ESE 2024

Main Exam Detailed Solutions

Mechanical Engineering

PAPER-I

EXAM DATE : 23-06-2024 | 09:00 AM to 12:00 PM

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ANALYSIS

Mechanical Engineering Paper-I
ESE 2024 Main Examination

Sl.	Subjects	Marks
1	Thermodynamics	52
2	Fluid Mechanis	84
3	Heat Transfer	74
4	IC Engine	52
5	Refrigeration and Air Conditioning	52
6	Turbo Machinery	62
7	Power Plant	62
8	Renewable Sources of Energy	42
	Total	**480**

Scroll down for detailed solutions

SECTION : A

Q.1 (a) (i) Explain with examples the concept of Rheology of fluids.

(ii) Clearly mention the basic difference between the Euler's equations of motion and the Navier-Stokes equations.

[8 + 4 = 12 Marks : 2024]

Solution:

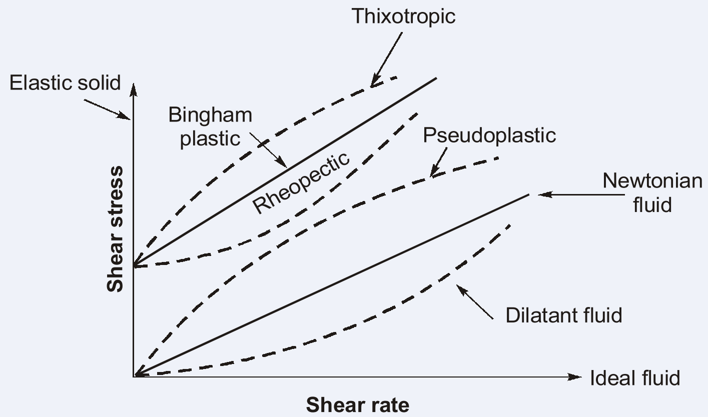
(i)

While most of the common fluids like water, air, petrol, ethanol and benzene follow Newton's law of viscosity as given by, there exists a large number of fluids which do not follow this linear relationship between the shear stress τ and the rate of deformation, $\frac{du}{dy}$. Such fluids which do not obey Newton's law of viscosity are known as Non-Newtonian fluids. In the non-Newtonian fluids, such as the one mentioned above, the relationship between rate of deformation, $\frac{du}{dy}$, and the shear stress τ can in general be expressed as a power law relation like

$$\tau = m\left(\frac{du}{dy}\right)^n$$

In this, m is known as consistency index and the power n is the flow index.

- When $n < 1$, the fluid is fluid is known as non-Newtonian pseudoplastic fluid. Gelatine, milk and blood are typical examples of pseudoplastic fluids.
- When $n > 1$, the fluids is known as non-Newtonian, dilatant fluid. Starch suspension, sugar solution and high-concentration sand suspension are typical examples of dilatant fluids.
- It may be noted that in equation, the case of $n = 1$ represents a Newtonian fluid, with $m = \mu$.
- The relationship between τ and $\frac{du}{dy}$ is known as rheological behaviour and figure is a schematic representation of rheological classification of fluids.



In figure, the x-axis also represents a Newtonian fluid with $\mu = 0$, that is a fluid with zero viscosity, such fluid called an ideal fluid or inviscid fluid. When $\frac{du}{dy}$ is zero for all τ, the situation represents an elastic solid.

Some Non-Newtonian fluids can be modeled as

$$\tau = \tau_y + \mu_p \left(\frac{du}{dy}\right)$$

Such fluids which require a yield stress τ_y for the flow to be established, are known as Bingham plastic.

There exist some non-Newtonian fluids which are time dependent, that is the shear stress and corresponding deformation rate are functions of time.

Thixotropic fluids are those which show an increase in apparent viscosity with time. Lipstic and certain paints and enamels exhibit thixotropic behaviour. The apparent viscosity may be defined as $\mu_{app} = \frac{\tau}{du/dy}$. Those fluids which show a decrease in the apparent viscosity with time are called rheopectic. Rheopectic fluids are much less common than thixotropic fluids. Gypsum suspensions in water and bentonite solutions are examples of rheopectic fluids. Thixotropy is an important property of paints and enamels. When subjected to high shear by the brush during application of paint, the apparent viscosity is so that the paint covers the surface smoothly, and brush marks disappear subsequently.

(ii)

- When compressibility, turbulence and surface tension forces are neglected and only gravity, pressure and viscous forces are taken into account then the equation of motion is called Navier-Stoke's equation of motion

  i.e. $$ma = F_g + F_p + F_v$$

- If the flow is assumed to be ideal, viscous forces is zero and only gravity and pressure forces are considered then the equation of motion is called Euler's equation of motion.

  i.e. $$ma = F_g + F_p$$

End of Solution

Q.1 (b) Explain the concept of mixing length introduced by Prandtl and state the relationship that exists between the turbulent shearing stress and the mixing length.

[12 Marks : 2024]

Solution:

Prandtl's mixing-length theory: Prandtl has developed a very useful theory of turbulence based on the concept of mixing-length using the analogy of mean-free-path of kinetic theory of gases. He introduced a linear dimension called the mixing length. In kinetic theory of gases the mean free path is defined as the average distance which a gas molecule travels before it strikes another molecule. Analogous to mean-free-path, the mixing length may be considered to represent the average distance perpendicular to the flow a small fluid mass will travel before its momentum is changed by the new environment. The turbulent shear stress expressed by equation, may be written in the form

$$(\tau_{xy})_t = \rho\varepsilon\frac{d\bar{u}}{dy} = -\rho\overline{u'v'}$$

From the analogy of kinetic theory of gases, it is reasonable to assume the following relation for turbulent flow:

$$\varepsilon = l u_*$$

where u_* is known as the shear or friction velocity; and l represents a unique length characterizing the local intensity of turbulent mixing at any level. The magnitude of l may be much larger than the mean free-path, and may also be a function of space (that is y), the mean velocity etc.

Prandtl's mixing-length theory is based on the assumption that there is complete correlation between the fluctuating velocity components u' and v'. From equation we obtain.

$$u_*^2 = \frac{(\tau_{xy})_t}{\rho} = -\overline{u'v'} = \sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}}$$

The turbulent shear stress may now written using equation,

$$(\tau_{xy})_t = \rho u_*^2 = \rho l u_*\frac{d\bar{u}}{dy}$$

which yields,

$$u_* = l\frac{d\bar{u}}{dy}$$

The Reynolds stress (i.e. the turbulent shear stress), substituting this value of u_* in equation is obtained as

$$(\tau_{xy})_t = \rho l^2\left(\frac{d\bar{u}}{dy}\right)^2$$

As the Reynolds stress depends on the sign of the mean velocity gradient, equation can be expressed as

$$(\tau_{xy})_t = \rho l^2\left|\frac{d\bar{u}}{dy}\right|\frac{d\bar{u}}{dy}$$

in which $\left|\frac{d\bar{u}}{dy}\right|$ represents the magnitude or absolute value of the mean velocity gradient while $\frac{d\bar{u}}{dy}$ is used with its actual sign (negative or positive). Equation contains length l which is known as the Prandtl's mixing length. This equation is valid so long as $u' \sim v'$. Prandtl argued that the length l must be proportional to distance y as the turbulent fluctuations u' and v' must vanish at the solid boundary.

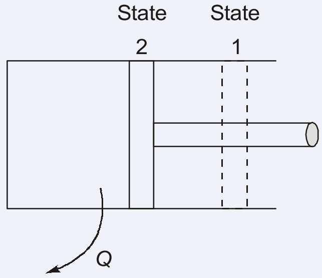
End of Solution

Q.1 (c) A stationary mass of gas is compressed without friction from an initial state of 0.45 m^3 and 0.12 MPa to a final state of 0.15 m^3 and 0.12 MPa, the pressure remaining constant during the process. There is a transfer of 57.6 kJ of heat from the gas during the process. How much does the internal energy of the gas change?

[12 Marks : 2024]

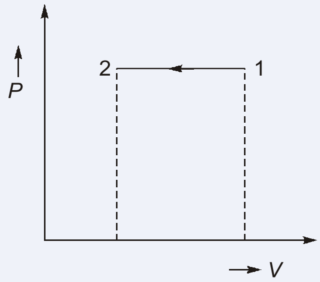
Solution:

Given: $P_1 = P_2 = 0.12$ MPa, $V_1 = 0.45$ m^3, $V_2 = 0.15$ m^3, $Q = -57.6$ kJ



Apply first law of thermodynamics or Energy conservation for process 1-2:

$$Q_{1\text{-}2} = \Delta U_{1\text{-}2} + W_{1\text{-}2} \quad \text{... (i)}$$



Now, process 1-2 is isobaric process,

$$\therefore \quad W_{1\text{-}2} = P(V_2 - V_1) = 0.12 \times 10^3\ (0.15 - 0.45)$$

$$= -36 \text{ kJ}$$

$$Q_{1\text{-}2} = -57.6 \text{ kJ} \quad \text{[Given]}$$

From equation (i)

$$\Rightarrow \quad -57.6 = \Delta U_{1\text{-}2} - 36$$

$$\therefore \quad \Delta U_{1\text{-}2} = -57.6 + 36 = -21.6 \text{ kJ} \quad \textbf{Ans.}$$

End of Solution

Q.1 (d) What is condensation? Explain the terms filmwise condensation and dropwise condensation. Which one is more preferred? Justify.

[12 Marks : 2024]

Solution:

Condensation: Condensation is the process by which a vapor turns into a liquid. This phase change occurs when the vapor is cooled to its dew point temperature or comes into contact with a surface at a temperature below the dew point. During condensation, the latent heat of vaporization is released.

Filmwise Condensation: Filmwise condensation occurs when the liquid condensate and forms a continuous film over the surface. The characteristics of filmwise condensation are:

- The liquid film grows in thickness as more vapor condenses.
- The heat transfer through the liquid film occurs by conduction, which is generally less efficient due to the thermal resistance of the film.
- The film can create a barrier, reducing the overall heat transfer coefficient.

Dropwise Condensation: Dropwise condensation occurs when the vapor condenses into discrete droplets on the surface rather than forming a continuous film. The characteristics of dropwise condensation are:

- The droplets form and grow in size until they are large enough to be pulled away by gravity or other forces.
- The surface remains partially exposed, allowing for higher heat transfer rates.
- The heat transfer is more efficient due to the direct contact between the vapor and the cooled surface.

Dropwise condensation is generally more preferred over filmwise condensation for the following reasons:

1. **Higher Heat Transfer Coefficient:** Dropwise condensation has a significantly higher heat transfer coefficient compared to filmwise condensation. This is because the direct contact between the vapor and the cooled surface reduces thermal resistance.
2. **Efficiency:** The absence of a continuous liquid film allows for more effective heat transfer, making dropwise condensation more efficient for practical applications.
3. **Surface Exposure:** In dropwise condensation, the surface remains exposed to the vapor as droplets form and fall away. This continuous renewal of the surface helps maintain high heat transfer rates.

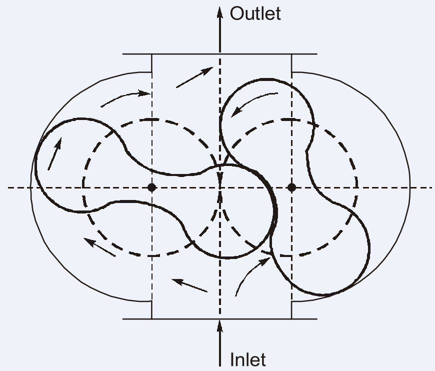
End of Solution

Q.1 (e) Explain the working of roots blower supercharger with diagram and mention its advantages.

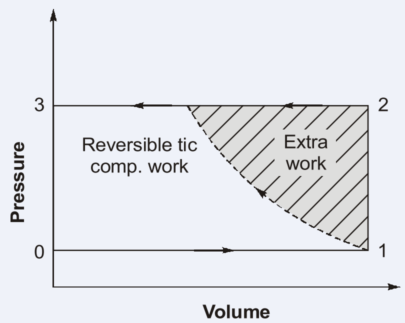
[12 Marks : 2024]

Solution:

Root Blower: Roots blower figure (a) consists of two cylindrically shaped lobes roating in opposite directions in a common housing. Air enters the space between the rotor lobes at inlet and is carried around the rotor to discharge port. It should be noted that no compression occurs in this process which is shown by 0-1 in figure (b).



(a) Roots blower



(b) p-v diagram of roots blower

Compression takes place only when the discharge port is opened and the pressure rises almost instantaneously [1-2 in figure (b)]. When the exhaust port opens, back-flow of air occurs and more work is required to compress the air. The extra work is required to compress the air. This extra work is represented by the shaded area of figure (b). This reduces the efficiency of the blower.

Moreover, there is always certain leakage between the lobes and the housing of the blower and noise is also produced. Nowadays three lobe rotors of helical shape are also used. This gives an air supply at reduced noise levels, and pulsations in the air supply are also reduced. The blower is run at about 1.5 times the engine speed.

The roots blower is highly suitable for pressure ratios up to 1.1 to 2.0 due to simplicity, low cost and good mechanical efficiency. The rotors do not require lubrication. However, due to moderate speed of about 4000 rpm the blower is bulky and heavy. The volumetric efficiency decreases rapidly with an increase in pressure ratio. The combined effect of the compressibility of air and the back-flow cause volumetric efficiency to reduce to values as low as 50-60% at pressure ratio of 3 : 1 as compared to efficiency values of 80-90% for pressure ratio of 1.5 : 1.

Roots blower is suitable for low and medium speed engines for stationary and marine installations.

End of Solution

Q.2(a) (i) The velocity distribution in the boundary layer over a high spillway face was found to have the following form:

$$\frac{u}{U_\infty} = \eta^{0.22}, \text{ where } \eta = \frac{y}{\delta}$$

The free stream velocity U_∞ at a certain section was observed to be 20 m/s and boundary layer thickness of 5 cm was estimated from the velocity distribution measured at the section. The discharge passing over the spillway was 5 m^3/s per metre length of spillway. Determine:

I. Displacement thickness,
II. Momentum thickness,
III. Energy thickness, and
IV. Loss of energy up to section under consideration.

(ii) A plate 4 m long and 20 cm wide is immersed in a fluid of density 1.2 kg/m^3 and kinematic viscosity 10^{-4} m^2/s. The fluids is moving with a velocity of 5 m/s. Calculate:

I. Boundary layer thickness, and
II. Drag force on both sides of the plate

Assume Blasius's solution.

[14 + 6 = 20 Marks : 2024]

Solution:

(i)

Given: $U_\infty = 20$ m/s; $\delta = 5$ cm $= 50$ mm; $\dfrac{Q}{\text{Length}} = 5\ \text{m}^3/\text{s}$; $\dfrac{U}{U_\infty} = \eta^{0.22}$; $\eta = \dfrac{y}{\delta}$

(i) Displacement thickness

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{U}{U_\infty}\right) d\eta = \int_0^1 \left(1 - \eta^{0.22}\right) d\eta$$

$$= \left(\eta - \frac{\eta^{1.22}}{1.22}\right)_0^1 = \left[1 - \frac{1}{1.22}\right]$$

$$\delta^* = 9.0164 \text{ mm} \qquad \textbf{Ans.}$$

(ii) Momentum thickness

$$\frac{\delta^{**}}{\delta} = \int_0^1 \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) d\eta$$

$$= \int_0^1 \left(\eta^{0.22} - \eta^{0.44}\right) d\eta = \left[\frac{\eta^{1.22}}{1.22} - \frac{\eta^{1.44}}{1.44}\right]_0^1$$

$$= \left(\frac{1}{1.22} - \frac{1}{1.44}\right)$$

$$\delta^{**} = 6.2614 \text{ mm} \qquad \textbf{Ans.}$$

(iii) Energy thickness

$$\frac{\delta^{***}}{\delta} = \int_0^1 \frac{U}{U_\infty}\left(1 - \left(\frac{U}{U_\infty}\right)^2\right) d\eta$$

$$= \int_0^1 \left(\eta^{0.22} - \eta^{0.66}\right) d\eta = \left[\frac{\eta^{1.22}}{1.22} - \frac{\eta^{1.66}}{1.66}\right]_0^1$$

$$= \left(\frac{1}{1.22} - \frac{1}{1.66}\right)$$

$$\delta^{***} = 10.8631 \text{ mm} \qquad \textbf{Ans.}$$

(iv)

$$\text{Loss of energy} = \frac{1}{2}(\rho)\left(Q \times \delta^{***}\right)U_\infty^2$$

$$= \frac{1}{2}(10^3)\left(\frac{10.8631}{10^3} \times 5\right)(20)^2 = 10863.1 \text{ W} \qquad \textbf{Ans.}$$

(ii)

Given: $L = 4$ m; $W = 0.2$ m; $\rho = 1.2$ kg/m^3; $\nu = 10^{-4}$ m^2/s; $U_\infty = 5$ m/s;

$$Re_L = \frac{U_\infty L}{\nu} = \frac{5 \times 4}{10^{-4}} = 2 \times 10^5$$

(i) Boundary layer thickness

$$\delta_L = \frac{5L}{\sqrt{Re_L}} = \frac{5 \times 4}{\sqrt{2 \times 10^5}} = 0.0447 \text{ m}$$

$$= 44.72 \text{ mm}$$ **Ans.**

(ii) Drag force on both side

$$C_D = 2C_{fL} = \frac{2 \times 0.664}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{2 \times 10^5}} = 2.9695 \times 10^{-3}$$

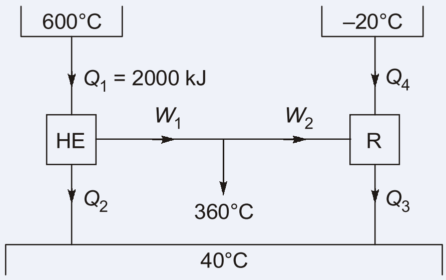
$$F_D = 2\left[\frac{C_D}{2}\rho U_\infty^2 A\right] = \frac{2 \times 2.9695 \times 10^{-3}}{2} \times 1.2 \times 5^2 \times (4 \times 0.2) = 0.07127 \text{ N}$$ **Ans.**

End of Solution

Q.2 (b) A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperature of 40°C and –20°C. The heat transfer to the heat engine is 2000 kJ and net work output of the combined engine refrigerator plant is 360 kJ.

(i) Evaluate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C.

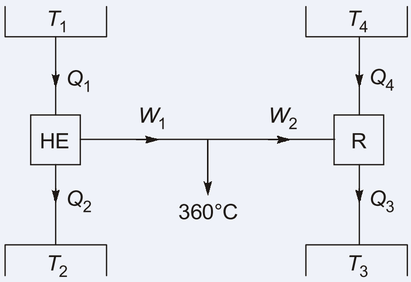
(ii) Evaluate heat transfer to the refrigerant and the net heat transfer to the reservoir when the efficiency of the heat engine and COP of the refrigerator are each 40% of their maximum possible values.



[20 Marks : 2024]

Solution:

Given: T_1 = 600°C = 873 K, T_2 = 40°C = 313 K, T_3 = 40°C = 313 K, T_4 = –20°C = 253K, W_{net} = 360 kJ, Q_1 = 2000 kJ



We know, $$\eta_{HE} = 1-\frac{T_2}{T_1} = 1-\frac{313}{873} = \frac{W_1}{Q_1}$$

or, $W_1 = 0.64146 \times Q_1 = 1282.932$ kJ

$\because$ $W_{net} = W_1 - W_2 = 360$ [Given]

$\therefore$ $W_2 = W_1 - 360 = 922.932$ kJ

We know, $$(COP)_R = \frac{T_4}{T_3 - T_4} = \frac{253}{313-253} = \frac{Q_4}{W_2}$$

$\therefore$ $Q_4 = 4.2167 \times W_2 = 3891.696$ kJ

Now, $Q_3 = Q_4 + W_2 = 3891.696 + 922.932$

or, $Q_3 = 4814.628$ kJ

and, $Q_2 = Q_1 - W_1 = 2000 - 1282.932 = 717.068$ kJ

(i) Heat transfer to the refrigerant,

$Q_4 = 3891.696$ kJ **Ans.**

Net heat transfer to the reservoir at 40°C

$= Q_2 + Q_3 = 717.068 + 4814.628$

$= 5531.696$ kJ **Ans.**

(ii) Now, Given, $$\eta_{HE} = 0.4 \times \eta_{max} = 0.4\times\left(1-\frac{313}{873}\right) = 0.2565$$

$$(COP)_R = 0.4 \times (COP)_{max} = 0.4\times\frac{253}{313-253} = 1.6867$$

Then, $$\eta_{HE} = 0.2565 = \frac{W_1}{Q_1}$$

or, $W_1 = 0.2565 \times Q_1 = 513$ kJ

$\because$ $W_{net} = W_1 - W_2 = 360$ kJ [Given]

$\therefore$ $W_2 = 513 - 360 = 153$ kJ

And, $Q_2 = Q_1 - W_1 = 2000 - 513 = 1487$ kJ

We know, $$(COP)_R = \frac{Q_4}{W_2} = 1.6867$$

$\therefore$ $Q_4 = 1.6867 \times W_2 = 258.065$ kJ

$Q_3 = W_2 + Q_4 = 153 + 258.065 = 411.065$ kJ

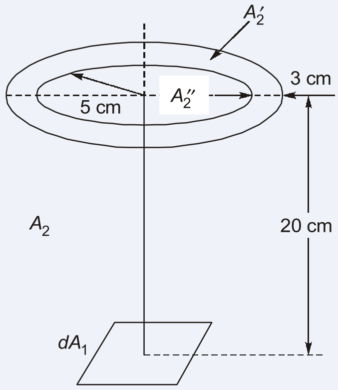
Now, Heat transfer to refrigerant, $Q_4 = 258.065$ kJ **Ans.**

Net heat transfer to reservoir $= Q_2 + Q_3 = 1487 + 411.065$

$= 1898.065$ kJ **Ans.**

End of Solution

Q.2 (c) Write down the properties of a black body. How do we define spectral emissivity of a real surface? Consider a small plane surface area dA_1 placed parallel to a circular ring A_2' of inner radius 5 cm and width 3 cm as shown in the figure. Calculate the fraction of the radiation emitted by the surface dA_1 that is intercepted by the ring (A_2') and also the fraction that passes through the hole (Area = A_2'') in the ring if the surfaces are placed 20 cm apart. Assume dA_1 is a very small (differential) surface element.



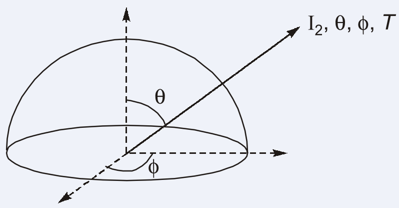
[20 marks : 2024]

Solution:

Properties of Blackbody

1. A blackbody absorbs all incident radiation, regardless of wavelength and direction.
2. For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody.
3. Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction. i.e., the blackbody is a diffuse emitter.

1. **Spectral Directional Emissivity:** Spectral directional emissivity of a surface at temperature T is defined as the ratio of the intensity of the radiation emitted at the wavelength λ and in the direction of θ and ϕ to the intensity of the radiation emitted by a blackbody at the same values of T and λ. Hence,



$$\varepsilon_{\lambda,\theta,\phi,T} = \frac{I_{\lambda,\theta,\phi,T}}{I_{b,\lambda,T}}.$$

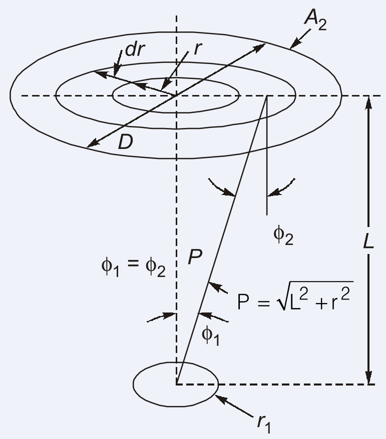
Subscripts λ and θ designate interest in a specific wavelength and direction for the emissivity.

$$A_1 F_{21} = \frac{1}{\pi}\int_{A_1}\int_{A_2} \frac{\cos\phi_1 \cos\phi_2}{r^2} dA_1 dA_2$$

Considering an elementary ring dA_2 of width dr at a radius r,

$$dA_2 = 2\pi r\, dr$$

Now, $$r = L\tan\phi_1$$



$$dr = L\sec^2\phi_1\, d\phi_1$$

$$dA_2 = 2\pi L^2 \tan\phi_1 \sec^2\phi_1\, d\phi_1$$

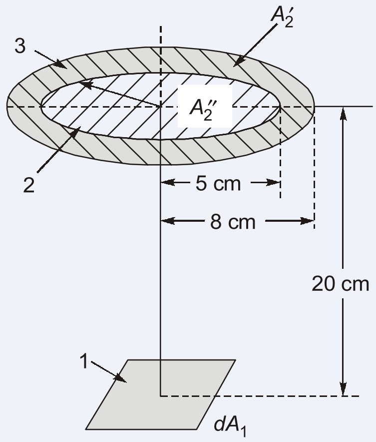
$$\frac{L}{P} = \cos\phi_1 \text{ or } P = \frac{L}{\cos\phi_1}$$

$$F_{12} = \frac{1}{\pi}\int_{A_2} \frac{\cos\phi_1\cos\phi_2}{P^2}\, dA_2$$

$$= \frac{1}{\pi}\int_{\phi_1=0}^{\alpha} \frac{\cos^2\phi_1 2\pi L^2 \tan\phi_1 \sec^2\phi_1 d\phi_1}{L^2}\cos^2\phi_1$$

$$F_{12} = \int_0^{\alpha} \sin^2\phi_1 d\phi_1 = \frac{1-\cos 2\alpha}{2}$$

$$\therefore \quad F_{12} = \sin^2\alpha = \frac{D^2/4}{D^2/4 + L^2} = \frac{D^2}{D^2 + 4L^2}$$



Let area -2 (A_2'') and area-3 (A_2') is area-4. Now, the fraction of the radiation emitted by the surface dA_1 that intercepted by area-2 (A_2'') is given as

$$F_{12} = \frac{D_2^2}{D_2^2 + 4L^2} = \frac{(10)^2}{(10)^2 + 4(20)^2} = \frac{1}{17} = 0.0588 \qquad \textbf{Ans.}$$

Also, the fraction of the radiation emitted by the surface dA_1 that intercepted by area-4(A_2' + A_2'') is given as;

$$F_{14} = \frac{D_4^2}{D_4^2 + 4L^2} = \frac{(16)^2}{(16)^2 + 4(20)^2} = \frac{4}{29} = 0.1379$$

Now, using principle of superposition, we can write

$$F_{14} = F_{12} + F_{13}$$

$$\Rightarrow \quad 0.1379 = 0.0588 + F_{13}$$

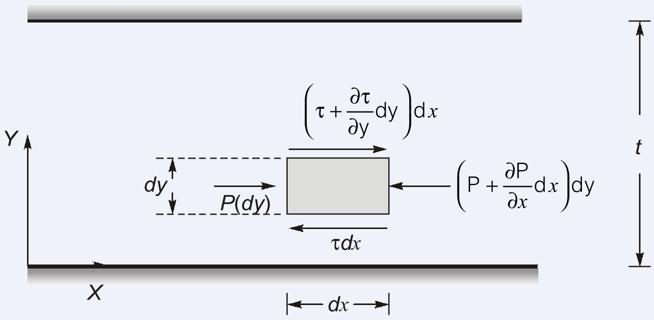
$$\Rightarrow \quad F_{13} = 0.0791 \qquad \textbf{Ans.}$$

Hence, the fraction of the radiation emitted by the surface dA_1 that is intercepted by the ring (A_2') is 0.0791 and the fraction that passes through the hole (Area = A_2'') in the ring is 0.0588.

End of Solution

Q.3 (a) Prove that the velocity distribution for viscous flow between two parallel plates, when both plates are fixed across a section is parabolic in nature. Also, prove that the maximum velocity is equal to one and a half times the average velocity.
[20 marks : 2024]

Solution:



Applying Newtonian law of motion on control volume

$$\because\; P(dy)-\left(P+\frac{\partial P}{\partial x}dx\right)dy+\left(\tau+\frac{\partial \tau}{\partial y}dy\right)dx-\tau dx = m\cancel{a}^{\,o}$$

$$\Rightarrow \quad -\frac{\partial P}{\partial x}dxdy+\frac{\partial \tau}{\partial y}dxdy = 0$$

$$\therefore \quad \frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x} \qquad \text{... (i)}$$

Newton's law of viscosity

$$\tau = \mu\frac{du}{dy}$$

$$\frac{\partial \tau}{\partial y} = \mu\frac{d^2u}{dy^2}$$

$$\frac{\partial P}{\partial x} = \mu\frac{d^2u}{dy^2} \qquad \text{[From equation (i)]}$$

or,
$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\times\frac{\partial P}{\partial x}$$

Integrating onboth sides, we get

$$\frac{du}{dy} = \frac{1}{\mu}\times\frac{\partial P}{\partial x}y+C_1$$

Again integrating on both sides, we get ... (A)

$$u = \frac{1}{\mu}\times\frac{\partial P}{\partial x}\times\frac{y^2}{2}+C_1y+C_2 \qquad \text{... (ii)}$$

Boundary condition,

(i) at $y = 0$, $u = 0$

$\Rightarrow C_2 = 0$

(ii) at $y = t$, $u = 0$

$\therefore$ $$0 = \frac{1}{\mu}\times\frac{\partial P}{\partial x}\times\frac{t^2}{2}+C_1 t$$

or $$C_1 = \frac{1}{\mu}\times\frac{\partial P}{\partial x}\times\frac{t}{2}$$

Put C_1 in equation (ii), $$u = -\frac{1}{2\mu}\times\frac{\partial P}{\partial x}y^2-\frac{1}{2\mu}\times\frac{\partial P}{\partial x}ty$$

$$u = -\frac{1}{2\mu}\times\frac{\partial P}{\partial x}t^2\left(\frac{y}{t}-\frac{y^2}{t^2}\right) \quad \text{... (iii)}$$

For maximum velocity, $$\frac{\partial u}{\partial y} = 0$$

From equation (A), we have $$0 = \frac{1}{\mu}\times\frac{\partial P}{\partial x}y-\frac{1}{\mu}\times\frac{\partial P}{\partial x}\times\frac{t}{2}$$

$\Rightarrow$ $$y = \frac{t}{2}$$

At $$y = \frac{t}{2} \Rightarrow U = U_{max}$$

Putting value $y = \frac{t}{2}$ in equation (iii)

$$U_{max} = -\frac{1}{2\mu}\times\frac{\partial P}{\partial x}t^2\left(\frac{t/2}{t}-\frac{(t/2)^2}{t^2}\right)$$

$$= -\frac{1}{2\mu}\times\frac{\partial P}{\partial x}t^2\left(\frac{1}{2}-\frac{1}{4}\right) = \frac{-1}{8\mu}\times\frac{\partial P}{\partial x}t^2$$

Average velocity $\bar{u}$, $$\bar{u}A = \int_o^t u(1\times dy)$$

From equation (iii), we have $$\bar{u} = \frac{1}{A}\int_o^t -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)t^2\left(\frac{y}{t}-\frac{y^2}{t^2}\right)dy$$

$$= \frac{1}{t\times 1}\left[-\frac{1}{2\mu}\frac{\partial P}{\partial x}t^2\right]\int_o^t\left(\frac{y}{t}-\frac{y^2}{t^2}\right)dy$$

$$= -\frac{1}{2\mu}\times\frac{\partial P}{\partial x}t\left[\frac{y^2}{2t}-\frac{y^3}{3t^2}\right]_o^t = -\frac{1}{2\mu}\frac{\partial P}{\partial x}\times t^2\left[\frac{1}{2}-\frac{1}{3}\right]$$

or, $$\bar{u} = -\frac{1}{2\mu}\frac{\partial P}{\partial x}\times t^2\times\frac{1}{6} = -\frac{1}{12\mu}\frac{\partial P}{\partial x}t^2$$

Taking ratio of $\bar{u}$ and u_{max}

$$\frac{\bar{u}}{u_{max}} = \frac{-\frac{1}{12\mu}\times\frac{\partial P}{\partial x}t^2}{-\frac{1}{8\mu}\times\frac{\partial P}{\partial x}t^2} = \frac{8}{12} = \frac{2}{3}$$

or, $$\bar{u} = \frac{2}{3}u_{max}$$

$\therefore$ $$u_{max} = \frac{2}{3}\bar{u} = 1.5\bar{u}$$

End of Solution

Q.3 (b) A four-cylinder petrol engine has an output of 52 kW at 2000 rpm. A Morse test is carried out and the brake torque readings are 177, 170, 168 and 174 N-m respectively. For normal running at this speed the specific fuel consumption is 0.364 kg/KW hr. The calorific value of fuel is 44200 kJ/kg. Calculate the mechanical and brake thermal efficiency of the engine.

[20 marks : 2024]

Solution:

Given: P = 52 kW, N = 2000 rpm, SFC = 0.364 kg/kWhr, C.V. = 44200 kJ/kg

$B.T_1$ = Brake torque reading when cylinder 1 is kept off = 177 N-m

$B.T_2$ = 170 N-m, $B.T_3$ = 168 N-m, $B.T_4$ = 174 N-m

Now, Brake torque of engine when all cylinder working (T) = $\frac{60 \times P}{2\pi N}$

or, $$T = \frac{60 \times 52 \times 1000}{2\pi \times 2000} = 248.281\text{ N-m}$$

Now, Indicated torque of first cylinder ($I.T_1$) = T − BT_1 = 248.281 − 177 = 71.28 N-m.

Similarly, $I.T_2$ = T − BT_2 = 248.281 − 170 = 78.28 N-m

$I.T_3$ = T − BT_3 = 248.281 − 168 = 80.281 N-m

$I.T_4$ = T − BT_4 = 248.281 − 174 = 74.28 N-m

Net indicated torque when all cylinder working (I.T.)

= $I.T_1 + I.T_2 + I.T_3 + I.T_4$

= 304.122 N-m

So, Indicated power of engine (I.P.) = $\frac{2\pi N \times I.T.}{60}$

or, $$I.P. = \frac{2 \times \pi \times 2000 \times 304.122}{60} = 63.695\text{ kW}$$

$\therefore$ Mechanical efficiency $(\eta_m) = \frac{B.P.}{I.P.} = \frac{52}{63.695} = 0.8163$

or, $\% \ \eta_m$ = 81.63% **Ans.**

We know, $$SFC = \frac{\text{Fuel consumption } (\dot{m}_f)}{B.P.}$$

$$0.364 = \frac{\dot{m}_f}{52}$$

or, $\dot{m}_f$ = 18.928 kg/hr

$\therefore$ Brake thermal efficiency $(\eta_{bth}) = \frac{B.P.}{\dot{m}_f \times C.V.}$

or,
$$\eta_{bth} = \frac{52 \times 3600}{18.928 \times 44200} = 0.2237$$
$$\eta_{bth} = 22.37\%$$ Ans.

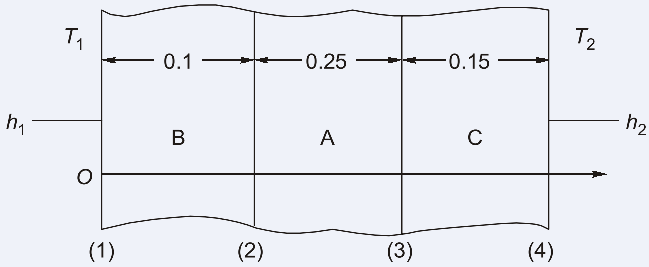
End of Solution

Q.3 (c) A heat generating slab A (thickness = 0.25 m, thermal conductivity K_A = 15 W/m°C) is sandwiched between two other slab B (thickness = 0.1 m, K_B = 10 W/m°C) and C (thickness = 0.15 m, K_C = 30 W/m°C) as shown in the figure. There is no heat generation in slab B or C. The temperature distribution in slab A is known to be $T_A = 90 + 4500x - 11000\,x^2$, where T is in °C and x is the distance in metres from the left surface of B. The wall B is in contact with a fluid at temperature T_1 = 40 °C, the wall heat transfer coefficient being h_1. Similarly, the free surface of C losses heat to a medium at temperature 35 °C, and the surface heat transfer coefficient is h_2. Assume steady state condition.

(i) Calculate the temperature at the surfaces of slab A. What is the maximum temperature in A and where does it occur?

(ii) Determine the temperature gradient at both the surfaces of each of the slabs A, B and C.

(iii) Find the temperature profiles in slabs B and C. Also, calculate the value of the heat transfer coefficients h_1 and h_2.



[20 marks : 2024]

Solution:

Given: For slab A:

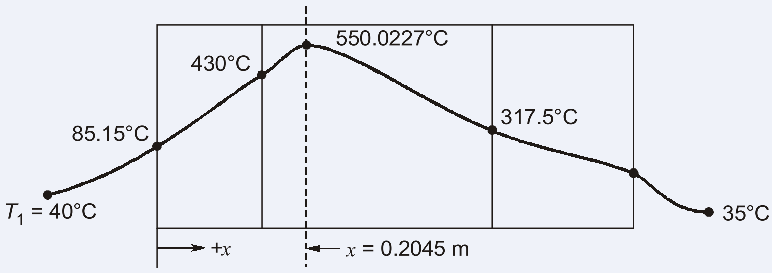
k_A = 15 W/m°C, t_A = 0.25 m, $T_A = (90 + 4500x - 11000x^2)$°C

For slab B

k_B = 10 W/m°C, t_B = 0.1 m

For slab C

k_C = 30 W/m°C, t_C = 0.15 m



(i) Calculation for surface temperature,

$$T_A = 90 + 4500x - 11000x^2$$

$$T_{A,\text{ left face}} = T(x = 0.1) = 90 + 4500 \times 0.1 - 11000\ (0.1)^2$$

$$= 430°\text{C} \qquad \textbf{Ans.}$$

$$T_{A,\text{ right face}} = T(x = 0.35) = 90 + 4500 \times 0.35 - 11000\ (0.35)^2$$

$$= 317.5°\text{C} \qquad \textbf{Ans.}$$

For maximum temperature in slab A

$$\frac{dT}{dx} = 0$$

$\Rightarrow$ $$0 = 4500 - 22000x$$

$\therefore$ $$x = 0.2045 \text{ m}$$

$$T(x = 0.2045) = T_{max}$$

$$T_{max} = 90 + 4500\ (0.2045) - 11000\ (0.2045)^2$$

$$= 550.0227°\text{C} \qquad \textbf{Ans.}$$

(ii) Applying steady state 1-D heat conduction equation for slab A

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k_A}$$

or, $$\dot{q}_A = -k_A\frac{d^2T}{dx^2} = -15 \times (-22000) = 330000 \text{ W/m}^3$$

Portion of heat flowing through slab B

$$330000\ (A)\ (0.1045) = q_B$$

or, $$\frac{q_B}{A} = 34485 \text{ W/m}^2$$

$$\frac{q_B}{A} = -\left(k\frac{dT}{dx}\right)_B$$

$$\left(\frac{dT}{dx}\right)_B = \frac{34485}{10} = 3448.5° \text{ C/m} \qquad \textbf{Ans.}$$

Position of heat transfer through C

$$\dot{q}_A \times A(0.1455) = q_C$$

or, $$\frac{\dot{q}_C}{A} = 330000 \times 0.1455 = 48015 \text{ W/m}^2$$

$$\frac{q_C}{A} = \left(-k\frac{dT}{dx}\right)_C$$

$\Rightarrow$ $$-\frac{48015}{30} = \left(\frac{dT}{dx}\right)_C$$

$$\left(\frac{dT}{dx}\right)_C = -1600.5°\text{C/m} \qquad \textbf{Ans.}$$

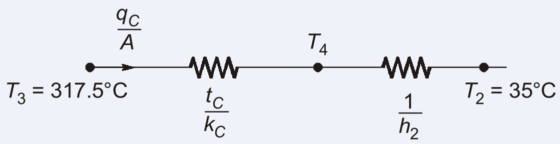
Temperature gradient for slab A.

$$\frac{dT}{dx} = 4500 - 22000x \text{ °C/m} \qquad \textbf{Ans.}$$

$$\left.\frac{dT}{dx}\right|_{A,k=0.1\,\text{m}} = 2300°\text{C/m}$$

$$\left.\frac{dT}{dx}\right|_{B,k=0.35\,\text{m}} = -3200°\text{C/m}$$

(iii)



$$48015 = \frac{T_3 - T_2}{\frac{t_C}{k_C} + \frac{1}{h_2}} = \frac{317.5° - 35}{\frac{0.15}{30} + \frac{1}{h_2}}$$

or, $h_2 = 1131.7619 \text{ W/m}^2\text{K}$ **Ans.**

We can write, $\frac{k_C}{t_C} \times (T_3 - T_4) = h_2(T_4 - T_2)$

or, $\frac{30}{0.15}(317.5 - T_4) = 1131.7619\,(T_4 - 35)$

$$T_4 = 77.425°\text{C}$$

In slab 'B' there is no heat generation,

∴ 1-D Heat transfer equation becomes

$$\frac{d^2T_B}{dx^2} = 0$$

or, $\frac{dT_B}{dx} = C_1$

or, $T_B(x) = C_1 x + C_2$ ... (i)

Using boundary condition,

At $x = 0$, $T_B(0) = T_{BL} = 85.15°\text{C}$

At $x = 0.1$, $T_B(0) = T_{AL} = 430°\text{C}$

Putting in equation (i)

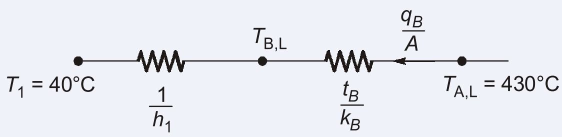
$$C_2 = 85.15$$

and $430°\text{C} = C_1 \times 0.1 + 85.15$

or, $C_1 = 3448.5$

∴ $T_B(x) = 3448.5x + 85.15$ **Ans.**

$$\frac{q_B}{A} = 34485 \text{ W/m}^2$$



$$\therefore \qquad \frac{1}{h_1} = \frac{T_{A,L} - T_1}{\left(\frac{q_B}{A}\right)} - \frac{t_B}{k_B} = \frac{430 - 40}{34485} - \frac{0.1}{10}$$

$$\therefore \qquad h_1 = 763.7873 \text{ W/m}^2\text{K} \qquad \textbf{Ans.}$$

We can write,
$$\frac{q_B}{A} = +k_B\left(\frac{430 - T_{BL}}{0.1}\right)$$

$$34485.85 = 10 \times \left(\frac{430 - T_{BL}}{0.1}\right)$$

$$T_{BL} = 85.15°\text{C}$$

In slab 'C' also there is no heat generation,

∴ 1-D heat transfer equation becomes

$$\frac{d^2T_C}{dx^2} = 0$$

or,
$$\frac{dT_C}{dx} = C_3$$

or,
$$T_C(x) = C_3x + C_4 \qquad \text{... (ii)}$$

Using boundary condition,

At $x = 0.35$, $T_C(0.35) = 317.5°\text{C}$

At $x = 0.5$, $T_C(0.5) = 77.425°\text{C}$

Putting in equation (ii)

$$317.5 = C_3 \times 85.15 \times C_4$$

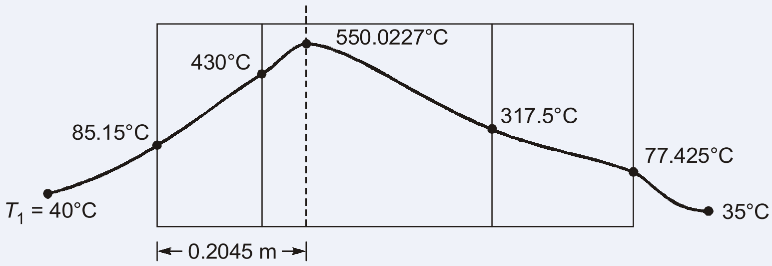
$$77.425 = C_3 \times 0.5 + C_4$$

On solving we get,

$$C_3 = -1600.5$$

$$C_4 = 877.675$$

Now,
$$T_C(x) = -1600.5x + 877.675 \qquad \textbf{Ans.}$$



End of Solution

Q.4 (a) (i) Show that the streamlines and equipotential lines form a net of mutually perpendicular lines.

(ii) Why do we use fins in heat exchanging devices? How do we define fin efficiency?

Give a few examples of different finned surfaces with sketches.

[20 marks : 2024]

Solution:

(i)

Lines of constant (ϕ) potential function are called equipotential lines and lines of constant (ψ) (stream function) are called streamlines.

For a two dimensional flow:

- Velocity in terms of potential function is represented by

$$u = \frac{-\partial \phi}{\partial x},\; v = \frac{-\partial \phi}{\partial y}$$

- Velocity in terms of stream function is represented by

$$u = \frac{-\partial \psi}{\partial y},\; v = \frac{\partial \psi}{\partial x}$$

The differential of potential function is

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$$

$$d\phi = -udx + (-v)dy$$

For equipotential lines ϕ is constant, so $d\phi = 0$

$$0 = -udx + (-v)dy$$

$$\left.\frac{dy}{dx}\right|_{\phi=c} = \frac{-u}{v}$$

Slope of equipotential lines $(m_1) = \left.\frac{dy}{dx}\right|_{\phi=c} = \frac{-u}{v}$

The differential of stream function:

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = vdx + (-u)dy$$

For stream lines ψ = Constant and $d\psi = 0$

$$0 = vdx - udy$$

$$\left.\frac{dy}{dx}\right|_{\psi=c} = \frac{v}{u}$$

Slope of constant function streamlines $(m_2) = \left.\frac{dy}{dx}\right|_{\psi=c} = \frac{v}{u}$

Since, $\quad m_1 \times m_2 = \frac{-u}{v} \times \frac{v}{u} = -1$

Equipotential lines and stream lines are orthogonal to each other.

(ii)

The heat transfer rate may be increased by increasing the surface area across which convection occurs. This may be done by using fins that extend from the wall into the surrounding fluid. The thermal conductivity of the fin material has a strong effect on the temperature distribution along the fin and thus the degree to which the heat transfer rate is enhanced.

Fin efficiency: In the limiting case of zero thermal resistance or infinite conductivity, the temperature of the fin will be uniform at the base value of T_b. The heat transfer from the fin will be maximum in this case and can be expressed as

$$\dot{Q}_{fin,\max} = hA_{\text{fin}}(T_b - T_\infty)$$

In reality, however, the temperature of the fin will drop along the fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference toward the fin tip as shown in figure below. To account for the effect of this decrease in temperature on heat transfer, we define a fin efficiency as the ratio of actual heat dissipated by the fin to the maximum heat transferred by the pin if the whole fin has temperature equal to the base temperature of the fin.

$$\eta_{\text{fin}} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin, if the entire fin were at base temperature}}$$

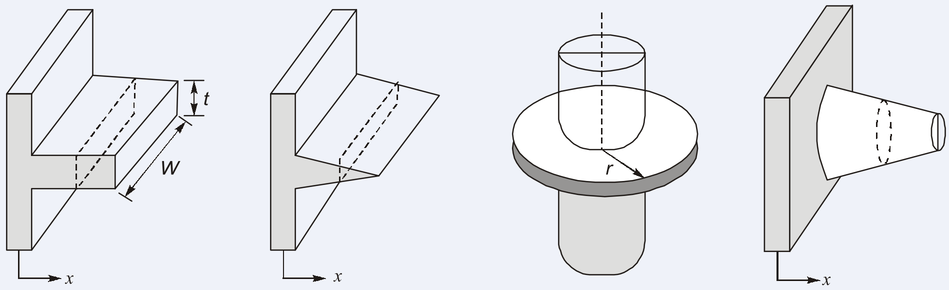
For the cases of constant cross section of very long fins and fins with adiabatic tips, the efficiency can be expressed as

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin,max}}} = \frac{\sqrt{hPkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hP}} = \frac{1}{mL}$$

$$\text{and } \eta_{\text{adiabatic tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin,max}}} = \frac{\sqrt{hPkA_c}\,(T_b - T_\infty)\tanh mL}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{\tanh mL}{mL}$$

since $A_{\text{fin}} = PL$ for fins with constant cross section.

Figure shows below different fin configurations. A straight fin is any extended surface that is attached to a plane wall. It may be of uniform cross-sectional area, or its cross-sectional may vary which with the distance x from the wall. An annular fin is one that is circumferentially attached to a cylinder. A pin fin or spine is an extended surface of circular cross-sections. Pin fins may also be of uniform or non-uniform cross-section.



(a) Straight fin of uniform cross-section
(b) Straight fin of non-uniform cross-section
(c) finAnnular
(d) Pin fin

Fin configurations

End of Solution

Q.4 (b) What is a polytropic process?

Give an expression between temperature and volume between two states of a polytropic process.

Find out entropy change between the two states of an ideal gas.

A mass of 0.25 kg of an ideal gas has a pressure of 300 kPa, a temperature of 90°C and a volume of 0.07 m^3. The gas undergoes an irreversible adiabatic process to a final pressure of 300 kPa and final volume of 0.10 m^3, during which the work done on the gas is 25 kJ. Evaluate C_P and C_V of the gas and the increase in entropy of the gas.

[20 marks : 2024]

Solution:

Polytropic Process

- Any general process in nature can be considered as a polytropic process.
- The process is represented by the equation $PV^n = C$, where, n is known as polytropic index whose value can vary from $-\infty$ to ∞.
- The expression for work done in case of polytropic process can be derived in a similar way as for adiabatic process and it is given by,

$$\boxed{W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1}}$$

And, for an ideal gas, $$W_{1-2} = \frac{mR(T_1 - T_2)}{n-1}$$

Polytropic Process: For this process, $PV^n = C$

Now, for ideal gas, $P^{1-n}\,T^n = C$ and $TV^{n-1} = C$

Expression between temperature and volume.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

Given: $m = 0.25$ kg, $P_1 = 300$ kPa, $T_1 = 90°\text{C} = 363$ K, $V_1 = 0.07$ m^3, $P_2 = 300$ kPa, $V_2 = 0.10$ m^3

$$W_{1-2} = -25 \text{ kJ}$$

$$P_1V_1 = mRT_1$$

$$R = \frac{P_1V_1}{mT_1} = \frac{300 \times 0.07}{0.25 \times 363} = 0.2314 \text{ kJ/kgK}$$

$$\frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2}$$

$$\therefore \quad T_2 = 518.57 \text{ K}$$

Applying energy conservation

$$\cancel{\delta Q}^{\,o} = du + \delta W$$

$$-\delta W = du$$

$$-W_{1-2} = mC_V(T_2 - T_1)$$

$$C_V = \frac{-W_{1-2}}{m(T_2 - T_1)} = -\frac{(-25)}{0.25 \times (518.57 - 363)} \quad \text{Ans.}$$

$\therefore \quad C_V = 0.6428$ kJ/kgK

$C_P = C_V + R = 0.6428 + 0.2314$

$= 0.8742$ kJ/kgK **Ans.**

Now,
$$(\Delta s)_{2-1} = m\left[C_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right] = mC_P \ln\frac{T_2}{T_1}$$
$$= 0.25 \times 0.8742 \times \ln\left(\frac{518.57}{363}\right)$$

or, $(\Delta s)_{2-1} = 0.07795$ kJ/K **Ans.**

End of Solution

Q.4 (c) What is catalytic converter package? Show the arrangement of catalytic converter package with the help of a diagram for HC, CO and NO_x. Explain its functioning also.
[20 marks : 2024]

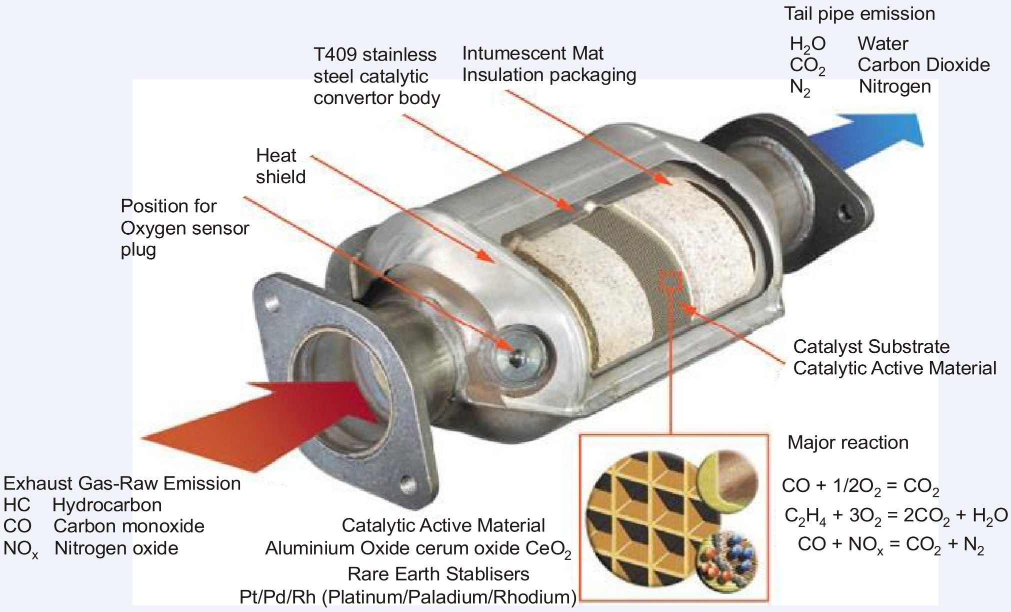
Solution:

A catalytic converter is an exhaust emission control device that reduces toxic gases and pollutants in exhaust gas from an internal combustion engine into less-toxic pollutants by catalyzing a redox reaction (an oxidation and a reduction reaction). Catalytic converters are usually used with internal combustion engines fueled by either gasoline or diesel. A catalytic converter is a large metal box, bolted to the underside of car, that has two pipes coming out of it. One of them (the converter's "input") is connected to the engine and brings in hot, polluted fumes from the engine's cylinders (where the fuel burns and produces power). The second pipe (the converter's "output") is connected to the tail pipe (exhaust). As the gases from the engine fumes blow over the catalyst, chemical reactions take place on its surface, breaking apart the pollutant gases and converting them into other gases that are safe enough to blow harmlessly out into the air.

There are two types of catalytic convertors:

- Two-way catalytic convertor (oxidation)
- Three-way catalytic convertor (oxidation-reduction)

The three-way catalyst, is generally a multicomponent material, containing the precious metals rhodium, platinum and (to a lesser extent) palladium, ceria (CeO_2), γ-alumina (Al_2O_3), and other metal oxides. It typically consists of a ceramic monolith of cordierite ($2Mg.2Al_2O_3.\ 5SiO_2$) with strong porous walls enclosing an array of parallel channels. This design allows a high rate of flow of exhaust gases. Cordierite is used because it can withstand the high temperatures in the exhaust, and the high rate of thermal expansion encountered when the engine first starts – typically, the exhaust gas temperature can reach several hundred degrees in less than a minute. Metallic monoliths are also used, particularly for small converters, but these are more expensive.



A catalytic Convertor

The catalytic converter, in a metal canister, is placed in the exhaust system of the vehicle. As the exhaust gases pass through it, they flow through the channels in the ceramic monolith, where they encounter the particles of alumina impregnated with the metal catalysts.

To achieve a large surface area for catalysis, the internal surfaces of the monolith are covered with a thin coating (30–50 μm) of a highly porous material, known as the wash coat. The total surface area is now equivalent to that of about two or three football pitches. The wash coat generally consists of alumina (70–85%) with a large surface area, with oxides, such as BaO, added as structural promoters (stabilisers to maintain surface area) and others, for example CeO_2, as chemical promoters. This system becomes the support for the precious metal components (Pt, Pd and Rh). These metals constitute only a small fraction (1-2%) of the total mass of the wash coat, but they are present in a highly dispersed form. They are generally applied by deposition from solution, although they may instead be introduced during formation of the wash coat itself.

$$CO + \frac{1}{2}O_2 \rightarrow CO_2$$

$$C_xH_y + zO_2 \rightarrow xCO_2 + yH_2O$$

Palladium and platinum promote the oxidation of CO and HC as in above equation, with platinum especially active in the hydrocarbon reaction. Rhodium promotes the reaction of NO_x in one or more of the following reactions:

$$NO + CO \rightarrow \frac{1}{2}N_2 + CO_2$$

$$2NO + 5CO + 3H_2O \rightarrow 2NH_3 + 5CO_2$$

$$2NO + 5CO \rightarrow N_2O + CO_2$$

$$NO + H_2 \rightarrow \frac{1}{2}N_2 + H_2O$$

$$2NO + 5H_2 \rightarrow 2NH_3 + 2H_2O$$
$$2NO + H_2 \rightarrow N_2O + H_2O$$

Also often used in cerium oxide, which promotes the so-called water gas shift

$$CO + H_2O \rightarrow CO_2 + H_2$$

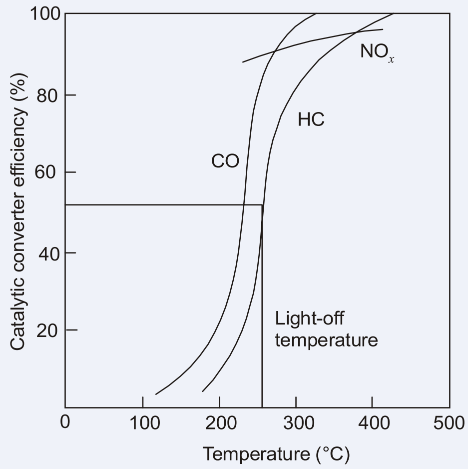


Figure: Conversion efficiency of catalytic converters as a function of converter temperature

They reduces CO by using water vapor as an oxidant instead of O_2, which is very important when the engine is running rich.

The figure given above shows that the efficiency of a catalytic converter is very much dependent on temperature. When a converter in good working order is operating at a fully warmed temperature of 400°C or above, it will remove 98-99% of CO, 95% of NO_x, and more than 95% of HC from exhaust flow emissions.

End of Solution

SECTION : B

Q.5 (a) (i) What is the significance of specific speed of a centrifugal pump?

(ii) A centrifugal pump operates at its optimal efficiency and delivers 3 cubic metre per second over a height of 22 m. The pump has a 36 cm diameter impeller and rotates at 3250 rpm. Compute the specific speed of the pump

(1) in terms of discharge, and

(2) in terms of power, if maximum efficiency of the pump is 80%.

[12 marks : 2024]

Solution:

(i)

The specific speed (N_s) of a centrifugal pump is characterizes the pump's design and performance. It is defined as:

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

where, N = Rotational speed (RPM)
Q = Flow rate (m^3/s)
H = Head (m)

The significance of specific speed lies in its' ability to:

1. Predict the pump's behavious under different operating condition.
2. Classify pumps into different design categories.
3. Facilitate comparisin and selection of pumps for specific applications.
4. Guide design improvement and optimization.

(ii)

$Q = 3\ m^3/s$; $H = 22$ m; $D_2 = 0.36$ m; $N = 3250$ rpm

$$\text{Specific speed, } N_s = \frac{N\sqrt{Q}}{H^{3/4}} = \frac{3250\times\sqrt{3}}{(22)^{3/4}}$$

$$N_s = 554.15 \qquad \text{Ans.(i)}$$

Now
$$\eta_{mano} = \frac{P}{\rho g Q H_m}$$

$\therefore$
$$H_m = \frac{P}{\rho g Q \eta_{mano}}$$

$\therefore$ Specific speed,
$$N_s = \frac{3250\times\sqrt{3}}{\left(\dfrac{P}{10^3\times 9.81\times 3\times 0.8}\right)^{3/4}} = \frac{10699270.69}{P^{3/4}} \qquad \text{Ans.(ii)}$$

End of Solution

Q.5 (b) A shell and coil type of evaporator is to be designed for a refrigerator. Coil is bare tube of copper. Refrigerant flows inside the tube and shell side water in stagnant condition. Coil is maintained at –5°C. Heat transfer coefficient on water side is 4100 W/m²K. The load on the evaporator is 2.8 kW. The LMTD is 18°C. The tube side heat transfer coefficient is given by

$$h_i = 0.555\left[\frac{9.81(\rho_f - \rho_g)k_f^3\cdot h_{fg}}{\mu_f\cdot \text{evaporator temp}\times D_i}\right]$$

The properties of refrigerant are as given below at –5°C.

Dynamic viscosity = 0.000191 kg/ms, Density of liquid = 1136 kg/m³

Density of vapour = 14.43 kg/m³, Tube inner diameter D_i = 0.005715 m

and thickness of tube is 0.001905 m.

Thermal conductivity of refrigerant = 0.0857 W/mK.

Thermal conductivity of tube is 400 W/mK.

Thermal conductivity of ice is 2.25 W/mK.

Latent heat of vapourization is 173.1 kJ/kg.

Find the tube length required.

[12 marks : 2024]

Solution:

Given : h_0 = 4100 W/m^{2}K; Q = 2.8 kW; LMTD = 18°C,

$$D_o = D_i + 2t = 0.005715 + 2 \times 0.001905$$
$$= 9.525 \times 10^{-3} \text{ m}$$

Inside heat transfer coefficient,

$$h_i = 0.555\left[\frac{9.81(\rho_f - \rho_g)K_f^3 h_{fg}}{\mu_f \cdot T_{evap} \cdot D_i}\right] = 0.555\left[\frac{9.81(1136-14.43)\times 0.0857^3 \times 173.1\times 10^3}{0.000191\times 268\times 0.005715}\right]$$

$$h_i = 2.274 \times 10^9 \text{ W/m}^2\text{K}$$

Overall heat transfer coefficient based on outer area

$$\frac{1}{U_0A_0} = \frac{1}{h_0A_0} + \frac{\ln\frac{r_0}{r_i}}{2\pi kL} + \frac{1}{h_iA_i}$$

or,

$$\frac{1}{U_0} = \frac{1}{h_0} + \frac{\ln\frac{r_0}{r_i}}{2\pi kL} \times A_0 + \frac{A_0}{h_iA_i}$$

$$\frac{1}{U_0} = \frac{1}{h_0} + \frac{r_0}{k} \times \ln\frac{r_0}{r_i} + \frac{r_0}{r_i \times h_i}$$

$$\frac{1}{U_0} = \frac{1}{4100} + \frac{4.7625\times 10^{-3}}{400}\ln\frac{(4.7625\times 10^{-3})}{2.8575\times 10^{-3}} + \frac{(4.7625\times 10^{-3})}{2.8575\times 10^{-3}\times 2.274\times 10^9}$$

$$\frac{1}{U_0} = 2.5 \times 10^{-4}$$

or $$U_0 = 4000 \text{ W/m}^2\text{K}$$

$\therefore$ $$Q = UA_0LMTD$$

$$2.8 \times 10^3 = 4000 \times (2\pi \times 4.7625 \times 10^{-3}) \times L \times 18$$

$\therefore$ $$L = 1.3 \text{ m}$$ **Ans.**

End of Solution

Q.5 (c) Air at DBT 35°C and WBT 23°C is passed over a coil and air comes out of the coil at DBT 25°C and WBT 20°C. Show the procedure on skeletal psychrometric chart.

(i) Mention the name of the process taking place.

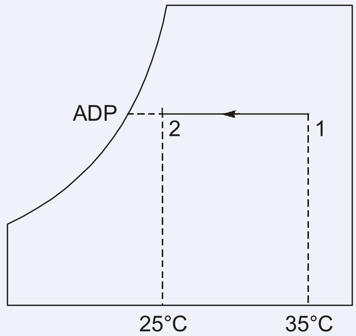
(ii) Find the coil surface temperature.

(iii) Find the bypass factor of the coil.

[12 marks : 2024]

Solution:

Refer figure,



- It is a sensible cooling process.
- From Psychrometric chart, coil surface temperature, we get

$$t_{ADP} = 17.5°C \qquad \textbf{Ans.}$$

By pass factor,

$$BPF = \frac{t_2 - t_{ADP}}{t_1 - t_{ADP}} = \frac{25 - 17.5}{35 - 17.5} = 0.428 \qquad \textbf{Ans.}$$

End of Solution

Q.5 (d) (i) What are the performance parameters of cooling towers? Define them.

(ii) What are the main advantages and disadvantages of mechanical draught cooling towers?

[12 marks : 2024]

Solution:

(i)

Performance Parameters : A cooling tower is characterized by three performance parameters namely (a) approach, (b) range, and (c) cooling efficiency. The approach temperature (A) is defined as the difference between the exit temperature of cooling water and the wet bulb temperature of the ambient air. Thus,

$$A = T_{c_2} - T_{wb}$$

where, T_{wb} = the wet bulb temperature of air; T_{c_2} = cooling water exit temperature from cooling tower.

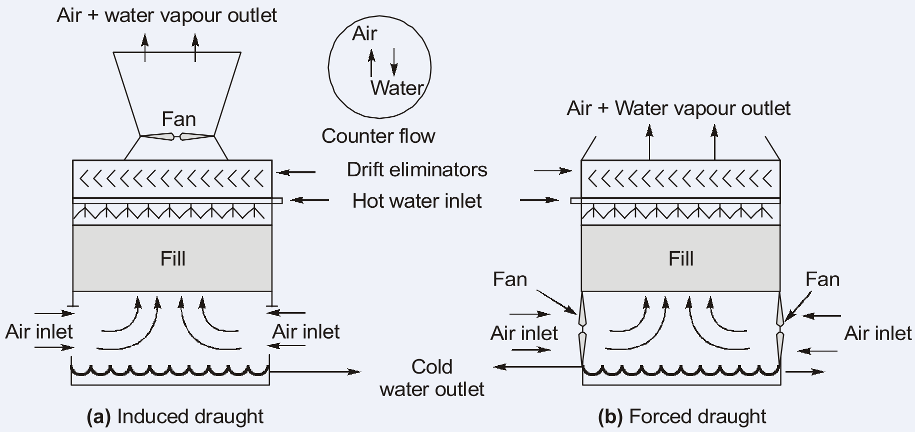
This approach (A) varies from 6 to 8°C.

The cooling range (R) is defined as the difference in temperature of the incoming warm water (T_{c_1}) and exiting cooler water (T_{c_2}). Thus,

$$\eta_{cooling} = \frac{\text{Actual cooling}}{\text{Maximum cooling possible}} = \frac{T_{c_1} - T_{c_2}}{T_{c_1} - T_{wb}}$$

(ii)

Mechanical Draught Cooling Towers: Figure shows the schematic of mechanical draught cooling tower. In this case, the air is moved by one or more mechanically driven fans. The fan could be of the forced-draught (FD) type or induced draught (ID) type. The FD fan is mounted on the lower sides to force air into the tower while ID fan is located on the top of the tower. Though, FD fan is thermodynamically superior as it handles cold air but it has shown some disadvantages because of air distribution problems, leakage, recirculation of the hot and moist exit air back to the tower and frost accumulation at fan inlets during winter operation. As a result, the majority of mechanical-draught cooling towers for utility application are therefore of the induced-draught (ID) type. In this case, air enters the sides of the tower through large openings at low velocity and passes through the fill. The fan located at the top of the tower exhausts the hot, humid air to the atmosphere. Induced-draught cooling towers are usually in the form of multi-cell with a number of fan stacks on the top and built in various arrangement like rectangular, octagonal, circular, etc. The fans are propeller type and driven by electric motor The blades of fans are usually made of cast aluminium, stainless steel or fibre glass to safeguard against corrosion.



Mechanical Draught (Induced and forced) Counter-flow Cooling Tower

The advantages of mechanical-draught cooling towers include the assurance of moving the required quantity of air at all loads and climatic conditions, low initial capital costs, and low physical profile. The main disadvantage include power consumption, operating and maintenance costs and greater noise generated from fans.

End of Solution

Q.5 (e) With the help of sketches, define Horizontal Axis Wind Turbine (HAWT) and Vertical Axis Wind Turbine (VAWT). Compare HAWT and VAWT in respect of

(i) Efficiency (ii) Space requirement

(iii) Cost (iv) Design and installation

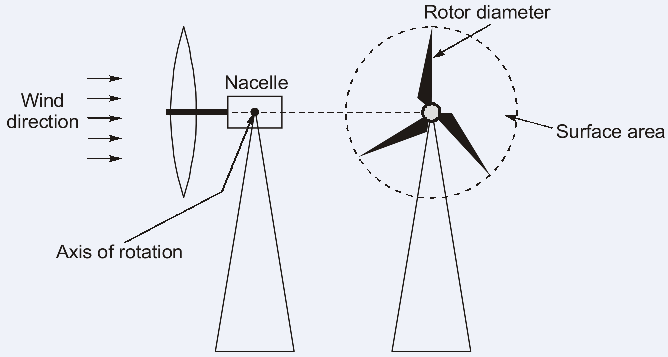
(v) Noise (vi) Self-starting

[12 marks : 2024]

Solution:

Horizontal Axis Wind Turbine (HAWT):

When the axis of rotation is parallel to the air stream (i.e. horizontal). The rotor is attached to the nacelle, and mounted at the top of a tower. It contains rotor brakes, gearbox, generator and electrical switch gear and control. Brakes are provided to stop the rotor when power generation is not desired and also at high wind conditon. Gearbox set up the shaft rpm to suit the generator. The generated electrical power is conducted to ground terminals through a cable.



Vertical Axis Wind Turbine (VAWT):

When the axis of rotation is vertical to the air stream (i.e. vertical) e.g. savonius rotor. These are in under develping stage. The main attraction of VAWT are:

(i) It can accept wind from any direction, eliminating the need of yaw contol.

(ii) The gearbox, generator, etc. are located at the ground, thus eliminating the heavy nacelle at the top of tower.

(iii) The inspection and maintenance also get easier.

(iv) It also reduces overall cost.

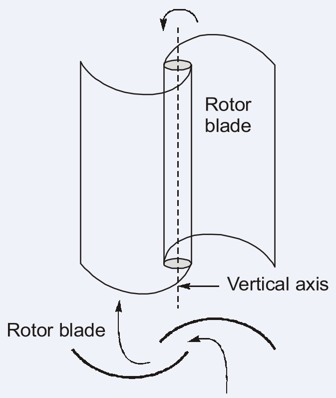


Figure: Savonious vertical-axis rotor

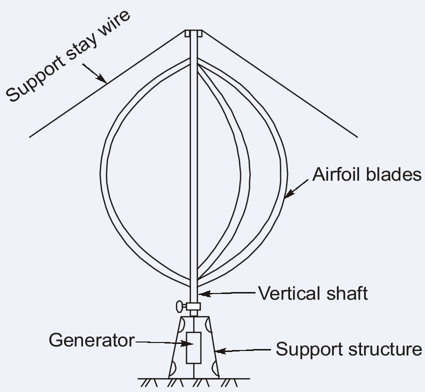


Figure: Darrieus rotor

A Vertical-axis Wind Turbine (VAWT) features a main rotor shaft arranged transversely to the wind and perpendicular to the ground. This design, also known as a “transverse axis wind turbine” or “cross-flow wind turbine,” eliminates the need for orientation mechanisms to face the wind, simplifying its design and reducing the need for wind-sensing equipment, unlike the more common horizontal-axis wind turbines.

Horizontal-axis wind turbines (HAWTs) dominate global use, chiefly known for their efficiency and high power output. These turbines feature a rotor with aerodynamically designed blades, which are parallel to the ground, allowing them to capture wind energy effectively.

	VAWT	HAWT
Efficiency	Lower practical efficiency due to aerodynamic drag, higher under ideal conditions	Generally higher efficiency, capturing 40-50% of wind energy
Design and Installation	Simpler, less costly, easier installation	More complex, costly due to need for precise alignment and substantial structural support
Noise Levels	High	Low
Cost	Less expensive because their design and installation is quite simple	More expensive due their complex design and installation
Self-start	Many of VAWT requires some self-start mechanism (as they are generally unable to produce sufficient aerodynamic starting torque)	It does not requires any self-starting mechanism.
Space requirement	Low	High

End of Solution

Q.6 (a) (i) Explain the working of counterflow Ranque-Hilsch tube refrigeration system with the help of a sketch.
Also define the following terms:
(I) Cold mass flow ratio
(II) Normalised temperature drop
(III) Cold orifice diameter ratio
(IV) Isentropic efficiency

(ii) Explain how the critical temperature of a refrigerant affects the performance of a refrigeration system represented on T-s diagram.

[20 marks : 2024]

Solution:

(i)

The vortex tube or Ranque-Hilsch tube, consists of a straight length of a tube with a concentric orifice located in a diaphragm near one end and a nozzle located tangentially near the outer radius adjacent to the orifice plate (shown in figure). Compressed gas enters the tube tangentially through a nozzle forming a vortex kind of motion. The diaphragm prevents leftward motion of the vortex which, therefore, travels towards the righthand side of the tube called the hot end. A hot stream at temperature T_h which is above the temperature of supply, say, T_3 ejects from the hot end through the throttle valve, while the cold stream at temperature T_c below the temperature of supply is received at the cold end through the orifice. The throttle-valve opening controls the temperature and proportion of the cold stream with respect to the hot stream; the larger the throttle valve opening, the lower the temperature of the cold stream and the smaller its fraction and vice versa. The throttle valve is placed sufficiently distant from the nozzle and the diaphragm immediately close to it.

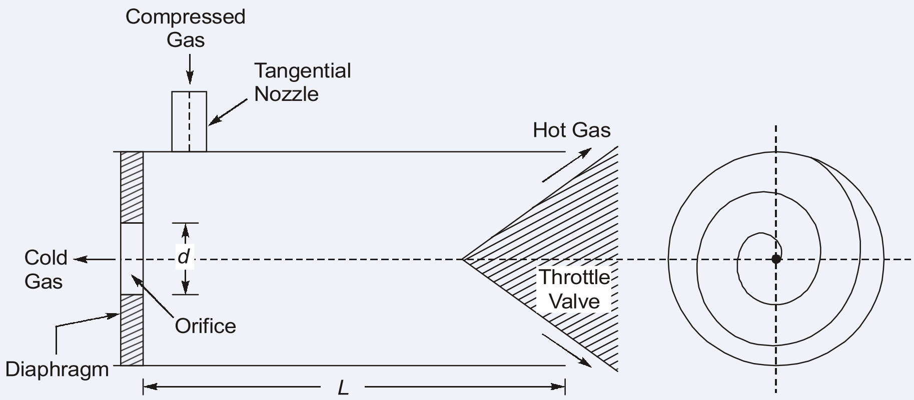


Fig. : Vortex tube

(i) **Cold mass flow ratio:** Cold mass fraction is the most important parameter indicating the vortex tube performance and the temperature/energy separation inside the vortex tube. Cold mass fraction is defined as the ratio of cold air mass flow rate to inlet air mass flow rate. The cold mass fraction can be controlled by the cone valve, which is placed at the hot tube end. This can be expressed as follows:

$$\mu_c = \frac{M_C}{M_i}, \qquad \text{...(i)}$$

where M_c is the mass flow rate of cold air and M_i is the mass flow rate of the entry air.

(ii) **Cold air temperature drop :** Cold air temperature drop or temperature reduction is defined as the difference in temperature between entry air temperature and cold air temperature:

$$\Delta T_c = T_i - T_c \qquad \text{...(ii)}$$

in which T_i is the entry air temperature and T_c is the cold air temperature.

(iii) **Cold orifice diameter ratio :** Cold orifice diameter ratio (β) is defined as the ratio of cold orifice diameter (d) to vortex tube diameter (D):

$$\beta = \frac{d}{D} \qquad \text{...(iii)}$$

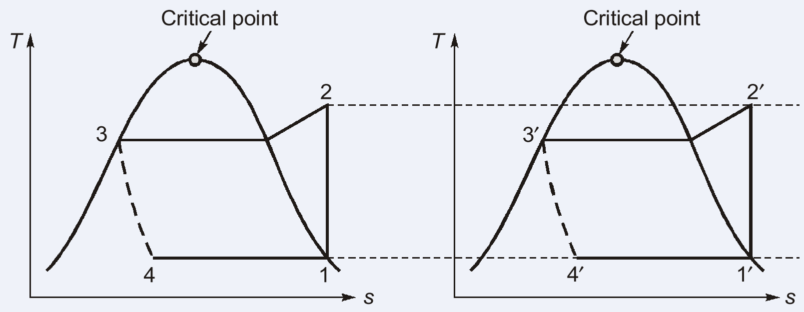
(iv) **Isentropic efficiency :** To calculate the cooling efficiency of the vortex tube, the principle of adiabatic expansion of ideal gas is used. As the air flows into the vortex tube, the expansion in isentropic process occurs. This can be written as follows:

$$\eta_{is} = \frac{T_i - T_C}{T_i\left(1 - \left(\frac{P_a}{P_i}\right)^{(\gamma-1/\gamma)}\right)}, \qquad \text{...(iv)}$$

where η_{is}, P_i, P_a and γ are the isentropic efficiency, inlet air pressure, atmosphere pressure and specific heat ratio, respectively.

(ii)

For high COP, in general, the critical temperature should be very high so that the condenser temperature line on the p-h diagram is far removed from the critical point. This ensures reasonable refrigerating effect which becomes very small if the state of the liquid before expansion is near the critical point. Also, the critical pressure should be low so as to result in low condensing pressure, except for carbon dioxide for which the critical temperature is 31°C for most of the common refrigerants, critical temperature is much above the normal condensing temperature.



Refrigerant 1 **Refrigerant 2**

In the above figure, we can see that both refrigerant but the refrigerant effect for 1 (i.e. temperature ($h_1 - h_4$)) is more than refrigerant effect for 2 (i.e. temperature ($h'_1 - h'_4$)) thus $(COP)_1$, would be more as compared to $(COP)_2$.

$\therefore$ We can say that, refrigerant with more critical temperature will perform better than refrigerant with less critical temperature.

End of Solution

Q.6 (b) In an air conditioning system two streams are mixed adiabatically. One stream is at DBT 15°C and WBT 12°C and the flow rate is 20 m^3/min. The second stream volume flow rate is 30 m^3/min. After mixing the two streams the condition of air is found to be 30°C DBT and 23°C WBT. Find the 2nd stream condition before mixing. Also, find the DBT, WBT, enthalpy, mass flow rate, volume flow rate, specific humidity, and relative humidity of the 2nd stream before mixing. On the skeletal chart, show the procedure to get the 2nd stream condition.

[20 marks : 2024]

Solution:

Given: For the 1st stream, $t_{a_1} = 15°C;\ t_{wbt,1} = 12°C;\ \dot{V}_{a1} = 20\ m^3/min$

From Psychrometric chart, $\omega_1 = 0.0075$ kg/kg of da, $\nu_1 = 0.825\ m^3/kg$;
$h_1 = 34$ kJ/kg of da

For the 2nd stream, $\dot{V}_2 = 30\ m^3/min$

For the final mixture, $t_{a,3} = 30°C;\ t_{wbt,3} = 23°C$

From Psychrometric chart, $\omega_3 = 0.0148$ kg/kg of da; $h_3 = 69$ kJ/kg of da;
$v_3 = 0.88\ m^3/kg$

Assuming incompressible flow

$$\dot{V}_1 + \dot{V}_2 = \dot{V}_3$$

$$20 + 30 = \dot{V}_3$$

or, $$\dot{V}_3 = 50 \text{ m}^3/\text{min}$$

$$\dot{m}_3 = \frac{\dot{V}_3}{v_3} = \frac{50}{0.88} = 56.81 \text{ kg/min}$$

and, $$\dot{m}_1 = \frac{V_1}{v_1} = \frac{20}{0.825} = 24.24 \text{ kg/min}$$

$\therefore$ $$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 32.57 \text{ kg/min}$$

Applying energy balance,

$\because$ $$(\dot{m}h)_1 + (\dot{m}h)_2 = (\dot{m}h)_3$$

$\Rightarrow$ $$h_2 = \frac{\dot{m}_3 h_3 - \dot{m}_1 h_1}{\dot{m}_2} = \frac{56.81 \times 69 - 24.24 \times 34}{32.57}$$

$\therefore$ $$h_2 = 95.05 \text{ kJ/kg of da}$$ **Ans.**

Conservation of vapour mass

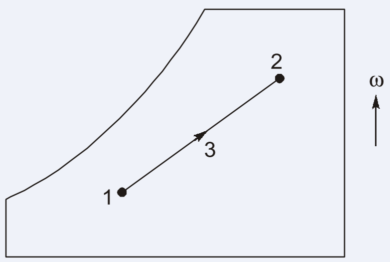
$$\dot{m}_1\omega_1 + \dot{m}_2\omega_2 = \dot{m}_3\omega_3$$

$$\omega_2 = \frac{\dot{m}_3\omega_3 - \dot{m}_1\omega_1}{\dot{m}_2} = \frac{56.81 \times 0.0148 - 24.24 \times 0.0075}{32.57}$$

$$\omega_2 = 0.02023 \text{ kg/kg of da}$$ **Ans.**

From Psychrometric chart, we obtain

$$t_2 = 42.5°\text{C};\ t_{wbt} = 29°\text{C};\ \phi_2 = 39\%$$ **Ans.**



End of Solution

Q.6 (c) In a direct contact counterflow splash-type filled cooling tower of a thermal power station, air enters at the bottom and leaves at the top of the cooling tower. Water is sprayed from the top of the cooling tower. The water enters at 35°C and leaves at 28°C. The ambient conditions are 25°C DBT and 55% RH. Water and air specific heats are 4.1867 and 1 kJ/kgK and mass flow rates are 18.5 and 15.5 kg/s respectively. Density of water is 1000 kg/m^3. Find the following:

(i) Range (ii) Approach

(iii) Cooling capacity of tower (iv) Evaporation loss

Evaporation loss (m^3/hr) is given by

Evaporation loss = 0.00085 × 1.8 × circulation rate (m^3/hr) × ΔT

ΔT = Difference in water entry and exit temperature.

[20 marks : 2024]

Solution:

Given : T_{w_1} = 35°C; T_{w_2} = 28°C; T_{a_1} = 25°C DBT; ϕ_1 = 55% RH; C_{pw} = 4.187 kJ/kgK; C_{pa} = 1 kJ/kgK; $\dot{m}_w$ = 18.5 kg/s; $\dot{m}_a$ = 15.5 kg/s; ρ = 1000 kg/m^3

We know, Range, $R = T_{w_1} - T_{w_2}$

$= 35 - 28 = 7°C$ **Ans.(i)**

Approach, $A = T_{w_2} - T_{WBT}$

From Psychrometric chart,

At 25°C DBT and 55% RH, we have

$T_{WBT} = 19°C$

$\therefore$ $A = 28 - 19 = 9°C$ **Ans.(ii)**

Cooling capacity, $\dot{Q} = \dot{m}_w C_{pw}\left(T_{w_2} - T_{w_2}\right) = 18.5 \times 4.187 \times (35 - 28)$

$= 542.21$ kW **Ans.(iii)**

Evaporation loss = 0.00085 × 1.8 × Circulation rate (m^3/hr) × ΔT

$$= 0.00085 \times 1.8 \times \frac{18.5}{10^3} \times 3600 \times (35 - 28)$$

$= 0.713$ m^3/hr **Ans.**

End of Solution

Q.7 (a) (i) A jet of velocity of 20 m/s strikes a flat plate inclined at 30° with the axis of the jet. If the cross-sectional area of the jet is 20 cm^2, find the force exerted by the jet on the plane. Also calculate the components of the force in the direction normal to the jet. Find also the ratio in which the discharge gets divided after striking the plate. Take density of water as 1000 kg/m^3.

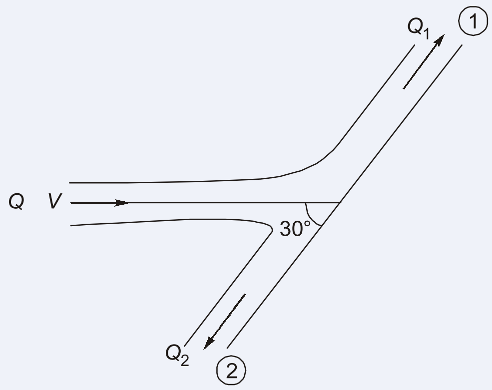
(ii) What is tidal range?

A simple single-basin type tidal power plant has a basin area of 20 km^2. The tide has a range of 8 m. The turbine stops operation when the head on it falls below 2 m. Determine the average power generated during one filling/emptying process in MW if the turbine generator efficiency is 70%. Assume specific gravity of sea water as 1.025.

[10 + 10 marks : 2024]

Solution:

(i)



Given: V_1 = 20 m/s; θ = 30°; a = 20 cm² = 20 × 10⁻⁴ m²; ρ = 1000 kg/m³

$$a = 20 \times 10^{-4}\ \text{m}^2$$

$$\dot{m} = \rho a V_1$$

Force normal to plate. $$(F_N) = \dot{m}V_1\sin\theta - (\dot{m}_1 \times 0 + \dot{m}_2 \times 0) = \rho a V_1^2 \sin\theta$$

Component of force in the direction of jet,

$$(F_x) = F_N \sin\theta = \rho a V_1^2 \sin^2\theta$$
$$= 1000 \times (20 \times 10^{-4}) \times 20^2 \times \sin^2 30° = 200\ \text{N}$$
$$= 200\ \text{N}$$

Component of force normal to jet, $$F_y = F_N \cos\theta = \rho a V_1^2 \sin\theta \cos\theta$$
$$= 1000 \times (20 \times 10^{-4}) \times 20^2 \sin 30° \cos 30°$$
$$= 346.41\ \text{N} \qquad \textbf{Ans.}$$

$$\Sigma F_n = \dot{m}U_{in} - \dot{m}U_{out}$$
$$0 = \rho Q V \cos 30 - [\rho Q_1 V - \rho Q_2 V]$$
$$0 = Q\cos 30 - Q_1 + Q_2$$
$$Q_1 = Q\cos 30 + Q_2 \qquad \text{....(i)}$$

From conservation of mass, $$\rho Q = \rho Q_1 + \rho Q_2$$
$$Q = Q_1 + Q_2 \qquad \text{....(ii)}$$

From (i) and (ii),

$\because$ $$Q - Q_2 = Q\cos 30 + Q_2$$

$\Rightarrow$ $$2Q_2 = Q(1 - \cos 30)$$

$\therefore$ $$Q_2 = \frac{Q}{2}(1 - \cos 30)$$

Put value of Q_2 in equation (i)

$$Q_1 = Q\cos 30° + \frac{Q}{2}(1 - \cos 30°) = Q\left(\cos 30° + \frac{1}{2} - \frac{\cos 30°}{2}\right)$$
$$= Q\left(\frac{1}{2} + \frac{\cos 30°}{2}\right) = \frac{Q}{2}(1 + \cos 30°)$$

$$\frac{Q_1}{Q_2} = \frac{1+\cos 30°}{1-\cos 30°}$$

$$\frac{Q_1}{Q_2} = 13.9282 \qquad \textbf{Ans.}$$

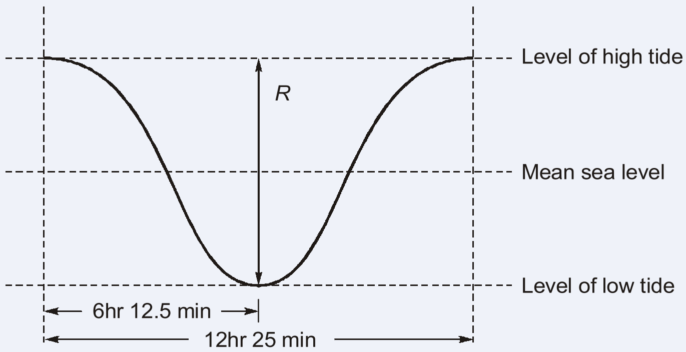
(ii)

Tidal range: The difference between high tide and low tide at a location is called the range of the tide. The tidal range R is defined as:

R = Water elevation at high tide – Water elevation at low tide

Note: The tidal range of moon is around 2.2 times more than that of the sun.

Tidal cycle: Time duration between two high or two low tides is called as one tidal cycle. There are two tidal cycle in a lunar day. One tidal cycle is 12 hr 25 min



$$\text{Energy potential, } E_F = \rho A g \int_2^8 z\,dz = \frac{1}{2}\rho A g\left(8^2 - 2^2\right)$$

where, $A = 20 \times 10^6\ m^2$; $\rho = 1025\ kg/m^3$; $g = 9.81\ m/s^2$ [Given]

$$\text{Average power, } P = \frac{W}{\text{Time}} = \frac{1}{2\times 22350}\times 20\times 10^6\times 1025\times 9.81\times\left(8^2-2^2\right)$$

$= 269.93$ MW [Given]

Turbine generator efficiency is 70%.

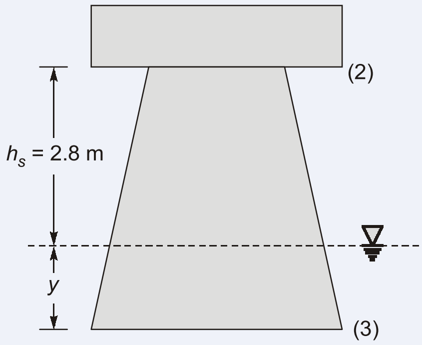
$\therefore \qquad P_{avg} = 269.93 \times 0.7 = 188.95$ MW **Ans.**

End of Solution

Q.7 (b) The draft tube of a Kaplan turbine has inlet diameter 2.5 m and inlet is set at 2.8 m above the tail race. When the turbine develops 2200 metric horsepower under a net head of 6.2 m, it is found that the vacuum gauge fitted at the inlet to the draft tube indicates a negative head of 4 m. If the turbine efficiency is 88%, calculate the draft tube efficiency. Further, if the turbine output is reduced to 50% with the same head, speed and draft tube efficiency, calculate the reading of the vacuum gauge. Assume, atmospheric pressure is 10.3 m of water and specific weight of water is 1000 kg/m^3.

[20 marks : 2024]

Solution:



Given: $d_2 = 2.5$ m, $kP_1 = 2200 \times 735.499$ W $= 1618.0987$ kW, $H = 6.2$ m,

$\eta_{overall} = 0.88$, $\eta_{DT} = ?$, $\frac{p_2}{\rho g} = -4\text{ m}$, $WP_1 = \frac{kP_1}{\eta_o}$, $Q = \left(\frac{kP_1}{\eta_o}\right) \times \left(\frac{1}{\rho g H}\right) = 30.23162\text{ m}^3/\text{s}$

$$a_2 = \frac{\pi}{4} \times (2.5)^2 = 4.9087\text{ m}^2$$

$\therefore$ $$V_2 = 6.1587\text{ m/s}$$

or, $$\frac{V_2^2}{2g} = 1.9332\text{ m}$$

Applying hydrostatic law between free surface and section (3)

$$\frac{p_3}{\rho g} = y$$

Applying energy balance between 2 and 3

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_s + y = \frac{p_3}{\rho g} + \frac{V_3^2}{2g} + \cancel{h_f}^{\,0}$$

or, $$\frac{V_3^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_s = -4 + 1.9332 + 2.8 = 0.7332\text{ m}$$

$$\eta_{draft\ tube} = \frac{\frac{V_2^2}{2g} - \frac{V_3^2}{2g}}{\frac{V_2^2}{2g}} = 0.6207$$ **Ans.**

$$kP_2 = \frac{1}{2} kP_1 = 809.04935\text{ kW}$$ [Given]

Assuming same overall efficiency of turbine,

$\therefore$ $$WP_2 = \frac{kP_2}{\eta_o} = \frac{809.04935}{0.88} = 919.374\text{ kW}$$

We know, $$WP_2 = \rho g Q H = 919.374$$

$\therefore$ $$Q = \frac{919.374 \times 1000}{1000 \times 9.81 \times 6.2}$$

$$Q = 15.1158 \text{ m}^3\text{/s}$$

or,
$$V_2 = \frac{Q}{a_2} = 3.0794 \text{ m/s}$$

and
$$\frac{V_2^2}{2g} = 0.4833 \text{ m}$$

$$\eta_{DT} = \frac{\frac{V_2^2}{2g} - \frac{V_3^2}{2g}}{\frac{V_2^2}{2g}} = 1 - \frac{\left(\frac{V_3^2}{2g}\right)}{\left(\frac{V_2^2}{2g}\right)}$$

$\therefore$
$$\frac{V_3^2}{2g} = \frac{V_2^2}{2g}(1 - \eta_{DT}) = 0.1833 \text{ m}$$

Applying energy balance between 2 and 3

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_s + y = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + \cancel{h_L}^{\,o}$$

$$\frac{P_2}{\rho g} = \frac{V_3^2}{2g} - \frac{V_2^2}{2g} - h_s = -3.09998 \text{ m}$$ **Ans.**

End of Solution

Q.7 (c) (i) Explain with the aid of illustrative sketch the working of a Ramjet engine. What are its applications?

(ii) Mention the various industrial wastes and by-products used as boiler fuels. Briefly explain them.

[10 + 10 marks : 2024]

Solution:

(i)

Ram jet is a steady combustion or continuous flow engine and has the simplest construction of any propulsion engine. Fig. (a) shows a schematic diagram of a ram jet engine. It consists of an inlet diffuser, a combustion chamber and an exit nozzle or tailpipe,

As the ram jet has no compressor hence the entire compression depends upon the ram compression. The ram pressure ratio increases very slowly in the subsonic speed range. That is why ram jet is boosted upto a speed of 290 km/h by a suitable means such as a turbojet or a rocket before the ram jet will produce any thrust and must be boosted to even higher speeds before the thrust produced exceeds the drag.

After the boosting of ram jet, velocity of air passing through the diffuser decreases and hence pressure increases. This is called ram compression and a "pressure barrier" is created after the end of the diffuser. The fuel is injected through injection nozzles into the combustion chamber where it is ignited by means of a spark plug. The expansion of gases toward the diffuser entrance is restricted by the pressure barrier at the end of the diffuser and as a result the gases are constrained to expand through the tail pipe and out of the exit nozzle at a high velocity.

As the ram jet engine has no turbine, the temperature of the gases of combustion is not limited to a relatively low value as in the turbojet engine. The air-fuel ratio is 15:1, the exhaust temperature ranges 1800°C to 2100°C. The jet action gives the necessary forward thrust to the engine.

Fig. (b) shows the representation of ram jet cycle on T-s diagram. The basic characteristics of the ram jet engines are:

(i) Simple in construction.

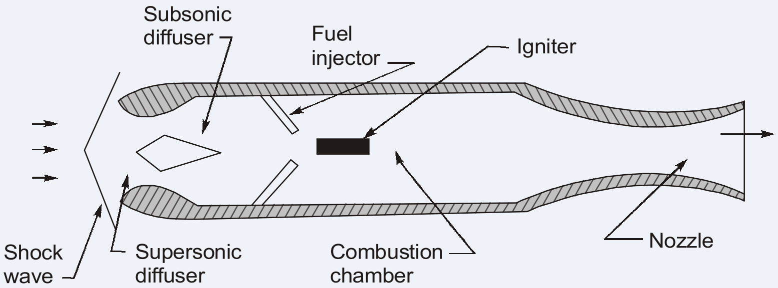


Fig. : (a) Ram Jet

(ii) No moving parts and hence free from unbalancing..

(iii) Greater thrust per unit engine weight than any other propulsion engine at supersonic speed except rocket.

(iv) The thrust per unit frontal area increases both with the efficiency and the air flow through the engine, therefore, much greater thrust per unit area is obtainable at high super- sonic speeds.

(v) The best performance of ram jet engine is obtained at flight speed of 1700 km/h to 2200 km/h.

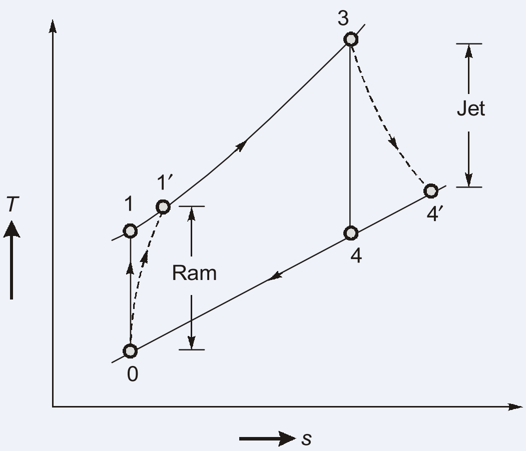


Fig. (b) : T-s diagram of Ram Jet Cycle

(vi) For successful operation the diffuser has to be carefully designed so that the kinetic energy associated with high entrance velocity is efficiently converted into pressure.

(vii) At low moderate speeds, the fuel consumption is too large. However, the fuel consumption decreases with flight speed.

(viii) The performance is independent of fuel technology and a wide range of fuels can be burned.

(ii)

Various industrial wastes adn by products used as boiler fuel are as follows:

1. Bagasse (Sugarcane waste): Residual fibers from sugar production.
2. Cofee grounds: Used coffee beans from coffee processing.
3. Rice husks: Hard outer layers of rice grains from rice milling.
4. Coconut shells: hard outer shells from coconut processing.
5. Palm oil waste (PKS, EFB, etc.): Various waste streams from palm oil production.
6. Wood chips/sawdust: Residual wood pieces from wood working.
7. Pulp/paper mill waste: Black liquor, bark, and other paper production residues.
8. Textile mill waste: Fabric scraps, cotton linters, and other textile residues.
9. Plastic waste: Recycled plastics converted into fuel.
10. Tire-derived fuel (TDF): Shredded tires used as fuel.
11. Used oil/grease: Recycled oil and grease from various industries.
12. Food processing waste: Nut shells, fruit pits, and other food processing residues.
13. Brewery/distillery waste: Spent grains and other residues from alcohol production.
14. Municipal solid waste (MSW): Household and business waste converted into fuel.
15. Refinery waste: Petroleum coke and other refinery residues.
16. Chemical plant waste: Various chemical residues and by products.
17. Pharmaceutical waste: Residuel materials from pharmaceutical production.
18. Hazardous waste: Solvents, pesticides, and other hazardous materials used as fuel in specialized boilers.
19. Construction/demolition waste: Wood drywall, and other construction residues.
20. Agricultural waste: Corn cobs, wheat straw, and other agricultural residues.

These industrial wastes and by-products are used as fuel in boilers to generate steam, reducing waste disposal costs and generating renewable energy.

End of Solution

Q.8 (a) A flat plate collector is of size 2 m length and 1 m width with one glass cover and 0° slope with horizontal. The gap between absorber plate and glass cover is 0.05 m and absorber plate to back plate is 0.015 m. Ambient air is passed between absorber plate and back plate at the rate of m = 0.1 kg/s. The solar radiation is I = 800 W/m^2,

Transmittance-Absorptance Product is T_A = 0.8, Overall loss coefficient U_L = 9.65 W/m^{2}K, Ambient temperature T_a = 300 K, Specific heat of air C_p = 1006.4 J/kgK, Natural convection heat transfer coefficient is 12.08 W/m^{2}K. Find the plate temperature T_p, and heat removal factor F_R. Use the correlations given below:

(i) Plate temperature

$$T_p = T_a + \left[\frac{I + T_A}{U_L}\right]\left[1 - G\xi\frac{C_p}{U_L}\right]$$

where $\xi = 1 - \exp\left[\left(\frac{-U_L}{GC_p}\right)\left(1 + \frac{U_L}{h}\right)^{-1}\right]$ Heat removal factor is given by

$$F_R = \left(G\frac{C_P}{U_L}\right)\left[1 - \exp\left(\frac{-F'U_L}{GC_P}\right)\right]$$

where $F' = \left(1 + \frac{U_L}{h}\right)^{-1}$

where G is the mass flow rate per unit area of absorber plate area (kg/s/m^2).

(ii) A company wanted 2 hours of buffer storage for a 1.5 MW solar thermal power plant that operates between 230°C and 380°C. Estimate the amount of material that is needed if Lithium Nitrate is used. Lithium Nitrate has the properties at melting point 252°C, latent heat 530 kJ/kg, specific heat of solid 2.02 and liquid 2.041 kJ/kg K respectively. Density of solid 2310 kg/m^3 and liquid 1776 kg/m^3, energy density 261 kWh/m^{2}K and thermal conductivity 1.35 W/mK.

[20 marks : 2024]

Solution:

(i)

Given: Plate temperature, $T_P = T_a + \left[\frac{I + T_A}{U_L}\right]\left(1 - G\xi\frac{C_P}{U_L}\right)$

where, $\xi = 1 - \exp\left[\left(\frac{-U_L}{GC_P}\right)\left(1 + \frac{U_L}{h}\right)^{-1}\right]$

Now, $A = 2 \times 1 = 2$ m^2, $\dot{m} = 0.1$ kg/s; $I = 800$ W/m^2

$T_A = 0.8$; $U_L = 9.65$ W/m^2-K; $T_a = 300$ K;
$C_p = 1006.4$ J/kgK; $h = 12.08$ W/m^{2}K,

$$G = \frac{\dot{M}}{A} = \frac{0.1}{2} = 0.05 \text{ kg/s.m}^2$$

$$\therefore \qquad \xi = 1 - \exp\left[\left(\frac{-9.65 \times 2}{0.1 \times 1006.4}\right)\left(1 + \frac{9.65}{12.08}\right)^{-1}\right]$$

$$\xi = 0.1011$$

$$\therefore \qquad T_P = 300 + \left[\frac{800 \times 0.8}{9.65}\right]\left[1 - \frac{0.05 \times 0.1011 \times 1006.4}{9.65}\right]$$

$$= 300 + 31.35 = 331.35 \text{ K}$$ **Ans.**

Now,
$$F' = \left(1 + \frac{U_L}{h}\right)^{-1} = \left(1 + \frac{9.65}{12.08}\right)^{-1}$$

$$F' = 0.5559$$

$$\therefore \qquad \text{Heat removal factor, } F_R = \left(G\frac{C_P}{U_L}\right)\left[1 - \exp\left(\frac{-F'U_L}{GC_P}\right)\right]$$

$$\therefore \qquad F_R = \left(\frac{0.05 \times 1006.4}{9.65}\right)\left(1 - \exp\left(\frac{-0.5559 \times 9.65}{0.05 \times 1006.4}\right)\right)$$

$$F_R = 5.2145 \times 0.1011$$

$$\therefore \qquad F_R = 0.5273$$ **Ans.**

(ii)

Total energy required = 1.5 × 2 × 3600 = 10800 MJ

Now, energy that can be obtained, is given by

$$E_a = m[C_{ps}(252 - 230) + 530 + C_{pl}(380 - 252)]$$

or

$$E_a = m[(2.02 \times 22)] + 530 + (2.041 \times 128)]$$

$$= m \times 835.688 \text{ kJ}$$

So, the total energy required = Energy obtained by the lithium nitrate

$$\Rightarrow \qquad 10800 \times 10^3 = m \times 835.688$$

$$\Rightarrow \qquad m = \frac{10800 \times 10^3}{835.688} = 12923.48 \text{ kg}$$ **Ans.**

End of Solution

Q.8 (b) The percentage composition of a solid fuel used in boiler of a power station is as follows:

Carbon 90%, Hydrogen 3.5%, Oxygen 3%, Nitrogen 1%, Sulphur 1% and the remaining being ash.

Determine the excess air supplied for the combustion of coal if the volumetric analysis of dry flue gases shows the following composition:

CO_2 : 10%, CO : 1%, N_2 : 82%, O_2 : 7%

Take oxygen as 23% in the air by mass.

[20 marks : 2024]

Solution:

Let us consider 100 kg of fuel. Let x moles of oxygen are supplied for combustion. The combustion equation can be written as

$$\Rightarrow \quad \frac{90}{12}C + \frac{3.5}{2}H_2 + \frac{1}{32}S + \frac{3}{32}O_2 + \frac{1}{28}N_2 + xO_2 + 3.76xN_2 \rightarrow$$

$$bCO_2 + dSO_2 + eO_2 + fN_2 + gH_2O + hCO$$

Equating coefficients,

Carbon : $$\frac{90}{12} = b + h$$

Hydrogen: $$\frac{3.5}{2} = g$$

$$\Rightarrow \quad g = 1.75$$

Oxygen: $$\frac{3}{32} + x = b + d + e + \frac{g}{2} + \frac{h}{2}$$

Sulphur: $$\frac{1}{32} = d$$

$$\Rightarrow \quad d = 0.03125$$

Nitrogen : $$\frac{1}{28} + 3.76x = f$$

Volumetric analysis:

$$\frac{b}{b+d+e+f+h} = 0.1$$

$$\frac{h}{b+d+e+f+h} = 0.01$$

$$\therefore \quad \frac{b}{h} = \frac{0.1}{0.01}$$

$$\Rightarrow \quad b = 10h$$

$$\therefore \quad \frac{90}{12} = 10h + h$$

$$\Rightarrow \quad h = 0.681$$

and $$b = 10 \times 0.681 = 6.81$$

Now, $$\frac{f}{b+d+e+f+h} = 0.82 \qquad \text{...(i)}$$

and $$\frac{e}{b+d+e+f+h} = 0.07 \quad ...(ii)$$

or $$\frac{e}{0.07} = 6.81 + 0.0321 + e + f + 0.681$$

$\therefore$ $$14.285e - e = 7.5231 + f$$
$$13.285e = 7.5231 + f \quad ...(iii)$$

From equation (i) and (iii)

$$\frac{13.285e - 7.5231}{6.81 + 0.03215 + e + (13.285e - 7.5231) + 0.681} = 0.82$$

$$\frac{13.285e - 7.5231}{14.285e + 5\times10^{-5}} = 0.82$$

On solving, we get

$$e = 4.78$$

$\therefore$ $$f = 13.285 \times 4.78 - 7.5231$$
$$= 55.98$$

Substituting in O_2 balance

$$\frac{3}{32} + x = 6.81 + 0.03125 + 4.78 + \frac{1.75}{2} + \frac{0.681}{2}$$

$$x = 12.836 - \frac{3}{32}$$

$\Rightarrow$ $$x = 12.74$$

Mass of oxygen supplied = 12.74 × 32 = 407.68 kg

$\therefore$ Mass of air supplied for 100 kg fuel

$$W_A = \frac{407.68}{0.23 \times 100} = 17.725 \text{ kg}$$

Theoretical air required per kg fuel

$$W_{Th} = 11.5C + 34.5\left(H - \frac{0}{8}\right) + 4.3N$$

$$= 11.5 \times 0.9 + 34.5\left(0.035 - \frac{0.03}{8}\right) + 4.3 \times 0.01$$

$$= 11.47 \text{ kg}$$

Percentage excess air $$= \frac{W_A - W_{Th}}{W_A} \times 100$$

$$= \frac{17.725 - 11.47}{17.725} \times 100 = 35.29\%$$ **Ans.**

End of Solution

Q.8 (c) A single stage, single acting air compressor 30 cm bore and 40 cm stroke runs at 200 rpm. The suction pressure is 1 bar at 15°C and delivery pressure is 5 bar. Determine the indicated mean effective pressure and the ideal power required to run it, when

(i) Compression is isothermal,

(ii) Compression follows the law $PV^{1.25}$ = Constant,

(iii) Compression is reversible adiabatic ($\gamma = 1.4$), and

(iv) Compression is irreversible adiabatic ($n = 1.5$).

Neglect clearance.

Determine the isothermal efficiency for (ii), (iii) and (iv). Assume isentropic or reversible adiabatic index, $\gamma = 1.4$ and $R = 0.287$ kJ/kgK.

[20 marks : 2024]

Solution:

Given: $D = 0.3$ m; $L = 0.4$ m; $N = 200$ rpm; $P_1 = 1$ bar; $T_1 = 15°$C; $P_2 = 5$ bar; $\gamma = 1.4$; R = 0.287 kJ/kgK

Now, Swept volume, $$\dot{V}_1 = \dot{V}_s = \frac{\pi}{4}D^2 L \times \frac{N}{60}$$

$$= \frac{\pi}{4}\times 0.3^2 \times 0.4 \times \frac{200}{60} = 0.0942 \text{ m}^3/\text{s}$$

Theoretical power, $$\dot{W}_{iso} = P_1\dot{V}_1 \ln\left(\frac{P_2}{P_1}\right) = 100\times 0.0942 \ln\left(\frac{5}{1}\right)$$

$$= 15.16 \text{ kW}$$ **Ans.(i)**

Mean effective pressure, $$P_m = \frac{\dot{W}_{iso}}{\dot{V}_s} = \frac{15.16\times 10^3}{0.0942}$$

$$P_m = 160.93 \text{ kPa}$$ **Ans.(i)**

For $n = 1.25$

$$\dot{W}_{poly} = \frac{n}{n-1}P_1\dot{V}_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.25}{0.25}\times 100\times 0.0942\left[(5)^{\frac{0.25}{1.25}} - 1\right]$$

$$= 17.88 \text{ kW}$$ **Ans.(ii)**

Also, $$P_{imep} = \frac{\dot{W}_{poly}}{\dot{V}_s} = \frac{17.88\times 10^3}{0.0942}$$

$$= 189.80 \text{ kPa}$$ **Ans.(ii)**

For reversible adiabatic,

$$\dot{W}_{isen} = \frac{\gamma}{\gamma-1} P_1 \dot{V}_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{1.4}{0.4} \times 100 \times 0.0942 \left[5^{0.4/1.4} - 1 \right]$$

$$= 19.248 \text{ kW}$$ **Ans.(iii)**

$$P_{imep} = \frac{\dot{W}_{isen}}{\dot{V}_s} = \frac{19.248}{0.0942}$$

$$= 204.33 \text{ kPa}$$ **Ans.(iii)**

For irreversible adiabatic

$$\dot{W}_{irrev} = \frac{n}{n-1} \times P_1 \dot{V}_1 \left[\left(\frac{P_2}{P_1} \right)^{n-1/n} - 1 \right]$$

$$= \frac{1.5}{0.5} \times 100 \times 0.0942 \left[(5)^{0.5/1.5} - 1 \right]$$

$$= 20.064 \text{ kW}$$

$$P_{imep} = \frac{\dot{W}_{irrev}}{\dot{V}_s} = \frac{20.064}{0.0942} = 212.99 \simeq 213 \text{ kPa}$$

Isothermal efficiency, $$\eta_{iso} = \frac{\text{Isothermal work}}{\text{Actual work}} \times 100$$

(ii) For $n = 1.25$,

$$\eta_{iso} = \frac{15.16}{17.88} \times 100 = 84.78\%$$

(iii) For reversible adiabatic,

$$\eta_{iso} = \frac{15.16}{19.248} \times 100 = 78.76\%$$

(iv) For irreversible adiabatic, i.e. $n = 1.5$

$$\eta_{iso} = \frac{15.16}{20.064} \times 100 = 75.55\%$$

End of Solution