MADE EASY

ESE 2024

Main Exam Detailed Solutions

Mechanical Engineering

PAPER-II

EXAM DATE : 23-06-2024 | 02:00 PM to 05:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/error/typo or want to contest the solution given by us, kindly send your suggested answer(s) with detailed explanation(s) at: **info@madeeasy.in**

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ANALYSIS

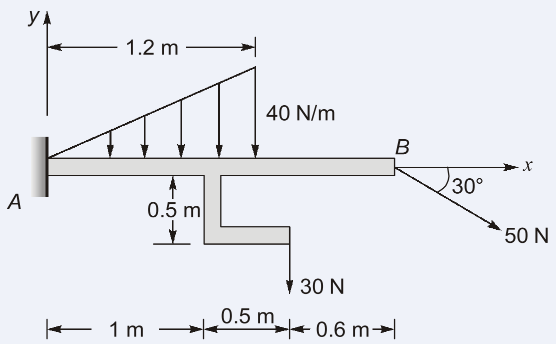
Mechanical Engineering Paper-II
ESE 2024 Main Examination

Sl.	Subjects	Marks
1	Engineering Mechanics	20
2	Strength of Materials	76
3	Theory of Machines	104
4	Machine Design	40
5	Engineering Materials	32
6	Manufacturing Engineering	82
7	Industrial and Maintenance Engineering	34
8	Mechatronics and Robotics	92
		Total 480

Scroll down for detailed solutions

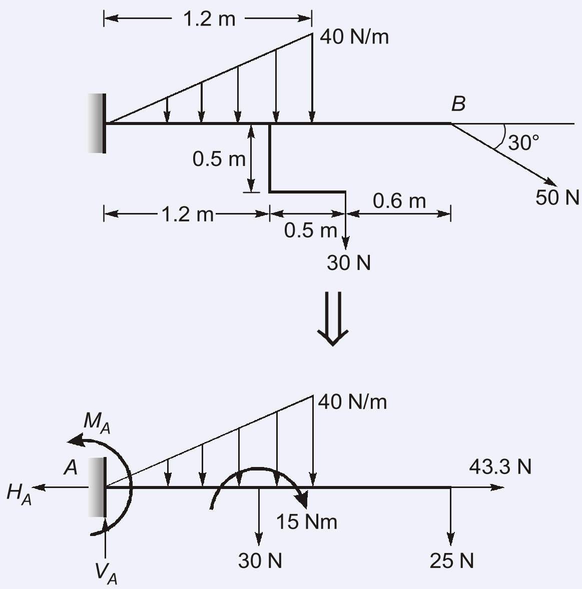
SECTION : A

Q.1 (a) Find the support reaction at the fixed end A of the loaded beam:



[12 marks : 2024]

Solution:



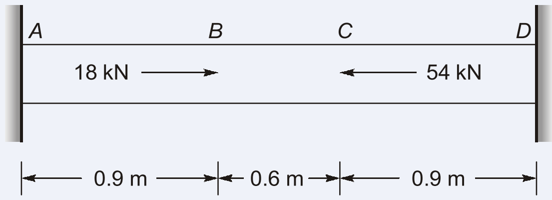
$\Sigma F_x = 0,$ $\quad H_A = 43.3\text{ N}$ **Ans.**

$\Sigma F_y = 0,$ $\quad V_A = \left(\frac{1}{2}\times 1.2\times 40\right)+30+25 = 79\text{ N}$ **Ans.**

$\Sigma M_A = 0,\ M_A = \left(\frac{1}{2}\times 1.2\times 40\times\frac{2}{3}\times 1.2\right)+15+(30\times 1)+(25\times 2.1) = 116.7\text{ Nm}$ **Ans.**

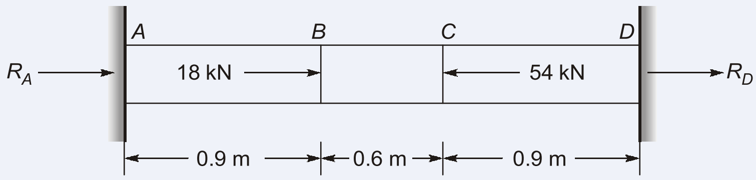
End of Solution

Q.1 (b) The straight bar AD of uniform cross-section is attached to the rigid end supports. Find the force acting on any cross-section in the regions AB, BC and CD:

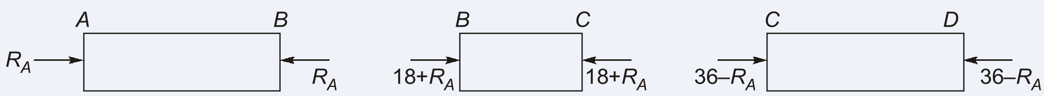


[12 marks : 2024]

Solution:



$\Sigma F = 0$, +ve(→)



$$R_A + R_D = 54 - 18 = 36 \text{ kN} \quad \text{... (i)}$$

As end support is rigid

$$\Delta_{AB} + \Delta_{BC} + \Delta_{CD} = 0$$

$$\frac{F_{AB} \times L_{AB}}{AE} + \frac{F_{BC} \times L_{BC}}{AE} + \frac{F_{CD} \times L_{CD}}{AE} = 0$$

$$(-R_A) \times 0.9 + (-R_A - 18) \times 0.6 + (54 - 18 - R_A) \times 0.9 = 0$$

$$-0.9R_A - 0.6R_A - 10.8 + 32.4 - 0.9R_A = 0$$

$$2.4R_A = 21.6$$

$$R_A = 9 \text{ kN}$$

$$R_D = 27 \text{ kN} \quad \text{[From equation (i)]}$$

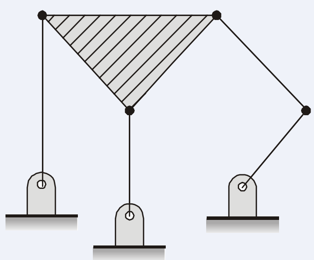
Force in AB $= -R_A = 9$ kN (Compression)

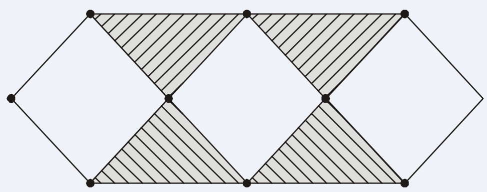
Force in BC $= -(R_A - 18) = 27$ kN (Compression)

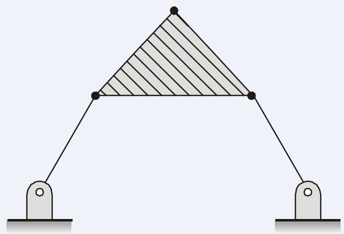
Force in CD $= R_D = 27$ kN (Tensile) Ans.

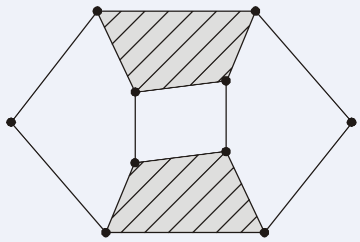
End of Solution

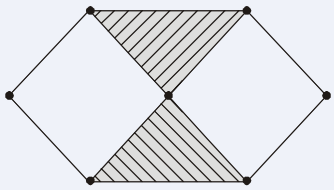
Q.1 (c) (i) Determine the degrees of freedom of the following planar linkages/ kinematic chains:

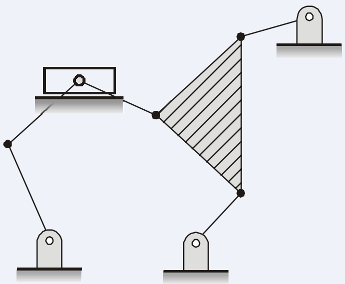
1. 

2. 

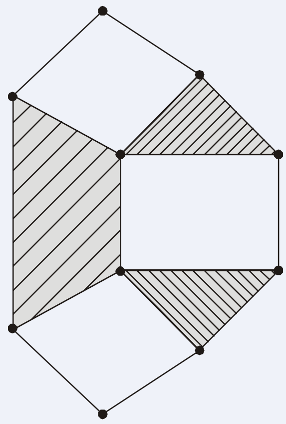
3. 

4. 

5. 

6. 

(ii) Determine and the show the structurally distinct (unique) inversions of the following 8-link kinematic chain:

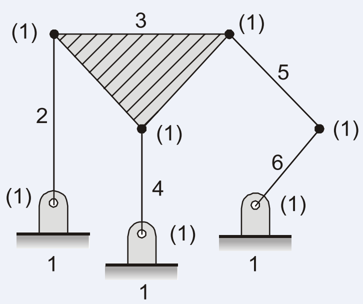


[6+6 = 12 marks : 2024]

Solution:

(i)

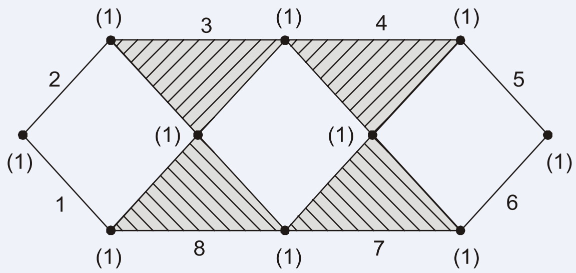
1.



$l = 6, j = 7, h = 0$

$$F = 3(l-1) - 2j - h = 3(6-1) - 2(7) - 0 = 1$$

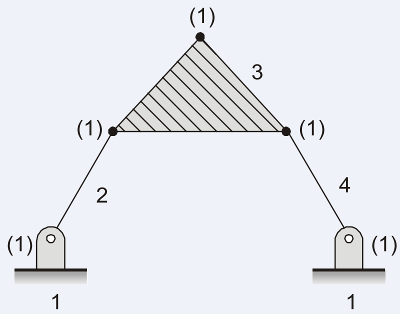
2.



$l = 8,\ j = 10,\ h = 0$

$$F = 3(l - 1) - 2j - h = 3(8 - 1) - 2\,(10) - 0 = 1$$

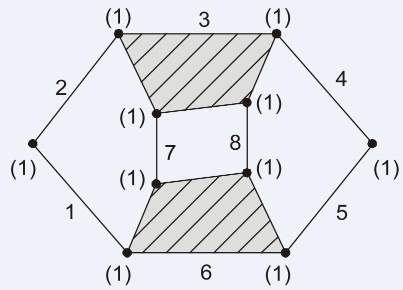
3.



$l = 4,\ j = 4,\ h = 0$

$$F = 3(l - 1) - 2j - h = 3(4 - 1) - 2\,(4) - 0 = 1$$

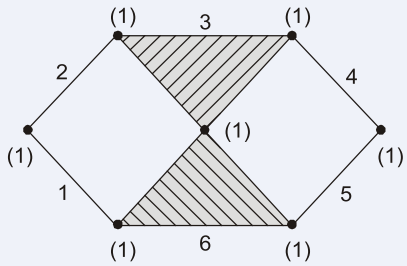
4.



$l = 8,\ j = 10,\ h = 0$

$$F = 3(l - 1) - 2j - h = 3(8 - 1) - 2\,(10) - 0 = 1$$

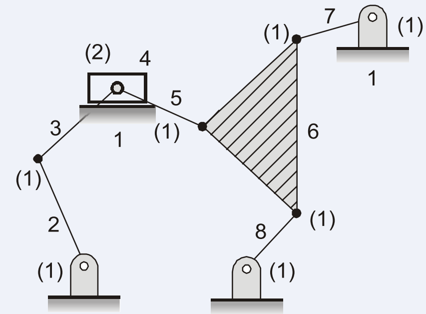
5.



$l = 6,\ j = 7,\ h = 0$

$$F = 3(l - 1) - 2j - h = 3(6 - 1) - 2\,(7) - 0 = 1$$

6.

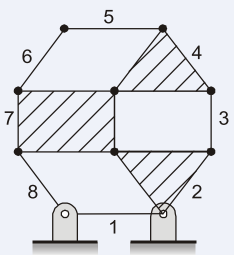


$l = 8$, $j = 10$, $h = 0$

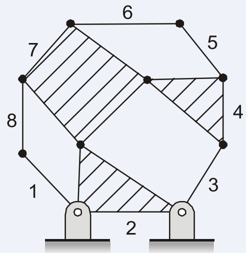
$$F = 3(l - 1) - 2j - h = 3(8 - 1) - 2\,(10) - 0 = 1$$

(ii) The kinematic chain has 8 links in all. A unique mechanism is obtained by fixing one of the links to the ground each time and retaining only one out of the symmetric mechanisms thus obtained.

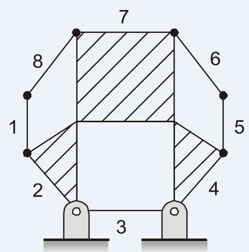
The given kinematic chain is symmetric about links 3 or 7. Thus, identical inversions (mechanisms) are obtained if the links 2, 1, 8 or 4, 5, 6 are fixed. In addition, two more unique mechanism can be obtained from the 8-link kinematic chain as shown in below figure.



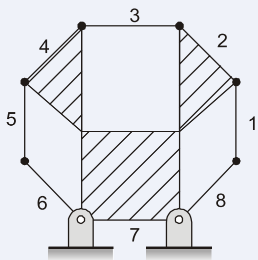
(a)



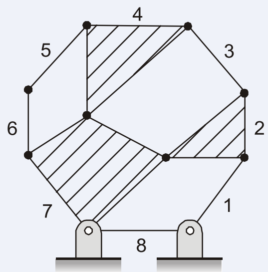
(b)



(c)



(d)



(e)

End of Solution

Q.1 (d) The turbine rotor of a ship has a mass of 2.5 tonnes and rotates at 1750 rpm clockwise when viewed from the aft. The radius of gyration of the rotor is 320 mm. Determine the gyroscopic couple and its effect when

(i) the ship turns right at a radius of 250 m with a speed of 30 km/hr,

(ii) the ship pitches with the bow rising at an angular velocity of 0.7 rad/s and

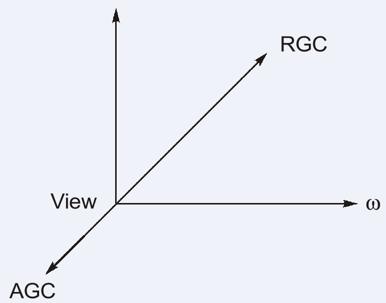
(iii) the ship rolls at an angular velocity of 0.2 rad/s.

[12 marks : 2024]

Solution:

Given: $m = 2.5 \times 10^3$ kg = 2500 kg, N = 1750 rpm (CW), $\omega = \frac{\pi \times 1750}{30} = 183.2596$ rad/s ,
k = 0.32 m, $I = mk^2$ = 256 kg-m^2

(i)



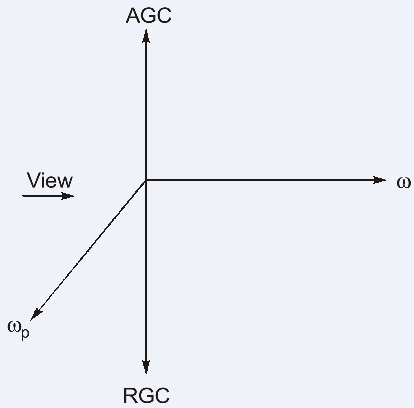
$$\omega_p = \frac{V}{R} = \frac{30 \times 5}{18 \times 250} = 0.0333 \text{ rad/s}$$

Gyroscopic couple, $C = I\omega\omega_p = 256 \times 183.2596 \times 0.0333$

$= 1562.25$ N-m **Ans.**

Effect: To lower the bow and raise the stern, when the ship turns right.

(ii)



$\omega_p = 0.7$ rad/s [Given]

Gyroscopic couple, $C = I\omega\omega_p = 256 \times 183.2596 \times 0.7$

$= 32840.12$ N-m

Effect: The effect of the reaction couple when bow is rising, is to turn the ship towards right or towards starboard.

(iii) $\omega_p = 0.2$ rad/s

Gyroscopic couple, $C = I\omega\omega_p = 256 \times 183.2596 \times 0.2$

$= 9382.89$ N-m

Effect: As the axis of spin is always parallel to the axis of precession for all positions, there is no gyroscopic effect on the ship.

End of Solution

Q.1 (e) What is the relationship between tensile and shear yield stresses as per:

(i) Von Mises' criterion and

(ii) Tresca's criterion?

The above relationship are to be derived by considering yielding under uniaxial tensile loading and under pure torsion.

[12 marks : 2024]

Solution:

(i)

Von Misses criteria

$$u_s = \frac{\sigma_y^2}{6G}$$

In 2D stress, $$u_s = \frac{1}{6G}\left[\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2\right] = \frac{\sigma_y^2}{6G}$$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2 \quad \text{... (i)}$$

For pure shear stress, $\sigma_1 = \tau$ and $\sigma_2 = -\tau$

∴ From equation (i),

$$\tau^2 + \tau^2 + \tau^2 = \sigma_y^2$$

$$\tau = \frac{\sigma_y}{\sqrt{3}}$$

(ii)

According to Tresca criteria, $\tau_{max} \leq \dfrac{\sigma_y}{2}$ ... (ii)

For pure torsion, $\sigma_1 = \tau$ & $\sigma_2 = -\tau$

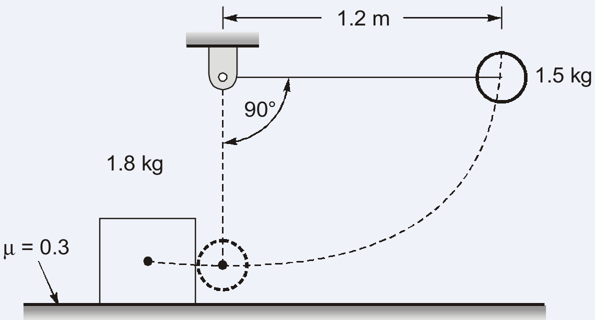
∴ $$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\tau - (-\tau)}{2} = \tau$$

∴ From equation (i), $$\tau = \frac{\sigma_y}{2}$$

Tresca criteria, $$\tau = \frac{\sigma_y}{2}$$

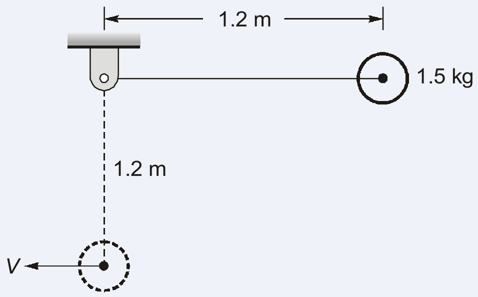
End of Solution

Q.2 (a) A smooth sphere of mass 1.5 kg is released from rest in the position when the flexible string attached to it is horizontal. It hits centrally a stationary block of mass 1.8 kg kept on a surface, with the coefficient of friction between the block and the surface being 0.3. If the coefficient of restitution is 0.8, how far would the block move after impact?



[20 marks : 2024]

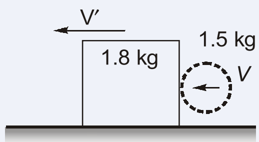
Solution:



By conservation of energy,

$$mgh = \frac{1}{2}mV^2$$

$$V = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1.2} = 4.8522 \text{ m/s}$$



$$P_x = \text{Constant}$$

Say V'' is final velocity of 1.5 kg in backward direction

$$1.5V + 0 = 1.5\,V'' + 1.8\,V'$$

$$1.5 \times 4.8522 = 1.5\,V'' + 1.8\,V'$$

$$7.2783 = 1.5\,V'' + 1.8\,V' \quad \text{... (i)}$$

As we all know, Velocity of separation = e (Velocity of approach)

$$(V' - V'') = 0.8\,(V - 0)$$

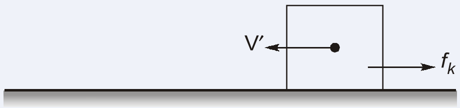
$$V' - V'' = 0.8 \times 4.8522 = 3.8817$$

$$V'' = V' - 3.8817 \quad \text{... (ii)}$$

From (i) and (ii),

$$7.2783 = 1.5\,(V' - 3.8817) + 1.8V'$$

$$V' = 3.9699 \text{ m/s} = u$$



Retardation, $a = \frac{f_k}{m} = \frac{\mu N}{m} = \frac{\mu mg}{m} = \mu g$

or, $a = 0.3 \times 9.81 = 2.943 \text{ m/s}^2$

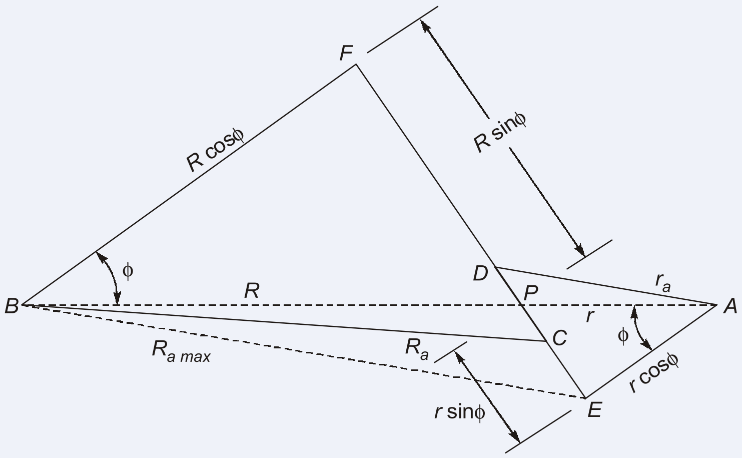
We know, $V^2 = u^2 + 2as$

$0 = (3.9699)^2 + 2(-2.943)S$

$S = 2.6775 \text{ m}$ **Ans.**

End of Solution

Q.2 (b) (i) Derive the formulation for path of contact of two gears A and B in contact as shown in the figure:



(ii) Each of two gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum and contact ratio.

[10+10 = 20 marks : 2024]

Solution:

(i)

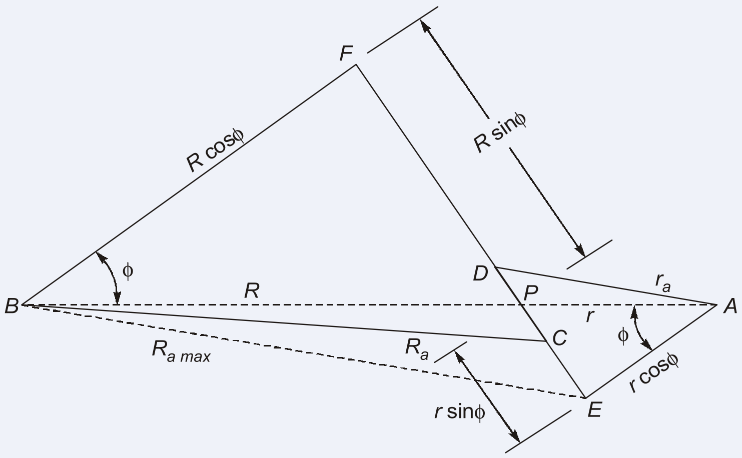


Figure given above shows two gear wheels with centre A and B in contact. The pinion 1 is the driver and is rotating clockwise. The wheel 2 is driven in the counter-clockwise direction. EF is their common tangent to the base circles. Contact of the two teeth is made where the addendum circle of the wheel meets the line of action EF, i.e. at C and is broken where the addendum circle of the pinion meets the line of action, i.e., at D. CD is then the path of contact.

Let
ϕ = pressure angle
r = pitch circle radius of pinion
r_a = addendum circle radius of pinion
R = pitch circle radius of wheel
R_a = addendum circle radius of wheel

Path of contact = Path of approach + Path of recess

$$CD = CP + PD$$
$$= (CF - PF) + (DE - PE)$$

Path of approach, $CP = \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$

Path of recess, $PD = \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$

Path of contact $= \sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)\sin\phi$

(ii)

Given: $T_1 = T_2 = 48$, $\phi = 20°$, $R = \frac{MT}{2} = \frac{8\times48}{2} = 192\text{ mm}$, $m = 8$ mm, AOC = 2.25 (πm)

Path of contact, $$\text{POC} = 2\left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right]$$

or, $$(\text{AOC})(\cos\phi) = 2\left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right] \quad \left[\because \text{AOC} = \frac{\text{POC}}{\cos\phi}\right]$$

$$\left[\frac{(2.25)(\pi m)(\cos\phi)}{2} + R\sin\phi\right]^2 = R_a^2 - R^2\cos^2\phi$$

or, $$R_a^2 = R^2\cos^2\phi + \left[\frac{(2.25)(\pi m)(\cos\phi)}{2} + R\sin\phi\right]^2$$

$$R_a^2 = 192^2 \times \cos^2 20° + \left[\frac{2.25\times\pi\times8\times\cos20°}{2} + 192\times\sin20°\right]^2$$

$\Rightarrow$ $$R_a^2 = 41059.40$$
$$R_a = 202.6312\text{ mm}$$

$\therefore$ $$\text{Addendum} = R_a - R = 10.6312\text{ mm} \quad \textbf{Ans.}$$

$$\text{Contact ratio} = \frac{AOC}{\pi m} = \frac{2.25\times\pi m}{\pi m} = 2.25 \quad \textbf{Ans.}$$

End of Solution

Q.2 (c) A cam having a lift of 1.2 cm operates the suction valve of a four-stroke petrol engine. The least radius of the cam is 2 cm and nose radius is 0.3 cm. The crank angle of the engine when suction valve opens is 4° after t.d.c. and it is 50° after b.d.c when the suction valve closes. The camshaft has a speed of 960 rpm. The cam is of circular type with circular nose and flanks. It is integral with camshaft and operates a flat-faced follower. Calculate

(i) the maximum velocity of the valve,

(ii) the maximum acceleration and retardation of the valve and

(iii) the minimum force to be exerted by the spring to overcome inertia of the valve parts which weigh 250g.

[20 marks : 2024]

Solution:

Given: Lift, $h = 1.2 \text{ cm} = 12 \text{ mm}$

Least radius of cam, $r_c = 2 \text{ cm} = 20 \text{ mm}$

Rotational speed, $N = 960 \text{ rpm}$

Nose radius, $r_n = 0.3 \text{ cm} = 3 \text{ mm}$

Mass of moving part, $m = 250\text{g} = 0.250 \text{ kg}$

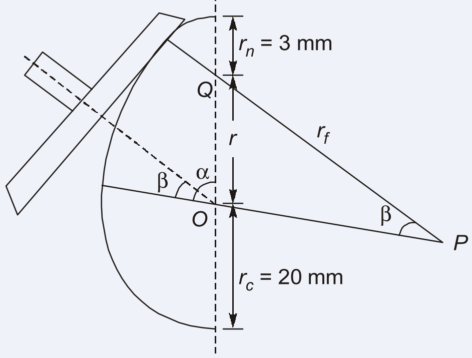
Crank rotation duration of the exhaust valve = 180° − 4° + 50° = 226°

In four-stroke engines, the camshaft speed is half that of the crankshaft.

Angle of action of the camshaft,

$$2\alpha = \frac{226°}{2} = 113°$$

$$\Rightarrow \quad \alpha = 56.5°$$



From the figure, $r + r_n = r_c + h$

$\Rightarrow \quad r = r_c + h - r_n = 20 + 12 - 3 = 29 \text{ mm}$

In ΔOPQ, we can write,

$$(PQ)^2 = (OP)^2 + (OQ)^2 - 2(OP)(OQ)\cos(\angle POQ)$$

$$\Rightarrow \quad (r_f - 3)^2 = (r_f - 20)^2 + (29)^2 - 2(r_f - 20)(29)\cos(180° - 56.5°)$$

$$\Rightarrow \quad r_f^2 + 9 - 6r_f = r_f^2 + 400 - 40r_f + 841 + (32.0123)(r_f - 20)$$

$$\Rightarrow \quad 1.9877 r_f = 1232 + 640.246$$

$$\Rightarrow \quad r_f = 297.707 \text{ mm}$$

Applying sine rule to ΔOPQ,

$$\frac{OQ}{\sin\beta} = \frac{PQ}{\sin(180° - \alpha)}$$

$$\Rightarrow \quad \frac{r}{\sin\beta} = \frac{r_f - r_n}{\sin(180° - 56.5°)}$$

$$\Rightarrow \quad \frac{29}{\sin\beta} = \frac{297.707 - 3}{\sin(123.5°)}$$

$$\Rightarrow \quad \sin\beta = 0.08205$$

$$\Rightarrow \quad \beta = 4.7067°$$

Velocity is maximum when the contact is on the point where the circular flank meets the circular nose.

$$v_{max} = \omega(r_f - r_c)\sin\beta$$

$$= \left(\frac{2\pi \times 960}{60}\right) \times (297.707 - 20) \times \sin(4.7067°)$$

$$= 2290.832 \text{ mm/s} = 2.2908 \text{ m/s} \qquad \textbf{Ans.}$$

Maximum acceleration is when $\theta = 0$,

$$f_{max} = \omega^2(r_f - r_c) = \left(\frac{2\pi \times 960}{60}\right)^2 \times (297.707 - 20)$$

$$= 2806638.827 \text{ mm/s}^2 = 2806.6388 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Maximum retardation is when $\alpha - \theta = 0$

$$f_{max} = \omega r^2 = \left(\frac{2\pi \times 960}{60}\right)^2 \times 29 = 293087.77 \text{ mm/s}^2$$

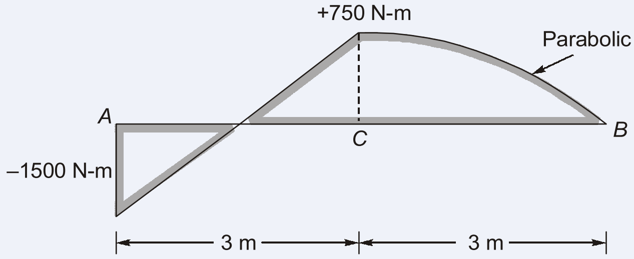
$$= 293.08 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Spring force is needed to maintain contact during the retardation of the follower.

Minimum force, $F = m \times f = 0.250 \times 293.088 = 73.272$ N **Ans.**

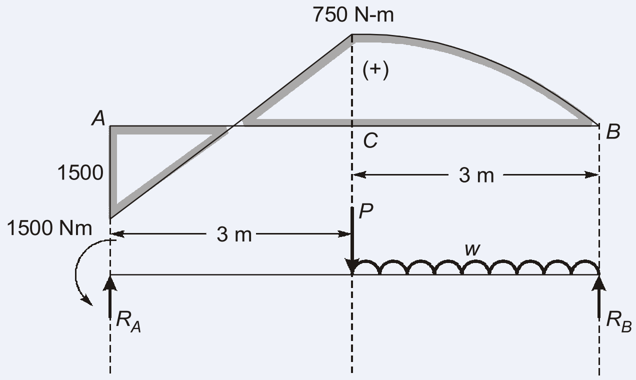
End of Solution

Q.3 (a) Show the loading on the beam corresponding to the bending moment diagram shown in the figure. The beam is simply supported at A and B:



[20 marks : 2024]

Solution:



From above diagram,

$$M_{C\ \text{left}} = +750 \text{ Nm} \qquad \text{[Given]}$$

or, $$3R_A - 1500 = 750$$

$$R_A = 750 \text{ N}$$

$$M_{C\ \text{right}} = 750\text{[Given]}$$

or, $$3R_B - w \times 3 \times 1.5 = 750$$

$$R_B - 1.5w = 250 \qquad \ldots \text{(i)}$$

$M_B = 0$

or, $6R_A - 1500 - 3P - w \times 3 \times 1.5 = 0$

$$3000 - 3P - 4.5w = 0$$

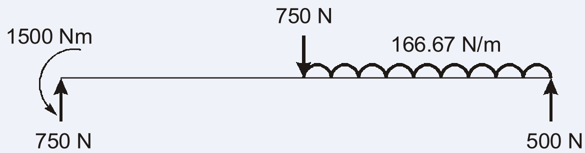
$$P + 1.5w = 1000 \qquad \ldots \text{(ii)}$$

By hit and trail, $$R_B = 500 \text{ N}$$

$$w = 166.67 \text{ N/m}$$

$$P = 750 \text{ N}$$

Loading diagram



End of Solution

Q.3 (b) A shaft 1.7 cm diameter and 1.2 m long is held in long bearings. The weight of a disc at the centre of the shaft is 20 kg. The eccentricity of the centre of gravity of the disc from centre of rotor is 0.03 cm. The Young's modulus of material of the shaft is 2×10^6 kg/cm^2. The permissible stress in the shaft material is 750 kg/cm^2. Calculate:

(i) the critical speed of the shaft and

(ii) the range of speed over which it is unsafe to run the shaft. Neglect weight of the shaft.

[20 marks : 2024]

Solution:

Given: m = 20 kg, l = 1.2 m, d = 0.017 m, e = 0.0003 m, σ_{per} = 750 kg/cm^2 = 750 × 9.81 × 10^4 = 73.575 MPa, E = 2 × 10^6 kg/cm^2 = 2 × 10^6 × 10^4 × 9.81 = 196.2 GPa

(i)

As the shaft is held in long bearing, it may be assumed to be fixed at the ends.

$$\delta = \text{Static deflection} = \frac{mgl^3}{192EI}$$

or,
$$\delta = \frac{20\times 9.81\times 1.2^3}{192\times 196.2\times 10^9\times \frac{\pi}{64}\times (0.017)^4} = 0.002195 \text{ m}$$

∴
$$\text{Natural frequency, } \omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.002195}} = 66.837 \text{ rad/s}$$

$$\text{Critical speed } (N_C) = \frac{66.837\times 60}{2\pi} = 638.253 \text{ rpm}$$ **Ans.**

(ii)

When the shaft rotates, additional dynamic load on the shaft can be obtained from the relation,

$$\frac{M}{I} = \frac{\sigma_{per}}{y}$$

or,
$$\frac{\frac{w_1 l}{8}}{\frac{\pi}{64}\times d^4} = \frac{\sigma_{per}}{d/2}$$

or,
$$\frac{w_1\times 1.2}{8\times \frac{\pi}{64}\times (0.017)^4} = \frac{73.575\times 10^6}{\left(\frac{0.017}{2}\right)}$$

∴
$$w_1 = 236.584 \text{ N}$$

$$\text{Additional deflection due to this load} = \frac{w_1}{w}\times\delta = \frac{w_1}{mg}\times\delta = \frac{236.584}{20\times 9.81}\times 0.002195$$

$$= 0.002647 \text{ m}$$

$$\text{We know, Additional deflection, } y = \frac{\pm e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$

$$0.002647 = \frac{\pm 0.0003}{\left(\frac{N_C}{N}\right)^2 - 1}$$

or,
$$\left(\frac{638.253}{N}\right)^2 = 1 \pm 0.11333$$

or, $$\left(\frac{638.253}{N_1}\right)^2 = 1.11333$$

or, $$N_1 = 604.894 \text{ rpm}$$

Now, $$\left(\frac{638.253}{N_2}\right)^2 = 1 - 0.11333 = 0.88667$$

$$N_2 = 677.816 \text{ rpm}$$

Thus, the range of unsafe speed is from 604.894 rpm to 677.816 rpm. **Ans.**

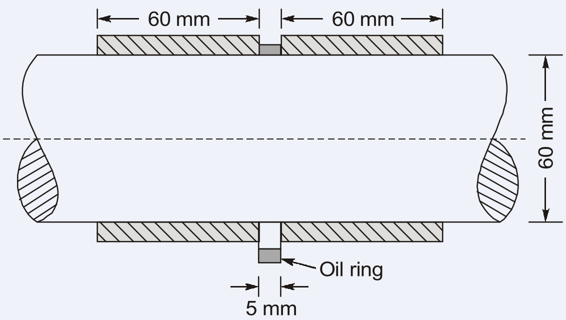
End of Solution

Q.3 (c) (i) A bolted joint is used to connect two components. The combined stiffness of the two components is twice the stiffness of the bolt. The initial tightening of the nut results in a preload of 10 kN in the bolt. The external force of 7.5 kN creates further tension in the bolt. The bolt is made of plain carbon steel 30C8, having tensile yield strength of 400 N/mm^2. There are coarse threads on the bolt. Calculate the tensile stress area of the bolt. The factor of safety specified is 3.

(ii) An oil ring of a shaft transmitting power is shown in the figure. There is no hydrodynamic action over 5 mm width of the oil ring. The total radial load on the journal is 21 kN and the journal rotates at 1440 rpm.

$$\frac{c}{r} = 0.8 \times 10^{-3}; \frac{h_o}{c} = 0.2$$

where c = radial clearance, r = radius and h_o = minimum oil thickness.
For the instant case, Sommerfeld number (S) = 0.0446.
For this case, calculate the viscosity of lubricant:



[10+10 = 20 marks : 2024]

Solution:

(i) Preload (P_i) = 10 kN [Given]

External load (P) = 7.5 kN

Let k_c be the stiffness of component and k_b be the stiffness of bolt.

$$k_c = 2k_b$$

$$S_{yt} = 400 \text{ MPa, FOS} = 3 \quad \text{[Given]}$$

$$(\sigma_t)_{per} = \frac{400}{3} \text{ MPa}$$

Bolt load increased by an amount, $\Delta p = p\left(\frac{k_b}{k_b + k_c}\right) = 7.5\left(\frac{k_b}{3k_b}\right) = \frac{7.5}{3}\text{ kN} = 2.5\text{ kN}$

Resultant load on the bolt, $P_b = p_i + \Delta p = 10 + 2.5 = 12.5$ kN

Size of bolt, $P_b \le (\sigma_t)_{per} A$

$$A \ge \frac{12.5 \times 10^3}{\left(\frac{400}{3}\right)} \ge 93.75\text{ mm}^2$$ **Ans.**

(ii) Given: Total radial load (W) = 21 kN

Journal rotational speed $(n) = \frac{1440}{60} = 24$ rps

Length of journal bearing, (L) = 0.06 m

Diameter of journal (D) = 0.06 m

Pressure induced $(P) = \frac{W}{LD} = \frac{10^3 \times 21}{0.06 \times 0.06} = 5.833$ MPa

Sommerfelf number (S) = 0.0446 [Given]

$$S = \left(\frac{Zn}{p}\right)\left(\frac{r}{C}\right)^2$$

$$Z = S \times \frac{p}{n} \times \left(\frac{C}{r}\right)^2 = 0.0446 \times \frac{5.833 \times 10^6}{24} \times \left(0.8 \times 10^{-3}\right)^2$$

$= 6.938 \times 10^{-3}$ Pa-s **Ans.**

End of Solution

Q.4 (a) A thick cylinder of 225 mm internal diameter has to be designed for a safe internal pressure of 50 MPa. Calculate the thickness of the cylinder wall using maximum shear stress theory. The axial stress may be neglected in the calculation. The yield stress of the cylinder material is 260 MPa and the factor of safety is 2.

[20 marks : 2024]

Solution:

Given: R_i = 112.5 mm, P = 50 MPa, σ_y = 260 MPa, FOS = 2

For thick cylinder with internal fluid, critical point is the inner surface

At inner surface.

$$\sigma_1 = \sigma_{h\,max} = \frac{P\left(R_i^2 - R_o^2\right)}{R_o^2 - R_i^2}$$

$$\sigma_2 = -P$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_y}{2FOS}$$

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_y}{2FOS}$$

$$P\left(1 + \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2}\right) = 130$$

$$\Rightarrow \quad 50\left(1+\frac{R_o^2+R_i^2}{R_o^2-R_i^2}\right)=130$$

$$\Rightarrow \quad \frac{R_o^2+R_i^2}{R_o^2-R_i^2} = 1.6$$

$$R_o^2 + 112.5^2 = 1.6R_o^2 - 1.6 \times 112.5^2$$

$$R_o = 234.18 \text{ mm}$$

$$\text{Thickness of cylinder} = R_o - R_i = 121.68 \text{ mm}$$ **Ans.**

End of Solution

Q.4 (b) A riveting machine is driven by a motor of 4 kW. The actual time to complete one riveting operations 1.5 seconds and it absorbs 12 kN-m of energy. The moving parts including the flywheel are equivalent to 220 kg at 0.5 m radius. Determine the speed of the flywheel immediately after riveting, if it is 380 rpm before riveting. Also determine the number of rivets closed per minutes

[20 marks : 2024]

Solution:

Given: Power = 4 kW, t_{opr} = 1.5 sec, E/hole = 12 kJ, m = 220 kg, k = 0.5 m,

$I = mk^2 = 220 \times (0.5)^2 = 55$ kg-m², N_{max} = 380 rpm, $t_{cycle} = \frac{\text{E/hole}}{\text{Power}} = 3$ sec

Energy fluctuation, $(\Delta E) = \text{E/hole} - pt_{opr} = 12 - 4 \times 1.5 = 6$ kJ

We know, $$\Delta E = \frac{I}{2}\times\left(\frac{2\pi}{60}\right)^2\left(N_{max}^2 - N_{min}^2\right)$$

or, $$6 \times 1000 = \frac{55}{2}\times\left(\frac{2\pi}{60}\right)^2\left(380^2 - N_{min}^2\right)$$

$$N_{min} = 352.8515 \text{ rpm}$$ **Ans.**

$$\text{No. of rivet per minute} = \frac{60}{t_{cycle}} = \frac{60}{3} = 20 \text{ rivet per min}$$

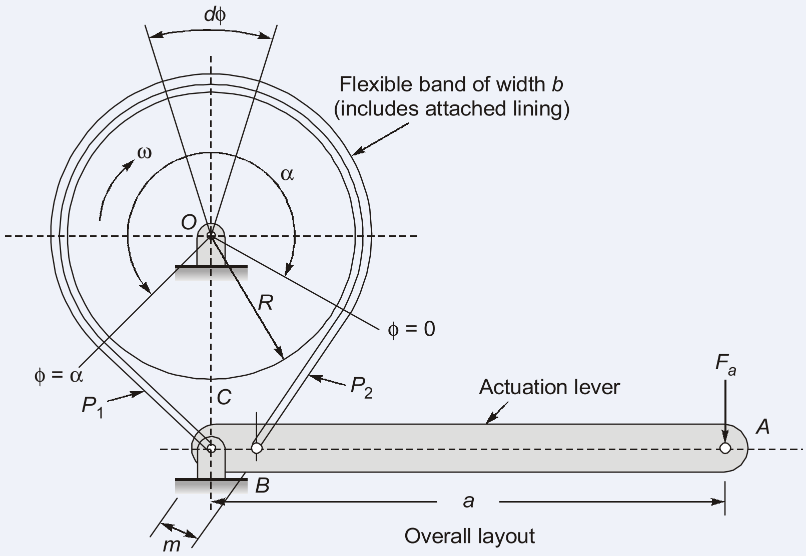
Now, energy supplied by the motor in one minute = 4000 × 60 = 240000 Nm

Energy required / riveting = 1200 Nm

$$\therefore \text{ Number of rivets closed per minute} = \frac{240000}{12000} = 20$$ **Ans.**

End of Solution

Q.4 (c) Refer to the following figure of the drum brake.
Prove that the braking torque (T_f) can be expressed as
$T_f = P_2R(e^{\mu\alpha} - 1)$ and $T_f = bR^2p_{max}(1 - e^{-\mu\alpha})$
where, R = drum radius, b = Width, p = Pressure at any point in the arc of contact and P = Tensile force in the band at the same point:



[20 marks : 2024]

Solution:

Where, P_1 = Tension on the tight side of the band (N)

P_2 = Tension on the loose side of the band (N)

μ = Coefficient of friction between the friction lining and the brake drum

ϕ = Angle of wrap (rad)

The torque M_t absorbed by the brake is given by,

$$M_t = (P_1 - P_2)R$$

where, M_t = Torque capacity of the brake (N-mm)

R = Radius of the brake drum (mm)

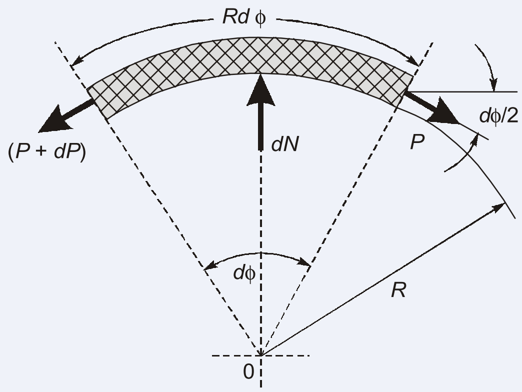
Considering the forces acting on the lever and taking moments about the pivot,

$$P_2 \times m = F_a \times a$$

$$\Rightarrow \quad F_a = \frac{P_2 \times m}{a}$$

An element of the band subtending an angle ($d\phi$) is shown in figure. The elemental area of the friction lining is ($R\,d\,\phi\,b$), where b is the width of the lining parallel to the axis of the brake drum. In the figure , (P) and ($P + dP$) are tensions in the band in the loose and tight sides respectively. If p is the intensity of pressure, the normal reaction (dN) is given by

$$dN = pR\,bd\phi \quad \text{... (i)}$$



Considering equilibrium of vertical forces on the element,

$$dN = P\sin\left(\frac{d\phi}{2}\right) + (P + dP)\sin\left(\frac{d\phi}{2}\right)$$

For small angles, $\sin\left(\frac{d\phi}{2}\right) = \left(\frac{d\phi}{2}\right)$

Neglecting higher order differentials,

$$dN = Pd\phi$$

From equation (i), $p = \frac{P}{Rb}$

The intensity of pressure is maximum, when the band tension P is equal to P_1,

or, $P_{max} = \frac{P_1}{Rb}$

$$\text{Torque } (T_f) = (P_1 - P_2)R = P_2R\left(\frac{P_1}{P_2} - 1\right) = P_2R\left(e^{\mu\alpha} - 1\right)$$

$$\text{Torque } (T_f) = (P_1 - P_2)R = P_1R\left(1 - e^{-\mu\alpha}\right) = (P_{max}Rb)R\left(1 - e^{-\mu\alpha}\right)$$

$$= P_{max}R^2b\left(1 - e^{-\mu\alpha}\right)$$

End of Solution

SECTION : B

Q.5 (a) Zirconium has an HCP crystal structure and a density of 6.51 g/cm^3. The atomic weight of zirconium is 91.22 g/mol. Answer the following:

(i) What is the volume of its unit cell in cubic metres?

(ii) If the c/a ratio is 1.593, compute the values of c and a.

[12 marks : 2024]

Solution:

(a)

The volume of the Zr unit cell may be computed using equation,

$$V_C = \frac{nA_{Zr}}{\rho N_A}$$

Now, for HCP, n = 6 atoms/cell, and for Zr, A_{Zr} = 91.22 g/mol. Thus,

$$V_C = \frac{(6 \text{ atoms/unit cell})(91.22 \text{ g/mol})}{(6.51 \text{ g/cm}^3)(6.022\times10^{23} \text{ atoms/mol})}$$

$$= 1.396 \times 10^{-22} \text{ cm}^3\text{/unit cell}$$

$$= 1.396 \times 10^{-28} \text{ m}^3\text{/unit cell}$$ **Ans.**

(b)

From equation of the solution, for HCP

$$V_C = 6R^2c\sqrt{3}$$

But, since a = $2R$, (i.e. $R = a/2$) then,

$$V_C = 6\left(\frac{a}{2}\right)^2 c\sqrt{3} = \frac{3\sqrt{3}a^2c}{2}$$

But, since c = 1.593a, $$V_C = \frac{3\sqrt{3}(1.593)a^3}{2} = 1.396\times10^{-22} \text{ cm}^3\text{/unit cell}$$

Now, solving for a, $$a = \left[\frac{(2)(1.396\times10^{-22}\text{ cm}^3)}{(3)(\sqrt{3})(2.593)}\right]^{1/3}$$

$$= 3.23 \times 10^{-8} \text{ cm} = 0.323 \text{ nm}$$

And finally, c = 1.593a = (1.593) (0.323 nm) = 0.515 nm **Ans.**

End of Solution

Q.5 (b) Give at least four comparisons between honing and lapping. Also list at least three functions performed by electrolyte in electrochemical machining (ECM) process.

[12 marks : 2024]

Solution:

Lapping process is used to attain a superior surface finish and parallelism on flat or spherical components.

Conversely, honing is employed to enchance the geometric precision of cylindrical components such as bores and tubes.

Difference are as follows:

Lapping	Honing
1. Abrasive particles are freely suspended in liquids.	1. Abrasive particles are bonded together.
2. Used for super finishing outside flat surface.	2. Used for super-finishing inside bore diameter.
3. No burr is produced.	3. Burr is produced.
4. Surface finish produced is 0.08 to 0.25 μm.	4. Surface finish produced is 0.13 μm to 1.25 μm.

Functions of electrolyte in electrochemical machining (ECM) process:

1. It carries the current between the tool and the workpiece.
2. It removes the product of the reaction from the inter-electrode gap.
3. It removes the heat produced by the current flow in the operation.

End of Solution

Q.5 (c) List five causes of service failure giving example of at least one mechanical component in which it occurs. Also list at least five causes of vibration in mechanical system.

An automobile has four tyres. The constant failure rates of tyres 1, 2, 3 and 4 are 0.00001 failure/hour, 0.00002 failure/hour, 0.00003 failure/hour and 0.00003 failure/hour respectively. The automobile cannot be driven when any one of the tyres punctures. Find the mean time to failure of the automobile with respect to tyres and reliability for operating the automobile for 500 hours without failure of tyres.

[12 marks : 2024]

Solution:

Although numerous conditions may cause or lead to service failures, the responsibility for a failure can generally be assigned to one of five classifications. Those classifications are broadly described as:

1. Design
2. Materials
3. Base metal defects
4. Fabrication
5. Service

"Most of the machines we use in our day to day life like the Mixer, Washing Machine, Vacuum Cleaner, etc. tend to indicate if something is wrong in them by means of vibration and noise (a major by-product caused by vibration). Some of the major contributors, which cause the change in vibration level of a machine are:

1. **Unbalance:** This is basically in reference to the rotating bodies. The uneven distribution of mass in a rotating body contributes to the unbalance.
2. **Misalignment:** This is an other major cause of vibration particularly in machines that are driven by motors or any other prime movers. A attack
3. **Bent Shaft:** A rotating shaft that is bent also produces the vibrating effect since it losses its rotation capability about its center.
4. **Wear and tear:** Friction and wear on moving parts, causing irregular motion and vibration.
5. **Bearings:** It is a major contributor for vibration. In majority of the cases every initial problem starts in the bearings and propagates to the rest of the members of the machine.

$$\lambda_1 = 0.00001 \text{ failure/hour}$$
$$\lambda_2 = 0.00002 \text{ failure/hour}$$
$$\lambda_3 = 0.00003 \text{ failure/hour}$$
$$\lambda_4 = 0.00003 \text{ failure/hour}$$

$$\lambda_s = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$$
$$= 0.00001 + 0.00002 + 0.00003 + 0.00003$$
$$= 9 \times 10^{-5} \text{ failure/hour}$$

$$\text{MTTF} = \frac{1}{\lambda_o} = \frac{1}{9 \times 10^{-5}} = 11111.11 \text{ hours} \qquad \textbf{Ans.}$$

and, Reliability, $(R_s) = e^{-\lambda_s t} = e^{-9\times10^{-5}\times500} = 0.95599$ **Ans.**

End of Solution

Q.5 (d) You are a consultant for operations of a firm that deals with just one item that costs Rs.45. The firm buys the item wholesale from a supplier and sells retail. You have compiled the following details for the item:

Parameters	Values
Annual demand	4380
Workdays/year	365
Opportunity cost of investment in inventory	12.5%
Fixed cost of order generation per order	₹22
Cost of inspecting items received	₹3
Cost due to breakage or spoilage	9.5%
Warehouse rental	6.5%
Insurance costs	1.5%

The following two options are available to you:

Option 1: The supplier can supply all items at once

Option 2: The supplier can supply 15 items per day

Which of the options would you recommend to the firm and why?

[12 marks : 2024]

Solution:

Given: D = 4380 units/yr, C = Rs.45/unit, Working day/yr = 365, i% = 12.5 + 9.5 + 1.5 + 6.5 = 30%

Handling cost, $C_h = 45 \times 0.3$ = Rs. 13.5/unit/yr

Ordering cost, C_0 = ₹22 + Rs 3 (Inspection cost) = Rs.25/order

Option 1: Q = 4380 unit/order

$$\text{TIC} = \frac{D}{Q} \times C_o + \frac{Q}{2} \times C_h = \text{Rs. } 29590$$

Option 2: p = 15/day, d = 12/day

We know,
$$\text{TIC} = \frac{D}{Q} \times C_o + \frac{Q}{2}\left(\frac{p-d}{p}\right) \times C_h$$
$$= \frac{4380}{15} \times 25 + \frac{15}{2}\left(\frac{15-12}{15}\right) \times 13.5 \times 292$$
$$= \text{Rs. } 13213$$

As the cost is less for 2nd option, we should prefer 2nd option.

End of Solution

Q.5 (e) Compare between hydraulic and electrical actuators' characteristics in the following points in brief:

(i) Stiffness of the actuators
(ii) Need of reduction gear
(iii) Need of braking device
(iv) Working in low and high temperature
(v) Working of the actuators
(vi) Maintenance need of the actuators

[12 marks : 2024]

Solution:

Characteristics	Hydraulic actuators	Electrical Actuators
(i) Stiffness	Highly stiff system with accuracy and better response	Low stiffness of system
(ii) Need of reduction gear	No reduction gear needed	Reduction gears reduce inertia on the motor
(iii) Maintenance need	Can be expensive and noisy so requires maintenance.	Reliable and low maintenance required
(iv) Working temperature	Operate at higher temperature as fluid act as heat sink	Operate at lower temperature
(v) Working	Good for large robots and heavy payload	Good for all sizes of robots
(vi) Need of braking device	No need of braking device	Motors needs braking device when not pavered, otherwise the arm will fall

End of Solution

Q.6 (a) (i) Three jobs are to be processed in a job shop consisting of three machines. Each job requires three operations and they are to be carried out in $1 \rightarrow 2 \rightarrow 3$ order. The following table indicates the machines required as well as processing time (in hours) required for each operation. Initially, all jobs and machines are available. Compute the makespan by drawing Gantt chart indicating every operation of each job using shortest processing time dispatching rule and break ties with least work remaining rule:

	Machine required for operation			Processing time of operation		
	1	**2**	**3**	**1**	**2**	**3**
Job 1	M1	M2	M3	4	2	3
Job 2	M1	M3	M2	2	4	4
Job 3	M3	M2	M1	3	5	3

(ii) Explain the physics of arc initiation in arc welding. Why is arc initiation difficult in plasma arc welding? Why is plasma arc welding called as plasma arc welding despite the fact that plasma is present in all other arc welding processes?

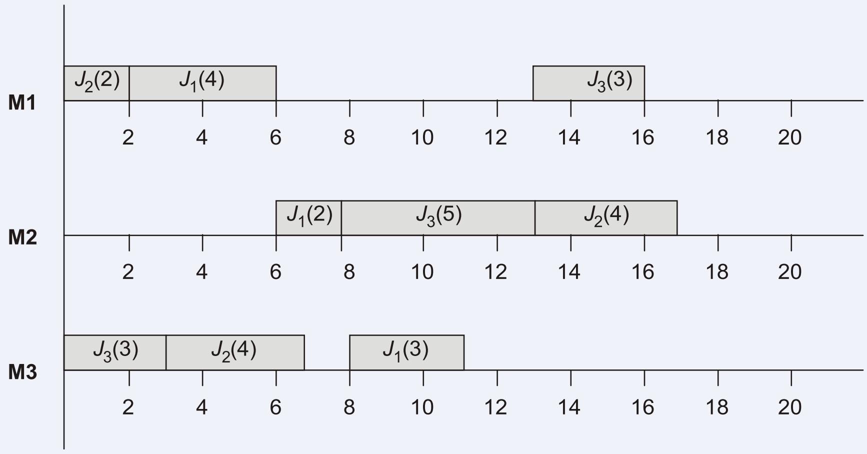
[10 + 10 marks : 2024]

Solution:

(i)

	Machine required for operation			Processing time of operation		
	1	**2**	**3**	**1**	**2**	**3**
Job 1	M1	M2	M3	4	2	3
Job 2	M1	M3	M2	2	4	4
Job 3	M3	M2	M1	3	5	3

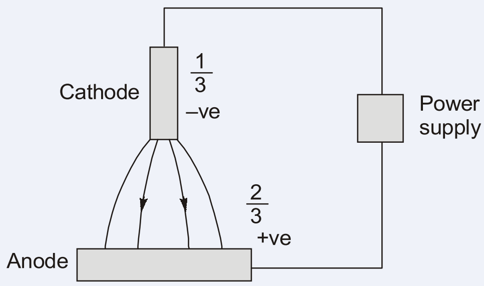
Gantt chart is shown below



From chart, make span, MST = 17 hours **Ans.**

(ii)

Physics of Arc initiation in Arc welding



By giving the power supply to the Cathode and Anode. Initially to generate the Arc between Cathode and Anode, Cathode will be in contact with Anode, due to short circuit, Arc is generated between Cathode and Anode. To continue the Arc further same gap is maintained between Cathode and Anode Known as Arc length.

When the electrons are coming from Cathode to Anode, $\frac{2}{3}$ of heat will be generated in the Anode and due to movement of +ve Ions from Anode to Cathode, $\frac{1}{3}$ of heat will be generated on Cathode.

Between Cathode and Anode, Initially atmospheric air will be there, when the electrons are coming from Cathode to Anode because of Ionization –ve and +ve charges are produced known as plasma which is a good conductor of electrons and which is having higher energy through which electrons are moving to continue the Arc.

Arc initiation in plasma arc welding is difficult, because there is a long gap between the electrode and workpiece and to generate the Arc a pilot Arc is generated between electrode and workpiece using this the air supplied through the nozzles will be ionized and become plasma which will transfer the arc towards workpiece which is having high energy and with this more heat will be generated on the workpiece.

In any Arc welding process, by getting the energy from the electrons air between electrode and workpiece will get ionized and converted to plasma, it is a state of matter present between electrode and workpiece.

In case of plasma arc welding gap between the electrode and workpiece is more, so there is no contact between electrode and workpiece.

To generate the arc between electrode and workpiece, a low current pilot arc obtained between the electrode and the nozzle. Because of pilot arc, the air which is supplied through inside nozzle at high pressure will be ionised and it will become plasma, which is restricted to pass through a nozzle due to which kinetic energy of plasma is very high and it will transfer the arc towards workpiece known as plasma arc.

End of Solution

Q.6 (b) (i) Briefly describe the techniques that may be used for galvanic protection. Also explain why cold-worked metals are more susceptible to corrosion than non-cold-worked metals.

(ii) Write the possible oxidation and reduction half-reactions that occur when magnesium is immersed in each of the following solutions:

(1) HCl

(2) HCl solution containing dissolved oxygen

(3) HCl solution containing dissolved oxygen and in addition Fe^{2+} ions

In which of the above solutions would you expect the magnesium to oxidize rapidly and why?

[10 + 10 marks : 2024]

Solution:

(i)

Techniques used for galvanic protection:

(a) Make anode surface area larger than cathode: This is because the anode is sacrified to protect the cathode, and a larger surface area provides a greater capacity for corrosion. The general rule of thumb is to have an anode surface area at least 5-10 times larger than cathode surface area.

(b) Sacrificial anodes : In this method, a metal that is more prone to corrosion, called the sacrificial anode, is attached to the structure that we want to protect. The sacrificial anode will corrode instead of the protected structure, common material used as sacrificial anodes include Zinc, Mg, Al, etc.

(c) **Impressed current Cathodic Protection:** Impressed current cathodic protection (ICCP) is another technique used for galvanic protection. In this method, an external direct current (DC) power source is connected to the metal structure that requires protection. The structure is made the cathode (negative electrode) while an inert, non-corrodable auxiliary anode is used as the positive electrode. The impressed current from the power source creates an externally driven electrochemical cell. this electrochemical cell produces a cathodic (negative) curent in the structure, protecting it from corrosion. In the presence of the impressed current, the structure will not corrode since it remains in a reduced state. ICCP systems are beneficial because they provide a consistent leval of protection and can protect larger and more complex structure compared to sacrificial anodes.

Cold worked metal are more suceptible to corrosion, than non-cold worked metals, because in cold working due to strain hardening internal stresses will develop and these stresses may lead to corrosion, local variation in electrochemical activity. These factors increase the metals chemical reactivity and create regions with different corrosion potentials, leading to a faster overall corrosion rate.

(ii)

1. **Magnesium in HCL**

In hydrochloric acid (HCl), magnesium can undergo the following reactions:

Oxidation Half-Reaction:

$$Mg \rightarrow Mg^{2+} + 2e^-$$

Reduction Half-Reaction:

$$2H^+ + 2e^- \rightarrow H_2$$

So, the overall reaction is: $Mg + 2H^+ \rightarrow Mg^{2+} + H_2$

2. **Magnesium in HCl with dissolved oxygen**

When dissolved oxygen is present in HCl solution, oxygen can also participate in the reduction reaction. The possible reactions are:

Oxidation Half-Reaction (Same as above)

$$Mg \rightarrow Mg^{2+} + 2e^-$$

Reduction Half-Reactions:

1. Reduction of H^+

$$2H^+ + 2e^- \rightarrow H_2$$

2. Reduction of O_2 in acidic solution:

$$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$$

So, possible overall reactions are:

1. For hydrogen ion reduction:

$$Mg + 2H^+ \rightarrow Mg^{2+} + H_2$$

2. For oxygen reduction:

$$2Mg + O_2 + 4H^+ \rightarrow 2Mg^{2+} + 2H_2O$$

3. **Magnesium in HCl solution containing dissolved oxygen and Fe^{2+} Ions**

When Fe^{2+} ions are present along with dissolved oxygen in HCl solution, the possible reactions expand further:

Oxidation Half-reaction (Same as above)

$$Mg \rightarrow Mg^{2+} + 2e^-$$

Reduction Half-Reactions:

Reduction of H^+

$$2H^+ + 2e^- \rightarrow H_2$$

Reduction of O_2 in acidic solution:

$$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$$

Reduction of Fe^{3+} (formed from oxidation of Fe^{2+}):

$$Fe^{3+} + e^- \rightarrow Fe^{2+}$$

So, possible overall reactions are:

For hydrogen ion reduction:

$$Mg + 2H^+ \rightarrow Mg^{2+} + H_2$$

For oxygen reduction:

$$2Mg + O_2 + 4H^+ \rightarrow 2Mg^{2+} + 2H_2O$$

3. For iron reduction (considering iron gets oxidized to Fe^{3+} and then reduced back):

$$Mg + 2Fe^{3+} \rightarrow Mg^{2+} + 2Fe^{2+}$$

Magnesium is most likely to corrode in the solution where it encounters the most favorable reduction reaction. Dissolved oxygen provides a strong oxidizing environment, making it more aggressive than HCl alone. When Fe^{2+} ions are added, the presence of Fe^{3+}/Fe^{2+} redox couple provides another pathway for reduction, possibly accelerating the corrosion of magnesium.

Therefore, the solutions rank in terms of increasing corrosion potential for magnesium as follows:

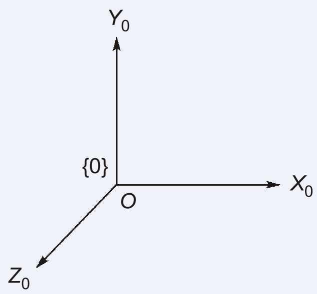
1. HCl (Least corrosive)
2. HCl with dissolved oxygen
3. HCl with dissolved oxygen and Fe^{2+} ions (most corrosive)

End of Solution

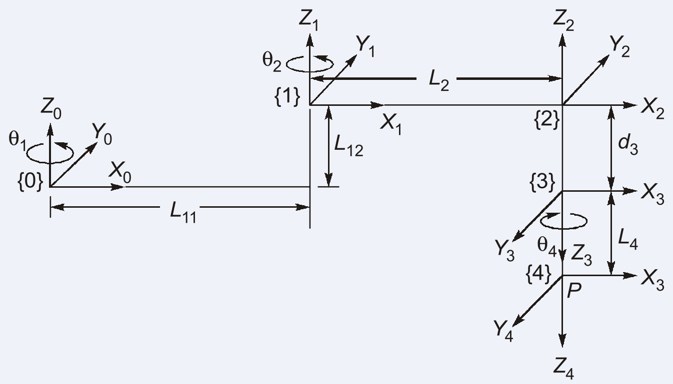
Q.6 (c) (i) The forward kinematic model of a planar 2 DOF (RR) manipulator with link lengths $a_1 = a_2 = 10$ units, is given by the matrix

$$^0T_2 = \begin{bmatrix} 0 & -1 & 0 & \frac{10}{\sqrt{2}} \\ 1 & 0 & 0 & 10 + \frac{10}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Draw the last frame {2}, with respect to {0} frame, by locating its position and its orientation. The initial frame, frame {0} is given as



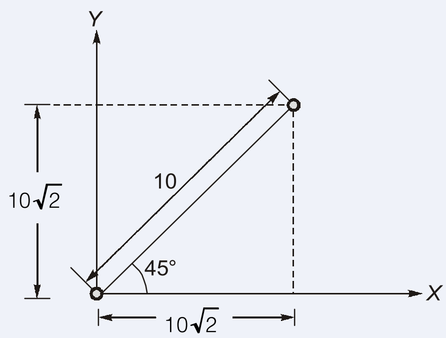
(ii) For the given frames of SCARA manipulator, generate the DH parameters table:



[10 + 10 marks : 2024]

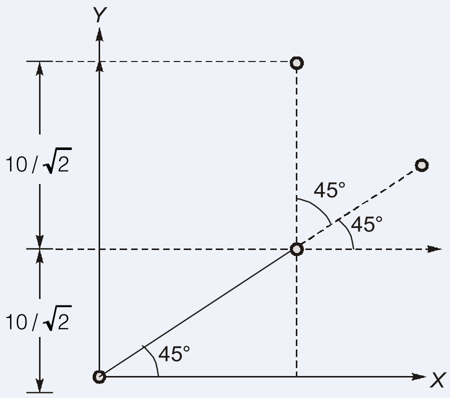
Solution:

(i)



1st transformation ${}^0T_1$ involves rotation of link a_1 about x-y plane by 45° CCW with respect to x-axis.

$$\therefore \quad {}^0T_1 = \begin{bmatrix} \cos 45° & -\sin 45° & 0 & 10\cos 45° \\ \sin 45° & \cos 45° & 0 & 10\sin 45° \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \frac{10}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{10}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



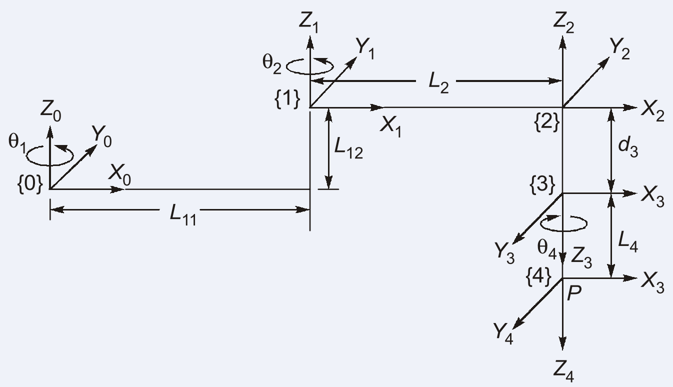
2nd transformation ${}^1T_2$ involves rotation of link a_2 by 45° CCW with respect to a_1.

$$\therefore \quad {}^1T_2 = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 & 10\cos 45^\circ \\ \sin 45^\circ & \cos 45^\circ & 0 & 10\sin 45^\circ \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \frac{10}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{10}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad {}^0T_2 = {}^0T_2 \times {}^1T_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \frac{10}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{10}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \frac{10}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{10}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & \frac{10}{\sqrt{2}} \\ 1 & 0 & 0 & 10+\frac{10}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii)

Frame assignment



θ & α_i → ACW → +ve

θ & α_i → CW → –ve

D-H parameters table

Parameter / Positions	θ	d	a	α_i
0 - 1	θ_1	L_{12}	L_{11}	0
1 - 2	θ_2	0	L_2	0
2 - 3	0	d_3	0	180°
3 - 4	θ_4	L_4	0	0

End of Solution

Q.7 (a) (i) How does permeability of molding sand vary with the moisture content? Explain with the help of neat sketches. Also explain the role of adding organic additives in the molding sand.

(ii) A dimension 57.975 mm is required to be set with the help of slip gauge blocks as accurately as possible. Two slip gauge block sets M45 (Grade 0) and M112 (Grade II) are available. The range and number of pieces in each set are given below:

Set M45 (Grade 0)			Set M112 (Grade II)		
Range (mm)	**Steps (mm)**	**Number of blocks**	**Range (mm)**	**Steps (mm)**	**Number of blocks**
1.001 to 1.009	0.001	9	1.0005	-	1
1.01 to 1.09	0.01	9	1.001 to 1.009	0.001	9
1.1 to 1.9	0.1	9	1.01 to 1.49	0.01	49
1.0 to 9.0	1.0	9	0.5 to 24.5	0.5	49
10.0 to 90.0	10.0	9	25.0 to 100.0	25.0	4

The permissible errors in $\frac{1}{100000}$ mm units in the mean length of Grade 0 and Grade II are given below:

Length (mm)	0 to 20	20 to 60
Grade II	+50 –20	+80 –50
Grade 0	±10	±15

Find the slip gauge that you will prefer, with reasons.

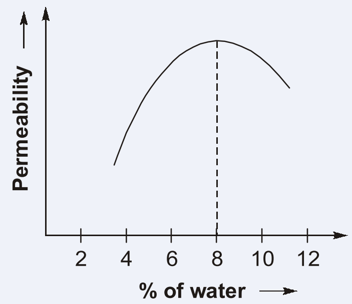
[10 marks : 2024]

Solution:

(i)

Permeability : Ability of moulding sand to allow the gases to escape is known as permeability. It is expressed by permeability number.

$$p_n = \frac{VH}{pAT}$$



- Permeability of moulding sand is varying with % of water in the above figure. It is obvious that at optimum water content permeability is optimum.
- At lower water content, dry clay powder, being finer than sand grains, fills up the voids between the sand particles, and thus reduce the permeability.
- With higher water content, moist clay forms a coating over the sand particles keeping them further away. Thus enhancing the permeability.
- Beyond the optimum water content, water itself fills up the void and reduce the permeability.

Role of adding organic additives to moulding sand:

Organic binders such as dextrin, molasses, cereal binders, linseed oil etc. all added to moulding sand, these increase resistance to deformation, skin hardness and expansion defects such as scab.

Organic binders also increase the hardness of mould, dry strength and hot strength of the mould.

(ii)

Given : Dimension = 57.975 mm

For set M45 (Grade 0): Dimension =

57.975 mm
–1.005 mm
56.97 mm
–1.07 mm
55.9 mm
–1.9 mm
54 mm
–4 mm
50 mm
–50 mm
0

Maximum upper tolerance obtainable $= 10 + 10 + 10 + 10 + 15 = 55$

$$= 55 \times \frac{1}{100000} = 0.00055 \text{ mm}$$

Maximum dimension $= 57.975 + 0.00055 = 57.97555$ mm

Minimum dimension $= 57.975 - 0.00055 = 57.97445$ mm

Minimum range = 57.97555 to 57.97445 mm [for grade 0 M45]
= 0.0011 mm

Set M112 (Grade II): Dimension =

$$
\begin{array}{r}
57.975 \text{ mm} \\
-1.005 \text{ mm} \\
\hline
56.97 \text{ mm} \\
-1.47 \text{ mm} \\
\hline
55.5 \text{ mm} \\
-5.5 \text{ mm} \\
\hline
50 \text{ mm} \\
-50 \text{ mm} \\
\hline
0 \\
\hline
\end{array}
$$

Maximum upper tolerance obtainable = 50 + 50 + 50 + 80 = 230

$$= 230 \times \frac{1}{100000} = 0.0023 \text{ mm}$$

Maximum lower tolerance = 20 + 20 + 20 + 50 = 110

$$= 110 \times \frac{1}{100000} = 0.0011 \text{ mm}$$

Maximum dimension = 57.975 + 0.0023 = 57.9773 mm

Minimum dimension = 57.975 – 0.0011 = 57.9739 mm

Range = 57.9773 – 57.9739 = 0.0034 mm

[For grade II M112]

Since, the range in M45 is less than M112, so it is clear that M45 should be selected for more accuracy as compared to grade 2 of M112.

End of Solution

Q.7 (b) (i) It is possible to drill a 25 mm nominal hole to an accuracy of $25^{+0.02}_{-0.02}$ mm using standard drill and drilling machine available. A shaft is to be machined to obtain a clearance fit in the above hole such that minimum allowance should be 0.01 mm and maximum clearance should not be more than 0.07 mm. Find the tolerance on the shaft. Also state why hole basis system of fits is generally preferred over shaft basis system of fits.

(ii) List the manufacturing situations where FMS technology can be successfully employed. Also give at least four differences between dedicated and random-order FMS

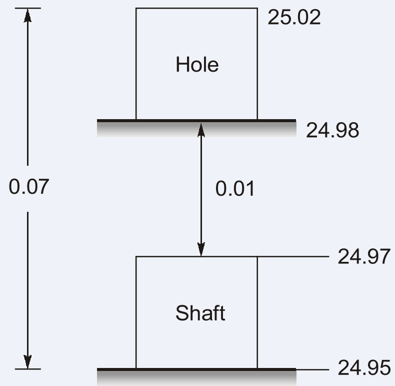
[10 + 10 marks : 2024]

Solution:

(i) Given:

Minimum allowance = 0.01 mm

Maximum clearance = 0.07 mm



Staff dimension $= 25^{-0.03}_{-0.05}$ mm

Shaft Tolerance $= 24.97 - 24.95 = 0.02$ mm

Hole basis system is preferred over shaft basis system because internal machining is more complex than external machining. Moreover tools for internal machining such as reamers, boring tools corresponding to higher degree of accuracy are difficult to manufacture.

Therefore first hole is produced and all the modifications are done on shaft involving external machining which is rather simple in operating.

(ii)

Situations where FMS is employed:

(i) To process different part styles in a non-batch mode.

(ii) To accept changes in production schedule.

(iii) To respond gracefully to equipment malfunctions and breakdowns in the system.

(iv) To accomodate the introduction of new part designs.

Dedicated FMS:

(i) A dedicated FMS is less flexible than random FMS.

(ii) Instead of manufacturing a large variety of parts, it will only produce a limited mix but will do so continuous for long period of time.

(iii) It has high production rate.

(iv) Scheduling is less complex.

Random FMS:

(i) Manufactures random mix of parts with varied configurations at any given time.

(ii) It uses machines that are able to handle a mix of product specifications.

(iii) It can also progress through parts in random order.

(iv) Scheduling is complex.

End of Solution

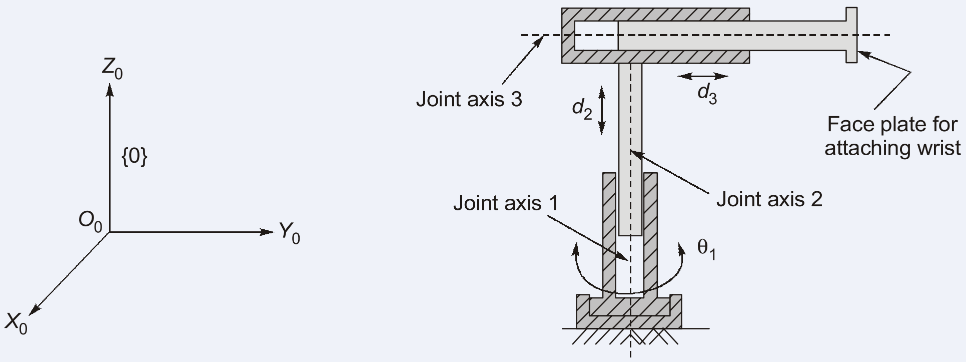
Q.7 (c) Formulate the forward kinematic model of the 3 DOF (RPP) manipulator arm, shown in the figure, by-

(i) generating and drawing the frames using DH rules;

(ii) generating the DH parameters table from the assigned frames;

(iii) generating the individual transformation matrices ${}^0T_1$, ${}^1T_2$, ${}^2T_3$ and the overall transformation matrix ${}^0T_3$.

Also draw the last frame {3}, if θ_1, d_2 and d_3 are given respectively as 0°, 10 units and 10 units, with reference to the given initial frame:



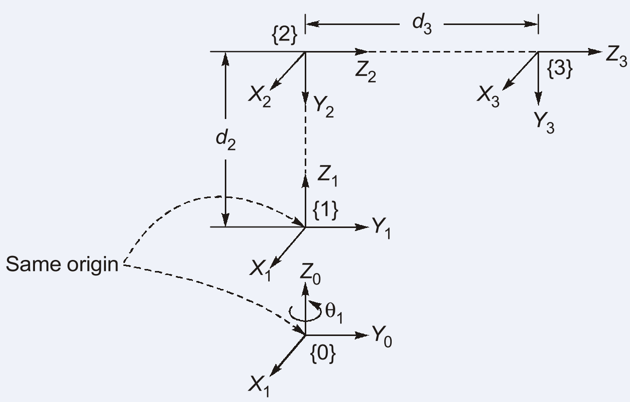
The homogeneous transformation matrix ${}^{i-1}T_i$ is given as

$$
{}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

[20 marks : 2024]

Solution:

The final frame assignment is shown in figure below:



Frame assignment for the cylindrical manipulator arm

Next, the joint-link parameters are identified and these are tabulated in table.

Link i	a_i	α_i	d_i	θ_i	q_i	$C\theta_i$	$S\theta_i$	$C\alpha_i$	$S\alpha_i$
1	0	0	0	θ_1	θ_1	C_1	S_1	1	0
2	0	–90°	d_2	0	d_2	1	0	0	–1
3	0	0	d_3	0	d_3	1	0	1	0

The transformation matrices for transformation of each link (frame) with respect to the previous one is obtained as:

$$^0T_1(\theta_1) = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2(d_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3(d_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation matrix for the manipulator is obtained by multiplying the link transformation matrices. Thus,

$$^0T_3 = {}^0T_1 {}^1T_2 {}^2T_3 = \begin{bmatrix} C_1 & 0 & -S_1 & -d_3S_1 \\ S_1 & 0 & C_1 & d_3C_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{Ans.}$$

Now, given, $\theta_1 = 0°$, $d_2 = 10$ units, $d_3 = 10$ units

then,

$$^0T_3 = \begin{bmatrix} C_1 & 0 & -S_1 & -d_3S_1 \\ S_1 & 0 & C_1 & d_3C_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End of Solution

Q.8 (a) A 12.7 mm diameter steel wire is drawn to obtain 35.5% reduction in area by drawing through a conical die of 6° semi-cone angle. The coefficient of friction between the wire material and die material at conical portion of die is 0.1 and there is no back pull. The tensile yield strength of the original specimen is 207 MPa and is 414 MPa at a strain of 0.5. Assuming linear stress relationship for the wire material and efficiency of electrical motor as 98%, find the drawing power and maximum possible reduction.

[20 marks : 2024]

Solution:

Given: d_0 = 12.7 mm; %Reduction in area = 35.5%; α = 6°; μ = 0.1; σ_y = 207 MPa; σ_y = 414 at a strain of 0.5, efficiency of electrical motor as 98%, drawing power = ? Maximum possible reduction = ?

$$\%\text{Reduction in area} = \left(\frac{A_0 - A_f}{A_0}\right) = 1 - \frac{A_f}{A_0} = 1 - \left(\frac{d_f}{d_0}\right)^2$$

$$0.355 = 1 - \left(\frac{d_f}{12.7}\right)^2$$

$\Rightarrow$ $$d_f = 10.2 \text{ mm}$$

$$\epsilon = \ln\left(\frac{A_0}{A_f}\right) = \ln\left(\frac{d_0}{d_f}\right)^2 = \ln\left(\frac{12.7}{10.2}\right)^2 = 0.4384$$

$$B = \mu\cot\alpha = 0.1 \times \cot 6° = 0.9514$$

Assuming

Linear stress-strain relationship

$$\sigma_0 = A + B\epsilon$$

At $\epsilon = 0$, $\sigma_0 = \sigma_y = 207$

$$207 = A + B \times 0$$

$\Rightarrow$ $$A = 207$$

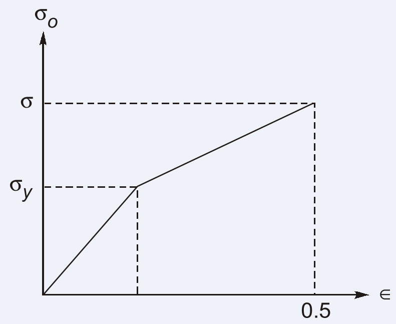
At $\epsilon = 0.5$, $\sigma_0 = \sigma = 414$

$$414 = 207 + B \times 0.5$$

$$B = 414$$

$$\sigma_0 = 207 + 414\epsilon$$

$$\sigma_0 = \text{Flow stress}$$



$$\sigma_{avg} = \frac{1}{\epsilon}\int_0^{\epsilon} \sigma_0 d\epsilon = \frac{1}{\epsilon}\int_0^{\epsilon}(207 + 414\epsilon)d\epsilon = \frac{1}{\epsilon}\left(207\epsilon + \frac{414\epsilon^2}{2}\right)$$

$$= 207 + \frac{414\epsilon}{2} = 207 + \frac{414}{2} \times 0.4384$$

or, $$\sigma_{avg} = 297.75 \text{ MPa}$$

We know, $$\sigma_d = \sigma_{avg}\left(\frac{1+\beta}{\beta}\right)\left(1 - \left(\frac{d_f}{d_0}\right)^{2\beta}\right) \quad (\because \text{No back tension})$$

$$= 297.75 \times \left(\frac{1 + 0.9514}{0.9514}\right)\left(1 - \left(\frac{10.2}{12.7}\right)^{2\times 0.9514}\right)$$

or, $$\sigma_d = 208.288 \text{ MPa}$$

$\therefore$ $$\text{Power} = (\sigma_d \times A_f) \times U = \left(208.29 \times \frac{\pi}{4}(10.2)^2\right) \times U$$

Drawing speed is not given.
Assuming drawing speed as 90 m/min.

$$\text{Power} = \left(208.29 \times \frac{\pi}{4}(10.2)^2\right) \times \frac{90}{60} = 25.53 \text{ kW}$$

$$P_{theoretical} = 25.53 \text{ kW}$$

$$\eta_{motor} = 98\% \quad \text{[Given]}$$

$$P_{actual} = \frac{P_{theoretical}}{\eta_{motor}} = \frac{25.53}{0.98} = 26.05 \text{ kW} \quad \textbf{Ans.}$$

Max possible reduction in diameter
Max reduction will be at $\sigma_d \le \sigma_0$
$\sigma_d = \sigma_0$

$$\sigma_0 = 207 + 414\epsilon = 207 + 414 \times 0.4384 = 388.5 \text{ MPa}$$

$$\therefore \quad \sigma_d = \sigma_0 = \sigma_{avg}\left(\frac{1+\beta}{\beta}\right)\left[1-\left(\frac{d_{f.min}}{d_0}\right)^{2\beta}\right]$$

or,
$$388.5 = 297.75\left(\frac{1+0.9514}{0.9514}\right)\left[1-\left(\frac{d_{f.min}}{12.7}\right)^{2\times0.9514}\right]$$

$$d_{f,\ min} = 7.466 \text{ mm}$$

∴ Maximum Percentage Reduction

$$= \left(\frac{d_0 - d_{f\min}}{d_0}\right) \times 100 = \left(1-\left(\frac{7.466}{12.7}\right)\right) \times 100 = 41.21\% \quad \textbf{Ans.}$$

End of Solution

Q.8 (b) (i) The transformation of frame $\{i-1\}$ to frame $\{i\}$ consists of four basic transformations as following:

(1) A rotation about Z_{i-1} axis by an angle θ_i
(2) A translation along Z_{i-1} axis by distance d_i
(3) A translation along X_i axis by distance a_i
(4) A rotation about X_i axis by an angle α_i

Generate the individual transformation matrices and also the composite transformation matrix ${}^{i-1}T_i$, due to the above successive transformations. If all the above parameters (DH) are zero, what will be the composite transformation matrix?

(ii) Explain the following sensor characteristics in brief:

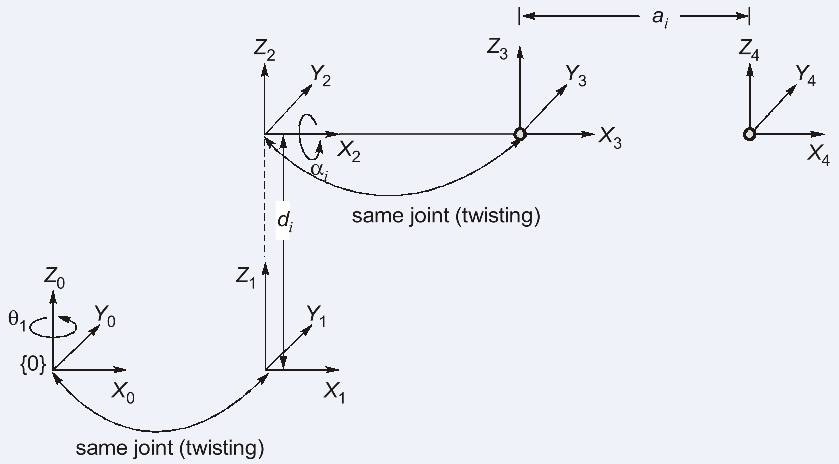
(1) Range
(2) Resolution
(3) Reliability
(4) Repeatability
(5) Sensitivity

[10 + 10 marks : 2024]

Solution:

(i) We know,

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 1 & 1 \end{bmatrix}$$



(i) Rotation about z-axis by angle θ_i,

$$^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(ii) Translation about z-axis by distance d_i,

$$^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iii) A rotation about x-axis by an angle α_i,

$$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iv) Translation along x-axis by distance, α_i

$$^3T_4 = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_4 = {}^0T_1 \times {}^1T_2 \times {}^2T_3 \times {}^3T_4$$

If all DH parameters are ‘zero’ then

$$^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1T_2 = {}^2T_3 = {}^3T_4$$

$$\therefore \qquad {}^0T_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii)

Sensitivity : It indicates how much a sensor's output reaction changes in response to a change in the quantity being measured.

If a sensor is represented mathematically as a function, then sensitivity would be the function's derivative with regard to input.

Sensitivity for instance is defined as the change in electrical resistance (output) per degree change in celcius) (input) in an electrical temperature sensor.

Range : It is a spectrum of values of a physical characteristic that a sensor can measure with reliability from min to max.

A temperature sensor may have a range of –50°C to 150°C.

Resolution : Its the sensor's capacity to detect even the smallest shift in the quantity being measured and react accordingly.

For example, a thermometer with a resolution of 0.1°C may detect temp changes as little as 0.1°C.

Reliability is the ability of a sensor to perform it's required functions under stated conditions for a specified period.

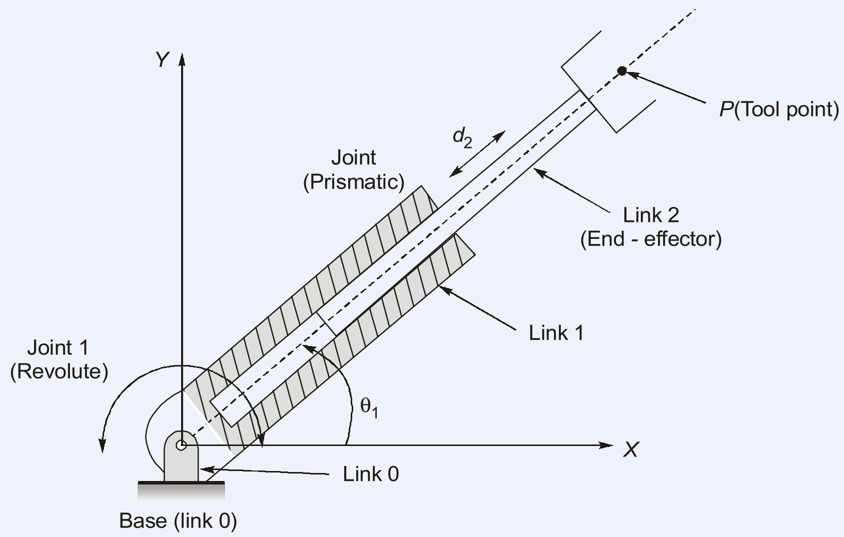
Repeatability is the ability of a sensor to provide the same O/P, for the same inputs over and over again. It's similar to uniformity except that it deals with how consistent a particular sensor is against itself.

End of Solution

Q.8 (c) (i) Generate a forward kinematic model of the given two degrees of freedom (RP) planar manipulator.

(ii) Determine the joint variables (θ_1, d_2) for the above manipulator using inverse kinematic model, if the position and orientation of the end-effector are given by the following matrix:

$$T_E = \begin{bmatrix} 0.707 & 0 & 0.707 & 70.71 \\ 0.707 & 0 & -0.707 & -70.71 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Given that

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[10 + 10 marks : 2024]

Solution:

(i)

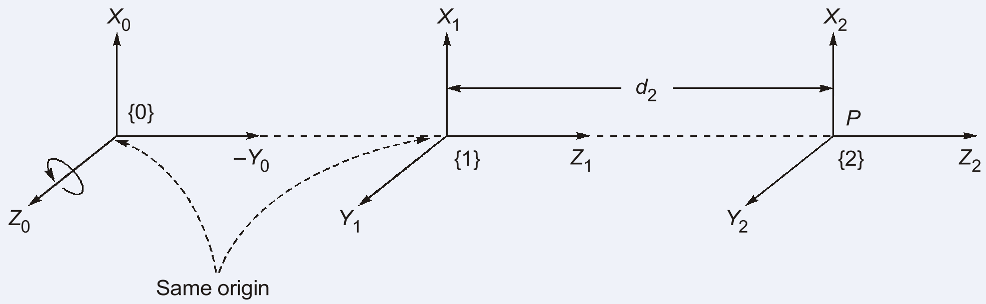


Fig. : Frame assignment for 2-DOF planar manipulator

The complete frame assignment is shown above in figure. The coinciding frames, frame {0} and frame {1} are drawn away from each other for clarity but marked as "same origin" and there is zero distance between their origins.

The assigned frames define the four DH-parameters for each link so as to completely specify the geometric structure of the given manipulator. The joint- link parameters are tabulated in Table. For each link, the displacement variable q_i is identified and placed in the displacement variable column. It is important to note that each row of the joint-link parameter table has exactly one variable and there is no row without a variable. Any deviation from these conditions indicates an error in frame assignment and/or joint-link parameter identification. Note that out of six constant joint-link parameters, five are zero and the sixth is 90°. The two displacement variables are θ_1 and d_2.

Joint-link parameters for the RP manipulator arm

Link i	a_i	α_i	d_i	θ_i	Displacement variable, q_i	$C\theta_i$	$S\theta_i$	$C\alpha_i$	$S\alpha_i$
1	0	90°	0	θ_1	θ_1	C_1	S_1	0	1
2	0	0	d_2	0	d_2	1	0	1	0

The next step is to obtain the individual transformation matrices ${}^0T_1$ and ${}^1T_2$ for relating successive links. These are obtained by substituting the values of the joint-link parameters. The two transformation are, therefore

$$ {}^0T_1(\theta) = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1T_2(d_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Each of the above transformation matrices is a function of only one variable, the displacement variable for the link. Finally, The forward kinematic model is obtained by combining the individual transform matrices. Thus, ${}^0T_2$ the transformation of tool frame, frame (2), with respect to base frame, frame {0} is obtained by substituting individual matrices. The final result after simplifying is:

$$ {}^0T_2 = {}^0T_1\,{}^1T_2 = \begin{bmatrix} C_1 & 0 & S_1 & d_2S_1 \\ S_1 & 0 & -C_1 & -d_2C_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

This is overall transformation, is equal to the end-effector transformation matrix, and the direct kinematic model in matrix form is:

$$ \begin{bmatrix} n_x & o_x & a_x & d_x \\ n_y & o_y & a_y & d_y \\ n_z & o_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1 & 0 & S_1 & d_2S_1 \\ S_1 & 0 & -C_1 & -d_2C_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(ii)

Equating the overall transformation with the position and orientation of the end effector as given:

$$ {}^0T_2 = \begin{bmatrix} C_1 & 0 & s_1 & d_2s_1 \\ s_1 & 0 & -C_1 & -d_2C_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$= \begin{bmatrix} 0.707 & 0 & 0.707 & 70.71 \\ 0.707 & 0 & -0.707 & -70.71 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From 1, 1 and 2, 1 elements we get

$$\theta_1 = \text{A TAN2}\,(a_y, a_x)$$

$$= \text{A TAN2}\,(0.707, 0.707)$$

$$\theta_1 = 45^\circ$$ **Ans.**

From elements 1, 4 and 2, 4 elements we get

$$(d_2 s_1)^2 + (-d_2 C_1)^2 = (70.71)^2 + (-70.71)^2$$

$$\therefore \quad d_2 \simeq 100 \text{ units}$$ **Ans.**

End of Solution

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