ESE 2024

Main Exam Detailed Solutions

Electrical Engineering

PAPER-II

EXAM DATE : 23-06-2024 | 02:00 PM to 05:00 PM

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ANALYSIS

Electrical Engineering Paper-II
ESE 2024 Main Examination

Sl.	Subjects	Marks
1.	Analog and Digital Electronics	40
2.	Power Systems	84
3.	Systems & Signal Processing	84
4.	Control Systems	84
5.	Electrical Machines	94
6.	Power Electronics	74
7.	Microprocessor	20
		Total 480

Scroll down for detailed solutions

SECTION : A

Q.1 (a) An LTI system has the property that

$$a^n u(n) \rightarrow na^n u(n)$$

Considering the system as causal and stable, find the frequency response $H(\omega)$ of the system. Also, determine the difference equation that relates the input $x(n)$ and the output $y(n)$.

[12 marks : 2024]

Solution:

Given informations for the LTI system are :

(i) Input : $x(n) = a^n.u(n)$

(ii) Output : $y(n) = na^n.u(n)$

(iii) System is causal and stable.

By applying DTFT on $x(n)$,

$$X(\omega) = \frac{1}{1-ae^{-j\omega}}$$

By applying DTFT of $y(n)$

$$Y(\omega) = \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^2}$$

The frequency response of the system is

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^2}\cdot(1-ae^{-j\omega})$$

$$\Rightarrow \quad H(\omega) = \frac{ae^{-j\omega}}{(1-ae^{-j\omega})}$$

Now,

$$\frac{Y(\omega)}{X(\omega)} = \frac{ae^{-j\omega}}{(1-ae^{-j\omega})}$$

$$\Rightarrow \quad Y(\omega)[1-ae^{-j\omega}] = ae^{-j\omega}X(\omega)$$

$$\Rightarrow \quad Y(\omega) - aY(\omega)e^{-j\omega} = ae^{-j\omega}X(\omega)$$

By applying the inverse DTFT, the difference equation of system is

$$y(n) - a.y(n-1) = ax(n-1)$$

End of Solution

Q.1 (b) Consider a unity feedback system with open-loop transfer function given as

$$G(s) = \frac{(s-2)(s-4)}{(s+20)(s-3)(s^2+s+1)}$$

Sketch the root locus diagram for $G(s)$ and show that $G(s)$ cannot be stabilized by a stable controller.

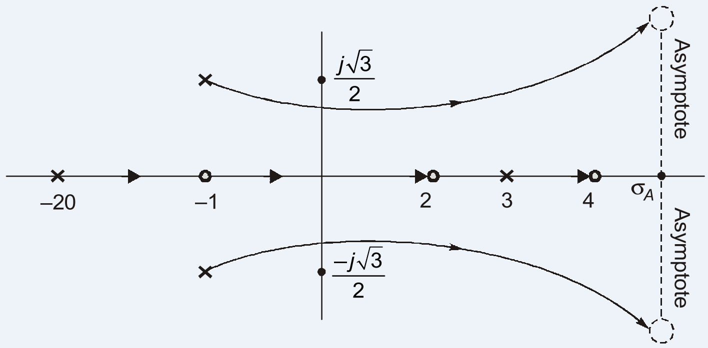
[12 marks : 2024]

Solution:

$$GH = \frac{K(s-2)(s-4)}{(s+20)(s-3)(s^2+s+1)}$$

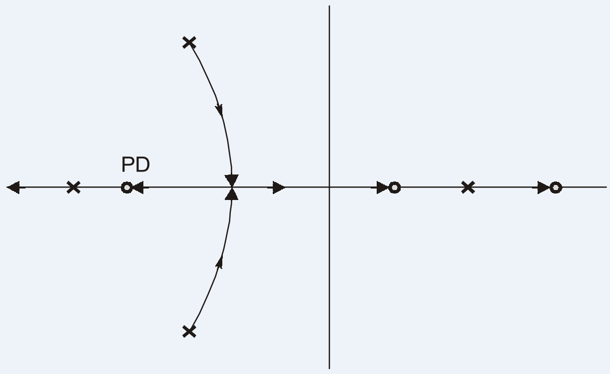
Root locus diagram:

Case 1 :



System is unstable.

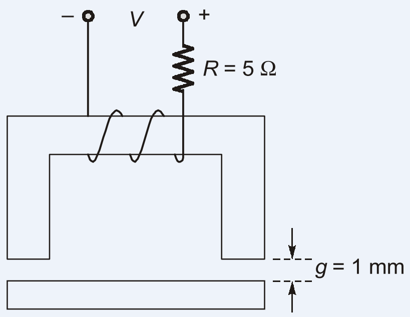
Case 2 : After using a PD controller



Still unstable.

End of Solution

Q.1 (c) The electromagnet shown in the figure below is used to lift a sheet of steel. The coil has 400 turns and a resistance of 5 ohms. The reluctance of the magnetic material is negligible. The magnetic core has a square cross-section of 5 cm by 5 cm. When the sheet of steel is fitted to the electromagnet, air gaps, each of length g = 1 mm, separate them. An average force of 550 newtons is required to lift the sheet of steel. Considering the fact that the same r.m.s. current is required to produce the same average torque when the supply is DC or 50 Hz AC, find the ratio of the two supply voltages (i.e., AC voltage r.m.s. value to DC voltage) to lift the sheet :



[12 marks : 2024]

Solution:

$$\text{Assume the lifting force} = \frac{B_g^2}{2\mu_o} \times \text{(Total air gap area)} \times N$$

First, we need to calculate the magnetic field strength (H) required to produce the force of 550 newton. We can use the formula :

$$F = \frac{B^2 A}{2\mu_o} \quad ...(1)$$

where
F = Force
B = Magnetic field strength
A = Cross sectional area of the core
$\mu_o = 4\pi \times 10^{-7}$

From eqn. (1),
$$B = \sqrt{\frac{2\mu_o F}{A}}$$

On substituting the given values, we get

$$B = \sqrt{\frac{2 \times 550 \times 4\pi \times 10^{-7}}{5 \times 5 \times 10^{-4}}} = 0.74357$$

We calculate the magnetomotive force (MMF) required to produce this magnetic field strength across the air gaps.

$$\text{MMF} = H \times g$$

where
H = Magnetic field strength
g = Length of air gap

$$\text{MMF} = 0.7435 \times 1 \text{ mm}$$
$$= 0.7435 \times 10^{-3}$$

and also
$$\text{MMF} = N \times I$$

$$I = \frac{0.7435 \times 10^{-3}}{400}$$

$$I = 1.859 \times 10^{-6} \text{ A}$$

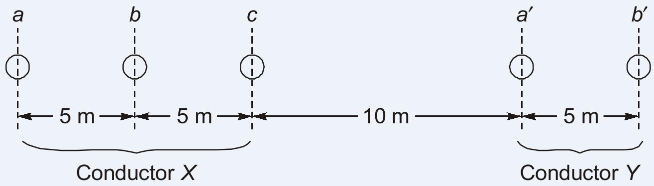
$$\text{AC voltage required} = I \times R \times \sqrt{2}$$

$$\text{DC voltage required} = I \times R$$

$$\frac{V_{AC}}{V_{DC}} = \frac{IR\sqrt{2}}{IR} = \sqrt{2} = 1.41$$

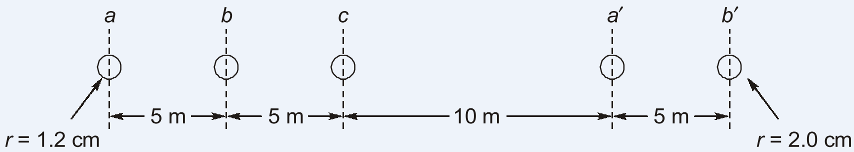
End of Solution

Q.1 (d) GO circuit of single-phase transmission line is composed of three solid conductors of radius 1.2 cm. The return circuit is composed of two solid conductors of radius 2 cm. The conductors layout is as shown in the figure below. Find the inductance per km of the complete line :



[12 marks : 2024]

Solution:



As the configuration is unsymmetric inductive per circuit is

$$L = L_1 + L_2$$

$$L_1 = 2\times10^{-7}\ln\frac{D_{m1}}{D_{s1}}$$

$$L_2 = 2\times10^{-7}\ln\frac{D_{m2}}{D_{s2}}$$

Here, $D_{m1} = D_{m2}$

$$D_{m1} = \sqrt[3]{D'_{m_1}D''_{m_1}D'''_{m_1}}$$

$$Dm' = \sqrt{Daa'Dab'} = \sqrt{20+25} = 22.36m_1$$

$$Dm'' = \sqrt{Dba'Dbb'} = \sqrt{10+15} = 12.24$$

$$Dm_1 = \sqrt[3]{22.36+17.32+12.24} = 16.8 \text{ m}$$

Here, $D_{m1} = D_m = 16.8 \text{ m}$

$$D_{s1} = \sqrt[9]{(D_{aa}D_{ab}D_{ac})(D_{ba}D_{bb}D_{bc})(D_{ca}D_{cb}D_{cc})}$$

$$Daa = Dbb = Dcc = 0.7788\times1.2 - 0.9345$$

$$= \sqrt[9]{(0.9345)^3(500)^4(1000)^2} = 71.845 \text{ cm}$$

$$D_{s2} = \sqrt[4]{Da'a'Da'b'Db'a'Db'b'}$$

Here, $Daa' = Db'b' = 0.7788\times20 = 1.5576$

$$D_{s2} = \sqrt{1.5576 \times 500} = 27.906 \text{ cm}$$

$$L_1 = 2 \times 10^{-7} \ln \frac{1680}{71.845} = 6.3 \times 10^{-7} \text{ H/mt}$$

$$= 0.63 \text{ mH/km}$$

$$L_2 = 2 \times 10^{-7} \ln \frac{1680}{27.906} = 8.1954 \times 10^{-7} \text{ H/mt}$$

$$= 0.81954 \text{ mH/km}$$

Total inductance, $L = L_1 + L_2$

$$= 1.45 \text{ mH/km}$$

End of Solution

Q.1 (e) An ideal step-down (buck) DC-DC converter has an input voltage of 15 V. The required output voltage is 5 V and the peak-to-peak inductor current ripple is limited to 0.5 A. The output voltage ripple (peak-to-peak) is limited to 10 mV. The switching frequency of the converter is kept at 10 kHz. Find the value of the (i) inductor and (ii) capacitor.

[12 marks : 2024]

Solution:

Buck converter : Assuming continuous conduction

No data to check whether it is continous or discontinous

$$V_o = \alpha V_s$$

$$s = 0.15$$

$$\alpha = \frac{1}{3}$$

$$\Delta I_L = \frac{\alpha(1-\alpha)V_s}{fL}$$

$$0.5 = \frac{\frac{1}{3}\left(1-\frac{1}{3}\right) \times 15}{10 \times 10^3 L}$$

$$L = \frac{2}{3} \times \frac{2}{3} \times \frac{15}{10 \times 10^3}$$

$$L = 0.66 \text{ mH}$$

$$\Delta V_o = \frac{\Delta I_L}{8fC}$$

$$10 \times 10^{-3} \text{ V} = \frac{0.5}{8 \times 10 \times 10^3 \times C}$$

$$C = \frac{0.5}{8 \times 10 \times 10^3 \times 10 \times 10^{-3}}$$

$$C = 625 \text{ µF}$$

End of Solution

Q.2 (a) Design a digital sequence detector circuit to detect the sequence 0110 in a serial input signal, using D flip-flops. The sequence detector should produce an output 1 whenever it detects the sequence 0110 in the serial input signal, e.g.,

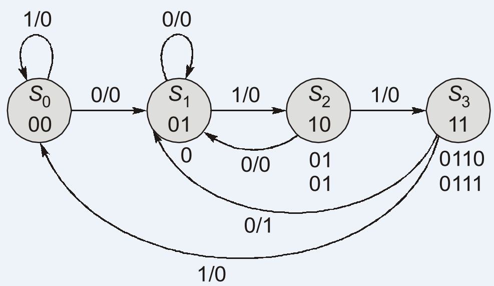
Serial Input X : 00110101101
Output Y : 00001000010

[20 marks : 2024]

Solution:

Serial input X : 00110101101, Output Y : 00001000010

Detected sequence = 0110



For the aboive sequence detector, the non-overlapping sequence has been given.

Table from State Diagram :

State	Next state		Output (y)	
	X = 0	X = 1	X = 0	X = 1
s_0	s_1	s_0	0	0
s_1	s_1	s_2	0	0
s_2	s_1	s_3	0	0
s_3	s_1	s_0	1	0

A simple state assignment can be assuming the outputs of flip flops or Q_1 and Q_0 as given below :

State	Q_1	Q_0
S_0	0	0
S_1	0	1
S_2	1	0
S_3	1	1

Excitation Table :

Present state			Next State		Output (Y)		
Q_1	Q_0	X	Q_1^+	Q_0^+		D_1	D_0
0	0	0	0	1	0	0	1
0	0	1	0	0	0	0	0
0	1	0	0	1	0	0	1
0	1	1	1	0	0	1	0
1	0	0	0	1	0	0	1
1	0	1	1	1	0	1	1
1	1	0	0	1	1	0	1
1	1	1	1	0	0	0	0

D_1

Q_1 \ Q_0X	00	01	11	10
0	0	0	1	0
1	0	1	0	0

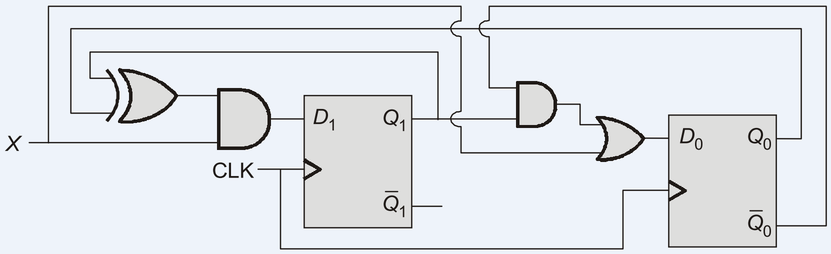
D_0

Q_1 \ Q_0X	00	01	11	10
0	1	0	0	1
1	1	1	0	1

$$D_0 = \bar{X} + Q_1\bar{Q}_0$$

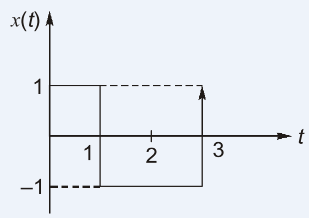
$$D_1 = Q_1\bar{Q}_0X + \bar{Q}_1Q_0X$$

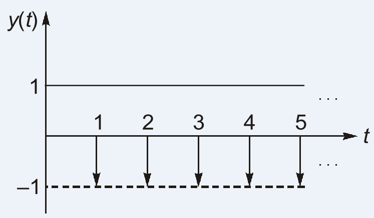
$$= (Q_1 \oplus Q_0)X$$



End of Solution

Q.2 (b) For the signals $x(t)$ and $y(t)$ given below, determine and sketch $\int_{-\infty}^{t} x(\tau)d\tau$ and $\int_{-\infty}^{t} y(\tau)d\tau$:



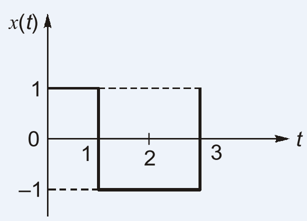


[20 marks : 2024]

Solution:

Part 1 :

Given waveform of $x(t)$ is

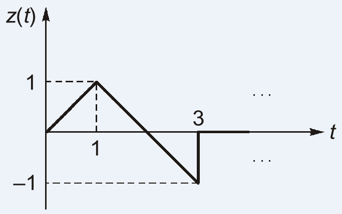


We can express $x(t)$ as :

$$x(t) = u(t) - 2u(t-1) + u(t-3) + \delta(t-3)$$

Let $$z(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

then
$$z(t) = \int_{-\infty}^{t} u(\tau)d\tau - 2\int_{-\infty}^{t} u(\tau-1)d\tau \; + \int_{-\infty}^{t} u(\tau-3)d\tau + \int_{-\infty}^{t} \partial(\tau-3)d\tau$$
$$= r(t) - 2r(t-1) + r(t-3) + u(t-3)$$



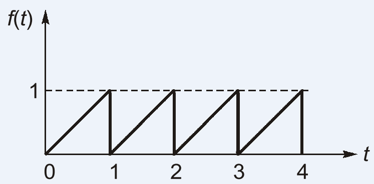
Part 2 :

We can express $y(t)$ as,

$$y(t) = u(t) - \sum_{n=1}^{\infty} \delta(t-n)$$

Let
$$f(t) = \int_{-\infty}^{t} y(\tau)d\tau$$

then
$$f(t) = \int_{-\infty}^{t} u(\tau)d\tau - \sum_{n=1}^{\infty}\left[\int_{-\infty}^{t} \delta(-n)d\tau\right]$$
$$= r(t) - \sum_{n=1}^{\infty} u(t-n)$$
$$= r(t) - u(t-1) - u(t-2) - u(t-3) - .....$$



Note : 1. $\int_{-\infty}^{t} u(\tau)d\tau = r(t)$ where $r(t)$ = ramp-signal = $t.u(t)$

2. $\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$

3. $\int_{-\infty}^{t} u(\tau - t_o)d\tau = r(t - t_o)$

4. $\int_{-\infty}^{t} \delta(\tau - t_o)d\tau = u(t - t_o)$

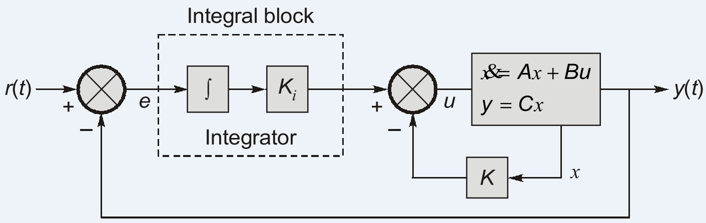
End of Solution

Q.2 (c) The state-space representation of a linear system is given as :

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -6 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u, \quad y = [1 \quad 0]\boldsymbol{x}$$

(i) Using Ackermann's formula, design a state feedback control law such that the controlled system admits peak overshoot $M_p \le 10\%$ and settling time $t_s \le 1$ sec.

(ii) Further, to achieve zero steady-state error for unit step inputs, an integral control block is inserted into the system as shown in the figure below :



For this enhanced configuration, find the state feedback control gain K and integral block gain K_i such that the closed-loop system achieves same performance as in part (i), i.e., ($M_p \le 10\%$ and $t_s \le 1$ sec) with zero steady-state error.

[20 marks : 2024]

Solution:

(i)

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -6 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad C = [1 \quad 0]$$

$$q(s) = |sI - (A - BK)| = 0$$

$$K = [K_1 \quad K_2]$$

$$BK = \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix}$$

$$A - BK = \begin{bmatrix} 0 & 1 \\ -2 & -6 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2-K_1 & -6-K_2 \end{bmatrix}$$

$$sI - (A - BK) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2-K_1 & -6-K_2 \end{bmatrix}$$

$$= \begin{bmatrix} S & -1 \\ K_1+2 & s+6+K_2 \end{bmatrix}$$

$$q(s) = s^2 + s(6 + K_2) + (K_1 + 2) = 0$$

Given : $M_P \le 10\% \Rightarrow q \ge 0.6$

$$t_s = \frac{4}{\xi\omega_n} \le 1 \quad \Rightarrow \quad \omega_n \ge 6.66$$

∴ $q(s) = s^2 + 2(0.6)(6.66) + (6.66)^2 = 0$

$$q(s) = s^2 + 8s + 44.35 = 0$$

Upon comparing $K_1 \approx 42; \quad K_2 \approx 2$

∴ Control law, $K = [42 \quad 2]$

(ii) $$\text{OLTF} = \frac{K_i}{s} \times [C(sI - (A - BK))^{-1}B]$$

where K is same as in part (i), i.e., $K = [42 \quad 2]$

$$A - BK = \begin{bmatrix} 0 & 1 \\ -44 & -8 \end{bmatrix}$$

∴ $$\text{OLTF} = \frac{K_i}{s(s^2 + 8s + 44)}, \text{ type} = 1, \text{ order} = 3$$

∴ e_{ss} of type = 1 system for step = 0.

But order = 3 system is conditionally stable.

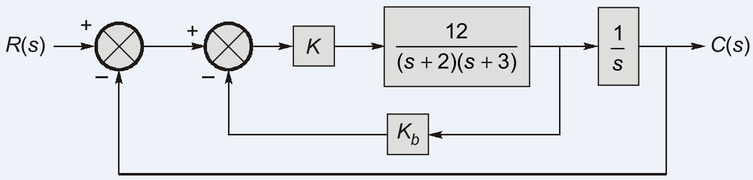
$$q(s) = s^3 + 8s^2 + 44s + K_i = 0$$

s^3	1	44
s^2	8	K_i
s^1	$\frac{8 \times 44 - K_i}{8}$	
s^0	K_i	

∴ $0 < K_i < 352$

End of Solution

Q.3 (a) A servo system with tachometer feedback is shown below. Assuming K_b to be positive, determine the ranges of K and K_b for which the system is stable :



[20 marks : 2024]

Solution:

From given diagram,

$$Tf = \frac{12K}{s^3 + 5s^2 + (6 + 12KK_b)s + 12K}$$

For stability

s^3	1	$6 + 12KK_b$
s^2	5	$12K$
s^1	$\frac{30 + 60KK_b - 12K}{5}$	
s^0	$12K$	

$\therefore\ 0 < K < \infty$ and $K_b > \dfrac{12K - 30}{60K}$

Let $K = 1 \Rightarrow K_b > -0.3 \Rightarrow$ Let $K_b = -0.2$

$$q(s) = s^3 + 5s^2 + 3.6s + 12 = 0$$

s^3	1	3.6
s^2	5	12
s^1	1.2	
s^0	12	

$\therefore$ Stable

End of Solution

Q.3 (b) (i) A 240 V separately excited DC motor has an armature resistance of $r_a = 0.06\ \Omega$. When it is connected to a 240 V supply, it draws 90 A from the supply and rotates at 1200 r.p.m.

(1) Find the torque developed by the motor at this operating condition.

(2) For the same excitation and same supply voltage, if the torque developed is 280 N-m, find the speed and armature current.

(ii) Explain briefly the various types of stepper motors. A three-stack stepper motor is used to produce a step size of 2°.

(1) Determine the steps per revolution.

(2) Determine the number of rotor teeth.

[10 + 10 marks : 2024]

Solution:

(i) Separately Excited Motor :

$$V = 240\text{ V},\quad R_a = 0.06$$
$$I_{a1} = 90\text{ A},\quad N_1 = 1200\text{ rpm}$$
$$E_b = V - I_a R_a = 240 - 90(0.06) = 234.6\text{ V}$$

(1)

$$T_{dev} = \frac{60}{2\pi N} P_{dev} = \frac{60}{2\pi N} E_b I_a$$

$$T_{dev} = \frac{60}{2\pi(1200)}(234.6)(90) = 168.1\text{ N-m}$$

(2) Same excitation
Same voltage (supply)

If $T_{dev} = 280$ N-m

$$T_{dev} = \frac{60}{2\pi}\left(\frac{E_b}{N}\right) I_a$$

$$\phi = \text{Constant}$$

$$E_b = \frac{\phi ZNP}{60\ A}$$

$$\frac{E_b}{N} = \frac{\phi ZP}{60A} = \text{Constant}$$

$$\therefore \quad \frac{E_{b1}}{N_1} = \frac{E_{b2}}{N_2}$$

$$\frac{E_{b1}}{N_1} = \frac{234.6}{1200} = 0.1955$$

$$\therefore \quad 280 = \frac{60}{2\pi}(0.1955)I_{a2}$$

$$\therefore \quad I_{a2} = 149.906 \text{ A}$$

$$E_{b2} = V - I_{a2}R_a = 240 - 149.906(0.06) = 231 \text{ V}$$

$$\frac{E_{b2}}{N_2} = \frac{E_{b1}}{N_1} = 0.1955$$

$$\therefore \quad N_2 = 1181.6 \text{ rpm}$$

$$N_2 = \frac{E_{b2}}{0.1955}$$

(ii)
$$\alpha = \frac{360}{m.N_\gamma}$$

where
α = Step Angle
m = Number of stacks
N_γ = Rotor poles/teeth

$$\therefore \quad N_\gamma = \frac{360}{m \cdot \alpha}$$

$$N_\gamma = \frac{360}{3 \times 2} = 60 \text{ (Number of rotor teeth)}$$

Step per revolution = A full cycle (360°) divided by step angle

$$\therefore \quad \text{Steps per revolution} = \frac{360°}{2} = 180 \text{ steps}$$

End of Solution

Q.3 (c) (i) Explain, on the basis of their controllability, the different types of semiconductor switches used in power electronic circuits. Also, draw their output characteristics.

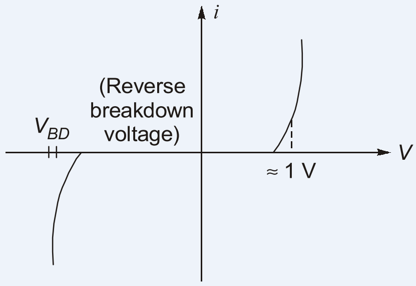
(ii) Explain the 'variable voltage variable frequency' method of speed control of a three-phase induction motor. Draw the relevant characteristics showing the variation of torque and speed below the base speed and above it.

[10 + 10 marks : 2024]

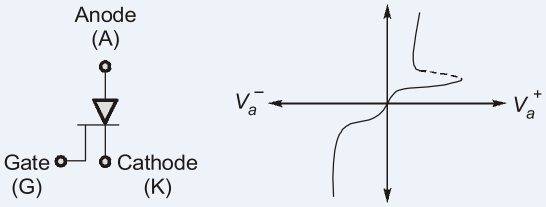
Solution:

(i) Power semiconductor devices can be classified into three categories according to their degree of controllability. The categories are :

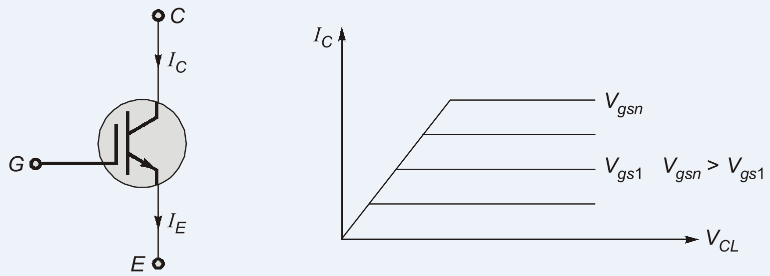
(a) Uncontrolled turn-on and off devices :
e.g. Diode : The on and off states of diodes are controlled by power circuit.



(b) Controlled turn-on and uncontrolled turn-off :
e.g. SCR (Silicon Controlled Rectifier)
SCR turned-on by a control signal and are turned-off by the power circuit.



(c) Controlled turn-on and off characteristics :
e.g., Insulated-gate bipolar transistors
IGBT are turned-on and off by controlled signals.



(ii)
$$V \simeq E_1 = \sqrt{2}\pi f N_1 \phi_r K\omega_1$$

$$\phi_r = \frac{1}{\sqrt{2}\pi N_1 K\omega_1}\left(\frac{V}{f}\right)$$

$$\phi_r \propto \frac{V}{f} \Rightarrow \text{Constant}$$

Stator impedance R_1, $x_1 \simeq 0$

$$T \simeq \frac{3}{\omega_s} \times \frac{V^2\left(\frac{R_2'}{s}\right)}{\left(\frac{R_2'}{s}\right) + x_2'^2}$$

Nominal voltage V_o is rated nominal frequency f_o

$$\frac{V}{f} = \frac{V_o}{f_o}$$

$$V = \left(\frac{f}{f_o}\right)V_o$$

$$\omega_s = \frac{2\pi}{60}N_s$$

$$\Rightarrow \qquad \frac{2\pi}{60}\times\frac{120f}{P} = \frac{4\pi f}{P}$$

Speed,

$$\omega_s = \frac{4\pi f}{P}$$

$$\omega_{so} = \frac{4\pi f_o}{P}$$

$$\frac{\omega_s}{\omega_{so}} = \frac{f}{f_o}$$

$$\omega_s = \frac{f}{f_o}\omega_{so}$$

Rotor reactance,

$$x'_{2o} = 2\pi f_o L'_{2o}$$

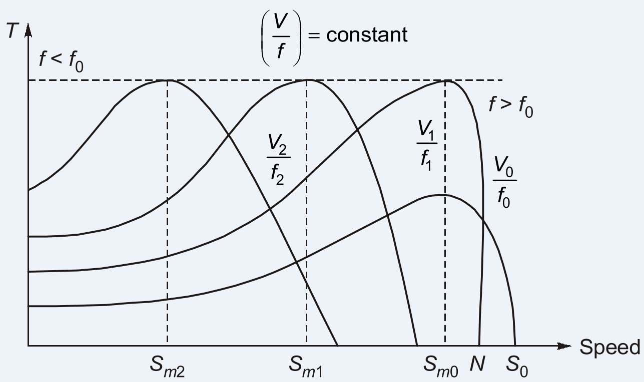
$$x'_2 = 2\pi f L'_2$$

$$x'_2 = \left(\frac{f}{f_o}\right)x'_{2o}$$

Slip for maximum torque,

$$S_{mo} \simeq \frac{R'_2}{x'_{2o}}$$

$$S_m = \frac{R'_2}{x'_2} = \left(\frac{f_o}{f}\right)\frac{R'_2}{x'_{2o}}$$



$$S_m = \frac{f_o}{f}S_{mo}$$

$$T_{max} \simeq \frac{3}{\omega_s}\frac{V^2}{2x'_2}$$

At nominal V_o, f_o

$$T_{mo} = \frac{3}{\omega_{so}} \times \frac{V_o^2}{2x'_{2o}}$$

$$T_m = \frac{3}{\omega_s} \cdot \frac{V^2}{2x'_2}$$

$$T_m = T_{mo}$$

Wide range of speed control

End of Solution

Q.4 (a) Determine the time signals for the bilateral Laplace transforms given below. Specify the properties used :

(i) $X(s) = \frac{1}{s^2} \cdot \frac{d}{ds}\left(\frac{e^{-3s}}{s}\right)$, ROC : Re($s$) > 0

(ii) $X(s) = s\left(\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}\right)$, ROC : Re($s$) > 0

[20 marks : 2024]

Solution:

(i)

$$X(s) = \frac{1}{s^2}\frac{d}{ds}\left[\frac{e^{-3s}}{s}\right], \quad \sigma > 0$$

$$= \frac{1}{s^2}\left[\frac{-3e^{-3s}s - 1e^{-3s}}{s^2}\right]$$

$$= \frac{-3}{s^3} \cdot e^{-3s} - \frac{e^{-3s}}{s^4}, \quad \sigma > 0 \qquad \text{...(i)}$$

As we know,
$$t^n u(t) \rightleftharpoons \frac{\lfloor n}{s^{n+1}}, \quad \sigma > 0$$

Put $n = 2$:
$$t^2.u(t) \rightleftharpoons \frac{\lfloor 2}{s^3} = \frac{2}{s^3}, \quad \sigma > 0$$

$$\frac{t^2}{2} \cdot u(t) \rightleftharpoons \frac{1}{s^3}, \quad \sigma > 0$$

$$-3\frac{(t-3)^2}{2}u(t-3) \rightleftharpoons -3\frac{e^{-3s}}{s^3}, \quad \sigma > 0$$

Put $n = 3$:
$$t^3 u(t) \rightleftharpoons \frac{\lfloor 3}{s^4}, \quad \sigma > 0$$

$$\frac{t^3}{6}u(t) \rightleftharpoons \frac{1}{s^4}, \quad \sigma > 0$$

$$\frac{(t-3)^3}{6}u(t-3) \rightleftharpoons \frac{e^{-3s}}{s^4}, \quad \sigma > 0$$

By applying invese LT on (i)

$$x(t) = \frac{-3}{2}(t-3)^2 u(t-3) - \frac{(t-3)^3}{6} \cdot u(t-3)$$

(ii)

$$X(s) = s\left[\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}\right], \quad \sigma > 0$$

$$= \frac{1}{s} - \frac{e^{-s}}{s} - e^{-2s}, \quad \sigma > 0 \qquad \text{...(i)}$$

$$u(t) \rightleftharpoons \frac{1}{s}, \quad \sigma > 0$$

$$u(t-1) \rightleftharpoons \frac{e^{-s}}{s}, \quad \sigma > 0$$

$$\delta(t-2) \rightleftharpoons e^{-2s}$$

By applying inverse LT on (i)

$$x(t) = u(t) - u(t-1) - \delta(t-2)$$

End of Solution

Q.4 (b) A 50 MVA generating station is connected to a three-phase line having impedance $Z = 300\angle 75°\ \Omega$ and admittance $Y = 0.0010j\ \mho$. The power at the generating station is 50 MVA at upf, at a voltage of 220 kV. There is a load of 25 MW at upf at the midpoint of the line Find the :

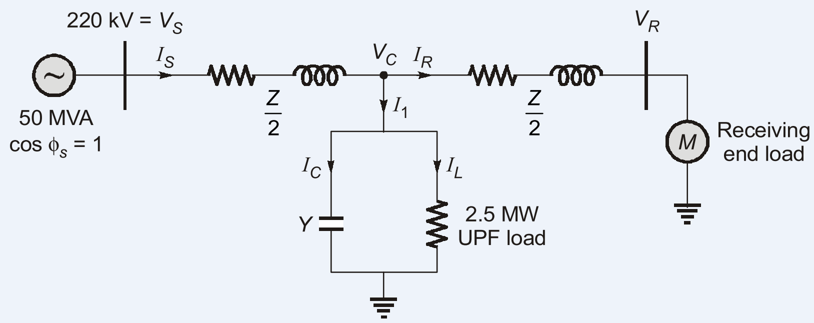
(i) voltage at the receiving end;

(ii) complex power at the receiving end.

Use nominal-T model for the lines.

[20 marks : 2024]

Solution:



Given :

$$Z = 300\angle 75°\ \Omega$$

$$Y = j0.001\ \mho$$

From sending end :

$$S_S = \sqrt{3} V_S I_S$$

$$50 \times 10^6 = \sqrt{3} \times 220 \times 10^3 \times I_S$$

$$I_S = 131 \text{ A} = 131\angle 0° \text{ A}$$

At mid point :

$$V_C = V_S - I_S \cdot \frac{Z}{2}$$

$$= \frac{220 \times 10^3}{\sqrt{3}} - 131 \times \frac{300\angle 75°}{2}$$

$$= 124\angle{-8.85°} \text{ kV/ph}$$

$$I_C = V_C Y$$

$$= 124 \times 10^3 \angle{-8.85°} \times 0.001\angle 90°$$

$$= 124\angle 81.15° \text{ A}$$

$$\text{Mid point load} = 25 \text{ MW}$$

$$\text{Per phase load} = \frac{25}{3} = 8.33 \text{ MW}$$

$$I_L = \frac{8.33 \times 10^6}{124 \times 10^3} = 67.17 \text{ A}$$

$$I_1 = I_C + I_L = 124\angle 81.15° + 67.17$$

$$= 149.83\angle 54.85° \text{ A}$$

$$I_S = I_1 + I_R$$

$$I_R = I_S - I_1$$

$$= 131 - 149.83\angle 54.85°$$

$$= 130.42\angle{-70°} \text{ A}$$

$$V_R = V_C - I_R \cdot \frac{Z}{2}$$

$$= 124 \times 10^3 \angle -8.85° - (130.42\angle -70°) \times \frac{300}{2}\angle 75°$$

$$= 105.11\angle{-12°} \text{ kV/Ph}$$

$$V_{RL} = 182\angle{-12°} \text{ kV}$$

Complex power at receiving end :

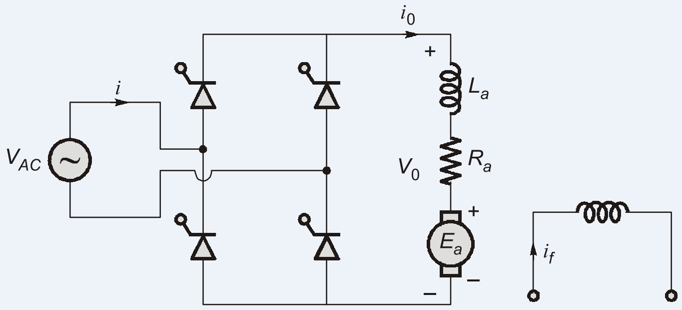
$$S_R = \sqrt{3} V_R I_R^*$$

$$= \sqrt{3} \times 182 \times 10^{-3} \angle -12° \times 130.42\angle -70°$$

$$= 41.125\angle{-82°} \text{ MVA}$$

End of Solution

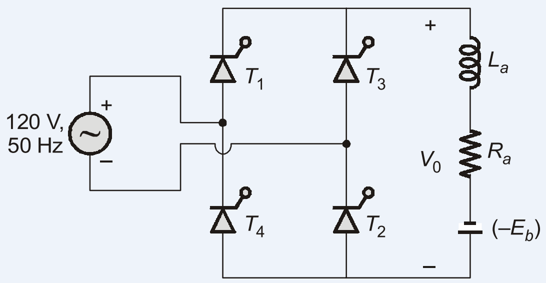
Q.4 (c) The speed of a DC motor is controlled by a single-phase AC-DC full converter as shown in the figure below. The AC supply is 1-ϕ, 120 V, 50 Hz. The armature resistance R_a = 1 Ω and armature circuit inductance is L_a = 25 mH. The motor voltage constant is 0.055 V/r.p.m. The field current of the motor is adjusted such that the armature voltage E_a is negative. The firing angle is α = 60° and the speed of the DC machine is 200 r.p.m. Assume that the DC machine current is continuous and ripple-free :



(i) Determine the average value of the DC machine current.
(ii) Determine whether the power is delivered or absorbed by the DC machine and also find this power.
(iii) Determine whether the power is delivered or absorbed by the AC supply and also find this power.
(iv) Explain where the power is going. Also, find the supply power factor.

[20 marks : 2024]

Solution:



Source delivers power to load ($I_o^2R_a$).
Here, since E_b is negative. Even back EMF also delivers power to ($I_o^2R_a$)

$$\frac{2V_m}{\pi}\cos\alpha = -11 + I_oR_a$$

$$V_o = -E_b + I_oR_a \qquad [\because E_b \rightarrow \text{Negative}]$$

$$I_o = \frac{V_o + E_b}{R_a} = \frac{54+11}{1} = 65\text{ A}$$

$$E_b = kN$$

$$E_b = 0.055 \times 200$$

$$E_b = 11\text{ V}$$

$$V_o = \frac{2V_m}{\pi}\cos\alpha = 2\times\frac{120\sqrt{2}}{\pi}\cos\alpha$$

$$V_o = 54\text{ V}$$

(i) $\text{Power} = V_oI_o$

$$= 54 \times 65 = 3510\text{ W}$$

(ii) Here DC Machine delivers power [$\because E_b$ is negative]

$$\text{Power delivered by machine} = E_b I_o = 11 \times 65$$
$$= 715 \text{ W}$$

and this power is also dissipated in Armature as copper loss.

(iii) AC $\xrightarrow{\text{Power}}$ DC

$\because \alpha < 90°$, rectification mode.

Here, $\alpha = 60°$ and ($\alpha < 90°$)

$\therefore$ Power delivered by AC source

$$= V_o I_o \text{ (which is also dissipated as Cooper (Cu)}$$
$$= 54 \times 65 = 3510 \text{ W}$$

(iv) Here, power delivered AC source and also power delivered by E_b is dissipated as copper loss in Amatibe windings.

$\therefore$ Total power dissipated in windings

$$= 3510 + 715 = 4225 \text{ W}$$
$$\text{PF} = g.\text{FDF}$$
$$= \frac{2\sqrt{2}}{\pi}\cos\alpha$$
$$= \frac{2\sqrt{2}}{\pi}\cos(60°)$$
$$\text{PF} = 0.45$$
$$P_{in} = P_o$$
$$V_{sr} I_{sr} \text{PF} = V_o I_o$$
$$\text{PF} = \frac{V_o I_o}{V_{sr} I_{sr}}$$
$$\text{PF} = \frac{V_o}{V_{sr}} = \frac{54}{120} = 0.45$$

End of Solution

SECTION : B

Q.5 (a) A first-order system with unity feedback has forward path transfer function $G(s) = \dfrac{28}{s + \alpha}$. The system is connected in cascade with an integral controller whose transfer function is $G_c(s) = \dfrac{K_i}{s}$. If the required natural frequency $\omega_n = 12$ rad/sec and the permissible steady-state error for unit ramp input is 0.1, find the values of K_i and α.

[12 marks : 2024]

Solution:

$$GH = \frac{K_i \times 28}{s(s+\alpha)};\ \text{Type} = 1$$

Given : $\omega_n = 12$

$\therefore$ $28K_i = \omega_n^2 = 144$

$\therefore$ $K_i = 5.14$

$$e_{ss} \text{ for ramp input} = \frac{1}{K_V}$$

$$K_V = \lim_{s\to 0} s\frac{\frac{144}{28}}{s(s+\alpha)} = \frac{5.14}{\alpha}$$

$$e_{ss} = \frac{\alpha}{5.14} = 0.1$$

$\therefore$ $\alpha = 0.514$

End of Solution

Q.5 (b) Let $z = re^{j\omega}$ and $s = \sigma + j\Omega$. Use bilinear transformation to show that if $r < 1$, then $\sigma < 0$, and if $r > 1$, then $\sigma > 0$, and when $r = 1$, then $\sigma = 0$.

[12 marks : 2024]

Solution:

Bilinear transformation relationship is

$$S = \frac{2}{T}\left[\frac{z-1}{z+1}\right]$$

$$\Rightarrow \quad \sigma + j\Omega = \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right] = \frac{2}{T}\frac{re^{j\omega}-1}{re^{j\omega}+1}\times\frac{re^{-j\omega}+1}{re^{-j\omega}+1}$$

$$= \frac{2}{T}\cdot\frac{r^2-1+r[e^{j\omega}-e^{-j\omega}]}{r^2+1+r(e^{j\omega}+e^{-j\omega})}$$

$$= \frac{2}{T}\cdot\frac{r^2-1+j2r\sin\omega}{r^2+1+2r\cos\omega}$$

$$\Rightarrow \quad \sigma + j\Omega = \frac{2}{T}\cdot\frac{r^2-1}{r^2+1+2r\cos\omega} + j\frac{4}{T}\cdot\frac{r\sin\omega}{r^2+1+2r\cos\omega}$$

By comparison of LHS and RHS

$$\sigma = \frac{2}{T}\cdot\frac{r^2-1}{r^2+1+2r\cos\omega} \quad \text{...(a)}$$

and

$$\Omega = \frac{4}{T}\cdot\frac{r\sin\omega}{r^2+1+2r\cos\omega} \quad \text{...(b)}$$

From eqn. (a),

If $r = 1$, then $\sigma = 0$

If $r < 1$, then $\sigma < 0$

and if $r > 1$, then $\sigma > 0$

End of Solution

Q.5 (c) A single-phase, 10 kVA, 460/120 V, 50 Hz transformer has an efficiency of 96% when it delivers 9 kW at 0.9 power factor. This transformer is connected as an auto-transformer to supply load to a 460 V circuit from a 580 V source.

(i) Show the autotransformer connection with appropriate dot convention.

(ii) Determine the maximum kVA the autotransformer can supply to the 460 V circuit.

(iii) Determine the efficiency of the autotransformer for full load at 0.9 power factor.

[12 marks : 2023]

Solution:

1-Ph, 10 kVA, 460/120, 2W T/F, η = 96% at 9 kW/0.9 p.f.

Given :

$$O/P = 9 \text{ kW}$$

$$P.f = 0.9$$

$$\eta = 0.96$$

$$\eta = \frac{O/P}{I/P}$$

$$\therefore \quad I/P = \frac{9}{0.96} = 9.375 \text{ kW}$$

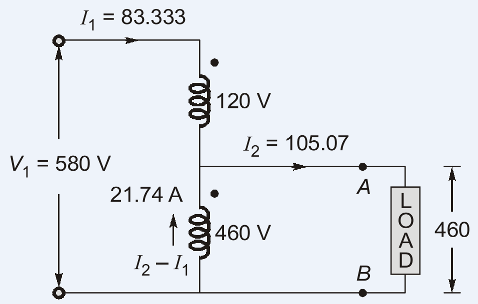
Total loss @ 9 kW, 0.9 p.f. = 9.375 − 9

Total loss @ 10 kVA = 0.375 kW

(i) 460/120 V, 2W T/F as 580/460 V

$$V_1I_1 = V_2I_2$$

$$I_2 = \frac{48.33 \times 10^3}{466}$$



(ii) HV winding as common additive polarity gives maximum kVA.

Actually, I_1 = 83.333 A

$$\therefore \quad V_1I_1 = 580 \times 83.33 = 48333.33$$

Maximum kVA = 48.333 kVA

$$V_1I_1 = V_2I_2$$

$$\therefore \quad I_2 = \frac{48.333 \times 10^3}{460}$$

(iii) η at F.L. 0.9 p.f :

$$\eta = \frac{1 \times 48.333 \times 0.9}{(1 \times 48.333 \times 0.9) + (0.375)} \times 100 = 99.14\%$$

Note : As V and I ratings are same in *pu*.
2ω T/F Case : Losses will be same in both cases.

End of Solution

Q.5 (d) A renewal energy-based power plant of 200 MW installed capacity has the following data :

$$\text{Capital cost} = \text{Rs. } 2{,}250/\text{kW}$$
$$\text{Interest + depreciation} = 10\%$$
$$\text{Annual load factor} = 0.7$$
$$\text{Annual capacity factor} = 0.6$$
$$\text{Annual running cost} = \text{Rs. } 40 \times 10^6$$
$$\text{Energy consumed by plant auxiliaries} = 6\%$$

Find the (i) reserve capacity and (ii) generation cost per unit

[12 marks : 2024]

Solution:

$$\text{Installed capacity I.C.} = 200 \text{ MW}$$
$$\text{Annual load factor, ALF} = 0.7, \text{ P.C.F.} = 0.6, \text{ IWD} = 10\%$$
$$\text{Reserve capacity} = \text{I.C.M.D.}$$
$$\text{M.D.} = \text{I.C.} \times \frac{\text{CF}}{\text{LF}}$$
$$= 200 \times \frac{0.6}{0.7} = 171.42 \text{ MW}$$

(i) $\text{R.C.} = 200 - 171.42 \text{ MW}$

$$\text{Number of units generated} = \text{M.D.} \times \text{ALF} \times \text{Hours in Year}$$
$$= 171.42 \times 0.7 \times 8760 \times 10^3$$
$$= 1051.15 \times 10^6$$

As auxiliary equipment is absorbing 6% generate.
Number of units reading to consumers

$$= 0.94 \times 1051.15 \times 10^6$$
$$= 988 \times 10^6$$
$$\text{Capital cost} = 2250 \times 200 \times 10^3 = 4.5 \times 10^8$$
$$\text{Interest and depreciation} = 10\% \Rightarrow 4.5 \times 10^7$$
$$\text{Total running charges} = 4 \times 10^7$$
$$\text{Overall cost of generation} = 4.5 \times 10^7 + 4.0 \times 10^7$$
$$= 8.5 \times 10^7$$

So, $$\text{Unit cost of generator} = \frac{8.5 \times 10^7}{988 \times 10^6} = 0.086$$
$$= 8.6 \text{ paise}$$

End of Solution

Q.5 (e) A controlled half-wave rectifier has an AC source of 240 V r.m.s. at 50 Hz. The load is a 30 Ω resistor.

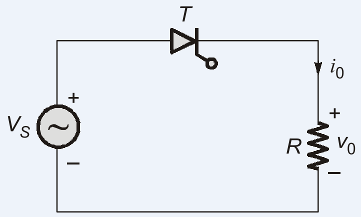
(i) Determine the delay angle such that the average load current is 2.5 A.

(ii) Determine the power absorbed by the load.

(iii) Determine the power factor.

[12 marks : 2024]

Solution:



$$I_o = \frac{V_o}{R}$$

$$2.5 = \frac{V_o}{30}$$

$$V_o = 2.5 \times 30$$

$$V_o = 75 \text{ V}$$

$$V_o = \frac{V_M}{2\pi}(1+\cos\alpha)$$

$$75 = \frac{240\sqrt{2}}{2\pi}(1+\cos\alpha)$$

(i) $$\alpha = 67.13° = 1.1716 \text{ rad}$$

$$V_{or} = \left\{\frac{1}{2\pi}\int_a^{\pi} V_m^2 \sin^2 \omega t\, d(\omega t)\right\}^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left\{(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right\}^{1/2}$$

$$= \frac{240\sqrt{2}}{2\sqrt{\pi}}\left\{(\pi-1.716)+\frac{1}{2}\sin(2\times 67.13)\right\}^{1/2}$$

$$= 95.746 \times 1.5257$$

$$V_{or} = 146.07 \text{ V}$$

$$I_{or} = \frac{V_{or}}{R}$$

$$P = I_{or}^2 R = \frac{V_{or}^2}{R} = \frac{(146.07)^2}{30} = 711.311 \text{ W}$$

(iii) $$\text{PF} = \frac{V_{or}}{V_{sr}} = \frac{146.07}{240} = 0.608 \text{ (lag)}$$

End of Solution

Q.6 (a) (i) What is meant by primary protection and backup protection? Give the reasons of primary protection failure. Also, define the terms (1) pick-up value and (2) reset value.

(ii) Explain the principle of Merz-Price system of protection used for power transformers.

[10 + 10 marks : 2024]

Solution:

(i) Primary Protection : Every component has its own relay fundamentally that relay operation is considered as primary protection. if any fault across within the vicinity relay operates.

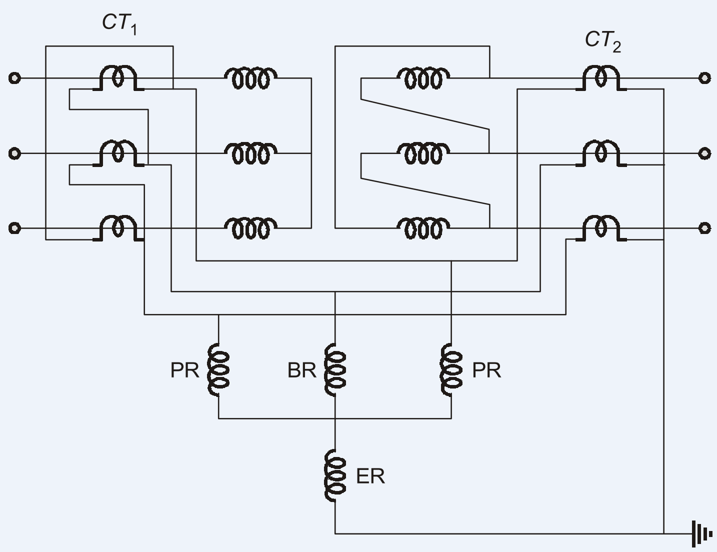
Back Up Protection : Under the maloperation of primary relay, the back up relay activates it is kept at the starting of sub section.

Pick Up Level : It is the minimum actuating quantity beyond which relay tends to operate.

Reset Level : It is the maximum value of actuating quantity that keeps the relay at off-state.

Fundamentally both sare same.

(ii) Merz-Price Differential Scheme of Protection :



Let us consider the star-delta T/F.

If there is any fault occurs the differential current is not equal to zero. Then, this differential component of current win activate the relay.

This scheme is considered as modified scheme consisting of 2-relays for phase protection and one relay for earth protection.

The balancing resistor with balance the voltage drops on the pilot wires. Otherwise the relay will operate for the differential voltage drops on the pilot wires.

Here the CT's on Y-side of T/F are connected in Δ and Δ-side of T/F on Y.

This is for :

1. Perfect balancing and relaying.
2. To compensage/nuetralise the 30° phase shift in Y-Δ connectivity.
3. To effective bypass of zero sequence currents in Y-Y connected T/F's.

End of Solution

Q.6 (b) In an 8085 microprocessor, an array of 34 bytes data is stored from memory location 8000H onwards. Certain six bytes data is available at memory location from 8100H to 8105H. Insert these six bytes into 34 bytes array starting from fourth location, i.e., 8003H onwards, such that the final array expands to 40 bytes. Write 8085 assembly language program with comments for this operation.

[20 marks : 2024]

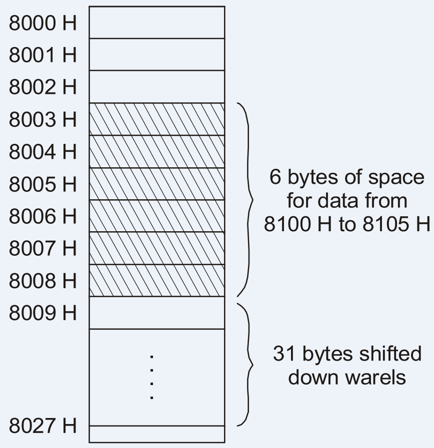
Solution:

Required Assembly Language Program :

- To insert 6B of data available from 8100H – 8105 in an array which already has 34B from 8000H. The data to be inserted from 4th memory location in existing array, i.e., 8003H.
- It can be noted that a space of 6B is to be formed from 8003H – 8008H.
- Hence, from 34B of data, 31B must shifted downwards, i.e., from 8009H – 8027H.

Algorithm :

1. Move 31 bytes from 8021H to 8003H towards downwards, i.e., to 8027H to 8009H. Hence, a space of 6 bytes be 8003H to 8008H is formed.
2. Copy the contents from 8100H – 8100H locations to 8003H – 8008H.



Required Assembly Program :

```
LXI H,     8021H; load HL pairs with source address
LXI D,     8027H; load 'DE' pair with destination address.
MVI C,     1FH; Count value 31 into register 'C'.
Repeat :   MOV A, H  ;   Data from source pointer to 'ACC'.
           STAX D    ;   'ACC' contents to be stored at destination.
           DCX H     ;   Decrement 'HL' pair, i.e., source pointer.
           DCX D     ;   Decrement 'DE' pair, i.e., destination pointer.
```

```
                DCR C       ;   decrement Deg-C by destination pointer
                MOVA, M
                JNZ Repeat  ;   Check if z = 0? in 'DCR C'. If true, program goes to Repeat
                                label.
                                If false, rest instrution is executed.
                                ; program to insert 6B from 8003H – 8008H, from the source
                                address 8100H to 8105H.
        LXI H,  8100H; load 'HL' pair with 8100H.
        LXI D,  8003H; load 'DE' pair with 8003H.
        MVI B,  06H; load count of 6 into Reg-B
        Again : MOV A, M    ;   data from memory to accumulator
                STAX D      ;   'ACC' data to destination at 8003H
                INX H       ;   Increment source memory pointer
                INX D       ;   Increment destination memory pointer
                DCR B       ;   decrement count in Reg-B
                JNZ Again   ;   if z = 0? True go to again else move
                HLT
```

End of Solution

Q.6 (c) Determine the signal $x[n]$ and rational z-transform $X(z)$ for the following cases :

(i) $x[n]$ is right-sided, $X(z)$ has a single pole, $x[0] = 4$, $x[2] = \frac{1}{4}$

(ii) $X[z]$ has poles at $z = \frac{1}{4}$ and $z = -1$, ROC includes the point $z = \frac{1}{2}$, $x[1] = 1$, $x[-1] = 1$.

[20 marks : 2024]

Solution:

(i) Given informations are :

(a) $x(n)$ is right-sided

(b) $X(z)$ has single pole

(c) $x(0) = 4$, $x(2) = \frac{1}{4}$

From (b), $$X(z) = \frac{K}{1 - az^{-1}}$$

By applying inverse ZT,

$$x(n) = K(a)^n u(n) \quad \text{...(i)}$$

Right sided inverse because of information (a).

Put $n = 0$: $x(0) = K = 4$ ...from information (c)

$\Rightarrow$ $K = 4$

Put $n = 2$:

$$x(2) = K.a^2 = \frac{1}{4} \quad \text{...from information (c)}$$

$\Rightarrow$ $$a^2 = \frac{1}{4K} = \frac{1}{16}$$

$\Rightarrow$ $$a = \frac{1}{4}$$

From (i), $$x(n) = K(a)^n u(n)$$

$$= 4\left(\frac{1}{4}\right)^n u(n)$$

(ii) Given informations are :

(a) Poles : $P_1 = \frac{1}{4}$, $P_2 = -1$

(b) ROC included : $Z = \frac{1}{2}$

(c) $x(1) = 1$, $x(-1) = 1$

From (a), $$X(z) = \frac{K}{(z-P_1)(z-P_2)}$$

$$= \frac{K}{\left(z-\frac{1}{4}\right)(z+1)}$$

$$= \left[\frac{K.z^2}{\left(z-\frac{1}{4}\right)(z+1)}\right]z^{-2}$$

$$= \left[\frac{K}{\left(1-\frac{1}{4}z^{-1}\right)(1+z^{-1})}\right]z^{-2}$$

$\Rightarrow$ $$X(z) = F(z)z^{-2} \quad \text{...(i)}$$

where, $$F(z) = \frac{K}{\left(1-\frac{1}{4}z^{-1}\right)(1+z^{-1})}$$

Now, $$F(z) = K\left[\frac{A}{1-\frac{1}{4}z^{-1}} + \frac{B}{1+z^{-1}}\right]$$

Residue calculation :

$$A = \left(\frac{1}{1+z^{-1}}\right)\Bigg|_{z=\frac{1}{4}} = \frac{1}{1+4} = \frac{1}{5}$$

$$B = \left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)\Bigg|_{z=-1} = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}$$

Thus, $$F(z) = K\left[\frac{1}{5}\cdot\frac{1}{1-\frac{1}{4}z^{-1}} + \frac{4}{5}\cdot\frac{1}{1+z^{-1}}\right] \quad ...(ii)$$

From information (b), ROC : $\frac{1}{4} < |z| < 1$

i.e., $x(n)$ will be a both sided sig.

By applying inverse ZT on (ii),

$$f(x) = K\left[\frac{1}{5}\cdot\left(\frac{1}{4}\right)^n u(n) - \frac{4}{5}\cdot(-1)^n u(-n-1)\right]$$

From (i), $X(z) = F(z)z^{-2}$

By applying inverse, $x(n) = f(n-2)$

$$\Rightarrow \quad x(n) = K\left[\frac{1}{5}\left(\frac{1}{4}\right)^{n-2} u(n-2) - \frac{4}{5}(-1)^{n-2}u\{-(n-2)-1\}\right]$$

$$= K\left[\frac{1}{5}\left(\frac{1}{4}\right)^{n-2} u(n-2) - \frac{4}{5}(-1)^{n-2}u(-n+1)\right]$$

Put $n = 1$: $$x(1) = K\left[-\frac{4}{5}(-1)^{-1}\right] = 1 \quad \text{...from info. (c)}$$

$$\Rightarrow \quad K = \frac{5}{4}$$

Since $K = \frac{5}{4}$, so

Put, $n = -1$

$$x(-1) = K\left[\frac{-4}{5},(-1)^{-3}\right] = 1 \quad \text{from info (c).}$$

$$x(n) = \frac{5}{4}\left[\frac{1}{5}\left(\frac{1}{4}\right)^{n-2}\cdot u(n-2) - \frac{4}{5}(-1)^{n-2}u(-n+1)\right]$$

$$\Rightarrow \quad x(n) = \left(\frac{1}{4}\right)^{n-1} u(n-2) - (-1)^{n-2}u(-n+1)$$

End of Solution

Q.7 (a) The open-loop transfer function of a system is given as :

$$G(s) = \frac{e^{-5s}}{s(s+1)}$$

Design a lead compensator such that the overall compensated system admits phase margin PM ≥ 40° and steady-state error due to ramp input $e_{ss} \le 0.1$.

[20 marks : 2024]

Solution:

$$GH = \frac{e^{-5s}}{s(s+1)}$$

Required $P_m \ge 40°$, $e_{ss} \le 0.1$

$$P_m = 180° + \left[-90° - \tan^{-1}(\omega) - 5\omega \times \frac{180°}{\pi}\right] = -172°$$

$$\omega = \omega_{gc} = 0.784$$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}} = 1 \quad \Rightarrow \quad \omega_{gc} = 0.784$$

∴ $$\phi_M = 40° - (-172°) + 5° \text{ (error)} = 217°$$

Single stage compensator should not be designed for $\phi_M > 80°$.

∴ 3 stages, each of $\frac{217°}{3} = 72.3°$

$$\alpha = \frac{1-\sin 72°3}{1+\sin 72°3} = 0.025$$

at $\omega_M \Rightarrow$ $$M = \sqrt{\alpha}$$

∴ $$\frac{1}{\omega\sqrt{\omega^2+1}} = \sqrt{0.025}$$

∴ $$\omega_M = \omega_{gc_{new}} = 2.41$$

∴ $$T = \frac{1}{\omega_M\sqrt{\alpha}} = 2.63$$

Findal design, $$GH = \frac{10e^{-5s}}{s(s+1)} \times \left(\frac{2.63s+1}{0.066s+1}\right)^3$$

End of Solution

Q.7 (b) A three-phase, 5 kVA, 208 volts, four-pole, 60 Hz, star-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 ohms per phase at rated terminal voltage. The machine is operated as a generator in parallel with a three-phase, 208 volts, 60 Hz power supply.

(i) Find the excitation voltage and the power angle when the machine is delivering rated kVA at 0.8 pf lagging. Draw the phasor diagram for this condition.

(ii) If the field excitation is now increased by 20 percent (without changing the prime mover power), find the stator current, power factor and reactive kVA supplied by the machine.

(iii) With the field current as in part (i), the prime mover power is slowly increased. What is the steady-state stability limit? What are the corresponding values of the stator (or armature) current and power factor at this maximum power transfer condition?

[20 marks : 2024]

Solution:

3-phase, 5 kVA, 208 V, 4*P*, 60 Hz, Y-alternator, R_a negligible, $X_s = 8\ \Omega$/ph

Rated : $$I_a = \frac{5000}{\sqrt{3}\times 208} = 13.879\text{ A}$$

(i) $$E\angle\delta = V\angle 0° + I_a\angle{-\phi}(R_a + jX_s)$$

$$= \frac{208}{\sqrt{3}} + 13.879\angle -\cos^{-1}(0.8)(j8)$$

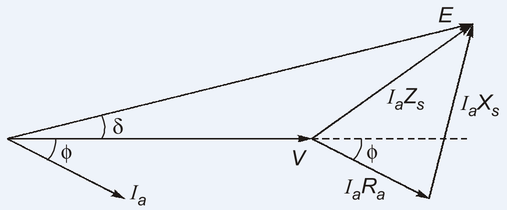
$$= 120 + (66.61 + j88.82)$$

$$E\angle\delta = 186.61 + j88.82 = 206.669\angle 25.45°$$

$$\text{Line value} = \sqrt{3}\times 206.669 = 323.208\text{ V}$$

$$\text{Power angle} = 25.45°$$

Phasor Diagram for Lagging Loads :



(ii) If I_f increased by 20%, *E* also increase by 20%.

$$E_1 \sin\delta_1 = 1.2\, E_1 \sin\delta_2$$

∴ $$\sin\delta_2 = \frac{\sin(25.45)}{1.2} = 0.3581$$

∴ $$\delta_2 = 20.983°$$

(Mech I/P Constant)

$$P = \frac{EV}{X_s}\sin\delta$$

$$E_1 \sin\delta_1 = E_2 \sin\delta_2$$

∵ *P*, *V* and X_s costant.

$E \sin\delta$ constant.

New I_a with increased excitation and new δ is

$$I_{a2} = \frac{E\angle\delta_2 - V\angle 0°}{Z_s\angle\theta} = \frac{1.2(206.669)\angle 20.983 - \frac{208}{\sqrt{3}}}{j8}$$

$$= \frac{248\angle 20.983 - 120\angle 0°}{j8} = \frac{111.55 + j88.8}{j8}$$

$$I_{a2} = 11.1 - j13.94 = 17.8\angle{-51.47}$$

New P.f. $= \cos(-51.47) = 0.622$ lagg

KVAR supplied $\Rightarrow \sqrt{3}\, VI\sin\phi$

$$= \sqrt{3} \times 208 \times 178 \times \sin(51.47)$$

$$= 5016.428 \text{ VARs}$$

(iii) With same I_f in (i), Mech I/P is increased.
Steady state stability limit is at $\delta = 90°$.
I_a at Max O/P condition, $\delta = 0$ condition

$$I_a = \frac{E\angle 90° - V}{Z\angle\theta} = \frac{1.2(206.669)(\angle 90 - 120\angle 0°)}{j8}$$

$$= \frac{-120 + j248}{j8} = \frac{275.5\angle 115.82}{j8}$$

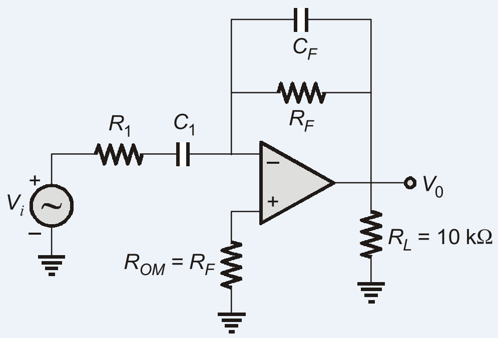
New current at max power, P.f. $= 30.999 + j14.999$

$$= 34.43\angle 25.82°$$

P.f. $= \cos(25.82) = 0.9$ leading

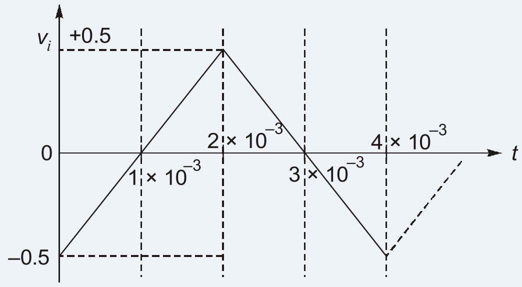
End of Solution

Q.7 (c) (i) The schematic diagram of a practical differentiator circuit using op-amp is as shown in the figure below :



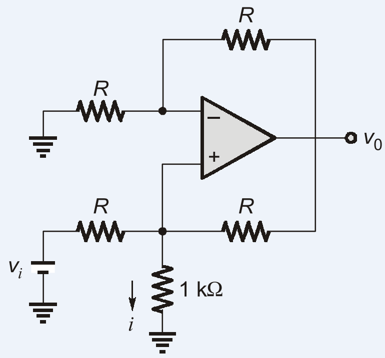
(1) Calculate the various component values, if the differentiator differentiates input signal from frequency 10 Hz to 2 kHz effectively.

(2) Also, sketch the output voltage (v_o) waveform, when the triangular wave as shown in the figure below is applied as input signal :



[10 marks : 2024]

(ii) For the op-amp circuit shown in the figure below, calculate the output voltage v_o and the current through 1 kΩ resistor, when the input voltage v_i = 2 V and R = 5 kΩ :

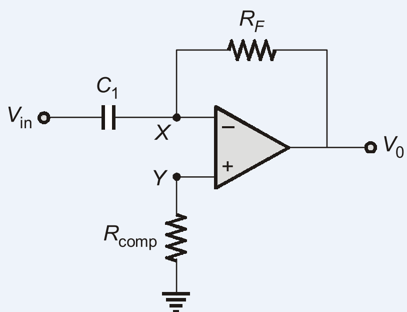


Also, compute the current i, if 1 kΩ resistor is replaced by 2 kΩ resistor.

[10 marks : 2024]

Solution:

(i) Ideal differentiator



$V^+ = 0$ then $\qquad V^- = 0 \quad [V^+ = V^-]$

Apply KCL at X

$$C_1 \frac{d}{dt}(-V_{in} + V_X) + \frac{V_X - V_o}{R_F} = 0$$

$$V_X = 0$$

$$C_1 \frac{d}{dt}(-V_{in}) = \frac{V_o}{R_F}$$

$$-C_1 R_F \frac{dV_{in}}{dt} = V_o$$

$$V_o = -R_F C_1 \frac{dV_i}{dt}$$

$$V_o(t) = -R_F C_1 \frac{d}{dt} V_i(t)$$

Apply Laplace transform

$$V_o(s) = -R_F C_1 s V_i(s)$$

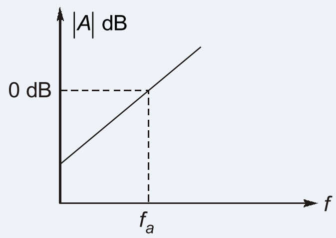
Gain of differentiator

$$\frac{V_o(s)}{V_i(s)} = -R_F C_1 (j\omega)$$

$$\frac{V_o(s)}{V_i(s)} = -jR_F C_1 2\pi F$$

$$|A| = \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = |-j\omega R_F C_1| = |-jR_F C_1 2\pi F|$$

$$A = 2\pi F R_F C_1$$

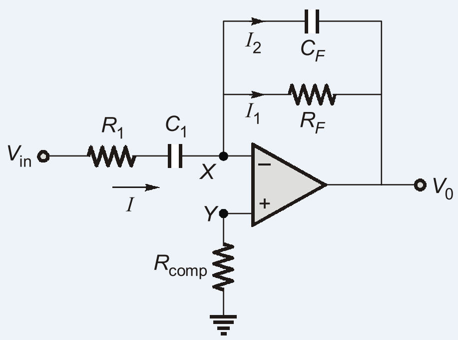


Assume f_a is the frequency at which gain becomes 0 dB (unity gain)

$$1 = 2\pi f_a R_f C_1$$

$$f_a = \frac{1}{2\pi R_f C_1}$$

Practical differentiator circuit :



KCL at X :

$$I = I_1 + I_2$$

$$\frac{V_i(s)}{R_1 + \frac{1}{sC_1}} = \frac{-V_o(s)}{R_f} - \frac{V_o(s)}{\frac{1}{sC_f}}$$

$$V_o(s)\left[\frac{1}{R_f} + sC_f\right] = \frac{-V_i(s)sC_1}{1 + R_1 sC_1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{-sC_1R_f}{(1+sR_1C_1)(1+sR_fC_f)}$$

If $R_1C_1 = R_fC_f$,
$$\frac{V_o(s)}{V_i(s)} = \frac{-sC_1R_f}{(1+sR_1C_1)^2}$$

If $R_fC_1 >> R_1C_1$,
$$\frac{V_o(s)}{V_i(s)} = -sR_fC_1 \text{ (neglect denominator)}$$

$$V_o(s) = -sR_fC_1V_i(s)$$

$$V_o(t) = -R_fC_1\frac{d}{dt}V_i(t) \text{ (It will behave as differentiator)}$$

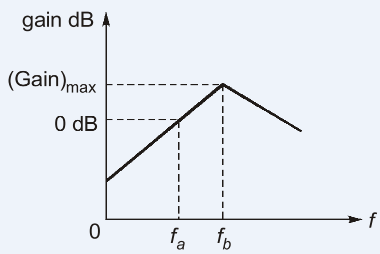
$$\frac{V_o(s)}{V_i(s)} = \frac{-sC_1R_f}{(1+sR_1C_1)^2} \qquad (R_1C_1 = R_fC_f)$$

Put $s = j\omega$,
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-j\omega C_1R_f}{(1+j\omega R_1C_1)^2} = \frac{-j2\pi FR_fC_1}{(1+j2\pi fR_1C_1)^2}$$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-j\left(\frac{F}{F_a}\right)}{\left(1+j\left(\frac{F}{F_b}\right)\right)^2}$$

where,
$$f_a = \frac{1}{2\pi R_fC_1} \qquad \text{[0 dB frequency]}$$

and
$$f_b = \frac{1}{2\pi R_1C_1} \qquad \text{[gain limiting frequency]}$$



Design condition :

$$f_b > f_a$$
$$f_b > 10f_a$$

$$\frac{1}{2\pi R_1C_1} = 10 \times \frac{1}{2\pi R_fC_1}$$

$$R_f = 10R_1$$
$$R_fC_f = R_1C_1$$
$$R_{comp} = R_1 \parallel R_f$$

Select $C_1 = 0.1\ \mu F$

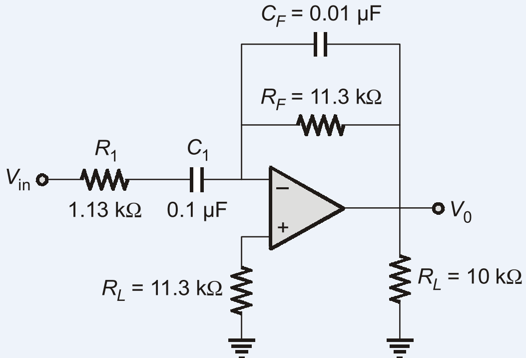
Now calculate R_f, $f_a = \frac{1}{2\pi R_f C_1}$

Select $f_a = \sqrt{10 \times 2000} = 141$ Hz (geometric mean)

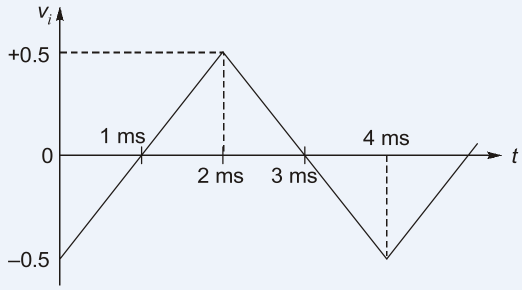
$$R_f = \frac{1}{2\pi f_c C_1} = \frac{1}{2\pi 141\ Hz \times 0.1\ \mu F}$$
$$R_f = \frac{1}{2\pi \times 141 \times 0.1 \times 10^{-6}} = 11.3\ k\Omega$$
$$R_f = 10R_1$$
$$11.3\ k\Omega = 10R_1$$
$$R_1 = \frac{11.3\ k\omega}{10} = 1.13\ k\Omega$$
$$R_1C_1 = R_fC_f$$
$$1.13\ k\Omega \times 0.1\ \mu F = 11.3\ k\Omega \times C_f$$
$$C_f = \frac{1.13 \times 10^3 \times 0.1 \times 10^{-6}}{11.3 \times 10^3}$$
$$= 0.01\ \mu F$$

Design components are :

$$R_1 = 1.13\ k\Omega$$
$$C_1 = 0.1\ \mu F$$
$$R_f = 11.3\ k\Omega$$
$$C_f = 0.01\ \mu F$$



2. Given ip/signal :



0 to 2 ms the slope $\frac{dV_{in}}{dt} = \frac{1\text{ V}}{2\text{ mS}}$

$= 0.5 \times 10^3$ V/sec

$V_{out} = -R_f C_1 \frac{dV_{in}}{dt}$

$= -11.3\text{ k}\Omega \times 0.1\ \mu\text{F} \times 0.5 \times 10^3$ Volts

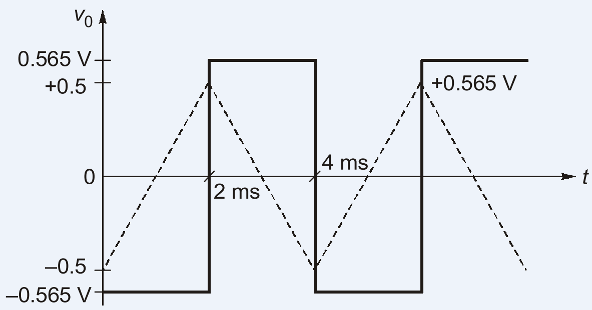
$= -0.565$ Volts

2 ms to 4 ms, the slope is $\frac{dV_{in}}{dt} = -\frac{1\text{ V}}{2\text{ ms}}$

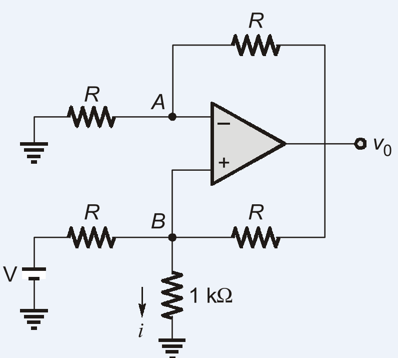
$= -0.5 \times 10^3$ V/sec

$V_{out} = -R_f C_1 \frac{dV_{in}}{dt}$

$= -11.3\text{ k}\Omega \times 0.1\ \mu\text{F} \times (-0.5 \times 10^3\text{ V/sec})$



(ii) $R = 5K$



$$V_A = \frac{V_o \times R}{R+R} = \frac{V_o}{2}$$

KCL at B :

$$\frac{2-V_B}{5} + \frac{V_o - V_B}{5} + \frac{0-V_B}{1} = 0$$

$$2 - V_B + V_o - V_B - 5V_B = 0$$

$$V_o - 7V_B = -2$$

Due to virtual S/C,

$$V_B = V_A = \frac{V_o}{2}$$

$$V_o - \frac{7V_o}{2} = -2$$

$$V_o = 0.8 \text{ V}$$

$$V_B = \frac{V_o}{2} = 0.4 \text{ V}$$

$$I = \frac{V_B}{1k} = \frac{0.4}{1k} = 0.4 \text{ mA}$$

If $1k$ is replaced by $2k$, then

$$\frac{2-V_B}{5} + \frac{V_o - V_B}{5} + \frac{0-V_B}{2} = 0$$

$$4 - 2V_B + 2V_o - 2V_B - 5V_B = 0$$

$$2V_o - 9V_B = -4$$

$$2V_o - \frac{9V_o}{2} = -4$$

$$V_o = 1.6 \text{ V}$$

$$V_B = \frac{V_o}{2} = 0.8 \text{ V}$$

$$I = \frac{V_B}{2k} = \frac{0.8}{2k} = 0.4 \text{ mA}$$

End of Solution

Q.8 (a) A three-phase induction machine is mechanically coupled to a DC shunt machine. The ratings and parameters of the two machines are as follows :

Induction machine :

Three-phase, 20 kW, 400 V, four-pole, 50 Hz, 1425 r.p.m.

$R_1 = 0.4\ \Omega$, $X_1 = 0.55\ \Omega$, $R'_2 = 0.35\ \Omega$, $X'_2 = 1.1\ \Omega$, $X_m = 38\ \Omega$

DC Machine :

220 V, 15 kW, 1500 r.p.m.

$r_a = 0.4\ \Omega$

The induction machine is connected to a three-phase, 400 V, 50 Hz supply and the DC machine is connected to a 220 V DC supply. The rotational losses of each machine of the M-G set may be considered constant at 225 watts. the system rotates at 1425 r.p.m. in the direction of the rotating field of the induction machine

(i) Determine the mode of operation of the induction machine and also the current taken by it.

(ii) Determine the real and reactive powers at the terminals of the induction machine and indicate their directions.

(iii) Determine the copper loss in the rotor circuit of the induction machine.

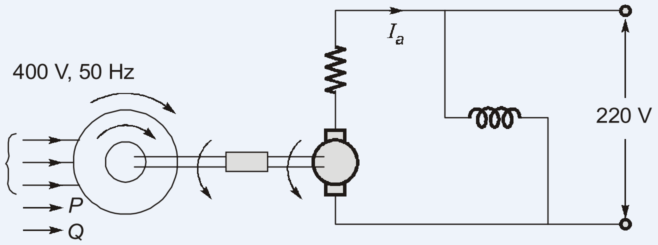
(iv) Determine the armature current (and its direction) of the DC machine.

[Given : R_1, X_1 are the stator winding resistance and leakage reactance, R'_2, X'_2 are the corresponding rotor values referred to the stator side and X_m is the magnetizing reactance]

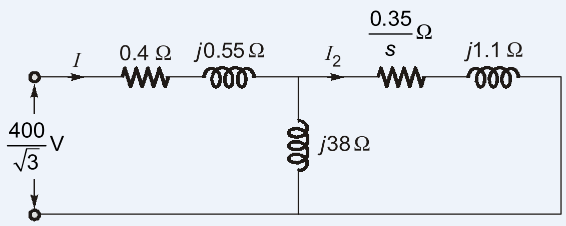
[20 marks : 2024]

Solution:

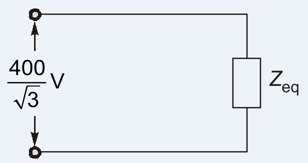
(i)



Since rotor rotates in the direction of rotation of rotating magnetic field at subsynchronous speed so induction machine operates in motoring mode.



Slip, $$s = \frac{1500 - 1425}{1500} = 0.05$$



$$Z_{eq} = 6.78\angle 19.08\ \Omega$$

$\Rightarrow$ $$I = \frac{400/\sqrt{3}\angle 0^\circ}{6.78\angle 19.08^\circ} = 34.052\angle -19.08^\circ$$

(ii) $$S = 3(VI^*) = 3\left(\frac{400}{\sqrt{3}} \times 34.062\right)\angle 19.08$$

$$= 23.6\angle 19.08 \text{ kVA}$$

$$P = 22.3 \text{ kW}$$

$$Q = 7.7 \text{ kVAr}$$

Real and reactive power both drawn from source to induction motor.

(iii) $$\text{Stator Cu loss} = \frac{3 \times 34.062^2 \times 0.4}{1000} \text{ kW} = 1.4 \text{ kW}$$

$$I_2 = \frac{j38}{(7 + j1.1) + j38} \times 34.062\angle -19.08$$

$$= 32.5856\angle -8.93$$

$\therefore$ $$\text{Rotor cu loss} = 3 \times 32.5856^2 \times 0.35$$

$$= 1.115 \text{ kW}$$

(iv) Full load current of DC generator,

$$I_{fl} = \frac{15000}{220} = 68.1818 \text{ A}$$

$\therefore$ $$E_a = 220 + I_{fl}.r_a = 247.2727 \text{ V}$$

$\therefore$ $$K_a\phi = \frac{247.2727 \times 60}{2z \times 1500}$$

Now, $$E'_a = K_a\phi \frac{2z \times 1425}{60} = 234.9 \text{ V}$$

$\therefore$ $$I'_a = \frac{E'_a V}{r_a} = 37.27 \text{ A}$$

End of Solution

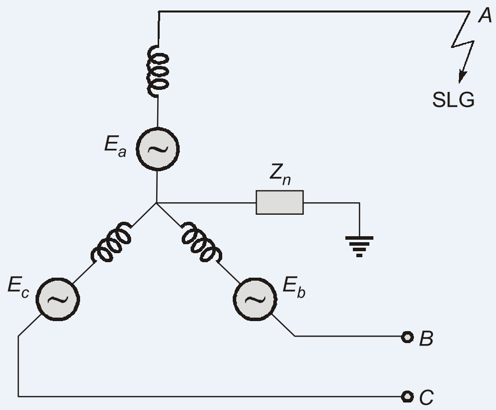
Q.8 (b) (i) Derive the expression for the fault current of the single line-to-ground fault, assuming that the fault is occurring in one of the phases of a three-phase, star-connected, unloaded generator with its neutral grounded through impedanze Z_n.

(ii) A three-phase, 20 MVA, 13.8 kV, star-connected, salient-pole generator has a direct-axis subtransient reactance of 0.25 pu. The negative and zero-sequence reactances are 0.35 pu and 0.10 pu respectively. The neutral of the generator is solidly grounded. Determine the subtransient current in the faulty phase and line-line voltage between non-faulty phases when a single line-to-ground fault occurs at the generator terminal with the generator operating unloaded at rated voltage. Neglect resistance.

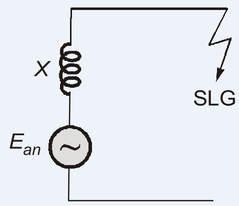
[20 marks : 2024]

Solution:

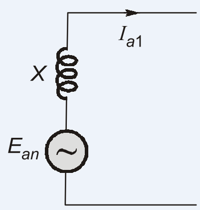
(i) Single line to ground fault in one of the phases of a 3-ϕ, Y-connected unloaded alternator



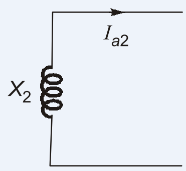
Let fault takes at phase 'A' at generator terminals per phase per unit equivalent circuit.



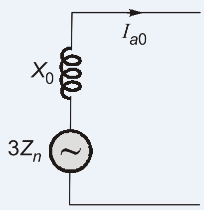
Thevenin equivalent of +ve sequence network diagram seen from the fault point.



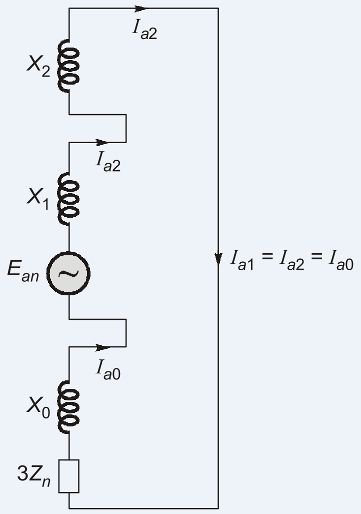
Thevenin equivalent of –ve sequence network as seen from the fault point :



Thevenin equivalent of zero sequence network as seen from the fault point



Equivalent circuit for single line to ground fault using +ve, –ve and zero sequence.



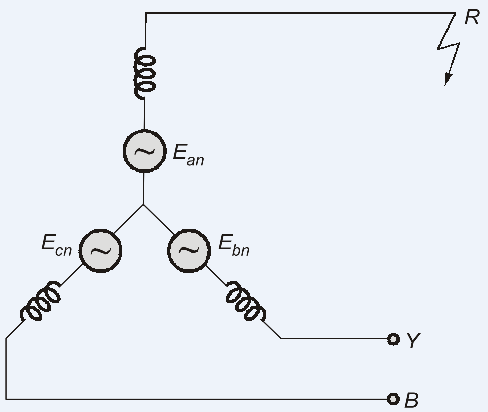
$$I_{a1} = I_{a2} = I_{a0} = \frac{E}{X_1 + X_2 + X_0 + 3Z_n}$$

In single line diagram

$$I_b = I_c = 0$$
$$I_{fault} = I_a$$
$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$
$$I_{a0} = \frac{1}{3}[I_a]$$
$$I_a = 3I_{a0}$$
$$I_f = 3I_{a0}$$

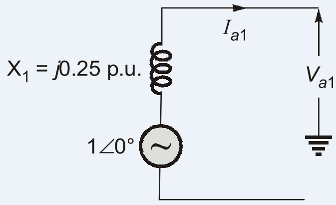
Fault current for single line to ground fault.

(ii)

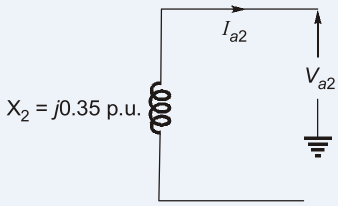


Let fault occurs at phase R :

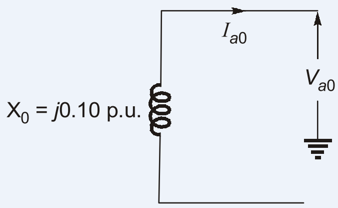
Thevenin's equivalent circuit of +ve sequence of phase 'A'



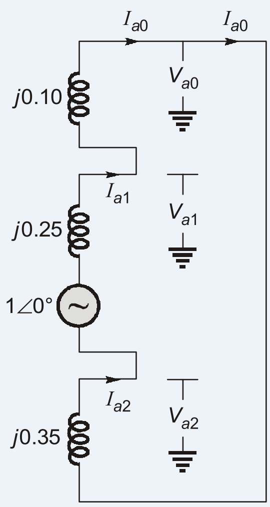
Thevenin's equivalent circuit of –ve sequence



Thevenin's equivalent circuit of zero sequence



Equivalent circuit of SLG fault :



$$I_{a0} = I_{a1} = I_{a2} = \frac{1\angle 0^\circ}{j0.10 + j0.25 + j0.35}$$

$$= -1.428i \text{ pu}$$

Fault current,

$$I_f = 3I_{a0}$$

$$I_f = 3 \times (-1.428)j$$

$$I_f = -4.285j \text{ pu}$$

$$I_f = -4.285 \times \frac{20 \times 10^3}{\sqrt{3} \times 13.8} \text{ Amp}$$

Sequence voltages of phase '*A*'

$$V_{a0} = -I_{a0} \times X_0$$
$$= -(-1.428j \times j0.10)$$
$$V_{a0} = -0.1428 \text{ pu}$$
$$V_{a1} = 1 - [j0.25 \times (-1.428i)] = 0.643 \text{ pu}$$
$$V_{a2} = -(-1.428j)(j0.35) = -0.4998 \text{ pu}$$

Phase voltage :

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$= -0.1428 + 0.643 - 0.4998$$
$$V_a \approx 0$$
$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$
$$= -0.1428 + (1\angle 240)(0.643) + (1\angle 120°)(-0.4998)$$
$$V_b = 1.0126\angle -102.223° \text{ pu}$$
$$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$$
$$= (-0.1428) + (1\angle 120°)|0.643) + (1\angle 240°)(-0.4998)$$
$$V_c = 1.012\angle 102.223 \text{ pu}$$

Line Voltage :

$$V_{ab} = V_a - V_b = 0 - (1.0126\angle -102.223°)$$
$$V_{ab} = 1.0126\angle 77.77° \text{ pu}$$
$$V_{bc} = V_b - V_c = (1.012\angle -102.223) - (1.012\angle 102.223)$$
$$V_{bc} = 1.978\angle -90° \text{ pu}$$
$$V_{ca} = V_c - V_a = 1.012\angle 102.223 \text{ pu}$$

End of Solution

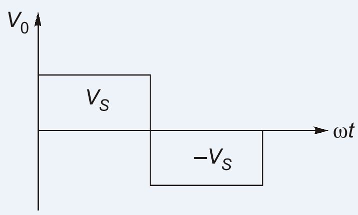
Q.8 (c) (i) A single-phase, full-bridge inverter has input DC voltage V_d = 250 V. The output voltage fundamental frequency is 50 Hz. Find the r.m.s. value of fundamental output voltage and its first two prominent harmonics for square-wave mode of operation.

(ii) Referring the inverter of part (i), if a motor-load of R = 10 Ω, L = 20 mH is connected to the output of the inverter, find the value of fundamental and its two prominent current harmonics for square-wave mode of operation. Assume no filtering.

[10 + 10 marks : 2024]

Solution:

(i) 1ϕ Full Bridge Inverter :



$$v_o = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$$

$$V_{o1} = \frac{2\sqrt{2}}{\pi} V_s = \frac{2\sqrt{2}}{\pi} \cdot 250 = 225.07 \text{ V}$$

First two predominant harmonics are 3rd and 5th.

$$V_{o3} = \frac{2\sqrt{2}}{3\pi} V_s = \frac{2\sqrt{2}}{3\pi} \cdot 250 = 75 \text{ V}$$

$$V_{o5} = \frac{2\sqrt{2}}{5\pi} V_s = \frac{2\sqrt{2}}{5\pi} \cdot 250 = 45.01 \text{ V}$$

(ii) RL Load :

$$R = 10\ \Omega,\ L = 20 \text{ mH}$$

$$I_{o1} = \frac{V_{o1}}{|Z_1|} = \frac{225.07}{\sqrt{R^2 + (2\pi.50.L)^2}}$$

$$= \frac{225.07}{\sqrt{10^2 + (100\pi.20.10^{-3})^2}}$$

$$= \frac{225.07}{11.81}$$

$$I_{o1} = 19 \text{ A}$$

First two predominant harmonics are 3rd and 5th.

$$I_{o3} = \frac{V_{o3}}{|Z_3|} = \frac{75}{\sqrt{10^2 + (100\pi \times 3 \times 20 \times 10^{-3})^2}}$$

$$I_{o3} = 3.514 \text{ A}$$

$$I_{o3} = \frac{V_{o5}}{|Z_5|} = \frac{45.01}{\sqrt{10^2 + (5 \times 100\pi \times 20 \times 10^{-3})^2}}$$

$$I_{o5} = 1.365 \text{ A}$$

End of Solution

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