Detailed Solutions

ESE-2025 Mains Test Series

CIVIL ENGINEERING

Section A : Solid Mechanics

Q.1 (a) Solution:

When walls yield by 0.2 mm.

Let the reaction developed at wall is R.

Now, total change in length of bar = 0.2 mm

Expansion due to temperature + contraction due to R = 0.20

$$\Rightarrow (L\alpha\Delta T)_{\text{Copper}} + (L\alpha\Delta T)_{\text{Brass}} + (L\alpha\Delta T)_{\text{Aluminium}} - \left(\frac{RL}{AE}\right)_{\text{Copper}} - \left(\frac{RL}{AE}\right)_{\text{Brass}} - \left(\frac{RL}{AE}\right)_{\text{Aluminium}} = 0.2$$

$$1000 \times 5 \times 10^{-6} \times 40 + 2000 \times 6 \times 10^{-6} \times 40 + 1000 \times 7.5 \times 10^{-6} \times 40$$

$$-R\left[\frac{1000}{200\times10^5} + \frac{2000}{300\times0.8\times10^5} + \frac{1000}{100\times0.5\times10^5}\right] = 0.2$$

$$\Rightarrow \quad 0.98 - R[3.33 \times 10^{-4}] = 0.2$$

$$\Rightarrow \quad 0.98 - 0.2 = R[3.33 \times 10^{-4}]$$

$$\Rightarrow \quad R = 2340 \text{ N}$$

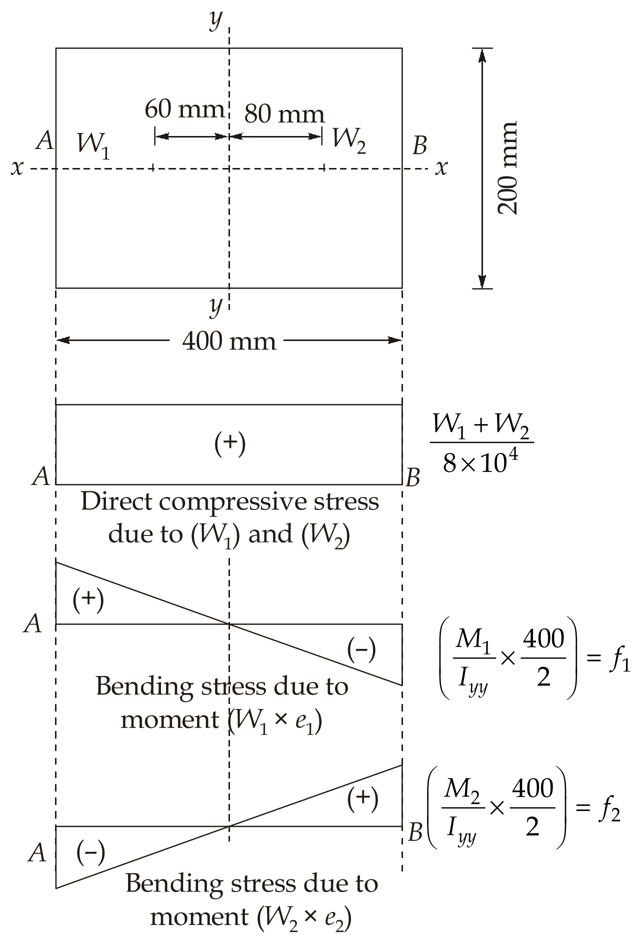
Now,

$$\text{Stress in copper} = \frac{R}{A_{\text{Copper}}} = \frac{2340}{200} = 11.7 \text{ N/mm}^2$$

$$\text{Stress in aluminium} = \frac{R}{A_{\text{aluminium}}} = \frac{2340}{300} = 7.8 \text{ N/mm}^2$$

$$\text{Stress in brass} = \frac{R}{A_{\text{Brass}}} = \frac{2340}{100} = 23.40 \text{ N/mm}^2$$

Q.1 (b) Solution:



Moment of inertia about y-axis, $I_y = \dfrac{200\times400^3}{12} = 10.67 \times 10^8 \text{ mm}^4$

Cross sectional area, $A = 200 \times 400 = 8 \times 10^4 \text{ mm}^2$

Direct compressive stress, $f_d = \dfrac{W_1+W_2}{A} = \dfrac{W_1+W_2}{8\times10^4}$

Now, bending moment due to W_1, $M_1 = W_1 \times 60 = 60W_1$

Similarly, bending moment due to W_2, $M_2 = W_2 \times 80 = 80W_2$

Now,bending stresses due to M_1, $f_1 = \dfrac{M_1}{I_y}\times\dfrac{400}{2} = \dfrac{60\,W_1}{10.67\times10^8}\times200 = \dfrac{W_1}{88916.67}$

(Compression at A and Tension at B)

Now, bending stress due to M_2, $f_2 = \dfrac{M_2}{I_y}\times\dfrac{400}{2} = \dfrac{80W_2}{10.67\times10^8}\times200 = \dfrac{W_2}{66687.5}$

Total stress at A and B,

$$\text{Stress at } A, f_A = f_d + f_1 - f_2$$

$$\Rightarrow \quad f_A = \frac{W_1 + W_2}{8 \times 10^4} + \frac{W_1}{88916.67} - \frac{W_2}{66687.5}$$

$$\text{Stress at } B, f_B = f_d - f_1 + f_2$$

$$\Rightarrow \quad f_B = \frac{W_1 + W_2}{8 \times 10^4} - \frac{W_1}{88916.67} + \frac{W_2}{66687.5}$$

Now, $f_A = 4f_B$

$$\Rightarrow \quad \frac{W_1 + W_2}{8 \times 10^4} + \frac{W_1}{88916.67} - \frac{W_2}{66687.5} = 4\left[\frac{W_1 + W_2}{8 \times 10^4} - \frac{W_1}{88916.67} + \frac{W_2}{66687.5}\right]$$

$$\frac{W_1}{88916.67} + \frac{4W_1}{88916.67} - \frac{W_2}{66687.5} - \frac{4W_2}{66687.5} = \frac{4(W_1 + W_2)}{8 \times 10^4} - \frac{(W_1 + W_2)}{8 \times 10^4}$$

$$\Rightarrow \quad \frac{5W_1}{88916.67} - \frac{5W_2}{66687.5} = \frac{3W_1}{8 \times 10^4} + \frac{3W_2}{8 \times 10^4}$$

$$\Rightarrow \quad \frac{5W_1}{88916.67} - \frac{3W_1}{8 \times 10^4} = \frac{5W_2}{66687.5} + \frac{3W_2}{8 \times 10^4}$$

$$\Rightarrow \quad W_1 \times 1.87324 \times 10^{-5} = 1.12476 \times 10^{-4} \times 50$$

$$W_1 = 300.22 \text{ kN}$$

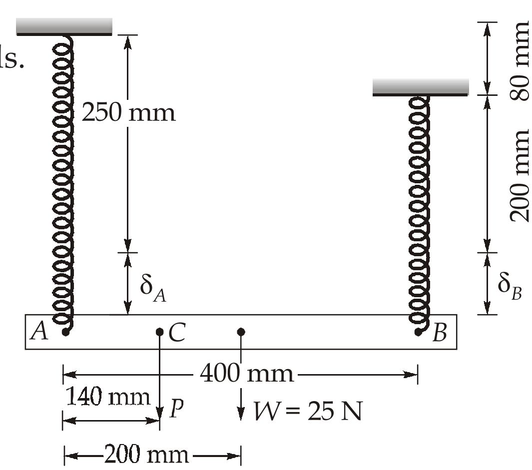
Q.1 (c) Solution:

Given: $k_A = 300 \text{ N/m} = 300 \text{ kN/mm}$

$k_B = 400 \text{ N/m} = 400 \text{ kN/mm}$

Let the load applied at C is P as shown in figure.

Let the reaction at A and B be V_A and V_B upwards.



$$\Sigma F_y = 0$$

$$\Rightarrow \quad V_A + V_B - P - W = 0 \quad \text{...(i)}$$

Also, $\Sigma M_B = 0$

$$\Rightarrow \quad V_A \times 0.4 - P \times 0.26 - W \times 0.2 = 0 \quad \text{...(ii)}$$

$$\Rightarrow \quad V_A = \frac{0.26P + 0.2W}{0.4} = 0.65P + 0.5W$$

Putting value in (i), we get

$$V_B = P + W - V_A$$
$$= P + W - 0.65P_A - 0.5W = 0.35P + 0.5\,W$$

As the bar is horizontal

$$0.250 + \delta_A = 0.080 + 0.200 + \delta_B \qquad \left(\because \delta_A = \frac{V_A}{k_A} \text{ and } \delta_B = \frac{V_B}{k_B}\right)$$

$$\Rightarrow \quad \delta_A = \delta_B + 0.03$$

$$\Rightarrow \quad \frac{V_A}{k_A} = \frac{V_B}{k_B} + 0.03$$

$$\Rightarrow \quad \frac{0.65P + 0.5W}{300} = \frac{0.35P + 0.5W}{400} + 0.03$$

$$\Rightarrow \quad \frac{0.65P + 0.5\times 25}{300} = \frac{0.35P + 0.5\times 25}{400} + 0.03 \qquad (\because W = 25 \text{ kN})$$

$$\Rightarrow \quad \frac{0.65P}{300} + 0.0417 = \frac{0.35P}{400} + 0.0312 + 0.03$$

$$\Rightarrow \quad P\left(\frac{0.65}{300} - \frac{0.35}{400}\right) = 0.0195$$

$$\Rightarrow \quad P = 15.16 \text{ N}$$

Q.1 (d) Solution:

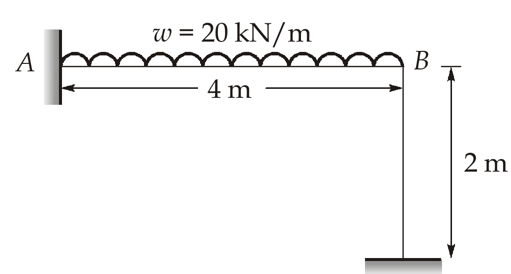
Given: $I = 1.2 \times 10^6 \text{ mm}^4$

$w = 20 \text{ kN/m} = 20 \text{ N/mm}$

Length of beam $l_B = 4$ m

Length of strut, $l_s = 2$ m

Let, the load taken by strut is P.



Now, deflection of B in beam, $\Delta_B = \dfrac{wl_B^4}{8EI} - \dfrac{Pl_B^3}{3EI}$

Deflection of B in strut, $\Delta_B = \dfrac{Pl_s}{AE}$

$$\Rightarrow \quad \frac{wl_B^4}{8EI} - \frac{Pl_B^3}{3EI} = \frac{Pl_s}{AE}$$

$$\Rightarrow \quad \frac{wl_B^4}{8I} - \frac{Pl_B^3}{3I} = \frac{Pl_s}{AE}$$

$$\frac{20\times(4000)^4}{8\times1.2\times10^6} - \frac{P\times(4000)^3}{3\times1.2\times10^6} = \frac{P\times2000}{400}$$

$$533.33\times10^6 = P\left(5+\frac{4000^3}{3\times1.2\times10^6}\right)$$

$$P = 29991.3 \text{ N}$$

$$P \simeq 30 \text{ kN}$$

Q.1 (e) Solution:

Let, N_s and N_h be rotational speed of solid and hollow shaft respectively.

J_s and J_h be polar moment of inertia of solid and hollow shaft respectively.

τ_s and τ_h be maximum shear stress of solid and hollow shaft respectively.

T_s and T_h be torque transmitted by solid and hollow shaft respectively.

P_s and P_h be power transmitted by solid and hollow shaft respectively.

Now, $$P_h = 1.35Ps$$

$$\Rightarrow \quad \frac{2\pi N_h T_h}{60} = \frac{1.35\times2\pi N_s T_s}{60}$$

$$\Rightarrow \quad N_h T_h = 1.35\times N_s T_s \quad ...(i)$$

Now, $$N_h = 1.03\times N_s \quad ...(ii)$$

On putting the value of N_h in equation (i)

$$1.03\, N_s T_h = 1.35 N_s \times T_s$$

$$\Rightarrow \quad T_h = 1.31\, T_s \quad ...(iii)$$

Now, $$\frac{T}{J} = \frac{\tau}{R}$$

$$\Rightarrow \quad \tau = \frac{TR}{J}$$

$$\Rightarrow \quad \frac{\tau_h}{\tau_s} = \frac{T_h\times J_s}{J_h\times T_s} \quad [\because R \text{ is same for both}]$$

But, $$\tau_h = 1.4\tau_s$$

$$\Rightarrow \quad 1.4 = 1.31\times\frac{J_s}{J_h} \quad \left(\because \frac{T_h}{T_s} = 1.31\right)$$

$$\Rightarrow \quad J_s = 1.069 J_h$$

$$\Rightarrow \quad \frac{\pi}{32} \times D^4 = 1.069 \times \frac{\pi}{32} \times (D^4 - d^4)$$

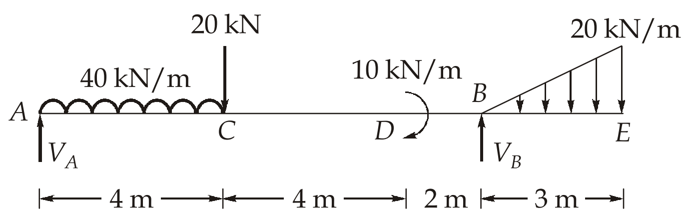
$$\Rightarrow \quad 30^4 = 1.069 \times (30^4 - d^4)$$

$$\Rightarrow \quad d^4 = 30^4 - 757717.49$$

$$\Rightarrow \quad d = 15.12 \text{ cm}$$

Q.2 (a) Solution:

Let, the vertical reactions at A and B be V_A and V_B respectively.



$\Sigma F_y = 0$

$$\Rightarrow \quad V_A + V_B - 40 \times 4 - 20 - \frac{1}{2} \times 3 \times 20 = 0$$

$$\Rightarrow \quad V_A + V_B = 210 \qquad \text{...(i)}$$

$\Sigma M_B = 0$

$$\Rightarrow \quad V_A \times 10 - 40 \times 4 \times 8 - 20 \times 6 + 10 + \frac{1}{2} \times 3 \times 20 \times 2 = 0$$

$$\Rightarrow \quad V_A \times 10 - 1280 - 120 + 10 + 60 = 0$$

$$\Rightarrow \quad V_A = 133 \text{ kN}$$

From (i), $\quad V_B = 210 - 133 = 77 \text{ kN}$

Shear force diagram:

Take a section x-x at x distance from A,

Portion AC:

$$S_x(x \text{ from } A) = V_A - 40x \qquad [0 \le x < 4]$$

$$= 133 - 40x$$

At $x = 0$, $\quad S_A = 133 - 40 \times 0 = 133 \text{ kN}$

At $x = 4$ m $\quad S_C$ (Just left of C) $= 133 - 40 \times 4 = -27$ kN

Portion CB:

$$S_x(x \text{ from } A) = V_A - 40 \times 4 - 20 \qquad [4 \le x < 10]$$

$= 133 - 160 - 20$

$= -47$

At $x = 4$ m, S_C (Just right of C) $= -47$ kN

At $x = 10$ m S_B (Just left of B) $= -47$ kN

Portion BE:

$$S_x(x \text{ from } A) = V_A - 40 \times 4 - 20 + V_B - \frac{1}{2} \times \frac{20}{3}(x-10) \times (x-10)$$

$[10 \le x < 13]$

$$= 133 - 160 - 20 + 77 - \frac{10}{3}(x-10)^2$$

$$= 30 - \frac{10}{3}(x-10)^2$$

At $x = 10$ m, S_B (Just right of B) $= 30 - \frac{10}{3}(10-10)^2 = 30$ kN

At $x = 13$ m S_E (Just left of E) $= 30 - \frac{10}{3}(13-10)^2 = 0$

Bending moment diagram:

Take a section x-x at x distance from A,

Portion AC:

$$M_x\ (x \text{ from } A) = V_A x - 40\frac{x^2}{2} \qquad [0 \le x < 4]$$

$= 133x - 20x$

At $x = 0$, $M_A = 0$

$x = 4$ m,

M_C (Just left of C) $= 133 \times 4 - 20 \times 4^2$

$= 212$ kN-m

Portion CD:

$$M_x(x \text{ from } A) = V_A x - 40 \times 4 \times (x-2) - 20\,(x-4) \qquad [4 \le x < 8]$$

$= 133x - 160(x - 2) - 20(x - 4)$

At $x = 4$ m,

M_C (Just right of C) $= 133 \times 4 - 160 \times 2$

$= 212$ kN-m

At $x = 8$ m,

M_D (Just left of D) $= 133 \times 8 - 160\,(8 - 2) - 20(8 - 4)$

$= 24$ kN-m

Portion DB:

$$M_x(x \text{ from } A) = V_A x - 40 \times 4 \times (x - 2) - 20(x - 4) + 10 \quad [8 \le x < 10]$$
$$= 133x - 160(x - 2) - 20(x - 4) + 10$$

At $x = 8$ m,

$$M_D \text{ (Just right of } D) = 133 \times 8 - 160(8 - 2) - 20(8 - 4) + 10$$
$$= 34 \text{ kN-m}$$

At $x = 10$ m,

$$M_B \text{ (Just left of } B) = 133 \times 10 - 160(10 - 2) - 20(10 - 4) + 10$$
$$= -60 \text{ kN-m}$$

Portion BE:

$$M_x(x \text{ from } A) = V_A x - 40 \times 4(x - 2) - 20(x - 4) + 10$$
$$+V_B(x-10) - \frac{1}{2} \times \frac{20}{3}(x-10) \times (x-10) \times \frac{(x-10)}{3}$$
$$[10 \le x < 13]$$
$$= 133x - 160(x-2) - 20(x-4) + 10 + 77(x-10) + 0$$
$$\frac{10}{9} \times 0$$

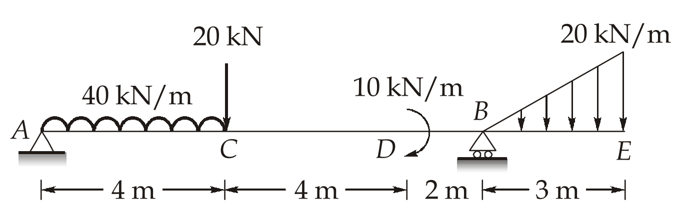
At $x = 10$ m,

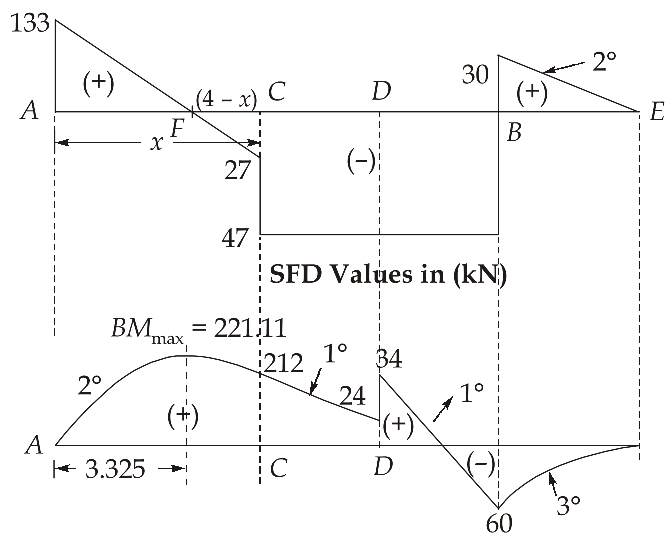
$$M_B \text{ (Just right of } B) = 133 \times 10 - 160(10 - 2) - 20(10 - 4) + 10 + 77 \times 0 + 0$$
$$= -60 \text{ kN-m}$$

At $x = 13$ m,

$$M_E \text{ (Just left of } E) = 133 \times 13 - 160(13 - 2) - 20(13 - 4) + 10 + 77(13 - 10)$$
$$- \frac{10}{9} \times (13-10)^3 = 0$$

Loading diagram





BMD Values in (kN-m)

From BMD

Maximum negative bending moment = 60 kNm

Now, maximum positive bending moment is in portion *AC* at *F* where shear force is zero.

Let, *SF* is zero at *x* from *A*.

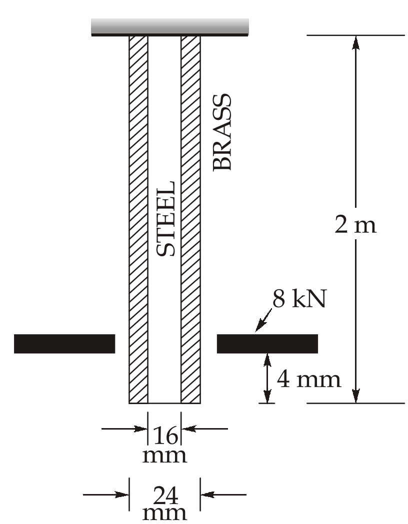
Now, $$\frac{133}{x} = \frac{27}{4-x}$$

$\Rightarrow$ $$532 - 133x = 27x$$

$\Rightarrow$ $$x = 3.325 \text{ m}$$

Now, $$M_F = 133x - 20x^2$$
$$= 133 \times 3.325 - 20 \times 3.325^2$$
$$= 221.11 \text{ kN-m}$$

Q.2 (b) Solution:



Area of steel bar, $$A_s = \left(\frac{\pi}{4}\right)16^2 = 64\pi$$

Area of brass bar, $$A_b = \left(\frac{\pi}{4}\right)(24^2 - 16^2) = 80\pi$$

Let x = Extension of bar in mm

Stress in steel bar, $$\sigma_s = \frac{E_s \cdot x}{L}$$

Stress in brass bar, $$\sigma_b = \frac{E_b \cdot x}{L}$$

$$\text{Total strain energy of the composite bar} = \frac{\sigma_s^2}{2E_s}A_sL + \frac{\sigma_b^2}{2E_b}A_bL$$

$$U = \frac{E_s^2x^2}{L^2 \cdot 2E_s}A_sL + \frac{E_b^2x^2}{L^2 \cdot 2E_b}A_bL$$

$$\Rightarrow \quad U = \frac{E_sx^2}{2L}A_s + \frac{E_bx^2}{2L}A_b$$

$$\Rightarrow \quad U = \frac{x^2}{2L}(E_sA_s + E_bA_b)$$

$$\Rightarrow \quad U = \frac{x^2}{2\times 2000}(205000\times 64\pi + 100000\times 80\pi)$$

$$\Rightarrow \quad U = 16587.61\, x^2 \text{ Nmm} \quad \text{...(i)}$$

Potential energy lost by the weight

$$E = W(h + x) = 8000\,(4 + x) \text{ Nmm} \quad \text{...(ii)}$$

From equation (i) and (ii)

$$16587.61\, x^2 = 8000(4 + x)$$

$$\Rightarrow \quad x^2 - 0.4823x - 1.9292 = 0$$

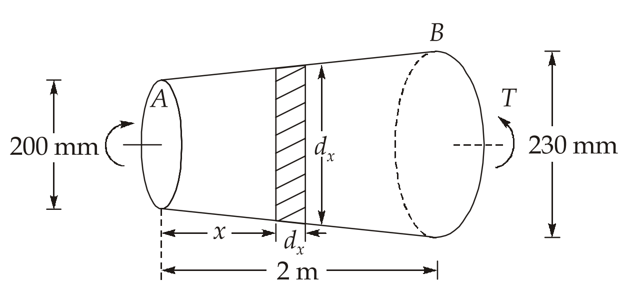
On solving

$$x = 1.6508 \text{ mm}, -1.168 \text{ mm}$$

$$\text{Stress in steel rod, } \sigma_{Steel} = \frac{E_{Steel}x}{L} = \frac{205000\times 1.6508}{2000} = 169.207 \text{ MPa}$$

$$\text{Stress in brass rod, } \sigma_B = \frac{E_{Brass}x}{L} = \frac{100000\times 1.6508}{2000} = 82.54 \text{ MPa}$$

Q.2 (c) Solution:



Consider a elemental strip of thicknes dx at a distance x from end A.

$$\text{Diameter of element strip, } d_x = 200+\frac{(230-200)}{2000}\times x = 200+\frac{3}{200}x$$

$$\text{Angle of twist of strip, } d\theta = \frac{Tdx}{GJ} = \frac{Tdx}{G\frac{\pi}{32}d_x^4} = \frac{32Tdx}{\pi G\left(200+\frac{3}{200}x\right)^4}$$

$$\text{Total angle of twist, } \theta = \int_0^{2000}\frac{32Tdx}{\pi G\left(200+\frac{3}{200}x\right)^4}$$

$$\Rightarrow \quad \theta = \frac{32T}{\pi G}\left[\frac{(-1)}{3\left(200+\frac{3}{200}x\right)^3}\right]_0^{2000}\times\frac{200}{3}$$

$$\Rightarrow \quad \theta = \frac{32T}{\pi G}\times\frac{200}{9}\left[\frac{1}{200^3}-\frac{1}{230^3}\right]$$

$$\Rightarrow \quad \theta = 9.69\times10^{-6}\left(\frac{T}{G}\right)\text{radian}$$

For a solid shaft of mean diameter

$$d_{\text{mean}} = \frac{1}{2}(200+230) = 215\,\text{mm}$$

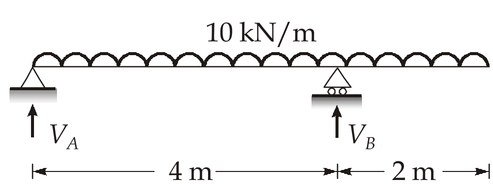
$$\text{Angle of twist,} \quad \theta' = \frac{Tl}{GJ} = \frac{T\times2000}{G\times\frac{\pi}{32}\times(215)^4}$$

$$\Rightarrow \quad \theta' = 9.534\times10^{-6}\left(\frac{T}{G}\right)\text{radian}$$

$$\therefore \quad \text{Percentage error} = \left(\frac{\theta-\theta'}{\theta}\right)\times100 = \left(\frac{9.69-9.534}{9.69}\right)\times100 = 1.61\%$$

Q.3 (a) Solution:

Let the vertical reactions at A and B be V_A and V_B.



$$\Sigma F_y = 0$$

$$\Rightarrow \quad V_A + V_B - 10\times6 = 0$$

$$\Rightarrow \quad V_A + V_B = 60 \quad \text{...(i)}$$

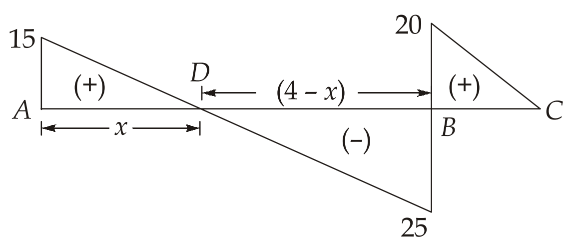
$$\Sigma M_B = 0$$

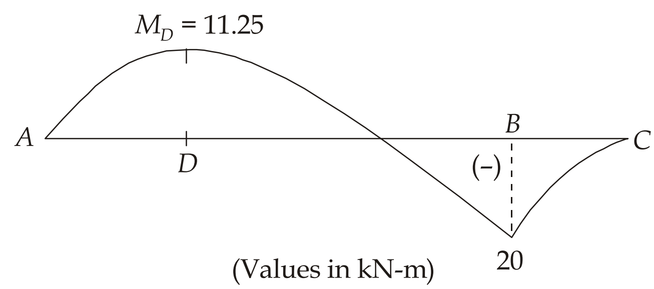
$$\Rightarrow \quad V_A \times 4 - 10 \times 6 \times 1 = 0$$

$$\Rightarrow \quad V_A = 15 \text{ kN}$$

Using (i), $V_B = 60 - 15 = 45$ kN

SFD and BMD are drawn below:





Position of D:

$$\frac{15}{x} = \frac{25}{4-x}$$

$$\Rightarrow \quad 60 - 15x = 25x$$

$$\Rightarrow \quad x = 1.5 \text{ m}$$

Now,
$$M_D = V_A \times x - 10 \times \frac{x^2}{2}$$

$$= 15 \times 1.5 - 10 \times \frac{1.5^2}{2} = 11.25 \text{ kNm}$$

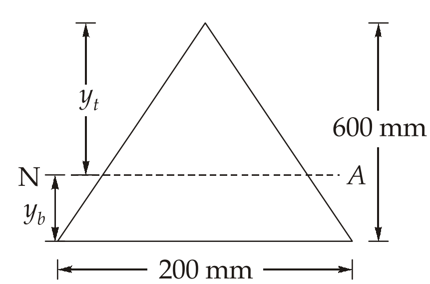
Calculation of bending stress:

From figure

Depth of neutral axis, from top y_t = 400 mm

So, Depth of neutral axis from bottom,

$$y_b = 600 - 400 = 200 \text{ mm}$$



$$\text{Moment of Inertia, } I = \frac{bh^3}{36}$$

$$= \frac{200 \times 600^3}{36} = 12 \times 10^8 \text{ mm}^4$$

Now, maximum positive bending moment at D, M_D = 11.25 kN-m

Maximum negative bending moment at B, M_B = 20 kN-m

At *D*:

Compressive stress and tensile stresses will be at top at bottom respectively.

So, compressive stress at $D, f_{top} = \frac{M_D}{I} \times y_{top}$

$$= \frac{11.25 \times 10^6}{12 \times 10^8} \times 400 = -3.75 \text{ N/mm}^2$$

Tensile stress at $D, f_{bottom} = \frac{M_D}{I} \times y_{bottom}$

$$= \frac{11.25 \times 10^6}{12 \times 10^8} \times 200 = 1.875 \text{ N/mm}^2 \quad \text{(Tensile)}$$

At *B*:

Tensile stresses and compressive stresses will be at top and bottom respectively.

So, tensile stresses at top, $f_{top} = \frac{M_B}{I} \times y_{top} = \frac{20 \times 10^6}{12 \times 10^8} \times 400$

$$= 6.67 \text{ N/mm}^2 \text{ (Tensile)}$$

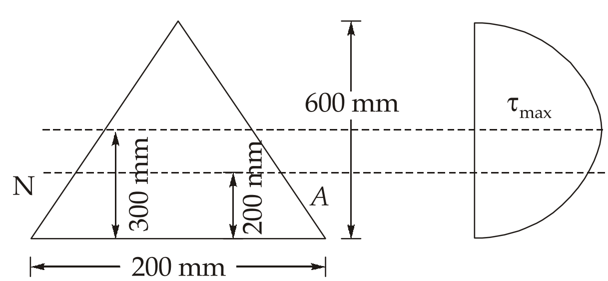
Compressive stresses at bottom,

$$f_{bottom} = \frac{M_B}{I} \times y_{bottom} = \frac{20 \times 10^6}{12 \times 10^8} \times 200$$

$$= +3.33 \text{ N/mm}^2$$

Hence, maximum tensile stresses will be 6.67 N/mm^2 at top fibre at B and maximum compressive stresses will be 3.75 N/mm^2 at *D* (1.5 m from *A*) at top fibre.

Maximum shearing stresses:



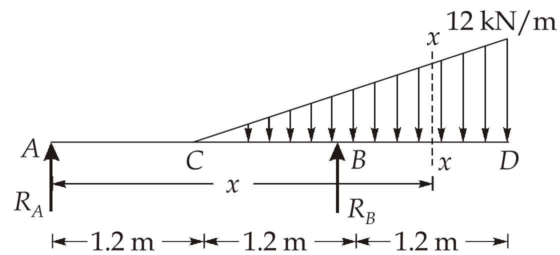
Maximum shearing force, $V_B = 45$ kNm

Now, Maximum shear stress, $\tau_{max} = \frac{3}{2} \times \frac{V_B}{A}$

$$= \frac{3}{2} \times \frac{45 \times 10^3}{\left(\frac{1}{2} \times 200 \times 600\right)} = 1.125 \text{ N/mm}^2 \text{ (Compsressive)}$$

Q.3 (b) Solution:

Let R_A and R_B be the vertical reactions at A and B respectively.



$\Sigma F_y = 0$

$$\Rightarrow \quad R_A + R_B - \frac{1}{2} \times 2.4 \times 12 = 0$$

$$\Rightarrow \quad R_A + R_B = 14.4 \qquad \text{...(i)}$$

$\Sigma M_A = 0$

$$\Rightarrow \quad -R_B \times 2.4 + \frac{1}{2} \times 2.4 \times 12 \times \left(1.2 + \frac{2}{3} \times 2.4\right) = 0$$

$$\Rightarrow \quad -R_B \times 2.4 + 40.32 = 0$$

$$\Rightarrow \quad R_B = 16.8 \text{ kN} \qquad \text{...(ii)}$$

$$\Rightarrow \quad R_A = 14.4 - 16.8 = -2.4 \text{ kN} = 2.4 \text{ kN} (\downarrow)$$

Using Macaulay's method

$$EI\left(\frac{d^2y}{dx^2}\right) = M_x$$

Where

$$M_x = R_A x + R_B \langle x - 2.4 \rangle - \frac{1}{2} \times \frac{12}{2.4} \times \langle x - 1.2 \rangle \times \langle x - 1.2 \rangle \frac{(x - 1.2)}{3}$$

$$= -2.4x + 16.8\langle x - 2.4 \rangle - \frac{5}{6}\langle x - 1.2 \rangle^3$$

$$\Rightarrow \quad EI\left(\frac{d^2y}{dx^2}\right) = -2.4x + 16.8\langle x - 2.4 \rangle - \frac{5}{6}\langle x - 1.2 \rangle^3 \qquad \text{...(iii)}$$

Integrating (iii), we get slope equation

$$\Rightarrow \quad EI\left(\frac{dy}{dx}\right) = -1.2x^2 + C_1 + \frac{16.8}{2}\langle x - 2.4 \rangle^2 - \frac{5}{6}\frac{\langle x - 1.2 \rangle^4}{4} \qquad \text{...(iv)}$$

Integrating (iv), we get deflection equation

$$\Rightarrow \quad EI(y) = -0.4x^3 + C_1 x + C_2 + 2.8\langle x - 2.4 \rangle^3 - \frac{1}{24}\langle x - 1.2 \rangle^5 \qquad \text{...(v)}$$

Now, At $x = 0, y = 0$

$\Rightarrow$ $0 = 0 + 0 + C_2 \Rightarrow C_2 = 0$

At $x = 2.4$ m, $y = 0$

$$\Rightarrow \quad 0 = -0.4 \times 2.4^3 + C_1 \times 2.4 - \frac{1}{24} \times 1.2^5$$

$$\Rightarrow \quad C_1 = 2.3472$$

$\Rightarrow$ Slope equation, $EI\left(\frac{dy}{dx}\right) = -1.2x^2 + 2.3472 + 8.4\langle x - 2.4\rangle^2 - \frac{5}{24}\langle x - 1.2\rangle^4$

$\Rightarrow$ Deflection equation, $EI \times (y) = -0.4x^2 + 2.3472x + 2.8\langle x - 2.4\rangle^3 - \frac{1}{24}\langle x - 1.2\rangle^5$

Slope at A and B,

$$\Rightarrow \quad \theta_A = \left(\frac{dy}{dx}\right)_{@x=0}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{@x=0} = \frac{2.3472}{EI} = \frac{2.3472}{3000} = 1.824 \times 10^{-4} \text{ radians}$$

Also, $\theta_B = \left(\frac{dy}{dx}\right)_{@x=2.4\text{m}}$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{@x=2.4} = \frac{1}{EI}\left[-1.2x^2 + 2.3472 - \frac{5}{24}(x - 1.2)^4\right]$$

$$= \frac{1}{3000}\left[-1.2 \times 2.4^2 + 2.3472 - \frac{5}{24}1.2^4\right]$$

$$= 1.6656 \times 10^{-3} \text{ radians}$$

Deflection at C and D, y

$$y_c = y_{@\,x\,=\,1.2\text{ m}}$$

$$\Rightarrow \quad y_{@\,x\,=\,1.2} = \frac{1}{EI}\left[-0.4x^3 + 2.3472\,x\right]$$

$$= \frac{1}{EI}\left[-0.4 \times 1.2^3 + 2.3472 \times 1.2\right]$$

$$= 5.52 \times 10^{-4} \text{ m} = 0.552 \text{ mm } (\uparrow)$$

$$y_D = y_{@x\,=\,3.6\text{ m}}$$

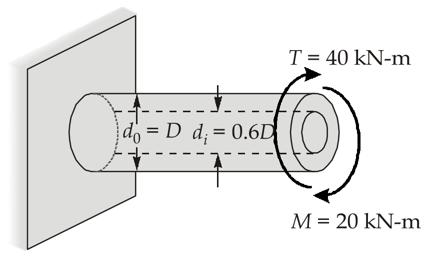
$$\Rightarrow \quad y_{@x\,=\,3.6\text{ m}} = \frac{1}{EI}\left[-0.4x^3 + 2.34772x + 2.8(x - 2.4)^3 - \frac{1}{24}(x - 1.2)^5\right]$$

$$= \frac{1}{3000}\left[-0.4\times3.6^3 + 2.3472\times3.6 + 2.8\times1.2^3 - \frac{1}{24}\times2.4^5\right]$$

$$= -2.897 \times 10^{-3} \text{ m} = -2.897 \text{ mm}$$

$$= 2.897 \text{ mm } (\downarrow)$$

Q.3 (c) Solution:



Given :

FOS = 2, μ = 0.3, f_y = 250 N/mm^2, M = 20 kN-m, T = 40 kN-m

Let, the outer diameter of shaft, $d_0 = D$

Let, the internal diameter of shaft, $d_i = 0.6\,D$

Now, section modulus of shaft, $Z = \dfrac{\pi\times\left(D^4-(0.6D)^4\right)}{64\times\dfrac{D}{2}}$

$$= \frac{\pi}{32D}\left(D^4 - 0.1296D^4\right) = 0.08545\,D^3$$

Polar section modulus of shaft, $Z_p = \dfrac{\pi\left(D^4-(0.6D)^4\right)}{32\times\dfrac{D}{2}}$

$$= \frac{\pi}{16D}\times\left(D^4 - 0.1296D^4\right) = 0.1709\,D^3$$

Now, Maximum bending stress, $\sigma_b = \dfrac{M}{Z} = \dfrac{20\times10^6}{0.08545\,D^3} = \dfrac{23.405\times10^7}{D^3}$

Maximum shear stress, $\tau = \dfrac{T}{Z_P} = \dfrac{40\times10^6}{0.1709\,D^3} = \dfrac{23.405\times10^7}{D^3}$

Principal stress ($\sigma_{1,2}$)

$$\sigma_{1,\,2} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$\Rightarrow \quad \sigma_{1,\,2} = \frac{23.405\times10^7}{2D^3} \pm \sqrt{\left(\frac{23.405\times10^7}{2D^3}\right)^2 + \left(\frac{23.405\times10^7}{D^3}\right)^2}$$

$$\Rightarrow \quad \sigma_{1,\,2} = \frac{23.405\times10^7}{D^3}\left[\frac{1}{2} \pm \sqrt{\frac{1}{4}+1}\right]$$

$$\Rightarrow \quad = \frac{23.405\times10^7}{D^3}[0.5 \pm 1.12]$$

$$\Rightarrow \quad \sigma_1 = \frac{37.87\times10^7}{D^3},\ \sigma_2 = -\frac{14.51\times10^7}{D^3}$$

1. Maximum principal stress theory

$$\sigma_1 \le \frac{f_y}{\text{FOS}}$$

$$\frac{37.87\times10^7}{D^3} \le \frac{250}{2}$$

$$\Rightarrow \quad D \ge 144.69 \text{ mm} \simeq 145 \text{ mm}$$

So, External diameter of shaft, d_0 = 145 mm

Internal diameter of shaft, $d_i = 0.6\, d_0$

$$= 0.6 \times 145 = 86.818 \text{ mm}$$

$$d_1 = 87 \text{ mm}$$

2. Maximum strain energy theory

$$U \le \frac{1}{2E}\left(\frac{f_y}{\text{FOS}}\right)^2$$

$$\Rightarrow \quad \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right) \le \frac{1}{2E}\left(\frac{f_y}{\text{FOS}}\right)^2$$

$$\Rightarrow 10^{14}\left[\left(\frac{37.87}{D^3}\right)^2 + \left(-\frac{14.51}{D^3}\right)^2 - 2\times0.3\times\left(\frac{37.87}{D^3}\right)\left(-\frac{14.51}{D^3}\right)\right] \le \left(\frac{250}{2}\right)^2$$

$$\Rightarrow \quad \frac{10^{14}}{D^6}(1452.98 + 212.87 + 333.68) \le 15625$$

$$\Rightarrow \quad D^6 \ge 0.128 \times 10^{14}$$

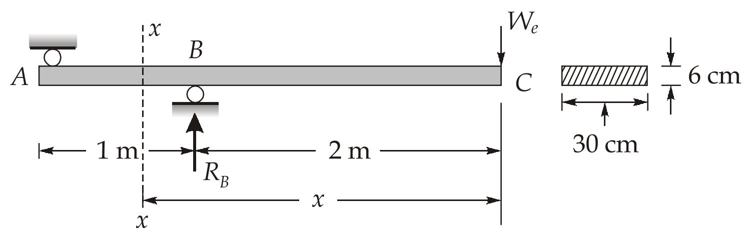
$$\Rightarrow \quad D \ge 152.58 \text{ mm}$$

So, external diameter of shaft, d_0 = 153 mm

Internal diameter of shaft, d_i = 91.8 mm

Q.4 (a) Solution:

Let W_e be equivalent gradually applied load which produced same deflection as produced by man.



Taking moment about A,

$$W_e \times 3 = R_B \times 1$$

$$R_B = 3W_e$$

Taking moment about x-x section

$$M_x = -W_e \times x + R_B(x-2)$$

$$EI\left(\frac{d^2y}{dx^2}\right) = -W_e x + 3W_e(x-2)$$

$$EI\left(\frac{dy}{dx}\right) = \frac{W_e \times x^2}{2} - \frac{3W_e(x-2)^2}{2} + C_1$$

$$EI(y) = \frac{W_e \times x^3}{6} - \frac{W_e(x-2)^3}{2} + C_1 x + C_2 \quad \text{...(I)}$$

At $x = 2, y = 0$

$$\Rightarrow \quad 0 = \frac{W_e \times 2^3}{6} + 2C_1 + C_2$$

$$\Rightarrow \quad 2C_1 + C_2 = \frac{-4W_e}{3} \quad \text{...(ii)}$$

At $x = 3, y = 0$

$$\Rightarrow \quad 0 = \frac{W_e \times 3^3}{6} - \frac{W_e(1)^3}{2} + 3C_1 + C_2$$

$$\Rightarrow \quad 3C_1 + C_2 = -4W_e \quad \text{...(iii)}$$

On solving equation (ii) and (iii)

$$C_1 = \frac{-8}{3}W_e, W_2 = 4W_e$$

On putting the value of C_1 and C_2 in equation (i)

$$EIy = \frac{W_e x^3}{6} - \frac{W_e(x-2)^3}{2} - \frac{8}{3}W_e x + 4W_e$$

At $x = 0$

$$EIy_c = 4W_e$$

$$y_c = \frac{4W_e}{EI}$$

$$I = \frac{1}{12}\times 300\times 60^3 = 540 \times 10^4 \text{ mm}^4$$

$$EI = 1 \times 10^4 \times 540 \times 10^4 \text{ N-mm}^2 = 54000 \text{ N-m}^2$$

$$\therefore \quad y_c = \frac{4W_e}{54000}$$

Work done be man when he jumps

$$= 600(h + y_c)$$

$$= 600\times\left(0.5+\frac{4W_e}{54000}\right) \quad \text{...(iv)}$$

Work done by equivalent applied load

$$= \frac{1}{2}W_e\times\delta = \frac{1}{2}W_e\times\frac{4W_e}{54000} = \frac{W_e^2}{27000} \quad \text{...(v)}$$

From equation (iv) and (v)

$$600\left(0.5+\frac{4W_e}{54000}\right) = \frac{W_e^2}{27000}$$

$$\Rightarrow \quad 8100000 + 1200\,W_e = W_e^2$$

$$\Rightarrow \quad W_e^2 - 1200W_e = 8100000$$

$$\Rightarrow \quad W_e^2 - 1200W_e - 8100000 = 0$$

$$W_e = 3508.6 \text{ N}, -2308.6$$

Take + ve sign $\quad W_e = 3508.6$ N

Maximum bending moment $= 3508.6 \times 2$ Nm

$= 7017.2$ Nm $= 7017200$ Nmm

Maximum stress produced

$$f_{max} = \frac{M_{max}}{y}\times y_{max}$$

$$\Rightarrow \quad f_{max} = \frac{7017200}{540\times 10^4}\times\left(\frac{60}{2}\right) = 38.97 \text{ N/mm}^2$$

Q.4 (b) Solution:

Given: External diameter, $D = 200$ mm

$f_c = 90$ N/mm^2, $L = 3$ m, $P = 800$ kN

Let internal diameter of hollow cast iron column is d.

$$\text{Effective length of column, } L_{eff} = \frac{L}{2} = \frac{3000}{2} = 1500 \text{ mm}$$

Using Rankine's formula

$$P = \frac{f_c A}{1 + \frac{1}{1600}\left(\frac{L}{r}\right)^2} \quad \text{...(i)}$$

Here, Area of cross section, $A = \frac{\pi}{4}(D^2 - d^2)$

$$\text{Area moment of inertia, } I_{min} = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$\text{Area moment of inertia, } I_{min} = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$\text{Radius of gyration, } r = \sqrt{\frac{I}{A}}$$

$$r = \sqrt{\frac{\frac{\pi}{64}\left(D^4 - d^4\right)}{\frac{\pi}{4}\left(D^2 - d^2\right)}}$$

$$\Rightarrow \quad r^2 = \frac{\left(D^2 + d^2\right)\left(D^2 - d^2\right)}{16\left(D^2 - d^2\right)}$$

$$\Rightarrow \quad r^2 = \frac{\left(D^2 + d^2\right)}{16}$$

From equation (i)

$$800 \times 10^3 = \frac{90 \times \frac{\pi}{4}\left(D^2 - d^2\right)}{1 + \frac{1}{1600} \times \frac{16(1500)^2}{D^2 + d^2}}$$

$$\Rightarrow \quad 800 \times 10^3 = \frac{70.686\left(D^2 - d^2\right)\left(D^2 + d^2\right)}{\left(D^2 + d^2\right) + 22500}$$

$\Rightarrow$ $800 \times 10^3(D^2 + d^2) + 800 \times 10^3(22500) = 70.686(D^4 - d^4)$

$\Rightarrow$ $800 \times 10^3(200^2 + d^2) + 800 \times 10^3(22500) = 70.686(200^4 - d^4)$

On simplification, we get $d^4 + 11318d^2 - 8.927 \times 10^8 = 0$

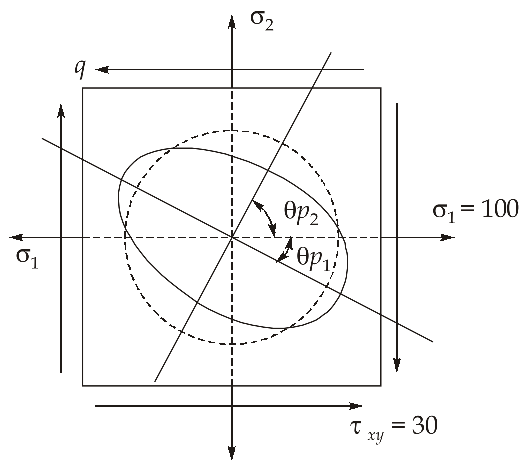
Let $d^2 = x$

$$x^2 + 11317.66x - 8.927 \times 10^8 = 0$$

From which $x = d^2 = 24750.417 \text{ mm}^2 \Rightarrow d = 157.32 \text{ mm}$

$\therefore$ $$\text{Metal thickness} = \frac{D-d}{2} = \frac{200-157.32}{2} = 21.34 \text{ mm}$$

Q.4 (c) Solution:



(All dimensions are in MPa)

$$\text{Principal stresses, } \sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{100+20}{2} \pm \sqrt{\left(\frac{100-20}{2}\right)^2 + (30)^2} = 60 \pm 50$$

$\therefore$ $$\sigma_1 = 110 \text{ N/mm}^2 \text{ and } \sigma_2 = +10 \text{ N/mm}^2$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -\frac{2 \times 30}{100-20} = -0.75$$

$$2\theta_p = -36.87°$$

$$\theta_{p1} = -18°44' \text{(–ve sign indicates anticlockwise direction)}$$

and $$\theta_{p2} = -18°44' + 90° = 71°56'$$

Major principal strain, $\in_x = \left(\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right) = (110 - 0.286 \times 10)\frac{1}{205 \times 1000}$

$\Rightarrow \quad \in_x = +5.226 \times 10^{-4}$

Increase in diameter, $\Delta d_x = \in_x \times d$

$\Rightarrow \quad \Delta d_x = 5.226 \times 10^{-4} \times 400$

$\Delta d_x = 0.209$ mm

Minor principal strain, $\in_y = \frac{\sigma_x}{E} - \frac{\mu\sigma_x}{E} = (10 - 110 - 0.286)\frac{1}{205 \times 1000}$

$\Rightarrow \quad \in_y = -1.047 \times 10^{-4}$

Length of major exes of ellipse = $d + \Delta d_x$

$= 400 + 0.209 = 400.209$ mm

Decrease in diameter, $\Delta d_y = -\in_y \times d$

$= -1.047 \times 10^{-4} \times 400 = -0.0418$ mm

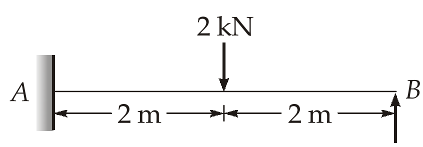
Length of minor exes of ellipse = $d - \Delta d_x$

$= 400 - 0.04188 = 399.958$ mm

Section B : Structural Analysis

Q.5 (a) Solution:

(i) When supports are at the same level.



Fixed end moments,

$$\overline{M}_{AB} = -\frac{wL}{8} = \frac{-2 \times 4}{8} = -1 \text{ kN-m}$$

$$\overline{M}_{BA} = \frac{wL}{8} = 1 \text{ kN-m}$$

$\because$ Support A is fixed, $\theta_A = 0$

$M_{BA} = 0$ [$\because$ support B is simply supported]

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right) + \overline{M}_{BA}$$

$$0 = \frac{2EI}{L}(2\theta_B) + 1$$

$$\therefore \qquad \theta_B = \frac{-L}{4EI}$$

$$\Rightarrow \qquad M_{AB} = \overline{M}_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$= -1 + \frac{2EI}{L} \times \left(-\frac{L}{4EI}\right) = -\frac{3}{2} \text{ kN-m}$$

(ii) When support 'B' sinks by 1 cm

$$\frac{\Delta}{l} = \frac{1}{400}$$

$$\frac{EI}{L} = \frac{2 \times 10^6}{400 \times 100} \text{ kN-m} = 50 \text{ kN-m}$$

$$M_{BA} = 0$$

$$0 = \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\Delta}{L}\right] + 1$$

$$\Rightarrow \qquad 0 = 2 \times 50 \times \left[2\theta_B - \frac{3}{400}\right] + 1$$

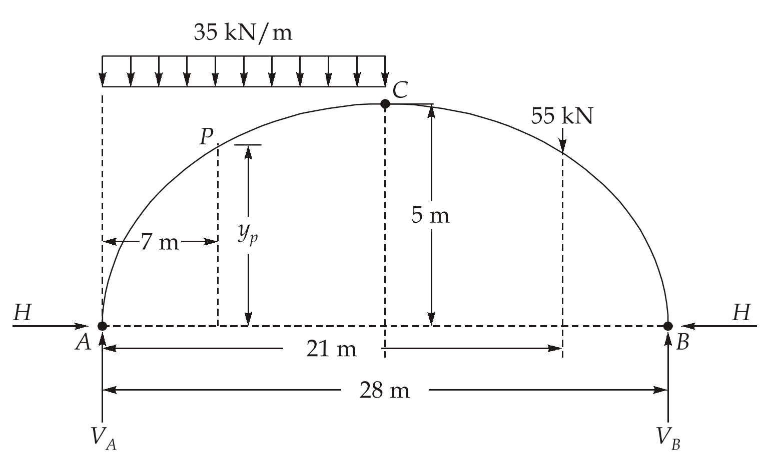
$$\Rightarrow \qquad \theta_B = -\frac{1}{800}$$

$$M_{AB} = -1 + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\Delta}{L}\right]$$

$$= -1 + 2 \times 50\left[-\frac{1}{800} - \frac{3}{400}\right] = -1 + 100 \times \left(-\frac{7}{800}\right)$$

$$= -1.875 \text{ kN-m}$$

5. (b)Solution:



$\Sigma M_B = 0$

$$\Rightarrow \quad V_A(28) = 55\,(28 - 21) + 35 \times 14 \times 21$$

$$\Rightarrow \quad V_A = 381.25 \text{ kN}$$

$$\therefore \quad V_B = (35 \times 14) + 55 - 381.25$$

$$= 163.75 \text{ kN} \qquad [\because V_A + V_B = 545 \text{ kN}]$$

$\Sigma M_C = 0$

$$\Rightarrow \quad V_A(14) = H(5) + 35 \times 14 \times 7$$

$$\Rightarrow \quad 381.25(14) = 5H + 3430$$

$$\therefore \quad H = 381.5 \text{ kN}$$

At section P which is located at 7 m from left support.

$$M = V_A(7) - Hy_p - 35 \times 7 \times \frac{7}{2}$$

$$= 381.25(7) - 381.5\, y_p - 857.5 \qquad \text{... (i)}$$

Taking left support 'A' as origin, equation of parabolic arch is,

$$y = \frac{4y_c}{l^2}\left(lx - x^2\right) = \frac{4 \times 5}{28^2}\left(28x - x^2\right)$$

$$= \frac{5}{196}\left(28x - x^2\right) \qquad \text{... (ii)}$$

$\therefore$ At $x = 7$ m, $y = y_p$ where

$$y_p = \frac{5}{196}\left(28 \times 7 - 7^2\right) = 3.75 \text{ m}$$

$\therefore$ From equation (i):

$$M = 381.25(7) - 381.5(3.75) - 857.5 = 380.625 \text{ kNm}$$

So, Required moment is 380.625 kNm.

Differentiating equation (i) w.r.t. x,

$$y' = \frac{dy}{dx} = \frac{5}{196}(28 - 2x) = \frac{5}{98}(14 - x)$$

At $x = 7$ m,

$$y' = \tan\theta = \frac{5}{98}(14 - 7) = 0.35714$$

$$\therefore \quad \theta = 19.65°$$

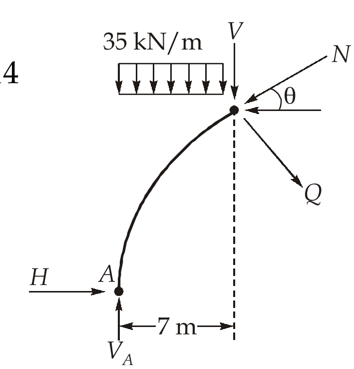
$\therefore$ Normal thrust at section P is

$$N = V\sin\theta + H\cos\theta$$

Now,

$$V = V_A - 35 \times 7 = 381.25 - 245$$

$$= 136.25 \text{ kN}$$



$\therefore$ $N = 136.25 \sin\theta + 381.5 \cos\theta$

$= 136.25 \sin 19.65° + 381.5 \cos 19.65°$

$= 405.1$ kN

Similarly, radial shear is given by,

$Q = V\cos\theta - H\sin\theta$

$= 136.25 \cos 19.65° - 381.5 \sin 19.65° = 0.027$ kN

Q.5 (c) Solution:

For overhang *AE*,

Support moment at *A*, $M_a = 20 \times 1.5 = 30$ kN-m (Hogging)

Applying theorem of three moments, for span *AB* and *BC* subjected to UDL,

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C + l_2 = \frac{-w_1 l_1^3}{4} - \frac{w_2 l_2^3}{4} + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

$$\Rightarrow \quad M_A(3) + 2M_B (3 + 3) + M_C (3) = \frac{-20 \times 3^3}{4} - \frac{30 \times 3^3}{4} + 0$$

$$\Rightarrow \quad -30(3) + 12 M_B + 3 M_C = -337.5$$

$$\Rightarrow \quad 4 M_B + M_C = -82.5 \qquad \text{...(i)}$$

Now consider spans BC and *CD*,

$$M_B l_1 + 2M_C (l_1 + l_2) + M_D (l_2) = \frac{-w_1 l_1^3}{4} - \frac{w_2 l_2^3}{4}$$

$$\Rightarrow \quad M_B (3) + 2M_C (3 + 3) + 0 = \frac{-30 \times 3^3}{4} - \frac{20 \times 3^3}{4}$$

$$\Rightarrow \quad M_B + 4M_C = -112.5 \qquad \text{...(ii)}$$

From (i) and (ii),

$M_B = -14.5$ kN-m

$M_C = -24.5$ kN-m

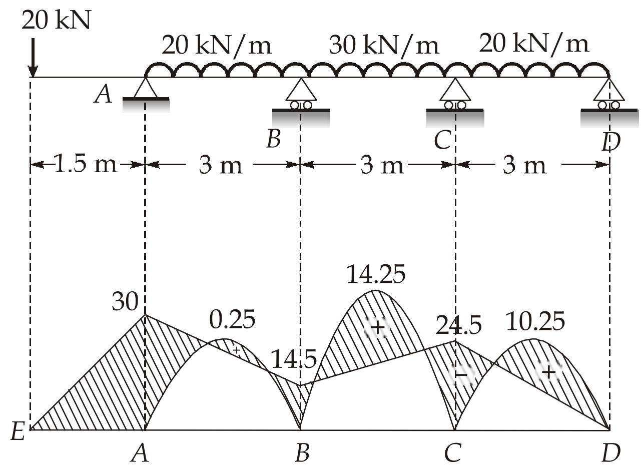
Maximum free B.M at the centre of span *AB* and *CD* is

$$= \frac{20 \times 3^2}{8} = 22.5 \text{ kN-m}$$

Maximum free B.M at the centre of span BC

$$= \frac{30 \times 3^2}{8} = 33.75 \text{ kN-m}$$

Bending moment diagram:



B.M. diagram (in kN-m)

Net positive bending moment at mid span of *AB*,

$$(+M_{net})_{AB} = 22.5 - \left[14.5 + \frac{(30-14.5)}{3} \times 1.5\right] = 0.25 \text{ kN-m}$$

Net positive bending moment at mid span of *BC*,

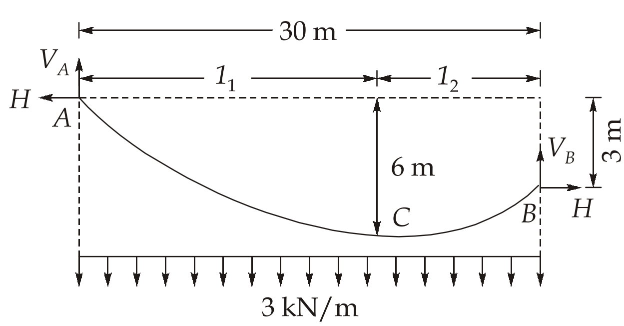
$$(+M_{net})_{BC} = 33.75 - \left[14.5 + \frac{(24.5-14.5)}{3} \times 1.5\right] = 14.25 \text{ kN-m}$$

Net positive bending moment at mid span of *CD*,

$$(+M_{net})_{CD} = 22.5 - \left[\frac{24.5}{3} \times 1.5\right] = 10.25 \text{ kN-m}$$

Q.5 (d) Solution:

Considering equilibrium of part *AC* of the cable,



$$l_1 = l\left(\frac{\sqrt{h_1}}{\sqrt{h_2}+\sqrt{h_2}}\right) = 30\left(\frac{\sqrt{6}}{\sqrt{6}+\sqrt{3}}\right) = 17.57 \text{ m}$$

$$l_2 = l\left(\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right) = 30\left(\frac{\sqrt{3}}{\sqrt{6}+\sqrt{3}}\right) = 12.43\,\text{m}$$

$\Sigma f_y = 0$

$\Rightarrow$ $$V_A + V_B = 3 \times 30 = 90 \quad ...(i)$$

Taking moments about C, (considering left portion)

$$V_A \times 17.57 - H \times 6 - 3 \times \frac{17.57^2}{2} = 0$$

$\Rightarrow$ $$V_A = \left(\frac{6H}{17.57} + \frac{3 \times 17.57}{2}\right) \quad ...(ii)$$

Taking moment aboutC, (Considering right portion)

$$V_B \times 12.43 - H \times 3 - 3 \times \frac{12.43^2}{2} = 0$$

$$V_B = \left(\frac{3H}{12.43} + \frac{3 \times 12.43}{2}\right) \quad ...(iii)$$

On putting the value of V_A and V_B in equation (i), we get

$$\left(\frac{6H}{17.57} + \frac{3 \times 17.57}{2}\right) + \left(\frac{3H}{12.43} + \frac{3 \times 12.43}{2}\right) = 90$$

$$0.5828H = 45$$

$$H = 77.21 \text{ kN}$$

From equation (ii)

$$V_A = \frac{6 \times 77.21}{17.57} + \frac{3 \times 17.57}{2} = 52.72 \text{ kN}$$

From equation (iii)

$$V_B = \frac{3 \times 77.21}{12.43} + \frac{3 \times 12.43}{2} = 37.28 \text{ kN}$$

Maximum tension occur at highest point i.e. (A)

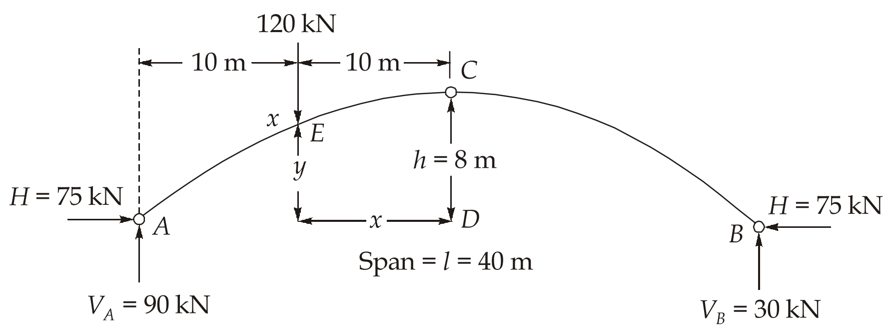
$$T_{max} = \sqrt{V_A^2 + H^2}$$

$$= \sqrt{(52.72)^2 + (77.21)^2} = 93.50 \text{ kN}$$

Minimum tension occur at lowest point i.e. (C)

$$T_{min} = H = 77.21 \text{ kN}$$

Q.5 (e) Solution:



Taking moments about A,

$$V_b \times 40 = 120 \times 10$$

$$\therefore \quad V_b = 30 \text{ kN} \uparrow$$

$$\therefore \quad V_a = 120 - 30 = 90 \text{ kN} \uparrow$$

Taking moments about C of the forces on the right side of C,

$$H \times 8 = 30 \times 20$$

$$H = 75 \text{ kN}$$

Let R be the radius of the arch

$$\therefore \quad 8(2R - 8) = 20 \times 20$$

$$\therefore \quad R = 29 \text{ m}$$

Let D be the middle point of AB.

The equation to the circular arch with D as origin is, $y = \sqrt{R^2 - x^2} - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$

$$y = \sqrt{29^2 - x^2} - \sqrt{29^2 - 20^2}$$

$$y = \sqrt{841 - x^2} - 21$$

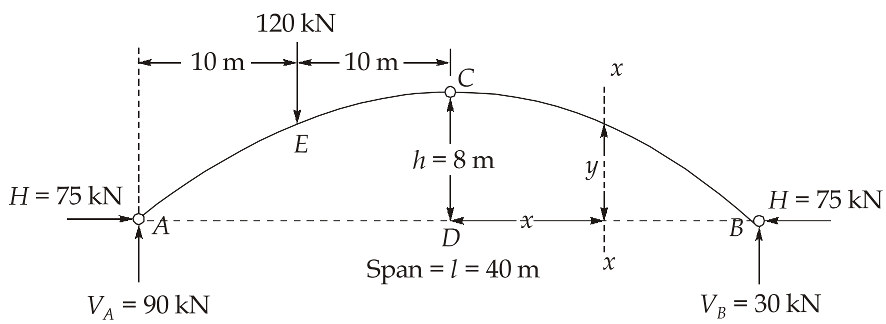
Maximum positive bending moment

The maximum positive bending moment occurs under the load, i.e.,

At E, $$y_E = \sqrt{841 - 10^2} - 21 = 6.221 \text{ m}$$

$$M_{max(+)} = 90 \times 10 - 75 \times 6.221 = +433.425 \text{ kNm}$$

For maximum negative bending moment



Taking moment about x-x section (Right side)

$$M_{xx} = 30 \times (20 - x) - 75y$$

$$\Rightarrow \quad M_{xx} = 30(20 - x) - 75\left(\sqrt{841 - x^2} - 21\right)$$

For maximum bending moment

$$\frac{dM_{xx}}{dx} = 0$$

$$\Rightarrow \quad -30 - 75 \times \frac{(-2x)}{2\sqrt{841 - x^2}} = 0$$

$$\Rightarrow \quad 2\sqrt{841 - x^2} = 5x$$

Squaring both sides

$$4(840 - x^2) = 25x^2$$

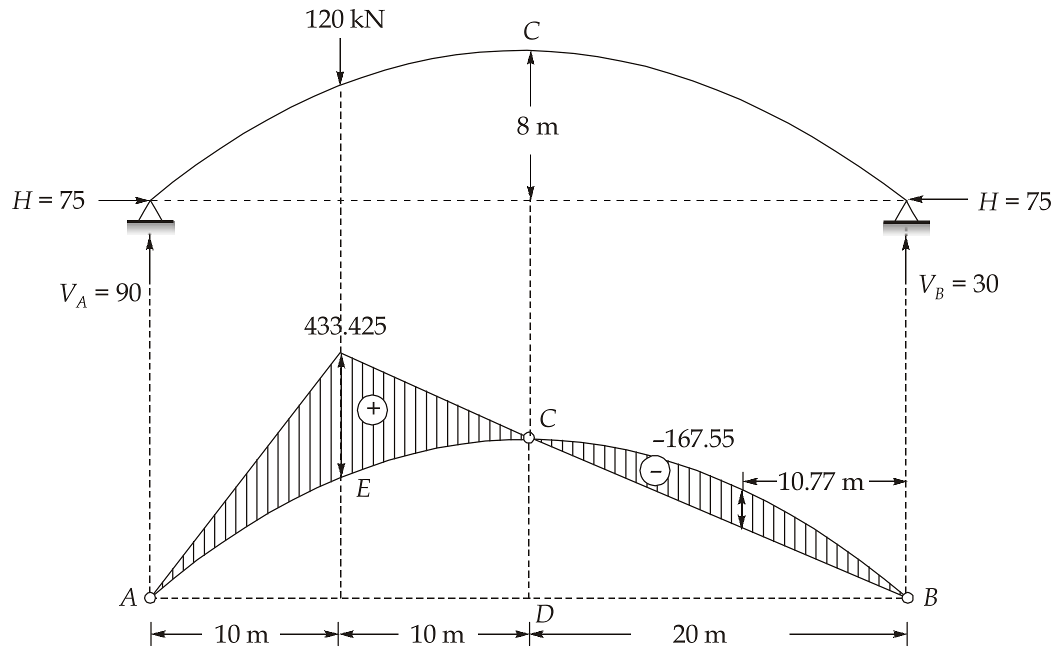
$$4 \times 841 = 29x^2$$

$$x = 10.77 \text{ m}$$

Maximum negative bending moment,

$$M_{\max(-ve)} = 30(20 - 10.77) - 75\left(\sqrt{841 - 10.77^2} - 21\right)$$

$$= -167.55 \text{ kNm}$$



BMD
(Values in kN-m)

Q.6 (a) Solution:

Since ends A and D are fixed. $\theta_A = \theta_D = 0$.

Let θ_B and θ_C be the rotations at 'B' and 'C' respectively and δ be the sway of the frame to the right.

The fixed end moments are,

$$\overline{M}_{AB} = \overline{M}_{BA} = 0, \quad \overline{M}_{BC} = \overline{M}_{CB} = 0$$

$$\overline{M}_{CD} = \overline{M}_{DC} = 0$$

$$\overline{M}_{BC} = 40 \times 1 = 40 \text{ kN-m}$$

$$\overline{M}_{CF} = 20 \times 1 = 20 \text{ kN-m}$$

Slope deflection equation:

$$M_{AB} = \overline{M}_{AB} + \frac{2EI_{AB}}{L_{AB}}\left(2\theta_A + \theta_B - \frac{3\delta}{L_{AB}}\right)$$

For span AB:

$$M_{AB} = \frac{2EI}{4}\left(0 + \theta_B - \frac{3\delta}{4}\right) = \frac{EI}{2}\left(\theta_B - \frac{3\delta}{4}\right)$$

$$M_{BA} = \frac{EI}{2}\left(2\theta_B - \frac{3\delta}{4}\right)$$

For span BC:

$$M_{BC} = \frac{EI}{2}(2\theta_B + \theta_C)$$

$$M_{CB} = \frac{EI}{2}(\theta_B + 2\theta_C)$$

For span *CD*:

$$M_{DC} = \frac{EI}{2}\left(\theta_C - \frac{3\delta}{4}\right)$$

$$M_{CD} = \frac{EI}{2}\left(2\theta_C - \frac{3\delta}{4}\right)$$

For equilibrium, sum of moments at joint *B* is 0.

$$-M_{BA} - M_{BC} = 0$$

$$\frac{EI}{2}\left(2\theta_B - \frac{3\delta}{4}\right) + \frac{EI}{2}(2\theta_B + \theta_C) + 40 = 0$$

$$4\,\theta_B + \theta_C - \frac{3\delta}{4} = -\frac{80}{EI} \quad \text{...(i)}$$

For equilibrium, the sum of moments at joint *C* is 0,

$$-M_{CB} - M_{CD} + 20 = 0$$

$$\frac{EI}{2}(2\theta_C + \theta_B) + \frac{EI}{2}\left(2\theta_C - \frac{3\delta}{4}\right) - 20 = 0$$

$$\theta_B + 4\theta_C - \frac{3\delta}{4} = \frac{40}{EI} \quad \text{...(ii)}$$

Total shear in vertical members is 0,

$$\therefore \quad \frac{M_{AB} + M_{BA}}{4} + \frac{M_{CD} + M_{DC}}{4} = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0$$

$$\frac{EI}{2}\left(\theta_B - \frac{3\delta}{4}\right) + \frac{EI}{2}\left(2\theta_B - \frac{3\delta}{4}\right) + \frac{EI}{2}\left(\theta_C - \frac{3\delta}{4}\right) + \frac{EI}{2}\left(2\theta_C - \frac{3\delta}{4}\right) = 0$$

$$3\theta_B + \theta_C - 3\delta = 0 \quad \text{...(iii)}$$

From (i), (ii) and (iii),

$$\theta_B = -\frac{25.7143}{EI}$$

$$\theta_C = \frac{14.2857}{3EI}$$

$$\delta = \frac{-11.4286}{9EI}$$

Final bending moments are

$$M_{AB} = \frac{1}{2}\times(-25.7143)-\frac{3}{8}(-11.4286) = -8.57 \text{ kN-m}$$

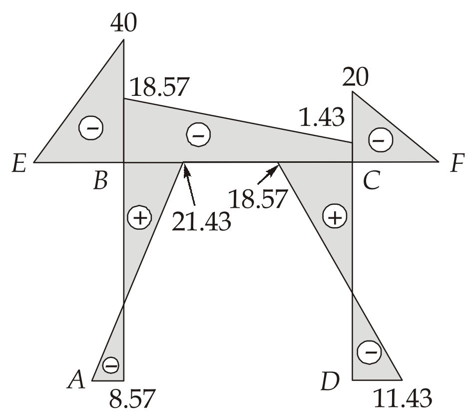
$$M_{BA} = -25.7143-\frac{3}{8}(-11.4286) = -21.43 \text{ kN-m}$$

$$M_{BC} = -25.7143+\frac{1}{2}(14.2857) = -18.57 \text{ kN-m}$$

$$M_{CB} = \frac{1}{2}\times(-25.7143)+(14.2857) = 1.43 \text{ kN-m}$$

$$M_{DC} = \frac{1}{2}(14.2857)-\frac{3}{8}(-11.4286) = 11.43 \text{ kN-m}$$

$$M_{CD} = (14.2857)-\frac{3}{8}(-11.4286) = 18.57 \text{ kN-m}$$



BMD (All value in kN-m)

Q.6 (b) Solution:

Let V_A and V_D be the vertical reactions at 'A' and 'D' respectively.

Taking moments about A,

$\Sigma M_A = 0, \Rightarrow$ $V_D \times 6 = 15 \times 1.5 + 30 \times 4.5$

$V_D = 26.25$ kN

$\Sigma f_y = 0, \Rightarrow$ $V_A + V_D = 45$

$V_A = 18.75$ kN

Forces in the members of the truss:

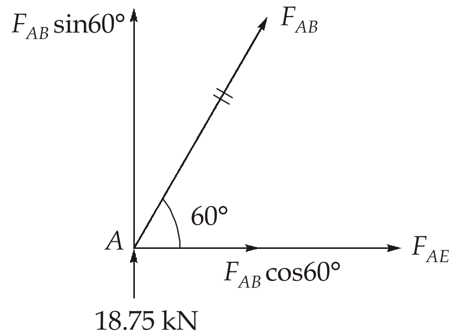
Joint A: Resolve vertically,

$\Sigma f_y = 0,$

$F_{AB} \sin 60° = -18.75$

$F_{AB} = -21.65$ kN = 21.65 kN (compressive)

Resolve horizontally,

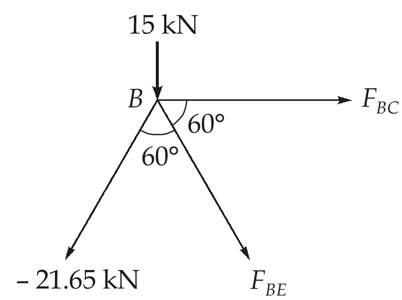


$\Sigma f_x = 0$

$$F_{AB} \cos 60° + F_{AE} = 0$$

$$F_{AE} = 10.83 \text{ kN (Tensile)}$$

Joint B:



$\Sigma f_y = 0,$

$$-F_{BE} \sin 60° - 15 - (-21.65 \sin 60°) = 0$$

$$F_{BE} = 4.33 \text{ kN (Tensile)}$$

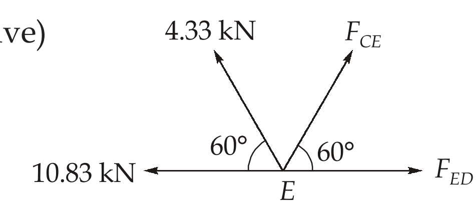
$\Sigma F_x = 0,$

$$F_{BC} + F_{BE} \cos 60° - (-21.65 \cos 60°) = 0$$

$$F_{BC} = -13 \text{ kN}$$

$$= 13 \text{ kN (compressive)}$$

Joint *E*:



$\Sigma f_y = 0, \Rightarrow \quad 4.33 \sin 60° + F_{CE} \sin 60° = 0$

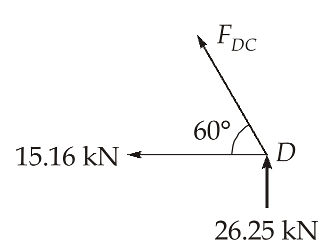
$$F_{CE} = -4.33 \text{ kN} = 4.33 \text{ kN (Compressive)}$$

$\Sigma f_x = 0,$

$$\Rightarrow \quad 10.83 + 4.33 \cos 60° = F_{CE} \cos 60° + F_{ED}$$

$$F_{ED} = 15.16 \text{ kN (Tensile)}$$

Joint D:

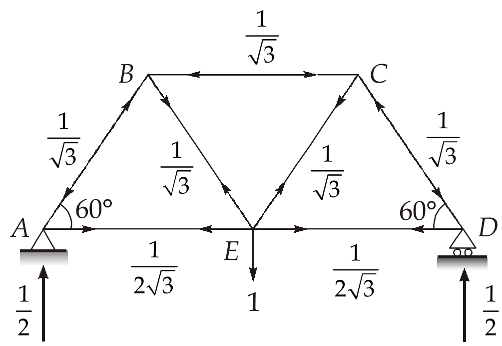


$\Sigma f_y = 0,$

$$F_{DC} \sin 60° = -26.25$$

$$F_{DC} = -30.32 \text{ kN} = 30.32 \text{ kN (Compressive)}$$

Now, applying a vertical load of 1 kN at *E*,



Joint A: $\Sigma f_y = 0, \Rightarrow \quad K_{AB} = \dfrac{1}{\sqrt{3}}$ (Compressive)

$\Sigma f_x = 0, \Rightarrow \quad K_{AE} = \dfrac{1}{2\sqrt{3}}$ (Tensile)

Joint B: $\Sigma f_y = 0, \Rightarrow \quad K_{BE} = \dfrac{1}{\sqrt{3}}$ (Tensile)

$\Sigma f_x = 0, \Rightarrow \quad K_{BC} = \dfrac{1}{\sqrt{3}}$ (Compressive)

Vertical deflection of $E = \Sigma\left(\dfrac{Pkl}{AE}\right)$

Member	P	K	PK
AB	– 21.65	$\frac{-1}{\sqrt{3}}$	+ 12.50
BC	– 13	$-\frac{1}{\sqrt{3}}$	+ 7.505
CD	– 30.32	$-\frac{1}{\sqrt{3}}$	+ 17.51
DE	15.16	$+\frac{1}{2\sqrt{3}}$	+ 4.38
EA	10.83	$+\frac{1}{2\sqrt{3}}$	+ 3.13
BE	4.33	$+\frac{1}{\sqrt{3}}$	+ 2.50
CE	– 4.33	$+\frac{1}{\sqrt{3}}$	– 2.50
		$\Sigma PK = 45.025$	

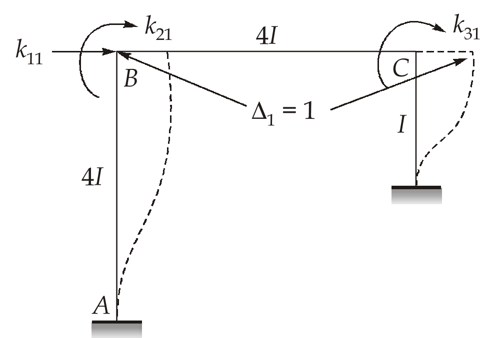
$\therefore$ Vertical deflection of joint $E = \Sigma\left(\dfrac{PKl}{AE}\right) = \dfrac{45.025 \times 10^3 \times 3000}{1200 \times 2 \times 10^5} \simeq 0.563$ mm

Q.6 (c) Solution:

The stiffness matrix can be developed by giving a unit displacement successively at coordinates 1, 2 and 3 without any displacement at other coordinates and determining the forces required at all the coordinates.

To generate the first column of the stifness matrix, give a unit displacement at coordinate. 1 as shown in figure.

For 1st column of stiffness matrix



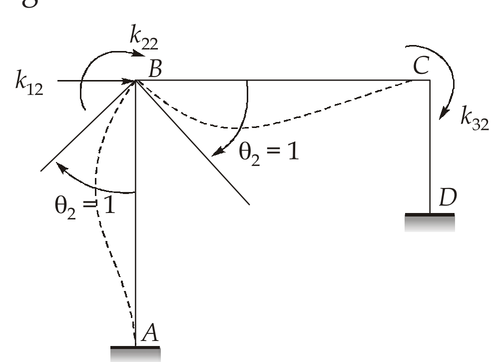
$$k_{11} = \frac{12E(4I)}{10^3} + \frac{12E(I)}{5^3} = 0.144EI$$

$$k_{21} = -\frac{6E(4I)}{10^2} = -0.24EI$$

$$k_{31} = -\frac{6E(I)}{5^2} = -0.24EI$$

- For 2nd column of stiffness matrix:

To generate the second column of the stiffness matrix, give a unit displacement at coordinate 3 as shown in figure.



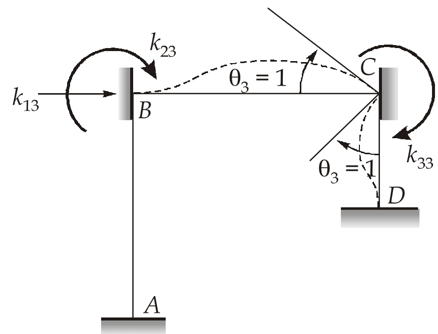
$$k_{12} = \frac{-3}{2}\left(\frac{M_{BA}}{l_{BA}}\right) = \frac{-3}{2}\frac{4E(4I)}{10 \times 10} = -0.24EI$$

$$k_{22} = \frac{4E(4I)}{10} + \frac{4E(4I)}{10} = 3.2EI$$

$$k_{23} = \frac{M_{BC}}{2} = \frac{1}{2} \times \frac{4E(4I)}{10} = 0.80EI$$

- For 3rd column of stiffness matrix

To generate the third column of the stiffness matrix, give a unit displacement at coordinate 3 as shown in figure.



$$k_{13} = \frac{-3}{2}\left(\frac{M_{CD}}{2}\right) = \frac{-3}{2}\frac{\left(\frac{4EI}{5}\right)}{5} = -0.24EI$$

$$k_{23} = \frac{1}{2}\left(\frac{4E(4I)}{10}\right) = 0.8EI$$

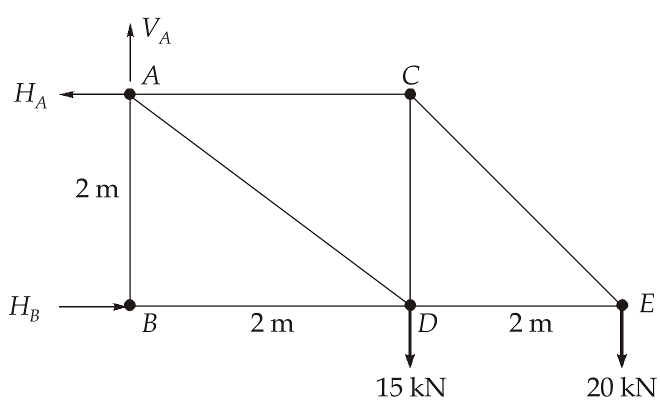
$$k_{33} = \frac{4E(4I)}{10} + \frac{4EI}{5} = 2.4EI$$

Hence, the required stiffness matrix $[k]$ is given by the equation

$$[k] = EI\begin{bmatrix} 0.144 & -0.240 & -0.240 \\ -0.240 & 3.200 & 0.800 \\ -0.240 & 0.800 & 2.400 \end{bmatrix}$$

Q.7 (a) Solution:

Consider diagonal member BC as redundant



Taking moments about A,

$$\Sigma M_A = 0 \quad \Rightarrow \quad H_B \times 2 = 15 \times 2 + 20 \times 4$$

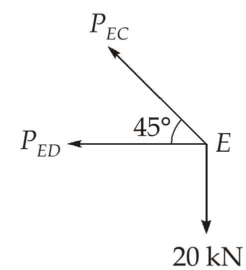
$$H_B = 55 \text{ kN } (\rightarrow)$$

$\therefore$ Horizontal reaction at A = $H_A = 55$ kN ($\leftarrow$)

And vertical reaction at A, V_A = 35 kN

Forces in the members :

Joint E,



$$\Sigma F_y = 0, \Rightarrow \quad P_{EC} \sin 45° = 20 \text{ kN}$$

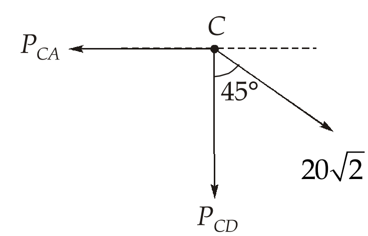
$$P_{EC} = 20\sqrt{2} \text{ kN (Tensile)}$$

$$\Sigma F_x = 0, \Rightarrow \quad -P_{ED} - P_{EC} \cos 45° = 0$$

$$P_{ED} = -20 \text{ kN}$$

$$P_{ED} = 20 \text{ kN (compressive)}$$

Joint C,



$$\Sigma F_x = 0, \Rightarrow \quad P_{CA} = 20\sqrt{2} \sin 45°$$

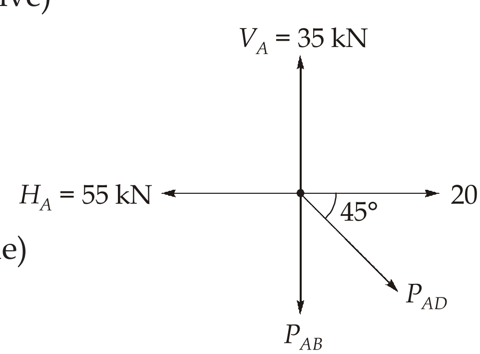
$$P_{CA} = 20 \text{ kN (Tensile)}$$

$$\Sigma F_y = 0, \Rightarrow \quad -P_{CD} - 20\sqrt{2} \cos 45° = 0$$

$$P_{CD} = -20 \text{ kN}$$

$$P_{CD} = 20 \text{ kN (compressive)}$$

Joint A,



$$\Sigma F_x = 0,$$

$$20 + P_{AD} \cos 45° = 55$$

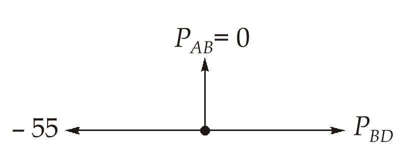
$$P_{AD} = 35\sqrt{2} \text{ kN (Tensile)}$$

$$\Sigma F_y = 0, \Rightarrow -P_{AB} - P_{AD} \sin 45° + 35 = 0$$

$$P_{AB} = 0$$

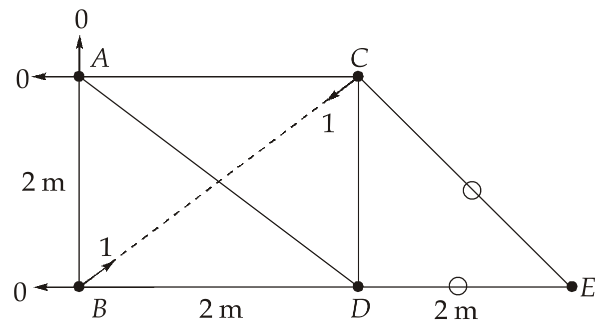
Joint B,

$$\Sigma F_x = 0,$$



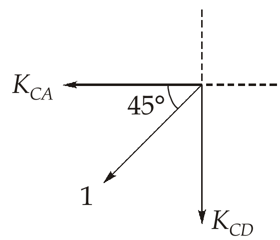
$$P_{BD} = 55 \text{ kN (Compressive)}$$

Now apply a pair of unit forces 1 kN each at B and C in the direction of member BC.



There will be no force in members EC and ED.

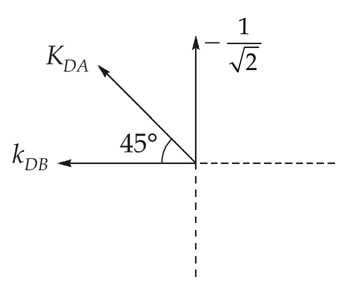
Joint C,



$$\Sigma f_x = 0 \quad \Rightarrow \quad K_{CA} = \frac{1}{\sqrt{2}} \text{ (compressive)}$$

$$\Sigma f_y = 0 \quad \Rightarrow \quad K_{CD} = -1 \sin 45° = \frac{1}{\sqrt{2}} \text{ (compressive)}$$

Joint D,

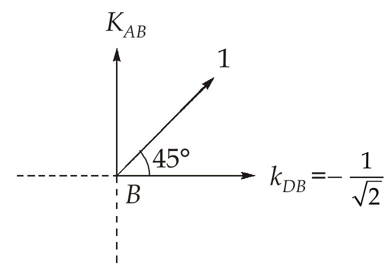


$$\Sigma f_y = 0, \quad \Rightarrow \quad K_{DA} \cos 45° = \frac{1}{\sqrt{2}}$$

$$K_{DA} = 1 \text{ (Tensile)}$$

$$\Sigma f_x = 0, \quad \Rightarrow \quad K_{DB} = -1 \cos 45°$$

$$K_{DB} = \frac{1}{\sqrt{2}} \text{ (compressive)}$$



Joint B,

$$\Sigma f_y = 0, \Rightarrow K_{AB} + 1 \times \sin 45° = 0$$

$$K_{AB} = \frac{1}{\sqrt{2}} \text{ (compressive)}$$

Actual force S in any member is,

$$S = P + XK$$

Where,
$$X = \frac{-\Sigma\left(\frac{PKL}{AE}\right)}{\Sigma\left(\frac{K^2L}{AE}\right)} = -\frac{\Sigma(PKL)}{\Sigma(K^2L)}$$

($\because$ Area A and Young's Modulus E is same for every member)

Member	P (kN)	K (kN)	l (m)	PKl	K^2l
AC	+ 20	$-\frac{1}{\sqrt{2}}$	2	$-20\sqrt{2}$	1
CE	$+20\sqrt{2}$	0	$2\sqrt{2}$	0	0
ED	– 20	0	2	0	0
DB	– 55	$-\frac{1}{\sqrt{2}}$	2	$+55\sqrt{2}$	1
DC	– 20	$-\frac{1}{\sqrt{2}}$	2	$+20\sqrt{2}$	1
AD	$+35\sqrt{2}$	+ 1	$2\sqrt{2}$	+ 140	$2\sqrt{2}$
BC	0	+ 1	$2\sqrt{2}$	0	$2\sqrt{2}$
AB	0	$-\frac{1}{\sqrt{2}}$	2	0	1
				Σ(PKL) = 217.78,	$\Sigma(K^2l)$ = 9.66

$\therefore$
$$x = \frac{-\Sigma\left(\frac{PKl}{AE}\right)}{\Sigma\left(\frac{K^2l}{AE}\right)} = \frac{-\Sigma(PKl)}{\Sigma(K^2l)} = \frac{-217.78}{9.66} = -22.54$$

Actual force in members,

$$F_{AC} = 20 - 22.54 \times \left(-\frac{1}{\sqrt{2}}\right) = 35.94 \text{ kN (Tensile)}$$

$$F_{CE} = 20\sqrt{2} - 22.54(0) = 28.28 \text{ kN (Tensile)}$$

$$E_D = -20 + 0 = 20 \text{ kN (Compressive)}$$

$$F_{DB} = -55 - 22.54 \times \left(-\frac{1}{\sqrt{2}}\right) = 39.06 \text{ kN (Compressive)}$$

$$F_{DC} = -20 - 22.54 \times \left(-\frac{1}{\sqrt{2}}\right) = 4.06 \text{ kN (Compressive)}$$

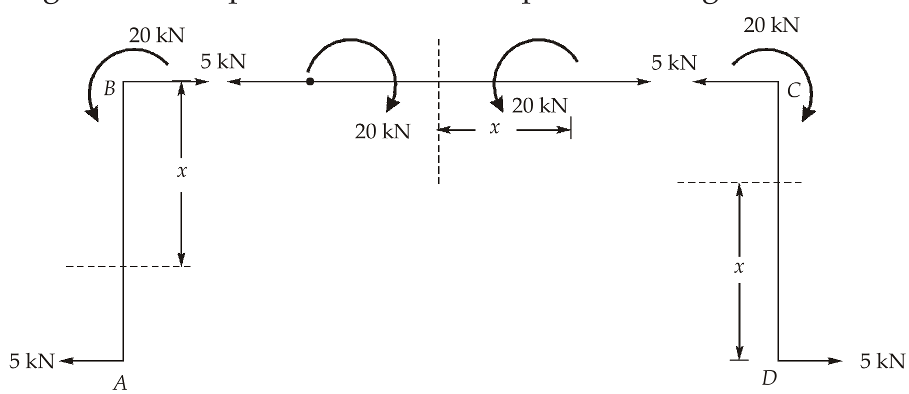
$$F_{AD} = 35\sqrt{2} - 22.54(1) = 26.96 \text{ kN (Tensile)}$$

$$F_{BC} = 0 - 22.54\ (1) = 22.54 \text{ kN (Compressive)}$$

$$F_{AB} = 0 - 22.54 \times \left(-\frac{1}{\sqrt{2}}\right) = 15.73 \text{ kN (Tensile)}$$

Q.7 (b) Solution:

The bending moment expression for various portions are given in table.



Portion	CD	BC	AB
Origin	D	C	B
Limit	$0-4$	$0-3$	$0-4$
M_x	$-5x$	-20	$-20+5x$

$$\text{Total strain energy, } U = \int_0^4 \frac{(-5x)^2}{2EI}dx + \int_0^3 \frac{(20)^2}{2EI}dx + \int_0^4 \frac{(-20+5x)^2}{2EI}dx$$

$$\Rightarrow \qquad U = \frac{25}{2EI}\left[\frac{x^3}{3}\right]_0^4 + \frac{400}{2EI}[x]_0^3 + \frac{1}{2EI}\left[400x - 200\frac{x^2}{2} + \frac{25x^3}{3}\right]_0^4$$

$$\Rightarrow \qquad U = \frac{266.67}{EI} + \frac{600}{EI} + \frac{1}{2EI}\left[1600 - 1600 + \frac{25 \times 64}{3}\right]$$

$$\Rightarrow \qquad U = \frac{1133.33}{EI} \qquad \text{...(i)}$$

Now, $$\text{Work done} = \frac{1}{2} \times P \times \Delta = \frac{1}{2} \times 5\Delta = 2.5\Delta \qquad \text{...(ii)}$$

From Equation (i) and (ii)

$$2.5\Delta = \frac{1133.33}{EI} = 56.7 \text{ mm}$$

$$\Rightarrow \qquad 2.5\Delta = \frac{1133.33}{8000} = 0.14166$$

$$\Rightarrow \qquad \Delta = 56.67 \text{ mm}$$

Q.7 (c) Solution:

The influence line diagram for moment at C, 10 m from the support A is as shown in figure.

where $$y_c = \frac{z(L-z)}{L} = \frac{10(25-10)}{25} = 6$$

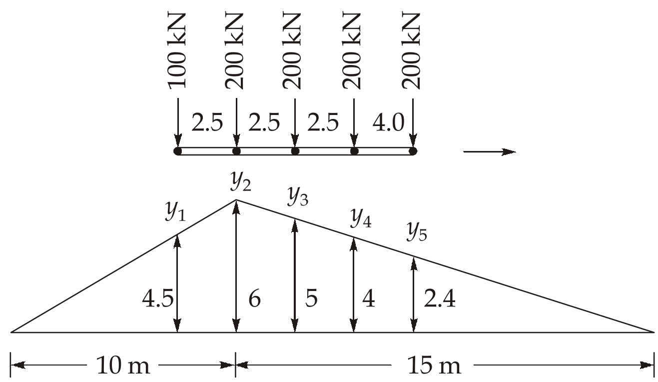
To find load position for maximum moment at C, we have to find average load on portion AC and CB and identify the load which when crosses makes the lighter portion heavier and heavier portion lighter.

When the train of load moves form left to right:

Load rolling on *C*	**Average load on *AC***		**Average load on *CB***
100	$\frac{800}{10}$	>	$\frac{100}{15}$
First 200	$\frac{600}{4}$	>	$\frac{300}{15}$
Second 200	$\frac{400}{10}$	>	$\frac{500}{15}$
Third 200	$\frac{200}{10}$	<	$\frac{700}{15}$

Therefore, when third 200 kN load is just on the section, moment is maximum at C.

Referring to figure.



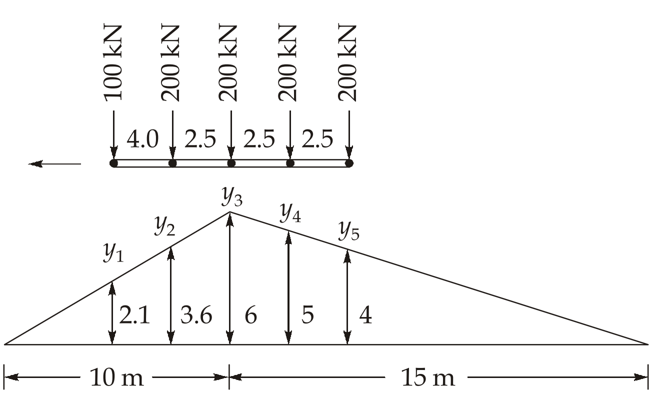
$$\text{Maximum moment at C} = 200y_1 + 200y_2 + 200y_3 + 200y_4 + 100y_5$$
$$= 200 \times 4.5 + 200 \times 6 + 200 \times 5 + 200 \times 4 + 100 \times 2.4$$
$$= 4140 \text{ kNm}$$

When load moves from right to left.

Calculations to find load position for maximum M_C when loads more from right to left

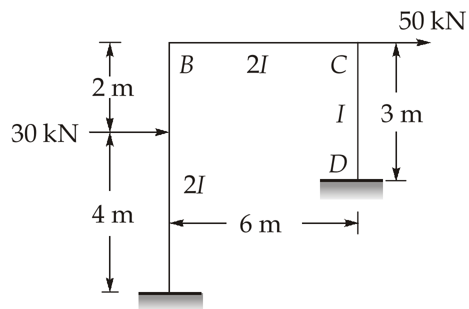
Load crossing	Average load on *AC*		Average load on *CB*
100	$\frac{100}{10}$	<	$\frac{800}{15}$
First 200	$\frac{300}{10}$	<	$\frac{600}{15}$
Second 200	$\frac{500}{10}$	>	$\frac{400}{15}$

Therefore, moment is maximum when second 200 kN load is on the section. Maximum moment is this case

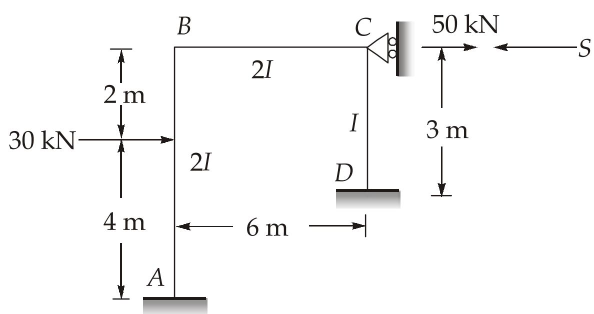


$$\text{Maximum moment at } C = 100y_1 + 200y_2 + 200y_3 + 200y_4 + 200y_5$$
$$= 100 \times 2.1 + 200 \times 3.6 + 200 \times 6 + 200 \times 5 + 200 \times 4$$
$$= 3930 \text{ kNm}$$

Q.8 (a) Solution:



Non-sway analysis:



Fixed end moment

$$M_{FAB} = -\frac{30 \times 4 \times 2^2}{6^2} = -13.33 \text{ kNm}$$

$$M_{FBA} = \frac{30 \times 4^2 \times 2}{6^2} = 26.67 \text{ kNm}$$

Distribution factors

Joints	Members	k	Σk	Distribution factors
B	BA	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	$\frac{8}{3}EI$	0.5
	BC	$\frac{4E(2I)}{6} = \frac{4}{3}EI$		0.5
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	$\frac{8}{3}EI$	0.5
	CD	$\frac{4}{3}EI$		0.5

Moment distribution for non-sway case

A	*B*		*C*		*D*
	0.5	0.5	0.5	0.5	
– 13.33	26.67				
	– 13.34	– 13.33			
– 6.67			–6.67		
			3.33	3.34	
		1.67			1.67
	– 0.84	– 0.83			
– 0.42			–0.42		
			0.21	0.21	
		0.10			0.10
	– 0.05	– 0.05			
– 20.42	12.44	–12.44	–3.55	3.55	1.77

Considering free body diagrams of columns, we get,

$$\vec{H}_A = \frac{M_{AB} + M_{BA} - 30 \times 2}{6} = \frac{-20.42 + 12.44 - 30 \times 2}{6}$$

$$= -11.33 \text{ kN}$$

$$\vec{H}_D = \frac{M_{CD} + M_{DC}}{3}$$

$$\vec{H}_D = \frac{3.55 + 1.77}{3} = 1.773 \text{ kN}$$

Consider the horizontal equilibrium ofthe frame, we get,

$$S = 30 + H_A + H_D + 50$$

$$= 30 - 11.33 + 1.773 + 50 = 70.443 \text{ kN}$$

Sway analysis:

Let the frame sway by D towards right. Fixed end moments developed in column AB be M_{F1} and that in CD be M_{F2}.

Then,
$$M_{F1} = -\frac{6E(2I)\Delta}{6^2} = -\frac{EI\Delta}{3}$$

$$M_{F2} = -\frac{6EI\Delta}{3^2} = -\frac{2}{3}EI\Delta$$

$\therefore$
$$\frac{M_{F1}}{M_{F2}} = \frac{1}{2}$$

Let $M_{F1} = -10$ kNm and $M_{F2} = -20$ kNm

i.e. $M_{FAB} = M_{FBA} = -10$ kNm and $M_{FCD} = M_{FDC} = -20$ kNm

Now, for this case, moment distribution may be carried out.

Moment distribution for arbitrary sway

A	*B*		*C*		*D*
	0.5	0.5	0.5	0.5	
– 10	–10			–20	–20
	5	5	10	10	
2.5		5.0	2.5		5.0
	–2.5	–2.5	–1.25	–1.25	
–1.25		–0.63	–1.25		–0.63
	0.31	0.32	0.62	0.63	
0.16		0.31	0.16		0.32
	–0.15	–0.16	–0.08	–0.08	
–0.08		–0.04	–0.08		–0.04
	0.02	0.02	0.04	0.04	
–8.67	–7.32	7.32	10.66	–10.66	–15.35

$$H_A = \frac{M_{AB} + M_{BA}}{6} \text{ and } H_D = \frac{M_{CD} + M_{DC}}{3}$$

$$S' + H_A + H_D = 0$$

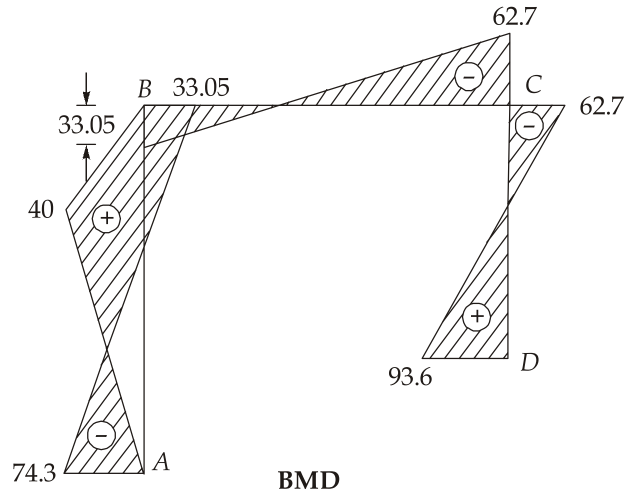
$$S' - \left(\frac{8.67 + 7.32}{6}\right) - \left(\frac{10.66 + 15.35}{3}\right) = 0$$

$$S' = 11.335 \text{ kN}$$

Therefore, sway correction factor $k = \frac{S}{S'} = \frac{70.443}{11.335} = 6.2146$

Final moment calculations

$M_{\text{Arbitrary sway}}$	– 8.67	– 7.32	7.32	10.66	–10.66	–15.35
$M_{\text{Sway}} = KM_{\text{Arbitrary sway}}$	– 53.88	– 45.49	45.49	66.25	– 66.25	– 95.39
$M_{\text{Non-sway}}$	– 20.42	12.44	– 12.44	– 3.55	3.55	1.77
$M_{\text{Final}} = (M_{\text{Non- sway}} + KM_{\text{Arbitrary sway}})$	74.30	– 33.05	33.05	62.70	–62.70	– 93.62

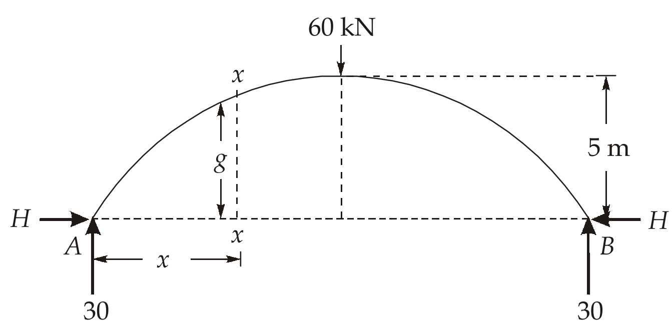


BMD
All dimension are in kN-m

Q.8 (b) Solution:

Given:

$$E = 200 \text{ kN/m}^2 = 200 \times 10^6 \text{ kN/m}^2$$
$$I = 5 \times 10^9 \text{ mm}^4 = 5 \times 10^{-3} \text{ m}^4$$
$$A_m = 10000 \text{ mm}^2 = 10^{-2} \text{ m}^2$$
$$\alpha = 10 \times 10^{-6}/°\text{C}$$
$$\Delta T = +20°\text{C}$$



Taking A as the origin, equation of parabolic arch is given by

$$y = \frac{4hx(L-x)}{L^2}$$

$$\Rightarrow \qquad y = \frac{4(5)\times x(L-x)}{50^2} = \frac{x(50-x)}{125}$$

Now,

$$\int_0^{50} y^2 \frac{dx}{EI} = \int_0^{50} \frac{x^2(50-x)^2}{125^2} dx$$

$$\Rightarrow \quad \int_0^{50} y^2 \frac{dx}{EI} = \int_0^{50} \left(\frac{2500x^2 - 100x^3 + x^4}{125^2} \right) dx$$

$$= \frac{1}{125^2} \left[2500\left(\frac{x^3}{3}\right) - 100\left(\frac{x^4}{4}\right) + \left(\frac{x^5}{5}\right) \right]_0^{50}$$

$$\Rightarrow \quad \int_0^{50} y^2 \frac{dx}{EI} = \frac{666.67}{EI} \quad \text{...(i)}$$

Beam moment is $M_x = 30x$

$$\therefore \quad \int_0^{50} My \frac{dx}{EI} = 2\int_0^{25} 30x\left(\frac{x}{125}\right)(50-x)\frac{dx}{EI} = \frac{60}{125EI}\int_0^{25} (50x^2 - x^3)dx$$

$$= \frac{60}{125EI}\left[50\frac{x^3}{3} - \frac{x^4}{4}\right]_0^{25} = \frac{78125}{EI} \quad \text{...(ii)}$$

Now, $EI = 200 \times 10^6 \times 5 \times 10^{-3} = 10^6 \text{ kNm}^2$

$EA_m = 200 \times 10^6 \times 10000 \times 10^{-2} = 2 \times 10^6 \text{ kN}$

(i) Neglecting the rib shortening, horizontal thrust developed is,

$$H = \frac{\int My\left(\frac{dx}{EI}\right) + L\alpha\Delta t}{\int y^2\left(\frac{dx}{EI}\right) + 0 + k}$$

$$= \frac{\left(\frac{78125}{10^6} + 50 \times 12 \times 10^{-6} \times 20\right)}{\left(\frac{666.67}{10^6} + 0 + 0.0001\right)} = \frac{0.090125}{7.6667 \times 10^{-4}}$$

$= 117.554$ kN

(ii) If the rib shortening is also considered, horizontal thrust is given by

$$H = \frac{\int My\left(\frac{dx}{EI}\right) + L\alpha\Delta t}{\int y^2\left(\frac{dx}{EI}\right) + \left(\frac{L}{EA_m}\right) + k}$$

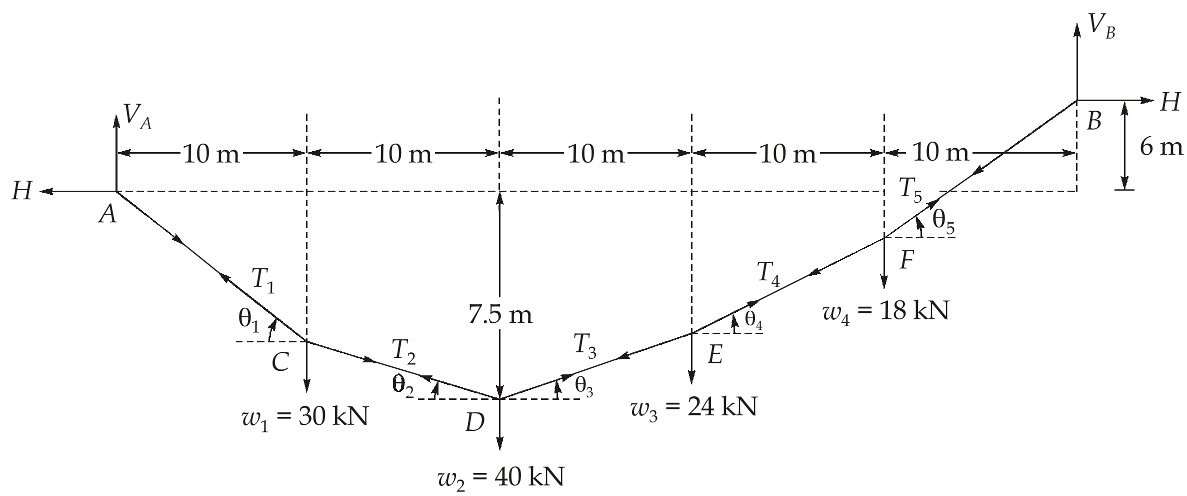
$$= \frac{\left(\frac{78125}{10^6} + 50 \times 12 \times 10^{-6} \times 20\right)}{\left(\frac{666.67}{10^6} + \frac{50}{2 \times 10^6} + 0.0001\right)} = \frac{0.090125}{7.9167 \times 10^{-4}}$$

$= 113.842$ kN

Q.8 (c) Solution:

Let tension in segments of cable are T_1, T_2, T_3, T_4 and T_5 and inclination are θ_1, θ_2, θ_3, θ_4 and θ_5 from horizontal as shown in free body diagram.

FBD of cable



$$\Sigma f_y = 0$$

$$\Rightarrow \quad V_A + V_B = 30 + 40 + 24 + 18$$

$$\Rightarrow \quad V_A + V_B = 112 \text{ kN} \quad \text{...(i)}$$

Taking moment about point D (Consider left side)

$$\Sigma M_D = 0$$

$$\Rightarrow \quad V_A \times 20 - H \times 7.5 - 30 \times 10 = 0$$

$$\Rightarrow \quad V_A = \left(\frac{7.5H}{20} + 15\right) \quad \text{...(ii)}$$

Taking moment about point D (Consider right side)

$$\Sigma M_D = 0$$

$$\Rightarrow \quad V_B \times 30 - H \times (7.5 + 6) - 24 \times 10 - 18 \times 20 = 0$$

$$\Rightarrow \quad V_B = \left(\frac{13.6H}{30} + 20\right) \quad \text{...(iii)}$$

On putting the value of V_A ann V_B in equation (i), we get

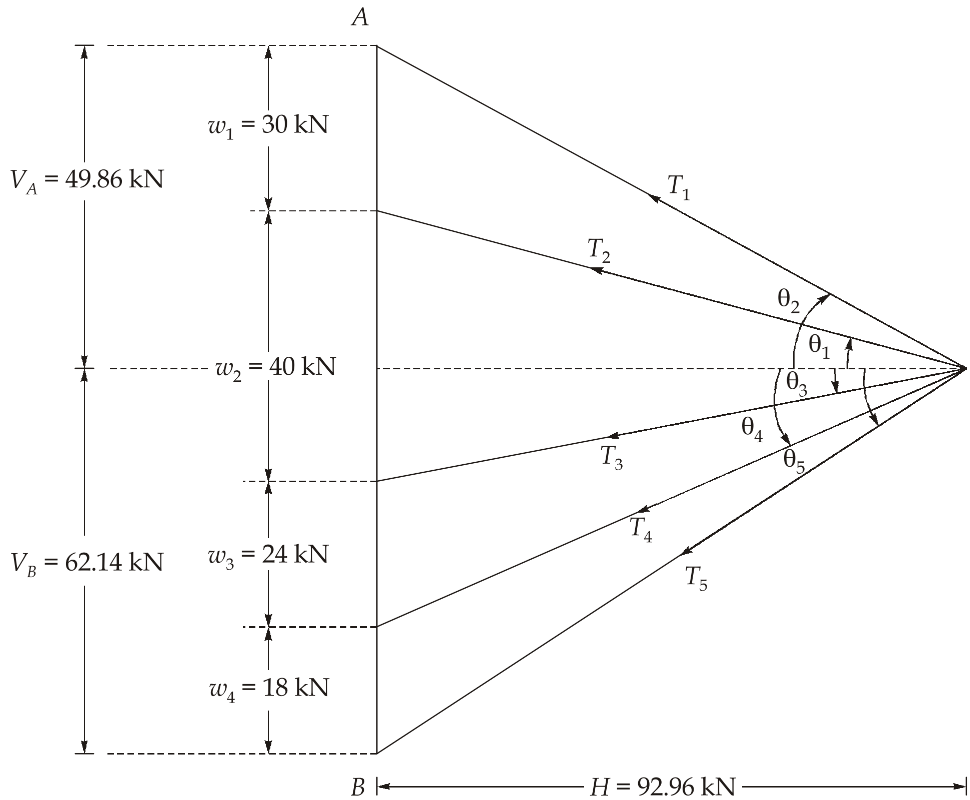
$$\left(\frac{7.5H}{20} + 15\right) + \left(\frac{13.6H}{30} + 20\right) = 112$$

$$H = 92.96 \text{ kN}$$

Now:

From equation (ii), $V_A = \frac{7.5 \times 92.96}{20} + 15 = 49.86 \text{ kN}$

From equaiton (iii), $V_B = \frac{13.6 \times 92.96}{30} + 20 = 62.14 \text{ kN}$



(Funicular polygon)

From function diagram

Tension in each segment of cable

$$T_1 = \sqrt{V_A^2 + H^2} = \sqrt{49.86^2 + 92.96^2} = 105.49 \text{ kN}$$

$$T_2 = \sqrt{(V_A - w_1)^2 + H^2} = \sqrt{(49.86 - 30)^2 + 92.96^2} = 95.06 \text{ kN}$$

$$T_3 = \sqrt{(V_B - w_3 - w_4)^2 + H^2} = \sqrt{(62.14 - 24 - 18)^2 + 92.96^2} = 95.12 \text{ kN}$$

$$T_4 = \sqrt{(V_B - w_4)^2 + H^2} = \sqrt{(62.14 - 18)^2 + 92.96^2} = 102.91 \text{ kN}$$

$$T_5 = \sqrt{V_B^2 + H^2} = \sqrt{62.14^2 + 92.96^2} = 111.82 \text{ kN}$$

Inclination of each segment of cable from horizontal.

$$\tan\theta_1 = \frac{V_A}{H} \Rightarrow \theta_1 = \tan^{-1}\left(\frac{49.86}{92.96}\right) \Rightarrow \theta_1 = 28.207°$$

$$\tan\theta_2 = \frac{(V_A - w_1)}{H} \Rightarrow \theta_2 = \tan^{-1}\left(\frac{19.86}{92.96}\right) \Rightarrow \theta_2 = 12.059°$$

$$\tan\theta_3 = \frac{(V_B - w_3 - w_4)}{H} \Rightarrow \theta_3 = \tan^{-1}\left(\frac{20.14}{92.96}\right) \Rightarrow \theta_3 = 12.224°$$

$$\tan\theta_4 = \frac{(V_B - w_4)}{H} \Rightarrow \theta_4 = \tan^{-1}\left(\frac{44.14}{92.96}\right) \Rightarrow \theta_4 = 25.40°$$

$$\tan\theta_5 = \frac{V_B}{H} \Rightarrow \theta_5 = \tan^{-1}\left(\frac{62.14}{92.96}\right) \Rightarrow \theta_5 = 33.761°$$

Final length of cable

$$\Rightarrow \quad L_{\text{final}} = \frac{10}{\cos\theta_1} + \frac{10}{\cos\theta_2} + \frac{10}{\cos\theta_3} + \frac{10}{\cos\theta_4} + \frac{10}{\cos\theta_5}$$

$$\Rightarrow \quad L_{\text{final}} = \frac{10}{\cos(28.207°)} + \frac{10}{\cos(12.059°)} + \frac{10}{\cos(12.224°)} + \frac{10}{\cos(25.40°)} + \frac{10}{\cos(33.761°)}$$

$$L_{\text{final}} = 54.903 \text{ m}$$

■■■■