MADE EASY

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Detailed Solutions

ESE-2025 Mains Test Series

Mechanical Engineering Test No : 2

Section A : Strength of Materials + Machine Design + Engineering Mechanics

1. (a)

Given : diameter of bar *ABC*, *D* = 70

Diameter of bar *MN* and *PQ*, *d* = 15 mm

Twisting moment, $T = 5000$ Nm

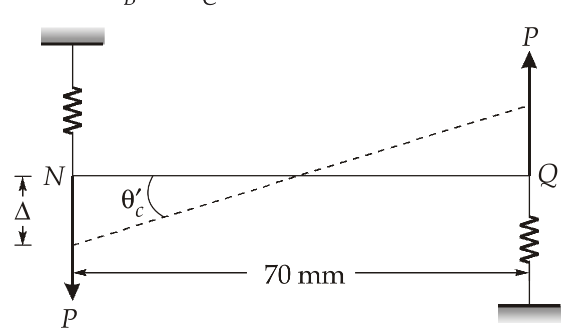
Let us consider that bars *MN* and *PQ* are temporarily disconnected from bar *ABC*, then the angle of twist at *B* relative to *A* is

$$\theta_B = \frac{TL}{GJ} = \frac{5000 \times 0.75}{\left(80 \times 10^9\right) \times \left(\frac{\pi}{32} \times 0.07^4\right)}$$

$$\theta_B = 0.0199 \text{ rad}$$

Since no additional twisting moments act between *B* and *C*, this same angle of twist exists at *C*,

$$\theta_B = \theta_C = 0.0199 \text{ rad}$$



As there is rotation at C, so there would be extension Δ of each of the vertical bar, which is accompanied by an axial force P in each bar.

For a small angle of rotation θ,

$$\Delta = 0.035\,\theta'_C$$

The axial force P constitute a couple of magnitude

$$P(0.07) = T_c$$

This couple must act in a sense opposite to the 5000 Nm load since the elastic vertical bars tend to restrain angular rotation.

The elongation of each vertical bar may be found to be

$$\Delta = \frac{PL}{AE} = \frac{P \times 1.5}{\frac{\pi}{4} \times (0.015)^2 \times E}$$

$$= \frac{\left(\frac{T_c}{0.07}\right) \times 1.5}{\frac{\pi}{4} \times (0.015)^2 \times 200 \times 10^9}$$

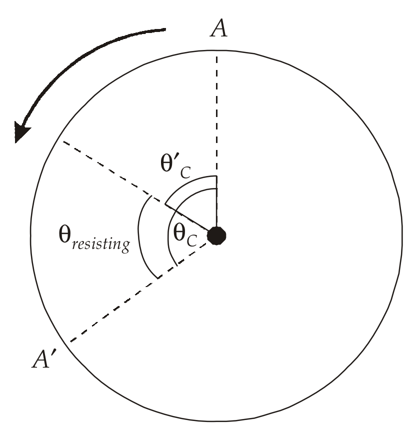
$$\Delta = 6.063 \times 10^{-7}\, T_c$$

$$\theta'_C = \frac{\Delta}{D/2} = \frac{6.063 \times 10^{-7} T_C}{0.035}$$

$$\theta'_C = 1.733 \times 10^{-5}\, T_C \text{ rad}$$

The angular rotation at end C is the due to twisting moment and angular rotation caused by the axial force P in vertical bars.

By compatibility equation,



$$\theta_C - \theta'_C = \theta_{resisting}$$

$$\theta_C - \theta'_C = \frac{T_C \times 1.5}{GJ}$$

$$0.0199 - 1.733 \times 10^{-5} T_C = \frac{T_C \times 1.5}{80 \times 10^9 \times \frac{\pi}{32} \times (0.07)^4}$$

$$\Rightarrow \quad 0.0199 - 1.733 \times 10^{-5} T_C = 7.958 \times 10^{-6} T_C$$

$$T_C = 786.76 \text{ Nm}$$

$$P = 11239.42 \text{ N}$$

The twisting moment between B and C is 786.76 Nm and between A and B is 5000 – 786.76 = 4213.24 Nm.

The peak torsional shearing stress occurs at the outer most fibres at all points between A and B.

$$\therefore \qquad \tau_{max} = \frac{4213.24 \times 0.035}{\frac{\pi}{32} \times (0.07)^4}$$

$$\tau_{max} = 62.56 \text{ MPa}$$

The axial stress in each of the vertical bars is

$$\sigma = \frac{11239.42}{\pi(0.0075)^2} = 63.59 \text{ MPa}$$

1. (b)

Let us first determine the reactions at A and C using the equilibrium equations

Taking moment at $A = 0$

$$\Sigma M_A = 0$$

$$2550 + R_C \times 6 - 280 \times 6 \times 6 = 0$$

$$R_C = \frac{280 \times 36 - 2550}{6} = 1255 \text{ N}$$

From equilibrium of forces,

$$\Sigma F_y = 0$$

$$R_A + R_C - 280 \times 6 = 0$$

$$R_A = 425 \text{ N}$$

For the coordinate x as shown, the shearing force at a distance x from point A is described by the three relations:

Shear force,

$$\text{SF} = 425, \qquad 0 < x < 3 \text{ m} \qquad ..(i)$$

$$\text{SF} = 425 - 280(x - 3), \qquad 3 < x < 6 \text{ m} \qquad ..(ii)$$

$$\text{SF} = 425 - 280(x - 3) + 1255, \qquad 6 < x < 9 \text{ m} \qquad ..(iii)$$

The bending moment in each case of three regions of the beam is described by

$$\text{BM} = 425x \qquad 0 < x < 3 \text{ m} \qquad ...(iv)$$

$$\text{BM} = 425x - 280(x-3)\frac{(x-3)}{2}$$

$$= 425x - \frac{280}{2}(x-3)^2 \qquad 3 < x < 6 \text{ m} \qquad ...(v)$$

$$\text{BM} = 425x - \frac{280}{2}(x-3)^2 + 1255 \times (x-6) \qquad 6 < x < 9 \text{ m} \qquad ...(v)$$

For shear force and bending moment diagrams we must find the values at point A, B, C and D.

Shear force is constant from A to B,

$$SF = 425 \text{ N}$$

$$\text{Shear force at } C,\ SF_c = 425 - 280(6-3) = -415 \text{ N}$$

As shear force is negative so between B and C there is a point where shear force is zero.

$$SF = 0$$

$$\Rightarrow \quad 425 - 280(x-3) = 0$$

$$x = 4.52 \text{ m}$$

The bending moment variation is linear between A to B.

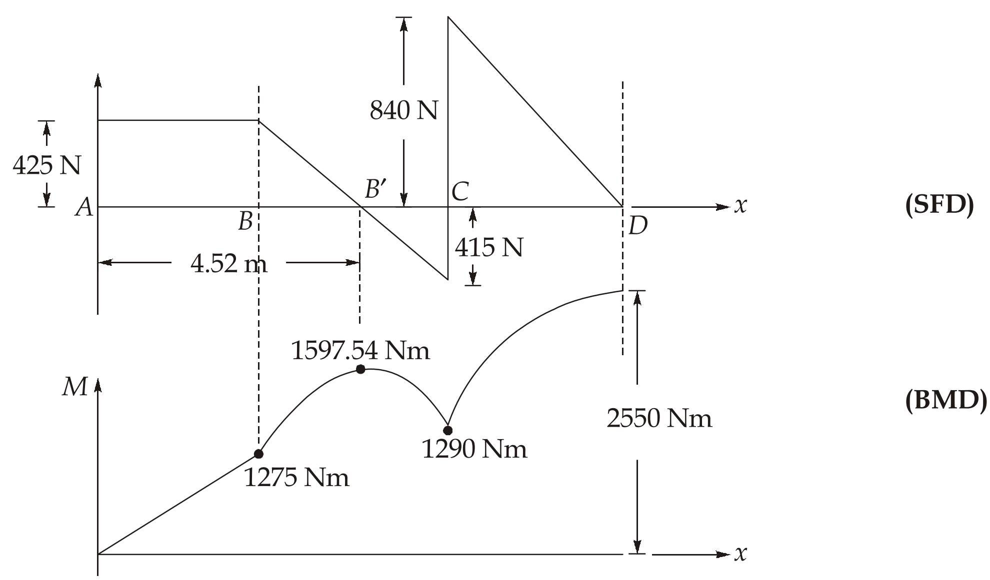
$$BM \text{ at } B = 425 \times 3 = 1275 \text{ Nm}$$

$$BM \text{ at } (x = 4.52),\ BM = 1597.544 \text{ Nm}$$

$$\text{BM at } C = 1290 \text{ Nm}$$

$$\text{BM at } D = 2550 \text{ Nm}$$

Making the shear force and bending moment diagrams using the above value.



1. (c)

Given : $P = 30$ kW; $N = 900$ rpm; $\mu = 0.3$; $p_a = 1.5$ N/mm^2; $D = 2d$

Torque transmitting capacity,

$$T = \frac{P \times 60}{2\pi N} = \frac{30 \times 60 \times 10^3}{2 \times \pi \times 900} \text{ N-m} = 318.30989 \text{ N-m}$$

$$T = 318309.89 \text{ N-mm}$$

Force required to engage the clutch,

$$F = \frac{\pi}{2} \times p_a \times d(D-d)$$

$$= \frac{\pi}{2} \times 1.5 \times d(2d-d) = \frac{\pi d^2}{2} \times 1.5 = 0.75\pi d^2$$

Also, for uniform wear theory,

$$T = \frac{\mu F}{4}(D+d)$$

$$\therefore \quad 318309.89 = \frac{0.3 \times 0.75 \times \pi \times d^2}{4}(2d+d)$$

$$1801265.51 = 3d^3$$

$$d^3 = 600421.84$$

$$d = 84.36 \text{ mm}$$

and $D = 2d = 2 \times 84.36$

Hence, the inner and outer diameters are 84.36 mm and 168.72 mm respectively.

1. (d)

Given : N = 1500 rpm; Z_P = 22; Z_G = 44; m = 3 mm ; b = 45 mm; S_{ut} = 600 N/mm^2; BHN = 400; C_s = 1.75; FOS = 2; Y = 0.33

Since the same material is used for the pinion and the gear, the pinion is weaker than the gear.

As, $$\sigma_b = \frac{1}{3}(S_{ut}) = \frac{1}{3} \times 600 = 200 \text{ N/mm}^2$$

Also, beam strength of gear tooth = $mb\sigma_b \times Y$

$$= 3 \times 45 \times 200 \times 0.33 = 8910 \text{ N}$$

$$\text{Wear strength} = d_p bQK \quad \text{...(i)}$$

where, $$Q = \frac{2z_G}{z_G + z_P} = \frac{2 \times 44}{44 + 22} = 1.33$$

$$k = 0.16 \times \left(\frac{BHN}{100}\right)^2 = 0.16 \times \left(\frac{400}{100}\right)^2 = 2.56$$

$$d_p = mz_P = 3 \times 22 = 66 \text{ mm}$$

$$\therefore \quad \text{Wear strength} = 45 \times 1.33 \times 66 \times 2.56 \quad \text{[From (i)]}$$

$$= 10112.256 \text{ N}$$

$$\text{Effective load} = \frac{C_s}{C_v} \times P_t \quad \text{...(ii)}$$

where, $$C_v = \frac{3}{3+v} = \frac{3}{3+\left(\frac{\pi d_P \times N_P}{60 \times 10^3}\right)} \quad [v < 10 \text{ m/s}]$$

$$C_v = \frac{3}{3+\left(\frac{\pi \times 66 \times 1500}{60 \times 10^3}\right)} = \frac{3}{3+5.184} = 0.366$$

$\therefore$ $$\text{Effective load} = \frac{1.75}{0.366} \times P_t \quad \text{[from (ii)]}$$

$$= 4.781\, P_t \text{ N}$$

As beam strength is lower than the wear strength. Therefore beam strength is the criterion of design.

$$\text{Beam strength} = \text{Effective load} \times \text{FOS}$$

$$8910 = 4.781\, P_t \times 2$$

$$P_t = 931.81 \text{ N}$$

$$\text{Rated power, } P = \frac{2 \times \pi \times N_P \times T}{60}$$

$$P = \frac{2 \times \pi \times 1500 \times \left(\frac{P_t \times d'_P}{2}\right)}{60}$$

$$= \frac{2 \times \pi \times 1500 \times 931.81 \times 66}{2 \times 60}$$

$$= 4830.156 \text{ N-m/s or Watt}$$

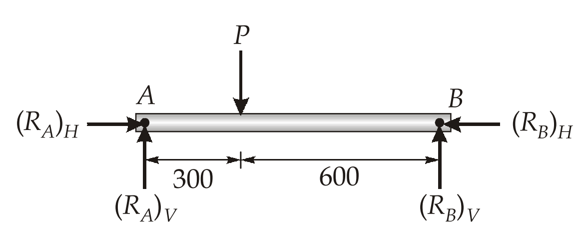
$$P = 4.83 \text{ kW}$$

Hence, the rated power that the gears can transmit is 4.83 kW.

1. (e)

Given : m_d = 40 kg; μ_s = 0.3; AB = 900 mm

From FBD of link AB



(All dimensions are in mm)

$$\Sigma F_H = 0$$

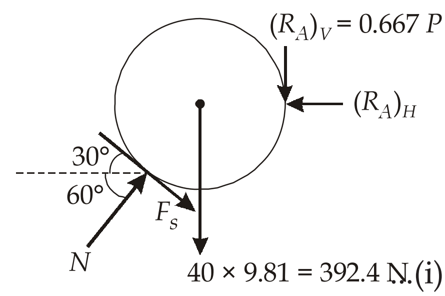
$\therefore$ $$(R_A)_H = (R_B)_H$$

$$\Sigma F_V = 0$$

$\therefore$ $$(R_A)_V + (R_B)_V = P$$

Taking moment about point B,

We get,



$$P \times 600 - (R_A)_V \times 900 = 0$$

$$(R_A)_V = 0.667P$$

From F.B.D. of disc

$$\Sigma F_V = 0$$

$$N\sin 60° - F_s \sin 30° - 0.667P - 392.4 = 0 \quad \text{...(i)}$$

Taking moment about centre point O,

We get,

$$F_s \times 200 - 0.667P \times 200 = 0$$

$$F_s = 0.667P$$

If the disc is on the verge of moving, slipping would have at point C. Hence,

$$F_s = \mu_s \times N = 0.3 \times N$$

$$\therefore \quad 0.3 \times N = 0.667P$$

$$N = 2.223P$$

On putting the value of F_s and N in equation (i)

$$2.223P \times \sin 60° - 0.667 \sin 30° - 0.667P - 392.4 = 0$$

$$1.925P - 0.3335P - 0.667P = 392.4$$

$$0.9245P = 392.4$$

$$P = 424.45 \text{ N}$$

Hence, the maximum vertical force P is 424.45 N.

2. (a)

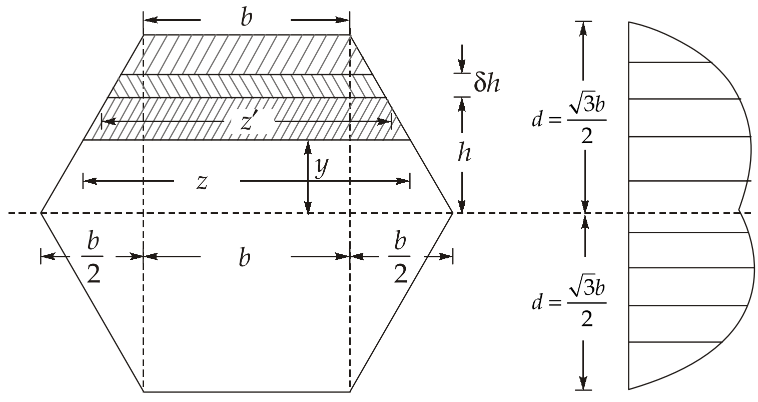
Given : F = 6 kN = 6000 N; b = 30 mm

Moment of inertia of the section, about the horizontal diagonal.

$$I = I \text{ of rectangle} + I \text{ of triangle}$$

$$= \left[\frac{b\left(\sqrt{3}b\right)^3}{12}\right] + \left[4\left(\frac{1}{12} \times \frac{b}{2} \times \left(\frac{\sqrt{3}b}{2}\right)^3\right)\right]$$

$$I = 0.5413\, b^4$$



(a) (b)

Now, let us consider a fiber of length z which as at a distance of y from the neutral axis.

As we know,

$$z = my + c \quad \text{...(i)}$$

at $y = 0$, $z = 2b$

We get, $c = 2b$ (from equation (i))

At $y = \frac{\sqrt{3}}{2}b,\ z = b$

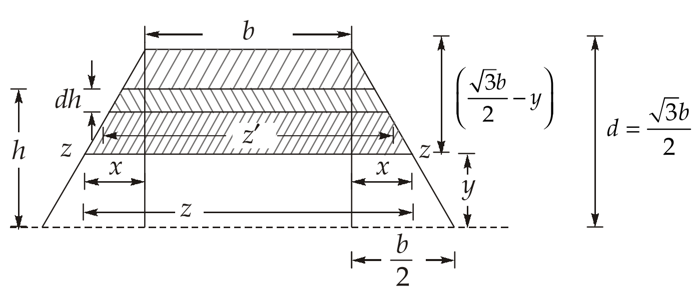
$$\Rightarrow \quad b = m \times \frac{\sqrt{3}b}{2} + 2b$$

$$\Rightarrow \quad m = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad z = -\frac{2}{\sqrt{3}}y + 2b$$

$$\Rightarrow \quad z = \frac{2}{\sqrt{3}}\left(\sqrt{3}b - y\right)$$

$$z = \frac{2}{\sqrt{3}}\left[\sqrt{3}b - y\right] \text{ and similarly } z' = \frac{2}{\sqrt{3}}\left(\sqrt{3}b - h\right)$$



Let consider a strip of thickness dh and length z' at a distance of h from the neutral axis.

$$A\bar{y} = \text{Moment of the shaded area about neutral axis}$$

$$A\bar{y} = \int_y^d \left[\frac{2}{\sqrt{3}}\left[\left(\sqrt{3}b - h\right)dh\right]\right]h = \frac{2}{\sqrt{3}}\int_y^d \sqrt{3}bh - h^2$$

$$A\bar{y} = \frac{2}{\sqrt{3}}\left[\frac{\sqrt{3}bh^2}{2} - \frac{h^3}{3}\right]_y^d$$

$$A\bar{y} = \frac{2}{\sqrt{3}}\left[\left(\frac{\sqrt{3}bd^2}{2} - \frac{d^3}{3}\right) - \left(\frac{\sqrt{3}by^2}{2} - \frac{y^3}{3}\right)\right]$$

On putting, $d = \dfrac{\sqrt{3}b}{2}$

$\Rightarrow$
$$A\bar{y} = \frac{2}{\sqrt{3}}\left[\left(\frac{\sqrt{3}}{2} \times \frac{3}{4}b^3 - \frac{3\sqrt{3}b^3}{24}\right) - \left(\frac{\sqrt{3}\,by^2}{2} - \frac{y^3}{3}\right)\right]$$

$$A\bar{y} = \frac{2}{\sqrt{3}}\left[\left(\frac{\sqrt{3}}{4}b^3\right) - \left(\frac{\sqrt{3}\,by^2}{2} - \frac{y^3}{3}\right)\right]$$

$\therefore$
$$\text{Shear stress} = \frac{FA\bar{y}}{Iz} = \frac{F \times \frac{2}{\sqrt{3}}\left[\frac{\sqrt{3}}{4}b^3 - \left(\frac{\sqrt{3}by^2}{2} - \frac{y^3}{3}\right)\right]}{0.5413b^4 \times \frac{2}{\sqrt{3}}\left(\sqrt{3}b - y\right)}$$

$$= \frac{F \times \left[\frac{\sqrt{3}}{4}b^3 - \left(\frac{\sqrt{3}by^2}{2} - \frac{y^3}{3}\right)\right]}{0.5413b^4 \times \left(\sqrt{3}b - y\right)}$$

We need shear stress at $y = 10$ mm,

Putting value of $F = 6000$ N, $b = 30$ mm and $y = 10$ mm in the above equation, we have

$$= \frac{6000\left[\frac{\sqrt{3}}{4}(30)^3 - \left(\frac{\sqrt{3} \times 30 \times 10^2}{2} - \frac{10^3}{3}\right)\right]}{0.5413(30)^4 \times \left(\sqrt{3} \times 30 - 10\right)}$$

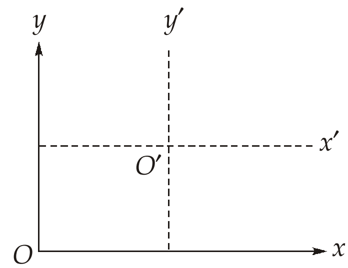
$$= 3.074 \text{ N/mm}^2$$

2. (b)

Given : $P_{\max} = 2000$ N, $N_{rpm} = 750$ rpm, $L_{10h} = 9000$ hr

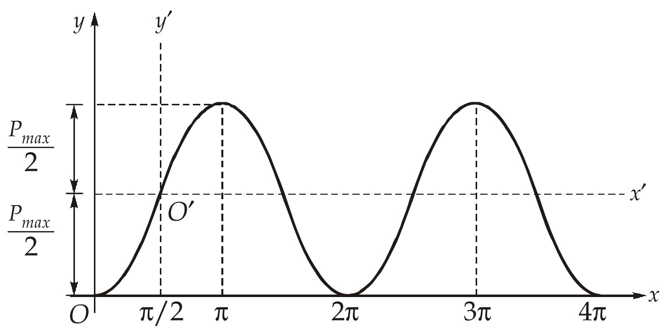
Equivalent load for complete work cycle:

In order to determining the equation of the sinusoidal curve as show in the figure, we use the basic relationship of plane analytical geometry.



The co-ordinate of origin O' w.r.t. origin O are (x_0, y_0). The relationships for transformation of co-ordinates with pure translation are as follows:

$$x' = x - x_0$$
$$y' = y - y_0$$



The equation of the above curve with respect to (x', y') co-ordinate system is given by

$$y' = \left(\frac{1}{2}P_{\max}\right)\sin x' \qquad \text{...(i)}$$

Also, $$x_0 = \frac{\pi}{2} \text{ and } y_0 = \frac{P_{\max}}{2}$$

∴ $$x' = x - \frac{\pi}{2} \text{ and } y' = y - \frac{P_{\max}}{2}$$

On putting the value of x' and y' in equation (i)

We get, $$\left(y - \frac{P_{\max}}{2}\right) = \frac{P_{\max}}{2}\sin\left(x - \frac{\pi}{2}\right)$$

$$\left(y - \frac{P_{max}}{2}\right) = -\left(\frac{P_{max}}{2}\right)\sin\left(\frac{\pi}{2} - x\right) \qquad [\text{As } \sin(-\theta) = -\sin\theta]$$

$$\left(y - \frac{P_{max}}{2}\right) = -\left(\frac{P_{max}}{2}\right)\cos x$$

$$y = \frac{P_{max}}{2} - \frac{P_{max}}{2}\cos x$$

$$y = \frac{P_{max}}{2}(1 - \cos x)$$

Replacing y by P and x by θ,

We get, $$P = \frac{P_{max}}{2}(1 - \cos\theta)$$

The equation for force P at an angle of rotation θ is given by $P = \frac{1}{2}P_{max}(1 - \cos\theta)$

then considering the work cycle from $\theta = 0$ to $\theta = 2\pi$

$$P_e = \left[\frac{1}{N}\int P^3 dN\right]^{1/3}$$

$$= \left[\frac{1}{2\pi}\int \frac{P_{max}^3}{8}(1 - \cos\theta)^3\, d\theta\right]^{1/3}$$

$$= \frac{P_{max}}{2}\left[\frac{1}{2\pi}\int_0^{2\pi}(1 - \cos\theta)^3\, d\theta\right]^{1/3}$$

Also, $$\int(1 - \cos\theta)^3 d\theta = \int(1 - 3\cos\theta + 3\cos^2\theta - \cos^3\theta)d\theta$$

$$\int(1 - \cos\theta)^3 d\theta = \int(1 - 3\cos\theta + \frac{3(1 + \cos 2\theta)}{2} - \cos\theta(1 - \sin^2\theta)d\theta$$

$$= \int(2.5 - 4\cos\theta + 1.5\cos 2\theta + \cos\theta\sin^2\theta)d\theta$$

$$\int(1 - \cos\theta)^3 d\theta = \left[2.5\theta - 4\sin\theta + 0.75\sin 2\theta + \frac{\sin^3\theta}{3}\right]$$

Since, $$\int_0^{2\pi}(1 - \cos\theta)^3\, d\theta = \left[2.5\theta - 4\sin\theta + 0.75\sin 2\theta + \frac{\sin^3\theta}{3}\right]_0^{2\pi}$$

$$= \left[2.5 \times 2\pi - 4\sin 2\pi - 0.75\sin 4\pi + \frac{\sin^3 2\pi}{3} - 2.5 \times 0 - 4\sin 0 + 0.75\sin 0° + \frac{\sin^3 0}{3}\right]$$

$$\int_0^{2\pi} (1-\cos\theta)^3 = 5\pi$$

$\therefore$
$$P_e = \frac{P_{max}}{2}\left[\frac{1}{2\pi}\times 5\pi\right]^{1/3}$$

$$P_e = \frac{P_{max}\times(2.5)^{1/3}}{2} = \frac{2000\times(2.5)^{1/3}}{2}$$

$$P_e = 1357.21 \text{ N}$$

As we know,
$$L_{10} = \frac{60\times N_{rpm}\times L_{10h}}{10^6}$$

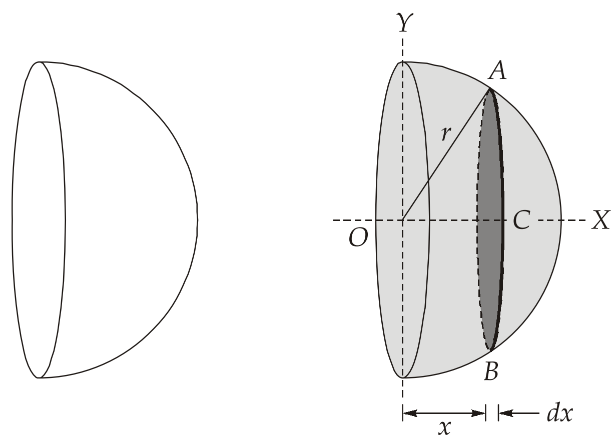
$$= \frac{60\times 750\times 9000}{10^6} = 405 \text{ million revolution}$$

$\therefore$
$$C = P_e(L_{10})^{1/3} = 1357.21 \times (405)^{1/3}$$

$$C = 10041.5 \text{ N}$$

2. (c)

Consider a hemisphere of radius r oriented as shown such that the YZ plane is the bounding plane. Consider a circular disc ACB at a distance x from the bounding plane and of infinitesimal thickness dx.



Its radius is given by,
$$AC = \sqrt{(OA)^2-(OC)^2} = \sqrt{r^2-x^2}$$

Therefore, volume of the disc is

$$dV = \pi\left(\sqrt{r^2-x^2}\right)^2 dx = \pi\left(r^2-x^2\right)dx$$

Hence, its weight is $dW = (dm)g$

$$= d(\rho V)g$$

Since over the disc, the density can be assumed to be constant,

$$dW = (\rho)(dV)g$$

Since the density varies as the distance from the bounding plane,

$$\rho = kx \qquad \text{(where } k \text{ is a constant)}$$

Hence, we can write $dW = [kx]\pi(r^2 - x^2)g\,dx$

Hence, the total weight of the hemisphere is obtained by integrating the above expression between limits:

$$W = \int_0^r dW = \int_0^r kx\pi\left(r^2 - x^2\right)g\,dx$$

$$= k\pi g\int_0^r \left(r^2 x - x^3\right)dx$$

$$= k\pi g\left[\frac{r^2x^2}{2} - \frac{x^4}{4}\right]_0^r = k\pi g\left[\frac{r^4}{4}\right]$$

The first moment of the circular disc ACB about the bounding plane is given as

$$x dW = k\pi g\left(r^2x^2 - x^4\right)dx$$

Hence, the first moment of the entire hemisphere about the bounding plane is obtained by integrating the above expression between limits,

$$\int_0^r x dW = \int_0^r k\pi g\left(r^2x^2 - x^4\right)dx$$

$$= k\pi g\left[\frac{r^2x^3}{3} - \frac{x^5}{5}\right]_0^r = k\pi g\left[\frac{2r^5}{15}\right]$$

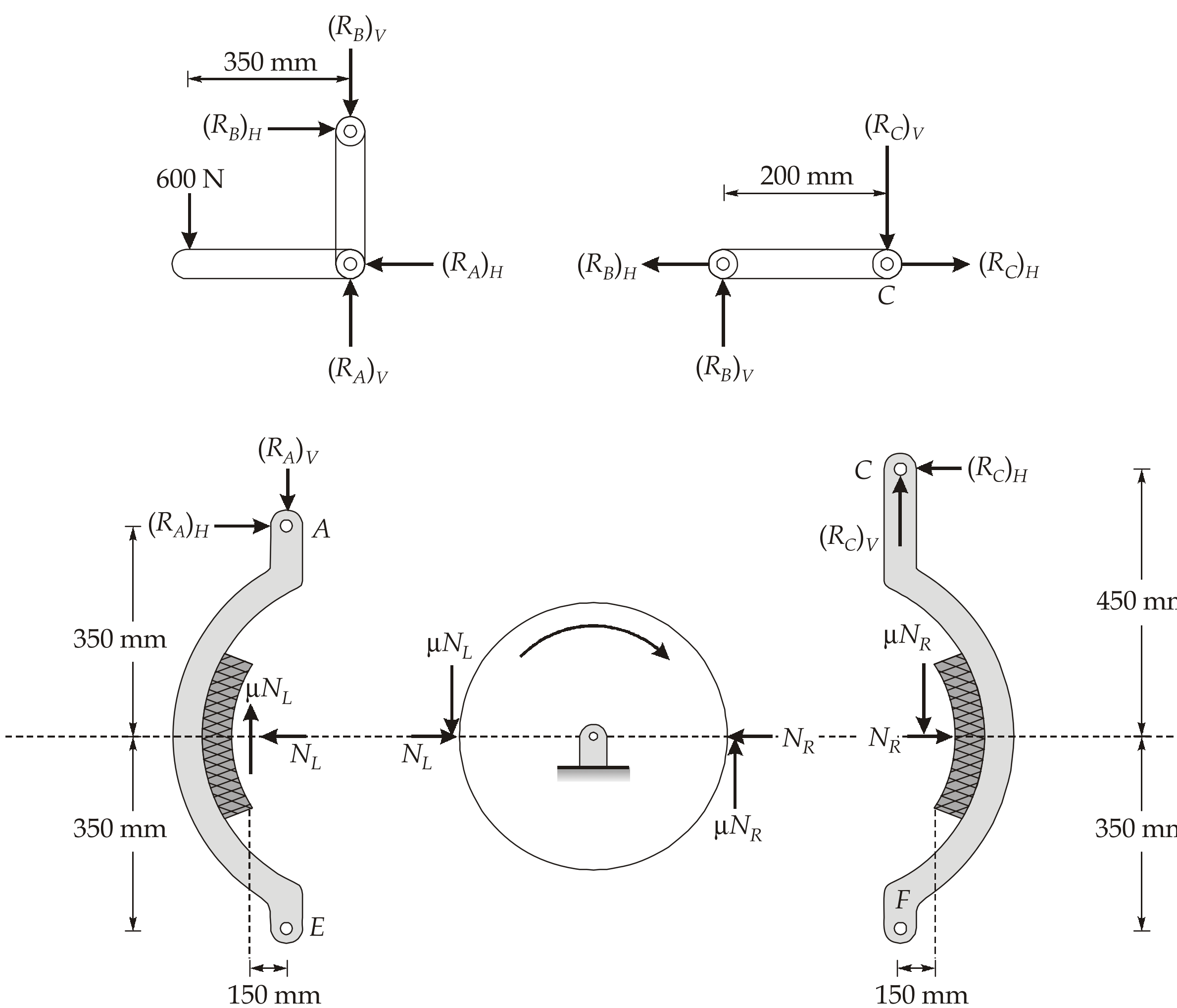
Therefore, the centre of gravity of the hemisphere is obtained as

$$\bar{x} = \frac{\int_0^r x\,dW}{\int_0^r dW} = \frac{k\pi g\left[\frac{2r^5}{15}\right]}{k\pi g\left[\frac{r^4}{4}\right]} = \frac{8}{15}r$$

3. (a)

Given : P = 600 N; μ = 0.30; R = 250 mm = 0.25 m; p = 1.2 N/mm^2; l = 2 W

The free body diagram of forces acting on various parts as shown in the figure below.



Considering the forces acting on the link DAB and taking moments about the pin A,

$$(R_B)_H \times 100 = 600 \times 350$$

$$(R_B)_H = 2100 \text{ N}$$

Considering equilibrium of horizontal and vertical forces on link DAB,

We get, $$(R_A)_H = (R_B)_H = 2100 \text{ N}$$

and $$(R_A)_V = 600 + (R_B)_V \quad \text{...(i)}$$

Again, considering forces acting on link BC,

$$(R_C)_H = (R_B)_H = 2100 \text{ N}$$

Taking moments about pin C,

$$(R_B)_V \times 200 = 0$$

$\therefore$ $$(R_B)_V = 0$$

Hence, $$(R_C)_V = (R_B)_V = 0$$

On putting the value of $(R_B)_V$ in equation (i)

$$(R_A)_V = 600 \text{ N}$$

Considering the free body diagram of forces acting on the right side link CF and taking moment of forces about pin F,

$$(R_C)_H (350 + 450) - N_R (350) - \mu N_R (150) = 0$$

$$2100 \times (350 + 450) - N_R (350) - 0.3 \times N_R (150) = 0$$

$$N_R = 4253.16 \text{ N}$$

Similarly, taking moments of forces about pin E of the left side link AE,

$$(R_A)_H (350 + 350) + \mu N_L (150) - N_L (350) = 0$$

$$2100 \times (700) + 0.3 \times N_L \times 150 - N_L \times 350 = 0$$

$$N_L = 4819.67 \text{ N}$$

(i) Torque absorbing capacity of brake is given by

$$T = \mu(N_R + N_L)R$$

$$= 0.3(4253.16 + 4819.67) \times 0.25$$

$$= 680.46 \text{ Nm}$$

(ii) Dimension of block

As $$N_L > N_R$$

$\therefore$ $$N_L = p \times l \times w$$

$$4819.67 = 1.2 \times 2w \times w$$

$$w^2 = 2008.196$$

$$w = 44.813 \text{ mm}$$

and $$l = 2w = 2 \times 44.813$$

$$= 89.63 \text{ mm}$$

3. (b)

The free-body diagram of the stick is shown below. Since the externally applied force does not pass through the point of suspension A, it alone contributes to the externally applied moment tending to rotate the stick about A.

(i) The angular acceleration of the stick

Applying the kinetic equation for rotational motion, we have

$$M_A = I_A\alpha$$

We know that the moment of the applied force about A is $M_A = 5 \times 0.7 = 3.5$ N and the mass moment of inertia of the stick about A is $I_A = \frac{ml^2}{3}$. Therefore, we can write

$$3.5 = \frac{0.5 \times (1)^2}{3}\alpha$$

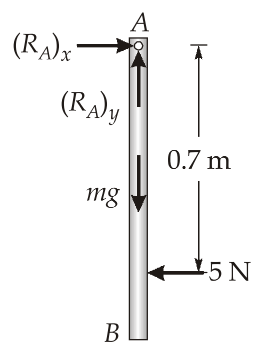
$$\Rightarrow \qquad \alpha = 21 \text{ rad/s}^2$$

Therefore, the tangential acceleration of the centre of mass is given as

$$(a_{cm})_x = r_{cm}\alpha = (0.5)(21) = 10.5 \text{ m/s}^2$$

(ii) The components of reaction at the hinge at A.

Applying the kinetic equations for translational motion, assuming all the forces are acting at the centre of mass, we have



$$\Sigma F_x = m(a_{cm})_x$$

$$\Rightarrow \qquad 5 - (R_A)_x = m(a_{cm})_x$$

$$5 - (R_A)_x = 0.5 \times 10.5$$

$$\Rightarrow \qquad (R_A)_x = -0.25 \text{ N}$$

$$\Sigma F_y = m(a_{cm})_y$$

$$\Rightarrow \qquad (R_A)_y - mg = 0$$

$$(R_A)_y = 0.5 \times 9.81 = 4.905 \text{ N}$$

(iii) The point of application of the horizontal force at which the horizontal component of the reaction at A is zero is determined by equation $(R_A)_x$ to zero. Then the horizontal component of acceleration of centre of mass is given as

$$5 - 0 = 0.5(a_{cm})_x$$

$$\Rightarrow \qquad (a_{cm})_x = 10 \text{ m/s}^2$$

Hence, the angular acceleration of the stick is given as

$$\alpha = \frac{(a_{cm})_x}{r_{cm}} = 20 \text{ rad/s}^2$$

Therefore, applying the kinetic equation of rotation, we have

$$M_A = I_A \alpha$$

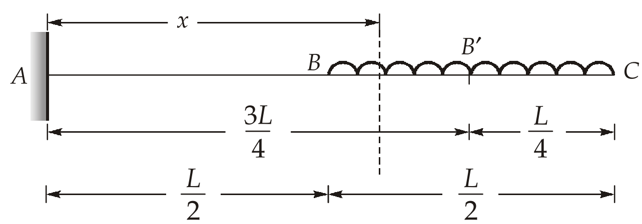
$$5 \times h = \frac{0.5 \times 1^2}{3} \times 20$$

$$h = 0.667 \text{ m}$$

3. (c)

First we need to find the reactions at A.

Taking moment at A,



$\Sigma M_A = 0$

$$M_A - \frac{wL^2}{3} - w\left(\frac{L}{2}\right)\left(\frac{3L}{4}\right) = 0$$

$$M_A = \frac{17wL^2}{24}$$

Summation of vertical forces,

$$\Sigma F_y = 0$$

$$R_A - \frac{wL}{2} = 0$$

$$R_A = \frac{wL}{2}$$

We may write the bending moment from left end for section in BC as

$$\text{BM} = \frac{wL}{2}x - \frac{17wL^2}{24} + \frac{wL^2}{3}\left(x - \frac{L}{2}\right)^0 - w\left(x - \frac{L}{2}\right)^1 \cdot \frac{\left(x - \frac{L}{2}\right)^1}{2}$$

$$EI\frac{d^2y}{dx^2} = \frac{wL}{2}x - \frac{17wL^2}{24} + \frac{wL^2}{3}\left(x - \frac{L}{2}\right)^0 - \frac{w}{2}\left(x - \frac{L}{2}\right)^2 \quad \text{...(i)}$$

Integrating the above equation, we have

$$EI\frac{dy}{dx} = \frac{wLx^2}{4} - \frac{17wL^2}{24}x + \frac{wL^2}{3}\left(x-\frac{L}{2}\right)^1 - \frac{w}{2}\frac{\left(x-\frac{L}{2}\right)^3}{3} + C_1 \quad \text{...(ii)}$$

The first boundary condition is

when, $x = 0, \frac{dy}{dx} = 0$

which gives, $C_1 = 0$

Putting value of $C_1 = 0$ in equation (ii) and integrating

$$EIy = \frac{wL}{4}\frac{x^3}{3} - \frac{17wL^2}{24}\frac{x^2}{2} + \frac{wL^2}{3}\frac{\left(x-\frac{L}{2}\right)^2}{2} - \frac{w}{6}\frac{\left(x-\frac{L}{2}\right)^4}{4} + C_2$$

The second boundary condition, when $x = 0$, $y = 0$

So, $C_2 = 0$

$$y = \frac{wLx^3}{12EI} - \frac{17wL^2x^2}{48EI} + \frac{wL^2}{6EI}\left(x-\frac{L}{2}\right)^2 - \frac{w}{24}\left(x-\frac{L}{2}\right)^4 \quad \text{...(iii)}$$

The maximum deflection will be at the tip of the bar i.e. at $x = L$

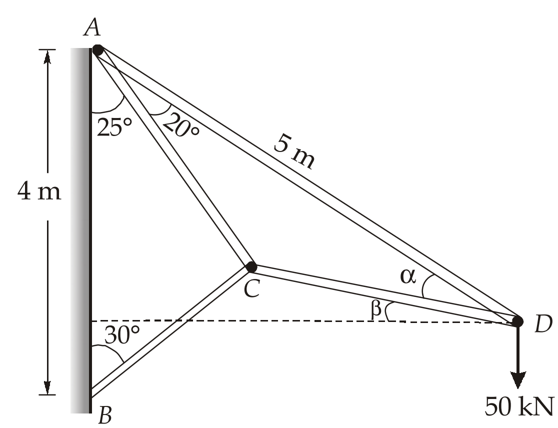
$$y_{max} = \frac{wL^4}{12EI} - \frac{17wL^4}{48EI} + \frac{wL^2}{6EI}\left(\frac{L}{2}\right)^2 - \frac{w}{24}\left(\frac{L}{2}\right)^4$$

$$= \frac{wL^4}{12EI} - \frac{17wL^4}{48EI} + \frac{wL^2}{24EI} - \frac{wL^4}{384EI}$$

$$y_{max} = -\frac{89}{384}\frac{wL^4}{EI}$$

Ans.

4. (a)



In ΔABC,

By sine rule, we have

$$\frac{AB}{\sin[180-(30+25)]} = \frac{AC}{\sin 30°}$$

$$AC = \frac{4 \times \sin 30°}{\sin 125°}$$

$$AC = 2.44 \text{ m}$$

In ΔACD,

By cosine rule, we have

$$CD = \sqrt{(AC)^2 + (AD)^2 - 2(AC)(AD)\cos 20°}$$

$$CD = \sqrt{(2.44)^2 + (5)^2 - 2 \times 2.44 \times 5 \times \cos 20°}$$

$$CD = 2.83 \text{ m}$$

In ΔACD, by sine rule we have

$$\frac{CD}{\sin 20°} = \frac{AC}{\sin\alpha}$$

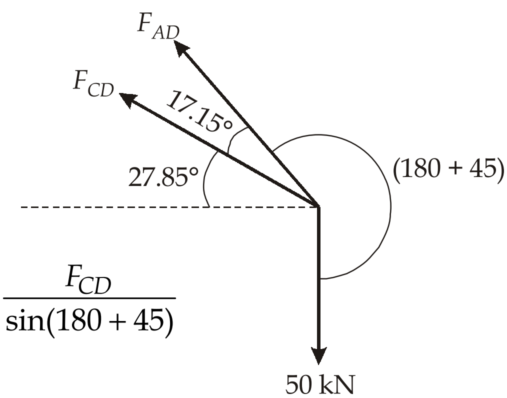
$$\frac{2.83}{\sin 20°} = \frac{2.44}{\sin\alpha}$$

$$\sin\alpha = \frac{2.44}{2.83} \times \sin 20°$$

$$\alpha = 17.15°$$

By geometry $\alpha + \beta = 45°$

$\therefore \quad \beta = 27.85°$



Considering the F.B.D. of joint D

By Lami's theorem

$$\frac{50}{\sin 17.15} = \frac{F_{AD}}{\sin(90 + 27.85)} = \frac{F_{CD}}{\sin(180 + 45)}$$

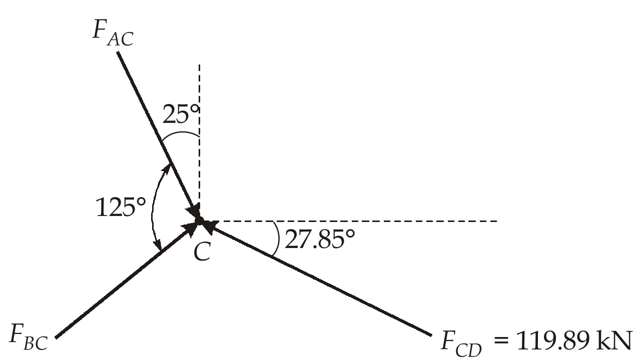
$$\therefore \quad F_{AD} = \frac{50 \times \sin 117.85}{\sin 17.15}$$

$$F_{AD} = 149.92 \text{ kN (T)}$$

$$F_{CD} = \frac{50}{\sin 17.15} \times \sin 225 = -119.89 \text{ kN}$$

or $\quad F_{CD} = 119.89 \text{ kN (C)}$

Consider the F.B.D. of joint C



By Lami's theorem,

$$\frac{F_{CD}}{\sin 125^\circ} = \frac{F_{BC}}{\sin(25+90+27.85)^\circ}$$

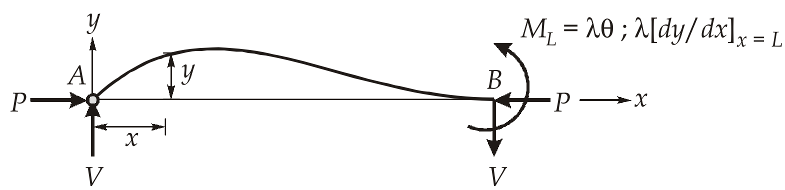
$$F_{BC} = \frac{119.89 \times \sin(142.85)}{\sin 125^\circ}$$

$$F_{BC} = 88.39 \text{ (C)}$$

Hence, the force in the member AD and BC are 149.92 kN (T) and 88.39 kN (C) respectively.

4. (b)

The buckled bar is shown in the figure below, where M_L represents the restoring moment.



Let the vertical reactions at A and B be V.

Moment in the bar, $M = Vx - Py$

and $$EI\frac{d^2y}{dx^2} = M$$

So, $$EI\frac{d^2y}{dx^2} = Vx - Py$$

The difference equation of buckling bar will be

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{V}{EI}x$$

Let $$\frac{P}{EI} = k^2$$

$$\frac{d^2y}{dx^2} + k^2y = \frac{V}{EI}x$$

The above non-homogeneous second order linear differential equation is to be solved first to obtain the solution.

Complementary solution,

$$D^2 + k^2 = 0$$

$$D = \pm ik$$

$$y_c = C_1 \cos kx + C_2 \sin kx$$

and particular solution:

$$PI = \frac{1}{D^2 + k^2} \times \frac{V}{EI} x$$

$$= \frac{1}{k^2}\left[\frac{1}{\left(\frac{D^2}{k^2}+1\right)}\right]\frac{Vx}{EI} = \frac{1}{k^2}\left(1+\frac{D^2}{k^2}\right)^{-1}\frac{Vx}{EI}$$

From Binomial equation of $\left(1+\frac{D^2}{k^2}\right)^{-1}$

$$= \frac{1}{k^2}\left[1-\frac{D^2}{k^2}+\frac{D^4}{k^4}----\right]\frac{Vx}{EI}$$

$$P.I. = \frac{1}{k^2}\left[\frac{Vx}{EI}\right]$$

Putting value of $k^2 = \frac{P}{EI}$

$$P.I. = \frac{V}{P} x$$

Complete solution of the differential equation,

$$y = y_c + PI$$

$$y = C_1 \cos kx + C_2 \sin kx + \frac{V}{P} x \quad \text{...(i)}$$

As the first boundary condition, when $x = 0$, $y = 0$; hence $C_1 = 0$
As the second boundary condition when $x = L$, $y = 0$
From equation (i), we obtain

$$0 = C_2 \sin kL + \frac{VL}{P}$$

$$\frac{V}{P} = -\frac{C_2}{L}\sin kL$$

Thus,
$$y = C_2\left[\sin kx - \frac{x}{L}\sin kL\right]$$

Differentiating the above equation we will get the slope.

$$\theta = \frac{dy}{dx} = C_2\left[k\cos kx - \frac{1}{L}\sin kL\right]$$

The restoring moment at the end B is

$$M_L = \lambda\theta_L$$

$$M_L = \lambda C_2\left[k\cos kL - \frac{1}{L}\sin kL\right] \quad \text{(restoring)} \quad \text{...(ii)}$$

Also, since in general equation $M = EI\dfrac{d^2y}{dx^2}$

$$M_L = EI\left(\frac{d^2y}{dx^2}\right)_{x=L}$$

$$= EI\left(-C_2k^2\sin kL\right) \quad \text{...(iii)}$$

From equation (ii) and (iii), we have

$$-\lambda C_2\left[k\cos kL - \frac{1}{L}\sin kL\right] = -EIC_2k^2\sin kL$$

[Negative because the restoring moment is ACW]

$$\frac{\lambda k\cos kL}{\sin kL} - \frac{\lambda}{L}\frac{\sin kL}{\sin kL} = EI\times\frac{P}{EI}$$

$$P = \lambda k\cot kL - \frac{\lambda}{L} \quad \textbf{Ans.}$$

4. (c)

Given : $P = \pm 1100$ N; $S_{ut} = 540$ N/mm^2; $q = 0.85$; R = 90%; $N = 11000$ cycles; $k_a = 0.78$; $k_b = 0.85$; $k_c = 0.897$

The failure will occur either at section A or at section B. At section A, although the bending moment is maximum there is no stress concentration and the diameter is also more compared with that of section B. It is therefore assumed that the failure will occur at section B.

$$S_e' = 0.5\times S_{ut} = 0.5\times 540 = 270 \text{ N/mm}^2$$

At section B,

$$\left(\frac{D}{d}\right) = 1.2 \text{ and } \left(\frac{r}{d}\right) = 0.20$$

∴ From stress concentration chart,

Theoretical stress concentration factor, $k_t = 1.4$

As we know, $k_f = 1 + q(k_t - 1)$

$$= 1 + 0.85(1.4 - 1)$$

$$= 1.34$$

Modifying factor to account for stress concentration $(k_d) = \dfrac{1}{k_f}$

$$k_d = \frac{1}{1.34} = 0.746$$

$$S_e = k_a \times k_b \times k_c \times k_d \times S_e'$$

$$= 0.78 \times 0.85 \times 0.897 \times 0.746 \times 270$$

$$S_e = 119.786 \text{ N/mm}^2$$

Also,

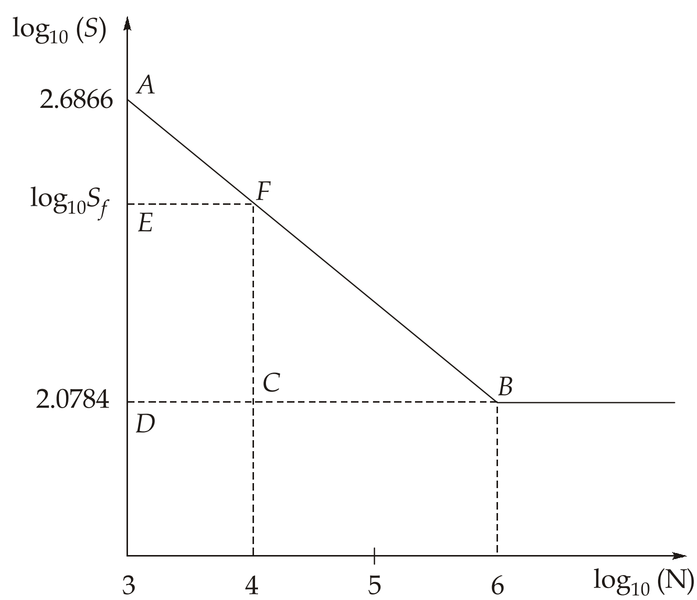
$$0.9S_{ut} = 0.9 \times 540 = 486 \text{ N/mm}^2$$

$$\log_{10}(0.9S_{ut}) = \log_{10}(486) = 2.6866$$

$$\log_{10}(S_e) = \log_{10}(119.786) = 2.0784$$

$$\log_{10}(11000) = 4.0414$$

The S-N curve for the problem is shown below



From similar triangles,

$$\frac{AE}{AD} = \frac{EF}{DB}$$

From figure,

$$AE = \frac{AD \times EF}{DB}$$

$$AE = \frac{(2.6866 - 2.0784) \times (4.0414 - 3)}{(6 - 3)} = 0.211$$

$$\therefore \qquad \log_{10}S_f = 2.6866 - AE = 2.6866 - 0.211$$

$$\log_{10}S_f = 2.4756$$

$$\log S_f = 298.951 \text{ N/mm}^2$$

$$S_f = \sigma_b = \frac{32M_b}{\pi d^3}$$

$$d^3 = \frac{32M_b}{\pi S_f} = \frac{32\times(1100\times200)}{\pi\times298.951} = 7495.88$$

$$d = 19.57 \text{ mm}$$

Section B : Strength of Materials + Machine Design + Engineering Mechanics

5. (a)

Given : $P = 20$ kN $= 20 \times 10^3$ N; $d_c = 16.5$ mm

$$\text{Area of core, } A_c = \frac{\pi}{4}\times(d_c)^2 = \frac{\pi}{4}\times(16.5)^2 = 213.82 \text{ mm}^2$$

$$\text{Stress in screw portion} = \frac{20\times1000}{213.82} = 93.54 \text{ N/mm}^2$$

$$\text{Stress in the shank} = \frac{20\times1000}{\frac{\pi}{4}\times(20)^2} = 63.66 \text{ N/mm}^2$$

$$\text{Total strain energy} = \frac{\Sigma\sigma^2}{2E}\times V$$

$$= \frac{1}{2\times2\times10^5}\left[(63.66)^2\times314.16\times50+(93.54)^2\times213.82\times20\right]$$

$$= 252.69 \text{ N-mm}$$

If now the shank is turned down to 16.5 mm diameter, the stress in the bolt will be 93.54 N/mm^2 throughout.

$$\therefore \qquad U = \frac{\sigma^2}{2E}\times V = \frac{(93.54)^2}{2\times2\times10^5}\times213.82\times70$$

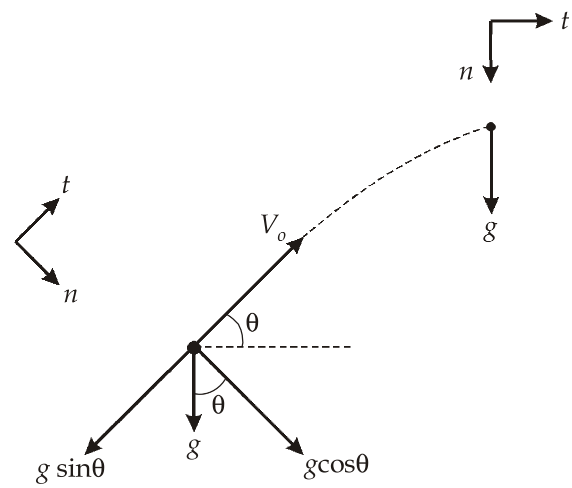
$$U = 327.40 \text{ N-mm}$$

$$\%\text{ increase in strain energy} = \frac{327.40-252.69}{252.69}\times100 = 29.57\%$$

Thus, the strain energy is increased by 29.57% by turning down the shank of the bolt to the root diameter of the thread.

5. (b)

Given: V_o = 75 m/s, θ = 15°



Let a_n and a_t be the normal and tangential components of acceleration.

∴ Just after launch,

$$a_n = g\cos\theta$$

$$a_n = 9.81\cos 15° = 9.476 \text{ m/s}^2$$

Also, $$a_n = \frac{V_o^2}{\rho};$$ where, ρ is radius of curvature

$$\Rightarrow \quad 9.476 = \frac{(75)^2}{\rho}$$

$$\Rightarrow \quad \rho = 593.6 \text{ m}$$

Now, time rate of change of speed is given by tangential acceleration,

i.e. $$a_t = \frac{dV}{dt} = -g\sin\theta$$

$$a_t = -9.81\sin 15° = -2.539 \text{ m/s}^2$$

At apex:

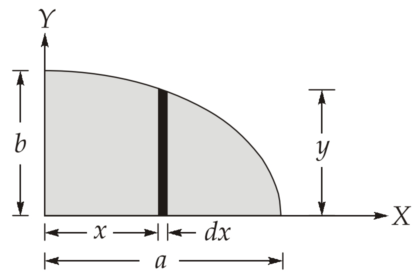
$$a_n = g$$

$$\Rightarrow \quad \frac{(V_o\cos 15)^2}{\rho} = g$$

$$\Rightarrow \quad \rho = \frac{(V_o\cos 15)^2}{g} = \frac{(75\cos 15)^2}{9.81} = 534.98 \text{ m}$$

Since tangential component of acceleration is zero at apex, time rate of change of speed is zero.

5. (c)



From the equation of the ellipse, we get the expression for y as

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

Consider a thin strip of infinitesimal thickness dx parallel to the Y-axis at a distance x from the origin. Its area dA is then given as

$$dA = ydx = \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

Hence, the area of the quadrant of the ellipse is obtained by integrating the infinitesimal area between the limits:

$$A = \int_0^a dA = \int_0^a ydx$$

$$= \int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

Upon integration by parts, we get

$$= \frac{b}{a}\left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a = \frac{\pi ab}{4}$$

Taking the first moment of the infinitesimal area about the Y-axis, we have

$$dM_y = xdA = x\frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

Hence, the first moment of the entire area about the Y-axis is obtained as

$$M_y = \int_0^a xdA = \frac{b}{a}\int_0^a x\sqrt{a^2 - x^2}\,dx$$

Let, $a^2 - x^2 = t$

$dt = -2xdx$

$$= \frac{b}{-2a}\int \sqrt{t}\,dt$$

$$= \frac{-b}{2a}\times\frac{1}{3/2}\left[\left(a^2 - x^2\right)^{3/2}\right]_0^a$$

$$= -\frac{b}{a}\left[\frac{\left(a^2 - x^2\right)^{3/2}}{3}\right]_0^a = \frac{a^2 b}{3}$$

Therefore, the x-coordinate of the centroid of the quadrant of the ellipse is given as

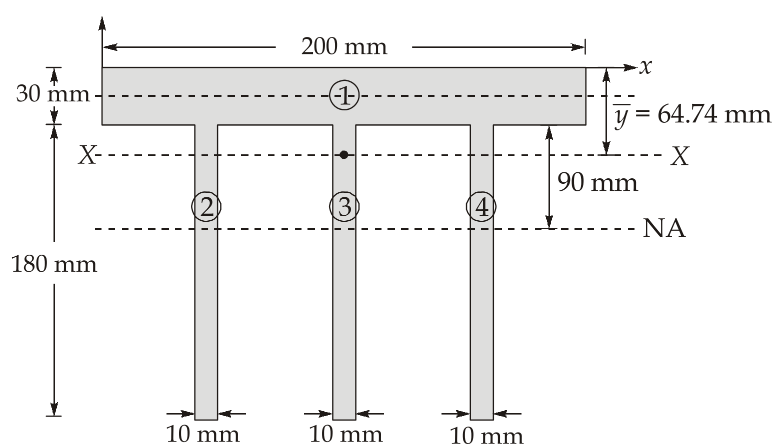
$$\bar{x} = \frac{M_y}{A} = \frac{\frac{a^2 b}{3}}{\frac{\pi ab}{4}} = \frac{4a}{3\pi}$$

In a similar manner, we can consider a thin horizontal strip and obtain the y-coordinate of the centroid is

$$\bar{y} = \frac{4b}{3\pi}$$

5. (d)

First we need to locate the centroid of the cross-section.



$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4}{A_1 + A_2 + A_3 + A_4}$$

Since 2, 3, 4 are identical.

$\therefore \quad A_2 = A_3 = A_4$

$$\bar{y} = \frac{A_1 y_1 + 3 \times A_4 y_4}{A_1 + 3A_4}$$

$$= \frac{(30 \times 200)(15) + 3 \times (180 \times 10)(90 + 30)}{(30 \times 200) + 3 \times (180 \times 10)}$$

$$\bar{y} = 64.74 \text{ mm}$$

The moment of inertia about X-X axis will be

$$I_{XX} = (\text{MOI of section (1)}) + (\text{MOI of section } (2 + 3 + 4))$$

$$= \left[\frac{1}{12} \times 200 \times 30^3 + (200 \times 30)(64.74 - 15)^2\right] + 3\left[\frac{1}{12} \times 10 \times 180^3 + (180 \times 10)(120 - 64.74)^2\right]$$

$$I_{XX} = 46.364 \times 10^6 \text{ mm}^4$$

The maximum bending moment occurs at the supporting wall and is

$$M_{max} = \frac{wL^2}{2}$$

We know the bending stress, $\sigma_b = \dfrac{M}{I} y$

$$(\sigma_b)_{max} = \frac{M_{max} y_{max}}{I}$$

y_{max} is the distance of most distant fiber from the neutral axis.

$$y_{max} = (180 + 30) - (64.74)$$

$$y_{max} = 145.26 \text{ mm}$$

$$(\sigma_b)_{max} = \frac{M_{max} y_{max}}{I}$$

$$90 \times 10^6 = \frac{\left(\dfrac{w \times 3^2}{2}\right)\left(\dfrac{145.26}{1000}\right)}{46.364 \times 10^6 \times 10^{-12}}$$

$$w = 6383.59 \text{ N/m}$$

5. (e)

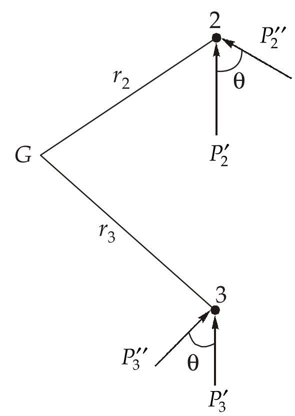
Given: $P = 80$ kN, $e = 400$ mm, $\tau = 100$ N/mm^2

Primary shear force, $P_2' = P_3' = \dfrac{80 \times 10^3}{5} = 16000 \text{ N}$

As, $r_5 = 0$

$\therefore \quad r_1 = r_2 = r_3 = r_4 = \sqrt{100^2 + 100^2} = 141.42 \text{ mm}$

The primary and secondary shear forces acting on rivets 2 and 3 are



$$\tan\theta = \frac{100}{100}$$

$$\theta = 45°$$

$$C = \frac{Pe}{(r_1^2 + r_2^2 + ...r_5^2)} = \frac{80 \times 10^3 \times (400)}{(4 \times 141.42^2 + 0)} = 400$$

$$P_3'' = Cr_3 = 400 \times 141.42 = 56569.085 \text{ N}$$

$$P_2'' = Cr_2 = 400 \times 141.42 = 56569.085 \text{ N}$$

The resultant P_2 is

$$P_2 = \sqrt{(P_2''\sin\theta)^2 + (P_2''\cos\theta + P_2')^2}$$

$$P_2 = \sqrt{(56569.085 \sin 45°)^2 + (56569.085 \sin 45 + 16000)^2}$$

$$P_2 = P_3 = 68819.13 \text{ N}$$

Equating the maximum shear force to the shear strength of the rivet,

$$P_3 = \frac{\pi}{4} d^2 \tau$$

$$68819.13 = \frac{\pi}{4} d^2 \times (100)$$

$$d = 29.6 \text{ mm}$$

6. (a)

Given : $m_1 = 60$ kg; $m_2 = 130$ kg; $V_1 = 5$ m/s

(i) Cart initially at rest

The components of velocity of the man in the horizontal and vertical direction are respectively 5 cos 35° and 5 sin 35°. Applying the principle of conservation of momentum along the horizontal direction, we have

$$m_1 v_{1x} + m_2 v_{2x} = (m_1 + m_2) v_x$$

$$60 \times 5\cos 35° + 130 \times 0 = (60 + 130) v_x$$

$$v_x = 1.293 \text{ m/s}$$

Initial kinetic energy of the system:

$$(\text{K.E.})_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} \times 60 \times 5^2 + \frac{1}{2} \times 130 \times 0^2 = 750 \text{ J}$$

Final kinetic energy of the system

$$(\text{K.E.})_f = \frac{1}{2}(m_1 + m_2) v^2$$

$$= \frac{1}{2}(60 + 130) \times (1.293)^2 = 158.83 \text{ J}$$

Therefore, loss in kinetic energy of the system = 750 – 158.83 = 591.17 J

(ii) Cart pushed with a velocity of 1 m/s away from the bridge: Applying the principle of conservation of momentum along the horizontal direction, we have

$$60 \times 5\cos 35° + 130 \times 1 = (60 + 130) v$$

$$v = 1.978 \text{ m/s}$$

Initial kinetic energy of the system,

$$(\text{K.E.})_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} \times 60 \times 5^2 + \frac{1}{2} \times 130 \times 1^2 = 815 \text{ J}$$

Final kinetic energy of the system = $\frac{1}{2}(m_1 + m_2) v^2$

$$= \frac{1}{2}(60 + 130) \times (1.978)^2 = 371.68 \text{ J}$$

Therefore loss in kinetic energy of the system = 815 – 371.68 = 443.32 J

(iii) Cart pushed with a velocity of 1 m/s towards the bridge. Applying the principle of conservation of momentum along the horizontal direction, we have

$$60 \times 5 \cos 35 - 130 \times 1 = (60 + 130) \times v$$

$$v = 0.61 \text{ m/s}$$

Initial kinetic energy of the system:

$$(KE)_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = 815 \text{ J}$$

Find kinetic energy of the system

$$(KE)_f = \frac{1}{2}(m_1 + m_2)v^2$$

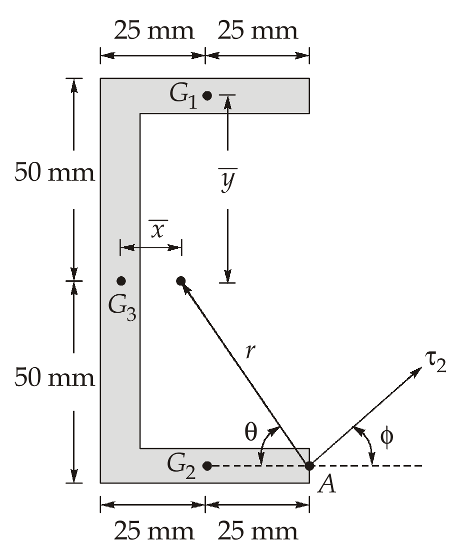
$$= \frac{1}{2}(60 + 130) \times (0.61)^2 = 35.35 \text{ J}$$

Therefore loss in kinetic energy of the system = 815 – 35.35 = 779.65 J

6. (b)

Given : P = 80 kN; τ = 120 N/mm^2

Primary shear stress,



There are two horizontal welds w_1 and w_2 and one vertical weld w_3, by symmetry, the centre of gravity G of three weld is midway between two horizontal welds,

i.e. $\bar{y}$ = 50 mm

Taking moment of three welds about the vertical line passing through G_3.

$$(50 + 100 + 50)\bar{x} = 50 \times 25 + 50 \times 25 + 100 \times 0$$

$$\bar{x} = 12.5 \text{ mm}$$

$$\text{Area of welds, } A_1 = (50t) \text{ mm}^2$$

$$A_2 = (50t) \text{ mm}^2$$

$$A_3 = (100t) \text{ mm}^2$$

$$A = A_1 + A_2 + A_3 = 200\,t \text{ mm}^2$$

Primary shear stress in the used is

$$\tau = \frac{P}{A} = \frac{80 \times 1000}{200t} = \frac{400}{t} \text{ N/mm}^2$$

Secondary shear stress

A is the farthest point from the centre of gravity G and its distance r is given by,

$$r = \sqrt{(50)^2 + (50 - 12.5)^2} = 62.5 \text{ mm}$$

Also,
$$\tan\theta = \frac{50}{(50 - 12.5)}$$

$$\Rightarrow \quad \theta = 53.13°$$

$$\phi = 90 - \theta = 36.87°$$

The secondary shear stress is inclined at 36.87° with horizontal.

$$e = (50 - \bar{x}) + 150 = (50 - 12.5) + 150 = 187.5 \text{ mm}$$

$$M = P \times e = (80 \times 10^3) \times 187.5$$

$$= 15000 \times 10^3 \text{ Nmm}$$

G_1, G_2 and G_3 are the centres of gravity of three welds and their distances from common centre of gravity G are as follows.

$$\overline{G_1G} = \overline{G_2G} = \sqrt{(25 - 12.5)^2 + (50)^2} = 51.54 \text{ mm}$$

$$r_1 = r_2 = 51.54 \text{ mm}$$

$$r_3 = \overline{G_3G} = \bar{x} = 12.5 \text{ mm}$$

$$J_1 = J_2 = A_1\left[\frac{l_1^2}{12} + r_1^2\right]$$

$$= (50t)\left[\frac{(50)^2}{12} + (51.54)^2\right] = (143229.17t) \text{ mm}^4$$

$$J_3 = A_3\left[\frac{l_3^2}{12} + r_3^2\right] = (100t)\left[\frac{100^2}{12} + (12.5)^2\right]$$

$$= (98958.33t)\ \text{mm}^4$$

$$J = 2J_1 + J_3$$

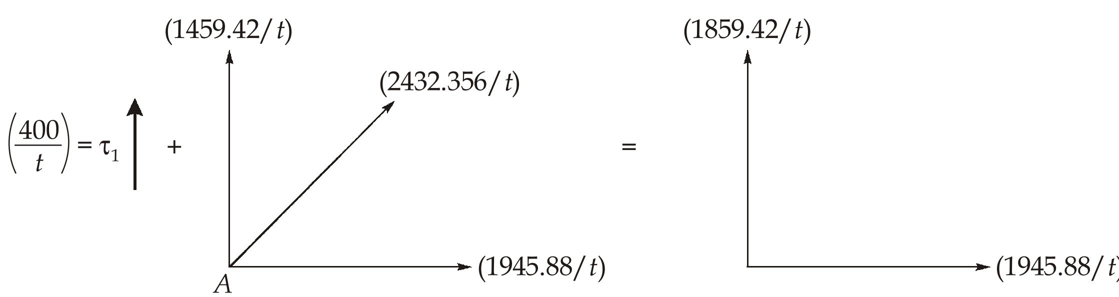
$$= 2(143229.17t) + (98958.33t) = 385416.66t\ \text{mm}^4$$

The secondary shear stress at point A is given by

$$\tau_2 = \frac{M}{J} r = \frac{(15000 \times 10^3) \times 62.5}{385428.83t}$$

$$= \frac{2432.356}{t} \text{N/mm}^2$$

Resultant shear stress,



$$\text{The vertical component} = \tau_2 \sin\phi + 400/t = \frac{1859.42}{t}$$

$$\text{Horizontal component} = \frac{2432.356}{t}\cos\phi = \frac{1945.88}{t}$$

$$\text{The resultant shear stress, } \tau = \sqrt{\left(\frac{1859.42}{t}\right)^2 + \left(\frac{1945.88}{t}\right)^2}$$

$$\tau = \frac{2691.45}{t} \text{N/mm}^2$$

Size of weld,

The permissible shear stress for the weld material is, 120 N/mm^2.

$$\therefore \quad 410 = \frac{2691.45}{t}$$

$$t = 6.56\ \text{mm}$$

$$h = \frac{t}{0.707}$$

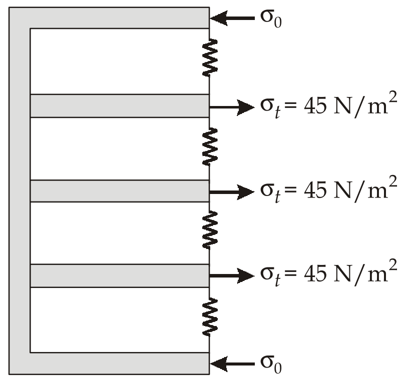
$$h = 9.285\ \text{mm}$$

6. (c)

Given : D = 1.6 m; L = 2.4 m; t = 10 mm; p = 2 N/mm^2; d = 35 mm

Initial condition : The tie bars are under tensile stress (σ_t) of 45 N/mm^2.

Let σ_0 be the longitudinal compressive stress in the cylinder walls.



(Initial condition)

From equilibrium conditions,

$$\sigma_0\,(\pi DT) = \sigma_t \times n \times \frac{\pi}{4} \times d^2$$

$$\sigma_0 \times \pi \times 1600 \times 10 = 45 \times 9 \times \frac{\pi}{4} \times 35^2$$

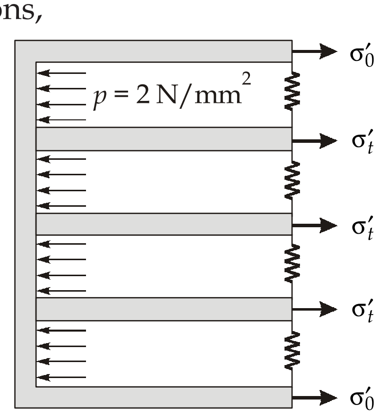
$$\sigma_0 = 7.752 \text{ N/mm}^2$$

There is no hoop stress initially.

Final condition:

Let σ_t' be the final stress in the tie bars and σ_0' be the final tensile longitudinal stress in the cylinder.

From equilibrium conditions,



$$\sigma_0' \pi \times D \times t + \sigma_t' \times n \times \frac{\pi}{4} \times d^2 = p\left(\frac{\pi}{4} D^2 - n\frac{\pi}{4} d^2\right)$$

or

$$\sigma_0' \times \pi \times 1600 \times 10 + \sigma_t' \times 9 \times \frac{\pi}{4} \times (35)^2 = 2\left(\frac{\pi}{4}(1600)^2 - 9 \times \frac{\pi}{4}(35)^2\right)$$

$$16000\pi\sigma_0' + 8659.014\sigma_t' = 4003920.567$$

$$\sigma_0' + 0.1723\sigma_t' = 79.66$$

$$\sigma_0' + 0.1723\sigma_t' - 79.66 = 0 \qquad \text{...(i)}$$

Hoop stress in the cylinder $= \frac{pD}{2t}$ [This hoop stress is not affected by the tie bars]

$$= \frac{2 \times 1600}{2 \times 10} = 160 \text{ N/mm}^2$$

From compatibility equation,

The increase in longitudinal strain for the bars = Increase in longitudinal strain for cylinder

i.e. $\varepsilon_t = \varepsilon_c$

where, ε_t = Increase in longitudinal strain in tie bars

$$\varepsilon_t = \frac{\sigma_t' - 45}{E}$$

ε_c = Increase in longitudinal strain of cylinder

= Final strain – Initial strain

$$\varepsilon_c = \frac{1}{E}(\sigma_0' - 0.3 \times 160) - \frac{1}{E}(-7.752)$$

$$\therefore \quad \frac{\sigma_t' - 45}{E} = \frac{1}{E}(\sigma_0' - 48 + 7.752)$$

$$\sigma_t' - 45 = \sigma_0' - 40.25$$

$$\sigma_0' - \sigma_t' + 4.75 = 0 \qquad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\sigma_0' = 67.25 \text{ N/mm}^2$$

$$\sigma_t' = 72 \text{ N/mm}^2$$

Increase in capacity = (2 × Increase in hoop strain + Increase in longitudinal strain) × Volume

$$= \frac{1}{2 \times 10^5}[2 \times (160 - 0.3 \times 67.25 - 0.3 \times 7.752) + 72 - 45] \times \left(\frac{\pi}{4} \times (1600)^2 \times 2400\right)$$

$$= 7.29 \times 10^6 \text{ mm}^3$$

7. (a)

Given : Major tensile stress, σ_x = 85 N/mm^2 ; Minor tensile stress, σ_y = 20 N/mm^2

Shear stress, τ_{xy} = 45 N/mm^2; $E = 2 \times 10^5$ N/mm^2

Major principal stress is given by

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{85+20}{2} + \sqrt{\left(\frac{85-20}{2}\right)^2 + (45)^2}$$

$$\sigma_1 = 108.01 \text{ N/mm}^2 \quad \text{(Tensile)}$$

$$\text{Minor principal stress, } \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{85+20}{2} - \sqrt{\left(\frac{85-20}{2}\right)^2 + (45)^2}$$

$$\sigma_2 = -3.01 \text{ N/mm}^2 \quad \text{or} \quad 3.01 \text{ N/mm}^2 \text{ (Compressive)}$$

It is clear that diagonal BD will be elongated and diagonal AC will be shortened. Hence the circle will become an ellipse whose major axis will be along BD and minor axis along AC. The major principal stress acts along BD and minor principal stress along AC.

$$\therefore \quad \text{Strain along BD} = \frac{\text{Major principal stress}}{E} - \frac{\text{Minor principal stress}}{mE}$$

$$= \frac{108.01}{2 \times 10^5} - \frac{(-3.01)}{4 \times 2 \times 10^5} = 5.438 \times 10^{-4}$$

∴ Increase in diameter along BD = Strain along BD × Diameter of hole

$$= 5.438 \times 10^{-4} \times 60 = 0.033 \text{ mm}$$

$$\text{Strain along AC} = \frac{\text{Minor principal stress}}{E} - \frac{\text{Major principal stress}}{mE}$$

$$= \frac{-3.01}{2 \times 10^5} - \frac{108.01}{4 \times 2 \times 10^5} = -1.501 \times 10^{-4}$$

∴ Decrease in length of diameter along AC = Strain along AC × Diameter of hole

$$= 1.501 \times 10^{-4} \times 60 = 0.00901 \text{ mm}$$

Hence, the circle will become an ellipse whose major axis will be 60 + 0.033 = 60.033 mm and minor axis will be 60 – 0.00901 = 59.99 mm.

7. (b)

Given : $W = 1250$ N; $N = 1500$ rpm; $d = 50$ mm; static load = 400 N; $P_i = 2$ N/mm²

1. Length of the bearing

Starting condition

The starting load is static load i.e. 400 N

$$P = \frac{W}{ld}$$

$$l = \frac{W}{Pd} = \frac{400}{2 \times 50} = 4 \text{ mm} \quad \text{...(a)}$$

Running condition

During running condition, the radial load on the bearing is 1250 N,

Permissible bearing pressure = 1 N/mm²

$$P = \frac{W}{ld}$$

$$l = \frac{W}{Pd} = \frac{1250}{1 \times 50} = 25 \text{ mm} \quad \text{...(b)}$$

From (a) and (b), the minimum length of bearing is 25 mm.

$$\left(\frac{l}{d}\right) = \frac{25}{50} = 0.5$$

We will assume standard value for (l/d) ratio as 0.5

$$= 0.5 \quad \text{...(i)}$$

2. Radial clearance

The standard value of radial clearance in case of Babbitt bearing is given by

$$c = (0.001)r = 0.001 \times 25 = 0.025 \text{ mm} \quad \text{..(ii)}$$

3. Minimum oil film thickness

The minimum oil film thickness is given by,

h_0 = 5(Sum of value of surface roughness of journal and bearing)

$$h_0 = 5(2 + 1) = 15 \text{ micrometer} = 0.015 \text{ mm} \quad \text{...(iii)}$$

4. Viscosity of lubricant

$$\left(\frac{l}{d}\right) = \frac{1}{2} \text{ and } \left(\frac{h_0}{c}\right) = \frac{0.015}{0.025} = 0.6$$

Referring Table

for the above mentioned values,

$$s = 0.779 \text{ and } \frac{Q}{rcn_s l} = 4.29$$

Also,
$$\left(\frac{r}{c}\right) = \frac{25}{0.025} = 1000$$

$$n_s = \frac{1500}{60} = 25 \text{ rev/s}$$

$$P = \frac{W}{ld} = \frac{1250}{25 \times 50} = 1 \text{ N/mm}^2$$

$\therefore$
$$s = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{P}$$

$$0.779 = (1000)^2 \times \frac{\mu \times 25}{1}$$

$$\mu = 31.16 \times 10^{-9} \text{ N-sec/mm}^2$$

Selection of lubricant

Referring fig, it is observed that the values of viscosity for SAE = 30 and SAE - 40 oils are 30 and 38 cP respectively at operating temperature of 65°C. We will select SAE - 40 oil for the application, which will satisfy the minimum viscosity of 31.16 cp.

Flow of lubricant

$$\frac{Q}{rcn_s l} = 4.29$$

$$Q = 4.29\, rcn_s\, l$$

$$= 4.29 \times 25 \times 0.025 \times 25 \times 25$$

$$= 1675.78 \text{ mm}^3\text{/s}$$

7. (c) (i)

Given :
- Number of wires = 30
- Diameter of each wire = 1.6 mm
- Weight of cage = 1.5 kN
- Weight of the rope = 4.6 N/m
- Length of the rope = 40 m
- E_{rope} = 70 GPa
- $\sigma_{allowable}$ = 120 MPa

Total area of cross-section, $A = \frac{\pi}{4} \times (1.6)^2 \times 4 \times 30 = 241.27 \text{ mm}^2$

The maximum stress occurs at the top of the wire rope where the weight of the rope is maximum.

$$\text{So, maximum load} = \text{Weight of the cage} + \text{Weight of the rope}$$
$$= 1500 + 4 \times 4.6 \times 40 = 2236 \text{ N}$$

$$\text{Initial stress in the rope, } \sigma = \frac{2236}{241.27} = 9.267 \text{ MPa}$$

So, equivalent static stress available for carrying the load = 120 – 9.267 = 110.73 MPa

The equivalent static load that can be carried,

$$P_e = 110.73 \times 241.27$$
$$= 26716.4 \text{ N}$$

$$\text{Extension of the rope, } \Delta = \frac{110.73 \times 40 \times 1000}{70 \times 1000} = 63.27 \text{ mm}$$

(i) With no drop, let W_1 be the weight which can be applied suddenly

So,
$$W_1\Delta = \frac{1}{2}P_e\Delta$$

$$\Rightarrow \quad W_1 = \frac{26716.4}{2} = 13358.2 \text{ N}$$

(ii) With 100 mm drop, let W_2 be the weight so,

$$W_2 \cdot (h + \Delta) = \frac{1}{2} P_e\Delta$$

$$\Rightarrow \quad W_2(100 + 63.27) = \frac{1}{2} \times 26716.4 \times 63.27$$

$$\Rightarrow \quad W_2 = 5176.53 \text{ N}$$

7. (c) (ii)

$$\text{Velocity of cooler, } V = 8 \text{ m/s}$$
$$\text{Time, } t = 3\text{s}$$

Acceleration of collar,

$$\therefore \quad V = u + at$$
$$\Rightarrow \quad 8 = 0 + a \times 3$$
$$\Rightarrow \quad a = \frac{8}{3} = 2.67 \text{ m/s}^2$$

FBD of collar

$$\therefore \quad F - 2T = m \times a$$

$$\Rightarrow \quad F - 2T = 8 \times \frac{8}{3} = \frac{64}{3} \text{ N} \quad \text{...(i)}$$

If the acceleration of collar is a, then acceleration of mass A will be $2a$ because if collar moves a distance x in the horizontal direction then during same time, mass A goes up by $2x$ distance.

For mass A:

$$T - 3g = m_A \times a'$$

$$T - 3g = 3 \times 2 \times a = 6a = 6 \times \frac{8}{3}$$

$$\Rightarrow \quad T - 3g = \frac{48}{3}$$

$$\Rightarrow \quad T = \frac{48}{3} + 3 \times 9.81 = 45.43 \text{ N}$$

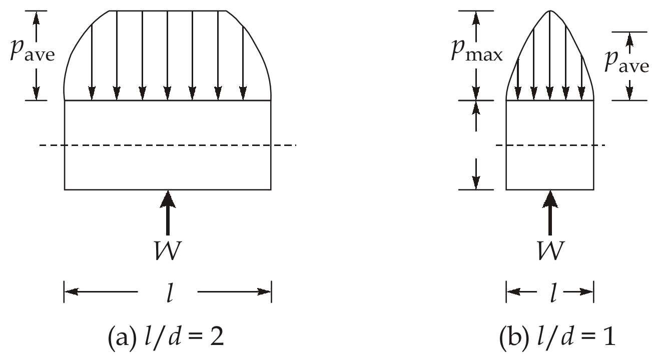
So, from equation (i),

$$F = \frac{64}{3} + 2 \times 45.43 = 112.19 \text{ N}$$

8. (a) (i)

The design and selection of journal bearings in mechanical systems require careful consideration of several interrelated parameters to ensure optimal performance, reliability and lifespan. The key parameters are

(i) Length to diameter ratio : The length to diameter ratio (l/d) affects the performance of the bearing. As the ratio increases, the resulting film pressure increases as shown in figure. A long bearing, therefore, has more load carrying capacity compared with a short bearing. A short bearing, on the other hand, has greater side flow, which improves heat dissipation. The long bearings are more susceptible to metal to metal contact at the two edges, when the shaft is deflected under load. The longer the bearing, more difficult it is to get sufficient oil flow through the passage between the journal and the bearing. Therefore, the design trend is to use (l/d) ratio as 1 or less than 1. When the shaft and the bearing are precisely aligned, the shaft deflection is within the limit and cooling of lubricant and bearing does not present a serious problem, the (l/d) ratio can be taken as more than 1. In practice, the (l/d) ratio varies from 0.5 to 2.0, but in the majority of applications, it is taken as 1 or less than 1. Following terminology is used in relation to (l/d) ratio,



Effect of (l/d) ratio on average bearing pressure

- When (l/d) ratio is more than 1, the bearing is called 'long' bearing.
- When (l/d) ratio is less than 1, the bearing is called 'short bearing.
- When (l/d) ratio is equal to 1, the bearing is called 'square' bearing.

(ii) Unit Bearing Pressure : The unit bearing pressure is the load per unit of projected area of the bearing in running condition. It depends upon number of factors, such as bearing material, operating temperature, the nature, and frequency of load and service conditions. The values of unit bearing pressure, based on past experience.

(iii) Start-up Load : The unit bearing pressure for starting conditions should not exceed 2 N/mm^2. The start-up load is static load when the shaft is stationary. It mainly consists of the dead weight of shaft and its attachments. The start-up load can be used to determine the minimum length of the bearing on the basis of starting conditions.

(iv) Radial Clearance : The radial clearance should be small to provide the necessary velocity gradient. However, this requires costly finishing operations, rigid mountings of the bearing assembly, and clean lubricating oil without any foreign particles. This increases the initial and maintenance costs.

(v) Minimum Oil Film Thickness : The surface finish of the journal and the bearing is governed by the value of the minimum oil film thickness selected by the designer and vice versa. There is a lower limit for the minimum oil film thickness, below which metal-to-metal contact occurs and hydrodynamic film breaks. This lower limit is given by,

$h_0 = (0.0002)r$

(vi) Maximum Oil Film Temperature : The lubricating oil tends to oxidize when the operating temperature exceeds 120°. Also, the surface of babbitt bearing tends to soften at 125°C (for bearing pressure of $7\ N/mm^2$) and at 190°C (for bearing pressure of $1.4\ N/mm^2$) . Therefore, the operating temperature should be kept within these limits.

8. (a) (ii)

Given: N = 200 rpm; C_s = 0.02; t = 60 mm; ρ = 7800 kg/m^3

The fluctuating terms (sin 2θ) and (cos 2θ) have a zero mean.

∴ Mean torque is given by

$$T_m = 15000 \text{ Nm}$$

where $T = T_m$

$$2000 \sin 2\theta - 1500 \cos 2\theta = 0$$

$$\tan 2\theta = \frac{1500}{2000}$$

$$\Rightarrow \quad 2\theta = 36.87° \text{ or } (180 + 36.87)$$

$$\theta = 18.44° \text{ or } 108.435°$$

The maximum and minimum angular velocities will occur at point A and B respectively.

$$\therefore \quad U_0 = \int_A^B (T - T_m)\, d\theta$$

$$= \int_{18.44}^{108.435} (200 \sin 2\theta - 1500 \cos 2\theta)\, d\theta$$

$$U_0 = \left[-2000\frac{\cos 2\theta}{2} - \frac{1500 \sin 2\theta}{2}\right]_{18.44}^{108.435}$$

$$= -1000(-0.799 - 0.799) - 750(-0.6 - 0.6)$$

$$= 1599.997 + 900$$

$$= 2499.997 \text{ N-m or J}$$

$$\omega = \frac{2\pi N}{60} = \left(\frac{2 \times \pi \times 200}{60}\right) \text{ rad/s} = 20.933 \text{ rad/s}$$

$$I = \frac{U_0}{\omega^2 c_s} = \frac{2499.997}{(20.933)^2 \times 0.02} = 285.254 \text{ kg-m}^2$$

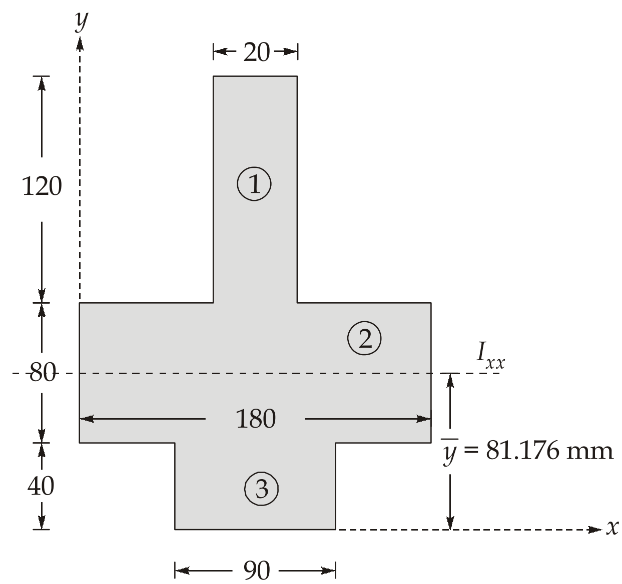
$$R^4 = \frac{2I}{\pi \rho t} = \frac{2 \times 285.254}{\pi \times 7800 \times \frac{60}{1000}} = 0.38803$$

$$R = 0.78926 \text{ m}$$

$$R = 789.26 \text{ mm}$$

8. (b)

Given : σ_t = 40 N/mm^2; σ_c = 140 N/mm^2



(All dimensions are in mm)

$$A_1 = 20 \times 120 = 2400 \text{ mm}^2$$
$$A_2 = 180 \times 80 = 14400 \text{ mm}^2$$
$$A_3 = 90 \times 40 = 3600 \text{ mm}^2$$
$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$
$$= \frac{2400 \times 180 + 14400 \times 80 + 3600 \times 20}{2400 + 14400 + 3600} = 81.176 \text{ mm}$$
$$I_{xx} = \frac{90 \times 40^3}{12} + 3600 \times (81.176 - 20)^2 + \frac{180 \times 80^3}{12}$$
$$+ 14400 \times (81.176 - 80)^2 + \frac{20 \times 120^3}{12} + 2400 \times (180 - 81.176)^2$$
$$= 1.395 \times 10^7 + 7.699 \times 10^6 + 2.632 \times 10^7$$
$$= 4796.9 \times 10^4 \text{ mm}^4$$

(a) For bottom face to be in tension

$$y_t = \bar{y} - 81.176,\ y_c = 158.824 \text{ mm}$$

Taking σ_t = 40 N/mm^2

Corresponding, $\sigma_c = \frac{\sigma_t}{y_t} \times y_c = \frac{40}{81.176} \times 158.824$

$$\sigma_c = 78.26 \text{ N/mm}^2$$

Which is much less than the permissible value of 140 N/mm^2.

Hence, $\sigma_t = 40 \text{ N/mm}^2$ is the governing stress

Now, $$M = \left(\sigma_t \times \frac{I}{y_t} = 40 \times \frac{4796.9 \times 10^4}{81.176} \text{ N-mm}\right)$$

$$M = 23.64 \text{ kN-m}$$

For bottom face to be in compression

$$y_c = 81.176 \text{ mm}; \ y_t = 158.824 \text{ mm}$$

If σ_c is allowed to reach the permissible value of 140 N/mm^2 corresponding σ_t will be much higher that the permissible value of 40 N/mm^2. Hence $(\sigma_t)_{man}$ should be taken as 40 N/mm^2.

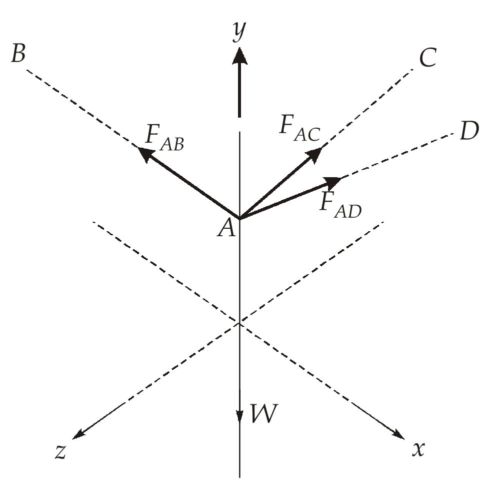
$\therefore$ $$\sigma_c = \frac{\sigma_t}{y_t} \times y_c = \frac{40}{158.824} \times 81.176$$

$$\sigma_c = 20.44 \text{ N/mm}^2$$

$$M = \sigma_t \times \frac{I}{y_t} = 40 \times \frac{4796.9 \times 10^4}{158.824}$$

$$M = 12.08 \text{ kN-m}$$

8. (c)



$$\text{Weight, } W = 20 \times 9.81 = 196.2 \text{ N}$$

Coordinate of A, B, C, D are

$$A \equiv (0, 1.2, 0)$$

$$B \equiv (-0.3, 2, 1)$$

$$C \equiv (0, 2, -1)$$
$$D \equiv (2, 2, 0)$$

Unit vectors, $e_{AB} = \dfrac{\overline{AB}}{|\overline{AB}|} = \dfrac{[-0.3, (2-1.2), (1-0)]}{\sqrt{0.3^2 + 0.8^2 + 1^2}}$

$$= \frac{-0.3\vec{i} + 0.8\vec{j} + 1\vec{k}}{1.3153} = -0.228\vec{i} + 0.608\vec{j} + 0.760\vec{k}$$

Similarly,

$$e_{AC} = 0.625\vec{j} - 0.781\vec{k}$$
$$e_{AD} = 0.928\vec{i} + 0.371\vec{j}$$

Forces are:

$$\overrightarrow{F_{AB}} = -0.228F_{AB}\vec{i} + 0.608F_{AB}\vec{j} + 0.760F_{AB}\vec{k}$$
$$\overrightarrow{F_{AC}} = 0.625F_{AC}\vec{j} - 0.781F_{AC}\vec{k}$$
$$\overrightarrow{F_{AD}} = 0.928F_{AD}\vec{i} + 0.371F_{AD}\vec{j}$$
$$\vec{W} = -196.2\vec{j}$$

Equations of equilibrium are

$\Sigma F_x = 0 \Rightarrow \quad -0.228F_{AB} + 0.928F_{AD} = 0$ ...(i)

$\Sigma F_y = 0 \Rightarrow \quad 0.608F_{AB} + 0.625F_{AC} + 0.371F_{AD} = 196.2$ ...(ii)

$\Sigma F_z = 0 \Rightarrow 0.760F_{AB} - 0.781F_{AC} = 0$ ...(iii)

Solving equations (i), (ii) and (iii), we get

$$F_{AB} = 150.1 \text{ N}$$
$$F_{AC} = 146.04 \text{ N}$$
$$F_{AD} = 36.9 \text{ N}$$

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