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Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Electrical Engineering Test No : 2

Section A : Digital Electronics + Microprocessors

Q.1 (a) Solution:

(i) The rules for BCD addition are as follows:

1. Add the corresponding digits in 4-bit group (BCD group) starting from LSB.
2. If each group 4-bit sum is less than or equal to 9, sum is correct and is in BCD form.
3. If any group 4-bit sum is greater than 9 or there generates a carry-out from any 4-bit sum, then sum is invalid. Add 6 (0110) to that invalid group. If after adding 0110 carry is generated then this carry is added to the next higher order BCD group and carry out of MSB is considered as part of the final sum.

$$\begin{array}{r l} & 0100\ 0110\ 1001\ .\ 0110 \\ + & 1000\ 0001\ 0111\ .\ 1000 \\ \hline & 1100\ 1000\ 0000\ .\ 1110 \\ & 0110\ \ 0110\ .\ 0110 \quad \text{(Correction)} \\ \hline 1 & 0010\ 1000\ 0111\ .\ 0100 \end{array}$$

Result: 10010100000111.0100

(ii) In BCD subtraction, the output is invalid if borrow occurs or result is greater than $(9)_{10}$ for a 4-bit subtraction. To correct this result we subtract $(6)_{10}$ from the corresponding 4-bit group.

$$\begin{array}{r} 0110\ 0101\ 0100\ .\ 0101 \\ -\ 1001\ 0111\ 1001\ .\ 0110 \\ \hline 1100\ 1101\ 1010\ .\ 1111 \\ -\ 0110\ 0110\ 0110\ .\ 0110 \\ \hline 0110\ 0111\ 0100\ .\ 1001 \end{array}$$

Here, minuend is less than subtrahend then result will be in 10's complement. So take 10's complement of the result.

$$\begin{array}{r} 1001\ 1001\ 1001\ .\ 1010 \\ 0110\ 0111\ 0100\ .\ 1001 \\ \hline (0011\ 0010\ 0101\ .\ 0001) \end{array}$$

Result: 001100100101.0001 (Negative)

(iii)

$$\begin{array}{r} 0110\ 0110\ 0110\ .\ 0110 \\ 0100\ 1001\ 1001\ .\ 1001 \\ \hline 0001\ 1100\ 1100\ .\ 1101 \\ 0000\ 0110\ 0110\ .\ 0110 \\ \hline (0001\ 0110\ 0110\ .\ 0111) \end{array}$$

Result: 000101100110.0111

Q.1 (b) Solution:

Writing in signed 10's complement form

$$+9286 = 009286$$

$$+801 = 000801$$

$$-9286 = \begin{array}{r} 999999 \\ -9286 \\ \hline 990713 \\ +1 \\ \hline 990714 \\ \hline \end{array} \begin{array}{l} \\ \\ \Rightarrow \text{9's complement form} \\ \\ \Rightarrow \text{10's complement form} \end{array}$$

$$-801 = \begin{array}{r} 999999 \\ -801 \\ \hline 999198 \\ +1 \\ \hline 999199 \\ \hline \end{array} \begin{array}{l} \\ \\ \Rightarrow \text{9's complement form} \\ \\ \Rightarrow \text{10's complement form} \end{array}$$

(i) $(+9286) + (+801) = 009286 + 000801$

$= 010087$

Hexadecimal equivalent of result

$$= (2767)_{Hex} = (2767)_{16}$$

(ii) $(+9286) + (-801) = 009286 + 999199$

$= \text{①}008485$ → Discard

$\therefore$ 008485

Hexadecimal equivalent of result

$= (2125)_{Hex} = (2125)_{16}$

(iii) $(-9286) + (+801) = 990714 + 000801$

$= 991515$

Hexadecimal equivalent of result

$= (F211B)_{16}$

(iv) $(-9286) + (-801) = 990714 + 999199$

$= \text{①}989913$ → Discard

$\therefore$ 989913

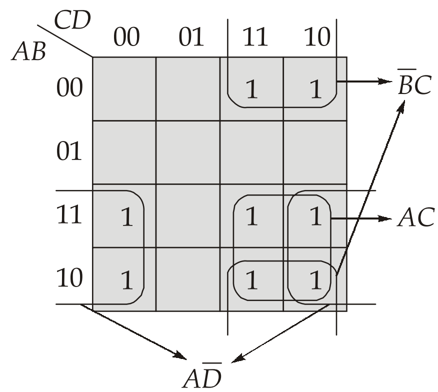
Hexadecimal equivalent of result = $(F1AD9)_{16}$

Q.1 (c) Solution:

We have,

$$F(A, B, C, D) = \Pi(0, 1, 4, 5, 6, 7, 9, 13).$$

$$\therefore \quad F(A, B, C, D) = \Sigma m(2, 3, 8, 10, 11, 12, 14, 15)$$

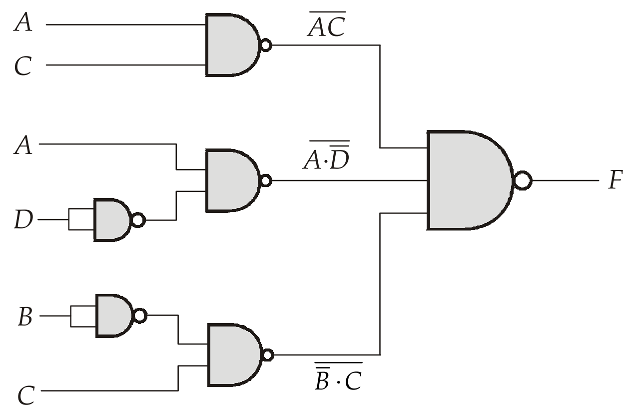


$$F = AC + A\bar{D} + \bar{B}C$$

Implementation using NAND gate:

$$F = \overline{\overline{AC + A\bar{D} + \bar{B}C}}$$

$$= \overline{\overline{AC} \cdot \overline{A\bar{D}} \cdot \overline{\bar{B}C}}$$



Q.1 (d) Solution:

Let output of logic circuit is Y.

$$Y = \bar{C}\bar{D} + A \oplus B$$

$$\therefore \quad Y = (A \oplus B) + \overline{C + D}$$

Also, serial input is same as output Y.

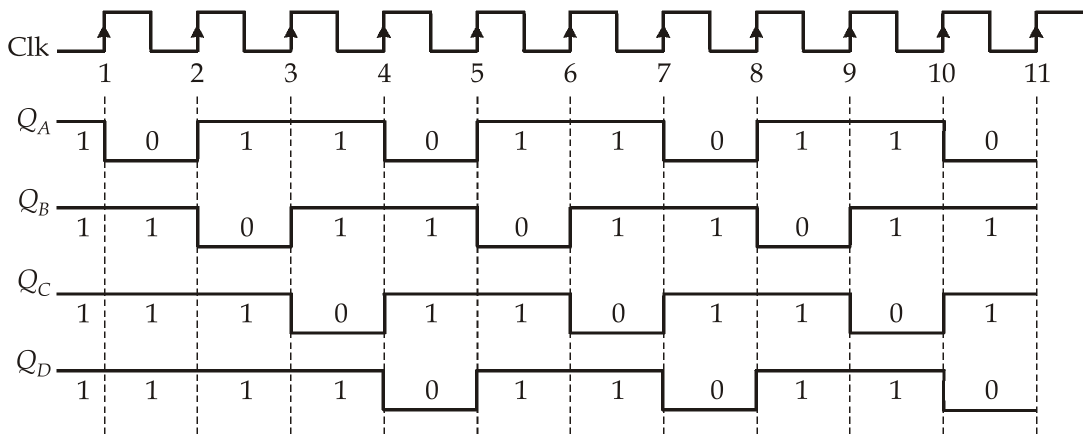
Initially, $ABCD = 1111$

$$\therefore \quad Y = (1 \oplus 1) + \overline{1 + 1} = 0 + 0 = 0$$

$\therefore$ Serial In = 0, in SIPO register serial input is applied to MSB flip-flop.

Clock	Serial In	A	B	C	D	Output, Y
–	X	1	1	1	1	0
1st	0	0	1	1	1	1
2nd	1	1	0	1	1	1
3rd	1	1	1	0	1	0
4th	0	0	1	1	0	1
5th	1	1	0	1	1	1
6th	1	1	1	0	1	0
7th	0	0	1	1	0	1
8th	1	1	0	1	1	1
9th	1	1	1	0	1	0
10th	0	0	1	1	0	1
11th	1	1	0	1	1	1

The output waveform is



Q.1 (e) Solution:

Assuming 10 bytes of data from starting address 2000H

To be transferred to starting address 4000H

```
          MVI   C, 0AH    ; Initialize count 10 in register C.
          LXI   H, 2000H  ; Initialize starting address of source
          LXI   D, 4000H  ; Initialize destination address.
REPEAT:   MOV   A, M      ; Byte into 'Accumulator'.
          STAX  D         ; Data transferred to reference of DE.
          INX   H         ; Increment source and destination pointers,
          INX   D
          DCR   C         ; Decrement count C
          JNZ   REPEAT    ; [check for no zero by jump instruction]
          HLT             ; Stop
```

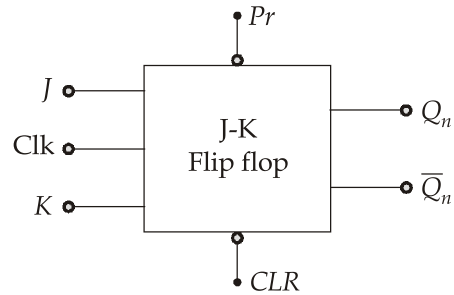
Q.2 (a) (i) Solution:

Truth table of J-K flip flop:

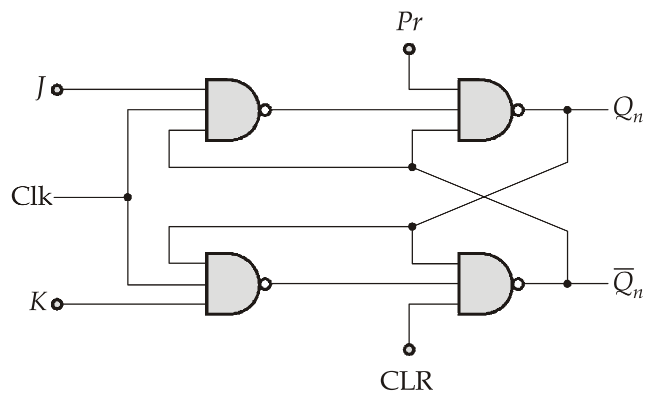
J	K	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	$\bar{Q}_n$

$$Q_{n+1} = J\bar{Q}_n + \bar{K}\cdot Q_n$$

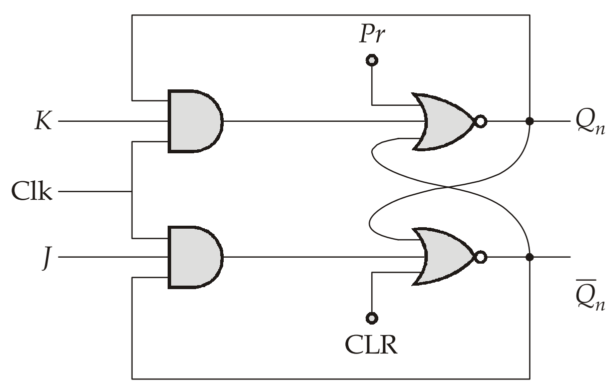
Logic Symbol:



JK flip-flop using NAND latch:



JK flip-flop using NOR latch:



Q.2 (a) (ii) Solution:

SR flip flop into JK flip flop:

J	K	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	$\bar{Q}_n$

Q_n	Q_{n+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

J	K	Q_n	Q_{n+1}	S	R
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	1	X	0
1	1	0	1	1	0
1	1	1	0	0	1

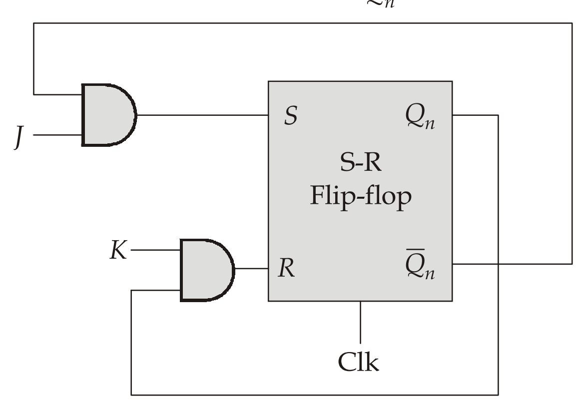
K-Map:

J \ KQ_n	00	01	11	10
0		X		
1	1	X		1

$S = J\bar{Q}_n$

J \ KQ_n	00	01	11	10
0	X		1	X
1			1	

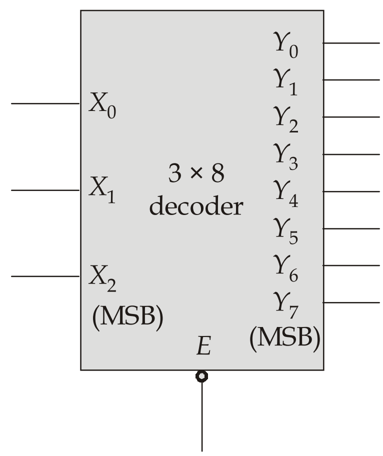
$R = KQ_n$



Race around condition:

- When level triggered clock is given to JK flip flop, then for $J = K = 1$, the output of flip flop may toggle between 0 and 1 for given clock pulse and hence, at the end of the clock pulse, the value of output is uncertain. This situation is known as Race-around condition.
- To avoid Race-around condition, we use Master-slave configuration in J-K flip-flop.

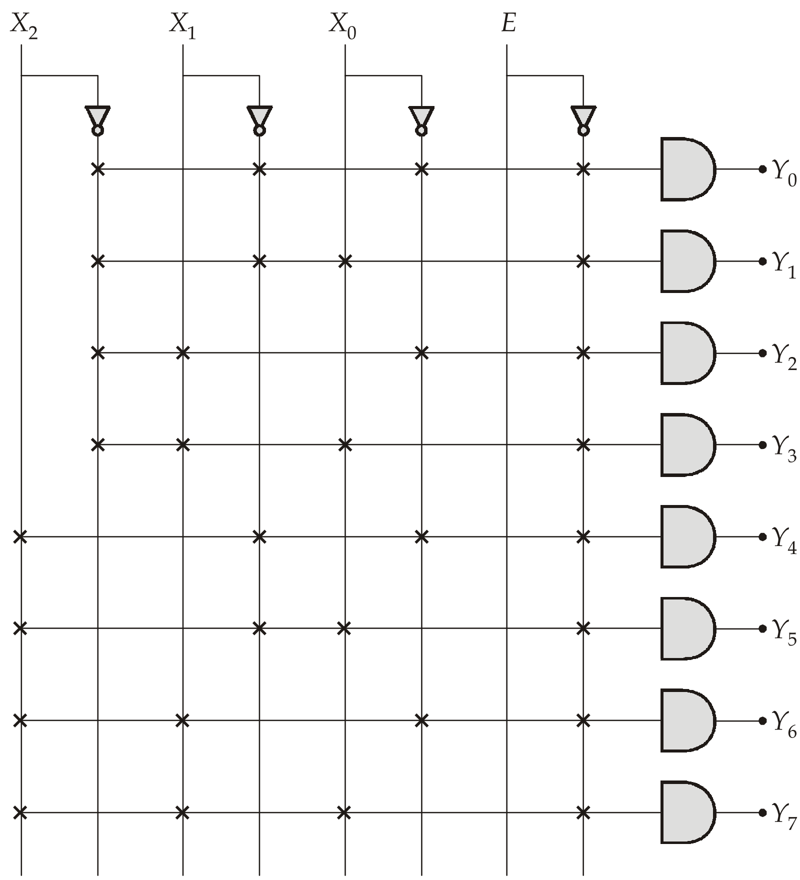
Q.2 (b) Solution:



⇒ **Truth table:**

E	X_2	X_1	X_0	Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1	Y_0
1	X	X	X	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	0	1	0	0
0	0	1	1	0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	1	0	0	0	0
0	1	0	1	0	0	1	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0	0	0
0	1	1	1	1	0	0	0	0	0	0	0

Logic diagram:

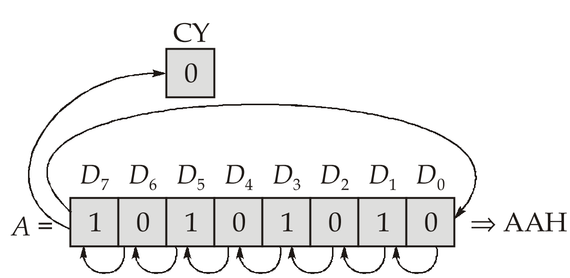


Q.2 (c) Solution:

(i) RLC : Rotate accumulator left:

- Each bit is shifted to the adjacent left position. Bit D_7 becomes D_0.
- CY flag is modified according to bit D_7.

A will be rotated with RLC instruction as shown below,



After the first RLC, A will be 55H with CY set.

CY: 1

A =	0	1	0	1	0	1	0	1	⇒ 55H

After the second RLC, A will be AAH again with CY reset.

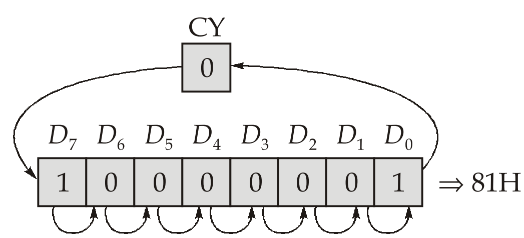
CY: 0

A =	1	0	1	0	1	0	1	0	⇒ AAH

(ii) RAR : Rotate accumulator right through carry:

- Each bit is shifted right to the adjacent position. Bit D_0 becomes the carry bit, and the carry bit is shifted into D_7.

A will be rotated with RAR instruction as shown below,



After the execution of the RAR instruction (A) will be 40H with the CY flag set.

CY: 1

0	1	0	0	0	0	0	0	⇒ 40H

(iii) CALL addr (label):

Unconditional call: Call the subroutine identified by the address.

- CALL instruction is used to call a subroutine before the control is transferred to the subroutine, the address of the next instruction of the main program is saved in the stack. The content of the stack pointer is decremented by two to indicate the new stack top. Then the program jumps to subroutine starting at address specified by the label.

$$[SP - 1] \leftarrow [PC]_H$$
$$[SP - 2] \leftarrow [PC]_L$$
$$[SP] \leftarrow [SP] - 2$$
$$[PC] \leftarrow \text{addr (label)}$$

(iv) RET (Return from subroutine):

- RET instruction is used at the end of a subroutine. Before the execution of a subroutine the address of the next instruction of the main program is saved in the stack. The execution of RET instruction brings back the saved address from the stack to the program counter. The content of the stack pointer is incremented by 2 to indicate the new stack top. Then the program jumps to the instruction of the main program next to CALL instruction which called the subroutine.

(v) CMP R/M (Compare Register or Memory with Accumulator):

- This is a 1-byte instruction.
- It compares the data byte in register or memory with the contents of the accumulator.
- If (A) < (R/M), the CY flag is set and the zero flag is reset.
- If A = R/M, the zero flag is set and the CY flag is reset.
- If A > (R/M), the CY and zero flags are reset.
- When memory is an operand, its address is specified by (HL).
- No contents are modified, however, all remaining flag (S, P, AC) are affected according to the result of the subtraction.

Q.3 (a) Solution:

(i) Given: $R = 10\ \text{k}\Omega$, $V_R = 10$ V, $n = 4$

1. The value of 1 LSB = 0.5 V

i.e., Resolution = 0.5 V

$$\text{Resolution} = \frac{V_R}{2^n} \times \frac{R_f}{R}$$

$$R_f = \frac{2^n \times R}{V_R} \times 0.5 = \frac{2^4 \times 10^4}{10} \times 0.5 = 8\ \text{k}\Omega$$

$$R_f = 8\ \text{k}\Omega$$

2. For Binary input = 1000; Output = 6 V

$$V_{out} = (\text{Resolution}) \times (\text{Decimal equivalent of binary})$$

$$6 = \left(\frac{10}{2^4} \times \frac{R_f}{10 \times 10^3}\right) \times 8$$

$$\Rightarrow \quad R_f = 12 \times 10^3$$

$$R_f = 12\ \text{k}\Omega$$

3.

$$V_{FS} = 12\text{ V}$$

$$12 = \frac{10 \times R_f}{10 \times 10^3} \Rightarrow R_f = 12\text{ k}\Omega$$

4. For the maximum output voltage,

$$S_1 = S_2 = S_3 = S_4 = 1.$$

Thus, for $V_{out} = 10\text{ V}$

$$V_{out} = \frac{V_R \cdot R_f}{R}\left(\frac{S_1}{2^1} + \frac{S_2}{2^2} + \frac{S_3}{2^3} + \frac{S_4}{2^4}\right)$$

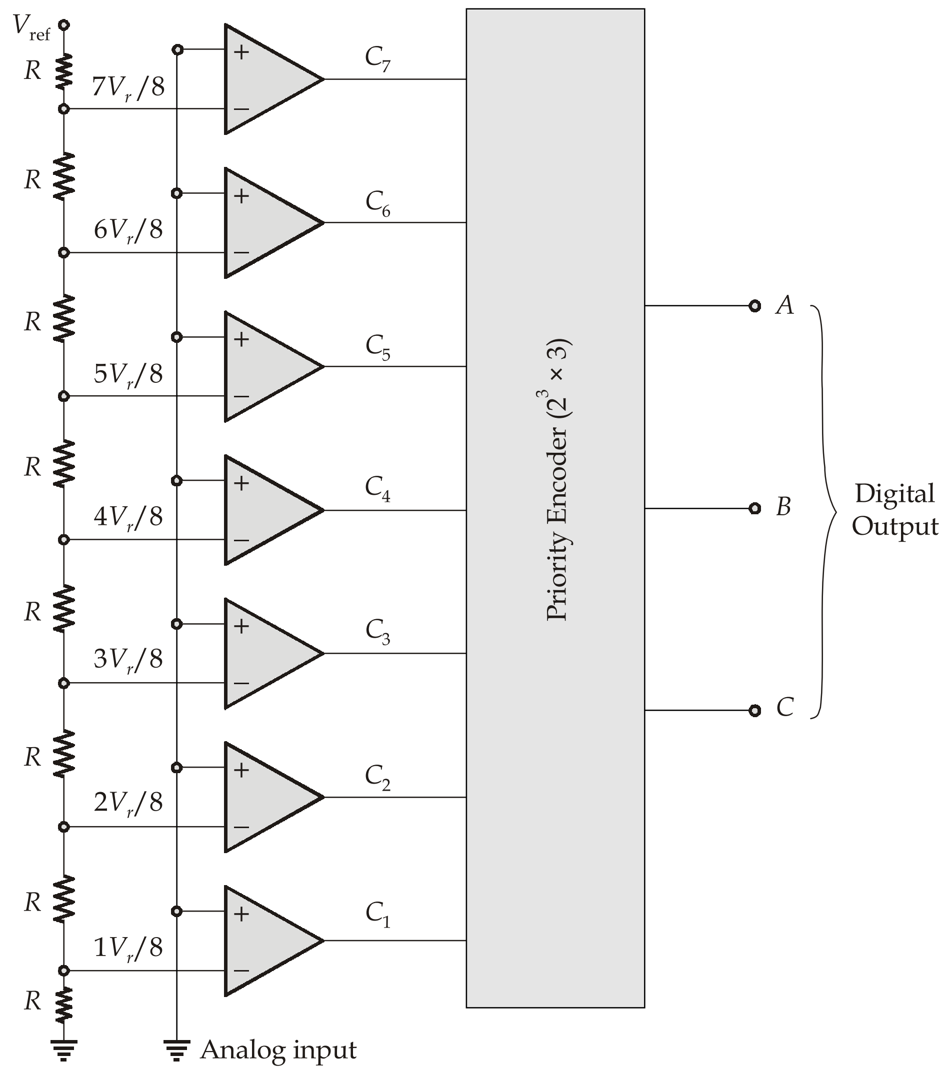
$$10 = \frac{10}{10^4} \times R_f (2^{-1} + 2^{-2} + 2^{-3} + 2^{-4})$$

$$R_f = 10.667\text{ k}\Omega$$

(ii) Flash Type ADC

- The flash or parallel comparator type ADC also called as simultaneous ADC is the fastest ADC, but it require much more circuitry than the others.
- This type of converter utilizes the parallel differential comparators that compare reference voltages with the input analog voltage.

  To convert an analog signal to digital signal of 'N' bits, it requires $(2^N - 1)$ comparator, 2^N resistors, and a $(2^N \times N)$ priority encoder.
- Figure given below shows a 3 bit flash type A/D converter which requires $(2^3 - 1) = 7$ comparators. The analog input which is to be converted is connected to the non-inverting input terminal of comparator where as the inverting input terminals of Op-amps are connected to a set of reference voltage provided by voltage divider that divides it into seven equal increment levels.
- Each level is compared to the analog input by a voltage comparator.
- All comparator outputs are connected to a priority encoder, which produces a digital output corresponding to the input having highest priority. Thus, the digital output represents the voltage that is closest in value to the analog input as shown in the truth table below:



Input Analog Voltage (V_a)	Comparator Output							Encoder Output		
	C_7	C_6	C_5	C_4	C_3	C_2	C_1	A	B	C
$V_a > 7V_r/8$	1	1	1	1	1	1	1	1	1	1
$6V_r/8 < V_a < 7V_r/8$	0	1	1	1	1	1	1	1	1	0
$5V_r/8 < V_a < 6V_r/8$	0	0	1	1	1	1	1	1	0	1
$4V_r/8 < V_a < 5V_r/8$	0	0	0	1	1	1	1	1	0	0
$3V_r/8 < V_a < 4V_r/8$	0	0	0	0	1	1	1	0	1	1
$2V_r/8 < V_a < 3V_r/8$	0	0	0	0	0	1	1	0	1	0
$V_r/8 < V_a < 2V_r/8$	0	0	0	0	0	0	1	0	0	1
$V_a < V_r/8$	0	0	0	0	0	0	0	0	0	0

- The flash converter uses no clock signal. The conversion takes place continuously. The only delays in the conversion are in comparators and priority encoders. Thus, the maximum number of clock pulse required for conversion is '1'.

Q.3 (b) Solution:

State table:

Present state			Next state			F.F Inputs				
A	B	C	A^+	B^+	C^+	J_A	K_A	T_B	S_C	R_C
0	0	1	0	1	1	0	X	1	X	0
0	1	1	1	0	0	1	X	1	0	1
1	0	0	0	1	0	X	1	1	0	X
0	1	0	0	0	0	0	X	1	0	X
0	0	0	0	0	1	0	X	0	1	0

K-map for J_A:

A \ BC	00	01	11	10
0	0	0	1	0
1	X	X	X	X

$$J_A = BC$$

K-map for K_A:

A \ BC	00	01	11	10
0	X	X	X	X
1	1	X	X	X

$$K_A = 1$$

K-map for T_B:

A \ BC	00	01	11	10
0	0	1	1	1
1	1	X	X	X

$$T_B = A + B + C$$

K-map for S_C:

A \ BC	00	01	11	10
0	1	X	0	0
1	0	X	X	X

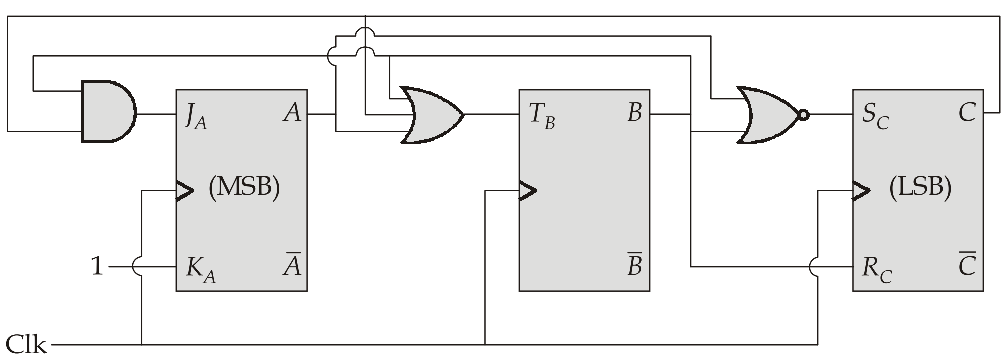
$$S_C = \overline{A}\overline{B} \text{ or } S_C = \overline{A+B}$$

K-map for R_C:

A \ BC	00	01	11	10
0	0	0	1	X
1	X	X	X	X

$$R_C = B$$

Circuit:



Q.3 (c) Solution:

(i) Excitation table for S-R flip flop:

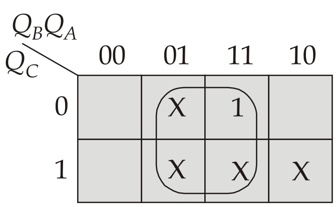
Q_n	Q_{n+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

The state table for the counter can be written as below:

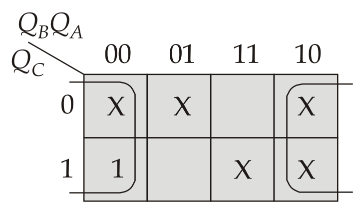
Q_C	Q_B	Q_A	Q_C^+	Q_B^+	Q_A^+	S_C	R_C	S_B	R_B	S_A	R_A
0	0	0	0	1	0	0	X	1	0	0	X
0	1	0	0	1	1	0	X	X	0	1	0
0	1	1	1	0	1	1	0	0	1	X	0
1	0	1	1	0	0	X	0	0	X	0	1
1	0	0	0	0	0	0	1	0	X	0	X

Unused states are: 001, 110, 111

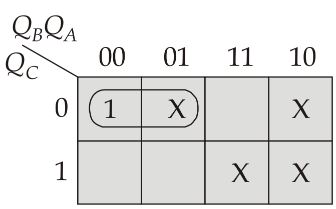
K-Map



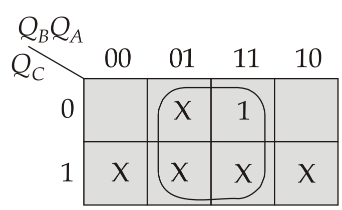
$S_C = Q_A$



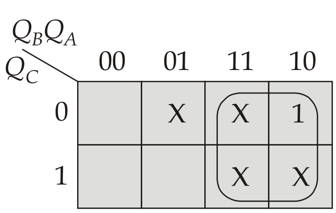
$R_C = \overline{Q}_A$



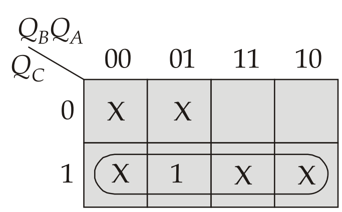
$S_C = \overline{Q}_C\overline{Q}_B$



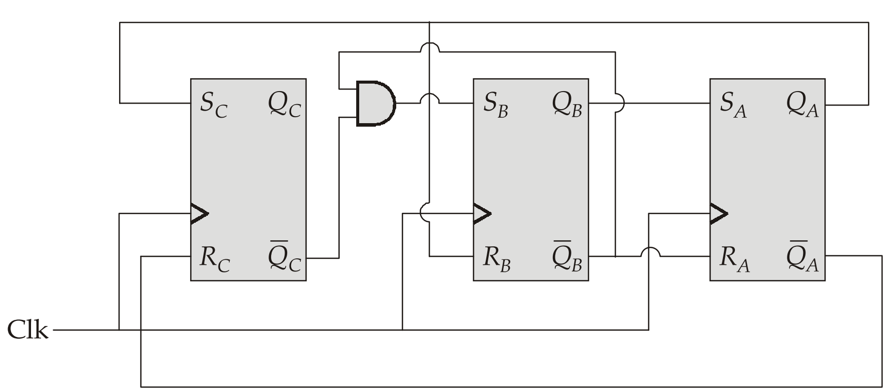
$R_B = Q_A$



$S_A = Q_B$

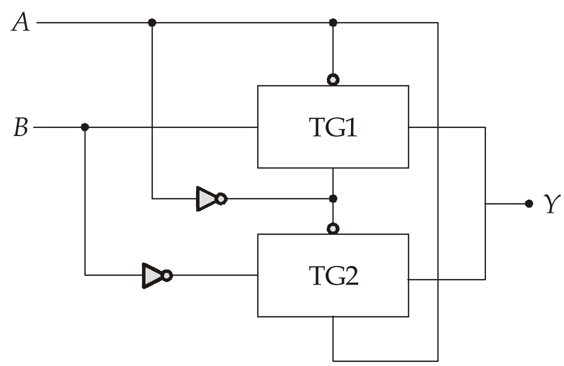


$R_A = Q_C$ or $\overline{Q}_B$



(ii) In the below circuit, when $A = 0$, TG1 is closed and passes the logic B to the output. When $A = 1$, TG2 is closed and the complement of logic B is passed to the output. The truth table of the logic circuit can be drawn as below which represents the XOR Gate.

A	B	TG1	TG2	O/P(Y)
0	0	Close	Open	0
0	1	Close	Open	1
1	0	Open	Close	1
1	1	Open	Close	0



Q.4 (a) Solution:

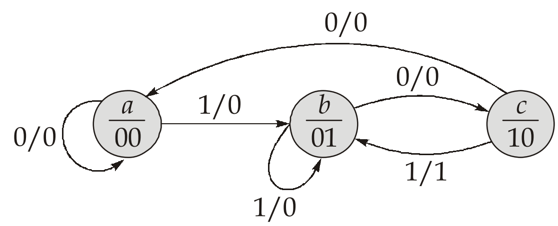
(i) State table for the given state diagram

Present state	Next state		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	a	b	0	0
b	c	b	0	0
c	a	b	0	1
d	a	b	0	1

Thus, 'c' and 'd' are redundant states. The reduced state table is thus obtained as below:

Present state	Next state		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	a	b	0	0
b	c	b	0	0
c	a	b	0	1

Reduced state diagram,



(ii) Excitation table for *JK* FF:

Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Since there are 4-states in the given state diagram, i.e., $N = 4$

The number of flip-flops,

$$2^n \geq N$$
$$2^n \geq 4$$
$$n \geq 2$$

Therefore, 2 JK flip flops are required to implement the sequential circuit.

Input	Present State		Next State		FF-inputs				Output
X	A	B	A^+	B^+	J_A	K_A	J_B	K_B	Y
0	0	0	0	0	0	X	0	X	0
1	0	0	0	1	0	X	1	X	0
0	0	1	1	0	1	X	X	1	0
1	0	1	0	1	0	X	X	0	0
0	1	0	0	0	X	1	0	X	0
1	1	0	0	1	X	1	1	X	1
0	1	1	X	X	X	X	X	X	X
1	1	1	X	X	X	X	X	X	X

For J_A:

X \ AB	00	01	11	10
0	0	1	X	X
1	0	0	X	X

$\therefore \quad J_A = \bar{X}B$

For K_A:

X \ AB	00	01	11	10
0	X	X	X	1
1	X	X	X	1

$\therefore \quad K_A = 1$

For J_B:

X \ AB	00	01	11	10
0		X	X	
1	1	X	X	1

$\therefore \quad J_B = X$

For K_B:

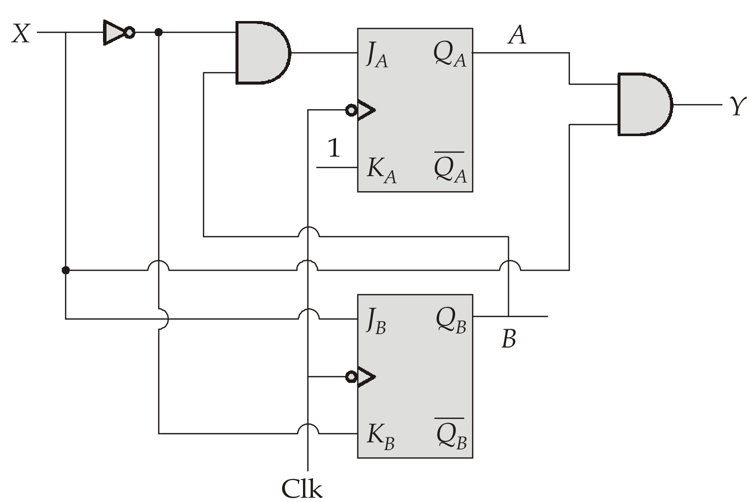
X \ AB	00	01	11	10
0	X	1	X	X
1	X		X	X

$\therefore \quad K_B = \bar{X}$

For output Y:

X \ AB	00	01	11	10
0			X	
1			X	1

$\therefore \quad Y = XA$



Q.4 (b) Solution:

For truth table of seven segment display when inputs $b_3\, b_2\, b_1\, b_0$ = 0000, a decimal 0 will be displayed i.e., all segments lit except X_5.

$$\therefore \quad X_0 = X_1 = X_2 = X_3 = X_4 = X_6 = 1;\ X_5 = 0$$

and $E = 0$ because 0000 is a valid input combination.

The values at the outputs for all other input combinations can be derived in similar manner as below:

Decimal digit	b_3	b_2	b_1	b_0	E	X_6	X_5	X_4	X_3	X_2	X_1	X_0
0	0	0	0	0	0	1	0	1	1	1	1	1
1	0	0	0	1	0	0	0	1	1	0	0	0
2	0	0	1	0	0	1	1	0	1	1	0	1
3	0	0	1	1	0	1	1	1	1	1	0	0
4	0	1	0	0	0	0	1	1	1	0	1	0
5	0	1	0	1	0	1	1	1	0	1	1	0
6	0	1	1	0	0	1	1	1	0	1	1	1
7	0	1	1	1	0	0	0	1	1	1	0	0
8	1	0	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	0	1	1	1	1	1	1	0
Invalid	1	0	1	0	1	1	1	0	0	1	1	1
Invalid	1	0	1	1	1	1	1	0	0	1	1	1
Invalid	1	1	0	0	1	1	1	0	0	1	1	1
Invalid	1	1	0	1	1	1	1	0	0	1	1	1
Invalid	1	1	1	0	1	1	1	0	0	1	1	1
Invalid	1	1	1	1	1	1	1	0	0	1	1	1

From the above, we can write

$E = \Sigma m(10, 11, 12, 13, 14, 15)$

$X_6 = \Sigma m(0, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_5 = \Sigma m(2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_4 = \Sigma m(0, 1, 3, 4, 5, 6, 7, 8, 9)$

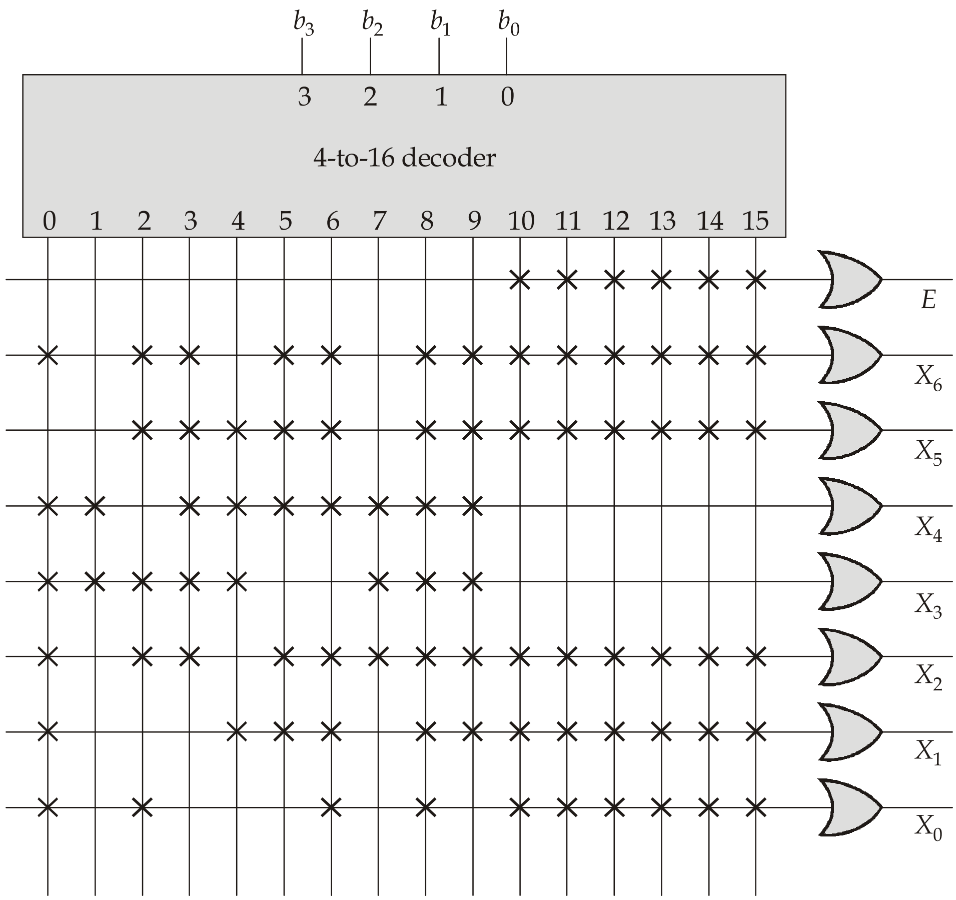
$X_3 = \Sigma m(0, 1, 2, 3, 4, 7, 8, 9)$

$X_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_1 = \Sigma m(0, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_0 = \Sigma m(0, 2, 6, 8, 10, 11, 12, 13, 14, 15)$

The above can be implemented using a ROM as below:



Q.4 (c) Solution:

Sensor port address → 30 H

LED port address → 50 H

+ve values stored from → 4000 H

No. of data bytes to be verified → 10

Logic of program:

For a signed 8-bit data D_7 bit is considered for sign. $D_6 - D_0$ → Magnitude

Sign bit → D_7

+ve ← 0

–ve ← 1

If MSB is 1 → –ve

If MSB is 0 → +ve

So using RLC and checking carry flag for '1'/'0'.

	LXI	D, 4000 H	; Initialize the memory pointer to 4000 H; so as to store +ve values
	MVI	C, 0AH	; Initialize count of 10 into register 'C'
REP:	IN	30H	; Read data from port 30H
	MOV	L, A	; Move content of 'Accumulator' to register 'L'.
	RLC		; Rotate 1 bit left to check '1'/'0'.
	JC	NEG	; Jump if carry flag is '1'.
	MOV	A, L	; If CY → 0, Move data to memory.
	STAX	D	
	INX	D	; Increment pointer
	JMP	NEXT	; Jump to next number.
NEG:	MVI	A, FFH	; If value is –ve send FFH to port 50 H to blink LED.
	OUT	50H	
NEXT:	DCR	C	; Check if count is non-zero after decrementing register 'C'.
	JNZ	REP	; If true repeat else stop.
	HLT		; Stop the program

Section B : Systems and Signal Processing

Q.5 (a) Solution:

(i)

$$y(n) = x(n) - 2x(n-1) + x(n+1)$$

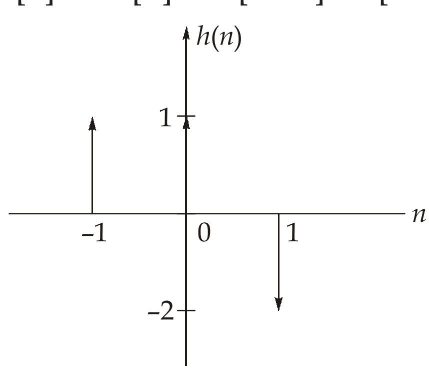
Taking z-transform,

$$Y(z) = X(z) - 2z^{-1}X(z) + zX(z)$$

$$H(z) = 1 - 2z^{-1} + z$$

Taking inverse z-transform

$$h[n] = \delta[n] - 2\delta[n-1] + \delta[n+1]$$



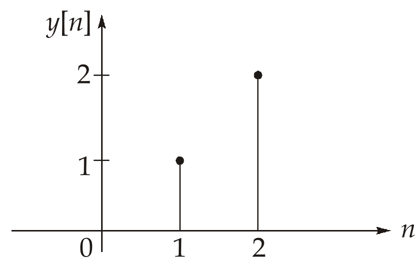
(ii)

$$y[n] = [\delta[n+1] + \delta[n] - 2\delta[n-1]] * x[n]$$

$$x[n] = u[n-2]$$

$$y[n] = [\delta[n+1] + \delta[n] - 2\delta[n-1]] * u[n-2]$$

$$y[n] = u[n-1] + u[n-2] - 2u[n-3]$$



(iii)

$$\sum_{n=-\infty}^{\infty} y[n] = \sum_{n=1}^{2} y[n]$$

$$= 1 + 2 = 3$$

Q.5 (b) Solution:

We desire the partial fraction expansion of $X(z)/z$, which is

$$F(z) = \frac{X(z)}{z} = \frac{z+2}{z(2z^2-7z+3)} = \frac{z+2}{2z\left(z-\frac{1}{2}\right)(z-3)}$$

$$= \frac{A_0}{z} + \frac{A_1}{z-\frac{1}{2}} + \frac{A_2}{z-3}$$

where,

$$A_0 = zF(z)|_{z=0} = \left.\frac{z+2}{2\left(z-\frac{1}{2}\right)(z-3)}\right|_{z=0} = \frac{2}{3}$$

$$A_1 = \left.\left(z-\frac{1}{2}\right)F(z)\right|_{z=\frac{1}{2}} = \left.\frac{z+2}{2z(z-3)}\right|_{z=\frac{1}{2}} = -1$$

$$A_2 = (z-3)F(z)|_{z=3} = \left.\frac{z+2}{2z\left(z-\frac{1}{2}\right)}\right|_{z=3} = \frac{1}{3}$$

Hence, by multiplying $X(z)/z$ by z, we obtain

$$X(z) = \frac{2}{3} - \frac{z}{z-\frac{1}{2}} + \frac{z/3}{z-3}$$

Here, the given function $X(z)$ has two poles, $p_1 = \frac{1}{2}$ and $p_2 = 3$ and the following three inverse transforms.

(a) In the region $|z| > 3$, all poles are interior, i.e., the signal $x(n)$ is causal, and therefore,

$$x(n) = \frac{2}{3}\delta(n) - \left(\frac{1}{2}\right)^n u(n) + \frac{1}{3}(3)^n u(n)$$

(b) In the region $|z| < \frac{1}{2}$, both the poles are exterior, i.e., $x(n)$ is anti-causal and hence

$$x(n) = \frac{2}{3}\delta(n) + \left(\frac{1}{2}\right)^n u(-n-1) - \frac{1}{3}(3)^n u(-n-1).$$

(c) In the region $\frac{1}{2} < |z| < 3$, $\left[i.e., |z| < 3 \text{ anti-causal and } |z| > \frac{1}{2} \text{ causal}\right]$, the pole $p_1 = \frac{1}{2}$ is interior and $p_2 = 3$ is exterior, and hence

$$x(n) = \frac{2}{3}\delta(n) - \left(\frac{1}{2}\right)^n u(n) - \frac{1}{3}(3)^n u(-n-1).$$

Q.5 (c) Solution:

By definition,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[a^n u(n) - a^n u(n-1)\right] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n} - \sum_{n=-\infty}^{\infty} a^n u(n-1) z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n - \sum_{n=1}^{\infty} (az^{-1})^n$$

$$= \frac{1}{1-az^{-1}} - \frac{az^{-1}}{1-az^{-1}} \quad \text{for } |az^{-1}| < 1 \rightarrow |z| > |a|$$

$$X(z) = \frac{1-az^{-1}}{1-az^{-1}} = 1 \quad \text{ROC is the entire } z\text{-plane}$$

The sequences $a^n u(n)$ and $a^n u(n-1)$ both have a region of convergence defined by $|z| > |a|$.

The signal, $x(n) = a^n u(n) - a^n u(n-1) = \delta(n)$ is a finite duration signal, and its z-transform is $X(z) = 1$, which has a ROC that is the entire z-plane.

Q.5 (d) Solution:

$$y(t) = (2 - 3e^{-t} + e^{-3t})\, u(t)$$

$$Y(s) = \frac{2}{s} - \frac{3}{s+1} + \frac{1}{s+3}$$

$$Y(s) = \frac{2(s^2 + 4s + 3) - 3s(s+3) + s(s+1)}{s(s+1)(s+3)}$$

$$= \frac{6}{s(s+1)(s+3)}$$

Consider the given unit step response,

$$s(t) = (1 - e^{-t} - te^{-t})\, u(t)$$

$$= u(t) - e^{-t}\, u(t) - te^{-t}\, u(t)$$

The Laplace transform of this equation yields

$$S(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$H(s) = s\, S(s) = \left(\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}\right)$$

$$H(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$= \frac{(s+1)^2 - s(s+1) - s}{(s+1)^2} = \frac{s^2 + 2s + 1 - s^2 - s - s}{(s+1)^2}$$

$$H(s) = \frac{1}{(s+1)^2} = \frac{Y(s)}{X(s)}$$

Therefore,

$$X(s) = \frac{Y(s)}{H(s)} = \frac{\dfrac{6}{(s(s+1)(s+3))}}{\dfrac{1}{((s+1)^2)}} = \frac{6(s+1)}{s(s+3)}$$

$$X(s) = \frac{2}{s} + \frac{4}{s+3}$$

The inverse Laplace transform of this equation yields

$$x(t) = 2\, u(t) + 4e^{-3t}\, u(t)$$

Q.5 (e) Solution:

(i) Given,

$$y(t) = e^{-2t}u(t)$$

$$\text{System function, } H(s) = \frac{Y(s)}{X(s)} = \frac{s-1}{s+1}$$

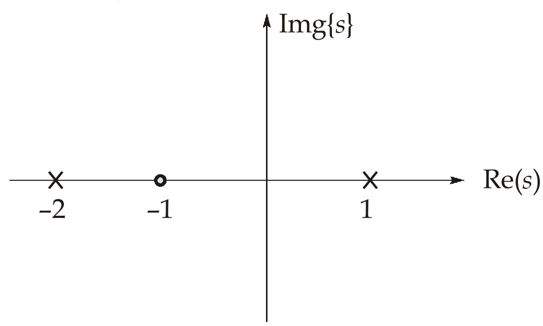
By taking Laplace transform of $y(t)$,

$$Y(s) = \frac{1}{s+2}; \text{Re}\{s\} > -2$$

$$\therefore \quad X(s) = \frac{Y(s)}{H(s)} = \frac{\frac{1}{s+2}}{\frac{s-1}{s+1}}$$

$$\therefore \quad X(s) = \frac{s+1}{(s-1)(s+2)}$$

The pole-zero diagram of $X(s)$ is



Since the given system is a causal system, the ROC of the impulse response is right-sided. Thus, from the given $H(s)$, the ROC is Re{s} > –1. We know that the ROC of $Y(s)$ is at least the intersection of the ROCs of $X(s)$ and $H(s)$.

In the above case, we can choose the ROC of $X(s)$ to be either –2 < Re{s} < 1 (or) Re{s} > 1.

In both cases, we get the same ROC for $Y(s)$ because the poles at s = –1 and s = 1 in $H(s)$ and $X(s)$ respectively are cancelled out by zeros.

The partial fraction expansion of $X(s)$ is

$$X(s) = \frac{\frac{2}{3}}{s-1} + \frac{\frac{1}{3}}{s+2}$$

$\therefore$ Taking the ROC of $X(s)$ to be –2 < Re{s} < 1

We get,
$$x(t) = \frac{2}{3}e^{t}\{-u(-t)\} + \frac{1}{3}e^{-2t}u(t)$$

Taking the ROC of $X(s)$ to be Re$\{s\} > 1$, we get

$$x(t) = \frac{2}{3}e^{t}u(t) + \frac{1}{3}e^{-2t}u(t)$$

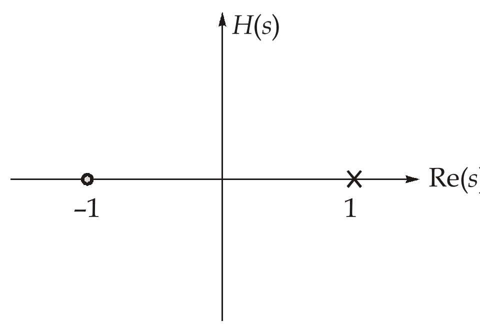
(ii) From the given information, we can write

$$H(s)Y(s) = X(s)$$

(Here, $X(s)$ is the output and $Y(s)$ is the input)

Clearly,
$$H(s) = \frac{X(s)}{Y(s)} = \frac{s+1}{s-1}$$

The pole-zero plot of $H(s)$ is



Since the system is given to be stable, the ROC of the impulse response mustt include the $j\omega$ axis. Thus, the ROC of $H(s)$ has to be Re$\{s\} < 1$.

We can write,

$$H(s) = \frac{s+1-2+2}{s-1} = \frac{s-1}{s-1} + \frac{2}{s-1}$$

$\therefore$
$$H(s) = 1 + \frac{2}{s-1}$$

$\therefore$
$$h(t) = \delta(t) + 2e^{t}\{-u(-t)\}$$

Also $Y(s)$ has the ROC: Re$\{s\} > -2$

$\therefore$ $X(s)$ must have the ROC: $-2 <$ Re$\{s\} < 1$

(i.e, the intersection of the ROCs of $Y(s)$ and $H(s)$).

$\therefore$
$$x(t) = -\frac{2}{3}e^{t}u(-t) + \frac{1}{3}e^{-2t}u(t)$$

Q.6 (a) Solution:

(i) We know that, $$\text{rect}(t/1) \xleftrightarrow{F.T} \sin c(f)$$

$$x(t) = \text{sinc } \alpha t$$

Using Duality property

$$x(t) \xleftrightarrow{F.T} X(f)$$

$$X(t) \longleftrightarrow x(-f)$$

$$\text{sinc }(t) \xleftrightarrow{F.T} \text{rect}(f/1)$$

Using scaling property of Fourier transform,

$$\text{sinc } \alpha t \xleftrightarrow{F.T} \frac{1}{\alpha}\text{rect}\left(\frac{f}{\alpha}\right)$$

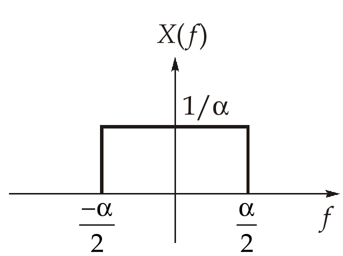
Fourier transform of $x(t)$

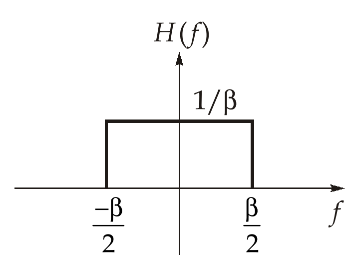
$$X(f) = \frac{1}{\alpha}\text{rect}\left(\frac{f}{\alpha}\right)$$

Similarly, $$y(t) \xleftrightarrow{F.T} Y(f)$$

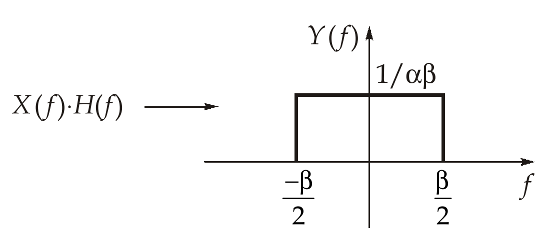
$$Y(f) = \frac{1}{\beta}\text{rect}\left(\frac{f}{\beta}\right)$$

Now, $$y(t) = x(t) * h(t)$$
$$Y(f) = X(f) \cdot H(f)$$





As $$\alpha > \beta$$



∴

$$Y(f) = \frac{1}{\alpha} \cdot \frac{1}{\beta} \text{rect}\left(\frac{f}{\beta}\right)$$

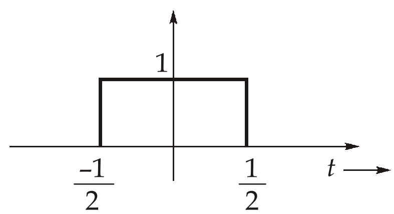
$$y(t) = \frac{1}{\alpha} \text{sin}\,c\;\beta t$$

(ii)

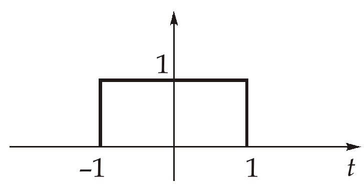
$$y(t) = h(t) * x(t)$$

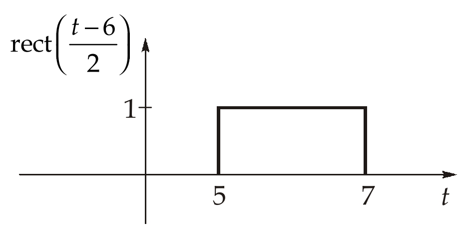
$$x(t) = \text{rect}\left(\frac{t-6}{2}\right)$$

rect (t) =



Put $t \to \frac{t}{2}$ $\quad \text{rect}\left(\frac{t}{2}\right) =$



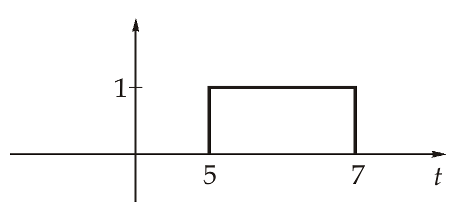


Put $t \to t - 6$

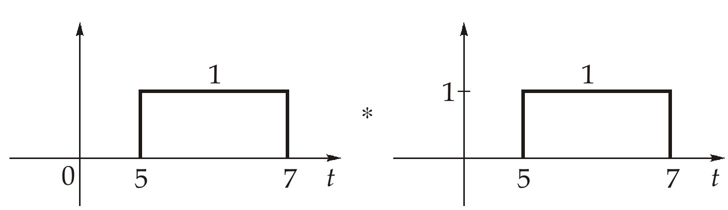
$$h(t) = \text{rect}\left(\frac{t}{2} - 3\right)$$

$$h(t) = \text{rect}\left(\frac{t-6}{2}\right) = x(t)$$

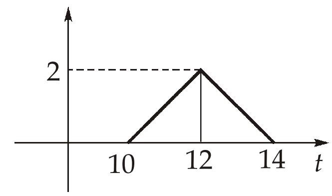
∴ $h(t) \to$



$y(t) = x(t) * h(t) =$



$y(t)$ =



Q.6 (b) Solution:

(i) Taking z-transform

$$H[z] = 5 - 7z^{-1} + 7z^{-3} - 5z^{-4}$$

$$z = re^{j\omega}$$

Assume, $r = 1$

$$|z| = 1$$

for low frequency, $\omega = 0, z = e^{j0}$

$$z = 1$$

for high frequency, $\omega = \pi, z = e^{j\pi}$

$$z = -1$$

for mid-frequency, $\omega = \frac{\pi}{2}, z = e^{j\pi/2}$

$$z = j$$

$$H(z) = 5 - 7z^{-1} + 7z^{-3} - 5z^{-4}$$

for low frequency, put $z = 1, H(z) = 5 - 7 + 7 - 5 = 0$

for high frequency, put $z = -1, H(z) = 5 + 7 - 7 - 5 = 0$

for mid frequency, put $z = j, H(z) = 5 + 7j + 7j - 5 = 14j$

∴ filter passes only the mid frequencies.

∴ The filter is band pass filter.

(ii)

$$h[n] = 5[\delta[n] - \delta[n-2]]$$

$$H(z) = 5(1 - z^{-2})$$

for low frequency, $z = 1 \quad \Rightarrow \quad H(z) = 0$

for high frequency, $z = -1 \quad \Rightarrow \quad H(z) = 5(1 - 1) = 0$

for mid frequency, $z = j, H(z) = 5[1 - (j)^{-2}]$

$$= 5[(1 - (-1))]$$

$$H(z) = 10$$

Filter is band pass filter.

Q.6 (c) Solution:

(i) The procedure used in impulse invariant method is

Step 1 :
$$H(s) = \sqrt{2}\left[\frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(s+\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}\right]$$

On taking inverse laplace transform of above expression,

Step 2 :
$$h(t) = \sqrt{2}e^{-t/\sqrt{2}}\sin\left(\frac{t}{\sqrt{2}}\right)u(t)$$

Step 3 :
$$t = nT_s = n \times 1 = n$$

$$h(n) = \sqrt{2}e^{-\frac{n}{\sqrt{2}}}\sin\left(\frac{n}{\sqrt{2}}\right)u(n)$$

As we know,

$$a^n \sin(\omega_o n)u(n) \xleftrightarrow{Z.T.} \frac{az^{-1}\sin(\omega_o)}{1-2az^{-1}\cos(\omega_o)+a^2z^{-2}}$$

where,
$$a = e^{-\frac{1}{\sqrt{2}}}, \ \omega_o = \frac{1}{\sqrt{2}}$$

Step 4 :
$$H(z) = \sqrt{2}\left[\frac{e^{-\frac{1}{\sqrt{2}}}z^{-1}\sin\left(\frac{1}{\sqrt{2}}\right)}{1-2e^{-\frac{1}{\sqrt{2}}}z^{-1}\cos\left(\frac{1}{\sqrt{2}}\right)+e^{-\sqrt{2}}z^{-2}}\right]$$

$$H(z) = \frac{0.453z^{-1}}{1-0.7497z^{-1}+0.2432z^{-2}}$$

(ii) Gaussian modulated signal,

$$x(t) = e^{-at^2}\cos\omega_c t$$

$$= e^{-at^2}\left[\frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2}\right]$$

$$= \frac{1}{2}e^{-at^2}e^{j\omega_c t}+\frac{1}{2}e^{-at^2}e^{-j\omega_c t}$$

Let $x(t) \xleftrightarrow{F.T.} X(\omega)$

$\therefore$ $$X(\omega) = \frac{1}{2}[\{F(e^{-at^2} e^{j\omega_c t})\} + \{F(e^{-at^2} e^{-j\omega_c t})\}] \quad ...(i)$$

As we know, $$F[e^{-at^2}] = \sqrt{\frac{\pi}{a}} e^{\frac{-\omega^2}{4a}}$$

or $$e^{-at^2} \xleftrightarrow{F.T.} \sqrt{\frac{\pi}{a}} e^{\frac{-\omega^2}{4a}}$$

By using frequency shifting property

$$x(t)e^{-j\omega_o t} \xleftrightarrow{F.T.} X(\omega + \omega_o)$$

We have from eqn. (i)

$$F(e^{-at^2} e^{j\omega_c t}) = F(e^{-at^2})|_{\omega \to \omega - \omega_c}$$

$$F(e^{-at^2} e^{-j\omega_c t}) = F(e^{-at^2})|_{\omega \to \omega + \omega_c}$$

$\therefore$ $$X(\omega) = \frac{1}{2}\left[\sqrt{\frac{\pi}{a}} e^{\frac{-(\omega-\omega_c)^2}{4a}} + \sqrt{\frac{\pi}{a}} e^{\frac{-(\omega+\omega_c)^2}{4a}}\right]$$

Q.7 (a) (i) Solution:

Taking z-transform of the given difference equation, we get

$$Y(z) = \frac{1}{6}X(z) + \frac{1}{3}z^{-1}X(z) + \frac{1}{6}z^{-2}X(z)$$

Therefore, $$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{6} + \frac{1}{3}z^{-1} + \frac{1}{6}z^{-2}$$

The frequency response is $$H(e^{j\omega}) = \frac{1}{6}\left[1 + 2e^{-j\omega} + e^{-j2\omega}\right]$$

$$= \frac{1}{6}e^{-j\omega}\left[e^{j\omega} + 2 + e^{-j\omega}\right]$$

$$= \frac{1}{3}(1 + \cos\omega)e^{-j\omega}$$

Hence, magnitude response is $|H(\omega)| = \frac{1}{3}(1 + \cos\omega)$, and

Phase response is $\phi(\omega) = -\omega$.

Q.7 (a) (ii) Solution:

$$H_d(e^{j\omega}) = \begin{cases} 0, & -\pi/4 \le \omega \le \pi/4 \\ e^{-j2\omega}, & \pi/4 \le |\omega| \le \pi \end{cases}$$

Therefore,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{-j2\omega}e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} e^{-j2\omega}e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{j\omega(n-2)}d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} e^{j\omega(n-2)}d\omega$$

$$= \frac{1}{\pi(n-2)}\left\{\left[\frac{e^{j(n-2)\pi} - e^{-j(n-2)\pi}}{2j}\right] - \left[\frac{e^{j(n-2)\pi/4} - e^{-j(n-2)\pi/4}}{2j}\right]\right\}$$

$$= \frac{1}{\pi(n-2)}\left[\sin\pi(n-2) - \sin(n-2)\pi/4\right], n \ne 2$$

$$h_d(2) = \frac{2}{2\pi}\int_{\pi/4}^{\pi} e^{-j2\omega}e^{j2\omega}d\omega = \frac{3}{4}$$

The filter coefficient are given by,

$$h_d(2) = \frac{3}{4}, h_d(0) = -\frac{1}{2\pi} = h_d(4)$$

and

$$h_d(1) = -\frac{1}{\sqrt{2}\pi} = h_d(3)$$

and by applying the window function, the new filter coefficients are

$$h(2) = \frac{3}{4}, h(0) = -\frac{1}{2\pi} = h(4) \text{ and } h(1) = -\frac{1}{\sqrt{2}\pi} = h(3)$$

The frequency response $H(e^{j\omega})$, is obtained as

$$H(e^{j\omega}) = \sum_{n=0}^{4} h(n)\cdot e^{-j\omega n}$$

$$H(e^{j\omega}) = e^{-j2\omega}\left[0.75 - \frac{\sqrt{2}}{\pi}\cos\omega - \frac{1}{\pi}\cos 2\omega\right]$$

Q.7 (b) Solution:

The waveform is periodic with period $T = 2\pi$ and fundamental frequency $\omega_0 = \frac{2\pi}{T} = 1$

The given waveform for one period may be written as

$$x(t) = \begin{cases} A\sin(t), & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The exponential Fourier series coefficient is

$$X_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt$$

$$= \frac{1}{2\pi}\left(\int_0^{\pi} A\sin(t)e^{-jnt}dt + \int_{\pi}^{2\pi} 0\,e^{-jnt}dt\right)$$

$$= \frac{A}{2\pi}\int_0^{\pi}\sin(t)e^{-jnt}dt = \frac{A}{2\pi}I$$

Where, $$I = \int_0^{\pi} e^{-jnt}\sin(t)\,dt$$

$$I = \left[-e^{-jnt}\cos(t)\right]_0^{\pi} - jn\int_0^{\pi} e^{-jnt}\cos(t)dt$$

$$I = \left[-e^{-jnt}\cos(t)\right]_0^{\pi} - jn\int_0^{\pi} e^{-jnt}\cos(t)dt$$

$$I = \left[-e^{-jnt}\cos(t)\right]_0^{\pi} - \left[-jne^{-jnt}\sin(t)\right]_0^{\pi} + n^2\int_0^{\pi} e^{-jnt}\sin(t)dt$$

$$I = \left[-e^{-jnt}\cos(t)\right]_0^{\pi} + \left[-jn\,e^{-jnt}\sin(t)\right]_0^{\pi} + n^2 I$$

$$I(1 - n^2) = \left[e^{-jnt}(-\cos(t) - jn\sin(t))\right]_0^{\pi}$$

$$I = \frac{e^{-jn\pi} + 1}{1 - n^2}$$

Therefore, $$X_n = \frac{A}{2\pi}I = \frac{A(e^{-jn\pi} + 1)}{2\pi(1 - n^2)}$$

$$= \begin{cases} \dfrac{A(e^{-jn\pi}+1)}{\pi(1-n^2)}, & n = \pm 2, \pm 4, \pm 6, \ldots \\ 0, & n = \pm 3, \pm 5, \pm 7 \ldots \end{cases}$$

For $n = \pm 1$, the expression for X_n becomes indeterminate, L'Hospital rule may be applied, i.e. the numerator and denominator are separately differentiated w.r.t. after which n is allowed to approach ± 1.

$$X_1 = \lim_{n \to 1} \frac{\frac{d}{dn} A(e^{-jn\pi}+1)}{\frac{d}{dn} 2\pi(1-n^2)} = -j\frac{A}{4}$$

Similarly, $$X_{-1} = \lim_{n \to -1} \frac{\frac{d}{dn} A(e^{-jn\pi}+1)}{\frac{d}{dn} 2\pi(1-n^2)} = j\frac{A}{4}$$

The average value is

$$X_0 = a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{2\pi}\left(\int_0^{\pi} A\sin(t)dt + 0\right)$$

$$= \frac{A}{2\pi}[-\cos(t)]_0^{\pi} = \frac{A}{\pi}$$

The exponential Fourier series for the given waveform is

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} X_n e^{jnt}$$

$$= \ldots. -\frac{A}{15\pi}e^{-j4t} - \frac{A}{3\pi}e^{-j2t} + j\frac{A}{4}e^{-jt} + \frac{A}{\pi} - j\frac{A}{4}e^{jt} - \frac{A}{3\pi}e^{j2t} - \frac{A}{15\pi}e^{j4t} \ldots.$$

$$C_n = \begin{cases} \dfrac{A}{\pi}, & n = 0 \\ -j\dfrac{A}{4}, & n = 1 \\ j\dfrac{A}{4}, & n = -1 \\ \dfrac{2A}{\pi(n^2-1)}, & n = \pm 2, \pm 4, \pm 6, \ldots. \\ 0, & n = \pm 3, \pm 5, \pm 7, \ldots. \end{cases}$$

Q.7 (c) (i) Solution:

Given,

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

where $$z(t) = e^{-t}u(t) + \delta(t)$$

By taking Fourier transform,

$$j\omega\, Y(\omega) + 10Y(\omega) = X(\omega)Z(\omega) - X(\omega)$$

$$Y(\omega)[10 + j\omega] = X(\omega)[Z(\omega) - 1]$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{Z(\omega)-1}{10+j\omega}$$

Taking Fourier Transform of $z(t)$,

$$Z(\omega) = \frac{1}{1+j\omega} + 1 = \frac{2+j\omega}{1+j\omega}$$

$$H(\omega) = \frac{\frac{2+j\omega}{1+j\omega} - 1}{10+j\omega} = \frac{(2+j\omega-1-j\omega)}{(1+j\omega)(10+j\omega)}$$

$$\therefore \quad H(\omega) = \frac{1}{(1+j\omega)(10+j\omega)}$$

By using partial fraction expansion,

$$\frac{1}{(1+j\omega)(10+j\omega)} = \frac{A}{1+j\omega} + \frac{B}{10+j\omega}$$

$$A = \left.\frac{1}{10+j\omega}\right|_{j\omega=-1} = \frac{1}{9}$$

$$B = \left.\frac{1}{1+j\omega}\right|_{j\omega=-10} = \frac{-1}{9}$$

$$\therefore \quad H(\omega) = \frac{\frac{1}{9}}{1+j\omega} + \frac{\frac{-1}{9}}{10+j\omega}$$

By taking inverse Fourier transform,

$$h(t) = \frac{1}{9}e^{-t}u(t) - \frac{1}{9}e^{-10t}u(t)$$

Q.7 (c) (ii) Solution:

Given that :
$$H(s) = \frac{s+0.1}{(s+0.1)^2+9}$$

From the above expression, we note that $\Omega_c = 3$. The sampling period T_s can be determined as

$$\Omega_c = \frac{2}{T_s}\tan\frac{\omega_r}{2}$$

$$T_s = \frac{2}{\Omega_c}\tan\frac{\omega_r}{2} \qquad \left(\text{Since } \omega_r = \frac{\pi}{4}\right)$$

$$= \frac{2}{3}\tan\frac{\pi}{8} = 0.276 \text{ sec}$$

Using bilinear transformation,

$$H(z) = H(s)\Big|_{s=\frac{2}{T_s}\frac{(z-1)}{(z+1)}}$$

$$H(z) = \frac{\frac{2}{T_s}\cdot\frac{(z-1)}{(z+1)}+0.1}{\left[\frac{2}{T_s}\frac{(z-1)}{(z+1)}+0.1\right]^2+9}$$

$$= \frac{\left(\frac{2}{T_s}\right)(z-1)(z+1)+(z+1)^2\times 0.1}{\left[\left(\frac{2}{T_s}\right)(z-1)+(0.1)(z+1)\right]^2+9(z+1)^2}$$

$$H(z) = \frac{\frac{2}{T_s}(z^2-1)+(z^2+2z+1)\times 0.1}{\frac{4}{T_s^2}(z-1)^2+(0.1)^2(z^2+2z+1)+\frac{4}{T_s}\times 0.1(z^2-1)+9z^2+18z+9}$$

Substitute $T_s = 0.276$ sec

$$H(z) = \frac{\frac{2}{0.276}(z^2-1)+0.1z^2+0.2z+0.1}{\frac{4}{(0.276)^2}(z^2+1-2z)+0.01z^2+0.02z+0.01+1.45(z^2-1)+9z^2+18z+9}$$

$$H(z) = \frac{7.246z^2-7.246+0.1z^2+0.2z+0.1}{52.51z^2+52.51-105.02z+0.01z^2+0.02z+0.01+1.45z^2-1.45+9z^2+18z+9}$$

$$H(z) = \frac{7.346z^2 + 0.2z - 7.146}{62.97z^2 - 87z + 60.07}$$

$$H(z) = \frac{1 + \dfrac{0.2z}{7.346z^2} - \dfrac{7.146}{7.346z^2}}{\dfrac{62.97z^2}{7.346z^2} - \dfrac{87z}{7.346z^2} + \dfrac{60.07}{7.346z^2}}$$

$$H(z) = \frac{1 + 0.027z^{-1} - 0.973z^{-2}}{8.572 - 11.84z^{-1} + 8.177z^{-2}}$$

Q.8 (a) Solution:

(i) $x(t) = x(-t)$

$x(t)$ is even signal.

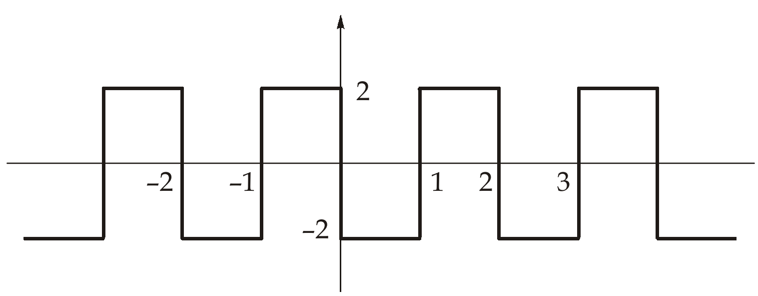
Also, $x(t)$ is half-wave symmetric, therefore only odd harmonics of $x(t)$ exist.

Let fourier series coefficient of $x(t)$ be C_n.

$$x(t) \xleftrightarrow{F.S} C_n$$

$$\frac{d}{dt}x(t) \longleftrightarrow jn\omega_0 C_n$$

$\frac{d}{dt}x(t)$



Let fourier series coefficient of $\frac{d}{dt}x(t)$ be d_n.

$$d_n = \frac{1}{T}\int_{-T/2}^{T/2} x'(t)e^{-jn\omega_0 t}\cdot dt$$

$$\omega_0 = \frac{2\pi}{T} = \pi \text{ rad/sec}$$

$$d_n = \frac{1}{2}\left[\int_{-1}^{0} 2e^{-jn\pi t}\cdot dt - \int_{0}^{1} 2e^{-jn\pi t}\cdot dt\right]$$

$$d_n = \frac{2}{2}\left[\frac{\left(e^{-jn\pi t}\right)_{-1}^{0}}{(-jn\pi)} - \frac{\left(e^{-jn\pi t}\right)_{0}^{1}}{(-jn\pi)}\right]$$

$$d_n = \frac{1}{(-jn\pi)}\left[1 - e^{jn\pi} - e^{-jn\pi} + 1\right] = \frac{2-(-1)^n-(-1)^n}{(-jn\pi)}$$

$$d_n = \frac{2\left[(-1)^n - 1\right]}{jn\pi}$$

Now using property of Fourier series,

$$d_n = jn\omega_0 C_n$$

$$C_n = \frac{d_n}{jn\omega_0} = \frac{2\left[(-1)^n - 1\right]}{jn\pi \cdot jn\pi}$$

$$C_n = \frac{2}{n^2\pi^2}\left[1-(-1)^n\right]$$

$$C_n = \frac{4}{n^2\pi^2}, \text{ for } n = \text{odd}$$

$$C_n = 0, \text{ for } n = \text{even}$$

(ii)

$$C_n = \frac{2}{n^2\pi^2}\left[1-(-1)^n\right]$$

Exponential Fourier series can be given as

$$x(n) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t} \quad ; \quad \text{where, } C_n = \begin{cases} \dfrac{4}{(n\pi)^2}; & n \rightarrow \text{odd} \\ 0; & n \rightarrow \text{even} \end{cases}$$

For 7th harmonic, $n = 7$

$$C_7 = \frac{2}{49\pi^2}\left[1-(-1)^7\right]$$

$$C_7 = \frac{4}{49\pi^2}$$

$$\text{Power in 7th harmonic} = 2|C_7|^2$$

$$= 2 \times \left(\frac{4}{49\pi^2}\right)^2 = 0.135 \text{ mW}$$

Q.8 (b) Solution:

Assume, $$y_1(t) = \cos^2(t) = \frac{1+\cos 2t}{2}$$

On taking fourier transform, we get

$$Y_1(\omega) = \frac{1}{2}[2\pi\delta(\omega)] + \frac{1}{2}\pi[\delta(\omega-2)+\delta(\omega+2)]$$

$$= \pi\delta(\omega) + \frac{\pi}{2}\delta(\omega-2) + \frac{\pi}{2}\delta(\omega+2)$$

Now, let $$y_2(t) = x(t)y_1(t) = x(t)\cos^2 t$$

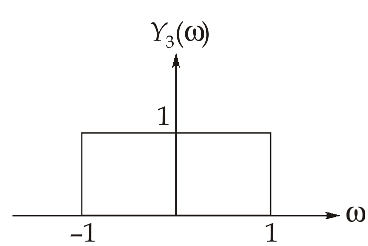
On taking fourier transform, we obtain

$$Y_2(\omega) = \frac{1}{2\pi}[X(\omega) * Y_1(\omega)]$$

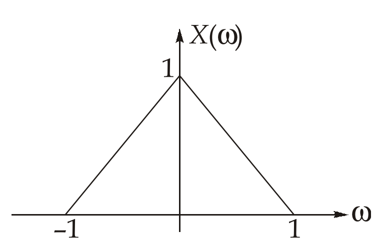
$$= \frac{1}{2\pi}\left[X(\omega) * \left\{(\pi\delta(\omega) + \frac{\pi}{2}\delta(\omega-2) + \frac{\pi}{2}\delta(\omega+2)\right\}\right]$$

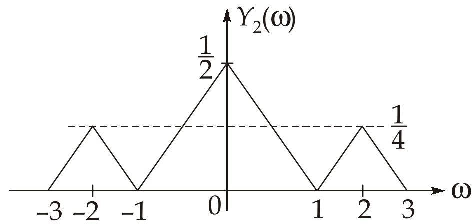
$$= \frac{1}{2}X(\omega) + \frac{1}{4}X(\omega-2) + \frac{1}{4}X(\omega+2)$$

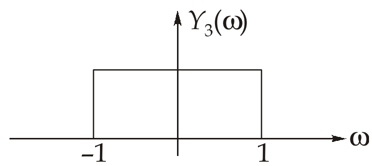
Now consider, $$y_3(t) = \frac{\sin t}{\pi t} \xleftrightarrow{F.T.} Y_3(\omega) = \text{rect}\left(\frac{\omega}{2}\right)$$



$$= \begin{cases} 1, & |\omega| < 1 \\ 0, & \text{otherwise} \end{cases}$$







$X(\omega)$, $Y_2(\omega)$ and $Y_3(\omega)$ are shown in above figures.

Now consider, $$g(t) = y_2(t)*y_3(t)$$

On taking F.T., we get

$$G(\omega) = Y_2(\omega).Y_3(\omega)$$

Now from figure (2) and (3), it is evident that

$$G(\omega) = \frac{1}{2}X(\omega)$$

$$\frac{G(\omega)}{X(\omega)} = H(\omega) = \frac{1}{2}$$

On taking inverse fourier transform, we have

$$h(t) = \frac{1}{2}\delta(t)$$

Q.8 (c) Solution:

(i) Given that,

$$x_1(t) = e^{j(2t+\pi/4)}$$

$$|x_1(t)|^2 = 1$$

Therefore, $$E_x = \int_{-\infty}^{\infty}(1)dt = \infty$$

$$P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|x_2(t)|^2\,dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}dt = \lim_{T\to\infty}\frac{1}{T}\times T = 1$$

Since, $E_x = \infty$ and $0 < P_x < \infty$, $x_2(t)$ is a power signal.

(ii) Given that,
$$x_2(t) = \left(\frac{1}{2}\right)^n u(n)$$
$$|x_2(n)|^2 = \left(\frac{1}{4}\right)^n u(n)$$

Therefore,
$$E_x = \sum_{n=-\infty}^{\infty} |x_2(n)|^2 = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u(n)$$
$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n u(n) = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$
$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_1(n)|^2$$
$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\frac{1}{4}\right)^n u(n)$$
$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$
$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\frac{1-\left(\frac{1}{4}\right)^{N+1}}{1-\frac{1}{4}}\right) = 0$$

Since $0 < E_x < \infty$ and $P_x = 0$, $x_2(n)$ is an energy signal.

(iii) Given that,
$$x_3(n) = e^{j\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)}$$
$$|x_3(n)|^2 = 1$$

Therefore,
$$E_x = \sum_{n=-\infty}^{\infty} |x_3(n)|^2 = \sum_{n=-\infty}^{\infty} 1 = \infty$$
$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_2(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1 = \lim_{N\to\infty} \frac{1}{2N+1}(2N+1) = 1$$

Since $E_x = \infty$ and $0 < P_x < \infty$, $x_3(n)$ is a power signal.

(iv) Given that,

$$x_4(n) = \cos\left(\frac{\pi}{4}n\right)$$

$$|x_4(n)|^2 = \cos^2\left(\frac{\pi}{4}n\right) = \frac{1+\cos\left(\left(\frac{\pi}{2}\right)n\right)}{2}$$

Therefore,

$$E_x = \sum_{n=-\infty}^{\infty} |x_4(n)|^2 = \sum_{n=-\infty}^{\infty} \frac{1+\cos\left(\left(\frac{\pi}{2}\right)n\right)}{2}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{\cos\left(\left(\frac{\pi}{2}\right)n\right)}{2} = \infty$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_4(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1+\cos\left(\frac{\pi}{2}\right)n}{2}$$

$$= \left(\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1}{2}\right) + \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1}{2}\cos\left(\frac{\pi}{2n}\right)$$

$$= \lim_{N\to\infty} \frac{1}{2}\frac{1}{(2N+1)}(2N+1) + 0 = \frac{1}{2}$$

Since $E_x = \infty$ and $0 < P_x < \infty$, $x_4(n)$ is a power signal.

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