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Detailed Solutions

ESE-2025
Mains Test Series

Mechanical Engineering
Test No : 9

Full Syllabus Test (Paper-I)

Section : A

1. (a) Solution:

Given: $p_1 = 1.013$ bar, $p_2 = 16$ bar, $p_3 = 16 - 0.3 = 15.7$ bar, $T_1 = 273 + 15 = 288$ K, $T_3 = 1350 + 273 = 1623$ K, $p_4 = 1.013$ bar, $\eta_C = \eta_T = 0.87$, $\eta_{CC} = 0.98$

For air and gases, $c_p = 1.005$ kJ/kgK

$\gamma = 1.4$

Power developed, $P = 200$ MW

For isentropic compression 1 - 2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{1.013}\right)^{\frac{1.4-1}{1.4}} = 2.2$$

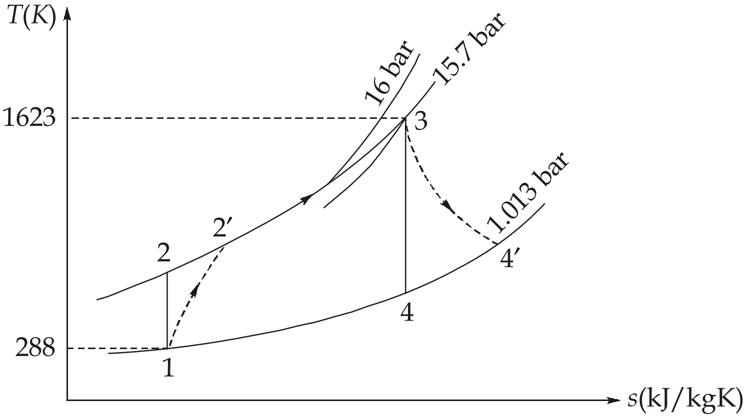
$$T_2 = 2.2 \times 288 = 633.6 \text{ K}$$

Now,

$$\eta_C = \frac{T_2 - T_1}{T_2 - T_1}$$

$$0.87 = \frac{633.6 - 288}{T_2 - 288}$$

$$T'_2 = 685.24 \text{ K}$$



For isentropic expansion process 3 - 4,

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.013}{15.7}\right)^{\frac{1.4-1}{1.4}} = 0.457$$

$$T_4 = 1623 \times 0.457 = 741.7 \text{ K}$$

Now, $$\eta_T = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.87 = \frac{1623 - T_4'}{1623 - 741.7}$$

$$T_4' = 856.3 \text{ K}$$

$$W_C = c_p(T_2' - T_1) = 1.005(685.24 - 288) = 399.23 \text{ kJ/kg}$$

$$W_T = c_p(T_3 - T_4') = 1.005(1623 - 856.3) = 770.53 \text{ kJ/kg}$$

$$W_{\text{net}} = W_T - W_C$$

$$= 770.53 - 399.23 = 371.3 \text{ kJ/kg}$$

If the mass of air flowing is $\dot{m}_a$ kg/s, then the power developed by the plant is given by

$$P = \dot{m}_a \times W_{net} \text{ kW} \times \eta_{cc}$$

$$200 \times 10^3 = \dot{m}_a \times 371.3 \times 0.98$$

$$\dot{m}_a = \frac{200 \times 10^3}{371.3 \times 0.98} = 549.64 \text{ kg/s}$$

1. (b) Solution:

$$\text{The weight of sea water displaced} = 4500 + 500 = 5000 \text{ N}$$

$$\therefore \quad \text{Volume of sea water displaced, } V = \frac{5000}{1.025 \times 9810} = 0.497 \text{ m}^3$$

$$\text{Depth of immersion} = \frac{0.497}{\frac{\pi}{4} \times (1.5)^2} = 0.281 \text{ m}$$

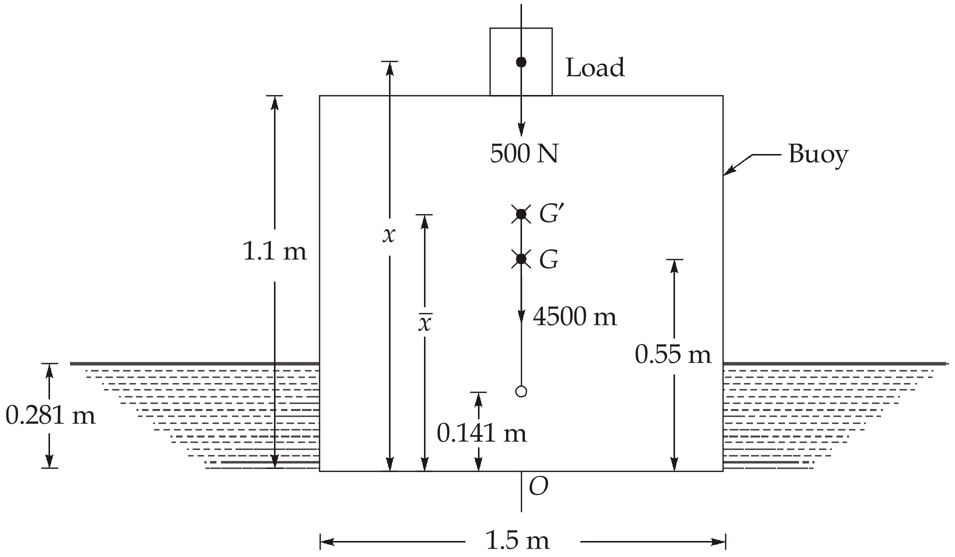
$$\therefore \quad \text{The height of the centre of buoyancy above the base} = \frac{0.281}{2} = 0.141 \text{ m}$$

The combined centre of gravity of the buoy and the load may be determined by taking the moments above the base of the buoy. Thus if the distance of the centre of gravity of the load from the base is x and that of the combined centre of gravity G' is $\bar{x}$, then by taking moments about the mid-point of base O, we get

$$5000\bar{x} = 4500 \times \frac{1.1}{2} + 500 \times x$$

$$\therefore \quad \bar{x} = \left(\frac{2475 + 500x}{5000}\right) = 0.495 + 0.1x$$

Thus,
$$\overline{BG'} = \overline{OG'} - \overline{OB}$$
$$= \bar{x} - 0.141$$
$$= (0.495 + 0.1x) - 0.141 = 0.354 + 0.1x$$



Since,
$$\overline{BM} = \frac{I}{V}$$

$$I = \frac{\pi}{64}(1.5)^4\,\text{m}^4\,;\ V = 0.497\ \text{m}^3$$

$$\therefore \qquad \overline{BM} = \frac{\pi}{64}\times\frac{(1.5)^4}{0.497} = 0.5\,\text{m}$$

For stable equilibrium of the floating buoy

$$\overline{BM} > \overline{BG'}$$

$$\text{or} \qquad 0.5 > 0.354 + 0.1x$$

$$\text{or} \qquad 0.1x < 0.146$$

$$\therefore \qquad x < 1.46\ \text{m}$$

i.e., the centre of gravity of the load must not be more than 1.46 – 1.1 = 0.36 m above the top of the buoy.

1. (c)

Regimes of boiling : Different boiling regimes are given below:

(i) Free Convection Boiling:

1. Free convection boiling exists when excess temperature (Δt_e) is less than 5°C.
2. In this region, the liquid near the surface is superheated slightly and convection currents circulate the liquid and evaporation takes place at the liquid surface.

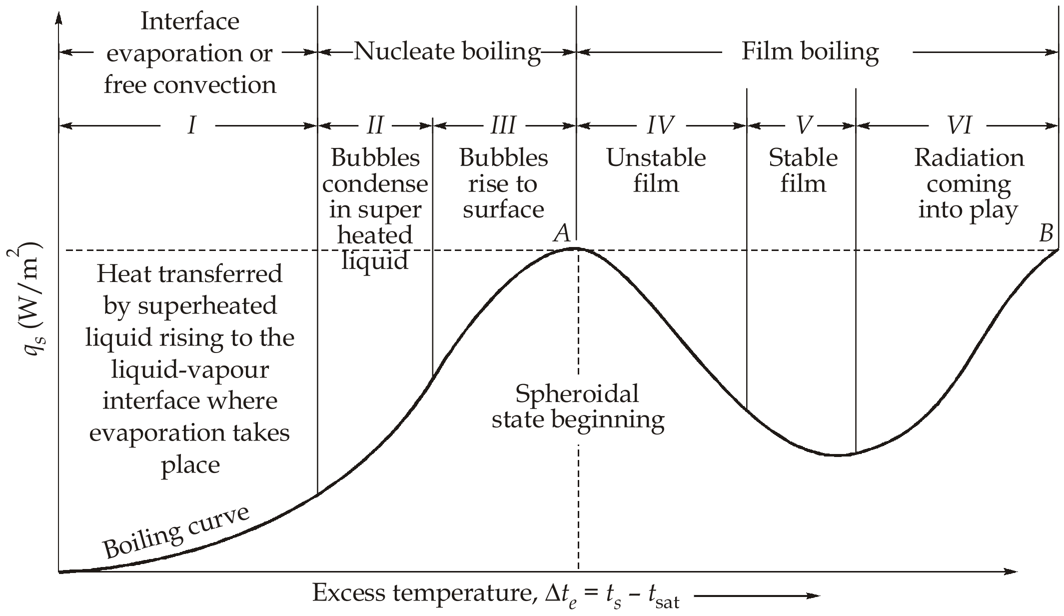
(ii) Nucleate Boiling :

1. Nucleate boiling exists when temperature (Δt_e) is between 5°C to 50°C ($5 \le \Delta t_e \le 50°\text{C}$).
2. This type of boiling exists in region II and III.
3. With the increase in Δt_e (excess temperature) the formation of bubbles on the surface of the wire at certain localised spots commences. The bubbles condense in the liquid without reaching the liquid surface. In fact, it is the region II where nucleate boiling starts.
4. With further increase in Δt_e the bubbles are formed more rapidly and rise to the surface of the liquid resulting in rapid evaporation, as indicated in the region III.
5. The nucleate boiling is thus characterised by formation of bubbles at the nucleation sites and the resulting liquid agitation.

(iii) Film Boiling:

1. Film boiling is also called unstable boiling or transition boiling which exists between 50°C to 150°C ($50°\text{C} \le \Delta t_e \le 150°\text{C}$).
2. In this region bubble formation is very high and vapour film form only on the fraction of surface.

3. The film acts as a thermal resistance in heat flow path, hence heat transfer rate decreases.



The boiling curve for water

1. (d) Solution:

$$\eta_o = 85\%;\ H = 8\text{ m};\ P = 160\text{ kW};\ N = 180\text{ rpm}$$

$$\text{Peripheral velocity, } u = 0.96\sqrt{2gH} = 0.96\sqrt{2\times 9.81\times 8} = 12.03\text{ m/s}$$

$$\text{Velocity of flow at inlet, } V_f = 0.36\sqrt{2gH} = 0.36\sqrt{2\times 9.81\times 8} = 4.51\text{ m/s}$$

$$\text{Hydraulic losses} = 25\%$$

$$\therefore \quad \eta_h = (100 - 25) = 75\%$$

For radial discharge,

$$\eta_h = \frac{V_w u}{gH}$$

or

$$0.75 = \frac{V_w \times 12.03}{9.81\times 8}$$

$$\therefore \quad V_w = 4.893\text{ m/s}$$

From inlet velocity triangle, we have

$$\tan\alpha = \frac{V_f}{V_w} = \frac{4.51}{4.893} = 0.922$$

$$\alpha = 42.67°$$

Also $$\tan(180-\theta) = \frac{V_f}{u-V_w} = \frac{4.51}{12.03-4.893} = 0.632$$

∴ $$\theta = 147.71°$$

$$u = \frac{\pi DN}{60}$$

or $$12.03 = \frac{\pi \times D \times 180}{60}$$

∴ $$D = 1.276 \text{ m}$$

Also, $$\eta_0 = \frac{P}{wQH}$$

or $$0.85 = \frac{160 \times 10^3}{9810 \times Q \times 8}$$

∴ $$Q = 2.398 \text{ m}^3/\text{s}$$

Neglecting the vane thickness, we have

$$Q = \pi D B V_f$$

or $$2.398 = \pi \times 1.276 \times B \times 4.51$$

∴ $$B = \frac{2.398}{\pi \times 1.276 \times 4.51}$$

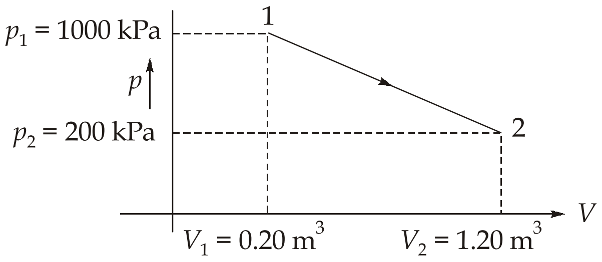
$$= 0.1326 \text{ m} = 132.6 \text{ mm}$$

1. (e) Solution:

Given : $m = 1.5$ kg; $p_1 = 1000$ kPa; $p_2 = 200$ kPa; $V_1 = 0.20$ m^3; $V_2 = 1.20$ m^3; $p = a + bV$, $\gamma = 1.20$

Specific internal energy, $u = 1.5pv - 85$ kJ/kg ...(i)

On plotting the p-V curve,



Net work done, $W_{1\text{-}2}$ = Area of $(1 - V_1 - V_2 - 2)$

$$= \frac{1}{2}\times(1.20-0.20)\times(1000-200)+200\times(1.20-0.20)$$

$$= 400 + 200 = 600 \text{ kJ} \qquad \text{...(ii)}$$

Now, from equation (i),

$$\text{Change in internal energy} = \Delta u$$

$$\Delta u = u_2 - u_1$$

$$= 1.5(p_2v_2 - p_1v_1) \qquad \left(\because\ v = \frac{V}{m}\right)$$

$$= 1.5\left(200\times\frac{1.20}{1.5}-1000\times\frac{0.2}{1.5}\right)$$

$$\Delta u = 40 \text{ kJ/kg}$$

$$\Delta U = 40 \times 1.5 \text{ kJ}$$

$$\Delta U = 60 \text{ kJ}$$

Net heat transfer,

$$\delta Q = dU + \delta W$$

$$= 60 + 600 = 660 \text{ kJ}$$

For maximum internal energy,

$$\frac{du}{dv} = 0$$

then,

$$\frac{d}{dv}(1.5pv-85) = 0$$

$$\frac{d}{dv}[1.5(a+bV)v-85] = 0 \qquad (\because\ p = a+bV)$$

$$\frac{d}{dv}[1.5(a+bmv)v-85] = 0 \qquad \left[\because\ v = \frac{V}{m}\right]$$

$$a + 2bmv = 0$$

$$\Rightarrow \qquad v = \frac{-a}{2bm} \qquad \text{...(iii)}$$

$\because$ It is given that

$$p = a + bV$$

Now putting the given values,

$$1000 = a + b \times 0.20 \qquad \text{...(iv)}$$

$$200 = a + b \times 1.20 \qquad \text{...(v)}$$

After solving equation (iv) and equation (v)

$$a = 1160$$

$$b = -800$$

Putting these values in equation (iii),

$$v = \frac{-1160}{2\times(-800)\times 1.5} = 0.4833 \text{ m}^3/\text{kg}$$

Since,

$$u = 1.5(a + bV)v - 85$$

$$= 1.5[(a + bmv)v] - 85$$

Now, putting the values of a, b and v for maximum internal energy.

$$u_{\max} = 1.5[1160 + (-800) \times 1.5 \times 0.4833] \times 0.4833 - 85$$

$$= 335.499 \text{ kJ/kg}$$

$$U_{\max} = 1.5 \times 335.499 \text{ kJ} = 503.25 \text{ kJ}$$

2. (a) Solution:

Given :

$$\mu_1 = \mu_2 = (\rho\nu) = (0.95 \times 1 \times 9) \text{ g/cm.s} = 8.55 \text{ g/cm.s}$$

$$L_1 = L_2 = 1000 \text{ m}; D_1 = 10 \text{ cm} = 0.10 \text{ m}; D_2 = 12 \text{ cm} = 0.12 \text{ m}$$

$$w_1 = w_2 = 0.95 \times 9810 \text{ N/m}^3 = 9319.5 \text{ N/m}^3$$

Assuming the flow in each pipe to be laminar, the head loss is given by

$$h_f = \frac{32\mu VL}{wD^2} = \frac{128\mu QL}{w\pi D^4}$$

Let Q_1 and Q_2 be the flow rates through each of the two pipes, which being in parallel, we have

$$h_{f_1} = h_{f_2}$$

or

$$\frac{128\mu_1 Q_1 L_1}{w_1 \pi D^2} = \frac{128\mu_2 Q_2 L_2}{w_2 \pi D^2}$$

∴

$$\frac{128\times 8.55\times Q_1\times 1000}{9319.5\times\pi\times(0.10)^4} = \frac{128\times 8.55\times Q_1\times 1000}{9319.5\times\pi\times(0.12)^4}$$

$$\frac{Q_1}{(0.10)^4} = \frac{Q_2}{(0.12)^4}$$

or

$$\frac{Q_1}{Q_2} = \left(\frac{0.10}{0.12}\right)^4 = 0.4823$$

or

$$Q_1 = 0.4823Q_2$$

But

$$Q_1 + Q_2 = 15 \; l/\text{s}$$

∴

$$Q_1 = 4.881 \; l/\text{s and } Q_2 = 10.119 \; l/\text{s}$$

The Reynolds number is given by

$$\text{Re} = \frac{VD}{\nu}$$

For pipe of diameter 10 cm, $V = \dfrac{4.881 \times 10^3}{(\pi/4)(10)^2} = 62.146 \text{ cm/s}$

$$\therefore \quad \text{Re} = \frac{62.146 \times 10}{9} = 69.051$$

Similarly for pipe of diameter 12 cm

$$V = \frac{10.119 \times 10^3}{(\pi/4)(12)^2} = 89.472 \text{ cm/s}$$

$$\therefore \quad \text{Re} = \frac{89.472 \times 12}{9} = 119.296$$

Thus the assumption of laminar flow in both the pipes is correct.

The head load, $h_f = \dfrac{128\mu QL}{w\pi D^4} = \dfrac{128\mu QL}{\rho g\pi D^4} = \dfrac{128\nu QL}{g\pi D^4}$

$$h_f = \frac{128 \times 9 \times 4.881 \times 10^3 \times 10^5}{981 \times \pi \times (10)^4}$$

$$= 1.824 \times 10^4 \text{ cm} = 182.45 \text{ m}$$

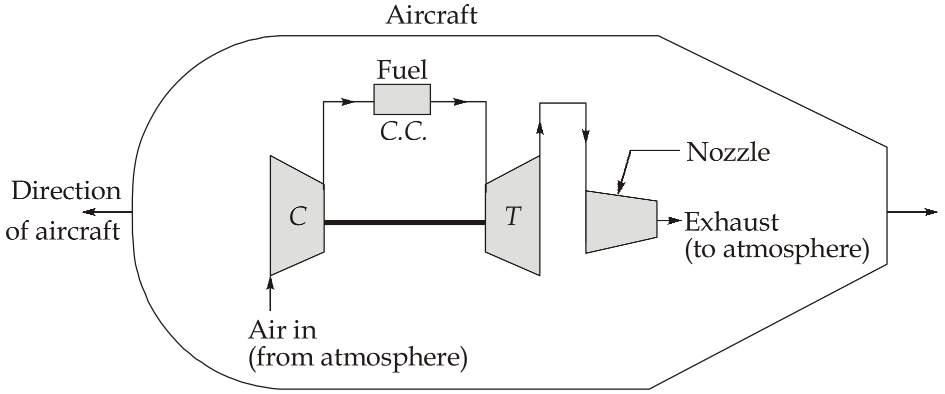
Power of the pump, $P = WQh_f$

$$= 0.95 \times 1000 \times 9.81 \times 15 \times 10^{-3} \times 182.45$$

$$= 25.51 \text{ kW}$$ **Ans.**

2. (b) Solution:

Construction: A turbojet engine consists of a diffuser at the entrance before the compressor (axial flow type rotary compressor) and a nozzle at exit after the turbine.



Gas turbine plant for turbojet

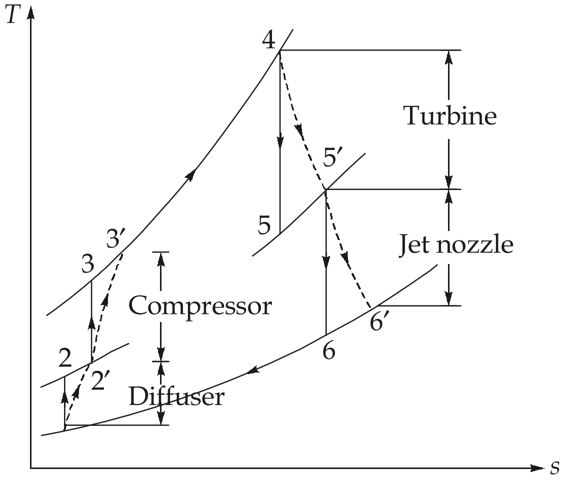
At entrance, diffuser slows down the air (because velocity of air entering is equal to the plane speed) and its kinetic energy is converted into pressure and this compression is known as ram compression. This compressed air is again compressed into compressor up to a pressure of 3 to 4 bar. This compressed air enters the combustion chamber where fuel is added and combustion occurs at constant pressure that results in rise of temperature rapidly. These hot gases from combustion chamber are expanded partially into turbine and developing the power from turbine. This power is just sufficient to drive the compressor, fuel pump and other auxiliaries.

The exhaust gases from turbine are at higher pressure than the atmospheric pressure. These gases are expanded in a nozzle and very high velocity jet is produced which provides a forward motion to the aircraft by the jet reaction according to Newton's third law of motion.

The turbojets are more suited to the aircrafts travelling above 800 km/h. It gives higher propulsion efficiency at higher speeds.

Overall efficiency of turbojet $= \eta_{\text{thermal}} \times \eta_{\text{propulsive}}$

T-s diagram:



T-s diagram of turbojet

Process 1-2: It shows ram compression with 100% efficient diffuser. In ideal diffuser velocity of air at point 2 is equal to zero ($C_2 = 0$) Process 1-2′ is real process.

Process 2-3: Isentropic compression of air and 2'-3' shows actual compression of air.

Process 3-4: Ideal addition of heat at constant pressure $P_3 = P_4$ Process 3′-4 shows actual addition of heat.

Process 4-5: Isentropic expansion of gas in turbine. Process 4-5 is actual expansion in turbine.

Process 5-6: Isentropic expansion of gas in nozzle and 5′-6′ is actual expansion of gas in nozzle.

Expressions: Let 1 kg of working fluid is flowing through the system.

1. **Diffuser:** For process 1-2, the energy equation is

$$\frac{C_1^2}{2} + h_1 + Q_{1-2} = \frac{C_2^2}{2} + h_2 + W_{1-2}$$

For ideal diffuser, $C_2 = 0,\ Q_{1\text{-}2} = 0,\ W_{1\text{-}2} = 0$

So, $$h_2 = h_1 + \frac{C_1^2}{2}$$

or $$T_2 = T_1 + \frac{C_1^2}{2C_p}$$

For diffuser efficiency,

$$\eta_d = \frac{h_2 - h_1}{h_{2'} - h_1} = \frac{T_2 - T_1}{T_{2'} - T_1}$$

$$h_{2'} = h_1 + \frac{C_1^2}{2\eta_d}$$

or $$T_{2'} = T_1 + \frac{C_1^2}{2C_p \cdot \eta_d}$$

2. **Compressor :** Ideal work of compressor

$$W_c = h_3 - h_2 = C_p(T_3 - T_2)$$

$$\text{Actual work} = h_{3'} - h_2 = \frac{h_3 - h_2}{\eta_c}$$

3. **Combustion chamber :** Ideal heat supplied per kg

$$Q = h_4 - h_3 = C_p(T_4 - T_3)$$

$$\text{Actual heat supplied} = C_{pg}\left(1 + \frac{m_f}{m_a}\right)T_4 - C_{pa} \cdot T_{3'}$$

4. **Turbine :**

$$\text{Turbine work, } W_t = h_4 - h_5$$
$$= C_p(T_4 - T_5)$$

$$\text{Actual turbine work} = h_4 - h_{5'} = C_p(T_4 - T_{5'})$$
$$= C_p(T_4 - T_5) \times \eta_t$$

5. **Jet nozzle :** For process 5 - 6,

$$h_5 + \frac{C_5^2}{2} = h_6 + \frac{C_6^2}{2}$$

For actual process, $h_{5'} + \frac{C_5^2}{2} = h_{6'} + \frac{C_{6'}^2}{2}$

Thermal efficiency, η_{th} is given by

$$\eta_{th} = \frac{(h_4 - h_{6'}) - (h_{3'} - h_1)}{(h_4 - h_{3'})} = \frac{(T_4 - T_{6'}) - (T_{3'} - T_1)}{(T_4 - T_{3'})}$$

2. (c) Solution:

Given : L = 6 cmm = 0.006 m; ρ = 7800 kg/m^3; C_p = 465 J/kgK; k = 54 W/m-°C; h = 3405 W/m^2-K; t_i = 88°C; t_∞ = 2200°C; t = 1095°C

$$\text{Characteristic length, } L_c = \frac{L}{2} = \frac{0.006}{2} = 0.003 \text{ m}$$

$$\text{Biot number, } B_i = \frac{hL_C}{k} = \frac{3400 \times 0.003}{51} = 0.2$$

As Bi > 0.1, therefore lumped analysis cannot be applied in this case.

Hence, Heisler charts are used to solve this problem.

Corresponding to $\frac{1}{B_i} = \frac{1}{0.2} = 5$ and $\frac{x}{L_c} = 1$ (outside surface from nose section), from Heisler charts.

$$\frac{t - t_\infty}{t_0 - t_\infty} = 0.9$$

Also, $$\frac{t - t_\infty}{t_i - t_\infty} = \left[\frac{t_0 - t_\infty}{t_i - t_\infty}\right] \times \left[\frac{t - t_\infty}{t_o - t_\infty}\right]$$

$$\frac{1095 - 2200}{88 - 2200} = \frac{t_0 - t_\infty}{t_i - t_\infty} \times 0.9$$

$$\frac{t_0 - t_\infty}{t_i - t_\infty} = \frac{1}{0.9} \times \frac{(-1105)}{(-2112)} = 0.5813$$

Now, again using the Heisler chart for $\frac{t_0 - t_\infty}{t_i - t_\infty} = 0.5813$ and $\frac{1}{B_i} = 5.$

We get, Fourier number, $F_0 = 3$

$$\frac{\alpha\tau}{L_c^2} = 3$$

$$\frac{k}{\rho C_p} \times \frac{\tau}{L_c^2} = 3 \qquad \left(\because \ \alpha = \frac{k}{\rho C_p}\right)$$

$$\frac{51}{7800 \times 465} \times \frac{\tau}{(0.003)^2} = 3$$

$$\tau = \frac{3 \times (0.003)^2 \times 7800 \times 465}{51}$$

$$\tau = 1.92 \text{ sec}$$

- The temperature at the inside ($x = 0$) is given by

$$\frac{t_0 - t_\infty}{t_i - t_\infty} = 0.5813$$

$$\frac{t_0 - 2200}{88 - 2200} = 0.5813$$

$$t_0 = 2200 + 0.5813(88 - 2200)$$

$$t_0 = 972.2944°\text{C}$$

3. (a) Solution:

Refrigerant : A refrigerant is defined as any substance that absorbs heat through expansion or vapourization and loses it through condensation in a refrigeration system. The basic function of refrigerant is take heat from the evaporator and loose the heat in the condenser with reasonable heat transfer rate and establishes the effective heat exchange in the system.

Desirable Properties of an Ideal Refrigerant:

1. Low boiling point.
2. Low specific heat of liquid.
3. Low specific volume of vapour.
4. Low cost.
5. High critical temperature.
6. High latent heat of vapourization.

7. Non-corrosive to metal.
8. Non-flammable and non-explosive.
9. Non-toxic.
10. Easy to liquefy at moderate pressure and temperature.

Classification of Refrigerants: The refrigerants are basically classified as follow:

(a) Primary Refrigerants:

1. Those refrigerants which directly take part in the refrigeration system and cool the substance by the absorption of latent heat.

   Example: Ammonia, Caron dioxide, Sulphur dioxide, Methyl chloride, Freon group etc.

2. The primary refrigerants are further classified into the following four groups:

i. Halo-carbon refrigerants,

ii. Azeotrope refrigerants,

iii. Inorganic refrigerants,

iv. Hydro-carbon refrigerant.

i. Halo-Carbon Refrigerants :

- The halocarbon compounds are obtained after replacing one or more of hydrogen atoms of hydrocarbon methane or ethane by one or more of the three halogens: chlorine, fluorine and bromine.
- Some of the commonly used halo-carbon compounds are R-11, R-12, R-13, R-21, R-22, etc.

ii. Azeotrope Refrigerant:

- The term 'azeotrope' refers to a stable mixture of refrigerants whose vapour and liquid phases retain identical compositions over a wide range of temperatures.
- Some of the azeotropes are R-500, R-502, R-503, etc.

iii. Inorganic Refrigerant :

- The inorganic refrigerants were most commonly used before the introduction of hydro-carbon group for all purposes.
- These refrigerants are still in use due to their inherent thermodynamic and physical properties.
- The important inorganic refrigerants are R-717, R-118, R-729, etc.

iv. **Hydro-Carbon Refrigerants:**

- Most of the refrigerants of this group are organic compounds and these are successfully used in industrial and commercial installations.
- Some of the important refrigerants of this group are R-600a, R-1150, R-1270, etc.

(b) Secondary Refrigerants:

1. Those refrigerants which are first cooled with the help of the primary refrigerants and are then employed for cooling purposes are known as secondary refrigerant.
2. These refrigerants cool substances by absorption of their sensible heat.
3. The commonly used secondary refrigerants are as follows:

   Water, Sodium chloride brine, Calcium chloride brine, Ethylene glycol, and Propylene glycol etc.

3. (b) Solution:

(i) The partial derivatives of the given functions are

$$\frac{\partial u}{\partial x} = 2 - 2xy; \ \frac{\partial v}{\partial y} = 2xy - 2$$

For a two-dimensional flow of incompressible fluid the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2xy + 2xy - 2 = 0$$

Therefore the functions represent a possible case of fluid flow.

The rotation component ω_z of any fluid element in the flow field is

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Now, $$\frac{\partial v}{\partial x} = (y^2 - x^2)$$

and $$\frac{\partial u}{\partial y} = (y^2 - x^2)$$

$$\omega_z = [(y^2 - x^2) - (y^2 - x^2)] = 0$$

which shows that the given functions represent an irrotational flow.

(ii)

As we know,
$$\frac{\partial \psi}{\partial x} = v = xy^2 - 2y - \frac{y^3}{3} \qquad \text{...(i)}$$

and
$$\frac{\partial \psi}{\partial y} = -u = -\left(\frac{y^3}{3} + 2x - x^2 y\right) \qquad \text{...(ii)}$$

Integrating equation (i), we get
$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} + f(y) \qquad \text{...(iii)}$$

Differentiating equation (iii) with respect to y, we get
$$\frac{\partial \psi}{\partial y} = x^2 y - 2x + f'(y) \qquad \text{...(iv)}$$

Equating the values of $\left(\frac{\partial \psi}{\partial y}\right)$ from equation (ii) and (iv), we get
$$-\left(\frac{y^3}{2} + 2x - x^2 y\right) = x^2 y - 2x + f'(y)$$

or
$$f'(y) = -\frac{y^3}{3}$$

By integrating both sides of the above expression, we get
$$f'(y) = -\frac{y^4}{12} + C$$

where C is a constant of integration which is a numerical constant only. Therefore
$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} - \frac{y^4}{12} + C$$

(iii)

Also,
$$-\frac{\partial \phi}{\partial x} = u = \frac{y^3}{3} + 2x - x^2 y \qquad \text{...(i)}$$

$$-\frac{\partial \phi}{\partial y} = v = xy^2 - 2y - \frac{x^3}{3} \qquad \text{...(ii)}$$

Integrating equation (i), we get
$$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + f(y) \qquad \text{...(iii)}$$

Differentiating equation (iii) with respect to y, we get

$$\frac{\partial \phi}{\partial y} = -xy^2 + \frac{x^3}{3} + f'(y) \qquad \text{..(iv)}$$

Equating the values of $\left(\frac{\partial \phi}{\partial y}\right)$ from equation (ii) and (iv), we get

$$-xy^2 + 2y + \frac{x^3}{3} = -xy^2 + \frac{x^3}{3} + f'(y)$$

or $$f'(y) = 2y$$

By integrating both sides of the above expression, we get

$$f(y) = y^2 + C$$

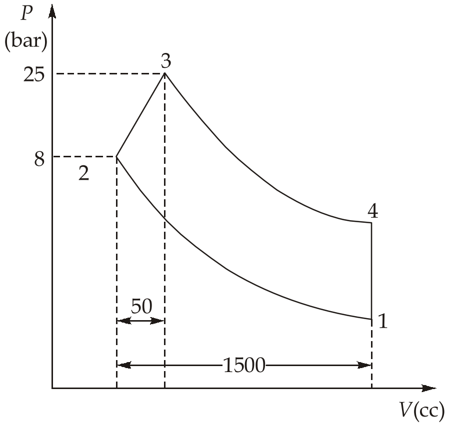
where C is a numerical constant of integration,

Therefore, $$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2 + C$$

3. (c) Solution:

Given :

$$r = 6,\ V_s = 0.0015 \text{ m}^3$$
$$P_2 = 8 \text{ bar},\ T_2 = 350°\text{C}$$
$$CV = 42 \text{ MJ/kg, AFR} = 16 : 1$$
$$P_3 = 25 \text{ bar}$$



Now $$V_1 - V_2 = 0.0015 \times 10^6 = 1500 \text{ cc}$$
$$V_1 = 6V_2$$

∴ $$V_2 = \frac{1500}{6-1} = 300 \text{ cc}$$

$$V_3 = \frac{1}{30}(\text{stroke volume}) + V_2 = \frac{1500}{30} + 300 = 350 \text{ cc}$$

$$T_2 = 350 + 273 = 623 \text{ K}$$

Now, $$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2}$$

So, $$T_3 = T_2 \times \frac{P_3V_3}{P_2V_2} = 623 \times \frac{25}{8} \times \frac{350}{300} = 2271.35 \text{ K}$$

We need to calculate work done and increase in internal energy between 2 and 3, so that we can find heat added by using first law of thermodynamics.

$$W_{23} = \text{Area under } 2-3 = \left(\frac{P_2+P_3}{2}\right) \times (V_3 - V_2)$$

$$= \left(\frac{25+8}{2}\right) \times 10^5 \times (350-300) \times 10^{-6} = 82.5 \text{ J}$$

$$[W_{2-3} = 82.5 \text{ J}]$$

$$\text{Mass of mixture, } m = \left(\frac{PV}{RT}\right)_2 = \frac{8 \times 10^5 \times 300 \times 10^{-6}}{287 \times 623}$$

$$m = 1.342 \times 10^{-3} \text{ kg}$$

$$\Delta U = U_3 - U_2 = mC_V(T_3 - T_2) = m(C_P - R)(T_3 - T_2)$$

$$\Delta U = 1.342 \times 10^{-3} \times (1 - 0.287)(2271.35 - 623)$$

$$= 1.5775 \text{ kJ}$$

$$Q_t = \Delta U + W = 1.5775 + \frac{82.5}{1000} = 1.66 \text{ kJ}$$

Actual heat liberated in one cycle, $Q_a = \left(\frac{m}{17}\right) \times CV$ $\left[m_{\text{gasoline}} = \frac{m}{17}\right]$

$$Q_a = \frac{1.342 \times 10^{-3}}{17} \times 42000 = 3.3155 \text{ kJ}$$

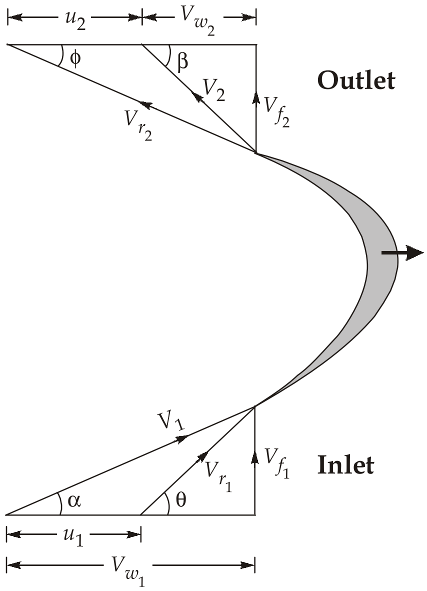
$$\text{Heat lost during explosion} = Q_a - Q_t$$

$$= 3.3155 - 1.66 = 1.655 \text{ kJ}$$

$$\text{Heat lost in kJ/kg} = \frac{1.655}{1.342 \times 10^{-3}} = 1233.628 \text{ kJ/kg of charge}$$

4. (a) Solution:

If H is the net head acting on the turbine then the work done by the runner per unit weight of water may be expressed as



$$\frac{V_{w_1}u_1 - V_{w_2}u_2}{g} = H - \text{losses}$$

The losses may be combinedly expressed as $(k_1 + k_2 + k_3 + k_4)\dfrac{V_f^2}{2g}$

Also $$V_{f_1} = V_{f_2} = V_f \text{ being constant}$$

Thus, $$\frac{V_{w_1}u_1 - V_{w_2}u_2}{g} = H - (k_1 + k_2 + k_3 + k_4)\frac{V_f^2}{2g} \quad \text{...(i)}$$

From inlet and outlet velocity triangles,

$$V_{w_1} = V_f \cot\alpha; \quad \frac{V_f}{V_{w_1} - u_1} = \tan\theta$$

or $$\frac{V_f}{V_f \cot\alpha - u_1} = \tan\theta$$

∴ $$u_1 = V_f(\cot\alpha - \cot\theta)$$

$$\frac{V_{f_1}}{V_{w_2} + u_2} = \tan\phi$$

or $$\frac{V_f}{V_{w_2} + u_2} = \tan\phi$$

Also, $$u_1 = \frac{\pi D_1 N}{60};\ u_2 = \frac{\pi D_2 N}{60}$$

$\therefore$ $$u_2 = \left(\frac{D_2}{D_1}\right)u_1$$

$$= nu_1 = nV_f(\cot\alpha - \cot\theta)$$

Introducing this value of u_2, we get

$$\frac{V_f}{V_{w_2} + nV_f(\cot\alpha - \cot\theta)} = \tan\phi = \frac{1}{\cot\phi}$$

or $$V_{w_2} = V_f\left[\cot\phi - n(\cot\alpha - \cot\theta)\right]$$

The hydraulic efficiency is given by

$$\eta_h = \frac{V_{w_1}u_1 - V_{w_2}u_2}{gH}$$

From equation (i), we have

$$gH = \left(V_{w_1}u_1 - V_{w_2}u_2\right) + \left(k_1 + k_2 + k_3 + k_4\right)\frac{V_f^2}{2}$$

Thus, $$\eta_h = \frac{\left(V_{w_1}u_1 - V_{w_2}u_2\right)}{\left(V_{w_1}u_1 - V_{w_2}u_2\right) + \left(k_1 + k_2 + k_3 + k_4\right)\frac{V_f^2}{2}}$$

$$\eta_h = \frac{2}{2 + \frac{\left(k_1 + k_2 + k_3 + k_4\right)V_f^2}{\left(V_{w_1}u_1 - V_{w_2}u_2\right)}}$$

$$\eta_h = \frac{2}{2 + \frac{\left(k_1 + k_2 + k_3 + k_4\right)V_f^2}{\left[V_f\cot\alpha \times V_f\left(\cot\alpha - \cot\theta\right)\right] - \left[V_f\left(\cot\phi - n(\cot\alpha - \cot\theta)\right) \times nV_f\left(\cot\alpha - \cot\theta\right)\right]}}$$

$$\eta_h = \frac{2}{2 + \frac{\left(k_1 + k_2 + k_3 + k_4\right)V_f^2}{V_f^2\left[\cot\alpha\left(\cot\alpha - \cot\theta\right) - n\left(\cot\alpha - \cot\theta\right)\left(\cot\phi - n\left(\cot\alpha - \cot\theta\right)\right)\right]}}$$

$$\eta_h = \frac{2}{2 + \frac{\left(k_1 + k_2 + k_3 + k_4\right)}{\left(\cot\alpha - \cot\theta\right)\left\{\cot\alpha\left(1 + n^2\right) - n\left(\cot\phi + n\cot\theta\right)\right\}}}$$

4. (b) Solution:

Given : $t_\infty = 25°\text{C}$; $t_s = 95°\text{C}$; $L = 4.5$ m; $U = 0.1$ m/s, $\nu = 0.65 \times 10^{-4}$ m^2/s, $\alpha = 7.2 \times 10^{-8}$ m^2/s, $k = 0.213$ W/m°C, $\rho = 956.8$ kg/m^3

Reynolds number at the end of the plate,

$$\text{Re} = \frac{UL}{\nu} = \frac{0.1 \times 4.5}{0.65 \times 10^{-4}} = 6923$$

$\because$ $\text{Re} < 5 \times 10^5$, hence flow is laminar

The hydrodynamic boundary layer thickness,

$$\delta = \frac{5L}{\sqrt{\text{Re}}} = \frac{5 \times 4.5}{\sqrt{6923}} = 0.2704 \text{ m} = 270.4 \text{ mm}$$

The thermal boundary layer thickness,

$$\delta_{th} = \frac{\delta}{(\text{Pr})^{1/3}}$$

where, $$\text{Pr} = \frac{\nu}{\alpha} = \frac{0.65 \times 10^{-4}}{7.2 \times 10^{-8}} = 902.77$$

$\therefore$ $$\delta_{th} = \frac{0.2704}{(902.77)^{1/3}} = 0.02798 \text{ m} = 27.98 \text{ mm}$$

The average skin friction coefficient is given by,

$$\bar{C}_f = \frac{1.328}{\sqrt{\text{Re}}} = \frac{1.328}{\sqrt{6923}} = 0.01596$$

The total drag force,

$$F_D = \bar{C}_f \times \frac{1}{2}\rho U^2 \times \text{Area of plate (for one side)}$$

$$= 0.01596 \times \frac{1}{2} \times 956.8 \times (0.1)^2 \times (4.5 \times 1)$$

$$= 0.3436 \text{ Nm}$$

$$Nu_x = \frac{h_x x}{k} = 0.332(Re_x)^{1/2}(\text{Pr})^{1/3}$$

$$= 0.332 \times (6923)^{1/2}(902.77)^{1/3} = 266.98$$

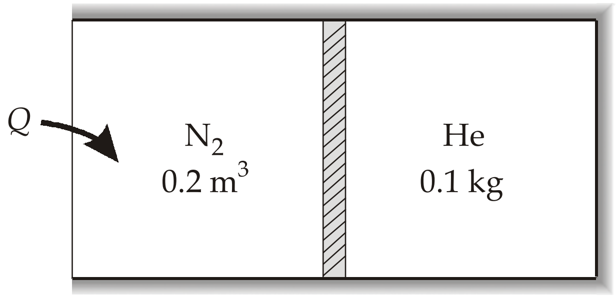
or $$h_x = \frac{266.98 \times k}{x} = \frac{266.98 \times 0.213}{4.5} = 12.64 \text{ W/m}^2\text{-°C}$$

$$\bar{h} = 2h_x = 2 \times 12.64 = 25.28 \text{ W/m}^2\text{°C}$$

Local transfer rate, $$Q = \bar{h}A_s(t_s - t_\infty)$$

$$= 25.28 \times (4.5 \times 1) \times (95 - 25) = 7963.2 \text{ W}$$

4. (c)



Helium undergoes an isentropic compression process and final temperature,

$$T_{\text{He},2} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (20+273)\left(\frac{120}{95}\right)^{0.667/1.667}$$

$$= 321.7 \text{ K}$$

The initial and final volumes of the helium are:

$$V_{\text{He},1} = \frac{mRT_1}{P_1} = \frac{0.1\times2.0769\times293}{95} = 0.6406 \text{ m}^3$$

$$V_{\text{He},2} = \frac{mRT_2}{P_2} = \frac{0.1\times2.0769\times321.7}{120} = 0.5568 \text{ m}^3$$

Final volume of nitrogen becomes:

$$V_{N_{2,2}} = V_{N_{2,1}} + V_{He,1} - V_{He,2}$$

$$= 0.2 + 0.6406 \;\; 0.5568$$

$$= 0.2838 \text{ m}^3$$

The mass and final temperature of nitrogen are:

$$m_{\text{N}2} = \frac{P_1V_1}{RT_1} = \frac{95\times0.2}{0.2968\times293} = 0.2185 \text{ kg}$$

$$T_{N_{2,2}} = \frac{P_2V_2}{M_{N2}} = \frac{120\times0.2838}{0.2185\times0.2968} = 525.1 \text{ K}$$

Heat transferred to the nitrogen can be determined as

$$Q_{\text{in}} = \Delta U_{N_2} + \Delta U_{\text{He}} \quad [W=0]$$

$$= [mc_v(T_2 - T_1)]_{N_2} + [mc_v(T_2 - T_1)]_{\text{He}}$$

$$= 0.2185 \times 0.743\,(525.1 - 293) + 0.1 \times 3.1156\,(321.7 - 293)$$

$$= 46.6 \text{ kJ}$$

As helium undergoes isentropic process, the entropy generation is zero for helium. Therefore,

$$S_{gen} = \Delta S_{N_2} + \Delta S_{He} + \Delta S_{surr} + = \Delta S_{N_2} + \Delta S_{surr}$$

where,

$$\Delta S_{surr} = -\frac{Q_{in}}{T_R} = -\frac{46.6}{500+273} = -0.060285 \text{ kJ/K}$$

$$\Delta S_{N_2} = m_{N_2}\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right)$$

$$= 0.2185\left(1.039\ln\frac{525.1}{293} - 0.2968\ln\frac{120}{95}\right)$$

$$= 0.2185\ (0.60617 - 0.06934)$$

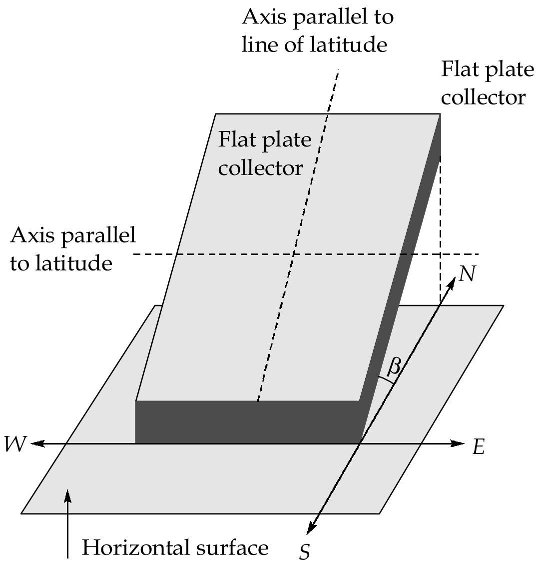
$$= 0.2185 \times 0.53683 = 0.1173 \text{ kJ/K}$$

$$S_{gen} = 0.1173 - 0.060285 = 0.057 \text{ kJ/K}$$

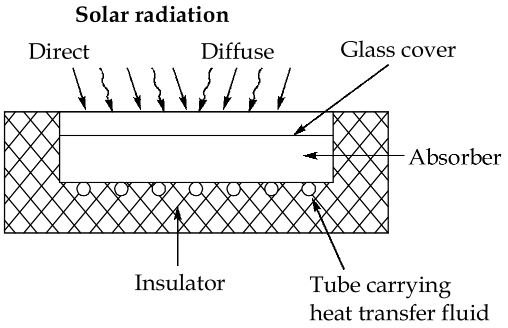
Section : B

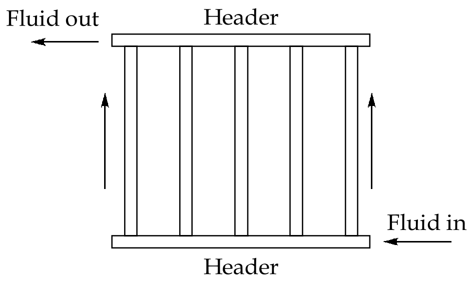
5. (a) Solution:

A flat-plate collector is placed in a position such that its length aligns with the line of longitude and is suitably tilted towards south to have maximum collection. The positioning of flat plate collector is shown in the figure below:



1. Basic elements are transparent cover; blackened absorber plate usually of copper, aluminium or steel; tubes, channels or passages in thermal contact with absorber plate and weather tight, insulated container to enclose the above components. The constructional details of flat plate collector is shown below.





2. A liquid, most commonly water or a mixture of water and ethylene glycol (if ambient temperature falls below 0°C during night) are used as the heat transport medium from collector to the next stage of system.

3. As solar radiation strikes on a specially treated metallic absorber plate, it is absorbed and raises the plate temperature. The absorber plate is usually made from a metal sheet ranging in thickness from 0.2 mm to 1 mm made of copper.

4. The heat is transferred to the heat-transfer liquid circulating the metallic tube diameter in range of 1 to 1.5 cm. These are soldered, brazed, welded or pressure bonded to the absorber plate.

5. Header pipes of diameter 2 to 2.5 cm lead the water in and out of the collector and distribute to tubes. They are mostly made of copper.

6. In the bottom and along the side walls, thermal insulation provided by a 2.5 to 8 cm thick layer of glass wool prevents heat loss from rear surface and side of the collector.

7. The glass cover permits the entry of solar radiation (transparent for incoming short wavelength) but largely opaque to the longer infrared radiation reflected from the absorber. Usually one or two covers with spacing ranging from 1.5 to 3 cm and plain or toughened glass of 4 to 5 mm thickness is most favoured material.

8. Pipe and fin type, rectangular or cylindrical full sandwich type and roll-bond or semi-sandwich type are basic types of absorber plate depending on the extent of wetted area relative to the absorbing surface area.

9. Liquid flat-plate collector efficiency is given as

$$\eta_i = \frac{\text{Useful heat gain}}{\text{Solar radiation incident on collection}}$$

5. (b) Solution:

The computations are performed in tabular form as shown below:

S.no	$u(\text{mm/s})$	$u' = (u - \bar{u})$	$v(\text{mm/s})$	$v' = (v - \bar{v})$	$u'v'$
1	+105	+8	–3	–3	–24
2	+110	+13	–16	–16	–208
3	+84	–13	+11	+11	–143
4	+89	–8	+25	+25	–200
5	+102	+5	–6	–6	–30
6	+94	–3	–20	–20	–280
7	+111	+14	–20	–20	–280
8	+101	+4	+4	+4	+16
9	+87	–10	+21	+21	–210
10	+95	–2	–2	–2	+4
11	+89	–8	+6	+6	–48
	$\sum u = 1067$ $\therefore\ \bar{u} = \frac{1067}{11}$ $= 97\ \text{mm/s}$		$\sum v = 0$ $\vec{v} = 0$		$\sum(u'v') = -1063$ $\overline{u'v'} = -\frac{1063}{11}$ $= -96.64$

$$\text{Reynold's shear stress, } \tau = \overline{\rho u'v'}$$

$$= (1.23 \times 96.64) = 118.87\ \text{N/m}^2$$

Q.5 (c) Solution:

Given :

$$r = 8,\ \frac{dc_v}{c_v} = 2\% = 0.02$$

Otto cycle efficiency is given by,

$$\eta = 1 - \frac{1}{r^{\gamma - 1}}$$

We know that, $$c_v = \frac{R}{\gamma - 1} \Rightarrow \gamma - 1 = \frac{R}{c_v}$$

$$\therefore \qquad \eta = 1 - \frac{1}{r^{R/c_v}} = 1 - (r)^{-R/c_v}$$

$$\Rightarrow \qquad 1 - \eta = (r)^{-R/c_v}$$

Taking log on both sides, $\log_e(1-\eta) = \dfrac{-R}{c_v}\log_e r,$

Differentiating both side, where R, r are constant

$$-\frac{d\eta}{1-\eta} = -R\log_e r \times \left[-\frac{1}{c_v^2}dc_v\right]$$

$$\Rightarrow \qquad -d\eta = (1-\eta)\frac{R}{c_v}\log_e r \times \left(\frac{dc_v}{c_v}\right)$$

$$\frac{d\eta}{\eta} = \left(\frac{\eta-1}{\eta}\right)(\gamma-1)\log_e r \times \left(\frac{dc_v}{c_v}\right) \qquad \left[\because \gamma-1=\frac{R}{c_v}\right]$$

Now at $r = 8$, $\qquad \eta_{\text{Otto}} = 1-\dfrac{1}{8^{1.4-1}} = 0.5647$

So, $\qquad \dfrac{d\eta}{\eta} = \left(\dfrac{0.5647-1}{0.5647}\right)\times(1.4-1)\times[\log_e 8]\times\dfrac{dc_v}{c_v}$

$$= \frac{0.5647-1}{0.5647}\times 0.4\times(2.0794)\times(0.02)$$

$$\mathbf{= -0.0128 \text{ or } -1.28\%}$$

Therefore, there is decrease of 1.28% of efficiency with increase in 2% specific heat at constant volume.

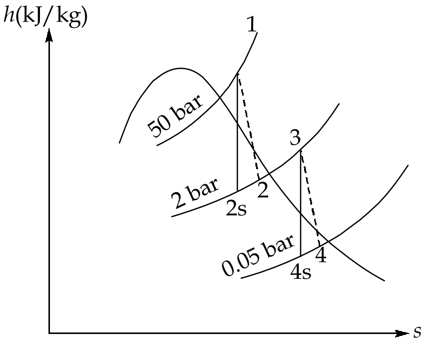
5. (d) Solution:

$\dot{m}$ = Steam flow rate through boiler (kg/s)

At 350°C and 50 bar:

h_1 = 3069.3 kJ/kg

s_1 = 6.4516 kJ/kgK



At 2.0 bar:

s_f = 1.5302 kJ/kgK, $\qquad h_f$ = 504.71 kJ/kg

s_{fg} = 5.5968 kJ/kgK, $\qquad h_{fg}$ = 2201.6 kJ/kg

At 0.05 bar:

s_f = 0.4762 kJ/kgK, $\qquad h_f$ = 137.75 kJ/kg

s_{fg} = 7.9176 kJ/kgK, $\qquad h_{fg}$ = 2423.0 kJ/kg

At 2.0 bar and 250°C:

$$h_3 = 2971.2 \text{ kJ/kg}$$

$$s_3 = 7.7100 \text{ kJ/kg.K}$$

Now,

$$s_1 = s_{2s} = (s_f + xs_{fg})_2$$

$$6.4516 = 1.5302 + x(5.5968)$$

$$x = 0.87932$$

$$h_{2s} = h_f + xh_{fg} = 504.71 + 0.87932 \times 2201.6$$

$$= 2440.6295 \text{ kJ/kg}$$

and

$$s_3 = s_{4s} = (s_f + xs_{fg})_4$$

$$7.7100 = 0.4762 + x \times 7.9176$$

$$x = 0.91363$$

$$h_{4s} = h_f + xh_{fg} = 137.75 + 0.91363 \times 2423$$

$$= 2351.488 \text{ kJ/kg}$$

Steam flow through low pressure stage = $\dot{m} - \dfrac{12000}{3600} = (\dot{m} - 3.333)$ kg/s

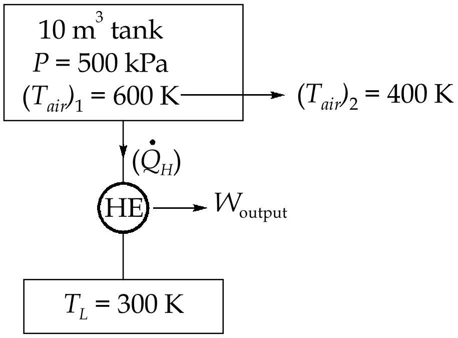
Turbine power output = 3750 kW

$$\dot{m}(h_1 - h_{2s}) \times \eta_{\text{isen}} + (\dot{m} - 3.333)(h_3 - h_{4s})\eta_{\text{isen}} = 3750 \text{ kW}$$

$$\dot{m}(3069.3 - 2440.6295) + (\dot{m} - 3.333)(2971.2 - 2351.488) = \frac{3750}{0.81}$$

$$\dot{m} = 5.363 \text{ kg/s}$$

5. (e) Solution:



$$T_H = T_{\text{air}} - 25°\text{C}$$

$$T_L = 300 \text{ K}$$

$$\dot{m}_{air} = \frac{PV_1}{RT_1} = \frac{500 \times 10}{0.287 \times 600} = 29.036 \text{ kg}$$

From first law of thermodynamics,

$$Q_H = -m_{air}c_v dT_{air}$$

$$\eta = \frac{W}{\dot{Q}_H}$$

$$\int_1^2 dW = \int_1^2 \left(1-\frac{T_L}{T_H}\right) d\dot{Q}_H$$

$$W = -\left[\int m_{air}c_v dT_{air} - \int \frac{T_L}{T_H} m_{air} \times c_v dT_{air}\right]$$

$$W = -\dot{m}_{air} \times c_v \left[\left[(T_H)_2 - (T_H)_1\right] - T_L \ln\left(\frac{T_{H_2}}{T_{H_1}}\right)\right]$$

$$T_{H_1} = (T_{air} - 25)_1 = 600 - 25 = 575 \text{ K}$$

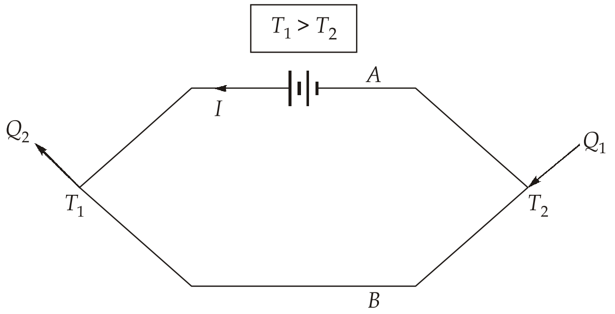
$$T_{H_2} = (T_{air} - 25)_2 = 400 - 25 = 375 \text{ K}$$

$$W = -29.036 \times 0.718\left[(375-575) - 300\ln\left(\frac{375}{575}\right)\right]$$

$$W_{output} = 1496.1832 \text{ kJ}$$

6. **(a) (i) Solution:**

Working principle: Thermo electric refrigeration works on the principle of "Peltier effect". When a battery is added in between the two conductors A and B whose junctions are initially at same temperature, and a current is made to flow through the circuit, the junction temperature will change, one junction becomes hot (T_1) and the other becomes cold (T_2). This effect is known as peltier effect. Refrigeration effect is obtained at the cold junction and heat is rejected to the surroundings at the hot junction. This is the basis for thermoelectric refrigeration system.



Peltier effect

The position of hot and cold junctions can be reversed by reversing the direction of current flow. The heat transfer rate at each junction is given by:

$$Q = \phi I$$

Where ϕ is the peltier coefficient is Volts and I is the current in Amperes.

6. (a) (ii) Solution:

Given: Latitude, $\phi = 22°$; $n = 148$; $a = 0.28$; $b = 0.48$; $\frac{L_a}{L_m} = 0.7944$

As we know,

$$\delta = 23.45 \sin\left\{\frac{360}{365} \times (284 + n)\right\} = 21.436°$$

$$I_n = I_{sc}\left\{1 + 0.033\cos\left(\frac{360}{365} \times n\right)\right\}$$

$$= 1329.61 \text{ W/m}^2 \text{ day} \qquad [\text{As } I_{sc} = 1367 \text{ W/m}^2]$$

$$I_n = 1329.61 \text{ W/m}^2 = 1329.61 \left(\frac{\text{J}}{\text{s} \times \text{m}^2}\right)$$

$$= 1329.61 \times 24 \times 60 \times 60 \text{ J/m}^2\text{day}$$

$$= \frac{1329.61 \times 24 \times 60 \times 60}{1000} \text{ kJ/m}^2\text{day}$$

$$= 114878.59 \text{ kJ/m}^2\text{day}$$

Also, sunshine hour angle, $\omega_s = \cos^{-1}(-\tan\phi \times \tan\delta) = 99.127° = 1.73$ rad

and,

$$H_0 = 3600 \times \frac{24}{\pi} \times I_{sc}\left[1 + 0.033\cos\left(\frac{360n}{365}\right)\right](\cos\phi\cos\delta\sin\omega_s + \omega_s\sin\phi\sin\delta)$$

where, I_{sc} in kW/m^2

$$\therefore \quad H_0 = 3600 \times \frac{24}{\pi} \times 1.367 \times \left[1 + 0.033\cos\left(\frac{360 \times 148}{365}\right)\right]\begin{pmatrix}\cos 22°\cos 21.436° \times \sin 99.127° \\ +1.73 \times \sin 22° \times \sin 21.436°\end{pmatrix}$$

$$H_0 = 37595.198 \times 0.9726 \times 1.08896$$

$$= 39817.92 \text{ kJ/m}^2\text{day}$$

Hence,

$$\frac{H_g}{H_o} = a + b\left(\frac{L_a}{L_m}\right) = 0.28 + 0.48 \times 0.7944 = 0.6613$$

$$H_g = 0.6613 \times 39817.92$$

$$\Rightarrow \quad H_g = 26331.59 \text{ kJ/m}^2\text{day}$$

Q.6 (b) Solution:

Let 100 kg of fuel or coal was supplied

$$\therefore \frac{84}{12}C+\frac{6}{2}H_2+a\left(O_2+\frac{79}{21}N_2\right) \rightarrow bCO_2 + d\,CO + e\,O_2 + f\,N_2 + gH_2O$$

Carbon balance, $b + d = 7$ ... (i)

Hydrogen balance $3 = g$

Oxygen balance $2a = 2b + d + 2e + g$

$$a = b+\frac{d}{2}+e+1.5 \quad \text{...(ii)}$$

Balancing nitrogen, $\frac{79}{21}a = f$ ...(iii)

Given : by dry gas analysis,

$$\frac{b}{b+d+e+f} = \frac{10.4}{100}$$

$$\frac{d}{b+d+e+f} = \frac{2}{100}$$

$$\therefore \quad \frac{b}{d} = 5.2$$

From (i) $5.2\,d + d = 7 \Rightarrow d = 1.129$

$$b = 5.2 \times 1.129 = 5.8709$$

$$b + d + e + f = \frac{100\times 5.8709}{10.4} = 56.4509$$

$$e + f = 49.4509 \quad \text{...(iv)}$$

By equation (ii)

$$a = 5.8709+\left(\frac{1.129}{2}\right)+e+1.5$$

$$a = 7.9354 + e \quad \text{...(v)}$$

By equation (iii)

$$f = 3.7619\text{ a} \quad \text{...(vi)}$$

By equation (iv)

$$e + f = 49.4509$$

$$a - 7.9354 + 3.7619\,a = 49.4509$$

$$a = 12.051$$

Now, $e = a - 7.9354 = 12.051 - 7.9354 = 4.1156$

$$f = (3.7619) \times a = 3.7619 \times 12.051 = 45.335$$

(i) Weight of dry flue gases $= 5.8709 \times 44 + 1.129 \times 28 + 4.1156 \times 32 + 45.335 \times 28$

$= 1691.0108$ kg for 100 kg of coal

Weight of dry flue gases per kg of coal = 16.91 kg

(ii) Weight of air supplied per kg of coal $= \left(\dfrac{a \times 32}{0.23}\right) \times \left(\dfrac{1}{100}\right)$ {$\because$ air contains 23% O_2 by weight}

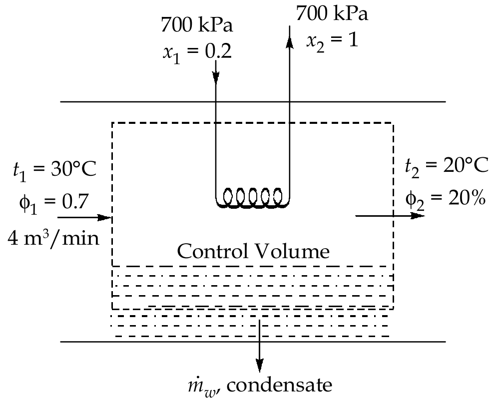
$$= \left(\frac{12.051 \times 32}{0.23}\right) \times \left(\frac{1}{100}\right)$$

Weight of air supplied per kg of coal = 16.7666 kg

(iii) Weight of water vapour formed per kg of fuel $= 3 \times \left(\dfrac{18}{100}\right)$

$= 0.54$ kg W.V./kg of fuel

6. (c) Solution:



At states-1 : $t_1 = 30°C$

$$\phi_1 = 0.7 = \frac{P_{v1}}{P_{vs}} = \frac{P_{v1}}{P_{sat@30°C}} = \frac{P_{v1}}{4.2469 \text{ kPa}}$$

$\Rightarrow$ $P_{v1} = 2.9728$ kPa

$$\omega_1 = 0.622\frac{P_{v1}}{P_t - P_{v1}} = 0.622 \times \frac{2.9728}{101.325 - 2.9728}$$

$$= 18.8 \times 10^{-3} \text{ kg/kg of d.a.}$$

$$h_1 = 1.005t_1 + \omega_1(2500 + 1.88t_1)$$

$$= 1.005 \times 30 + 18.8 \times 10^{-3}(2500 + 1.88 \times 30)$$

$$= 78.21 \text{ kJ/kg of d.a}$$

$$v_1 = \frac{RT_1}{P_{a1}} = \frac{0.287 \times (273 + 30)}{101.325 - 2.9728} = 0.8842 \text{ m}^3\text{/kg d.a}$$

at state 2 : $t_2 = 20°\text{C}$

$$\phi_2 = 20\%$$

$$0.2 = \frac{P_{v2}}{P_{\text{sat@20°C}}} = \frac{P_{v2}}{2.3392}$$

$\Rightarrow$ $P_{v2} = 0.4678$ kPa

$$\omega_2 = 0.622\frac{P_{v2}}{P_t - P_{v2}} = 0.622 \times \frac{0.4678}{101.325 - 0.4678}$$

$$= 2.885 \times 10^{-3} \text{ kg/kg of da}$$

$$h_2 = 1.005t_2 + \omega_2(2500 + 1.88t_2)$$

$$= 1.005 \times 20 + 2.885 \times 10^{-3}(2500 + 1.88 \times 20)$$

$$= 27.42 \text{ kJ/kg of d.a}$$

Mass flow rate of dry air $= \dfrac{4}{v_1}$

$$= \frac{4}{0.8842} = 4.524 \text{ kg/min}$$

So, mass flow rate of vapour at inlet and exit are

$$\dot{m}_{v1} = \dot{m}_a\omega_1 = 4.524 \times 18.8 \times 10^{-3}$$

$$= 0.08505 \text{ kg/min}$$

$$\dot{m}_{v2} = \dot{m}_a\omega_2 = 4.524 \times 2.885 \times 10^{-3} = 0.01305 \text{ kg/min}$$

(a) On applying mass balance on control volumes,

$$\dot{m}_w = \dot{m}_{v1} - \dot{m}_{v2}$$

$$= 0.08505 - 0.01305$$

$$= 0.072 \text{ kg/min}$$

(b) On applying energy balance on control volume

$$\dot{m}_a h_1 = \dot{Q}_{\text{out}} + \dot{m}_a h_2 + \dot{m}_w (h_2)_w$$

$$(h_2)_w = h_{f@20°C} = 83.915 \text{ kJ/kg}$$

$$\Rightarrow \quad 4.524 \times 78.21 = \dot{Q}_{\text{out}} + 4.524 \times 27.42 + 0.072 \times 83.915$$

$$\Rightarrow \quad \dot{Q}_{\text{out}} = 223.73 \text{ kJ/min} = 3.728 \text{ kW}$$

(c) $\because$

$$\dot{Q}_{\text{out}} = \dot{m}_R (h_{R2} - h_{R1})$$

$$\Rightarrow \quad \dot{m}_R = \frac{\dot{Q}_{\text{out}}}{h_{R2} - h_{R1}}$$

$\because$

$$h_{R2} = h_{g@700\text{kPa}} = 261.85 \text{ kJ/kg}$$

$$h_{R1} = (h_f + x_1 h_{fg})_{@700\text{kPa}}$$

$$= 86.78 + 0.2 \times 175.07$$

$$= 121.79 \text{ kJ/kg}$$

So, mass flow rate of refrigerant,

$$\dot{m}_R = \frac{\dot{Q}_{\text{out}}}{h_{R2} - h_{R1}} = \frac{3.728}{261.85 - 121.79}$$

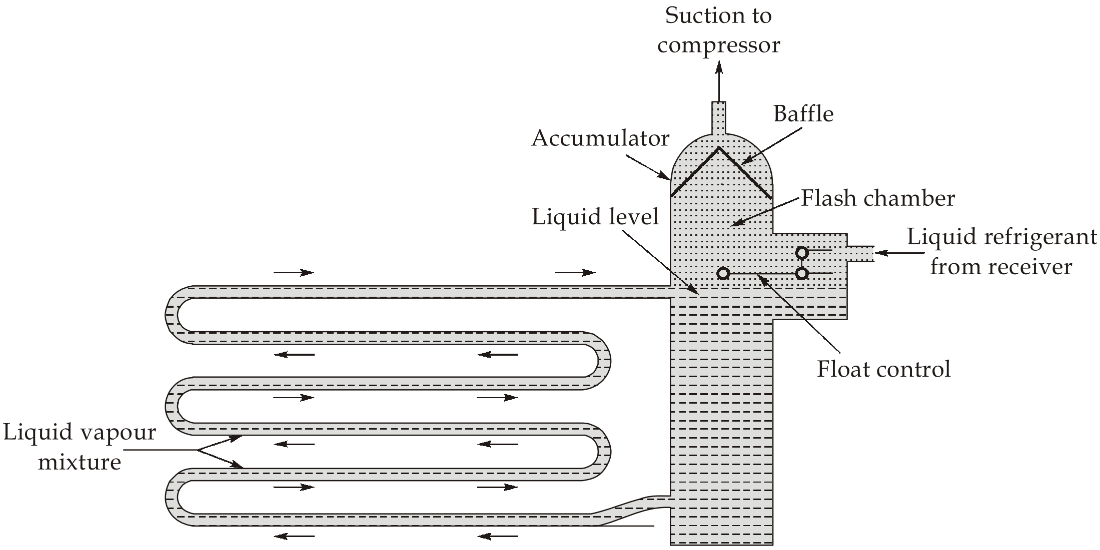
$$= 0.0266 \text{ kg/s} = 1.6 \text{ kg/min}$$

7. (a)(i) Solution:

Flooded Type Evaporators: A flooded or overfeed evaporator is one wherein the amount of liquid refrigerant circulated through the evaporator is considerably in excess of that which can be vaporised. Here a constant refrigerant liquid level is maintained. A float valve is used as the throttling device which maintains a constant liquid level in the evaporator. Due to the heat supplied by the substance to be cooled, the liquid refrigerant is vaporised and so the liquid level falls down. The float valve opens to admit more liquid and thus maintains a constant liquid level. As a result the evaporator is always filled with liquid to the level determined by the float adjustment and the inside surface is wetted with the liquid. Thus this type is called flooded evaporator.

To prevent liquid carry over to compressor, accumulator is generally used with a flooded evaporator. Accumulator also serves as the chamber for the liquid level float valve. The evaporator coil is connected to the accumulator and the liquid flow from the accumulator to the evaporator coil is generally by gravity.

The vapour formed by the vaporising of the liquid in the coil being lighter, rises up and passes on to the top of the accumulator from where it enters the suction line. Sometime liquid eliminators are provided in the accumulator top to prevent the possible carry over of the liquid particles from the accumulator to the suction line.



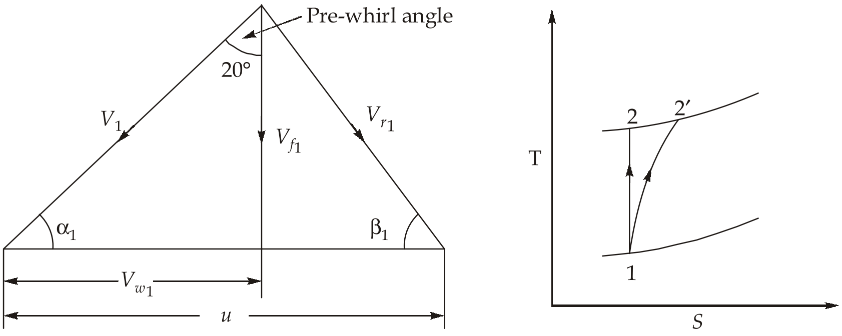
Flooded type evaporator

- These evaporators give high rate of heat transfer as the whole surface of the evaporator coil remains in contact with liquid refrigerant. These are bulky in construction and require large amount of refrigerant for their working.
- The flooded evaporators are used in large installations, especially in chemical and food processing industries.

7. (a)(ii) Solution:

$N = 18000$ rpm, $\eta_c = 80\%$, $D_1 = 225$ mm, $V_1 = 130$ m/s,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.225 \times 18000}{60} = 212.06 \text{ m/s}$$



$$V_{w1} = V_1 \sin 20° = 44.463 \text{ m/s}$$

Static condition at input, $T_1 = 298 \text{ K} = 25°\text{C}$, $P_1 = 1$ bar

$$T_2 = T_1\left(r_p\right)^{\frac{\gamma\ ?1}{\gamma}} = 298 \times 4^{\left(\frac{0.4}{1.4}\right)} = 442.83 \text{ K}$$

$$T_2' = T_1 + \frac{(T_2 - T_1)}{\eta_c} = 298 + \frac{(442.83 - 298)}{0.8} = 479.033 \text{ K}$$

$$\text{Work input to compressor} = m(u_2V_{w2} - u_1V_{w1}) \qquad \because \text{Slip factor, } \mu = \frac{V_{w2}}{u_2}$$

$$= m\left(\mu\, u_2^2 - u_1\, V_{w1}\right) \qquad \text{... (i)}$$

$$\text{From thermodynamic relation} = c_p\,(T_2' - T_1) \qquad \text{... (ii)}$$

$$\because \quad c_p\,(T_2' - T_1) = (\mu\, u_2^2 - u_1\, V_{w1}) \qquad (\text{Taking } m = 1 \text{ kg})$$

$$1005 \times (479.033 - 298) = 0.9\, u_2^2 - 212.06 \times 44.463$$

$$u_2 = 461.12 \text{ m/s}$$

also

$$u_2 = \frac{\pi\, D_2 N}{60}$$

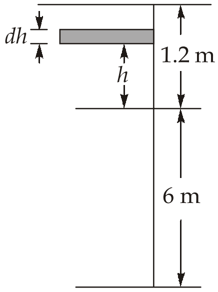
$$461.12 = \frac{\pi \times D_2 \times 18000}{60}$$

$$D_2 = 0.4893 \text{ m} = 489.3 \text{ mm}$$

7. (b) Solution:

Work done by pump to raise the infinitesimal mass of water '*dm*' to height *h*.

$$dW = \int_0^{1.2} \frac{\rho Aghdh}{\eta_{\text{pump}}}$$



$$\text{Pump work to raise water, } W = \frac{\rho Ag \times 1.2^2}{2 \times 0.5} = \rho Ag \times 1.2^2$$

Now, the height of water from datum is 7.2 m.

$$W = \eta_{gen} \frac{\rho Ag \times 7.2^2}{2}$$

When water is not pumped,

$$\text{Work potential,} \qquad W_1 = \rho Ag \times \frac{6^2}{2} \times \eta_{gen}$$

$$\therefore \text{ Increased in work potential} = \eta_{gen}\frac{\rho Ag 7.2^2}{2} - \eta_{gen}\frac{\rho Ag\times 6^2}{2} - \rho Ag\times 1.2^2$$

$$= \rho Ag\left[\frac{0.5}{2}\times(7.2^2-6^2)-1.2^2\right] = 2.52\ \rho Ag$$

$$\text{Net energy gain as \% of input energy} = \frac{2.52\rho Ag}{1.2^2\rho Ag}\times 100 = 175\%$$

7. (c) Solution:

$$\text{Brake power, B.P.} = \frac{\pi DN(W-S)}{60\times 1000} = \frac{\pi(60+0.5)\times 450\times(210-30)\times 10^{-2}}{60000}$$

$$= 2.566 \text{ kW}$$

Actual mean effective pressure is given by

$$P_m = \frac{\text{Area of indicator diagram}}{\text{Length of indicator diagram}}\times\text{Spring scale}$$

$$= \frac{4.15}{6.25}\times 11 = 7.3 \text{ bar}$$

$$\text{Indicated power, } I.P. = \frac{P_m LAN}{2\times 60\times 10^3} = \frac{7.3\times 10^5\times\frac{15}{100}\times\frac{\pi}{4}(0.1)^2\times\frac{450}{2}}{60\times 1000}$$

$$= 3.225 \text{ kW}$$

$$\text{Mechanical efficiency, } \eta_m = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{2.566}{3.225} = 0.795 \text{ or } 79.5\%$$

Total fuel consumption per hour = $B.P. \times m_s = 2.566 \times 0.3 = 0.77$ kg/hour

$$\text{Indicated thermal efficiency, } \eta_{ith} = \frac{3.225\times 60\times 60}{0.77\times 41800} = 0.36 \text{ or } 36\%$$

$$\text{Brake thermal efficiency, } \eta_{bth} = 0.36 \times \text{mechanical efficiency}$$

$$= 0.36 \times 0.795 = 0.286 \text{ or } 28.6\%$$

8. (a) Solution:

Thermo-chemical conversion: Thermo-chemical conversion is a process to decompose biomass with various combinations of temperatures and pressures.

(a) **Pyrolysis:** Biomass is heated in absence of oxygen, or partially combusted in a limited oxygen supply, to produce a hydrocarbon, rich in gas mixture (H_2, CO_2, CO, CH_4 and lower hydrocarbons), an oil like liquid and a carbon rich solid residue (charcoal).

The pyrolytic or 'bio-oil' produced can easily be transported and refined into a series of products similar to refining crude oil. There is no waste product, the conversion efficiency is high (82%) depending upon the feed stock used, the process temperature in reactor and the fuel air ratio during combustion.

(b) **Gasification:** Gasification is conversion of a solid biomass, at a high temperature with controlled air, into a gaseous fuel. The output gas is known as producer gas, a mixture of H_2 (15-20%), CO (10-20%), CH_4(1-5%), CO_2(9-12%) and N_2(45-55%). The gas is more versatile than the solid biomass, it can be burnt to produce process heat and steam, or used in internal combustion engines or gas turbines to generate electricity. The gasification process renders the use of biomass which is relatively clean and acceptable in environmental terms.

(c) **Liquefication:** Liquefication of biomass can be processed through 'fast' or 'flash' pyrolysis, called 'pyrolytic oil' which is a dark brown liquid of low viscosity and a mixture of hydrocarbons. Pyrolysis liquid is a good substitute for heating oil.

Another liquefaction method is through methanol synthesis. Gasification of biomass produces synthetic gas containing a mixture of H_2 and CO. The gas is purified by adjusting the hydrogen and carbon monoxide composition. Finally, the purified gas is subjected to liquefaction process, converted to methanol over a zinc chromium catalyst. Methanol can be used as liquid fuel.

Bio-chemical conversion: There are two types of biochemical conversions:

(a) **Anaerobic digestion:** This process converts the cattle dung, human wastes and other organic waste with high moisture content into biogas (gobar gas) through anaerobic fermentation in absence of air. Fermentation occurs in two stages by two different metabolic groups of bacteria. Initially the organic material is hydrolyzed into fatty acids, alcohol, sugars, H_2 and CO_2. Methane forming bacteria then converts the products of the first to CH_4 and CO_2, in the temperature range 30-55°C.

Biogas produced can be used for heating, or for operating engine driven generators to produce electricity. Fermentation occurs in a sealed tank called 'digester' where the sludge left behind is used as enriched fertilizer.

(b) **Ethanol fermentation:** Ethanol can be produced by decomposition of biomass containing sugar like sugarcane, cassava sweet sorghum, beet, potato, corn, grape etc, into sugar molecules such as glucose ($C_6H_{12}O_6$) and sucrose($C_{12}H_{22}O_{11}$).

Ethanol fermentation involves biological conversion of sugar into ethanol and CO_2.

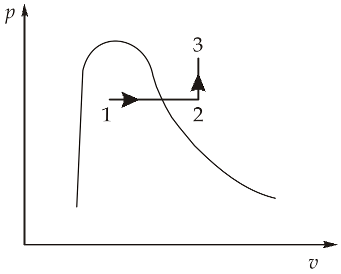
$$C_{12}H_{22}O_{11} + H_2O \longrightarrow 2C_6H_{12}O_6$$

$$C_6H_{12}O_6 \xrightarrow{\text{Fermentation}} 2C_2H_5OH + 2CO_2$$

Ethanol has emerged as the major alcohol fuel and is blended with petrol.

8. (b) Solution:

Given data:



$$x_1 = 0.25$$

$$\text{Mass of piston: } m_p = 40 \text{ kg}$$

$$d = 10 \text{ cm} = 0.10 \text{ m}$$

$$\therefore \quad A = \frac{\pi}{4}d^2 = \frac{3.14}{4} \times (0.10)^2 = 7.85 \times 10^{-3} \text{ m}^2$$

$$p_{\text{atm}} = 1 \text{ bar} = 100 \text{ kPa}$$

$$y_1 = 1 \text{ cm} = 0.01 \text{ m}$$

$$y_2 = 5 \text{ cm} = 0.05 \text{ m}$$

$$V_1 = Ay_1 = 7.85 \times 10^{-3} \times 0.01 = 7.85 \times 10^{-5} \text{ m}^3$$

$$V_2 = Ay_2 = 7.85 \times 10^{-3} \times 0.05 = 39.25 \times 10^{-5} \text{ m}^3$$

To fix the initial equilibrium state,

we have

$$p_1 A = m_p g + p_{\text{atm}} A$$

$$p_1 \times 7.85 \times 10^{-3} = \frac{40 \times 9.81}{1000} + 100 \times 7.85 \times 10^{-3}$$

$$p_1 = \frac{40 \times 9.81}{1000 \times 7.85 \times 10^{-3}} + 100$$

$$= 149.98 \text{ kPa} = 1.499 \text{ bar}$$

From saturated steam table,

At $p_1 = 1.5$ bar

$$v_f = 0.001053 \text{ m}^3/\text{kg},$$

$$v_g = 1.1593 \text{ m}^3/\text{kg}$$

$$h_f = 467.11 \text{ kJ/kg}$$

$$h_g = 2693.6 \text{ kJ/kg}$$

$$v_1 = v_f + x_1(v_g - v_f) = 0.001053 + 0.25\,(1.1593 - 0.001053)$$

$$= 0.29061 \text{ m}^3/\text{kg}$$

Thus $$m = \frac{V_1}{v_1} = \frac{7.85\times10^{-5}}{0.29061} = 2.70 \times 10^{-4} \text{ kg}$$

also $$m = \frac{V_2}{v_2}$$

$\therefore$ $$2.70 \times 10^{-4} = \frac{39.25\times10^{-5}}{v_2}$$

or $$v_2 = 1.4537 \text{ m}^3/\text{kg}$$

It may be noted that v_g at 1.5 bar = 1.1593 m^3/kg

Thus the vapour is in a super heated state when the piston touches the stops. The process is shown on the p-v diagram.

The work done during the constant pressure process,

$$W_{1-2} = p_1(V_2 - V_1) = 149.98 \times (39.25 \times 10^{-5} - 7.85 \times 10^{-5})$$
$$= 0.04709 \text{ kJ}$$

Net heat supplied, Q = Heat supplied at constant pressure + heat supplied at constant volume

$$= (U_2 - U_1) + W_{1-2} + (U_3 - U_2) + W_{2-3}$$
$$= U_2 - U_1 + W_{1-2} + U_3 - U_2 + 0$$
$$= W_{1-2} + U_3 - U_1 = W_{1-2} + m(u_3 - u_1)$$

where $$u_1 = u_f + x_1(u_g - u_f)$$
$$= h_f - p_1v_f + x_1(h_g - p_1v_g - h_f + p_1v_f)$$
$$= 467.11 - 149.98 \times 0.001053 + 0.25(2693.6 - 149.98 \times 1.1593 - 467.11 + 149.98 \times 0.001053) = 980.14 \text{ kJ/kg}$$

From superheated steam table,

At $$p_3 = 3 \text{ bar and } v_3 = v_2 = 1.4537 \text{m}^3/\text{kg}$$

By interpolation, $$h_3 = 3703.2 + \frac{(3927.1 - 3703.2)(1.4537 - 1.3414)}{1.4957 - 1.3414}$$
$$= 3866.15 \text{ kJ/kg}$$

$\therefore$ $$u_3 = h_3 - p_3v_3$$
$$= 3866.15 - 300 \times 1.4537 = 3430.04 \text{ kJ/kg}$$

Thus $$Q = W_{1-2} + m(u_3 - u_1)$$
$$= 0.04709 + 2.70 \times 10^{-4}(3430.04 - 980.14)$$
$$= 0.7085 \text{ kJ}$$

8. (c) Solution:

$$\eta_o = \frac{P}{wQH}$$

or $$0.85 = \frac{1545 \times 1000}{9.81 \times 1000 \times Q \times 6.5}$$

$\therefore$ $$Q = 28.505 \text{ m}^3/\text{s}$$

The velocity at inlet to draft tube is

$$V_2 = \frac{28.505}{\left(\frac{\pi}{4}\right) \times (2.6)^2} = 5.369 \text{ m/s}$$

As we know, $$\frac{p_2}{w} = \frac{p_a}{w} - \left[H_s + \left((1-k)\frac{V_2^2 - V_3^2}{2g}\right)\right]$$

Also, $$\frac{p_2}{w} = 10.3 - 4 = 6.3 \text{ m}$$

$$\frac{p_a}{w} = 10.6 \text{ m; and } H_s = 2.9 \text{ m}$$

$$6.3 = 10.3 - \left[2.9 + (1-k)\frac{V_2^2 - V_3^2}{2g}\right]$$

or $$(1-k)\frac{V_2^2 - V_3^2}{2g} = 1.1$$

Efficiency of draft tube is given as

$$\eta_d = \frac{(1-k)\frac{V_2^2 - V_3^2}{2g}}{\left(\frac{V_2^2}{2g}\right)}$$

$$\frac{V_2^2}{2g} = \frac{(5.369)^2}{2 \times 9.81} = 1.469 \text{ m}$$

$\therefore$ $$\eta_d = \frac{1.1}{1.469} = 0.749 = 74.9\%$$

When the turbine output is reduced to half,

$$P = \frac{1545}{2} = 772.5 \times 10^3 \text{W}$$

Thus with the same head and the efficiency of the turbine, we have

$$0.85 = \frac{772.5 \times 1000}{9.81 \times 1000 \times Q \times 6.5}$$

$$Q = 14.253 \text{ m}^3/\text{s}$$

and
$$V_2 = \frac{14.253}{\left(\frac{\pi}{4}\right) \times (2.6)^2} = 2.685 \text{ m/s}$$

∴
$$\frac{V_2^2}{2g} = \frac{(2.685)^2}{2 \times 9.81} = 0.367 \text{ m}$$

For the same draft tube efficiency, we have

$$0.749 = \frac{(1-k)\left[\left(\frac{V_2^2 - V_3^2}{2g}\right)\right]}{0.367}$$

or
$$(1-k)\left[\left(\frac{V_2^2 - V_3^2}{2g}\right)\right] = 0.275$$

Thus from equation, we have

$$\frac{p_2}{w} = 10.3 - [2.9 + 0.275] = 7.125 \text{ m (abs)}$$

∴ Reading of vacuum gage would be

$$7.125 - 10.3 = -3.175 \text{ m}$$

i.e. negative head of 3.175 m.

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