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Detailed Solutions

ESE-2025 Mains Test Series

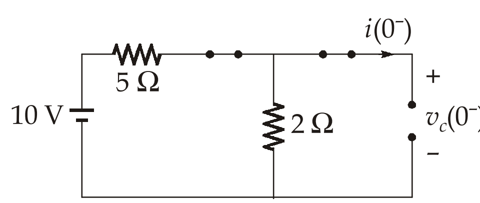
E & T Engineering Test No : 1

Section A : Network Theory + Electronic Devices and Circuits

Q.1 (a) Solution:

Method I:

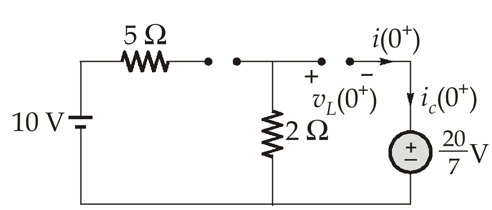
For $t < 0$, the switch is closed. At steady state, the capacitor acts as an open circuit to dc and the inductor acts as a short circuit to dc, as represented in the below figure.



$$v_c(0^-) = 10 \times \frac{2}{5+2} = \frac{20}{7}\text{ V}$$

$$i(0^-) = 0\text{ A}$$

At $t = 0^+$;



Since current through the inductor and voltage across the capacitor cannot change instantaneously,

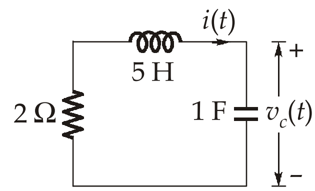
$$v_c(0^+) = v_c(0^-) = \frac{20}{7}\text{ V}$$

$$i(0^+) = i(0^-) = 0\text{ A}$$

From the circuit at $t = 0^+$, $v_L(0^+) = \frac{-20}{7}$ V, $i_c(0^+) = 0$ A

For $t > 0$:

$R = 2\ \Omega$, $L = 5$ H, $C = 1$ F



The characteristic equation of the system can be obtained using KVL as $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

The roots of the characteristic equation are given by

$$s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where, $$\alpha = \frac{R}{2L} = \frac{2}{2\times 5} = 0.2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times 1}} = \frac{1}{\sqrt{5}}\text{ rad/sec}$$

As $\alpha < \omega_0$, the response is underdamped.

$\therefore$ $$i(t) = e^{-\alpha t}\left[B_1 \cos\omega_d t + B_2 \sin\omega_d t\right] \quad \text{...(i)}$$

where, $$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 - (0.2)^2} = 0.4\text{ rad/sec}$$

$\therefore$ From equation (i),

$$i(t) = e^{-0.2t}[B_1 \cos 0.4t + B_2 \sin 0.4t] \quad \text{...(ii)}$$

As at $t = 0^+$, current $i(0^+) = 0$ A

$\Rightarrow$ $$B_1 = 0$$

On differentiating equation (ii),

$$\frac{di(t)}{dt} = B_1[e^{-0.2t}(-0.4 \sin 0.4t) - 0.2e^{-0.2t} \cos 0.4t] + B_2[e^{-0.2t}(0.4 \cos 0.4t) - 0.2e^{-0.2t} \sin 0.4t]$$

At $t = 0^+$;

$$\frac{di(0^+)}{dt} = -0.2B_1 + 0.4B_2$$

We know that,
$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{\left(\frac{-20}{7}\right)}{5} = \frac{-4}{7}$$

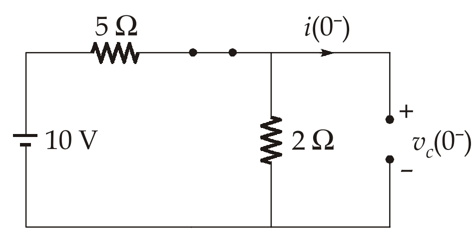
$$\Rightarrow \quad -0.2(0) + 0.4(B_2) = \frac{-4}{7}$$

$$\Rightarrow \quad B_2 = -1.429$$

The required values are, $\alpha = 0.2$, $\omega_d = 0.4$ rad/sec, $B_1 = 0$, $B_2 = -1.429$

Method-II:

For $t < 0$, the switch is closed. At steady state, the capacitor acts as an open circuit to dc and the inductor acts as a short circuit to dc, as represented in the figure below:



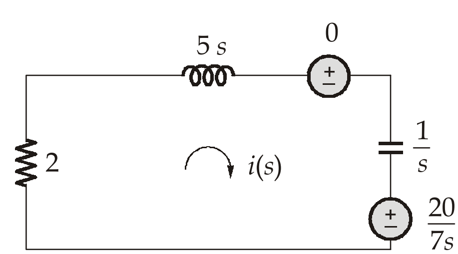
$$v_c(0^-) = \frac{10 \times 2}{5+2} = \frac{20}{7} \text{ Volt}$$

$$i(0^-) = 0 \text{ A}$$

Since current through the inductor and voltage across the capacitor cannot change instantaneously,

$$v_c(0^-) = v_c(0^+) = \frac{20}{7} \text{volt and } i(0^-) = i(0^+) = 0A.$$

Considering the initial conditions, the circuit can be drawn in s-domain for $t > 0$ as below:



On applying KVL in the loop, we get

$$i(s) = -\frac{(20/7s)}{\left(\frac{1}{s}+2+5s\right)}$$

$$i(s) = \frac{-20}{7+14s+35s^2} = \frac{-20}{35s^2+14s+7}$$

Note:

We know that,

$$35s^2 + 14s + 7 \quad \text{and} \quad s^2+\frac{14s}{35}+\frac{7}{35}$$

both get the same roots but $35s^2 + 14s + 7$ will affect the gain factor of current.

∴ **Make sure that coefficient of "s^2" is always 1 before taking inverse laplace transform.**

$$i(s) = \frac{-20}{35\left(s^2+\frac{14s}{35}+\frac{7}{35}\right)} = \frac{\left(\frac{-20}{35}\right)}{(s+0.2)^2+0.16}$$

$$i(s) = \frac{\left(\frac{-20}{35}\right)}{(s+0.2)^2+(0.4)^2}$$

$$\Rightarrow \qquad i(s) = \frac{\left(\frac{-20}{35}\right)}{(0.4)}\left[\frac{0.4}{(s+0.2)^2+(0.4)^2}\right]$$

On taking inverse Laplace of $i(s)$, we get

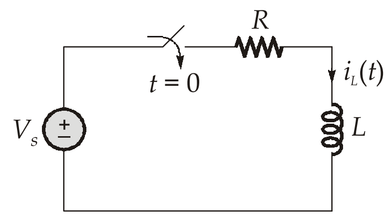
$$i(t) = -1.428\,[e^{-0.2t}\sin(0.4t)]\text{A}$$

On comparing we get,

$$\alpha = 0.2$$
$$B_1 = 0$$
$$B_2 = -1.428$$
$$\omega_d = 0.4 \text{ rad/sec}$$

Q.1 (b) Solution:

We have,



The inductor current is,

$$i_L(t) = \frac{V_s}{R}\left[1 - e^{-t/\tau}\right]; t \geq 0$$

where, time constant, $\tau = \frac{L}{R}$

The voltage across inductor is,

$$v_L(t) = L\frac{di_L(t)}{dt} = L\frac{d}{dt}\left(\frac{V_s}{R}[1 - e^{-t/\tau}]\right)$$

$$\Rightarrow \quad v_L(t) = \frac{LV_s}{R}\left[-\left(-\frac{1}{\tau}\right)e^{-t/\tau}\right] = V_s e^{-t/\tau}$$

$$\therefore \quad P_L(t) = V_L(t) \times i_L(t)$$

$$= V_s e^{-t/\tau} \times \frac{V_s}{R}[1 - e^{-t/\tau}] = \frac{V_s^2}{R}\left[e^{-t/\tau} - e^{-2t/\tau}\right]$$

Energy stored in the inductor,

$$E_L(t) = \int_0^\infty P_L(t)dt = \int_0^\infty \frac{V_s^2}{R}\left[e^{-t/\tau} - e^{-2t/\tau}\right]dt$$

$$= \frac{V_s^2}{R}\left[\frac{e^{-t/\tau}}{-1/\tau} - \frac{e^{-2t/\tau}}{-2/\tau}\right]_0^\infty$$

$$= \frac{V_s^2}{R}\left[\tau - \frac{\tau}{2}\right] = \frac{V_s^2}{R}\left[\frac{\tau}{2}\right] = \frac{V_s^2}{R}\left[\frac{L}{2R}\right]$$

$$\therefore \quad E_L(t) = \frac{1}{2}L\left[\frac{V_s}{R}\right]^2 \quad \text{...(i)}$$

The resistor current is,

$$i_R(t) = i_L(t) = \frac{V_s}{R}\left[1 - e^{-t/\tau}\right]; t \geq 0$$

$$v_R(t) = R \times i_R(t) = V_s[1 - e^{-t/\tau}]$$

$$\therefore \quad P_R(t) = v_R(t) \times i_R(t)$$

$$= V_s\left[1-e^{-t/\tau}\right]\times\frac{V_s}{R}\left[1-e^{-t/\tau}\right] = \frac{V_s^2}{R}\left[1-e^{-t/\tau}\right]^2$$

Energy dissipated in the resistor,

$$E_R(t) = \int_0^\infty P_R(t)dt = \int_0^\infty \frac{V_s^2}{R}\left[1-e^{-t/\tau}\right]^2 dt = \infty$$

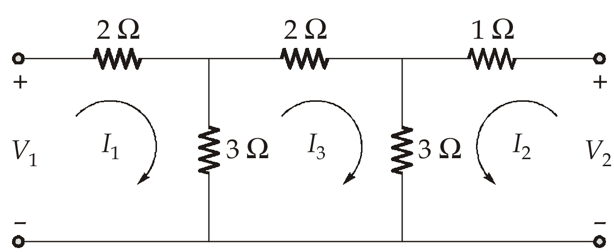
$$\therefore \quad \text{Efficiency, } \eta = \frac{E_0}{E_{in}}\times 100\% = \frac{E_0}{E_0 + \text{Losses}}\times 100\%$$

$$= \frac{E_L(t)}{E_L(t)+E_R(t)}\times 100\%$$

$$\Rightarrow \quad \eta = \frac{\frac{1}{2}L\left[\frac{V_s}{R}\right]^2}{\frac{1}{2}L\left[\frac{V_s}{R}\right]^2 + \infty}\times 100\% = 0\%$$

Q.1 (c) Solution:

Given circuit is



Applying KVL to Mesh 1,

$$V_1 = 2I_1 + 3(I_1 - I_3) = 5I_1 - 3I_3 \quad \text{...(i)}$$

Applying KVL to Mesh-2,

$$V_2 = 4I_2 + 3I_3 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-3I_1 + 3I_2 + 8I_3 = 0$$

$$I_3 = \frac{3}{8}I_1 - \frac{3}{8}I_2 \quad \text{...(iii)}$$

Substituting the equation (iii) in the equation (i)

$$V_1 = 5I_1 - 3\left(\frac{3}{8}I_1 - \frac{3}{8}I_2\right)$$

$$V_1 = \frac{31}{8}I_1 + \frac{9}{8}I_2 \quad \text{...(iv)}$$

Substituting the equation (iii) in the equation (ii),

$$V_2 = 4I_2 + 3\left(\frac{3}{8}I_1 - \frac{3}{8}I_2\right)$$

$$V_2 = \frac{9}{8}I_1 + \frac{23}{8}I_2$$

$$I_1 = \frac{8}{9}V_2 - \frac{23}{9}I_2 \quad \text{...(v)}$$

Substituting the equation (v) in the equation (iv),

$$V_1 = \frac{31}{8}\left(\frac{8}{9}V_2 - \frac{23}{9}I_2\right) + \frac{9}{8}I_2$$

$$V_1 = \frac{31}{9}V_2 - \frac{79}{9}I_2 \quad \text{...(vi)}$$

Comparing equation (v) and (vi) with *ABCD* parameter equations,

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

We get,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{31}{9} & \frac{79}{9}\Omega \\ \frac{8}{9}℧ & \frac{23}{9} \end{bmatrix}$$

Q.1 (d) Solution:

Given, doping concentrations,

Source/Drain Doping concentration,

$$N_d = 10^{19} \text{ cm}^{-3}$$

Channel doping concentration,

$$N_a = 10^{16} \text{ cm}^{-3}$$

$$\text{Channel length, } L = 1.2 \text{ μm}$$

$$|V_{SB}| = 0$$

At punch-through, $x_{d0} + x_{dr} = L$

Where, x_{d0} : Zero-biased source-substrate *pn* junction width.

x_{dr} : Reverse biased drain-substrate *pn* junction width.

The zero-biased source-substrate *pn* junction width is,

$$x_{d0} = \left(\frac{2\epsilon_s V_{bi}}{qN_a}\right)^{1/2}$$

Where,

$$\text{built-in potential, } V_{bi} = V_T \ln\left[\frac{N_a N_d}{n_i^2}\right]$$

$$V_{bi} = 0.0259 \ln\left[\frac{10^{16} \times 10^{19}}{(1.5 \times 10^{10})^2}\right]$$

$$V_{bi} = 0.874 \text{ V}$$

$$\therefore \quad x_{d0} = \left\{\frac{2 \times 11.7 \times 8.85 \times 10^{-14} \times 0.874}{1.6 \times 10^{-19} \times 10^{16}}\right\}^{1/2}$$

$$x_{d0} = 0.336 \ \mu\text{m}$$

The reverse biased drain-substrate *pn* junction width,

$$x_{dr} = \left\{\frac{2\epsilon_s (V_{bi} + V_{DS})}{qN_a}\right\}^{1/2} \quad \text{...(i)}$$

where V_{DS} is punch-through voltage.

$$\because \quad x_{d0} + x_{dr} = L$$

$$0.336 \ \mu\text{m} + x_{dr} = 1.2 \ \mu\text{m}$$

$$\therefore \quad x_{dr} = 0.864 \ \mu\text{m}$$

From equation (i), $V_{bi} + V_{DS} = \dfrac{x_{dr}^2 q N_a}{2\epsilon_s} = \dfrac{(0.864 \times 10^{-4})^2 (1.6 \times 10^{-19})(10^{16})}{2 \times 11.7 \times 8.85 \times 10^{-14}}$

$$= 5.77 \ V$$

The punch-through voltage,

$$V_{DS} = 5.77 - V_{bi}$$

$$= 5.77 - 0.874 \ V$$

$$\therefore \quad V_{DS} \simeq 4.9 \text{ V}$$

Q.1 (e) Solution:

(i) From the given plot, we can define common emitter current gain, β as

$$\therefore \quad \beta = \frac{\Delta I_C}{\Delta I_B} = \frac{10^{-2} - 10^{-6}}{10^{-4} - 10^{-8}} = \frac{10^{-2}\left[1 - 10^{-4}\right]}{10^{-4}\left[1 - 10^{-4}\right]}$$

$$\therefore \quad \beta = 100$$

(ii) In NPN BJT, the collector current density due to the excess electron concentration profile in the base region is given by

$$J_C = qD_B \frac{n'(0)}{W_B}$$

Where, $n'(0) = \frac{n_i^2}{N_B}\left(e^{qV_{BE}/kT} - 1\right)$ is the excess electron concentration at the base-emitter junction and W_B is the base width

$$\therefore \quad J_C = q\frac{D_B}{W_B}\frac{n_i^2}{N_B}\left(e^{qV_{BE}/kT} - 1\right)$$

At $V_{BE} = 0.3$ V; $J_C = 10^{-6}$ A/cm^2 (from the graph)

$$\therefore \quad 10^{-6} = 1.6 \times 10^{-19} \times \frac{10^{20}}{N_B} \times \frac{10}{0.2 \times 10^{-4}}\left(e^{\frac{0.3}{0.026}} - 1\right)$$

$$N_B = \frac{1.6 \times 10^{-19} \times 10^{21}}{0.2 \times 10^{-10}}\left(e^{\frac{0.3}{0.026}} - 1\right)$$

$$N_B = 8.2 \times 10^{17} \text{ cm}^{-3}$$

(iii) Base transit time, $\tau_B = \frac{W_B^2}{2D_B}$

$$\therefore \quad \tau_B = \frac{\left(0.2 \times 10^{-4}\right)^2}{2 \times 10} = \frac{0.04 \times 10^{-8}}{20}$$

$$= 2 \times 10^{-3} \times 10^{-8} = 2 \times 10^{-11} \text{ sec}$$

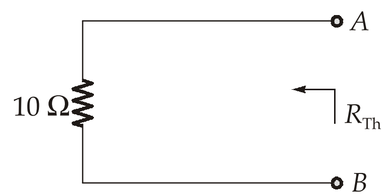
$$\tau_B = 20 \text{ psec.}$$

Q.2 (a) Solution:

To calculate Thevenin equivalent, R_{Th} and V_{Th} across AB, we remove 5 Ω resistor.

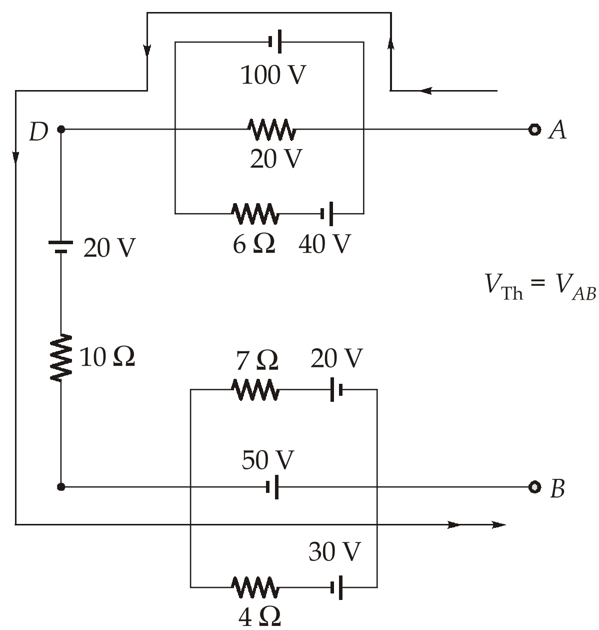
Calculation of R_{Th}:

All voltage source will be shorted,



Hence, $R_{Th} = 10\ \Omega$

Calculation of V_{Th}:

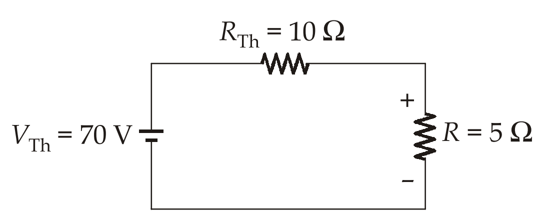


Applying KVL in the path shown,

$$V_A - 100 - 20 + 50 = V_B$$

$$V_{Th} = V_A - V_B = 70\ \text{V}$$

Thevenin's equivalent circuit:

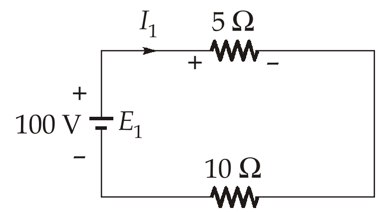


The current through 5 Ω resistor is given by,

$$I_R = \frac{70}{15} = 4.667\ \text{A}$$

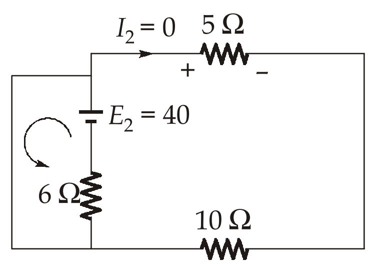
Verification by superposition theorem: According to superposition theorem, in any linear and bilateral network having multiple independent sources, the response of an element will be equal to the algebraic sum of the responses of that element by considering one source at a time and deactivating all other sources.

- **Current due to E_1,**



$$I_1 = \frac{100}{15} = 6.67 \text{ A}$$

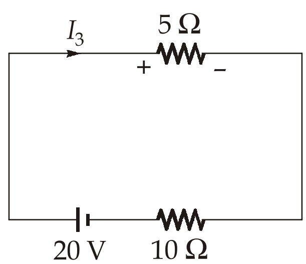
- **Current due to E_2,**



Current will flow through shorted branch. Hence,

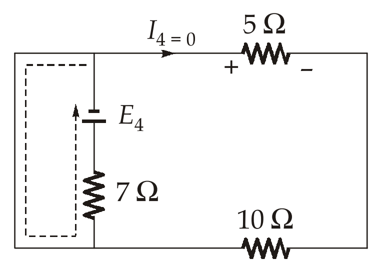
$$I_2 = 0 \text{ A}$$

- **Current due to E_3,**



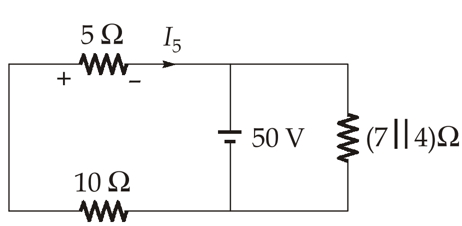
$$I_3 = \frac{20}{15} = 1.33 \text{ A}$$

- **Current due to E_4,**



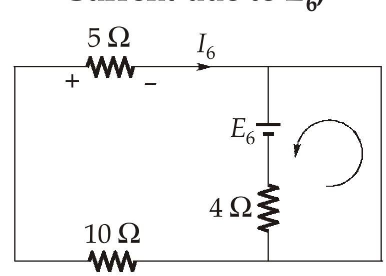
$$I_4 = 0 \text{ A}$$

- **Current due to E_5,**



$$I_5 = \frac{-50}{15} = -3.33 \text{ A}$$

- **Current due to E_6,**



$$I_6 = 0 \text{ A}$$

Note: Resistance in parallel to the ideal voltage source has no effect on current I.

Applying superposition theorem:

Current through 5 Ω resistor,

$$I = I_1 + I_2 + I_3 + I_4 + I_5 + I_6$$
$$= 6.67 + 0 + 1.33 + 0 - 3.33 + 0 = 4.67 \text{ A}$$

Thus, same results are obtained using both Thevenin and superposition theorem.

Q.2 (b) Solution:

Given,
$$N_a = 10^{15} \text{ cm}^{-3}$$
$$t_{ox} = 750 \text{ Å} = 750 \times 10^{-8} \text{ cm}$$
$$V_{FB} = -1.5\ V.$$

(i) Threshold voltage of n-channel MOSFET,

$$V_T = V_{FB} + 2\phi_f + \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

Where, Q'_{SD}(max) is the maximum amount of space charge in the channel, given by

$$|Q'_{SD}(\max)| = q\,N_a\,x_{dT}$$

Here, x_{dT} is the maximum space charge width

$$x_{dT}\Big|_{\phi_f = 2\phi_{fp}} = \left[\frac{4\in_s \phi_{fp}}{qN_a}\right]^{\frac{1}{2}}$$

Where,
$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = 0.026 \ln\left[\frac{10^{15}}{1.5\times 10^{10}}\right]$$

∴
$$\phi_{fp} = 0.288\ V$$

Thus,
$$x_{dT} = \left[\frac{4\times 11.7\times 8.85\times 10^{-14}\times 0.288}{1.6\times 10^{-19}\times 10^{15}}\right]^{\frac{1}{2}}$$

$$x_{dT} = 0.863\ \mu\text{m}$$

∴
$$|Q'_{SD}(\max)| = 1.6\times 10^{-19}\times 10^{15}\times 0.863\times 10^{-4}$$

$$|Q'_{SD}(\max)| = 1.38\times 10^{-8}\ \text{C/cm}^2$$

∴ Threshold voltage,
$$V_T = -1.5 + (2\times 0.288) + \frac{1.38\times 10^{-8}}{C_{ox}}$$

Where, $$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{3.9\times 8.85\times 10^{-14}}{750\times 10^{-8}} = 4.6 \times 10^{-8}\ \text{F/cm}^2$$

$\therefore$ $$V_T = -1.5 + 2\,(0.288) + \frac{1.38\times 10^{-8}}{4.6\times 10^{-8}} = -0.624\ V.$$

(ii) To achieve a threshold voltage of $V_T = 0.9$ V, acceptor impurities are implanted into the channel region to increase the threshold voltage by an amount,

$$\Delta V_T = 0.9 - (-0.624) = 1.52\ \text{V}$$

The threshold voltage shift due to ion implant is given by

$$\Delta V_T = \frac{qD_I}{C_{ox}}$$

Thus, Ion implant density required,

$$D_I = \frac{(\Delta V_T)\times C_{ox}}{q} = \frac{1.52\times 4.6\times 10^{-8}}{1.6\times 10^{-19}} = 4.37 \times 10^{11}\ \text{cm}^{-3}$$

(iii) Given, $$V_{SB} = 2\ \text{V}$$

Then the change in threshold voltage due to V_{SB},

$$\Delta V_T = \frac{\sqrt{2q\in_s N_a}}{C_{ox}}\left(\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right)$$

$$= \frac{\sqrt{2\times 1.6\times 10^{-19}\times 11.7\times 8.85\times 10^{-14}\times 10^{15}}}{4.6\times 10^{-8}}\left[\sqrt{2(0.288)+2} - \sqrt{2\times 0.288}\right]$$

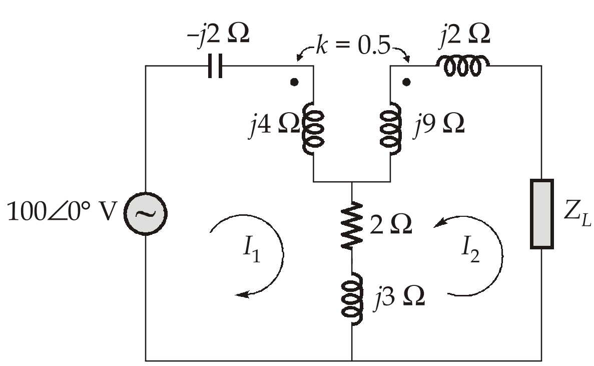
$$\Delta V_T = +0.335\ \text{V}$$

$\therefore$ The threshold voltage with $V_{SB} = 2$ V is,

$$V_T = +0.90 + 0.335 = 1.235\ \text{V}$$

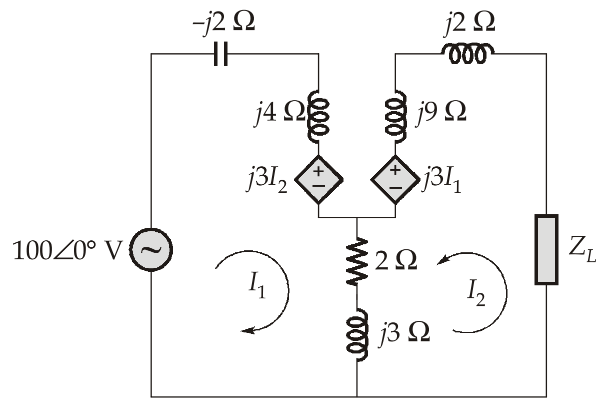
Q.2 (c) Solution:

(i)

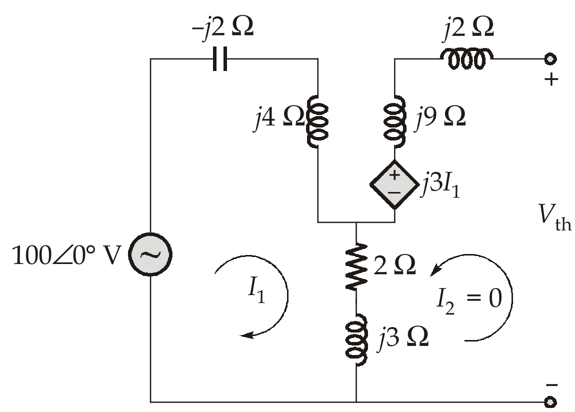


$$\text{Mutual inductance} = k\sqrt{L_1 L_2} = 0.5\sqrt{4\times 9} = 3\text{ H}$$

Using the dot convention, the equivalent circuit can be drawn as below:



Thevenin voltage: To calculate Thevenin voltage V_{th}, we disconnect the load impedance Z_L.



Applying KVL in loop 1, we get,

$$100 = (-j2 + 4j + 2 + j3)I_1$$

$$I_1 = \frac{100}{2+5j} = 18.57\angle -68.19°\text{A}$$

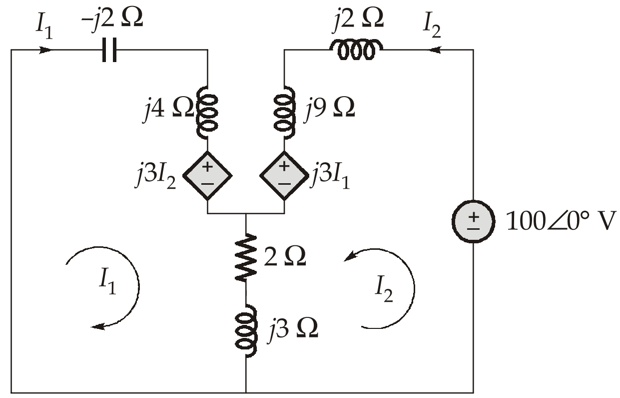
$$V_{th} = j3I_1 + (2 + j3)I_1$$

$$V_{th} = (2 + j6)I_1$$

$$V_{th} = (2 + j6) \times 18.57 \angle -68.19°$$

$$V_{th} = 117.44 \angle 3.36°\text{V}$$

Thevenin impedance: To calculate Thevenin impedance Z_{th}, we replace the independent voltage source by short-circuit. Applying 100 V at the terminals of Z_L, the Thevenin impedance is given by,



Thevenin impedance: $$Z_{th} = \frac{100}{I_2}$$

Applying KVL in loop 1, we get

$$(-j2 + j4)I_1 + j3I_2 + (2 + j3)(I_1 + I_2) = 0$$

$$(2 + j5)I_1 = -(2 + j6)I_2$$

$$I_1 = -\frac{(2+j6)}{(2+j5)}I_2$$

Applying KVL in loop 2, we get

$$(2 + j3)(I_1 + I_2) + j3I_1 + j11I_2 = 100$$

$$(2 + j6)I_1 + (2 + j14)I_2 = 100$$

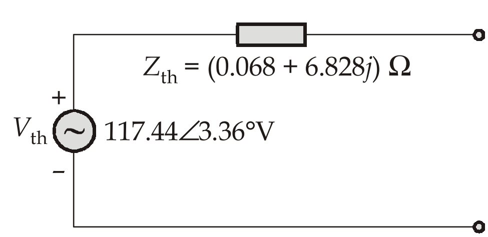
$$(2+j6)\left(\frac{2+j6}{2+j5}\right)I_2 + (2+j14)I_2 = 100$$

$$I_2 = \frac{-100}{\frac{(2+j6)^2}{2+j5} - (2+j14)}$$

$$Z_{th} = \frac{100}{I_2} = (2+j14) - \frac{-32+24j}{2+j5}$$

$$Z_{th} = (0.068 + 6.828j)\ \Omega$$

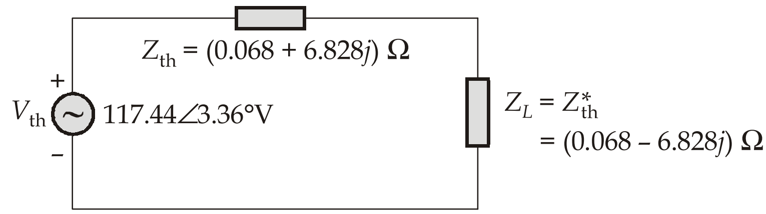
Thevenin equivalent circuit is



(ii) For maximum power transfer, according to maximum power transfer theorem,

$$Z_L = Z_{th}^*$$

$$Z_L = (0.068 - j6.828)\Omega$$



The maximum power transferred to the load Z_L is thus,

$$P_{max} = \frac{|V_{th}|^2}{4R_{th}} = \frac{117.44 \times 117.44}{4 \times 0.068} = 50706.45 \text{ W} = 50.7 \text{ kW}$$

Q.3 (a) Solution:

(i) In a MOS structure (with P-substrate), the electric field in the semiconductor is given by

$$E = \int_0^x \frac{\rho(x)}{\epsilon_{si}} dx$$

Where, $\rho(x)$ is charge density.

The charge density within space charge region in the semiconductor is

$$\rho(x) = -q N_A$$

$$\therefore \quad E(x) = \int_0^x \frac{-qN_A}{\epsilon_{si}} dx$$

$$= -q\frac{N_A}{\epsilon_{si}} x + c$$

at the end of the space charge region, electric field will be zero.

$$E(W) = -q\frac{N_A}{\epsilon_{si}} W + C = 0$$

$$\Rightarrow \quad C = q\frac{N_A}{\epsilon_{si}} W$$

$$\therefore \quad E(x) = -q\frac{N_A}{\varepsilon \epsilon_{si}} x + q\frac{N_A}{\epsilon_{si}} W$$

$$E(x) = q\frac{N_A}{\epsilon_{si}} [W - x]$$

From the given electric field graph, the maximum value of the field E_{max} = 30 kV/cm at x = 0.

$$\therefore \quad E_{max} = q\frac{N_A}{\in_{si}}W \quad \Rightarrow N_A = \frac{\in_{si}}{q}\frac{E_{max}}{W}$$

$$\therefore \quad N_A = \frac{10^{-12}}{1.6\times10^{-19}}\times\frac{30\times10^3}{0.6\times10^{-4}} = 3.125 \times 10^{15} \text{ cm}^{-3}$$

In the bulk, the hole concentration will be equal to N_A.

$$N_A = n_i\, e^{-\frac{\psi_B}{V_T}}$$

$$\Rightarrow \quad \psi_B = -V_T \ln\left(\frac{N_A}{n_i}\right) = -0.026 \ln\left(\frac{3.125\times10^{15}}{1.5\times10^{10}}\right)$$

$$\therefore \quad \psi_B = -0.318 \text{ V}$$

(ii) Potential drop in the semiconductor is the difference between the potential at the surface, ψ_S and the potential at the bulk semiconductor ψ_B.

i.e, $$V_{SC} = \psi_s - \psi_B$$

where, $$V_{SC} = \int E\cdot dx = \frac{E_{max}W}{2} + 0.2\ \mu\text{m} \times 20 \times 10^3$$

$$= \frac{1}{2}\times0.6\times10^{-4}\times10\times10^3 + 0.2\times10^{-4}\times20\times10^3$$

$$\therefore \quad V_{SC} = 0.7 \text{ V}$$

$$\therefore \quad \psi_S = V_{SC} + \psi_B = 0.7 \text{ V} + (-0.318 \text{ V})$$

$$\psi_S = 0.382 \text{ V}$$

(iii) Threshold voltage, V_T

$$V_T = -\frac{Q_B}{C_{ox}} + 2|\psi_B|$$

Q_B = Charge in the space charge region of semiconductor, is qN_AW and W is width of space charge region.

$$W = \sqrt{\frac{2\in_{si}}{q}\times\frac{2|\psi_B|}{N_A}}$$

$$\therefore \quad Q_B = -qN_A\sqrt{\frac{2\in_{si}\times2|\psi_B|}{qN_A}}$$

$$V_T = \frac{\sqrt{2\epsilon_{si} \times qN_A \times 2|\psi_B|}}{C_{ox}} + 2|\psi_B|$$

$$= \frac{\sqrt{2\times10^{-12}\times1.6\times10^{-19}\times3.125\times10^{15}\times2\times0.318}}{15\times10^{-9}} + 2\times0.318$$

$$V_T = 2.317 \text{ V}$$

(iv) The applied potential at the gate will be the addition of the potential drop at the dielectric and the potential drop in the semiconductor.

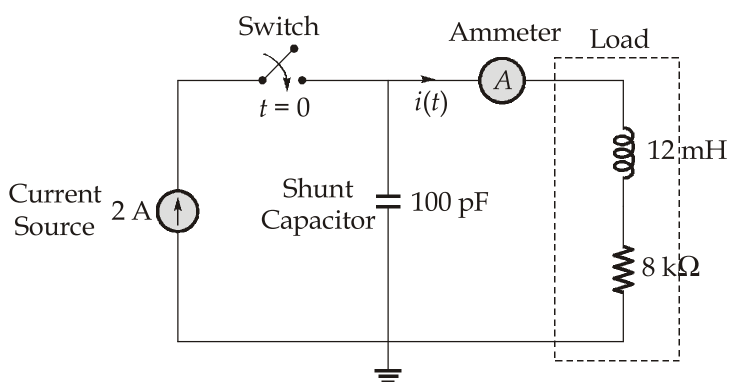
i.e, $$V_G = V_{ox} + V_{SC}$$

$$= 30\times10^3 \frac{V}{\text{cm}}\times0.2\times10^{-4}\text{cm} + \frac{1}{2}\times10\times10^3\times\frac{V}{\text{cm}}\times0.6\times10^{-4}\text{cm}$$

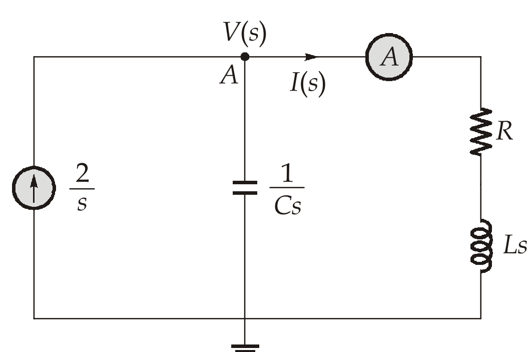
$\therefore$ $$V_G = V_{ox} + V_{SC} = 0.6 + 0.3 = 0.9 \text{ V}$$

Q.3 (b) Solution:

(i) Given circuit is



In s-domain, the circuit can be redrawn as below,



On applying KCL at node A,

We get, $$\frac{2}{s} = \frac{V(s)}{1/Cs} + \frac{V(s)}{R+Ls}$$

$$\frac{2}{s} = V(s)\left[Cs + \frac{1}{R+Ls}\right]$$

$$V(s) = \frac{2/s}{Cs + \frac{1}{R+Ls}}$$

$$V(s) = \frac{(2/s)(R+Ls)}{LCs^2 + RCs + 1}$$

$$I(s) = \frac{V(s)}{R+Ls} = \frac{(2/s)(R+Ls)}{LCs^2 + RCs + 1} \cdot \frac{1}{R+Ls}$$

$$I(s) = \frac{2/LC}{s\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

Given, L = 12 mH; C = 100 pF; $R = 8 \times 10^3\ \Omega$

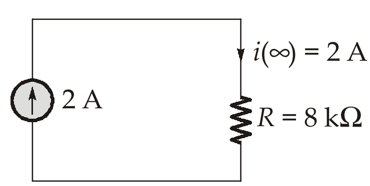
Comparing the characteristic equation $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$,

we get, $$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} = \frac{8\times10^3}{2}\sqrt{\frac{100\times10^{-12}}{12\times10^{-3}}} = 0.365$$

Thus, the response is under-damped. Hence,

$$\text{Maximum over shoot} = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} = e^{-\left(\frac{0.365\pi}{\sqrt{1-(0.365)^2}}\right)} = 0.29$$

At steady state, the inductor acts as short circuit and the capacitor acts as open circuit. Thus, the circuit at steady state is drawn as below,

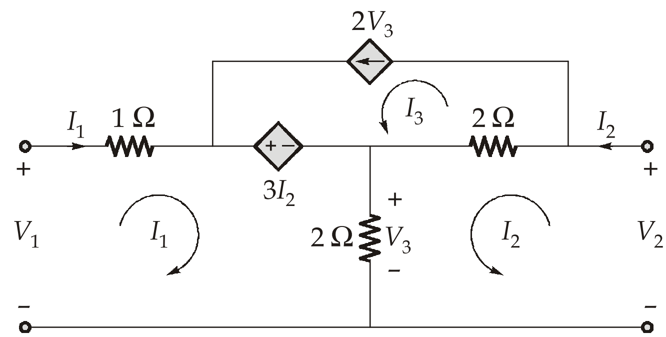


$$\text{Maximum overshoot} = \frac{i(t)_{max} - i(\infty)}{i(\infty)} = 0.29$$

$$i(t)_{max} = 2 + 2 \times 0.29 = 2.58 \text{ mA}$$

Thus, the maximum ammeter reading that will be observed after the switch is closed is 2.58 mA.

(ii) Given circuit is



Applying KVL to Mesh 1,

$$V_1 - I_1 - 3I_2 - 2(I_1 + I_2) = 0$$

$$V_1 = 3I_1 + 5I_2 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 - 2(I_2 - I_3) - 2(I_1 + I_2) = 0$$

$$V_2 - 2I_2 + 2I_3 - 2I_1 - 2I_2 = 0$$

$$V_2 = 2I_1 + 4I_2 - 2I_3 \quad \text{...(ii)}$$

Writing equation for Mesh 3,

$$I_3 = 2V_3 \quad \text{...(iii)}$$

From figure, $V_3 = 2(I_1 + I_2)$

$$I_3 = 2V_3 = 4I_1 + 4I_2 \quad \text{...(iv)}$$

Substituting the equation (iv) in the equation (ii),

$$V_2 = -6I_1 - 4I_2 \quad \text{...(v)}$$

The z-parameter equations of a two-port network are given by

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Comparing equation (i) and (v) with z-parameter equations, we get

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix}$$

Y-Parameters:

The Y parameters matrix of a two-port network is given by the inverse of the Z parameters matrix i.e. $[Y] = [Z]^{-1}$.

We have, $\Delta Z = Z_{11}\, Z_{22} - Z_{12}Z_{21}$

$$= 3(-4) - 5(-6) = 18$$

Thus,
$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{-4}{18} = \frac{-2}{9}\mho$$
$$Y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-(-6)}{18} = \frac{1}{3}\mho$$
$$Y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-5}{18}\mho$$
$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{3}{18}\mho = \frac{1}{6}\mho$$

Hence, Y-parameters are
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -\frac{2}{9} & \frac{-5}{18} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

Q.3 (c) Solution:

(i) A solar cell is a pn junction device with no voltage directly applied across the junction. The solar cell converts photon power into electrical power and delivers this power to a load.

Consider the pn junction solar cell shown in figure with a resistive load.

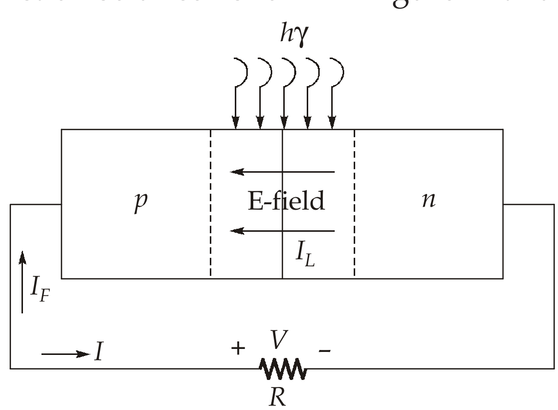


Fig.: A pn junction solar cell with resistive load.

Even with zero bias applied to the junction, an electric field exists in the space charge region as shown in the figure. Incident photon illumination can create electron-hole pairs in the space charge region that will be swept out producing the photo current I_L in the reverse-bias direction as shown.

The photo current I_L produces a voltage drop across the resistive load which forward biases the pn junction. The forward-bias voltage produces a forward-bias current I_F as indicated in the figure. The net *pn* junction current, in the reverse-bias direction, is

$$I = I_L - I_F = I_L - I_S\left[\exp\left(\frac{qV}{kT}\right) - 1\right] \quad \text{...(i)}$$

where the ideal diode equation has been used.

As the diode becomes forward biased, the magnitude of the electric field in the space charge region decreases, but does not go to zero or change direction. The photocurrent is always in the reverse-bias direction and the net solar cell current is also always in the reverse-bias direction.

There are two limiting cases of interest. The short-circuit condition occurs when $R = 0$, so that $V = 0$. The current in this case is referred to as the short-circuit current.

$$I = I_{SC} = I_L$$

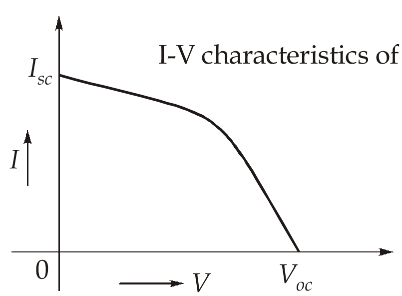
The second limiting case is the open-circuit condition and occurs when $R \to \infty$. The net current is zero and the voltage produced is the open-circuit voltage. The photo current is just balanced by the forward-biased junction current so we have,

$$I = 0 = I_L - I_S\left[\exp\left(\frac{qV_{oc}}{kT}\right) - 1\right]$$

We can find the open circuit voltage V_{oc} as,

$$V_{oc} = V_t \ln\left(1 + \frac{I}{I_S}\right)$$

A plot of the diode current I as a function of the diode voltage V from equation (i) is as shown in figure.



(ii) Let I_n represents the current due to electrons and I_p represents the current due to holes.

The total diode current is given by

$$I = I_n + I_p = \left(\frac{qAD_n n_{p0}}{L_n} + \frac{qAD_p p_{n0}}{L_p}\right)(e^{qV/kT} - 1)$$

where
$$n_{p0} = \frac{n_i^2}{N_a} \text{ and } p_{n0} = \frac{n_i^2}{N_d}$$

Given, $$\frac{I_n}{I_n + I_p} = 0.95$$

$$\Rightarrow \quad \frac{\left(\frac{qAD_n n_{p_0}}{L_n}\right)}{\frac{qAD_n n_{p_0}}{L_n} + \frac{qAD_p p_{n_0}}{L_p}} = \frac{\frac{D_n}{L_n N_a}}{\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}} = \frac{\frac{D_n}{L_n}}{\frac{D_n}{L_n} + \frac{D_p}{L_p}\cdot\frac{N_a}{N_d}} = 0.95$$

We have,

$$L_n = \sqrt{D_n \tau_{n_0}} = \sqrt{25 \times 0.1 \times 10^{-6}}$$
$$= 1.581 \times 10^{-3} \text{ cm}$$
$$L_p = \sqrt{D_p \tau_{p_0}} = \sqrt{10 \times 0.1 \times 10^{-6}}$$
$$= 1 \times 10^{-3} \text{ cm}$$

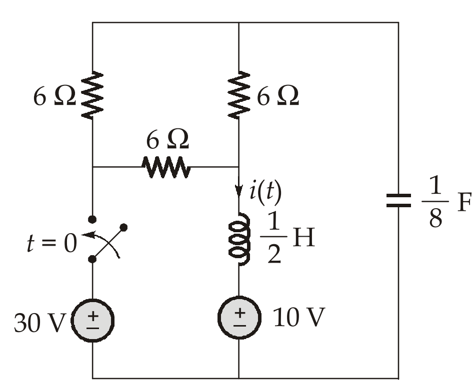
$$\frac{\frac{D_n}{L_n}}{\frac{D_n}{L_n} + \frac{D_p}{L_p}\cdot\frac{N_a}{N_d}} = \frac{\frac{25}{1.581 \times 10^{-3}}}{\frac{25}{1.58 \times 10^{-3}} + \frac{10}{1 \times 10^{-3}}\cdot\left(\frac{N_a}{N_d}\right)} = 0.95$$

$$\frac{15813}{15813 + 10000\left(\frac{N_a}{N_d}\right)} = 0.95$$

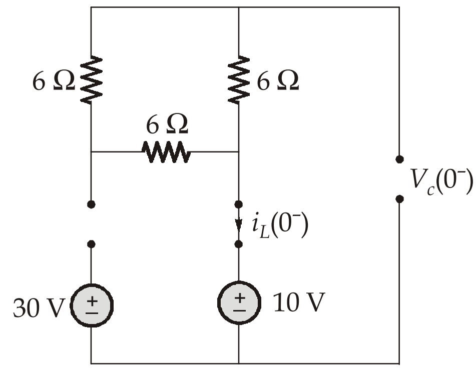
$$\Rightarrow \quad \left(\frac{N_a}{N_d}\right) = 0.0832$$

Q.4 (a) Solution:

Given circuit is



For $t < 0$: At steady state, the capacitor acts as open circuit and the inductor acts as short circuit. Thus,

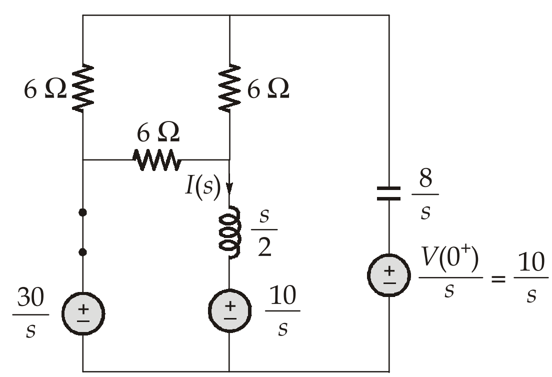


$$i_L(0^-) = 0 \text{ A}$$
$$V_c(0^-) = 10 \text{ V}$$

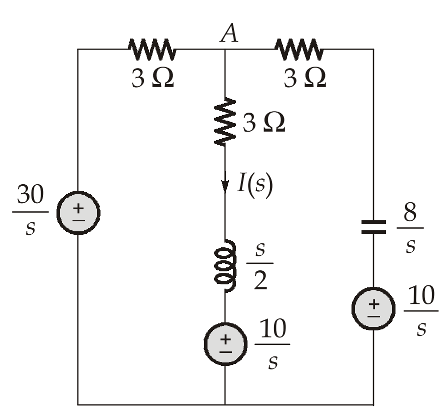
For $t > 0$: Since the inductor current and the capacitor voltage cannot change instantaneously, thus

$$i_L(0^+) = i_L(0^-) = 0 \text{ A}$$
$$V_c(0^+) = V_c(0^-) = 10 \text{ A}$$

For $t > 0$, the circuit in s-domain can be drawn as below,



Applying Delta to star transformation,



KCL at A

$$\frac{V_A(s)-\frac{30}{s}}{3}+\frac{V_A(s)-\frac{10}{s}}{3+\frac{s}{2}}+\frac{V_A(s)-\frac{10}{s}}{3+\frac{8}{s}}=0$$

$$V_A(s)\left[\frac{1}{3}+\frac{2}{s+6}+\frac{s}{3s+8}\right] = \frac{10}{s}+\frac{20}{s(s+6)}+\frac{10}{3s+8}$$

$$V_A(s)\left[\frac{(s+6)(3s+8)+6(3s+8)+3s(s+6)}{3(s+6)(3s+8)}\right] = \frac{10s(s+6)(3s+8)+20s(3s+8)+10s^2(s+6)}{s^2(s+6)(3s+8)}$$

$$V_A(s) = \frac{3}{s^2}\times\frac{10s(s+6)(3s+8)+20s(3s+8)+10s^2(s+6)}{(s+6)(3s+8)+6(3s+8)+3s(s+6)}$$

We have,

$$I(s) = \frac{V_A(s)-\frac{10}{s}}{3+\frac{s}{2}}$$

$$= \frac{\frac{3}{s^2}\times\frac{10s(s+6)(3s+8)+20s(3s+8)+10s^2(s+6)}{(s+6)(3s+8)+6(3s+8)+3s(s+6)}-\frac{10}{s}}{3+\frac{s}{2}}$$

$$= \frac{\begin{array}{c}30s^2(s+6)(3s+8)+60s^2(3s+8)+30s^3(s+6)\\ -10s^2(s+6)(3s+8)-60s^2(3s+8)-30s^3(s+6)\end{array}}{\frac{s^3}{2}(s+6)}$$

$$I(s) = \frac{40(3s+8)}{s(s+6)}$$

From partial fraction expansion,

$$I(s) = \frac{A}{s}+\frac{B}{s+6}$$

where

$$A = \lim_{s\to 0}\frac{40(3s+8)}{(s+6)}=\frac{40\times 8}{6}=\frac{160}{3}$$

$$B = \lim_{s\to -6}\frac{40(3s+8)}{s}=\frac{40\times 10}{6}=\frac{200}{3}$$

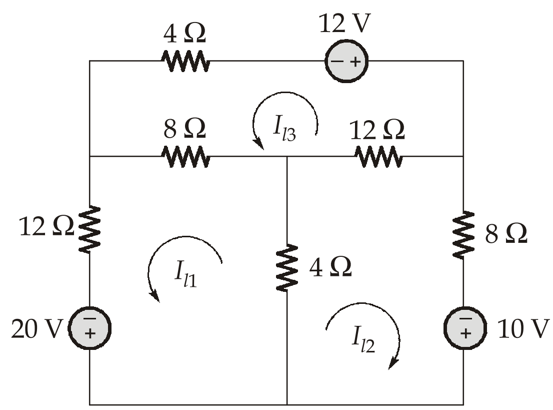
$$I(s) = \frac{40}{3}\left(\frac{4}{s}+\frac{5}{s+6}\right)$$

Taking Inverse Laplace transform, we get

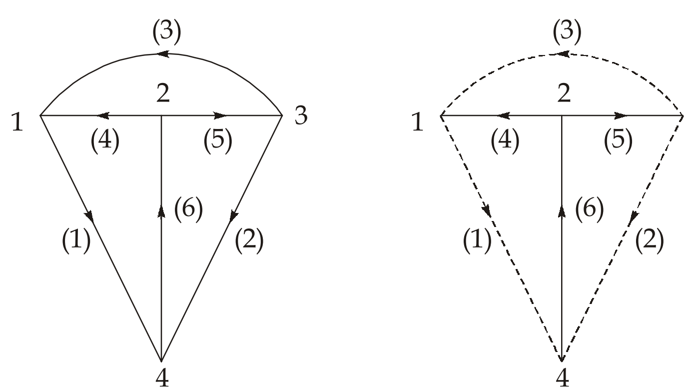
$$i(t) = \frac{40}{3}(4+5e^{-6t})u(t)A$$

Q.4 (b) Solution:

Given circuit is



The oriented graph and its selected tree are shown in figure.



Tie set : {1, 4, 6}

{2, 5, 6}

{3, 5, 4}

Tie set matrix (B),

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix} \end{array}$$

The KVL equations in matrix from is given by

$$BZ_b B^T I_l = BV_s - BZ_b I_s$$

Here, $I_s = 0,$

$$BZ_b B^T I_l = BV_s$$

where Impedance Matrix, $Z_b = \begin{bmatrix} 12 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$ $B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$V_s = \begin{bmatrix} 20 \\ 10 \\ -12 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We get, $BZ_b = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 12 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 12 & 0 & 0 & 8 & 0 & 4 \\ 0 & 8 & 0 & 0 & 12 & 4 \\ 0 & 0 & 4 & -8 & 12 & 0 \end{bmatrix}$$

$$BZ_b B^T = \begin{bmatrix} 12 & 0 & 0 & 8 & 0 & 4 \\ 0 & 8 & 0 & 0 & 12 & 4 \\ 0 & 0 & 4 & -8 & 12 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 4 & -8 \\ 4 & 24 & 12 \\ -8 & 12 & 24 \end{bmatrix}$$

$$BV_s = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \\ -12 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ -12 \end{bmatrix}$$

Hence, the KVL equation in matrix form is given by

$$\begin{bmatrix} 24 & 4 & -8 \\ 4 & 24 & 12 \\ -8 & 12 & 24 \end{bmatrix} \begin{bmatrix} I_{l1} \\ I_{l2} \\ I_{l3} \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ -12 \end{bmatrix}$$

Solving this matrix equations using Cramer's rule, the loop currents can be obtained as, we get,

$$I_{l1} = \frac{\begin{vmatrix} 20 & 4 & -8 \\ 10 & 24 & 12 \\ -12 & 12 & 24 \end{vmatrix}}{\begin{vmatrix} 24 & 4 & -8 \\ 4 & 24 & 12 \\ -8 & 12 & 24 \end{vmatrix}} = \frac{3840}{7680} = \frac{1}{2} = 0.5 \text{ A}$$

$$I_{l2} = \frac{\begin{vmatrix} 24 & 20 & -8 \\ 4 & 10 & 12 \\ -8 & -12 & 24 \end{vmatrix}}{\begin{vmatrix} 24 & 4 & -8 \\ 4 & 24 & 12 \\ -8 & 12 & 24 \end{vmatrix}} = \frac{5120}{7680} = \frac{2}{3} = 0.667 \text{ A}$$

$$I_{l3} = \frac{\begin{vmatrix} 24 & 4 & 20 \\ 4 & 24 & 10 \\ -8 & 12 & -12 \end{vmatrix}}{\begin{vmatrix} 24 & 4 & -8 \\ 4 & 24 & 12 \\ -8 & 12 & 24 \end{vmatrix}} = -\frac{5120}{7680} = \frac{-2}{3} = -0.667 \text{ A}$$

Q.4 (c) Solution:

(i) 1. We know, current density in an n-type semiconductor,

$$J = nq\mu_n E + qD_n \frac{dn}{dx}$$

At thermal equilibrium,

$$J = 0$$

$$\Rightarrow \quad 0 = nq\mu_n E + qD_n \frac{dn}{dx}$$

Here, $$n = N_d = N_{d_0} e^{-\alpha x}$$

$$\Rightarrow \quad 0 = N_{d_0} e^{-\alpha x} q\mu_n E + qD_n \frac{d}{dx}\left[N_{d_0} e^{-\alpha x}\right]$$

$$\Rightarrow \quad 0 = N_{d_0} e^{-\alpha x} q\mu_n E + qD_n \left[-\alpha N_{d_0} e^{-\alpha x}\right]$$

$$\Rightarrow \quad N_{d_0} e^{-\alpha x} q\mu_n E = qD_n \alpha N_{d_0} e^{-\alpha x}$$

$$\Rightarrow \quad \mu_n E = D_n \alpha$$

$$\Rightarrow \quad E = \frac{D_n}{\mu_n}\alpha = V_T \alpha = \frac{kT}{q}\alpha$$

2. The potential difference between $x = 0$ and $x = \frac{1}{\alpha}$ is,

$$V = -\int_0^{1/\alpha} E dx = -\int_0^{1/\alpha} \left(\frac{kT}{q}\right)\alpha \, dx$$

$$= -\left(\frac{kT}{q}\right)\alpha (x)_0^{1/\alpha} = -\frac{kT}{q}$$

$$\Rightarrow \quad V = -V_T = -\frac{kT}{q} = -\frac{D_n}{\mu_n}$$

(ii) Let us consider two materials in intimate contact at equilibrium.

There is no current, and therefore no net charge transport, at thermal equilibrium. There is also no net transfer of energy.

Therefore, for each energy E in the figure given, any transfer of electrons from material 1 to material 2 must be exactly balanced by the opposite transfer of electrons from material 2 to material 1.

$\therefore$ At energy E, the rate of transfer of electrons from material 1 to 2 is proportional to the number of filled states at E in material 1 times the number of empty states at E in material 2:

Fermi-Dirac distribution function $f(E)$ provides the probability of electron occupying a certain energy level E and $N(E)$ represents the number of available electron states at energy level E. Thus,

Rate of transfer of electrons from material 1 to material 2 $\propto N_1(E)f_1(E)\cdot N_2(E)[1-f_2(E)]$

Similarly,

Rate of transfer of electrons from material 2 to material 1 $\propto N_2(E)f_2(E)\cdot N_1(E)[1-f_1(E)]$

At equilibrium, these rates must be equal, i.e.

$$N_1(E)f_1(E)\cdot N_2(E)[1-f_2(E)] = N_2(E)f_2(E)\cdot N_1(E)[1-f_1(E)]$$

$$\Rightarrow N_1(E)N_2(E)f_1(E) - N_1(E)N_2(E)f_1(E)f_2(E) = N_1(E)N_2(E)f_2(E) - N_1(E)N_2(E)f_1(E)f_2(E)$$

$$\Rightarrow \quad N_1(E)N_2(E)f_1(E) = N_1(E)N_2(E)f_2(E)$$

$$\Rightarrow \quad f_1(E) = f_2(E)$$

$$\Rightarrow \quad \frac{1}{1+e^{(E-E_{F_1})/kT}} = \frac{1}{1+e^{(E-E_{F_2})/kT}}$$

$$\Rightarrow \quad E_{F_1} = E_{F_2}$$

$\therefore$ No discontinuity or gradient can arise in the equilibrium Fermi level E_F. More generally, we can state that the Fermi level at equilibrium must be constant throughout the materials in intimate contact, (or) no gradient exists in the Fermi level at equilibrium:

$$\frac{dE_F}{dx} = 0$$

Section B : Network Theory + Electronic Devices and Circuits

Q.5 (a) Solution:

(i) The Hall voltage,

$$V_H = E_H W$$
$$= -16.5\times10^{-3}\times5\times10^{-2}$$
$$= -0.825 \text{ mV}$$

(ii) As V_H is negative, it is n-type sample.

(iii) The hall voltage is given by

$$V_H = \frac{R_H I_x B_z}{d}, \qquad \text{where, } R_H = \frac{1}{nq}$$

The majority carrier concentration,

$$n = -\frac{I_x B_z}{q d V_H}$$

$$= -\frac{(0.5\times10^{-3})\times(6.5\times10^{-2})}{(1.6\times10^{-19})\times(5\times10^{-5})\times(-0.825\times10^{-3})}$$

$\Rightarrow$ $$n = 4.924 \times 10^{21}\ \text{m}^{-3}$$

$\Rightarrow$ $$n = 4.924 \times 10^{15}\ \text{cm}^{-3}$$

(iv) The majority carrier mobility,

The conductivity of n-type semiconductor is given by

$$\sigma = nq\mu_n$$

Also, we have $$J_x = \sigma E_x$$

$\Rightarrow$ $$\sigma = \frac{I_x}{Wd}\times\frac{L}{V_x}$$

We get, $$\mu_n = \frac{I_x L}{qnV_x Wd}$$

$$= \frac{(0.5\times10^{-3})\times(0.5\times10^{-2})}{(1.6\times10^{-19})\times(4.924\times10^{21})\times(1.25)\times(5\times10^{-4})\times(5\times10^{-5})}$$

$$= 0.1015$$

$\Rightarrow$ $$\mu_n = 0.1015\ \text{m}^2/\text{V-s} = 1015\ \text{cm}^2/\text{V-s}$$

Q.5 (b) Solution:

(i) The built-in potential for the photodiode is given by

$$V_{bi} = V_T \ln\left[\frac{N_a N_d}{n_i^2}\right]$$

$\Rightarrow$ $$V_{bi} = 0.0259\ \ln\left[\frac{2\times10^{16}\times10^{18}}{(1.5\times10^{10})^2}\right]$$

$\Rightarrow$ $$V_{bi} = 0.832\ V$$

The space charge width is,

$$W = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\left(\frac{N_a+N_d}{N_a N_d}\right)}$$

$$= \sqrt{\frac{2\times11.7\times8.85\times10^{-14}\times(0.832+5)}{(1.6\times10^{-19})}\left(\frac{2\times10^{16}+10^{18}}{2\times10^{16}\times10^{18}}\right)}$$

$$= 0.6205\ \mu\text{m}$$

∴ The prompt photocurrent density is,

$$J_{L_1} = qG_LW$$

$$= 1.6 \times 10^{-19} \times 10^{21} \times 0.6205 \times 10^{-4}$$

$$= 9.928\ \text{mA/cm}^2$$

Note: Prompt photocurrent is the component of photocurrent that responds very quickly to the photon illumination and arises from electron-hole pairs generated within the depletion region width W.

(ii) The total steady-state photocurrent density is,

$$J_L = q(W + L_n + L_p)\ G_L$$

where,

$$L_n = \sqrt{D_n\tau_n} = \sqrt{25\times2\times10^{-7}} = 22.4\ \mu\text{m}$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{10\times10^{-7}} = 10\ \mu\text{m}$$

∴

$$J_L = 1.6 \times 10^{-19} \times [0.6205 + 22.4 + 10] \times 10^{-4} \times 10^{21}$$

$$J_L = 0.53\ \text{A/cm}^2$$

Q.5 (c) Solution:

The Fermi dirac distribution function, $f(E)$ gives the probability that an energy level is occupied by an electron.

(i) Given,

$$f(E) = 10^{-6}$$

$$E - E_F = 0.55\ \text{eV}$$

The Fermi dirac distribution function is given by,

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$10^{-6} = \frac{1}{1+e^{0.55/kT}}$$

$$\Rightarrow \quad 1 + e^{0.55/kT} = 10^6$$

$$\Rightarrow \quad e^{0.55/kT} \simeq 10^6$$

$$\Rightarrow \quad \frac{0.55}{kT} = \ln(10^6)$$

$$\Rightarrow \quad \frac{0.55}{kT} = 13.82$$

$$\Rightarrow \quad T = \frac{0.55\text{ eV}}{8.62\times10^{-5}\text{ eV/K}\times13.82}$$

$$\Rightarrow \quad T = 461.69\ K$$

(ii) Built-in potential, $V_{bi} = kT\ln\left(\frac{N_aN_d}{n_i^2}\right)$

If the doping in the p-region increases by a factor of 2,

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT\ln\left(\frac{2N_aN_d}{n_i^2}\right)}{kT\ln\left(\frac{N_aN_d}{n_i^2}\right)} = \frac{kT\ln 2 + kT\ln\left(\frac{N_aN_d}{n_i^2}\right)}{kT\ln\left(\frac{N_aN_d}{n_i^2}\right)}$$

We can write this as,

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT\ln 2 + V_{bi}(N_a)}{V_{bi}(N_a)}$$

$$\Rightarrow \quad V_{bi}(2N_a) = kT\ln 2 + V_{bi}(N_a)$$

$$\Rightarrow \quad \Delta V_{bi} = kT\ln 2 = (0.0259)\ln 2$$

$$\Rightarrow \quad \Delta V_{bi} = 0.017953$$

$$\Rightarrow \quad \Delta V_{bi} \simeq 17.953\text{ mV}$$

5. (d)

(i) We have,

$$v = \begin{cases} V_m\sin\theta\ ; & 0<\theta<\frac{\pi}{3} \\ \frac{\sqrt{3}}{2}V_m\ ; & \frac{\pi}{3}<\theta<\frac{\pi}{2} \\ V_m\sin\theta; & \frac{\pi}{2}<\theta<\pi \end{cases}$$

The rms value of the voltage is given by,

$$V_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)d\theta}$$

$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/3} V_m^2 \sin^2\theta d\theta + \int_{\pi/3}^{\pi/2}\left(\frac{\sqrt{3}}{2}V_m\right)^2 d\theta + \int_{\pi/2}^{\pi} V_m^2 \sin^2\theta d\theta\right]}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left\{\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/3} + \left(\frac{\sqrt{3}}{2}\right)^2 [\theta]_{\pi/3}^{\pi/2} + \left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_{\pi/2}^{\pi}\right\}}$$

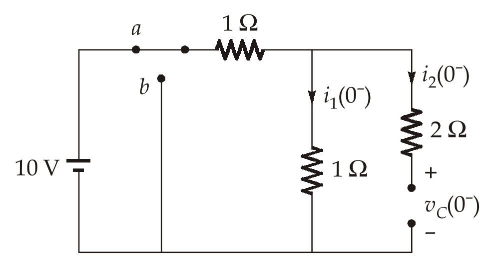
$$= \sqrt{\frac{V_m^2}{\pi}\left\{\left[\frac{\pi}{6}-\frac{\sqrt{3}}{8}\right] + \left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{\pi}{6}\right) + \left[\frac{\pi}{2}-\frac{\pi}{4}\right]\right\}}$$

$$= V_m\sqrt{\frac{1}{6}-\frac{\sqrt{3}}{8\pi}+\frac{1}{8}+\frac{1}{4}} = \sqrt{0.473}\, V_m$$

The power dissipated in the resistor is,

$$P = \frac{V_{rms}^2}{R} = 0.473 \text{ W}$$

(ii) For $t < 0$: The network has attained steady-state condition. Hence, the capacitor acts as open circuit and the inductor acts as short circuit.

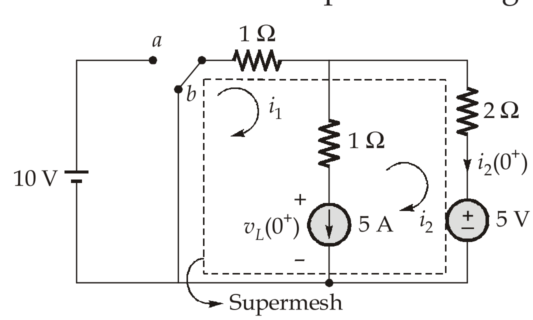


$$i_1(0^-) = \frac{10}{1+1} = \frac{10}{2} = 5 \text{ A}$$

$$i_2(0^-) = 0 \text{ A}$$

$$v_c(0^-) = 10\times\frac{1}{1+1} = \frac{10}{2} = 5 \text{ V}$$

At $t = 0^+$: Since the inductor current and capacitor voltage cannot change abruptly,



$$i_1(0^+) = i_1(0^-) = 5 \text{ A}$$
$$v_c(0^+) = v_c(0^-) = 5 \text{ V}$$

KVL around loop-1:

$$i_1 + (i_1 - i_2) + v_L(0^+) = 0$$

But $i_1 - i_2 = 5$ A

$$\Rightarrow \quad i_1 + 5 + v_L(0^+) = 0 \quad \text{...(i)}$$

KVL around the outer loop,

$$i_1 + 2i_2 + 5 = 0$$
$$\Rightarrow \quad i_1 = -(5 + 2i_2)$$
$$\Rightarrow \quad 5 + i_2 = -(5 + 2i_2) \quad \Rightarrow \quad i_2 = \frac{-10}{3}\text{A} = i_2(0^+)$$
$$\therefore \quad i_1 = -\left[5 + 2\times\left(\frac{-10}{3}\right)\right] = \frac{5}{3}\text{A}$$

From equation (i),

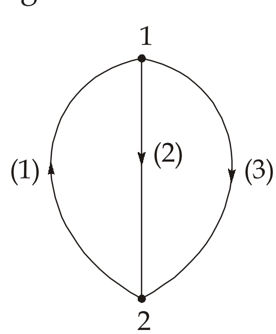
$$\frac{5}{3} + 5 + v_L(0^+) = 0$$
$$\Rightarrow \quad v_L(0^+) = \frac{-20}{3}\text{V}$$

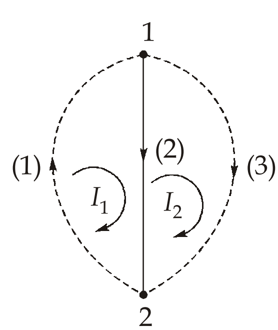
We know, $$v_L(0^+) = \frac{Ldi_L(0^+)}{dt} = \frac{Ldi_1(0^+)}{dt}$$

$$\Rightarrow \quad \frac{di_1(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{\left(\frac{-20}{3}\right)}{2} = \frac{-10}{3}\text{A/s}$$

Q.5 (e) Solution:

For the given branch currents, the current flows into the dotted terminals. From the dot convention, the mutual inductance is positive. The oriented graph and its selected tree are shown in fig.





Links : {1, 3}
Tie set 1 : {1, 2}
Tie set 2 : {3, 2}

Tie set matrix (B)

$$B = \begin{matrix} \\ I_1 \\ I_2 \end{matrix}\begin{matrix} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \end{matrix}$$

The KVL equation in matrix form is given by

$$BZ_b B^T I_l = BV_s - BZ_b I_s$$

Here, $I_s = 0$

$$BZ_b B^T I_l = BV_s$$

Where, Impedance matrix, $Z_b = \begin{bmatrix} 3+j4 & j3 & 0 \\ j3 & j5 & 0 \\ 0 & 0 & -j8 \end{bmatrix}$

$$B^T = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad V_s = \begin{bmatrix} 50\angle 45^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$BZ_b = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3+j4 & j3 & 0 \\ j3 & 5j & 0 \\ 0 & 0 & -j8 \end{bmatrix}$$

$$= \begin{bmatrix} 3+j7 & j8 & 0 \\ -j3 & -j5 & -j8 \end{bmatrix}$$

$$BZ_b B^T = \begin{bmatrix} 3+j7 & j8 & 0 \\ -j3 & -j5 & -j8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3+j15 & -j8 \\ -j8 & -j3 \end{bmatrix}$$

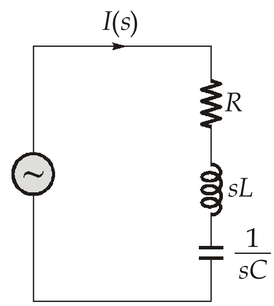
$$BV_s = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 50\angle 45^\circ \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 50\angle 45^\circ \\ 0 \end{bmatrix}$$

Hence, the network equilibrium equations in matrix form is given by,

$$\begin{bmatrix} 3+j15 & -j8 \\ -j8 & -j3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50\angle 45^\circ \\ 0 \end{bmatrix}$$

Q.6 (a) Solution:

(i) Series RLC Circuit:



***s*-domain**

For series RLC circuit,

$$Z(s) = R + sL + \frac{1}{sC} \quad ...(1)$$

Given,

$$Z(s) = \frac{20((s+1)^2 + 10^2)}{s} = \frac{20(s^2 + 2s + 101)}{s}$$

$$Z(s) = 20s + 40 + \frac{2020}{s} \quad ...(2)$$

Comparing (1) with (2),

$$R = 40\ \Omega$$

$$L = 20\ \text{H}$$

$$C = \frac{1}{2020} = 4.95 \times 10^{-4}\ \text{F}$$

- Resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\omega_0 = \frac{1}{\sqrt{20 \times 4.95 \times 10^{-4}}} = 10.05\ \text{rad/sec}$$

- Q-factor, $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$Q = \frac{1}{40} \times \sqrt{\frac{20}{4.95 \times 10^{-4}}} = 5.02$$

- Bandwidth, $B = \frac{R}{L} = \frac{40}{20} = 2\ \text{rad/sec}$

- For series RLC circuit, $Z = R + j(X_L - X_C)$.

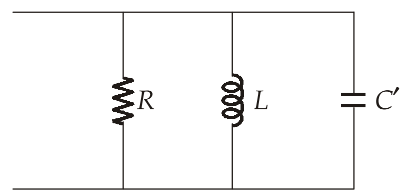
  At resonance, $X_L = X_C$

Thus,

Impedance under resonance condition,

$$Z_0 = R = 40\ \Omega$$

(ii) When a capacitance C_{ext} is added in series with the capacitor C, the equivalent parallel RLC circuit is given by



where

$$R = 40\ \Omega$$

$$L = 20\ \text{H}$$

$$C' = \left[\frac{(4.95\times10^{-4}C_{ext})}{4.95\times10^{-4}+C_{ext}}\right]F$$

With capacitor C_{ext}, new resonant frequency,

$$\omega_0' = 5\omega_0$$

$$\omega_0' = 5 \times 10.05 = 50.25\ \text{rad/sec}$$

Here,

$$Y = \frac{1}{R}+\frac{1}{j\omega L}+j\omega C$$

and at resonance,

$$\left|\frac{1}{\omega_0' L}\right| = |\omega_0' C'|$$

$$|\omega_0'|^2 = \frac{1}{LC'}$$

$$C' = \frac{1}{(\omega_0')^2 L}$$

On substuting the values, we get

$$C' = \frac{1}{((50.25)^2\times20)} = 19.8\ \mu\text{F}$$

As

$$C' = \frac{4.95\times10^{-4}C_{ext}}{4.95\times10^{-4}+C_{ext}}$$

$$\frac{(4.95\times10^{-4})(C_{ext})}{(4.95\times10^{-4}+C_{ext})} = 19.8\ \mu\text{F}$$

$$25C_{ext} = 4.95\times10^{-4} + C_{ext}$$

$$\Rightarrow \quad C_{ext} = 20.625\ \mu\text{F}$$

Q.6 (b) Solution:

(i) Given: $T = 550$ K

$n_i = 3.20 \times 10^{14}$ cm^{-3}

For the intrinsic carrier concentration to contribute no more than 5 percent of the total electron concentration, we set the minimum donor concentration as $n = 1.05\,N_d$. We have,

$$\text{Electron concentration, } n = \frac{N_d}{2}+\sqrt{\left(\frac{N_d}{2}\right)^2+n_i^2}$$

$$\Rightarrow \quad 1.05\,N_d = 0.5\,N_d + \sqrt{0.25N_d^2+\left(3.20\times10^{14}\right)^2}$$

$$(0.55\,N_d)^2 = 0.25N_d^2+\left(3.20\times10^{14}\right)^2$$

$$0.0525N_d^2 = \left(3.20\times10^{14}\right)^2$$

$$\Rightarrow \quad N_d = \frac{\left(3.20\times10^{14}\right)}{\sqrt{(0.0525)}}$$

$$\Rightarrow \quad N_d = 1.4\times10^{15}\ \text{cm}^{-3}$$

$\therefore$ If the temperature remains less than $T = 550$ K, then the intrinsic carrier concentration will contribute less than 5 percent of the total electron concentration for this donor impurity concentration. For $T > 550$ K, the intrinsic carrier concentration increases.

(ii) The depletion layer capacitance per unit area for a p-n junction is given by

$$C' = \frac{\epsilon_s}{\sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)(V_{bi}+V_R)}}$$

For a p^+n junction, $N_a >> N_d$

Thus,
$$C' = \frac{\epsilon_s}{\sqrt{\frac{2\epsilon_s}{qN_d}(V_{bi}+V_R)}}$$

We get,
$$\left(\frac{1}{C'}\right)^2 = \frac{2}{qN_d\epsilon_s}(V_{bi}+V_R) \quad \text{...(i)}$$

Hence, the slope of the curve is,
$$\text{Slope} = \frac{2}{q\varepsilon_s N_d}$$

$$\Rightarrow \quad N_d = \frac{2}{q\varepsilon_s \times \text{slope}} = \frac{2}{(1.6\times10^{-19})\times11.7\times8.85\times10^{-14}\times1.32\times10^{15}}$$

$$\Rightarrow \quad N_d = 9.15 \times 10^{15}\ \text{cm}^{-3}$$

Also, from equation (i),

We get the intercept of the curve on the voltage axis as V_{bi} = -0.855 V.

We know,
$$V_{bi} = V_t \ln\left(\frac{N_aN_d}{n_i^2}\right)$$

$$\Rightarrow \quad e^{\left(\frac{V_{bi}}{V_t}\right)} = \frac{N_aN_d}{n_i^2}$$

$$\Rightarrow \quad N_a = \frac{n_i^2}{N_d}e^{\left(\frac{V_{bi}}{V_t}\right)}$$

$$= \frac{(1.5\times10^{10})^2}{(9.15\times10^{15})}e^{\left(\frac{0.855}{0.0259}\right)}$$

$$\Rightarrow \quad N_a = 5.34 \times 10^{18}\ \text{cm}^{-3}$$

∴ For the given p^+n junction,

$$N_a = 5.34 \times 10^{18}\ \text{cm}^{-3}$$
$$N_d = 9.15 \times 10^{15}\ \text{cm}^{-3}$$

Q.6 (c) Solution:

Given,

Substrate doping concentration, $N_a = 10^{16}/\text{cm}^3$,

oxide thickness, t_{ox} = 50 nm

(i) At flatband,

The hole concentration at $x = 0$ is equal to the doping concentration in the semiconductor.

i.e., $$p(x = 0) = N_a = 10^{16}/\text{cm}^3$$

(ii) At threshold voltage,

the electron concentration at $x = 0$ is equal to the doping level of the substrate.

$$n(x = 0) = N_a = 10^{16}/\text{cm}^3$$

Using the mass action law,

$$np = n_i^2$$

$$p(x = 0) = \frac{n_i^2}{n(x=0)} = \frac{(10^{10})^2}{10^{16}} = 10^4/\text{cm}^3$$

(iii) Using the Boltzmann relation to relate carrier concentration across the depletion region of a MOS structure under bias, we get

$$\phi(x = 0) - \phi(x = x_d) = \frac{kT}{q}\ln\left(\frac{p(x_d)}{p(x=0)}\right)$$

$$p(x = 0) = p(x_d)\exp\left(\frac{-q}{kT}[\phi(x=0)-\phi(x=x_d)]\right)$$

$$= 10^{16}\exp\left(\frac{-0.5}{0.026}\right)$$

$\therefore$ $$p(x = 0) = 4.5 \times 10^7 \text{ cm}^{-3}$$

(iv) Given, capacitance per unit area of the MOS structure, $C = 50$ nF/cm^2

The capacitance of oxide,

$$C_{ox} = \frac{\epsilon_{\text{oxide}}}{t_{ox}} = \frac{3.45\times10^{-13}\text{F/cm}}{50\times10^{-7}\text{ cm}}$$

$\therefore$ $$C_{ox} = 6.9 \times 10^{-8} \text{ F/cm}^2$$

(or) $$C_{ox} = 69 \text{ nF/cm}^2$$

Since the oxide capacitance, C_{ox} is higher than the given capacitance of MOS structure. Hence, the MOS structure is in depletion mode.

If C_s is the depletion capacitance of the MOSFET,

We have, $$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_s}$$

$$C_s = \frac{1}{\frac{1}{C} - \frac{1}{C_{ox}}} = \frac{1}{\frac{1}{50} - \frac{1}{69}}$$

$\therefore$ $C_s = 182 \text{ nF/cm}^2$

Width of the depletion region,

$$x_d = \frac{\epsilon_{si}}{C_s} = \frac{1.05 \times 10^{-12}}{182 \times 10^{-9}} = 5.8 \times 10^{-6} \text{cm}$$

$$x_d = 58 \text{ nm}$$

The built-in potential across this depletion region (x_d) is

$$V_{bi} = \frac{qN_a x_d^2}{2\epsilon_{si}} = \frac{1.6 \times 10^{-19} \times 10^{16} \times (5.8 \times 10^{-6})^2}{2 \times 1.05 \times 10^{-12}}$$

$$V_{bi} = 2.6 \times 10^{-2} \text{ V}$$

$\therefore$ The hole concentration at oxide-semiconductor interface ($x = 0$) is

$$p(x=0) = p(x_d)e^{\frac{-qV_{bi}}{KT}}$$

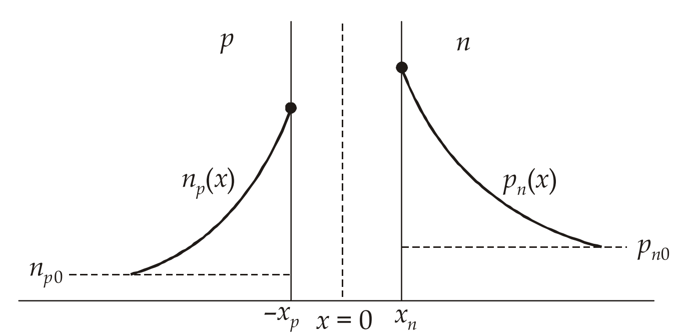
Since $p(x_d) = N_a = 10^{16} \text{ cm}^{-3}$

$$p(x=0) = 10^{16} \exp\left(\frac{-qV_{bi}}{kT}\right) = 10^{16} \exp\left(\frac{-2.6 \times 10^{-2}}{0.026}\right)$$

$$p(x=0) = 3.67 \times 10^{15}/\text{cm}^3$$

Q.7 (a) Solution:

The steady state excess carrier distribution in a *p-n* junction is given by



Neglecting the depletion layer width, at any distance, the hole concentation on *n*-side is,

$$p_n(x) = \Delta p_n(0)e^{-(x-x_n)/L_p} + p_{n_0} \quad \text{...(i)}$$

$$\approx \; p_n(0)e^{-x/L_p} + p_{n_0}$$

where $\Delta p_n(0)$ is the excess hole concentration at the junction ($x = 0$)

On differentiating equation (i) with respect to 'x',

$$\frac{dp_n(x)}{dx} = \frac{\Delta p_n(0)e^{-x/L_p}}{-L_p} \quad \text{...(ii)}$$

Diffusion current due to holes on n-side is,

$$I_{pn}(x) = -qAD_p \frac{dp_n(x)}{dx}$$

Using equation (ii),

$$I_{pn}(x) = -qAD_p \left[\frac{\Delta p_n(0)e^{-x/L_p}}{-L_p}\right]$$

At $x = 0$:

$$I_{pn}(0) = \frac{qAD_p}{L_p}\Delta p_n(0) = \frac{qAD_p}{L_p} p_{n_0}\left[e^{V/V_T} - 1\right] \quad \text{...(iii)}$$

$$\left(\text{Using Law of junction, } \Delta p_n(0) = p_{n_0}\left[e^{V/V_T} - 1\right]\right)$$

Similarly,

$$I_{np}(0) = \frac{qAD_n}{L_n} n_{p0}\left[e^{V/V_T} - 1\right] \quad \text{...(iv)}$$

Using equations (iii) and (iv)

$$\frac{I_{pn}(0)}{I_{np}(0)} = \frac{\frac{qAD_p}{L_p} p_{n_0}\left[e^{V/V_T} - 1\right]}{\frac{qAD_n}{L_n} n_{p0}\left[e^{V/V_T} - 1\right]}$$

$$= \left(\frac{D_p p_{n_0}}{D_n n_{p_0}}\right)\left(\frac{L_n}{L_p}\right)$$

$$= \left(\frac{\mu_p V_T \frac{n_i^2}{N_D}}{\mu_n V_T \frac{n_i^2}{N_A}}\right)\left(\frac{L_n}{L_p}\right) \qquad \left(\text{From Einstein's relation, } \frac{D}{\mu} = V_T\right)$$

$$= \frac{\mu_p N_A L_n}{\mu_n N_D L_p}$$

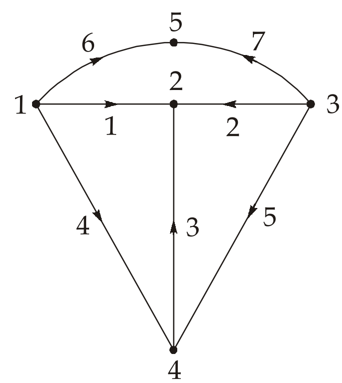
$$= \frac{q}{q} \times \frac{N_A \mu_p}{N_D \mu_n} \times \frac{L_n}{L_p} = \left(\frac{\sigma_p}{\sigma_n}\right) \times \left(\frac{L_n}{L_p}\right)$$

$$\therefore \quad \frac{I_{pn}(0)}{I_{np}(0)} = \left(\frac{\sigma_p}{\sigma_n}\right) \times \left(\frac{L_n}{L_p}\right)$$

Q.7 (b) Solution:

For drawing the oriented graph,

1. Replace all resistors, inductors and capacitors by line segments.
2. Replace all voltage sources by short circuits and current source by an open circuit.
3. Assume directions of branch currents arbitrarily and
4. Number all the nodes and branches.



(i) Complete Incidence Matrix (A_a):

The elements of the complete incidence matrix are obtained using,

$$a_{ij} = \begin{cases} 1, \text{ if branch } j \text{ is incident at node } i \text{ and is oriented away from node } j \\ -1, \text{ if branch } j \text{ is incident at node } i \text{ and is oriented towards node } j \\ 0, \text{ if branch } j \text{ is not incident at node } j \end{cases}$$

Nodes ↓	Branches → 1	2	3	4	5	6	7
1	1	0	0	1	0	1	0
2	–1	–1	–1	0	0	0	0
3	0	1	0	0	1	0	1
4	0	0	1	–1	–1	0	0
5	0	0	0	0	0	–1	–1

$$A_a = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

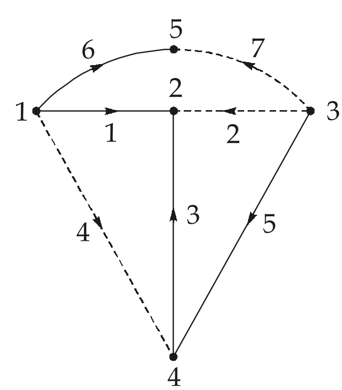
The reduced incidence matrix A is obtained from the complete incidence matrix A_a by eliminating one of the rows.

Eliminating the last row from the matrix A_a, we get the incidence matrix A as below,

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 & 0 & 0 \end{bmatrix}$$

(ii) Tie set Matrix (B)

Consider the tree with branches 1, 3, 5 and 6 i.e. branches {1, 3, 5, 6} represent Twigs and branches {2, 4, 7} represent links. The tie-set is defined as a set of branches that forms a closed loop in a graph containing one link and remaining twigs. Thus, for the given graph, we have three Tie sets or independent loops as given below,



Twigs : {1, 3, 5, 6}
Links : {2, 4, 7}
Tie set 2 : {2, 3, 5}
Tie set 4 : {4, 1, 3}
Tie set 7 : {7, 6, 1, 3, 5}

The elements of the tie set matrix are obtained using,

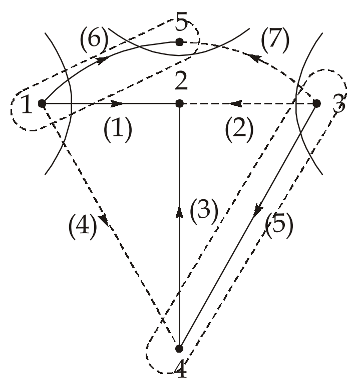
$$b_{ij} = \begin{cases} 1, \text{ if branch } j \text{ is in loop } i \text{ and their orientations coincide} \\ -1, \text{ if branch } j \text{ is in loop } i \text{ and their orientation do no coincide} \\ 0, \text{ if branch } j \text{ is not in loop } i \end{cases}$$

Tie set matrix,

$$B = \begin{array}{c} \\ 2 \\ 4 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 1 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & -1 & 1 \end{bmatrix} \end{array}$$

(iii) f-cut set matrix (Q)

A fundamental cut-set of a graph with respect to a tree is a cut-set formed by one twig and a set of links that, when removed, disconnects the graph into two or more parts. Thus, for the twigs {1, 3, 5, 6}, the fundamental cut-sets are as below,



f-cut set 1 : {1, 4, 7}
f-cut set 3 : {3, 4, 7, 2}
f-cut set 5 : {5, 2, 7}
f-cut set 6 : {6, 7}

The orientation of each cut set is given by the orientation of the corresponding connecting twig. Thus, the f-cutset matrix is obtained as,

$$Q = \begin{array}{c} \\ 1 \\ 3 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Q.7 (c) Solution:

(i) From circuit: $R_{eq} = R_1 + [R_2 \parallel 5]$

From voltage division rule:

$$V_0 = V_s \times \frac{R_2 \parallel 5}{1 + R_1 + R_2 \parallel 5}$$

$$\frac{V_0}{V_s} = \frac{R_2 \parallel 5}{1 + R_{eq}}$$

$$0.05 = \frac{\frac{R_2 \cdot 5}{R_2 + 5}}{1 + 39}$$

$$2 = \frac{5R_2}{R_2 + 5}$$

$$2R_2 + 10 = 5R_2$$

$$R_2 = \frac{10}{3} = 3.33\ \text{k}\Omega$$

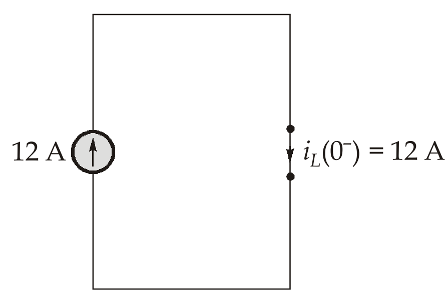
$$\because \qquad R_{eq} = R_1 + [R_2 \parallel 5] = R_1 + \frac{\frac{10}{3}\times 5}{\frac{10}{3}+5}$$

$$39 = R_1 + 2$$

$$R_1 = 37 \text{ k}\Omega$$

(ii) At $t = 0^-$,

Inductor was connected to source for a long time. So it acts as short circuit at steady state.



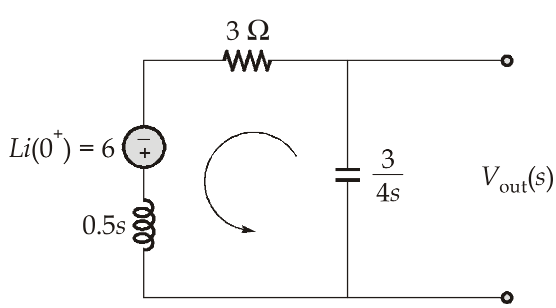
Given, $V_c(0^-) = 0$

Since the inductor current and capacitor voltage cannot change instantaneously, thus

$$i_L(0^+) = i_L(0^-) = 12 \text{ A}$$

$$V_c(0^+) = V_c(0^-) = 0 \text{ V}$$

For $t > 0$, the circuit in s-domain can be drawn as below,



$$V_{out}(s) = \frac{-6\times\frac{3}{4s}}{\frac{3}{4s}+0.5s+3} = \frac{-18}{3+2s^2+12s}$$

$$= \frac{-9}{s^2+6s+1.5} = \frac{-9}{(s+3)^2-7.5}$$

$$V_{out}(s) = \frac{-9}{\sqrt{7.5}} \frac{\sqrt{7.5}}{(s+3)^2-(\sqrt{7.5})^2}$$

$$V_{out}(t) = \frac{-9}{\sqrt{7.5}}\left(e^{-3t}\sinh\sqrt{7.5}t\right)$$

Thus, at t = 1.5 sec

$$V_{out}(1.5) = \frac{-9}{\sqrt{7.5}}\left(e^{-3.5\times1.5}\sinh\sqrt{7.5}\times1.5\right)$$

$$= -0.524 \text{ V}$$

Q.8 (a) Solution:

(i) Given,

$$A = 1 \text{ cm}^2$$

$$N_A = 10^{17} \text{ cm}^{-3}$$

$$\Delta p = 3 \times 10^{16} \text{ cm}^{-3}$$

$$x = 500 \text{ Å} = 0.5 \times 10^{-5} \text{ cm}$$

Diffusion constant, $D_p = \frac{kT}{q}\mu_p$ ... using the Einstein's relation

$$= 0.0259 \times 500$$

$$= 12.95 \text{ cm}^2/\text{s}$$

Diffusion length, $L_p = \sqrt{D_p\tau_p} = \sqrt{12.95\times10^{-10}} = 3.6 \times 10^{-5}$ cm

The excess carrier concentration will decay exponentially with distance due to recombination. Thus, at a distance 'x',

Hole concentration, $p = p_0 + \Delta p e^{-x/L_p}$

$$= 10^{17} + (3\times10^{16})e^{-\left(\frac{0.5\times10^{-5}}{3.6\times10^{-5}}\right)}$$

$$= 1.2611 \times 10^{17} \text{ cm}^{-3}$$

We know, $p = n_i e^{(E_i-E_{F_p})/kT}$

$\Rightarrow$ $(1.2611 \times 10^{17}) = (1.5\times10^{10})e^{(E_i-E_{F_p})/kT}$

$\Rightarrow$ $(E_i - E_{F_p}) = kT\left(\ln\frac{1.2611\times10^{17}}{1.5\times10^{10}}\right) = 0.0259\times15.945$

$\Rightarrow$ $(E_i - E_{F_p}) = 0.413$ eV

(ii) We know, conductivity in a semiconductor is given by,

$$\sigma = nq\,\mu_n + pq\,\mu_p$$

With excess carriers present,

$$n = n_0 + \delta n$$

$$p = p_0 + \delta p$$

For uniform excess generation rate,

$$\delta n = \delta p$$

$$\Rightarrow \qquad \sigma = q\mu_n(n_0 + \delta n) + q\mu_p(p_0 + \delta p)$$

$$= q\mu_n(n_0 + \delta p) + q\mu_p(p_0 + \delta p)$$

$$= q\mu_n n_0 + q\mu_p p_0 + q(\mu_n + \mu_p)\delta p$$

$$= q\mu_n n_0 + q\mu_p p_0 + \Delta\sigma$$

Where, $\Delta\sigma = q(\mu_n + \mu_p)\,\delta p$ is the change in the semiconductor conductivity.

In steady state, $\delta p = g'\tau_{p_0}$

Where g' is the generation rate and τ_{p_0} is the hole lifetime before recombination

$$\therefore \qquad \Delta\sigma = q(\mu_n + \mu_p)\,g'\tau_{p_0}$$

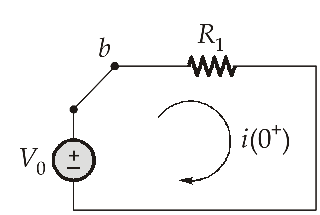
Q.8 (b) Solution:

(i) Since the switch in the position 'a' for a long time before $t = 0$,

At $t = 0^-$, the network has attained steady state condition. Hence, the capacitor C_1 acts as an open circuit and is charged to V_0 volt. Thus

$$V_1(0^-) = V_0$$

At $t = 0^+$:



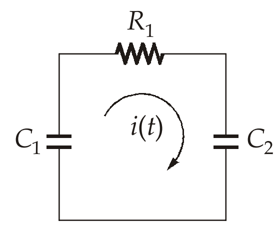
Since the voltage across the capacitor cannot change instantaneously. Thus,

$$V_1(0^+) = V_0$$

$$V_2(0^+) = 0$$

We have, $$i(0^+) = \frac{V_0}{R_1}$$

For $t > 0$:



Writing KVL equation for $t > 0$,

$$-\frac{1}{C_1}\int_0^t i\,dt + V_0 - R_1 i - \frac{1}{C_2}\int_0^t i\,dt = 0 \quad \text{...(i)}$$

Differentiating the equation (i), we get,

$$-\frac{i}{C_1} - R_1\frac{di}{dt} - \frac{i}{C_2} = 0$$

$$\frac{di}{dt} + \frac{1}{R_1}\left(\frac{C_1 + C_2}{C_1 C_2}\right)i = 0$$

The solution of this differential equation is given by

$$i(t) = ke^{-\frac{1}{R_1}\left(\frac{C_1+C_2}{C_1C_2}\right)t}$$

At $t = 0$, $$i(0) = \frac{V_0}{R_1}$$

We get, $$k = \frac{V_0}{R_1}$$

$\therefore$ $$i(t) = \frac{V_0}{R_1}e^{-\frac{1}{R_1}\left(\frac{C_1+C_2}{C_1C_2}\right)t}$$

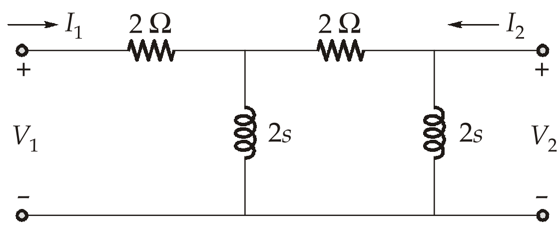
$$= \frac{V_0}{R_1}e^{-\frac{1}{R_1C}t} \quad \text{where, } C = \frac{C_1C_2}{C_1 + C_2}$$

$$V_2(t) = \frac{1}{C_2}\int_0^t i\,dt = \frac{1}{C_2}\int_0^t \frac{V_0}{R_1}e^{-\frac{t}{R_1C}}dt$$

$$= \frac{V_0}{R_1C_2}R_1C\left(1 - e^{-\frac{1}{R_1C}t}\right)$$

$$V_2(t) = \frac{V_0}{C_2} C\left(1 - e^{-\frac{t}{R_1 C}}\right) \qquad \text{where, } C = \frac{C_1 C_2}{C_1 + C_2}$$

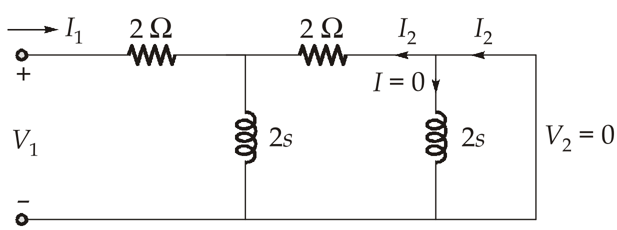
(ii) The given network in s-domain is drawn as below,



The h-parameters are described by the following equations:

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

To calculate h_{11} and h_{21}, we put $V_2 = 0$.



Applying KVL, we get

$$V_1 = (2 + 2s)I_1 + 2sI_2 \qquad ...(1)$$

and $$(2 + 2s)I_2 + 2sI_1 = 0$$

$$I_2 = \frac{-2sI_1}{2(s+1)} = \frac{-s}{s+1} I_1 \qquad ...(2)$$

Put value of I_2 in equation (1)

$$V_1 = (2+2s)I_1 - \frac{2s^2}{s+1} I_1$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{2(s+1)^2 - 2s^2}{s+1} = \frac{2s^2 + 4s + 2 - 2s^2}{s+1}$$

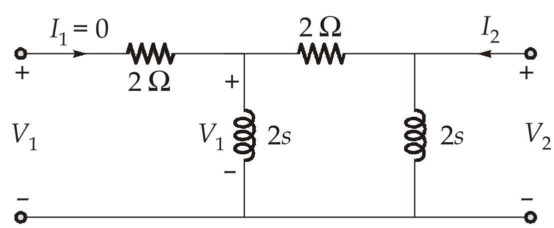
$$h_{11} = \frac{2(2s+1)}{s+1}$$

Using equation (2),

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-s}{s+1}$$

To calculate h_{12} and h_{22}, we put $I_1 = 0$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



Using KCL, we can write

$$I_2 = \frac{V_2 - V_1}{2} + \frac{V_2}{2s} \quad \text{...(3)}$$

Thus, $$V_1 = I_2 \times \frac{2s}{4s+2} \times 2s = \frac{4s^2}{2(2s+1)} I_2 \quad \text{...(4)}$$

Put value of V_1 in equation (3),

$$I_2 = \frac{V_2}{2} - \frac{s^2}{2s+1} I_2 + \frac{V_2}{2s}$$

$$I_2\left(1 + \frac{s^2}{2s+1}\right) = V_2\left(\frac{s+1}{2s}\right)$$

$$I_2\left(\frac{s^2+2s+1}{2s+1}\right) = V_2\left(\frac{s+1}{2s}\right)$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{(s+1)(2s+1)}{2s(s+1)^2} = \frac{(2s+1)}{2s(s+1)}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

From equation (4),

$$I_2 = \frac{2s+1}{2s^2} V_1$$

Put value of I_2 in equation (3), we get

$$\frac{2s+1}{2s^2}V_1 + \frac{V_1}{2} = V_2\left(\frac{s+1}{2s}\right)$$

$$V_1\left(\frac{2s+1+s^2}{2s^2}\right) = V_2\left(\frac{s+1}{2s}\right)$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{s(s+1)}{(s+1)^2} = \frac{s}{s+1}$$

In matrix form, the h-parameters can be represented as below:

$$[h] = \begin{bmatrix} \frac{2(2s+1)}{s+1} & \frac{s}{s+1} \\ \frac{-s}{s+1} & \frac{(2s+1)}{2s(s+1)} \end{bmatrix}$$

Q.8 (c) Solution:

(i) The cutoff frequency of the BJT is,

$$f_T = \frac{1}{2\pi\,\tau_{ec}}$$

Where,

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$

τ_{ec} = emitter-to-collector time delay

τ_e = emitter-base junction capacitance charging time.

τ_b = base transit time

τ_d = collector depletion region transit time

τ_c = collector capacitance charging time.

Given, τ_b = 100 ps,

τ_e = 25 ps

Collector depletion region transit time,

$$\tau_d = \frac{x_d}{v_d} = \frac{1.2\times10^{-4}}{10^7} = 1.2 \times 10^{-11} \text{ (or) } 12 \text{ ps.}$$

Emitter base junction capacitance charging time,

$$\tau_e = r_e\,c_e = (10) \times (0.1 \times 10^{-12}) = 10^{-12}s = 1 \text{ ps}$$

$$\therefore \quad \tau_{ec} = 25 + 100 + 12 + 1 = 138 \text{ ps}$$

Thus, Cut off frequency, $f_T = \dfrac{1}{2\pi \times 138 \times 10^{-12}}$

$$f_T = 1.15 \text{ GHz.}$$

(ii) Given,

doping concentrations, $N_E = 10^{18}\ \text{cm}^{-3}$

$$N_B = 10^{17}\ \text{cm}^{-3}$$

$$N_C = 10^{16}\ \text{cm}^{-3}$$

$$\tau_{E0} = \tau_{B0} = \tau_{C0} = 2 \times 10^{-7}\ \text{s.}$$

$$D_E = 10\ \text{cm}^2/\text{s};\ D_B = 20\ \text{cm}^2/\text{s};\ D_C = 15\ \text{cm}^2/\text{s.}$$

$$\text{Area, } A = 10^{-3}\ \text{cm}^2$$

The collector current, $I_c = (J_{nc} + J_{pc}).\ A$

where: J_{nc} : Collector junction current density due to electrons.

J_{pc} : Collector junction current density due to holes.

In the inverse-active mode, the collector base junction is forward biased. Thus, the diffusion current density due to electrons in the base region is given by

$$J_{nc} = \frac{qD_B n_{B0}}{x_B} \exp\left(\frac{V_{BC}}{V_t}\right)$$

Where, $n_{B0} = \dfrac{n_i^2}{N_B} = \dfrac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3\ \text{cm}^{-3}$

$$\therefore \quad J_{nc} = \frac{1.6 \times 10^{-19} \times 20 \times 2.25 \times 10^3}{10^{-4}} \times \exp\left(\frac{0.6}{0.0259}\right)$$

$$J_{nc} = 0.829\ \text{A/cm}^2$$

The electrons from collector diffuse through the base and towards the collector region. Since the recombination factor is unity, thus J_{nc} constitute the emitter current. Similarly, there is a component of collector current due to injection of holes from base to collector given by,

$$J_{pc} = \frac{q\,D_C p_{co}}{L_C} \exp\left(\frac{V_{BC}}{V_t}\right)$$

where p_{C0} is the concentration of holes in the collector at the edge of the depletion region, D_C is the diffusion coefficient for holes and L_C is the diffusion length for holes in the collector. We have,

$$p_{c0} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

and $$L_c = \sqrt{D_C\tau_{C0}} = \sqrt{15\times2\times10^{-7}} = 1.73 \times 10^{-3} \text{ cm}$$

$\therefore$ $$J_{pc} = \frac{1.6\times10^{-19}\times15\times2.25\times10^4}{1.73\times10^{-3}}\exp\left(\frac{0.6}{0.0259}\right)$$

$$J_{pc} = 0.359 \text{ A/cm}^2$$

$\therefore$ $$I_c = (J_{nc} + J_{pc})A = (0.829 + 0.359) \times 10^{-3}$$

$$I_c = 1.19 \text{ mA}$$

The emitter Current, $$I_E = J_{nc} \cdot A$$

$\therefore$ $$I_E = (0.829) \times 10^{-3}$$

$$I_E = 0.829 \text{ mA}$$

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