{e^x}{24}\left\{4x^3-6x^2+18x-9\right\}$$

The solution is

$$y = C_1 e^x + C_2 e^{-x} - \frac{1}{2}[x \sin x + \cos x] + \frac{e^x}{24}\left[4x^3 - 6x^2 + 18x - 9\right]$$

**Q.1 (d) Solution:**

Since $z = 0$ is the boundary, $\hat{k}$ is the normal to the boundary plane, and the normal component of the field is

$$E_{1n} = \vec{E}_1 \cdot \hat{k} = 70$$

$$\therefore \quad \vec{E}_{1n} = 70\hat{k}$$

$$\therefore \quad \vec{E}_{1t} = \vec{E}_1 - \vec{E}_{1n} = -30\hat{i} + 50\hat{j}$$

By the boundary conditions for dielectric-dielectric interface, we get

$$\vec{E}_{2t} = \vec{E}_{1t} = -30\hat{i} + 50\hat{j}$$

and

$$\varepsilon_{r2}\vec{E}_{2n} = \varepsilon_{r1}\vec{E}_{1n}$$

$$\Rightarrow \quad \vec{E}_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\vec{E}_{1n} = \frac{2.5}{4} \times 70\hat{k} = 43.75\hat{k}$$

$$\therefore \quad \vec{E}_2 = \vec{E}_{2n} + \vec{E}_{2t} = \left(-30\hat{i} + 50\hat{j} + 43.75\hat{k}\right)\text{V/m}$$

Hence, the flux density is

$$\vec{D}_2 = \varepsilon_2\vec{E}_2 = \varepsilon_0\varepsilon_{r2} \times (-30\hat{i} + 50\hat{j} + 43.75\hat{k})$$

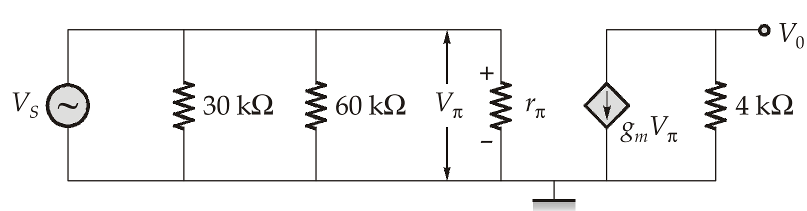
$$= 8.854 \times 10^{-12} \times 4 \times (-30\hat{i} + 50\hat{j} + 43.75\hat{k})$$

$$= -1.061\hat{i} + 1.768\hat{j} + 1.547\hat{k} \text{ nC/m}^2$$

The angle between electric field intensity in the second medium and the normal to the boundary surface is given as

$$\theta_2 = \tan^{-1}\left(\frac{\vec{E}_{2t}}{\vec{E}_{2n}}\right) = \tan^{-1}\left(\frac{\sqrt{30^2 + 50^2}}{43.75}\right) = 53.12°$$

**Q.1 (e) Solution:**



The amplifier equivalent model is

$$V_0 = -g_m V_\pi \times 4\text{ k}$$

Where,

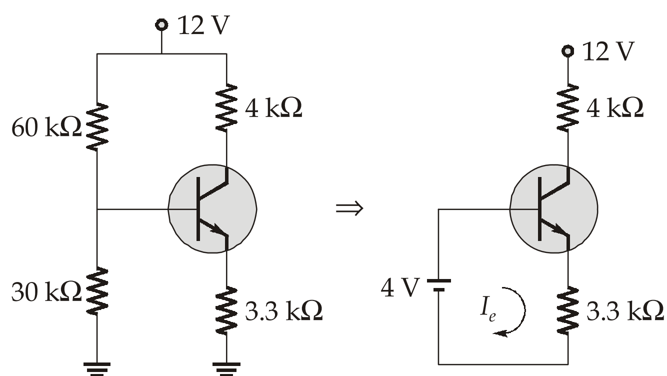
$$V_\pi = V_s$$

$$V_0 = -g_m V_s \times 4\text{ k}$$

$$g_m = \frac{|I_c|}{V_T}$$

$I_C$ is a DC current

The DC equivalent of the circuit is




$$I_e = \frac{4-0.7}{3.3} = 1\text{ mA},$$

$$I_c = I_e$$

$$g_m = \frac{1\text{ mA}}{25\text{ mV}} \qquad \text{(Where } V_T = 25\text{ mV)}$$

$$A_v = \frac{V_0}{V_s} = -\frac{1\text{ mA}}{0.025}\times 4\text{k} = -160$$

$$|A_v| = 160$$

**Q.2 (a) Solution:**

**(i)** The hybrid parameter of two port is given by

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$V_2 = h_{21}I_1 + h_{22}V_2$$

When we make,

$$V_2 = 0$$

$$V_x = \frac{(1-a)I_1R_2R_3}{(R_2+R_3)}$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_1 + V_x$$

$$= R_1 + \frac{(1-a)I_2R_2R_3}{(R_2+R_3)} = \frac{R_1R_2 + R_1R_3 + (1-a)R_2R_3}{R_2+R_3}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -a - \frac{(1-a)R_2}{R_2+R_3} = \frac{-(aR_3+R_2)}{(R_2+R_3)}$$

Hence, $$h_{21} = \frac{-(aR_3+R_2)}{(R_2+R_3)}$$

Now we open circuit port (1),

Then, $$I_1 = 0$$

Hence, $$h_{12} = \frac{V_1}{V_2} = \frac{R_2}{R_2+R_3}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{R_2+R_3}$$

Therefore, hybrid parameter are

$$h_{11} = \frac{R_1R_2 + R_1R_3 + (1-a)R_2R_3}{R_2+R_3}$$

$$h_{12} = \frac{R_2}{R_2+R_3}$$

$$h_{21} = \frac{-(aR_3+R_2)}{(R_2+R_3)}$$

$$h_{22} = \frac{1}{(R_2+R_3)}$$

**(ii)** Now $g$-parameters are given by the equations

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

When $I_2 = 0$, i.e. we open circuit port (2), then

$$g_{11} = \frac{I_1}{V_1} = \frac{1}{R_1+R_2}$$

$$g_{21} = \frac{V_2}{V_1} = \frac{R_2 + aR_3}{R_1 + R_2}$$

Now, short circuit port (1) then

$$g_{12} = \frac{I_1}{I_2} = \frac{-R_2}{R_1 + R_2}$$

$$g_{22} = \frac{V_2}{I_2} = \left(\frac{-aR_2}{R_1 + R_2} + 1\right)R_3 + \left(\frac{-R_2}{R_1 + R_2} + 1\right)R_2$$

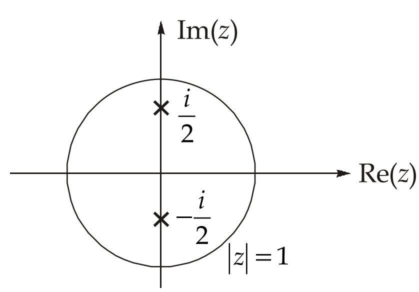
$$g_{22} = \frac{R_1R_2 + R_1R_3 + (1-a)R_2R_3}{R_1 + R_2}$$

**Q.2 (b) (i) Solution:**

Let, $$f(z) = \frac{\cosh z}{4z+1} = \frac{\cosh(z)}{4\left[z^2 + \frac{1}{4}\right]}$$

$$= \frac{\frac{\cosh(z)}{4}}{\left(z - \frac{i}{2}\right)\left(z + \frac{i}{2}\right)}$$

Then the singular points of $f(z)$ are $z = \frac{i}{2}, \frac{-i}{2}$




Here, the two singular points $z = \frac{i}{2}$ and $z = \frac{-i}{2}$ lie inside the circle $|z| = 1$.

Now, $$f(z) = \frac{\cosh z}{4}\left[\frac{1}{\left(z - \frac{i}{2}\right)\left[z - \left(\frac{-i}{2}\right)\right]}\right]$$

$$f(z) = \left[\frac{\cosh(z)}{4}\left[\frac{1}{\left(z-\frac{i}{2}\right)\left(\frac{i}{2}+\frac{i}{2}\right)}\right]+\frac{1}{\left(z+\frac{i}{2}\right)\left(\frac{-i}{2}-\frac{i}{2}\right)}\right]$$

$$f(z) = \frac{\left(\frac{\cosh(z)}{4i}\right)}{\left[z-\frac{i}{2}\right]}+\frac{\left(\frac{\cosh(z)}{-4i}\right)}{\left[z-\left(-\frac{i}{2}\right)\right]}$$

By Caychy's integral formula, we have

$$\oint_C f(z)dz = \frac{1}{4i}\oint\frac{(\cosh(z))}{\left[z-\frac{1}{2}\right]}dz+\frac{1}{-4i}\oint_C\frac{\cosh(z)}{\left[1-\left(\frac{-i}{2}\right)\right]}dz$$

$$= \left(\frac{1}{4i}\right)\left[2\pi i(\cos z)_{z=i/2}\right]+\left(\frac{-1}{4i}\right)\left[2\pi i(\cos z)_{z=-i/2}\right]$$

$$= \frac{\pi}{2}\left[\cosh\left(\frac{i}{2}\right)+\right]\left[\frac{-\pi}{2}\right]\left[\cosh\left(\frac{-i}{2}\right)\right]$$

$$= \frac{\pi}{2}\cosh\left(\frac{i}{2}\right)-\frac{\pi}{2}\cosh\left(\frac{-i}{2}\right) = 0 \qquad [\because \cosh(-z) = \cosh(z)]$$

**Q.2 (b) (ii) Solution:**

$$T = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$\Rightarrow$
$$T^{-1} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

Now,
$$D = T^{-1}AT = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}a & h\\ h & b\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$= \begin{bmatrix}a\cos\theta-h\sin\theta & h\cos\theta-b\sin\theta\\ a\sin\theta+h\cos\theta & h\sin\theta+b\cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$= \begin{bmatrix}a\cos^2\theta+b\sin^2\theta-2h\sin\theta\cos\theta & (a-b)\sin\theta\cos\theta+h\sin^2\theta+h\cos^2\theta\\ (a-b)\sin\theta\cos\theta+h\cos^2\theta-h\sin^2\theta & a\sin^2\theta+2h\sin\theta\sin\cos\theta+b\cos^2\theta\end{bmatrix}$$

$$= \begin{bmatrix} a\cos^2\theta - h\sin 2\theta + b\sin^2\theta & (a-b)\sin\theta\cos\theta + h\cos 2\theta \\ (a-b)\sin\theta\cos\theta + h\cos 2\theta & a\sin^2\theta + h\sin 2\theta + b\cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \text{ being diagonal matrix}$$

iff

$$(a-b)\sin\theta\cos\theta + h\cos^2\theta = 0$$

$$\frac{(a-b)}{2}\sin 2\theta + h\cos 2\theta = 0$$

$$\Rightarrow \quad \frac{(a-b)}{2}\sin 2\theta = -h\cos^2\theta$$

$$\tan 2\theta = \frac{2h}{b-a}$$

$$\Rightarrow \quad \theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{b-a}\right)$$

**Q.2 (c) Solution:**

**(i)** $$\sigma = ne\mu_d$$

Concentration of conduction electrons,

$$n = \frac{\sigma}{e\mu_d} = \frac{1}{\rho e\mu_d}$$

$$= \frac{1}{(8.37\times10^{-8}\times100)(1.602\times10^{-19})(6)}\ \text{cm}^{-3}$$

$$n = 1.243 \times 10^{23}\ \text{cm}^{-3}$$

Atomic concentration, $$n_{at} = \frac{dN_A}{M_{at}} = \frac{7.31\times6.022\times10^{23}}{114.82}\ \text{cm}^{-3}$$

$$n_{at} = 3.834 \times 10^{22}\ \text{cm}^{-3}$$

Effective number of conduction electrons donated per In atom ($n_{\text{eff}}$) is,

$$n_{\text{eff}} = \frac{n}{n_{at}} = \frac{1.243\times10^{23}}{3.834\times10^{22}} = 3.24$$

**(ii)** If $\tau$ is the mean scattering time, $l$ is the mean free path and $u$ = mean speed of conduction electrons, then

$$l = u\tau$$

Drift mobility, $\mu_d = \dfrac{e\tau}{m_e}$; $m_e$ = electron mass

So, $\tau = \dfrac{\mu_d m_e}{e} = \dfrac{(6\times10^{-4})(9.1094\times10^{-31})}{(1.602\times10^{-19})}$ s

$= 3.412 \times 10^{-15}$ s

Mean free path, $l = u\tau = (1.74 \times 10^{8})(3.412 \times 10^{-15})$ cm

$= 5.94$ nm

**(iii)** According to Wiedemann – Franz – Lorenz law,

Thermal conductivity, $K = \sigma T C_{WFL}$

Where, $C_{WFL}$ = Lorenz number (or) Wiedemann – Franz – Lorenz coefficient

$$C_{WFL} = \frac{\pi^2 k_B^2}{3e^2} = \frac{\pi^2\times(1.381\times10^{-23})^2}{3\times(1.602\times10^{-19})^2}$$

$= 2.443 \times 10^{-8}\ W\Omega K^{-2}$

So, $K = \dfrac{300\times2.443\times10^{-8}}{8.37\times10^{-8}}\ \text{W m}^{-1}\text{K}^{-1}$

$= 87.563\ \text{W m}^{-1}\text{K}^{-1}$

**Q.3 (a) (i) Solution:**

Given that series resistance,

$R_S = 5\ k\Omega = 5000\ \Omega$

Series inductance, $L_S = 0.8$ H

Series capacitance, $C_S = 0.08$ pF $= 8 \times 10^{-14}$ F

Parallel capacitance, $C_P = 1.0$ pF $= 1 \times 10^{-12}$ F

We know that,

Series resonant frequency, $f_s = \dfrac{L}{2\pi\sqrt{L_S C_S}} = \dfrac{1}{2\pi\sqrt{0.8\times8\times10^{-14}}} = 629$ kHz

Also, parallel resonant frequency,

$$f_p = \frac{1}{2\pi}\sqrt{\frac{1+\dfrac{C_S}{C_P}}{L_S C_S}} = \frac{1}{2\pi}\sqrt{\frac{1+\dfrac{8\times10^{-14}}{1\times10^{-12}}}{0.8\times8\times10^{-14}}} = 654 \text{ kHz}$$

**Q.3 (a) (ii) Solution:**

Given, $R_C = 4.7 \text{ k}\Omega$, $R_E = 3.3 \text{ k}\Omega$

$V_{CC} = 12 \text{ V}$, $V_{EE} = -12 \text{ V}$

We have,

$$I_E = \frac{V_{EE} - V_{BE}}{2R_E} = \frac{12 - 0.7}{2 \times 3.3 \times 10^3} = 1.712 \text{ mA}$$

Now, $I_C = I_E = 1.712 \text{ mA}$

Therefore, $I_{CQ} = 1.712 \text{ mA}$

Also we write,

$$V_{CEQ} = V_{CC} + V_{BE} - I_{CQ}R_C$$

$$= 12 + 0.7 - 1.712 \times 10^{-3} = 4.653 \text{ V}$$

Therefore $Q$ points is (1.712 mA, 4.653 V)

**Q.3 (b) Solution:**

An oscilloscope is a graph display device. It draws the graph of an electrical signal. Oscilloscope are mainly of two types:

1. Analog oscilloscope
2. Digital oscilloscope

- **Analog oscilloscope:** Works with continuous variable voltage whereas digital oscilloscope works with voltage samples.
- Analog oscilloscope could be of dual beam type in which two separate beams are produced by separate electron guns. It can also be of dual trace type.
- There can also be storage type analog oscilloscope. These oscilloscope allows the trace pattern to remain on screen even after several minutes of removing the signal.
- In analog sampling oscilloscope only samples of signals are taken. The samples are then reassembled to create waveform.
- **Digital oscilloscope:** Digital storage oscilloscope use digital memory, which can store data as long as it is required.
- Digital sampling oscilloscope (DSO) works on same principle as analog sampling oscilloscope.
- **Digital phosphor oscilloscope:** It combines the features of DSO and analog phosphor oscilloscope.
- **Mixed domain oscilloscope:** It combines RF spectrum analyzer, logic analyzer and digital oscilloscope to observe signal from each domain i.e. analog, RF and logic time correlated with each other.

**Application:** Measurement of phase and frequency using Lissajous pattern - when sinusoidal voltages are applied simultaneously on horizontal and vertical plates, Lissajous pattern are formed. Each type of pattern give information about frequencies and phase of applied signal.

- **Measurement of voltages and currents:** Deflection is proportional to applied voltage. Current can be measured by measuring the voltage drop across a known resistance.
- Display, measurement and analysis of waveforms.

**Velocity of electron:** Loss in potential energy of electron = Gain in kinetic energy

$e$ = Charge of electron = $1.6 \times 10^{-19}$ C

$V$ = Potential different between plates = 2000 V (Given)

$m$ = Mass of electron = $9.31 \times 10^{-31}$ kg

$v$ = Velocity of electron = ?

Potential energy of an electron at potential,

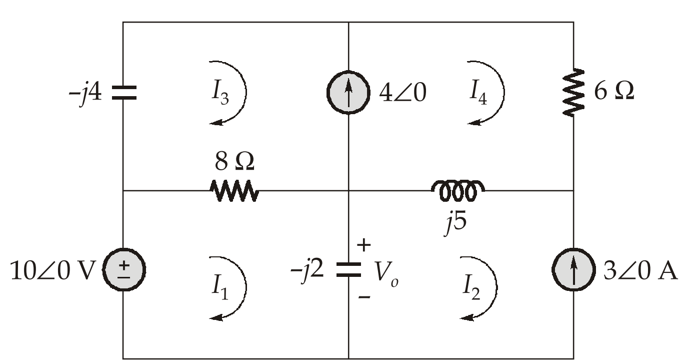
$$V = eV$$

$$eV = \frac{1}{2}\text{mv}^2$$

$$v = \left(\frac{2eV}{m}\right)^{1/2} = \left[\frac{(2)(2000)(1.6\times10^{-19})}{9.31\times10^{-31}}\right]^{1/2} = 26.22 \times 10^6 \text{ msec}^{-1}$$

**Q.3 (c) Solution:**

As shown in figure, meshes 3 and 4 form a supermesh due to the current source between the meshes.




For mesh 1, KVL gives

$$-10 + (8 - j2)I_1 - (-j2)I_2 - 8I_3 = 0$$

$$(8 - j2)I_1 + j2I_2 - 8I_3 = 10 \quad \text{...(i)}$$

$$I_2 = -3 \quad \text{...(ii)}$$

For the super mesh,

$$(8 - j4)I_3 - 8I_1 + (6 + j5)I_4 - j5I_2 = 0 \quad \text{...(iii)}$$

Due to the current source between meshes 3 and 4

$$-I_3 + I_4 = 4 \quad \text{...(iv)}$$

Substitute eqn. (ii) in eqn. (i) and eqn. (iii)

$$(8 - j2)I_1 - 8I_3 = 10 + j6 \quad \text{...(v)}$$

$$-8I_1 + (8 - j4)I_3 + (6 + j5)I_4 = -j15 \quad \text{...(vi)}$$

Substitute $I_4 = 4 + I_3$ in eqn. (vi)

$$-8I_1 + (8 - j4)I_3 + (6 + j5)(4 + I_3) = -j15$$

$$-8I_1 + (14 + j1)I_3 = -24 - j35 \quad \text{...(vii)}$$

From eqn. (v) and eqn. (vii), we obtain matrix equation.

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & (14 + j1) \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j1 \end{vmatrix} = (8 - j2)(14 + j1) - 64$$

$$= 112 + j8 - j28 + 2 - 64 = (50 - j20)$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j1 \end{vmatrix}$$

$$= 140 + j10 + j84 - 6 - 192 - j280$$

$$= -58 - j186$$

Current $I_1$ is obtained as

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\angle 274.5 \text{ A}$$

The required voltage $V_o$ is

$$V_o = -j2(I_1 - I_2) = -j2(3.618\angle 274.5 + 3)$$

$$V_o = 9.756\angle 222.32° \text{ V}$$

**Q.4 (a) (i) Solution:**

Given:

Ionic radius of $Fe^{++}$, $r^+ = 0.077$ nm ;

Ionic radius of $O^{--}$, $r^- = 0.140$ nm

Atomic weight of Fe, $A_{Fe} = 55.845$ g/mole ;

Atomic weight of O, $A_o = 16$ g/mole

Avogadro's number, $N_A = 6.022 \times 10^{23}$/mole

The structures of ionic crystals are governed by the ratio of the radius of cation ($r^+$) to that of anion ($r^-$), called as radius ratio. For the given crystal structure,

$$\frac{r^+}{r^-} = \frac{0.077}{0.140} = 0.55$$

The relation between radius radio ($r^+/r^-$), coordination numbers and the structure of compounds is given as below:

| Radius ratio ($r^+/r^-$) | Coordination Number | Geometry of Molecule | Structure Type |
|---|---|---|---|
| 0 to 0.155 | 2 | Linear | – |
| 0.155 to 0.225 | 3 | Planar triangular | – |
| 0.225 to 0.414 | 4 | Tetrahedral | Sphalenite, ZnS |
| 0.414 to 0.732 | 6 | Octahedral | Sodium chloride (Rock salt) |
| 0.732 to 1 | 8 | Body-centred cubic | Cesium chloride (CsCl) |

Since, the radius ratio of the given crystal structure lies between 0.414 to 0.732. Hence, the crystal structure is octahedral with coordination number of six. FeO has rock salt crystal structure.

The unit cell of FeO contains 8$O^-$ ions on corners and 6$O^-$ ions on the face-centres. Hence, the number of $O^-$ ions per unit cell,

$$N^- = 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$$

Similarly, there are twelve $Fe^+$ ions on the edge centres and one $Fe^+$ ion in the body centre. Hence, the number of $Fe^+$ ions per unit cell,

$$N^+ = 12 \times \frac{1}{4} + 1 = 4$$

Thus, a unit cell of FeO has 4 FeO units. The theoretical density for FeO is given as

$$\rho = \frac{n'(\Sigma A_C + \Sigma A_A)}{V_C \cdot N_A} \quad \text{...(i)}$$

where, $n'$ = Number of formula units in a unit cell = 4

$\Sigma A_C$ = Sum of the atomic weights of all cations in a formula unit

$\Sigma A_A$ = Sum of the atomic weights of all anions in a formula unit.

$V_C$ = Unit cell volume

For rock-salt crystal structure, the edge length of unit cell,

$$a = 2(r^+ + r^-) = 2(0.077 + 0.140) = 0.434 \text{ nm}$$

From equation (i), we get, $\rho = \dfrac{4(A_{Fe} + A_o)}{a^3 \cdot N_A}$

Substituting the values, $\rho = \dfrac{4(55.845+16)}{(0.434\times10^{-9})^3\times6.022\times10^{23}} = 5.8377\times10^6 \text{ g/m}^3$

$$\rho = 5.8377 \text{ g/cm}^3$$

**Q.4 (a) (ii) Solution:**

**Fabrication of ceramic products:**The various steps involved in the fabrication of ceramic products are as below:

1. **Milling:** The raw materials are prepared for ceramics processing through a number of different techniques. This stage is designed to separate the raw materials from any impurities that may exist and to prepare them for better mixing and forming.
2. **Sizing, Batching and Mixing:** Through sizing, the raw materials are refined even further depending on the intended application. Batching in ceramic manufacturing is the mixing of different raw materials into predetermined compounds. The batched materials are then mixed to give the resulting product a more homogeneous and uniform composition.
3. **Forming:** The raw materials have now been sized, batched and mixed into their desired amount. The ceramic materials are now formed into shape using any number of ceramic manufacturing processes. Slip casting, injection moulding and dry pressing are three kinds of techniques when forming raw material.
4. **Glazing:** After being formed and left to dry, the ceramic materials are then glazed. Each glaze has different properties that will help to determine the finish of the final product. In ceramic manufacturing, glazing is usually done with spray. In particular, dried powder pressing uses glazing techniques to support the physical properties of the pressed part.
5. **Firing:** The firing process in ceramic manufacturing involves placing your formed and glazed greenware into a sintering oven for heat treating. The sintering process creates density and hardens the material into its final result.

Role of Powder pressing and sintering in the fabrication of ceramic products:

- **Powder Pressing:** It is a *forming* technique which takes loose, granulated ceramic powders and compresses them inside of a die or press. The powder conforms and takes the shape of the die. Afterwards, the pressed part is heated to temper it.

- **Sintering:** It is the process of ***firing*** the agglomerated powder at high temperature but below the melting point of powder. It provides mechanical strength, eliminates pores, and increases the ceramic density.

**Q.4 (b) Solution:**

**(i)** Total secondary circuit resistance = 1.2 + 0.2 = 1.4 Ω

Total secondary circuit reactance = 0.5 + 0.3 = 0.8 Ω

Secondary circuit phase angle, $\delta = \frac{\tan^{-1} 0.8}{1.4} = 29°.42'$

or $\cos\delta = 0.8686$

and $\sin\delta = 0.4955$

Primary winding turns, $N_p = 1$

Secondary winding turns, $N_s = 200$

∴ Turns ratio, $n = 200$

Magnetizing current, $I_m = \frac{\text{magnetizing emf}}{\text{primary turns}} = \frac{100}{1} = 100\text{ A}$

Loss component, $I_e = \frac{\text{mmf equivalent to iron loss}}{\text{primary winding turns}} = \frac{50}{1} = 50\text{ A}$

∴ Actual ratio, $R = n + \frac{I_e \cos\delta + I_m \sin\delta}{I_s}$

$$= 200 + \frac{50 \times 0.8686 + 100 \times 0.4955}{5} = 218.6$$

Primary current, $I_p$ = Actual transformation ratio × secondary current

= 218.6 × 5 = 1093 A

$$\text{Ratio error} = \frac{K_n - R}{R} \times 100\%$$

$$\%\varepsilon_r = \frac{200 - 218.6}{218.6} \times 100\%$$

$$= -8.50\%$$

**(ii)** In order to eliminate the ratio error, we must reduce the secondary winding turns or in other words we must reduce the turns ratio.

The nominal ratio is 200 and therefore for zero ratio error the actual transformation ratio should be equal to the nominal ratio,

Nominal ratio, $K_n = 200$

Actual ratio, $R = n + \dfrac{I_e \cos\delta + I_m \sin\delta}{I_s}$

∴ For zero ratio error, $K_n = R$

or $200 = n + \dfrac{50 \times 0.8686 + 100 \times 0.4955}{5} = n + 18.6$

Turns ratio, $n = 181.4$

Hence secondary winding turns, $N_s = nN_p = 181.4 \times 1 = 181.4$

Reduction in secondary winding turns,

$= 200 - 181.4 \approx 19$

**Q.4 (c) (i) Solution:**

$$\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$$

$$\text{div}\vec{A} = (4 - 4y + 2z)$$

$$\oiint_s (\vec{A} \cdot \vec{n})ds = \iiint_v div\vec{A}\, dx\, dy\, dz$$

$$= \iiint_v (4 - 4y + 2z)dx\, dy\, dz$$

Converting to cylindrical polar co-ordinates

$$x = r\cos\theta,$$

$$y = r\sin\theta,$$

$$z = z$$

$$dx\, dy\, dz\, z = r\, dr\, d\theta\, dz$$

$r$ limits : 0 to 2

$\theta$ limits : 0 to $2\pi$

z limits : 0 to 3

$$\iiint_s \vec{A} \cdot \hat{n}\, ds = \int_{r=0}^{2} \int_{\theta=0}^{2\pi} \int_{0}^{3} (4 - 4r\sin\theta + 2z) r\, dr\, d\theta\, dz$$

$$= \int_{r=0}^{2} \int_{z=0}^{3} \int_{\theta=0}^{2\pi} (4r - 4r^2\sin\theta + 2rz)\, dr\, d\theta\, dz$$

$$= \int_{r=0}^{2} \int_{\theta=0}^{2\pi} (4rz - 4r^2 z\sin\theta + rz^2)_0^3\, dr\, d\theta$$

$$= \int_{r=0}^{2} \int_{\theta=0}^{2\pi} (12r - 12r^2 \sin\theta + 9r) dr\, d\theta$$

$$= \int_{r=0}^{2} (12r\theta + 12r^2 \cos\theta + 9r\theta)_{\theta=0}^{2\pi} dr$$

$$= \int_{r=0}^{2} \left[(24r\pi + 12r^2 \cos(2\pi) + 18r\pi) - (12r^2)\right] dr$$

$$= 84\pi$$

**Q.4 (c) (ii) Solution:**

$$x = 19.13 - 0.87y \quad \text{...(i)}$$

$$y = 11.64 - 0.50x \quad \text{...(ii)}$$

As equation (i), (ii) pass through $(\bar{x}, \bar{y})$:

$$\bar{x} = 19.13 - 0.87\bar{y} \quad \text{...(iii)}$$

$$\bar{y} = 11.64 - 0.50\bar{x} \quad \text{...(iv)}$$

On solving equation (iii) and (iv),

$$\bar{x} = 15.935$$

and $\bar{y} = 3.67$

Hence, mean of $x$ = $\bar{x} = 15.935$

mean of $y$ = $\bar{y} = 3.67$

From equation (i),

$$\frac{\sigma_x}{\sigma_y} = -0.87 \quad \text{...(v)}$$

From equation (ii),

$$r\frac{\sigma_y}{\sigma_x} = -0.50 \quad \text{...(vi)}$$

As $\sigma_x$ and $\sigma_y$ are always positive, so $r$ is negative multiplying equation (v) and (vi),

$$r\frac{\sigma_x}{\sigma_y} \cdot \frac{r\sigma_y}{\sigma_x} = (-0.87) \times (-0.50)$$

$$r^2 = 0.435$$

$\Rightarrow$ $r = -0.66$

Hence correlation coefficient, $r = -0.66$

## Section-B

**Q.5 (a) Solution:**

**For I-cache:**

$$\text{Number of lines} = \frac{4\text{K}}{4} = 1\text{ K}$$

← 30 bits →

| Tag | Line | Offset |
|---|---|---|
| | log (1 K) = 10 bits | log (4) = 2 bits |

$$\text{Tag bits} = 30 - (10 + 2) = 30 - (12) = 18 \text{ bits}$$

$$\text{Capacity of tag memory} = \text{Tag bits} \times \text{line size}$$
$$= 18 \text{ bits} \times 1\text{ K}$$

**For D-cache:**

$$\text{Number of lines} = \frac{4\text{K}}{4} = 1\text{ K}$$

$$\text{Number of sets} = \frac{1\text{K}}{2} = 2^9$$

← 30 bits →

| Tag | Sets | Offset |
|---|---|---|
| | $\log(2^9)$ = 9 bits | log (4) = 2 bits |

$$\text{Tag bits} = 30 - (9 + 2) = 30 - (11) = 19 \text{ bits}$$

Capacity of tag memory = Number of tag bits × Number of set × Number of lines in each set

$$= 19 \text{ bits} \times 2^9 \times 2 = 1\text{ K} \times 19 \text{ bits}$$

**For L2-cache:**

$$\text{Number of lines} = \frac{64\text{K}}{16} = 4\text{ K}$$

$$\text{Number of sets} = \frac{4\text{K}}{2^2} = 1\text{ K}$$

← 30 bits →

| Tag | Sets | Offset |
|---|---|---|
| | log (1 K) = 10 bits | log (16) = 4 bits |

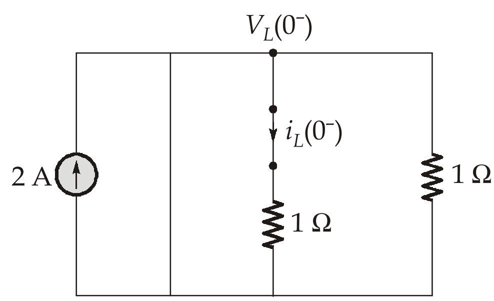
$$\text{Tag bits} = 30 - (10 + 4)$$
$$= 30 - (14) = 16 \text{ bits}$$

Capacity of tag memory = Number of tag bits × Number of set × Number of lines in each set

$$= 16 \text{ bits} \times 2^{10} \times 4$$
$$= 2^{12} \times 16 \text{ bits}$$
$$= 4\text{ K} \times 16 \text{ bits}$$

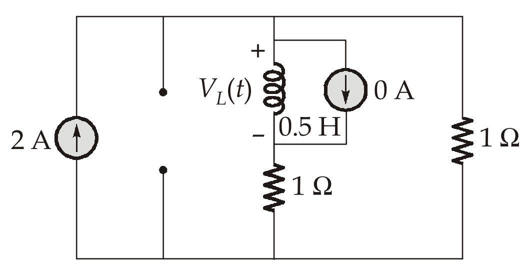
**Q.5 (b) Solution:**

At $t = 0^-$, the switch is closed and steady state condition is reached.

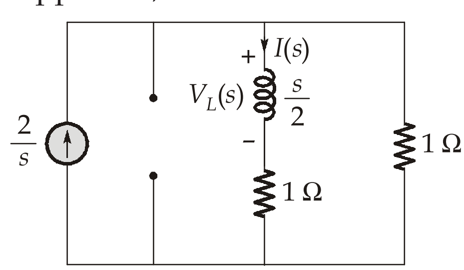



$$\therefore \quad i_L(0^-) = 0 = i_L(0^+)$$
$$V_L(0^-) = 0$$

for $t > 0$; the transformed network is,




By using Laplace transform approach,




$$I(s) = \frac{\frac{2}{s}\times 1}{1+1+\frac{s}{2}} = \frac{\frac{2}{s}}{\frac{s+4}{2}} = \frac{4}{s(s+4)}$$

$$V_L(s) = \frac{s}{2}I(s) = \frac{s}{2}\frac{4}{s(s+4)}$$

$$V_L(s) = \frac{2}{s+4} \Rightarrow v_L(t) = 2e^{-4t};\ t > 0$$

**Q.5 (c) Solution:**

**(i)** Deflection in radian, $\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$

Rate of change of self inductance,

$$\frac{dL}{d\theta} = \frac{2K\theta}{I^2}$$

or $$dL = \frac{2K}{I^2}\theta\, d\theta$$

Substituting the value $I = 4\,\theta^n$ in the above expression, we have

$$\frac{dL}{d\theta} = \frac{1}{8}K\theta^{1-2n}$$

Integrating the above expression, we have

$$L = \frac{K\theta^{2-2n}}{8(2-2n)} + A = \frac{K}{16(1-n)}\theta^{2-2n} + A$$

where $A$ = constant of integration

We have $I = 4\theta^n$ . Thus $\theta = 0$ when $I = 0$. It is given that when $I = 0$ the value of self-inductance $L = 10 \times 10^{-3}$ H.

Putting this in expression for $L$ ,

$$10 \times 10^{-3} = 0 + A$$

or $$A = 10 \times 10^{-3}$$

Hence the expression for self-inductance as a function of $\theta$ and $n$ is

$$L = \frac{K}{16(1-n)}\theta^{2-2n} + 10\times10^{-3}$$

$$= \frac{1}{100(1-n)}\theta^{2-2n} + 10\times10^{-3}\,\text{H}$$

as $$K = 0.16\ \text{Nm/rad}$$

**(ii)** With $n = 0.75$, the expression for self inductance is,

$$L = \frac{1}{100(1-0.75)}\theta^{2-2\times0.75} + 10\times10^{-3}$$

$$= 0.04\,\theta^{0.5} + 10 \times 10^{-3}\ \text{H}$$

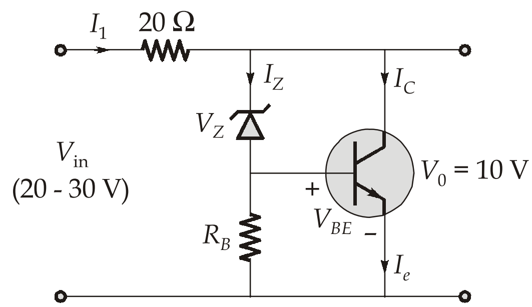
Putting, $L = 60 \times 10^{-3}$, we have $60 \times 10^{-3}$

$$= 0.04\ \theta^{0.5} + 10 \times 10^{-3}$$

$\therefore$ Deflection, $\theta = 1.56 \text{ rad} = 89.5°$

Current, $I = 4\ \theta^{n} = 4 \times 1.56^{0.75} = 5.58 \text{ A}$

**Q.5 (d) Solution:**




$$I_{1\ max} = \frac{V_{in\,max} - V_0}{20}$$

$$I_1 = \frac{30-10}{20} = 1 \text{ A} \qquad (I_Z = 0)$$

$$I_E = I_C + I_Z$$

$$I_B = I_Z \text{ (as no current flows in } R_B\text{)}$$

$$\beta = \frac{I_C}{I_B} = \frac{I_C}{I_Z}$$

$$I_C = \beta\ I_Z$$

$$I_E = \beta I_Z + I_Z = (99+1)I_Z$$

$$I_E = 100 I_Z$$

$$I_1 = I_E = 100 I_Z$$

$$I_Z = \frac{I_1}{100} = \frac{1}{100} = 0.01 \text{ A}$$

$$P_Z = V_Z I_Z = 9.5 \times 0.01 = 95 \text{ mW}$$

$$I_C = 99 I_Z = 99 \times 0.01 = 99 \text{ A}$$

$$P_T = V_C I_C = 10 \times 0.99 = 9.9 \text{ W}$$

**Q.5 (e) Solution:**

**(i)**

$$p + N_D = n$$

And $$np = n_i^2$$

So, $$(p + N_D)p = n_i^2$$

$$p^2 + 10^8 p - (1.5 \times 10^{10})^2 = 0$$

$$p = 1.495 \times 10^{10}/\text{cm}^3 \text{ and } 1.505 \times 10^{10}/\text{cm}^3$$

So we have,

$$p = 1.495 \times 10^{10}/\text{cm}^3$$

and $$n = 1.505 \times 10^{10}/\text{cm}^3$$

$$\rho = \frac{1}{q(n\mu_n + p\mu_p)}$$

$$\rho = \frac{1}{1.6 \times 10^{-19}\left(1.505 \times 10^{10} \times 1200 + 1.495 \times 10^{10} \times \frac{1200}{2.5}\right)}$$

$$\rho = 247662.07\ \Omega\text{-cm}$$

$$R = \frac{\rho l}{A}$$

$$R = \frac{247662.07 \times 20 \times 10^{-6} \times 10^{-2}}{2 \times 1 \times 10^{-12}}$$

$$R = 2.476 \times 10^{10}\ \Omega$$

**(ii)**

$$p = n + N_A$$

$$np = n_i^2$$

$$(n + N_A)n = n_i^2$$

$$n^2 + 10^{10} n - (1.5 \times 10^{10})^2 = 0$$

$$n = 1.08 \times 10^{10}/\text{cm}^3$$

$$p = 2.083 \times 10^{10}/\text{cm}^3$$

$$\rho = \frac{1}{1.6 \times 10^{-19}\left[2.083 \times 10^{10} \times \frac{1200}{2.5} + 1.08 \times 10^{10} \times 1200\right]}$$

$$\rho = 272231.5\ \Omega\text{-cm}$$

$$R = \frac{\rho l}{A} = \frac{272231.5 \times 20 \times 10^{-6} \times 10^{-2}}{2 \times 1 \times 10^{-12}}$$

$$R = 2.7 \times 10^{10}\ \Omega$$

**Q.6 (a) (i) Solution:**

**1.**

$$\int_{-\infty}^{\infty} f(x)dx = \int_0^1 6(1-x)dx = \int_0^1 (6x-6x^2)dx$$

$$= \left(3x^2 - 2x^3\right)_0^1 = 3-2 = 1$$

$f(x) > 0$ for $0 \le x \le 1$

Hence, the given function is a probability density function.

**2.**

$$\text{Mean} = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_0^1 x \cdot 6x(1-x)dx$$

$$= \int_0^1 (6x^2 - 6x^3)dx = \left(2x^3 - \frac{3}{2}x^4\right)_0^1 = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\text{Variance} = \int_{-\infty}^{\infty} (x-\bar{x})^2 \cdot f(x)dx = \int_0^1 \left(x - \frac{1}{2}\right)^2 \cdot 6x(1-x)dx$$

$$= \int_0^1 \left(x^2 - x + \frac{1}{4}\right)(6x - 6x^2)dx$$

$$= \int_0^1 \left(12x^3 - 6x^4 - \frac{15}{2}x^2 + \frac{3}{2}x\right)dx$$

$$= \left(3x^4 - \frac{6}{5}x^5 - \frac{5}{2}x^3 + \frac{3x^2}{4}\right)_0^1$$

$$= \left(3 - \frac{6}{5} - \frac{5}{2} + \frac{3}{4}\right) = \frac{1}{20}$$

**Q.6 (a) (ii) Solution:**

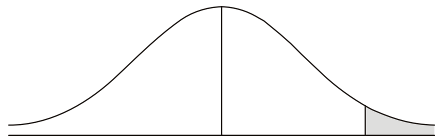
$$\text{Mean} = \bar{x} = 39.5$$

$$\text{Standard deviation, } \sigma = 12.5$$

$$\frac{x}{\sigma} = \frac{60-39.5}{12.5} = \frac{20.5}{12.5} = \frac{41}{25} = 1.64$$

We have to find out area for $\frac{x}{\sigma} = 1.64$

$$\text{Area for } \frac{x}{\sigma} = 1.6 \text{ is } 0.94520$$




$$\text{Area for } \frac{x}{\sigma} = 1.7 \text{ is } 0.95543$$

$$\text{Difference for } 0.1 = 0.01023$$

$$\text{Difference for } 0.04 = 0.004092$$

$$\therefore \quad \text{Area for } \frac{x}{\sigma} = 1.64 \text{ is } 0.94520 + 0.004092$$

$$= 0.949292$$

$$\text{Area right to ordinate } 1.64 = 1 - 0.949292 = 0.050708$$

Number of candidates who secured marks 60 or more

$$= 5000 \times 0.050708 = 253.54$$

$$\text{Candidates securing first division} = 254$$

**Q.6 (b) Solution:**

**(i) 1.** $(18)_{10} - (33)_{10} = (18)_{10} + (-33)_{10}$

2's complement of $(18)_{10}$ = 0010010

2's complement of $(-32)_{10}$ = 1011111

$$(18)_{10} + (-32)_{10}$$

```
  0 0 0 1 0 0 1 0
+ 1 0 1 1 1 1 1 1
-----------------
    1 1 1 0 0 0 1
```

Since MSB is 1. So, the result is –ve. So, to get the result, take 2's complement of the above result.

$$(18)_{10} - (33)_{10} = -(0001111)_2$$

$$= -(15)_{10}$$

**2.** $-(14)_{10} - (26)_{10} = (-14)_{10} + (-26)_{10}$

2's complement of $(-14)_{10}$ = 110010

2's complement of $(-26)_{10}$ = 100110

$(-14)_{10} + (-26)_{10}$

$$\begin{array}{r} 110010 \\ +100110 \\ \hline 1011000 \end{array}$$

Here, the carry is generated. So, discard the carry and the result is negative. To get the actual result take 2's complement of above result.

$$-(14)_{10} - (26)_{10} = (101000)_2$$
$$= -(40)_{10}$$

**(ii)** Given :
$$h_C = 85\% \Rightarrow h_C = 0.85$$
$$h_M = 9.5\% \Rightarrow h_M = 0.095,\ t_C = 8 \text{ ns}$$
$$t_M = 125 \text{ nsec},\ t_{VM} = 15 \text{ ms}$$

Hit ratio of virtual memory $= (100 - 85 - 9.5)\%$
$$h_{VM} = 5.5\% \Rightarrow h_{VM} = 0.055$$

Average access time,
$$t_{avg} = h_C t_C + h_M t_M + h_{VM} t_{VM}$$
$$t_{avg} = (0.85 \times 8 + 0.095 \times 125 + 0.055 \times 15 \times 10^6) \text{ nsec}$$
$$= 825018.675 \text{ nsec}$$
$$t_{avg} = 825.019\ \mu\text{sec}$$

**Q.6 (c) (i) Solution:**

Now,
$$R_1 = 20\ \Omega\ ; \quad L_1 = 0.22 \text{ H}$$
$$R_4 = 750\ \Omega\ ; \quad R_3 = 40\ \Omega$$
$$L_3 = 0.1 \text{ H}$$

At balance, $R_4(R_1 + j\omega L_1) = R_2(R_3 + j\omega L_1)$

Thus the two balance equations are:
$$R_1 = \frac{R_2 L_3}{4} \text{ and } L_1 = \frac{R_2 L_3}{4}$$

From above, we have
$$\frac{L_1}{L_3} = \frac{R_2}{R_4} = \frac{R_1}{R_3}$$

∴ Value of $R_2$ required for balance
$$R_2 = R_4 \frac{L_1}{L_3} = 750 \times \frac{0.22}{0.1} = 1650\ \Omega$$

and $$\frac{L_1}{L_3} = \frac{R_2}{R_4} = 2.2$$

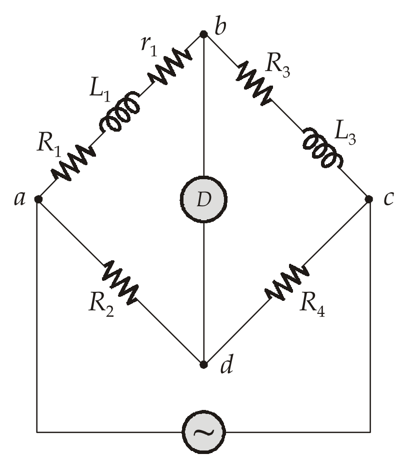
Now examine the value of ratio $\frac{R_1}{R_3}$ for the existing circuit, we have

The value of this ratio should be 2.2 for both resistive and inductive balance and therefore we must add a series resistance to arm *ab*. Let this series resistance be $r_1$ .

Therefore $$\frac{R_1 + r_1}{R_3} = 2.2$$

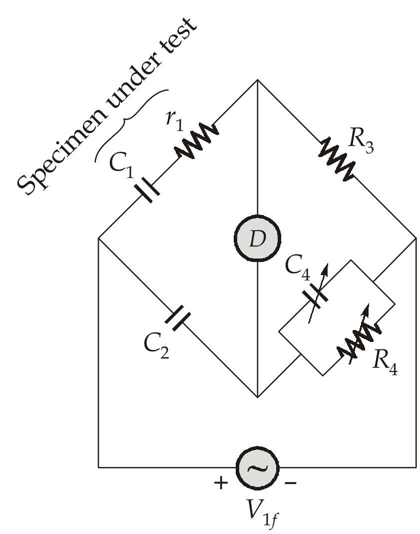
or $$r_1 = 2.2 \times 40 - 20 = 68\ \Omega$$

The modified circuit is shown in figure below,




**Q.6 (c) (ii) Solution:**

The Schering bridge is shown in figure below, where $C_1$ and $r_1$ represent the capacitance and series resistance of the capacitor,



$$r_1 = \frac{C_4}{C_2}R_3 = \frac{0.5\times10^{-6}}{106\times10^{-12}}260$$

$$= 1.23 \times 10^6\ \Omega$$

and $$C_1 = \frac{R_4}{R_3}C_3 = \frac{1000}{\pi\times260}\times106\times10^{-12}\text{ F}$$

$$= 130\text{ pF}$$

Power factor of sheet $= \omega C_1 r_1 = 2\pi \times 50 \times 130 \times 10^{-12} \times 1.23 \times 10^6 = 0.05$

Now capacitance, $$C_1 = \varepsilon_r\varepsilon_0\frac{A}{d}$$

∴ Relative permittivity, $$\varepsilon_r = \frac{C_1 d}{\varepsilon_0 A} = \frac{130\times10^{-12}\times4.5\times10^{-3}}{8.854\times10^{-12}\times\frac{\pi}{4}(0.12)^2} = 5.9$$

where $\varepsilon_0$ = permittivity of free space

$$= 8.84 \times 10^{-12}\text{ F/m}$$

**Q.7 (a) Solution:**

**(i)** Two sources of magnetic moments for electrons are:

- **Orbital Angular Moment:** Due to the orbital motion of the electron around the nucleus. This motion, in a sense, can be considered as a current loop, resulting in a magnetic moment along its axis of rotation.

$$\mu_{m0} = iA$$

$$= \frac{ev}{2\pi r}\cdot\pi r^2 \quad (v : \text{orbital speed}, r : \text{orbital radius}, e : \text{electron charge})$$

$$= \frac{evr}{2}$$

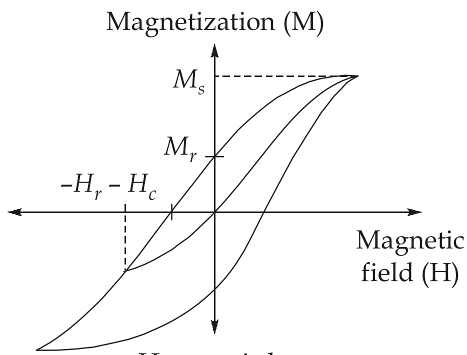
$$\mu_{m0} = \frac{e\omega r^2}{2} \quad (v = \omega r)\ (\omega\text{: Orbital angular speed})$$

- **Spin Magnetic Moment:** Due to electron spin on its own axis

$$\mu_{ms} = \frac{e}{2}\cdot\frac{nh}{m_e} \quad (n : \text{valence}, m_e : \text{Mass of electron. } h : \text{Planck's constant})$$

$$= n\,\mu_B \quad (\mu_B : \text{Bohr magneton} = 9.28 \times 10^{-24}\text{ Am}^2)$$

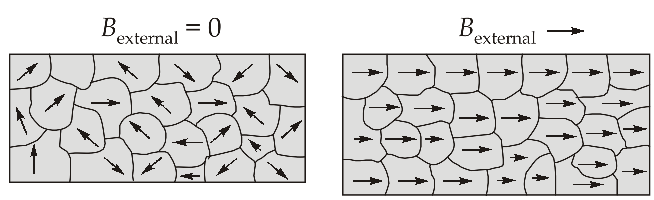
**(ii)** Certain magnetic materials can retain a memory of an applied field once it is removed. This behavior is called 'Hysteresis' and a plot of the variation of magnetization with magnetic field is called a 'Hysteresis loop'.



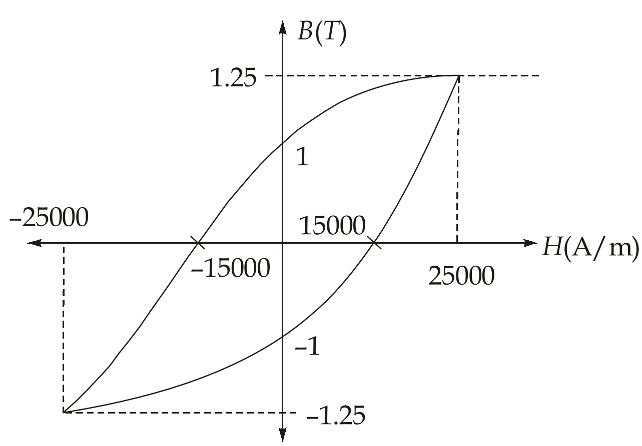

Hysteresis loop

$M_s$ : Magnetization Saturation
$M_r$ : Retentivity
$H_c$ : Coercivity
$H_r$ : Coercivity of Remanence

This property is related to 'magnetic domains' in the material-which are microscopic ordering of electron spin characteristic leads to the formation of regions of magnetic alignment. Both ferromagnetic and ferrimagnetic materials will retain an imposed magnetization indefinitely and find applications in permanent magnets, magnetic tape and storage devices, etc. Magnetic hysteresis occurs when the magnetic flux lags behind the magnetic because of higher flux density in these materials. Any ferromagnetic or ferrimagnetic materials, at a temperature below $T_c$, is composed of small volume regions in which there is a mutual alignment in same direction of all magnetic dipole moments.




**(iii)**




$$\text{Energy loss per cycle} = \int_{H=-25000}^{H=+25000} B\,dH = 2\times\frac{1}{2}\times(2T)(25000\text{ A/m})$$

$$= 50000\text{ J/s}$$

**Q.7 (b) (i) Solution:**

1. 16 bit

| opcode | register | register |
|---|---|---|
| 4 bit | 6 bit | 6 bit |

2. Number of 2-address instruction = $2^4 = 16$
3. Number of free opcodes = (16 – 12) = 4
4. Number of 1-address memory reference instruction

16 bit

| opcode | memory |
|---|---|
| 6 bit | 10 bit |

$= 4 \times 2^2 = 16$

5. Number of free opcodes = (16 – 12) = 4
6. Number of 0-address instruction opcodes = $4 \times 2^{10} = 4096$

**Q.7 (b) (ii) Solution:**

Deadlock has following four characteristics :

1. Mutual Exclusion
2. Hold and Wait
3. No preemption
4. Circular Wait

**Deadlock Prevention :** We can prevent deadlock by eliminating any of the below four conditions :

1. **Eliminate Mutual Exclusion :** It means more than one process can have access to a single resource at the same time. It is impossible because if multiple processes access the same resource simultaneously, there will be chaos and no process will be completed. So, this is not feasible. Hence, the OS can not avoid mutual exclusion.
2. **Eliminate Hold and Wait :** To avoid the hold and wait, there are many ways to acquire all the required resources before starting the execution. But this is also feasible because a process will use a single resource at a time. Here, the resource utilization will be very less.

   Another way is if a process is holding a resource and wants to have additional resources, then it must free the acquired resources. This way, we can avoid the hold and wait condition, but it can result in starvation.
3. **Eliminate No Preemption :** It means the CPU can not take acquired resources from any process forcefully even though that process is in a waiting state. We can remove

the no preemption and forcefully take resources from a waiting process, we can avoid the deadlock. This is an implementable logic to avoid deadlock.

4. **Elimination of Circular Wait :** In circular wait, two processess are stuck in the waiting state for the resources which have been held by each other. To avoid the circular wait, we assign a numerical integer value to all resources and a process has to access the resource in increasing or decreasing order.

   If the process acquires resources in increasing order, it will only have access to the new additional resource if that resource has a higher integer value. And if that resource has a lesser integer value, it must free the acquired resource before taking the new resource and vice-versa for decreasing order.

**Q.7 (c) (i) Solution:**

We find out the force on charge at $A$ due to the other two charges situated at point $B$ and point $C$. Force on charge at $A$ due charge at $B$ is:

$$\vec{F}_{BA} = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} = \frac{q^2}{4\pi\varepsilon d^2}$$

Force on charge at $A$ due to charge at $C$ is

$$\vec{F}_{CA} = \frac{q^2}{4\pi\varepsilon d^2}$$

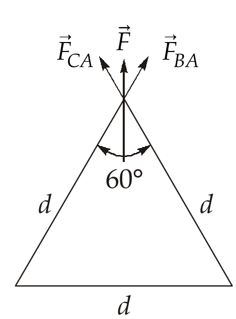
It is observed that the horizontal components of the total force cancel out. Hence, the total force acts along the upward direction and its magnitude is given as

$$F = F_{BA}\cos 30° + F_{CA}\cos 30°$$

$$= 2\times\frac{q^2}{4\pi\varepsilon d^2}\times\cos 30°$$

$$= 2\times\frac{q^2}{4\pi\varepsilon d^2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}q^2}{4\pi\varepsilon d^2}$$

$$F = \frac{\sqrt{3}q^2}{4\pi\varepsilon d^2}$$



**Q.7 (c) (ii) Solution:**

$$\rho_V = \rho_0 e^{-(\sigma/\varepsilon)t} = \rho_0 e^{-t/\lambda}$$

At $t = \frac{\varepsilon}{\sigma}$, $\rho_V = \rho_0\, e^{-1} = \frac{1}{e}\rho_0$

This shows that if at any instant a charge density ρ existed within a conductor, it would decrease to $\frac{1}{e}$ times this value in a time $\frac{\varepsilon}{\sigma}$ second

For copper, $\sigma = 5.8 \times 10^7$ mho/m and $\varepsilon_r = 1$

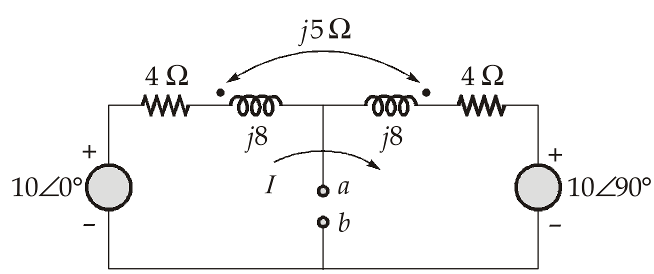
the value of this time is $\tau = \frac{\varepsilon}{\sigma} = \frac{\varepsilon_0 \varepsilon_r}{\sigma}$

$$= \frac{8.854 \times 10^{-12} \times 1}{5.8 \times 10^7}$$

$$= 1.53 \times 10^{-19} \text{ second}$$

**Q.8 (a) Solution:**

For finding open circuit voltage across *ab*, let *I* be the current in the circuit,




Now, applying KVL in the above circuit

$$4I + j8I + j8I + 4I - j5I - j5I = 10 - j10$$

or $$I(8 + j6) = 10(1 - j)$$

or, $$I = \frac{10 - j10}{8 + j6} = \frac{14.14\angle -45^\circ}{10\angle 36.86^\circ} = 1.414\angle{-81.8^\circ}$$

Now, $$v_{ab} = v' = 10 - (4 + j8 - j5)I$$

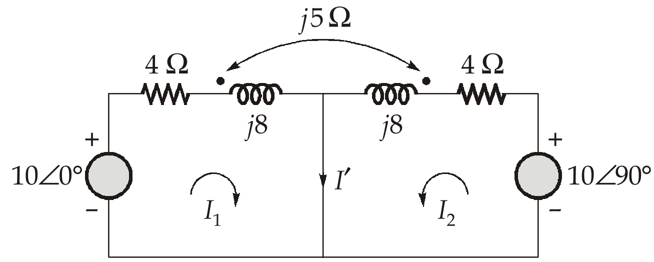
or $$v' = 10 - (4 + j3)(1.414\angle{-81.87^\circ})$$

$$v' = 7.07\angle 45^\circ$$

For finding short circuit current,

$I_{sc} = I'$, the circuit is

Let $I_1$ and $I_2$ be the loop current




Now, in loop-1

$$j10 = 4I_1 + j8\,I_1 + j5\,I_2$$

or, $$j10 = (4 + j8)I_1 + j5\,I_2 \quad \text{...(i)}$$

In loop-2:

$$j10 = 4I_2 + j8\,I_2 + j5\,I_2$$

or, $$j10 = j5\,I_1 + (4 + j8)I_2 \quad \text{...(ii)}$$

In matrix form,

$$\begin{bmatrix} 4+j8 & j5 \\ j5 & 4+j8 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ +j10 \end{bmatrix}$$

∴ $$I_1 = \frac{\begin{vmatrix} 10 & j5 \\ j10 & 4+j8 \end{vmatrix}}{\begin{vmatrix} 4+j8 & j5 \\ j5 & 4+j8 \end{vmatrix}} = \frac{120\angle 41.63°}{68\angle 109.75°}$$

$$I_1 = 1.77\angle{-68.12°}$$

$$I_2 = \frac{\begin{vmatrix} 4+j8 & 10 \\ j5 & j10 \end{vmatrix}}{\begin{vmatrix} 4+j8 & j5 \\ j5 & 4+j8 \end{vmatrix}} = \frac{80\angle -172.87°}{68\angle 109.75°}$$

or, $$I_2 = 1.185\angle 77.36°$$

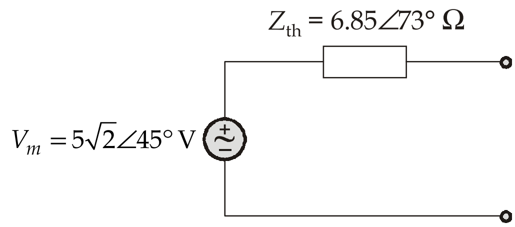
∴ $$I' = I_1 + I_2 = 1.77\angle{-68.12°} + 1.185\angle 77.36°$$

$$I' = 1.04\angle{-28.0°}\text{ A}$$

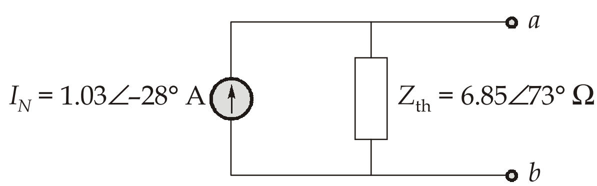
∴ $$Z' \text{ (Thevenin resistance)} = \frac{V'}{I'}$$

or, $$Z' = \frac{7.07\angle 45°}{1.04\angle 28°} = 6.80\angle 73°\ \Omega$$

Thevenin equivalent network,

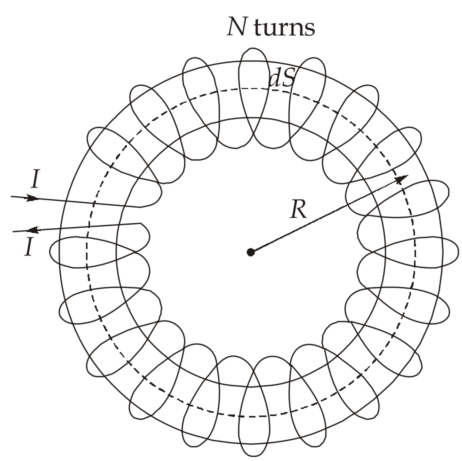



Norton equivalent network,




**Q.8 (b) Solution:**

**(i)** The magnetic field inside the toroid is




$$H = \frac{NI}{2\pi r}$$

$$B = \frac{\mu NI}{2\pi r}$$

The magnetic flux inside the toroid,

$$\phi = B.A = \frac{\mu NIA}{2\pi r}$$

As we know,

$$N.\phi = LI$$

$$L = \frac{\mu N^2 AI}{2\pi RI} = \frac{\mu N^2 A}{2\pi R} \text{ H}$$

Therefore, $$N = \sqrt{\frac{2\pi RL}{\mu_o \mu_r A}} = \sqrt{\frac{2\pi \times 50 \times 10^{-2} \times 2.5}{4\pi \times 10^{-7} \times 200 \times 12 \times 10^{-4}}}$$

No. of turns required, $N = 5103$ turns

**(ii)** Magnetic flux density at a radial distance ρ, due to infinite line current wire is given as

$$\vec{B} = \frac{\mu I_1}{2\pi\rho} \hat{a}_\phi$$

We know that the flux linkage in rectangular loop due to wire is given as

$$\phi_{12} = \iint \vec{B} \cdot \vec{ds} = \iint \frac{\mu I_1 \hat{a}_\phi}{2\pi\rho} (d\rho dz \hat{a}_\phi)$$

$$\phi_{12} = \frac{\mu I_1}{2\pi} \int_{z=0}^{z=b} \int_{\rho=\rho_o}^{\rho_o+a} \frac{1}{\rho} d\rho dz$$

$$\phi_{12} = \frac{\mu I_1 b}{2\pi} \ln \rho \Big|_{\rho_o}^{\rho_o+a}$$

$$\phi_{12} = \frac{\mu I_1 b}{2\pi} \ln \left[\frac{\rho_o + a}{\rho_o}\right]$$

Now, mutual inductance between wire and loop is

$$M_{12} = \frac{\phi_{12}}{I_1} = \frac{\mu b}{2\pi} \ln \left[\frac{\rho_o + a}{\rho_o}\right]$$

Now, $a = b = \rho_o = 1$ m

$$M_{12} = \frac{4\pi \times 10^{-7}}{2\pi} \ln \left[\frac{2}{1}\right] = 138.62 \text{ nH}$$

**Q.8 (c) Solution:**

Load power = 100 × 9 × 0.1 = 90 W

Load power factor cos ϕ = 0.1

∴ $\phi = 84.26°$

and $\sin\phi = 0.995$

and $\tan\phi = 9.95$

Resistance of pressure coil circuit,

$$R_p = 3000\ \Omega$$

Reactance of pressure coil circuit $= 2\pi \times 50 \times 30 \times 10^{-3} = 9.42\ \Omega$

As the phase angle of pressure coil circuit is small,

$$\beta = \tan\beta \approx \frac{9.42}{3000} = 0.00314 \text{ rad}$$

**(i)** When the pressure coil is connected on the load side, the wattmeter measures power loss in pressure oil circuit in addition to load power,

$$\text{True power} = VI\cos\phi = 100 \times 9 \times 0.1 = 90 \text{ W}$$

Let us consider only the effect of inductance

$$\text{Reading of wattmeter} = \text{True power } (1 + \tan\phi \tan\beta)$$

$$= 90(1 + 9.95 \times 0.00314) = 92.81 \text{ W}$$

Power loss in pressure coil circuit,

$$= \frac{V^2}{R_p} = \frac{(100)^2}{3000} = 3.33 \text{ W}$$

∴ Reading of wattmeter considering the power loss in pressure coil circuit

$$= 92.81 + 3.33 = 96.14 \text{ W}$$

$$\text{Percentage error} = \frac{96.14 - 90}{90} \times 100 = 6.82\%$$

**(ii)** When the current coil is on the load side, the wattmeter measures the power in the load plus the power loss in the current coil. In fact, the current coil acts as a load,

$$\therefore \quad \text{Total power} = \text{Power consumed in load} + I^2R_C$$

$$= 90 + (9)^2 \times 0.1 = 98.1 \text{ W}$$

$$\text{Impedance of load} = \frac{100}{9} = 11.1\ \Omega$$

$$\text{Resistance of load} = 11.1 \times 0.1 = 1.11\ \Omega$$

$$\text{Reactance of load} = 11.1 \times 0.995 = 11.05\ \Omega$$

Resistance of load plus resistance of current coil

$$= 1.11 + 0.1 = 1.21\ \Omega$$

Reactance of load plus reactance of current coil

$$= 11.05\ \Omega$$

Impedance of load plus reactance of current coil

$$= \sqrt{(1.21)^2 + (11.05)^2} = 11.1\ \Omega$$

Power factor of load including current coil

$$= \frac{1.21}{11.1} = 0.109$$

$\therefore \quad \phi = 83.74°$

and $\quad \tan\phi = 9.12$

$$\text{Reading of wattmeter} = 98.1(1 + 9.2 \times 0.00314) = 100.9 \text{ W}$$

$$\text{Percentage error} = \frac{100.9 - 90}{90} \times 100 = 12.1\%$$

It is clear from above that in low power factor circuits we should not use a connection wherein the current coil is on the load side as it results in greater errors.

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