ESE 2024

Main Exam
Detailed Solutions

Electrical Engineering

PAPER-I

EXAM DATE : 23-06-2024 | 09:00 AM to 12:00 PM

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ANALYSIS

Electrical Engineering Paper-II
ESE 2024 Main Examination

Sl.	Subjects	Marks
1.	Electric Circuits	40
2.	Electromagnetic Fields	44
3.	Electrical Materials	64
4.	Engineering Mathematics	80
5.	Basic Electronics Engineering	84
6.	Computer Fundamental	36
7.	Electrical and Electronic Measurements	132
		Total 480

Scroll down for detailed solutions

SECTION : A

Q.1 (a) Using double integral, find the volume in the positive octant of the ellipsoid.

$$\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{4} = 1$$

[12 marks : 2024]

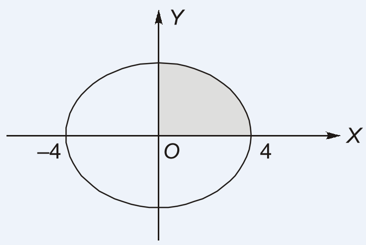
Solution:

The volume of ellipsoid is equal in upper and lower z-plane and same is the case for xy-plane.

So, the volume in all 8 octant is same.

$\therefore$ Volume, V = 8(Volume of S in 1st octant)

Volume of 'S' in 1st octant using double integral is given by $\iint_R (Z = F(x,y)dydx$



$$= \int_{x=0}^{4}\int_{y=0}^{3\sqrt{1-\frac{x^2}{16}}} 2\sqrt{1-\frac{x^2}{16}-\frac{y^2}{9}}\, dydx$$

$$= \int_{x=0}^{4}\int_{y=0}^{3\sqrt{1-\frac{x^2}{16}}} 2\sqrt{\left(\sqrt{1-\frac{x^2}{16}}\right)^2-\left(\frac{y}{3}\right)^2}\, dydx$$

$$= 2\int_0^4\left[\frac{y}{6}\sqrt{\left(\sqrt{1-\frac{x^2}{16}}\right)^2-\left(\frac{y}{3}\right)^2}+\left(\frac{1-\frac{x^2}{16}}{2}\right)\sin^{-1}\left(\frac{y}{3\sqrt{1-\frac{x^2}{16}}}\right)\right]$$

$$= 2\int_0^4 \frac{1-\frac{x^2}{16}}{2}\sin^{-1}(1)\,dx$$

$$= \frac{\pi}{2}\left[x-\frac{x^3}{48}\right]_0^4 = \frac{\pi}{2}\left[4-\frac{4}{3}\right] = \frac{8\pi}{6}$$

Hence, Total value of 'S' $= 8\times\frac{8\pi}{6} = \frac{64\pi}{6} = \frac{32\pi}{3}$

End of Solution

Q.1 (b) The strength of one bond of Magnesium Oxide (MgO) is 10.54 eV. How much joules of energy for vapourization will be needed by 0.35 kg of Magnesium Oxide? (Take Avogadro number = 6.022×10^{23} atoms/mol, charge of electron = 1.6×10^{-19} Coulombs, atomic mass of Mg = $24u$, atomic mass of Oxygen = $16u$)

[12 marks : 2024]

Solution:

Insufficient Data

End of Solution

Q.1 (c) What are Maxwell's equation in Point form and Integral Form? How do these equations take form in free space?

[12 marks : 2024]

Solution:

Maxwell's equations in Point and Integral form are :

(A) Maxwell's equation in static electric and magnetic fields :

Integral Form	Point Form
$\oint \bar{D} \cdot dS = Q_{enc}$	$\bar{\nabla} \cdot \bar{D} = \rho_v$
$\oint \bar{E} \cdot d\bar{l} = 0$	$\bar{\nabla} \times \bar{E} = 0$
$\oint \bar{B} \cdot d\bar{S} = 0$	$\bar{\nabla} \cdot \bar{B} = 0$
$\oint \bar{H} \cdot d\bar{l} = I_{enc}$	$\bar{\nabla} \times \bar{H} = \bar{J}$

(B) Maxwell's equation in time varying fields :

Integral Form	Point Form
$\oint \bar{D} \cdot dS = Q_{enc}$	$\bar{\nabla} \cdot \bar{D} = \rho_v$
$\oint \bar{E} \cdot d\bar{l} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -j\omega\mu\bar{H}$
$\oint \bar{B} \cdot d\bar{S} = 0$	$\bar{\nabla} \cdot \bar{B} = 0$
$\oint \bar{H} \cdot d\bar{l} = \int \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) \cdot d\bar{S}$	$\nabla \times H = \bar{J} + \frac{\partial \bar{D}}{\partial t} = (\sigma + j\omega\varepsilon)\bar{E}$

From Maxwell's equations :

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \equiv -\mu \frac{\partial \bar{H}}{\partial t} \quad \text{...(i)}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \equiv \sigma \bar{E} + \in \frac{\partial \bar{E}}{\partial t} \quad \text{...(ii)}$$

Taking curl of eqn. (i) on both sides,

$$\nabla \times \nabla \times \bar{E} = \nabla \times \left(-\mu \frac{\partial \bar{H}}{\partial t}\right)$$

$$\Rightarrow \quad \nabla(\nabla\cdot\bar{E})-\nabla^2\bar{E} = -\mu\frac{\partial}{\partial t}(\nabla\times\bar{H})$$

$$\Rightarrow \quad \nabla(\nabla\cdot\bar{E})-\nabla^2\bar{E} = -\mu\frac{\partial}{\partial t}\left(\sigma\bar{E}+\in\frac{\partial\bar{E}}{\partial t}\right)$$

$$\Rightarrow \quad \nabla(\nabla\cdot\bar{E})-\nabla^2\bar{E} = \mu\sigma\frac{\partial\bar{E}}{\partial t}-\mu\in\frac{\partial^2\bar{E}}{\partial t^2}$$

For free space, $\rho_v = 0,\ \in\ = \in_o,\ \mu = \mu_o,\ \sigma = 0$

As $\rho_v = 0 \Rightarrow$ $\nabla\cdot\bar{D} = 0 \Rightarrow \nabla\cdot\bar{E} = 0$

Hence, $$-\nabla^2\bar{E} = -\mu_o\in_o\frac{\partial^2\bar{E}}{\partial t^2}$$

$$\Rightarrow \quad \nabla^2\bar{E} = \mu_o\in_o\frac{\partial^2\bar{E}}{\partial t^2}$$

Similarly, $$\nabla^2\bar{H} = \mu_o\in_o\frac{\partial^2\bar{H}}{\partial t^2}$$

End of Solution

Q.1 (d) For an abrupt silicon p-n junction with acceptor ion concentration $N_A = 4\times10^{16}$ cm^{-3} and donor ion concentration $N_D = 10\times10^{15}$ cm^{-3}, if $T = 300$ K, intrinsic carrier concentration $n_i = 1\times10^{10}$ cm^{-3}, calculate the maximum electric field in the depletion region when $V_a = -3.5$ volts. Assume relative dielectric constant of silicon $\varepsilon_r = 11.8$, electron charge $q = 1.6\times10^{-19}$ C, Boltzmann's constant $k = 1.38\times10^{-23}$ J/K, permittivity of free space $\varepsilon_0 = 8.854\times10^{-12}$ F/m.

[12 marks : 2024]

Solution:

Given :
$$N_A = 4\times10^{16}\ \text{cm}^{-3}$$
$$N_D = 10\times10^{15}\ \text{cm}^{-3}$$
$$T = 300\ \text{K}$$
$$n_i = 1\times10^{10}\ \text{cm}^{-3}$$
$$V_a = -3.5\ \text{V}$$
$$\in_r = 11.8$$

The maximum electric field in the abrupt pn-junction

$$E_{max} = -\left\{\frac{2q(V_{bi}+V_R)}{\in_s}\left(\frac{N_aN_d}{N_a+N_d}\right)\right\}^{1/2}$$

where V_R is magnitude of reverse bias voltage.

where, $$V_{bi} = V_T\ln\left(\frac{N_aN_d}{n_i^2}\right)$$

$$= 0.026\ln\left(\frac{4\times10^{16}\times10\times10^{15}}{10^{20}}\right)$$

$$V_{bi} = 0.754 \text{ V}$$

$$\therefore \quad E_{max} = -\left\{\frac{2\times1.6\times10^{-19}(0.754+3.5)}{11.8\times8.854\times10^{-14}}\times\left(\frac{4\times10^{16}\times10^{16}}{4\times10^{16}+10^{16}}\right)\right\}^{1/2}$$

$$E_{max} = -32.3 \text{ V/am or } E_{max} = -3.23 \text{ kV/m}$$

End of Solution

Q.1 (e) Write a 'C' program to identify whether the given input word is a 'palindrome'. The program should read the word from the terminal and display the message whether the input word is a palindrome or not. If the input word is 'END' it should exit the program.

[12 marks : 2024]

Solution:

Algorithm Logic :

Step 1 : Create a function to check if the string is a palindrome.
Ispalindrome(str)

Step 2 : Initialize indexes for low and high levels to be 0 and (n – 1) respectively.

Step 3 : Until the low index (l) is lower than the high index (h), do the following :

(i) If str (l) is different from str (h) return false.

(ii) If str (l) and str (h) are same
then increment l, i.e., i++
and decrement h, i.e., h– –

Step 4 : If we reach this step, means there is no mismatch and the string is a Palindrome. Otherwise, step 3 (i) is true, it is not a Palindrome.

Code :

```
#include <stdio.h>
#incluce <string.h>
void Ispalindrome (char str[])
{
int l=0;
int h= strlen(str)–1;
while (h>l)
{
if [str[(l++]! = str[h--])
{
Printf("%S is not a Palindrome string\n", str);
return;
}
}
Printf("%S is a Palindrome string\n",str);
}
int main( )
```

```
{
    Is Palindrome("level");
    Is Palindrome("radar");
    Is Palindrome("END");
    return 0;
}
```

End of Solution

Q.2 (a) (i) Determine the eigen values and eigen vectors of $B = 2A^2 - \frac{1}{2}A + 3I$ where $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.

(ii) Using the method of Lagrange's multipliers, find the largest product of the numbers x, y and z, when $x + y + z^2 = 25$.

[10 + 10 marks : 2024]

Solution:

(i) Eigen value of $A_{2\times2}$ given by $|A - \lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^2 - 10\lambda + 24 = 0$$

$$\lambda_1 + \lambda_2 = 10$$

$$\lambda_1\lambda_2 = 24$$

$$\Rightarrow \quad 6, 4 = \lambda_A$$

Given : $$B = 2A^2 - \frac{1}{2}A + 3I$$

$$\therefore \quad \lambda_B = 2\lambda_A^2 - \frac{1}{2}\lambda_A + 3$$

For $\lambda_A = 6$: $$\lambda_B = 2(6)^2 - \frac{1}{2}(6) + 3 = 72$$

For $\lambda_A = 4$: $$\lambda_B = 2(4)^2 - \frac{1}{2}(4) + 3 = 33$$

Find Eigen Vectors of A :

(1) $\lambda_1 = 6$:

Sub λ_1 in $(A - \lambda I) = 0$

$$\begin{bmatrix} 2 & -4 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad 2x_1 - 4y_1 = 0$$

$$\Rightarrow \quad \frac{x_1}{2} = \frac{y_1}{1} = K$$

$$\therefore \quad X_1 = K\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

(2) $\lambda_2 = 4$:

Sub λ_2 in $(A - \lambda I) = 0$

$$\begin{bmatrix} 4 & -4 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad x_2 - y_2 = 0$$

$$\Rightarrow \quad x_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = K\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The Eigen vectors of B and A are same.

(ii) Find maximum value of xyz : f w.r.t. $x + y + z^2 = 25$: ϕ

By LMM :

Let
$$F = f + \lambda\phi$$
$$F = xyz + \lambda(x + y + z^2 - 25)$$
$$\frac{\partial F}{\partial x} = yz + \lambda(1) = 0 \Rightarrow \lambda = -yz$$
$$\frac{\partial F}{\partial y} = xz + \lambda(1) = 0 \Rightarrow \lambda = -xz$$
$$\frac{\partial F}{\partial z} = xy + \lambda(2z) = 0 \Rightarrow \lambda = \frac{-xy}{2z}$$
$$\therefore \quad \lambda = -yz = -xz = \frac{-xy}{2z}$$
$$\lambda = \frac{1}{x} = \frac{1}{y} = \frac{1}{2z^2}$$
$$\Rightarrow \quad x = \frac{1}{\lambda},\ y = \frac{1}{\lambda},\ z^2 = \frac{1}{2\lambda}$$
$$\therefore \quad \frac{1}{\lambda} + \frac{1}{\lambda} + \frac{1}{2\lambda} = 25$$
$$\frac{1}{\lambda}\left[2 + \frac{1}{2}\right] = 25$$
$$\frac{1}{\lambda} = \frac{50}{5} = 10$$
$$\lambda = \frac{1}{10}$$
$$\therefore \quad (x, y, z) = \left(10, 10, \sqrt{5}\right)$$
$$\therefore \quad f_{max} = f\left(10, 10, \sqrt{5}\right) = 10 \times 10 \times \sqrt{5} = 100\sqrt{5}$$

End of Solution

Q.2 (b) From atomic interpretations of spontaneous magnetization in ferromagnetic materials and the Curie-Weiss Law, determine Curie constant in terms of Bohr magneton (β) and N spins per m^3 in the material. Establish how spontaneous magnetization takes place below the Curie temperature.

[20 marks : 2024]

Solution:

Curie Weiss Law for Ferromagnetic Material :

The magnetization for ferromagnetic material is given by :

$$\vec{M} = N\beta_m\left(\frac{\mu_o\beta_m H_i}{KT}\right) = \frac{N\beta_m^2\mu_o H_i}{KT} \quad \text{...(1)}$$

where, β_m = magnetic dipole moment in Bohr-magnetron

and N = Number of dipole moment/m^3

H_i = Internal magnetic field

For ferromagnetic materials, $H_i = H + \gamma m$

where γ = internal field constant

$$\vec{M} = \frac{N\beta_m^2\mu_o(H+\gamma m)}{KT}$$

$$M\left[1-\frac{N\beta_m^2\mu_o\gamma}{KT}\right] = \frac{N\beta_m^2\mu_o H}{KT}$$

$$M = \frac{\frac{N\beta_m^2\mu_o H}{KT}}{1-\frac{N\beta_m^2\mu_o\gamma}{KT}}$$

$$\chi_m H = \frac{\frac{N\beta_m^2\mu_o H}{KT}}{1-\frac{N\beta_m^2\mu_o\gamma}{KT}} \quad \{M = \chi_m H\}$$

χ_m = Magnetic successptibility

$$\chi_m = \frac{\frac{N\beta_m^2\mu_o}{K}}{T-\frac{N\beta_m^2\mu_o\gamma}{K}}$$

Above equation can be rewritten as

$$\chi_m = \frac{C}{T-\theta}$$

where C = Curie constant $= \frac{N\beta_m^2\mu_o}{K}$

$$\theta = C.\gamma = \frac{N\beta_m^2\mu_o\gamma}{K}$$

End of Solution

Q.2 (c) (i) Give briefly the concept of precision in measurements. Explain with examples the roles of 'significant figures' on measurement of precision of a measuring tool.

(ii) Using a standard cell of 1.016 V, a simple potentiometer balances at 48.4 cm. Calculate : (I) the emf of a cell that balances at 72 cm, (II) the percentage error in a voltmeter, measuring a voltage which balances at 66 cm, when reading is 1.40 V.

[10 + 10 marks : 2024]

Solution:

(i) Precision is a measure of the reproductibility of the measurements, i.e., given a fixed value of a quantity. Precision is a measure of the degree of agreement within a group of measurements. The term 'precise' means clearly or sharply defined.
Precision is used in measurements to describe the consistency or the reproducibility of results. Precision instruments are not guaranteed for accuracy.
Precision depends upon number of significant figures. The more are significant figures the more is precision.
Significant figures convey actual information regarding the magnitude and the measurement precision of a quantity.
Example : Let us take an example, if a voltage is specified as 256 V, its value should be taken as closer to 256 V than to either 257 V or 255 V. If the value of voltage is described as 256.0 V, it means that the voltage is closer to 256.0 V, then it is to 256.1 V or 255.9 V. In 256, there are three significant figures while in 256.0, there are four. The latter, which more significant figures, expresses a measurement of greater precision than the former.

(ii) $\because$ Balance at 48.4 cm is corresponding to an emf of 1.016 V.
$\therefore$ Balance at 72 cm is corresponding to an emf of

$$\frac{1.016}{48.4} \times 72 = 1.5114 \text{ V}$$

$\therefore$ Balance at 66 cm is corresponding to an emf of

$$\frac{1.016}{48.4} \times 66 = 1.385 \text{ V}$$

$$\% \text{ error} = \frac{\text{Measured Value} - \text{True Value}}{\text{True Value}}$$

$$= \frac{1.40 - 1.385}{1.385}$$

$$= 1.08\%$$

End of Solution

Q.3 (a) (i) Using the Cauchy-Riemann equations, show that $f(z) = f(r, \theta) = r^4(\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta) + 4ir^2\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta)$ is analytic in the entire z-plane and hence find its derivative in terms of z.

(ii) The mutual inductance between two coils varies with the angle of displacement of the moving coil from its zero position as follows :

Angle (degree)(x)	0	15	30	60	90	105	120
Mutual Inductance (μH)(y)	–336	–275	–192	0	192	275	336

Determine the Pearson's Correlation Coefficient (r_{xy}) and the angle between the two regression lines formed by the above data.

[8 + 12 marks : 2024]

Solution:

(i) Given,

$$f(r,\theta) = r^4(\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta) + 4ir^2\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta)$$

$$= r^4(1 - 8\cos^2\theta\sin^2\theta) + 4ir^2\sin\theta\cos\theta(\cos 2\theta)$$

$$f(r,\theta) = r^4(\cos 4\theta) + ir^2\sin 4\theta$$

$$u = r^4\cos 4\theta;\ v = r^2\sin 4\theta$$

$f(z)$ is analytic iff C-R equation holds, i.e.,

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$$

$$\frac{\partial u}{\partial r} = 4r^3\cos 4\theta$$

$$\Rightarrow \quad \frac{1}{r}\frac{\partial v}{\partial \theta} = 4r\cos 4\theta$$

$$\Rightarrow \quad \frac{\partial u}{\partial r} \neq \frac{1}{r}\frac{\partial v}{\partial \theta}$$

$\therefore$ $f(z)$ is not analytic.

(ii)

$$V_{xy} = \frac{Cov(x,y)}{\sigma_x\sigma_y}$$

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
0	–336	–60	–336	20160	3600	1,12,896
15	–275	–45	–275	12375	2025	75625
30	–192	–30	–192	5760	900	36864
60	0	0	0	0	0	0
90	192	30	192	5760	900	36864
105	275	45	275	12375	2025	75625
120	336	60	336	20160	3600	1,12,896
$\Sigma x = 420$	$\Sigma y = 0$			$\Sigma(X-\bar{X})(y-\bar{y})$ = 76,596	$\Sigma(x-\bar{x})^2$ = 13050	$\Sigma(y-\bar{y})^2$ = 4507

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{420}{7} = 60$$

$$\bar{y} = \frac{\Sigma y}{n} = 0$$

$$\sigma_x = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}} = \sqrt{\frac{13050}{7}} = 43.18$$

$$\sigma_y = \sqrt{\frac{(Y-\bar{Y})^2}{n}} = \sqrt{\frac{450770}{7}} = 253.76$$

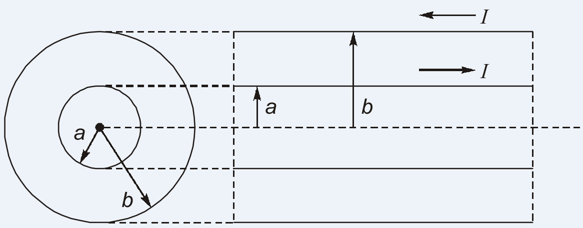
$$\sigma_{xy} = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{n} = \frac{76596}{7} = 10942$$

$$\gamma_{XY} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{10942.2}{43.18 \times 253.76} = 0.99$$

End of Solution

Q.3 (b) (i) Two point charges of Q_1 = 6 nC and Q_2 = 8 nC are placed at (2, 2) and (6, 8) respectively. Show that the equation of the locus on which the electric field intensities due to Q_1 and Q_2 are equal represents a circle. Find its centre and radius.

(ii) Determine the inductance per unit length of an air filled co-axial cable having a solid inner conductor of radius '*a*' metres and a very thin outer conductor of inner radius '*b*' metres. Assume that the current flows via the inner conductor and returns in the outer conductor and is uniformly distributed over the cross-section of inner conductor.



[10 + 10 marks : 2024]

Solution:

(i)

$$E_1 = \frac{Q_1}{4\pi\varepsilon_o r_1^2}$$

$\Rightarrow$
$$E_1 = \frac{6\times10^{-9}\times9\times10^{9}}{|(x,y)-(2,2)|^2} = \frac{54}{(x-2)^2+(y-2)^2}$$

Similarly,
$$E_2 = \frac{Q_2}{4\pi\varepsilon_o r_2^2}$$

$\Rightarrow$
$$E_2 = \frac{8\times10^{-9}\times9\times10^{9}}{|(x,y)-(6,8)|} = \frac{72}{(x-6)^2+(y-8)^2}$$

According to the question, $E_1 = E_2$

$\Rightarrow$
$$\frac{54}{(x-2)^2+(y-2)^2} = \frac{72}{(x-6)^2+(y-8)^2}$$

$\Rightarrow$
$$3\{(x-6)^2+(y-8)^2\} = 4\{(x-2)^2+(y-2)^2\}$$

$\Rightarrow 3\{x^2 - 12x + 36 + y^2 - 16y + 64\} = 4\{x^2 - 4x + 4 + y^2 - 4y + 4\}$

$3x^2 - 36x + 108 + 3y^2 - 48y + 192 = 4x^2 - 16x + 16 + 4y^2 - 16y + 16$

$\Rightarrow x^2 + 20x + y^2 + 32y - 268 = 0$

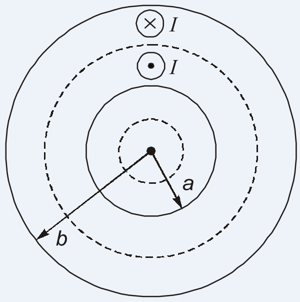
$\Rightarrow (x + 10)^2 + (y + 16)^2 - 624 = 0$

$\Rightarrow (x + 10)^2 + (y + 16)^2 = (24.97)^2$

Hence, the locus is a circle with centre (–10, –16) and radius is 24.97 m.

(ii) As we know that

Magnetic energy, $W_m = \frac{1}{2}LI^2$



Also, $W_m = \frac{1}{2}\int \frac{B^2}{\mu} dV$

$\therefore \quad \frac{1}{2}LI^2 = \frac{1}{2}\int \frac{B^2}{\mu} dV$

$\Rightarrow \quad L = \frac{1}{I^2}\int \frac{B^2}{\mu} dV$

Case (i) : ρ < *a* :

For $\rho < a$, $H = \frac{I\rho}{2\pi a^2}$

$\Rightarrow \quad B = \frac{\mu_o I\rho}{2\pi a^2}$

$\therefore \quad L_{int} = \frac{1}{I^2}\int \frac{\mu_o^2 I^2 \rho^2}{\mu_o 4\pi^2 a^4} \cdot \rho d\rho d\phi dz$

$= \frac{\mu_o}{4\pi^2 a^4} \int_{\rho=0}^{a} \rho^3 d\rho \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{l} d\tau$

$= \frac{\mu_o}{4\pi^2 a^4} \frac{\rho^4}{4}\Big|_0^a \phi\Big|_0^{2\pi} z\Big|_0^l$

$= \frac{\mu_o}{4\pi^2 a^4} \cdot \frac{a^4}{4} \cdot 2\pi l = \frac{\mu_o l}{8\pi}$

Case (ii):

For $a \le \rho < b$, $H = \frac{I}{2\pi\rho}$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi\rho}$$

$$\therefore \quad L_{ext} = \frac{1}{I^2}\int \frac{\mu_0^2 I^2}{4\pi^2\rho^2}\rho d\rho\, d\phi\, dz$$

$$= \frac{\mu_0^2}{4\pi^2}\int_{\rho=a}^{b}\frac{d\rho}{\rho}\int_{\phi=0}^{2\pi}\int_{z=0}^{l} dz$$

$$= \frac{\mu_0^2}{4\pi^2}\ln\rho\Big|_a^b \phi\Big|_0^{2\pi} z\Big|_0^l$$

$$= \frac{\mu_0^2}{4\pi^2}\ln\left(\frac{b}{a}\right)2\pi l$$

$$= \frac{\mu_0^2 l}{2\pi^2}\ln\left(\frac{b}{a}\right)$$

$$\therefore \quad L = L_{in} + L_{ext}$$

$$\Rightarrow \quad L = \frac{\mu_0 l}{2\pi}\left(\frac{1}{4}+\ln\left(\frac{b}{a}\right)\right)$$

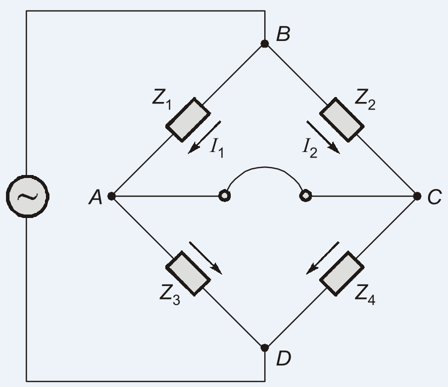
$\Rightarrow$ Inductance per unit length,

$$L_m = \frac{\mu_0}{2\pi}\left[\frac{1}{4}+\ln\left(\frac{b}{a}\right)\right]$$

End of Solution

Q.3 (c) (i) The impedance of the basic ac bridge shown in the following figure is given as follows :

$Z_1 = 100\ \Omega\angle 80°$ (inductive impedance)
$Z_2 = 250\ \Omega$ (pure resistance)
$Z_3 = 400\ \Omega\angle 30°$ (inductive impedance)
Z_4 = unknown



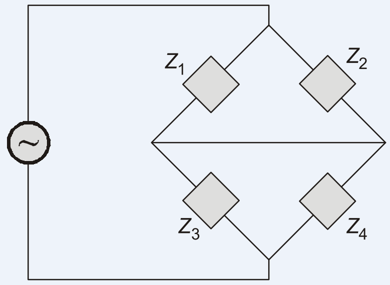
Determine the value of the constants of the unknown arm.

(ii) Derive the equations for balance in the case of Maxwell's inductance capacitance bridge. Draw the phasor diagrams for balance conditions.

[10 + 10 marks : 2024]

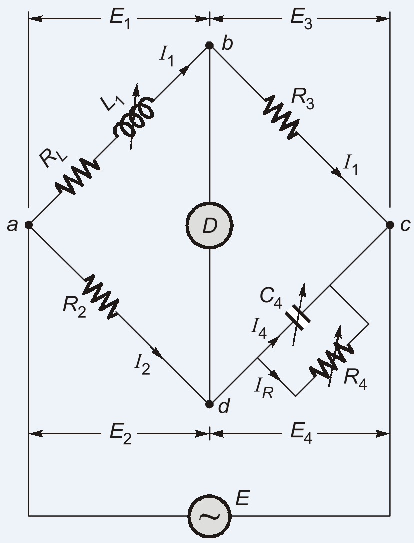
Solution:

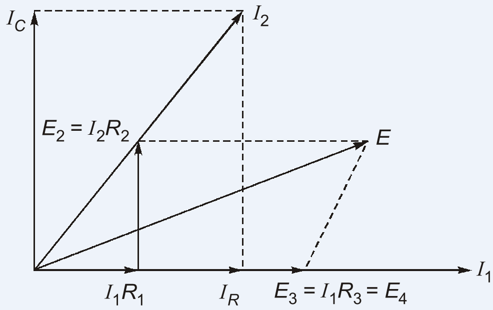
(i)



Balance bridge,
$$Z_1Z_4 = Z_2Z_3$$
$$(100\angle 80^\circ)(Z_4) = 250(400\angle 30^\circ)$$
$$Z_4 = 1000\angle{-50^\circ}$$

(ii) In this bridge, an inductance is measured by comparison with a standard variable capacitance. The connection and the phasor diagram at the balance condition are given in figure





Writing the equation for balance,
$$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4R_4}\right) = R_2R_3$$
or
$$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega R_2R_3C_4R_4$$
Separating the real and imaginary terms, we have
$$R_1 = \frac{R_2R_3}{R_4}$$
and
$$L_1 = R_2R_3C_4$$

Thus, we have two variables R_4 and C_4 which appear in one of the two balance equations and hence the two equations are independent.

The expression for Q factor

$$Q = \frac{\omega L_1}{R_1} = \omega C_4 R_4$$

Following are advantages of Maxwell's inductance capacitance bridge :

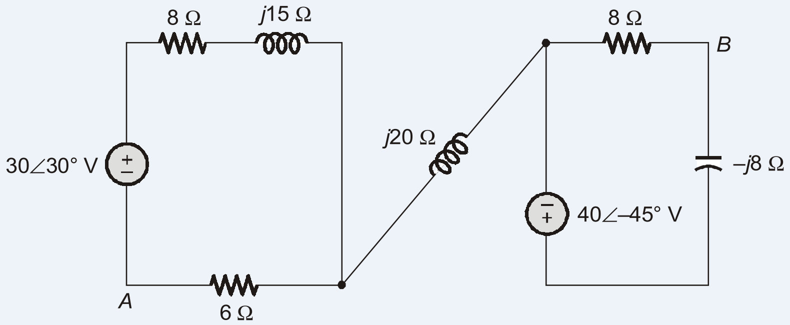
- The balance equations are independent if we choose R_4 and C_4 as variable elements.
- The value of L_1 and R_1 are independent of frequency.
- The bridge circuit gives simpler expressions for unknown R_1 and L_1.
- It is suitable for measurement of a wide range inductance at audio and power frequencies.

Disadvantages of Maxwell's inductance capacitance bridge :

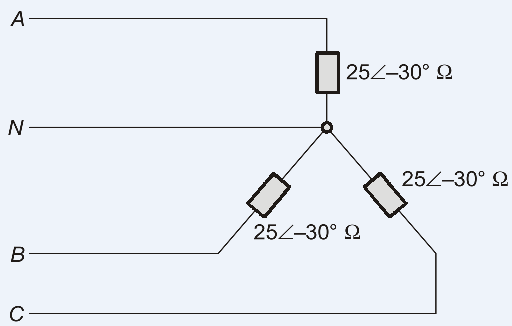
- This is not suitable for measurement of high Q coils.
- It is not suitable for measurement of low Q coils, because sliding balance problem occurs to satisfy the phase angle condition for bridge balance.
- The bridge circuit requires a variable standard capacitor which is expensive.

End of Solution

Q.4 (a) (i) Determine the phasor voltage V_{AB} in the circuit given in the following figure :



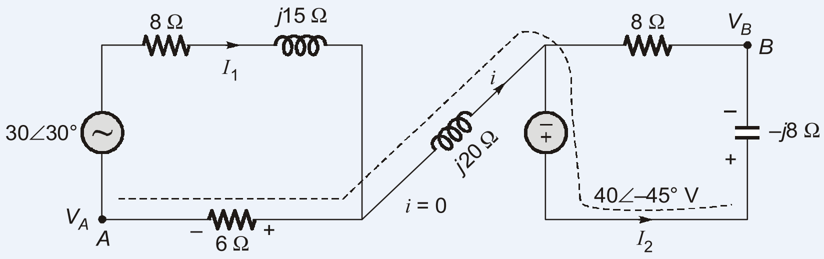
(ii) A three-phase, four-wire, CBA system has an effective line voltage of 140 V and it has three impedances of 25∠–30° Ω in a Y-connection, as shown in the figure. Determine the line currents and draw the voltage-current phasor diagram.



[10 + 10 marks : 2024]

Solution:

(i)

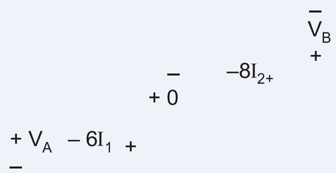


The two loop currents are independent to each other

$$I_1 = \frac{30\angle 30^\circ}{8+6+j15} = \frac{30\angle 30^\circ}{14+j15} = 1.462\angle{-16.975}$$

$$I_2 = \frac{40\angle -45^\circ}{8-j8} = 3.5355\angle 0^\circ$$

Apply KVL from A to B



$$V_A + 6I_1 + 8I_2 - V_B = 0$$

$$V_A - V_B = -6I_1 - 8I_2$$

$$= -6(1.462\angle{-16.975} - 8(3.5355)$$

$$= 36.76\angle 176 \text{ Volts}$$

(ii)

$$V_{Ph} = \frac{V_{Line}}{\sqrt{3}}\angle -30^\circ$$

$$V_{Ph} = \frac{140}{\sqrt{3}}\angle -30^\circ = 80.83\angle -30^\circ$$

$$I_{C'Phase} = \frac{80.83\angle -30^\circ}{25\angle -30^\circ} \Rightarrow 3.233 \text{ A}$$

$$I_{B'Phase} = \frac{80.83\angle -150^\circ}{25\angle -30^\circ} \Rightarrow 3.233\angle{-120^\circ} \text{ A}$$

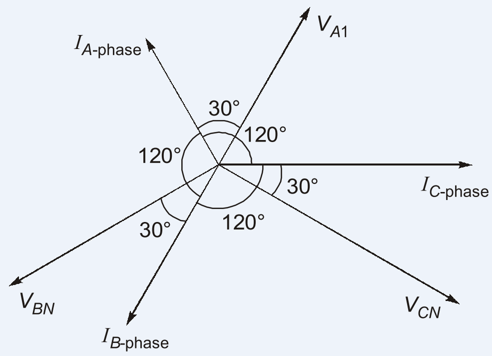
$$I_{A'Phase} = \frac{80.83\angle -270^\circ}{25\angle -30^\circ} \Rightarrow 3.233\angle{-240^\circ} \text{ A}$$

$I_{Line} = I_{Phase}$ (Line currents are)

$I_A = 3.233\angle{-240^\circ}$ A

$I_B = 3.233\angle{-120^\circ}$ A

$I_C = 3.233\angle 0^\circ$ A;



End of Solution

Q.4 (b) (i) A Hall voltage of 3.5×10^{-8} V in magnitude is generated for an aluminium specimen of 15 mm thickness with a current of 25 A and a magnetic field of 0.6 Tesla, imposed in a direction perpendicular to the current. Calculate electron mobility for aluminium. (Take electrical conductivity for aluminium as 3.7×10^7 Ω/m).

(ii) Six micrograms of antimony are thoroughly mixed in molten form with 200 g of pure germanium and antimony atoms substitute for germanium atoms uniformly throughout the solid material. Determine the density of antimony atoms, the density of donated electrons and the conductivity, if electron mobility is 3500 cm²/Vs, for the carriers.

(Take charge of electron = 1.6×10^{-19} Coulombs, density of germanium = 5.46 gm/cm³, atomic weight of antimony = 121.76 u, Avogadro number = 6.022×10^{23} atoms/mol)

[10 + 10 marks : 2024]

Solution:

(i) Given : Hall voltage $V_H = 3.5 \times 10^{-8}$ V

$W = 15$ mm

$I = 25$ A

$B = 0.6$ Tesla

$\sigma_{Al} = 3.7 \times 10^7$ Ω/m

Mobility, $\mu = \sigma R_H$

where Hall coefficient, $R_H = \dfrac{V_H W}{BI}$

$$\therefore \quad R_H = \frac{3.5 \times 10^{-8} \times 15 \times 10^{-3}}{0.6 \times 25}$$

$R_H = 3.5 \times 10^{-11}$ m³/A-sec

$\therefore \quad \mu = 3.7 \times 10^7 \times 3.5 \times 10^{-11}$ m²/V-sec

$\mu = 1.295 \times 10^{-3}$ m²/V-sec

$\mu = 1295$ cm²/V-sec

(ii) Densityo f antimony added,

$$\rho_{sb} = \frac{6\times10^{-6}\text{g}}{36.63\ \text{cm}^3} = 0.1638 \times 10^{-6}\ \text{g/cm}^3$$

Atomic weight of antimony, $A = 121.76$ amu

$= 121.76$ g/mol

Hence, number of antimony atoms per unit volume

$$N = \frac{N_A\rho}{A}$$

$$= \frac{6.022\times10^{23}\frac{\text{atoms}}{\text{mol}}\times0.1638\times10^{-6}\frac{\text{g}}{\text{cm}^3}}{121.76\ \text{g/mol}}$$

$$N = 0.0081 \times 10^{17}$$

Density of antimony atoms, $N = 8.1 \times 10^{14}$ atoms/cm^3

Each antimony atom donates one electron. So, density of donated electrons is

$$n = N = 8.1 \times 10^{14}\ \text{electrons/cm}^3$$

Conductivity, $\sigma = nq\mu_n$

$\Rightarrow$ $\sigma = 8.1 \times 10^{14} \times 1.6 \times 10^{-19} \times 3500\ (\Omega\text{-cm})^{-1}$

$\Rightarrow$ $\sigma = 0.4536\ (\Omega\text{-cm})^{-1}$

End of Solution

Q4 (c) (i) The dimensions of the coil of a moving coil voltmeter are 3 cm and 2.5 cm and the coil has 150 turns. The scale has 100 divisions. The air gap flux is 0.15 Wb/m^2. Determine the series resistance when the meter is to be used for 0-100 V. The spring constant is 2.5 × 10^{-6} Nm per division and the resistance of the coil is 1 Ω.

(ii) In a 10 A dynamometer type ammeter, the rate of change of mutual inductance with deflection is constant and is equal to 0.005 μH per degree. It has a full scale deflection of 90°. Find the deflection when the current to be measured is 5 A.

[10 + 10 marks : 2024]

Solution:

(i) Controlling torque at full scale deflection

$$T_c = 2.5 \times 10^{-6}\ \text{Nm} \times 100$$
$$= 250 \times 10^{-6}\ \text{Nm}$$

Deflecting torque at full scale deflection

$$T_d = NBIdI$$
$$= 150 \times 0.15 \times 3 \times 10^{-2} \times 2.5 \times 10^{-2} I$$
$$= 168.75 \times 10^{-4} I\ \text{N-m}$$

At final steady position,

$$T_d = T_c$$
$$168.75 \times 10^{-4} I = 250 \times 10^{-6}$$
$$I = \frac{250\times10^{-6}}{168.75\times10^{-4}}$$

$$= 14.81 \text{ mA}$$

Let the resistance of the voltmeter circuit be R_s.

$\therefore$ Voltage across the instrument $= 14.81 \times 10^{-3}(R_s + 1)$

$\therefore$ Voltage across the instrument $= 14.81 \times 10^{-3}(R_s + 1)$

$$14.81 \times 10^{-3}(R_s + 1) = 100 \text{ V}$$

$$R_s + 1 = \frac{100 \times 10^3}{14.81}$$

$$= 6.752 \text{ k}\Omega$$

$$R_s = 6.751 \text{ k}\Omega$$

(ii) Given that : EDM type ammeter

$$I = 10 \text{ Amp, FSD} = \theta = 90° = \frac{\pi}{2} \text{ rad}$$

Rate of change of mutual inductance $= \dfrac{dM}{d\theta} = 0.005 \text{ μH/degree}$

We know that, $$\theta = \frac{I^2}{K_c} \times \frac{dM}{d\theta}$$

$$90° = \frac{(10)^2}{K_c} \times 0.005 \times 10^{-6} \text{ H/degree}$$

$$K_c = \frac{100}{\pi/2} \times 0.005 \times 10^{-6} \text{ H/degree}$$

$$K_c = 0.3183 \times 10^{-6} \text{ N-m/degree}$$

For a current of $I = 5$ Amp $\Rightarrow \theta = ?$

$$\theta = \frac{(5)^2}{0.3183 \times 10^{-6}} \times 0.005 \times 10^{-6} \text{ H/degree}$$

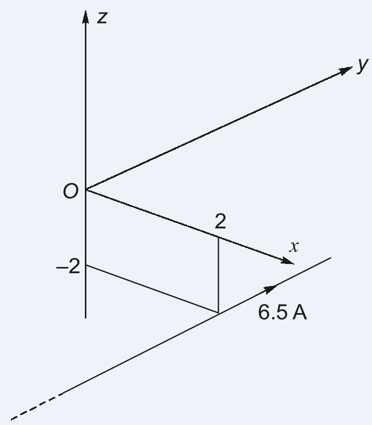
$$= 0.3927 \text{ rad}$$

$$\theta = 0.3927 \times \frac{180°}{\pi} = 22.5° \text{ degree}$$

End of Solution

SECTION : B

Q.5 (a) In the following figure, a current filament of 6.5 A in the $\vec{a}_y$ direction is parallel to the y-axis at x = 2 m, z = –2 m. Determine $\vec{H}$ at the origin.

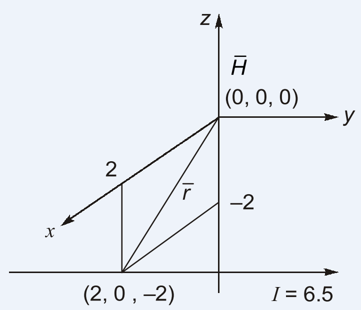


[12 marks : 2024]

Solution:

As we know that,

$$\bar{H} = \frac{I}{2\pi\rho}(\hat{a}_i \times \hat{a}_r) \text{ /R-H curl}$$



Direction:

$$\bar{r} = \bar{r}_f - \bar{r}_i = (0,0,0) - (2,0,-2)$$

$$\Rightarrow \quad \bar{r} = -2\hat{a}_x + 2\hat{a}_z$$

$$\Rightarrow \quad |\bar{r}| = 2\sqrt{2}$$

$$\therefore \quad \hat{a}_r = \frac{\bar{r}}{|\bar{r}|} = \frac{-\hat{a}_x + \hat{a}_z}{\sqrt{2}}$$

$$\hat{a}_I = \hat{a}_y$$

$$\therefore \quad \hat{a}_H = \hat{a}_I \times \hat{a}_r$$

$$= \hat{a}_y \times \frac{-\hat{a}_x + \hat{a}_z}{\sqrt{2}} = \frac{\hat{a}_z + \hat{a}_x}{\sqrt{2}}$$

Hence, $$\bar{H} = \frac{6.5}{2\pi \cdot 2\sqrt{2}} \cdot \frac{\hat{a}_x + \hat{a}_z}{\sqrt{2}}$$

$\Rightarrow$ $$\bar{H} = \frac{6.5(\hat{a}_x + \hat{a}_z)}{8\pi} \text{ A/m}$$

End of Solution

Q.5 (b) Derive an expression for the dielectric constant (ε_r) in elemental dielectrics, in terms of the atomic quantities.

[12 marks : 2024]

Solution:

(a) The elemental solid dielectrics are those materials which are build up of single type of atoms, e.g., diamond, silicon, germanium. Such materials contain neither ions nor dipoles.

(b) Hence, they exhibit only electronic polarization. So, for such materials

$$P_i = P_o = 0$$

$\therefore$ Total polarization is $$P = P_e$$

If α_e is electronic polarizability then

$$P = N\alpha_e . E_i \quad \text{...(i)}$$

where, E_i = internal field

for cubic symmetry structure atoms, internal field is given by

$$E_i = E + \frac{P}{3\varepsilon_o} \quad \text{...(ii)}$$

Put eqn. (ii) in eqn. (i)

$$P = N\alpha_e \left[E + \frac{P}{3\varepsilon_o}\right] \quad \text{...(iii)}$$

Also, we have $$P = \varepsilon_o(\varepsilon_r - 1).E \quad \text{...(iv)}$$

Equating eqn. (iii) and (iv)

$$\varepsilon_o(\varepsilon_r - 1).E = N\alpha_e \left[E + \frac{P}{3\varepsilon_o}\right]$$

$$\therefore \quad \varepsilon_o(\varepsilon_r - 1).E = N\alpha_e \left[E + \frac{\varepsilon_o(\varepsilon_r - 1)\cdot E}{3\varepsilon_o}\right]$$

$$\therefore \quad \varepsilon_o(\varepsilon_r - 1).E = N\alpha_e \cdot E\left[1 + \frac{(\varepsilon_r - 1)}{3}\right]$$

$$\therefore \quad \varepsilon_o(\varepsilon_r - 1) = N\alpha_e \left[\frac{3 + \varepsilon_r - 1}{3}\right]$$

$$\therefore \quad \varepsilon_o(\varepsilon_r - 1) = N\alpha_e \left[\frac{\varepsilon_r + 2}{3}\right]$$

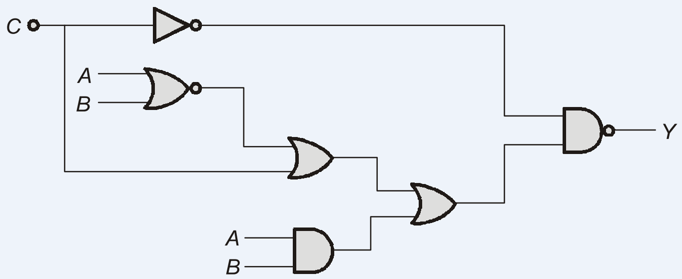
$$\therefore \quad \frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_o}$$

This is Clausius Mossotti equation.

End of Solution

Q.5 (c) (i) P is a 16 bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. Find the 2's complement representation of the product (8P).

(ii) In the circuit shown below if $C = 0$, find the expression for Y by minimization.



[6 + 6 marks : 2024]

Solution:

(i) Given : 2's compliment representation of number, $P = (F87B)_{16}$

Binary equivalent of 2's compliment of number $P = (1111\ 10000\ 0111\ 1100)_2$

The number P will be obtained by taking 2's compliment of above number

$$\text{2's compliment} = (0000\ 0111\ 1000\ 0100)_2$$

The above binary number is P

$$P = (0000\ 0111\ 1000\ 0100)_2$$

The decimal equivalent of number P

$$= (2^{10} + 2^9 + 2^8 + 1 \times 2^7 + 1 \times 2^2)$$

$$= (1924)_{10}$$

Product of 8 and number P

$$8P = (15392)_{10}$$

The binary equivalent of number $8P$

$$(8P) = 1111\ 0000\ 1000\ 00$$

To pad it into 16 bit format

$$8P = 0011\ 1100\ 0010\ 0000$$

$$\text{2's compliment of } 8P = (1100\ 0011\ 1110\ 0000)_2$$

Convert it into hexadecimal

$$= (C3E0)_{16}$$

(ii) Output expression Y,

$$Y = \overline{\overline{C} \cdot \overline{A+B} + AB}$$

$$= \overline{\overline{C}} + \left[\overline{(\overline{A+B+C}) + AB}\right]$$

$$= C + \left(\overline{\overline{A+B}+C}\right)\overline{AB}$$

$$= C + (A+B)(\overline{A}+\overline{B}) \cdot \overline{C}$$

$$Y = C + A\overline{B} + \overline{A}B \cdot \overline{C}$$

Given : $C = 0 \quad \Rightarrow \quad \bar{C} = 1$

$\therefore \quad Y = A\bar{B} + \bar{A}B = A \oplus B$

End of Solution

Q.5 (d) Explain with necessary diagrams how a dual slope integrating type ADC operates for conversion of analog input voltage into digital form.

[12 marks : 2024]

Solution:

Dual-Slope Integrating Type ADC : In dual slope ADC, the integrator two different ramps one with an unknown analog input voltage V_a as the input and another with a known reference voltage V_{ref} as the input. Hence, it is known as the dual slope ADC.

The dual-slope ADC has one of the slowest conversion time (typically 10 to 100 ms) but has the advantage of relatively low cost because it does not require precision components such as DAC or VCO, also they are insusceptible to noise and parameters variations due to the change in temperature.

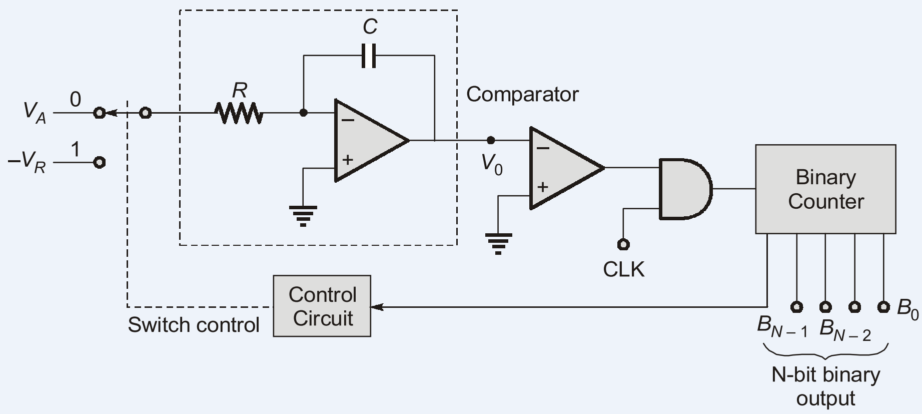
The figure shows the block diagram of a dual-slope integrating type ADC. It has four major building block as :

(i) An integrator

(ii) A voltage comparator

(iii) A binary counter

(iv) A switching/control circuit



The basic operation of this ADC involves the linear charging and discharging of capacitor '*C*', using constant currents.

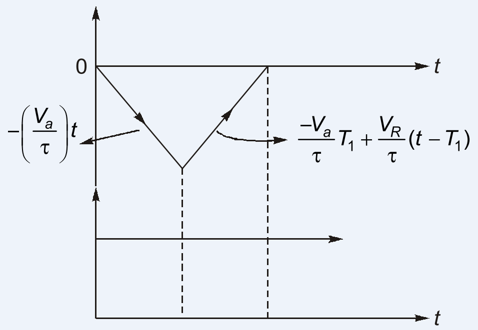
The conversion process starts at $t = 0$, when switch '*S*' is in position '*O*', connecting the analog input V_a to the input to the input of the integrator. The output of the integrator is

$$V_o = -\frac{1}{\tau}\int_0^t V_a dt = -\left(\frac{V_a}{\tau}\right)t$$

This result in HIGh V_c thus, enabling the AND gate and the clock pulses reach the Clk input terminal of the counter which was initially clear. The counter counts from 00 00

to 111 ..... 11 when $2^N - 1$ clock pulses are applied. At the next clock pulses 2^N, the counter is cleared and Q becomes 1. This controls the state of S_1 which now moves to position 1 at T_1, thereby connecting V_R to the input of the integrator. The output of the integrator now starts to move in the positive direction. The counter countinues to count until $V_o < 0$. As soon as V_o goes positive at T_2, V_C goes low disabling the AND gates. The counter will stop counting in the absence of the CLK pulses.

Waveforms of Dual-Slope ADC :



Since, $T_1 = 2N \times T_C$ and $T_2 = N \times T_C$

where, T_C = Time period of the CLK

when S_1 is at '1', then

$$V_o = -\frac{V_a}{\tau}T_1 + \frac{V_R}{\tau}(t - T_1)$$

From the figure above at T_2, $V_o = 0$.

Now,
$$(T_2 - T_1)\frac{V_R}{\tau} = \frac{V_a}{\tau} \times T_1$$

$$T_2 - T_1 = \frac{V_a}{V_R} \times T_1 = \frac{V_a}{V_R}2^N T_c$$

Let the count recorded in the counter be 'n' at T_2.

Then, $T_2 - T_1 = n \times T_C$

From the above equations,

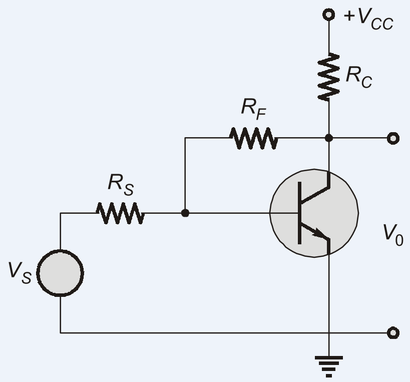
$$n \times T_C = \frac{V_a}{V_R}2^N T_C$$

$$n = \frac{V_a}{V_R}2^N$$

The above equation shows that the output of counter is proportional to the 'V_a'.
The count recorded in the counter is numerically equals to 'V_a' if $V_R = 2^N$.

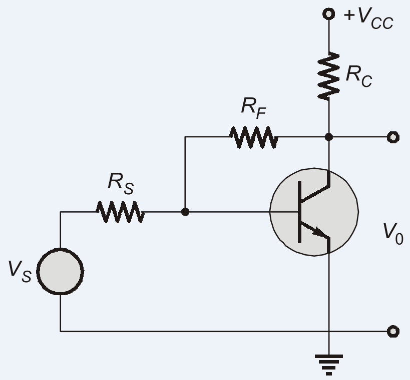
End of Solution

Q.5 (e) Identify the type of feedback in the BJT circuit shown below. Draw the general feedback model showing the basic amplifier and feedback network. Find the input impedance and output impedance with feedback of the circuit shown. Its parameters are R_C = 5 kΩ, R_F = 50 kΩ, R_S = 15 kΩ, h_{ie} = 1200 Ω, h_{fe} = 60 and $h_{re} = h_{oe} = 0$.



[12 marks : 2024]

Solution:



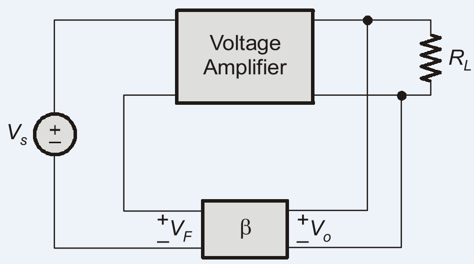
R_C = 5 kΩ, R_F = 50 kΩ, R_S = 15 kΩ, h_{ie} = 1200 Ω, h_{fe} = 60 and $h_{re} = h_{oe} = 0$

Input impedance, $Z'_1 = ?$

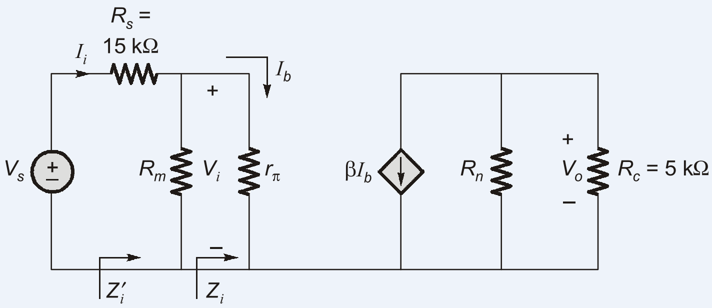
Output impedance, $Z'_0 = ?$

In this circuit R_F is feedback element and direct connected at output, i.e., voltage sampling and indirect connected to input, i.e., voltage mixing series-shunt feedback or voltage series feedback.

General Feedback Model :



Small Signal Analysis :



$$r_\pi = h_{ie} = 1.2 \text{ k}\Omega$$

$$R_m = \frac{R_F}{1-A_V}$$

$$R_n \cong 50 \text{ k}\Omega$$

$$V_o = -(R_n \parallel R_c)\beta I_b$$

$$V_i = h_{ie} I_b$$

$$\frac{V_o}{V_i} = A_V = \frac{-(R_n \parallel R_c)\beta}{h_{ie}} = \frac{-(50 \parallel 5)60}{1.2} = -227.272$$

$$R_m = \frac{50\times10^3}{1+227.272} = \frac{50\times10^3}{228.272} = 219.036 \ \Omega$$

$$Z_i = \frac{V_i}{I_b} = h_{ie}$$

$$Z'_i = \frac{V_s}{I_i} = R_m \parallel r_\pi + R_s$$

$$= 0.219 \parallel 1.2 + 15 = 0.185 + 15$$

$$Z'_i = 15.185 \text{ k}\Omega$$

Calculate Z'_o :

$$Z'_o = R_n \parallel R_c = \frac{50\times5}{55} = 4.545 \text{ k}\Omega$$

End of Solution

Q.6 (a) (i) Draw the circuit diagram of Hartley oscillator using FET. If L_1 = 15 mH and C = 50 pF, calculate L_2 for a frequency of oscillation of 168 kHz. The mutual inductance between L_1 and L_2 is 5 μH. Find the required value of μ of FET to be used for this circuit.

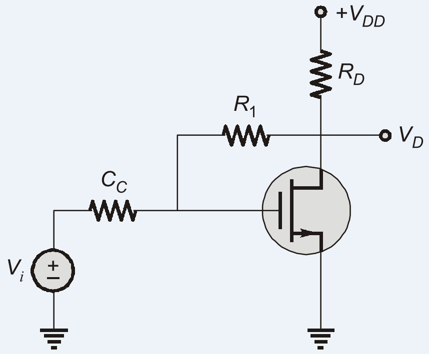
(ii) For the FET amplifier below, determine V_{DQ} and I_{DQ}. Assume that FET is operating in its saturation region.

$$V_{DD} = 12 \text{ V}$$

$$K_n = 0.24 \text{ mA/V}^2$$

$$V_{TN} = 3 \text{ V}$$

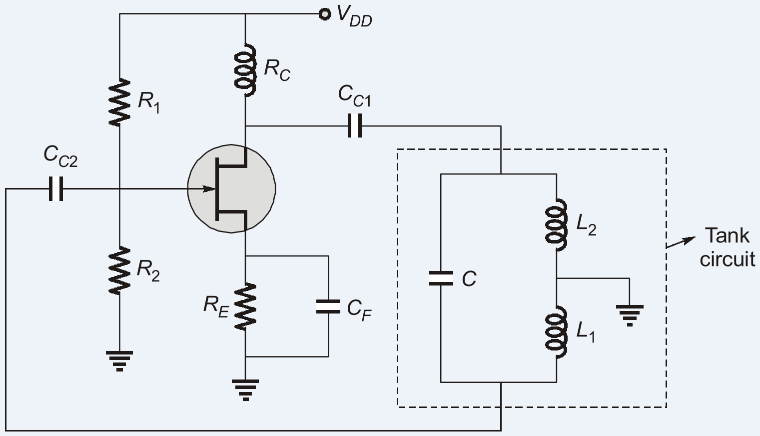
R_1 = 10 M
R_D = 2 K
λ = 0



[10 + 10 marks : 2024]

Solution:

(i)



FET hartly oscillator
We know that,
For Hartely oscillator the frequency of oscillation is given by

$$f = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

$$168 \times 10^3 = \frac{1}{2\pi\sqrt{50\times10^{-12}(15\times10^{-3} + L_2) + 2M}}$$

$$\sqrt{50\times10^{-12}(15\times10^{-3} + L_2) + 2M} = \frac{1}{2\pi\times168\times103}$$

$$50 \times 10^{-12}\,(15 \times 10^{-3} + L_2) + 2M = 8.97 \times 10^{-13}$$

$$\alpha_2 + 2M = 2.94 \times 10^{-3}$$

$$\alpha_2 = 2.94 \times 10^{-3} - 2(5 \times 10^{-6})$$

$$\alpha_2 = 2.93 \text{ mH}$$

$$g_m R_C \geq \frac{L_1}{L_2}$$

$$\mu \geq \frac{L_1}{L_2}$$

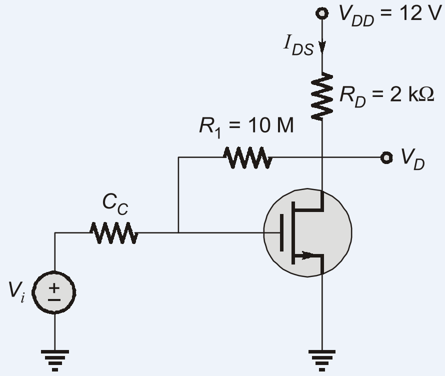
$$\mu \geq \frac{15 \times 10^{-3}}{2.93 \times 10^{-3}}$$

$$\mu \geq 5.12$$

(ii) $$-12 + I_{DS} \times 2K + V_{DS} = 0$$

$$I_{DS} = \frac{12 - V_{DS}}{2K} \quad \text{...(i)}$$

$$I_{DS} = K_N[V_{GS} - V_{TN}]^2$$
$$I_{DS} = 0.24[V_{GS} - 3]^2 \text{ mA} \quad \text{...(ii)}$$



From equation (i) and (ii)

$$\frac{12 - V_{DS}}{2K} = 0.24[V_{DS} - 3]^2 \times 10^{-3}$$

$$12 - V_{DS} = 0.48[V_{DS} - 3]^2$$
$$0.48(V_{DS}^2 + 9 - 6V_{DS}) = 12 - V_{DS}$$
$$0.48(V_{DS}^2 + 4.32 - 2.88V_{DS} + V_{DS} = 12$$
$$0.48(V_{DS}^2 - 1.88V_{DS} - 7.68 = 0$$
$$V_{DS} = 6.41 \text{ V}, -2.5 \text{ V}$$

For FET to work in saturation region, $V_{DS} > V_T$. Thus, $V_{DS} = -2.5$ V discarded.

Now, $$I_{DS} = 0.24[6.41 - 3]^2 \text{ mA}$$
$$I_{DS} = 2.8 \text{ mA}$$

Hence, $$V_{DS} = V_D - V_S \quad \Rightarrow \quad V_D = 6.41 \text{ V and } I_D = 2.8 \text{ mA}$$

End of Solution

Q.6 (b) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, compute $y(0.02)$ in steps of 0.02 using (i) Modified Euler's method and (ii) Fourth order Runge-Kutta method correct to four decimal places.

[20 marks : 2024]

Solution:

(i) By Euler's modified method :

Given : $h = 0.02,\ x_0 = 0$

So that $x_1 = x_0 = 0 + 0.02 = 0.02$

$y_1 = y_{(0.02)} = ?$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

where, $y_1^{(0)} = y_0 + hf(x_0, y_0)$

$$y_1^{(0)} = 1 + (0.02)\left[\frac{y_0 - x_0}{y_0 + x_0}\right] = 1 + 0.02 = 1.02$$

Sub in $y_1^{(1)}$:

$$y_1^{(1)} = 1 + \frac{0.02}{2}\left[\frac{y_0 - x_0}{y_0 + x_0} + \frac{y_1^{(0)} - x_1}{y_1^{(0)} + x_1}\right]$$

$$= 1 + 0.01\left[\frac{1-0}{1+0} + \frac{1.02 - 0.02}{1.02 + 0.02}\right]$$

$$= 1 + (0.01)\left[1 + \frac{1}{1.04}\right] = 1.0196$$

$$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1,\ y_1^{(1)})]$$

$$= 1 + 0.01\left[\frac{1-0}{1+0} + \frac{1.0196 - 0.02}{1.0196 + 0.02}\right] = 1.0196$$

$\therefore$ $y_1 = 1.0196 = y(0.02)$

(ii) By R-K 4th Order :

$$y_1 = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$K_1 = hf(x_0, y_0) = (0.02)\left[\frac{1-0}{1+0}\right] = 0.02$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$

$$= 0.02\left[\frac{y_0 + \frac{K_1}{2} - x_0 + \frac{h}{2}}{y_0 + \frac{K_1}{2} + x_0 + \frac{h}{2}}\right]$$

$$= 0.02\left[\frac{1+\frac{0.02}{2}-0+\frac{0.02}{2}}{1+\frac{0.02}{2}+0+\frac{0.02}{2}}\right] = 0.02$$

$$K_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}\right)$$

$$= (0.02)\left[\frac{1+\frac{0.02}{2}-0+\frac{0.02}{2}}{1+\frac{0.02}{2}+0+\frac{0.02}{2}}\right] = 0.02$$

$$K_4 = hf(x_{0,th}, y_0 + K_3)$$

$$= (0.02)\left[\frac{(1+0.02)-(0+0.02)}{1+0.02+(0+0.02)}\right] = 0.0192$$

$$y_1 = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$= 1+\frac{1}{6}[0.02 + 2(0.02 + 0.02) + 0.0192]$$

$$y_1 = 1.0198$$

End of Solution

Q.6 (c) (i) A digital computer has a memory unit with 32 bits per word. The instruction set consists of 270 different operations. All instructions have an operation code and an address part allowed for only one address. Each instruction is stored in one word of memory. Find the number of bits needed for op-code, for address part of instruction and the maximum allowable size of memory.

(ii) The distance between two stations is '*L*' kilometers and all the frames are '*K*' bits log, and the propagation delay per kilometer is '*t*' seconds and the channel capacity between them is 'R' bits/second. Find the minimum number of bits '*b*' for the sequence number field in a frame for maximum utilization, if the sliding window protocol is used. Assume processing delay to be negligible. Derive the equation for '*b*'.

[10 + 10 marks : 2024]

Solution:

(i)

Word Size = 32 bit
\# Instructions = 270
Instruction size = 1 word = 32 bit

Instruction format =

OPCODE	Addr
log 270	23 bit
$Log_2 2^9$	
9 bit	

$$\text{Opcode Size} = 9 \text{ bit}$$
$$\text{Address size} = 23 \text{ bit}$$
$$\begin{aligned}\text{Maximum Memory} &= 2^{23} \text{ cells}\\ &= 2^{23} * 32 \text{ bit}\\ &= 2^{23} * \text{word}\\ &= 8\text{M Word}\\ &= 8\text{M} * 32 \text{ bit}\\ &= 8\text{M} * 4\text{B}\\ &= 32 \text{ MB}\end{aligned}$$

(ii) Available sequence number ≥ Sender window size + Receiver window size

- For sliding window protocol, neglect the receiver window size.
- Available sequence number ≥ Sender window size
- The propagation delay $T_p = L * t$ sec
- The transmission delay, $T_d = \dfrac{\text{Frame size}}{\text{Bandwidth}}$

$$T_d = \frac{K}{R}\text{sec}$$

- The sender window size = $1 + 2a$

For maximum utilization, η = 100%, then the sender must send $(1 + 2a)$ frames.

$$1 + 2a = 1+2\left(\frac{Lt}{K/R}\right) = 1+2\left(\frac{LtR}{K}\right)$$

$$1 + 2a = \frac{K+2LRt}{K}$$

The minimum number of bits in a sequence is 'b'.

$$b \geq \log_2\left(\frac{K+2LRt}{K}\right)$$

End of Solution

Q.7 (a) (i) The reverse saturation currents IS1 and IS2 of transistors Q1 and Q2 respectively, which are shown in the circuit below are :

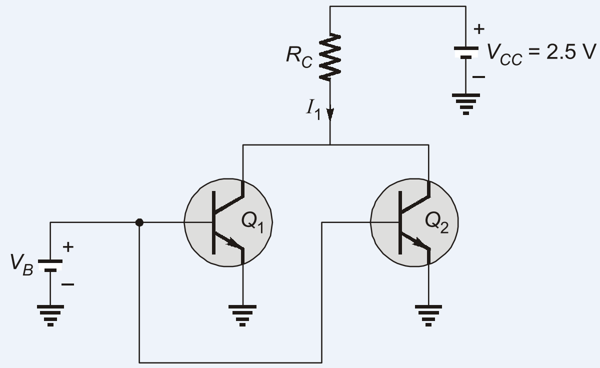
$$I_{S1} = 2I_{S2} = 5 \times 10^{-16} \text{ A.}$$

If $I_1 = 1.2$ mA, find :

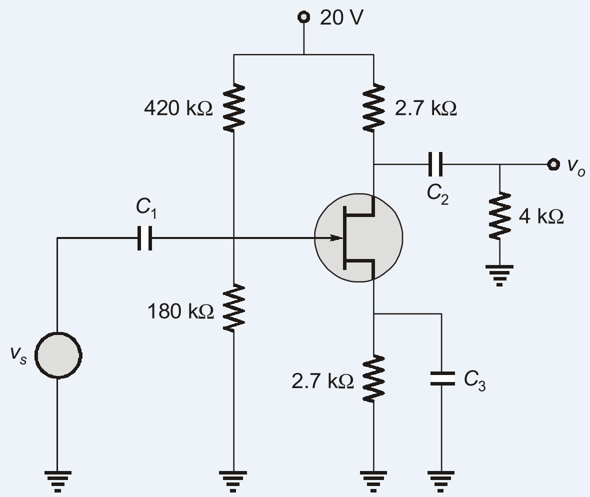
(a) the value of V_B, and

(b) the value of R_C which places transistors at the edge of active region.

Assume volt equivalent of temperature $V_T = 26 \times 10^{-3}$ V.



(ii) An n-channel JFET amplifier circuit is shown in the figure below. The JFET parameters are the drain to source saturation current I_{DSS} = 12 mA, pinch off voltage V_P = –4 V, the channel length modulation coefficient λ = 0.008 V^{-1}. Find the small signal transconductance g_m and voltage gain A_v.



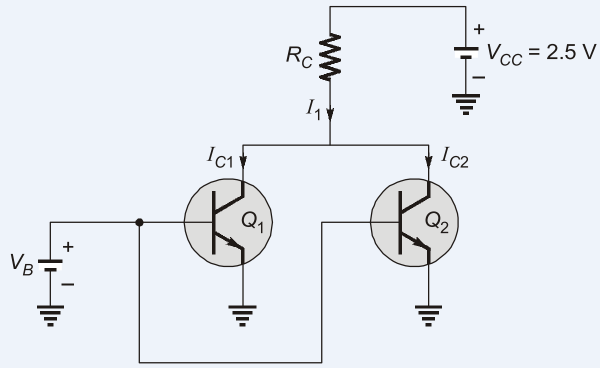
[10 + 10 marks : 2023]

Solution:

(i)

$$I_{S1} = 2I_{S2} = 5 \times 10^{-16} \text{ A}$$
$$I_1 = 1.2 \text{ mA}$$
$$V_T = 26 \text{ mV}$$

Given circuit :



Calculation of V_B :

$$I_1 = I_{C1} + I_{C2}$$

$$I_1 = I_{S1}e^{V_B/V_T} + I_{S2}e^{V_B/V_T}$$

$$I_1 = (I_{S1} + I_{S2})e^{V_B/V_T}$$

$$V_B = V_T \ln\left(\frac{I_1}{I_{S1} + I_{S2}}\right)$$

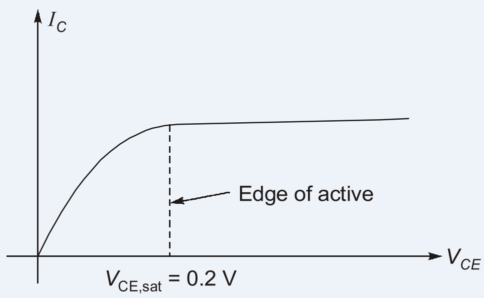
$$= V_T \ln\left(\frac{I_1}{I_{S1} + \frac{I_{S1}}{2}}\right)$$

$$= V_T \ln\left(\frac{I_1}{\frac{3}{2}I_{S1}}\right)$$

$$= 26 \text{ mV} \ln\left(\frac{1.2 \times 10^{-3}}{\frac{3}{2} \times 5 \times 10^{-16}}\right) = 730.6 \text{ mV}$$

$$V_B = 730.6 \text{ mV}$$

(ii) Transistors at edge of active mode



$$V_{CE\,sat} = V_C \text{ edge of active}$$

Applying KVL :

$$V_{CC} = I_1 R_C + V_C$$

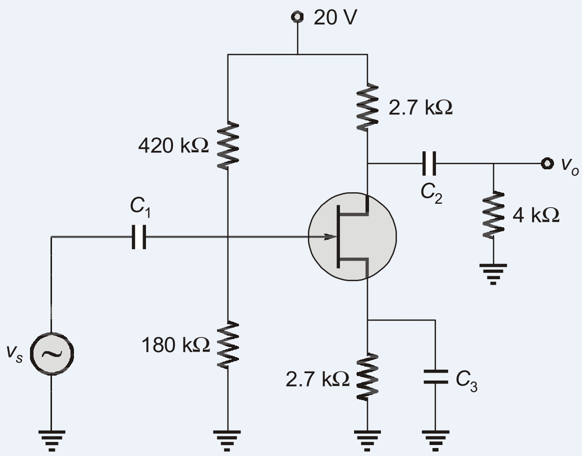
$$R_C \text{(Edge of active)} = \frac{V_{CC} - V_{C\,\text{edge}}}{I_1}$$

$$= \frac{25 - 0.2}{1.2 \times 10^{-3}}$$

$$R_C = \frac{2.3}{1.2}\text{k}\Omega = 1.91\ \text{k}\Omega$$

$$R_{C\,\text{edge}} = 1.91\ \text{k}\Omega$$

(ii) Given circuit :



Given n-channel JFET

$$I_{DSS} = 12\ \text{mA}$$
$$V_P = -4\ \text{V}$$
$$\lambda = 0.008\ \text{V}^{-1}$$

Calculation of g_m :

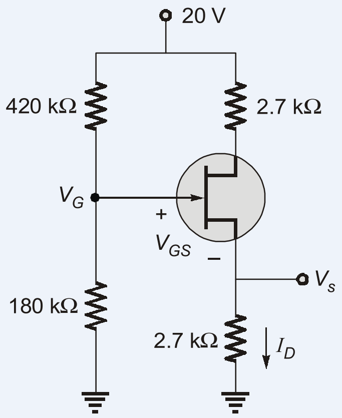
$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 (1 + \lambda V_{DS})$$

d.w.r.t. V_{GS}

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{-2I_{DSS}}{V_P}\left(1 - \frac{V_{GS}}{V_P}\right)(1 + \lambda V_{DS})$$

We have to calculate V_{GS} and V_{DS} to solve g_m

DC model:



$$V_G = V_{GS} + I_D R_S$$

$$V_G = 20 \times \frac{180\text{ k}}{600\text{ k}} = 6\text{ V}$$

$$6 = V_{GS} + I_D \times 2.7\text{ K}$$

$$I_D = \frac{6 - V_{GS}}{2.7\text{ K}} \quad \text{...(i)}$$

$$I_D = I_{DDS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 \quad \text{...(ii)}$$

$$\frac{6 - V_{GS}}{2.7\text{ K}} = 12\text{ mA}\left(1 - \frac{V_{GS}}{(-4)}\right)^2$$

$$6 - V_{GS} = 32.4\,[1 + V_{GS} \times 0.25]^2$$

$$= 32.4\,[1 + 0.5\,V_{GS} + 0.0625\,V_{GS}^2]$$

$$= 32.4 + 162\,V_{GS} + 2\,V_{GS}^2$$

$$2\,V_{GS}^2 + 163\,V_{GS} + 26.4 = 0$$

$$V_{GS}^2 + 81.5 V_{GS} + 13.2 = 0$$

$$V_{GS} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-81.5 \pm \sqrt{6642.25 - 4 \times 1 \times 13.2}}{2}$$

$$= \frac{-81.5 \pm \sqrt{6642.25 - 52.8}}{2}$$

$$= \frac{-81.5 \pm \sqrt{6589.45}}{2}$$

$$= \frac{-81.5 \pm 81.17}{2}$$

$$= \frac{-81.5 + 81.17}{2}, \frac{-81.5 - 81.17}{2}$$

$$= -0.165 \text{ V } (\checkmark), \quad -81.333 \,(\times)$$

$$V_{GS} = -0.165 \text{ V}$$

$$I_D = \frac{6 - V_{GS}}{2.7\text{ k}}$$

$$= \frac{6-(-0.165)}{2.7\text{ k}} = \frac{6.165}{2.7\text{ k}} = 2.28 \text{ mA}$$

$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$

$$= 20 - 2.28 \text{ mA } (2.7\text{ k} + 2.7\text{ k})$$

$$= 20 - 12.31 \text{ V}$$

$$= 7.69 \text{ V}$$

Finally,

$$V_{GS} = -0.165 \text{ V},$$

$$V_{DS} = 7.69 \text{ V}$$

$$g_m = \frac{-2I_{DSS}}{V_P}\left(1 - \frac{V_{GS}}{V_P}\right)(1 + \lambda V_{DS})$$

$$= \frac{-2 \times 12 \text{ mA}}{-4}\left[1 - \left(\frac{-0.165}{-4}\right)\right][1 + 0.008 \times 7.69]$$

$$= 6[1 - 0.041]\,[1 + 0.0615]$$

$$= 6[0.959]\,[1.0615]$$

$$g_m = 6.107 \text{ mS}$$

$$A_V \text{(By pass)} = -g_m R_L$$

$$R_L = 2.7\text{ k} \,||\, 4\text{ k}$$

$$= 1.61 \text{ k}\Omega$$

$$A_V = -6.107 \text{ mS} \times 1.61 \text{ k}\Omega$$

$$= -9.83$$

End of Solution

Q.7 (b) (i) A 1000/100 V potential transformer has the following parameters :

Primary resistance = 97.5 Ω
Primary reactance = 65.4 Ω
Secondary resistance = 0.86 Ω
Total equivalent reactance = 110 Ω
Magnetizing current at 0.4 p.f. = 0.02 A

Find (I) the phase angle error at no load, and (II) load in VA at unity power factor at which the phase angle error will be zero.

(ii) A Phantom loading arrangement is used to test a 220 V, 5 A dc energy meter at its marked ratings. The resistance of the pressure coil circuit is 8800 Ω and that of current coil is 0.1 Ω. Calculate the power consumed when testing the meter with :

(I) Direct loading arrangements

(II) Phantom loading with current coil circuit excited by a 6 V battery.

[12 + 8 marks : 2024]

Solution:

(i) No load power factor, $\cos\alpha = 0.4$

$$\sin\alpha = \sqrt{(1)^2 - (0.4)^2} = 0.917$$

$$I_m = I_o \sin\alpha$$

$$0.02 = I_o \times 0.917$$

$$I_o = 0.0218 \text{ A}$$

$$I_e = I_o \cos\alpha = 0.0218 \times 0.4$$

$$= 0.008724 \text{ A}$$

Turns ratio, $n = \dfrac{1000}{100} = 10$

Phase angle, $$\theta = \frac{\frac{I_s}{n}(X_P \cos\Delta - R_P \sin\Delta) + I_e x_P - I_m r_P}{nV_s}$$

At no load $I_s = 0$

$\therefore$ $$\theta = \frac{I_e x_P - I_m x_P}{nV_s}$$

$$= \frac{0.008724 \times 65.4 - 0.02 \times 97.5}{10 \times 100}$$

At unity powr factor, $\cos\Delta = 1$ and $\sin\Delta = 0$

$$\theta = \frac{\frac{I_s}{n} X_P - I_e x_P - I_m r_P}{nV_s}$$

For $\theta = 0$,

$$\frac{I_s}{n} X_P + I_e X_P - I_m r_P = 0$$

$$I_S = \frac{n}{X_P}(I_m r_P - I_e X_P) = 0.1254$$

$$= \frac{10}{110}(0.02 \times 97.5 - 0.008724 \times 65.4)$$

$$= 0.1254$$

Burden $= V_S I_S$

$= 100 \times 0.1254 = 12.54$ VA

(ii) Given : $V = 220$ V, $I = 5$ A

$R_P = 8800\ \Omega$, $R_C = 0.1\ \Omega$

$$P = \frac{V^2}{R_P} = \frac{220 \times 220}{8800} = 5.5 \text{ W}$$

(I) Direct loading, $V \times I = 220 \times 5 = 1100$ W

Total power $= 5.5 + 1100 = 1105.5$ W

(II) During phantom loading :

Power consumed by current coil = $6 \times 5 = 30$ W

Total power consumed = $5.5 + 30$

$= 35.5$ W

So, the phantom loading consume less power.

End of Solution

Q.7 (c) (i) If $\bar{F} = (2x^3 - 3z)\boldsymbol{i} - 2xyj - 4xk$, then evaluate $\iiint\limits_V \nabla \times \bar{F} dV$, where V is bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$ where $\nabla = \boldsymbol{i}\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$.

(ii) Find the Fourier series for $f(x) = x^2$, $0 < x < 4$. Also, draw the graph of its periodic extension.

[10 + 10 marks : 2024]

Solution:

(i) V is bounded by region under plane $2x + 2y + z = 4$ in the 1st octant.

$$\bar{\nabla} \times \bar{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^3 - 3z & -2xy & -4x \end{vmatrix}$$

$$= i(0 - 0) - j(-4 + 3) + k(-2y)$$

x : 0 to 2

y : 0 to $2 - x$

z : 0 to $4 - 2x - 2y$

$$\bar{\nabla} \times \bar{F} = \hat{j} - 2y\hat{k}$$

$$\iiint\limits_V (\hat{j} - 2y\hat{k})dxdydz = \int_0^2 \int_0^{2-x} \int_0^{4-2x-2y} (\hat{j} - 2y\hat{k})dzdydx$$

$$= \int_0^2 \int_0^{2-x} [j(4 - 2x - 2y) - 2y(4 - 2x - 2y)K]dydx$$

$$= \int_0^2 \left[j[(4 - 2x)y - y^2] - 2k\left[(4 - 2x)\frac{y^2}{2} - \frac{2y^3}{3}\right]_0^{2-x} \right] dx$$

$$= \int_0^2 \left[j[2(2 - x)^2 - (2 - x)^2] - 2k\left[(2 - x)^3 - \frac{2}{3}(2 - x)^3\right] \right] dx$$

$$= \int_0^2 \left[j(2 - x)^2 - \frac{2k}{3}(2 - x)^3 \right] dx = \frac{8}{3}[j - k]$$

(ii) Given : $f(x) = x^2 \quad (0 < x < 4)$

Fourier series of $f(x)$

$$f(x) = a_0 + \sum_{n=1,2,3}^{\infty} (a_n \cos n\omega_o x + b_n \sin n\omega_o x)$$

Given : $T_o = 4$: $\omega_o = \frac{2\pi}{4} = \frac{\pi}{2}$

$$a_o = \frac{1}{T_o}\int_{T_o} f(x)dx = \frac{1}{4}\int_0^4 x^2 dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

$$= \frac{1}{4}\left[\frac{64}{3}\right] = \frac{16}{3}$$

$$a_n = \frac{2}{T_o}\int_0^{T_o} f(x)\cos h\omega_o x dx = \frac{2}{4}\int_0^4 f(x)\cos\frac{h\pi}{2}x\,dx$$

$$= \frac{1}{2}\int_0^4 x^2\cos\frac{h\pi x}{2}dx$$

$$= \frac{1}{2}\left[\frac{x^2\sin\frac{h\pi}{2}x}{\left(\frac{h\pi}{2}\right)} + \frac{2x\cos\frac{h\pi}{2}x}{\left(\frac{h\pi}{2}\right)^2} - \frac{2\sin\left(\frac{h\pi}{2}\right)^2 x}{\left(\frac{h\pi}{2}\right)^3}\right]_0^4$$

$$= \frac{1}{2}\left[\frac{16\sin 2n\pi}{\frac{h\pi}{2}} + \frac{2\times 4\cos 2n\pi}{\left(\frac{n\pi}{2}\right)^2} - \frac{2\sin 2n\pi}{\left(\frac{n\pi}{2}\right)^3} + 0\right]$$

$$= \frac{1}{2}\left[16\times 0 + \frac{2\times 4\cos 2n\pi}{\frac{n^2\pi^2}{4}} - 0\right]$$

$$a_n = \frac{16}{\pi^2 n^2}\cos 2n\pi$$

$$b_n = \frac{2}{T_o}\int_{T_o} x^2\frac{\sinh\pi x}{2}dx$$

$$b_n = \frac{2}{4}\int_0^4 x^2\frac{\sin n\pi x}{2}dx$$

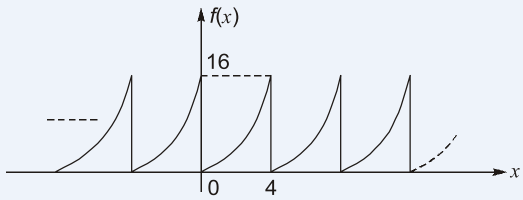
$$= \frac{1}{2}\int_0^4 x^2\frac{\sin n\pi x}{2}dx$$

$$= \frac{1}{2}\left[\frac{-x^2\cos\frac{h\pi}{2}x}{\frac{h\pi}{2}} + \frac{2x\sin\frac{h\pi}{2}x}{\left(\frac{h\pi}{2}\right)^2} + \frac{2\cos\frac{h\pi}{2}x}{\left(\frac{h\pi}{2}\right)^3}\right]_0^4$$

$$= \frac{1}{2}\left[\frac{-16\cos 2n\pi}{\left(\frac{h\pi}{2}\right)} + \frac{8\sin 2n\pi}{\left(\frac{h\pi}{2}\right)^2} + \frac{2\cos 2n\pi}{\left(\frac{h\pi}{2}\right)^3} - \frac{2}{\left(\frac{n\pi}{2}\right)^3}\right]$$

$$b_n = \frac{1}{2}\left[\frac{-16\cos 2n\pi}{\left(\frac{n\pi}{2}\right)} + \frac{2\cos 2n\pi}{\left(\frac{n\pi}{2}\right)^3} - \frac{2}{\left(\frac{n\pi}{2}\right)^3}\right]$$

Periodic extension of $x^2 : 0 < x < 4$



End of Solution

Q.8 (a) (i) A strain gauge of cross-sectional area 3.6 cm^2 has been bonded to a beam 0.1 m long. The gauge factor of the strain gauge is 2.2 and the unstrained resistance of the strain gauge is 220 Ω. Young's modulus of steel (beam) is 207 GN/m^2. Due to application of a load, the resistance of the gauge changes by 0.015 Ω.

Calculate the change in length of the steel beam and the amount of force applied to the beam.

(ii) Specify the reasons of using 'Sample and Hold' circuits in multi-channel data acquisition system.

Draw and explain briefly the working of a S/H circuit.

[10 + 10 marks : 2024]

Solution:

(i) Given data :

Cross sectional area, $A = 3.6$ cm^2

Length of beam, $l = 0.1$ m

Gauge factor, $G = 2.2$

Unstrained resistance of strain gauge,

$R = 220\ \Omega$

Young's modular, $Y = 207$ GN/m^2

Change in resistance, $\Delta R = 0.015\ \Omega$

Now, Gauge factor, $G = \dfrac{\Delta R / R}{\Delta l / l}$

$$2.2 = \frac{\frac{0.015}{220}}{\frac{\Delta l}{l}}$$

$$\frac{\Delta l}{l} = 3.1 \times 10^{-15}$$

Change in length, $\Delta l = 3.1 \times 10^{-5} \times l$

$$= 3.1 \times 10^{-5} \times (0.1)$$

$$\Delta l = 3.1 \times 10^{-6}\ \text{m}$$

Since, Young's Modular, $Y = \dfrac{\text{Stress}}{\text{Strain}}$

$$\text{Stress}\left(\frac{F}{A}\right) = Y \times \frac{\Delta l}{l}$$

$$= 207 \times 10^{9} \times 3.1 \times 10^{-6}$$

$$= 6.41528 \times 10^{6}\ \text{N/m}^2$$

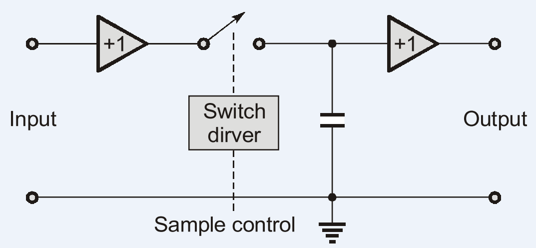
Amount of force applied, $F = 6.415 \times 10^{6} \times A$

$$= 6.415 \times 10^{6} \times 3.6 \times 10^{-4}$$

$$F = 230.95\ \text{N}$$

(ii) In data-acquisition systems, sample-holds are used either to "freeze" fast-moving signals during conversion or to store multiplexer outputs while the signal is being converted and the multiplexer is seeking the next signal to be converted. In analog data-reduction they may be used to determine peaks or valleys, establish amplitudes in resolver-to-digital conversion and facilitate analog computations involving signals obtained at different instants of time. In data-distribution systems, sample-holds are used for holding converted data between updates. Fast sample-holds may be used to acquire and measure fast pulses of arbitrary timing and width.

Sample-hold circuits are the device that store analog information and reduce the aperture time of an A/D converter. A sample hold is simply a voltage-memory device in which an input voltage is acquired and then stored on a high-quality capacitor.



A_1 is an input buffer amplifier with a high input impedance so that the source, which may be an analog multiplexer, is not loaded. The output of A_1 must be capable of driving the hold capacitor with stability and enough derive current to change it rapidly. S_1 is an electronic switch, generally an FET, which is rapidly switched on or off by a driver circuit that interfaces with TTL inputs.

C is a capacitor with low leakage and low dielectric absorption characteristics, it is a polystyrene, polycarbonate or Teflon type. In the case of hybrid sample-holds, the MOS-type capacitor is frequency used.

A_2 is the output amplifier that buffers the voltage on the hold capacitor. It must, therefore, have extremely low input bias current, and for this reason an FET input amplifier is required.

There are two modes of operation for a sample-hold sample (or tracking) mode when the switch is closed; and hold mode, when the switch is open.

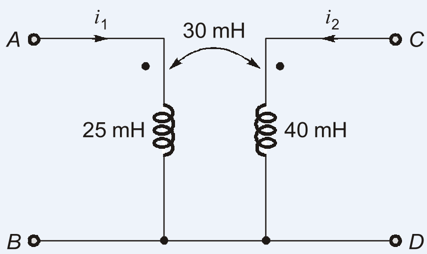
Sample-holds are usually operated in one of two basic ways. The device can continuously track the input signal and be switched into the hold mode only at certain specified times, spending most of the time in tracking mode. This is the case for a sample-hold employed as a degilitcher at the output of a D/A converter.

Alternatively, the device can stay in the hold mode most of the time and go to the sample mode just to acquire a new input signal level. This is the case for a sample-hold used in data-acquisition system following the multiplexer.

End of Solution

Q.8 (b) (i) A series RC circuit has R = 10 kΩ and C = 15 μF and the circuit has two voltage sources in series, given by $v_1 = 20u(-t)$ V and $v_2 = 20u(t - t')$ V. Determine the complete expression for the voltage across the capacitor and plot it as a function of time, assuming t' as a positive quantity.

(ii) Find the T equivalent of the linear network given in the following figure.

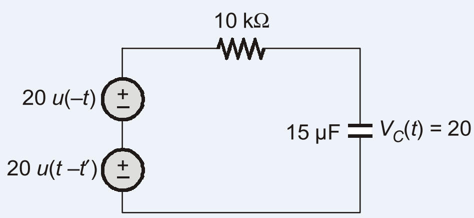


Establish the equivalence between the network and its T equivalent.

[12 + 8 marks : 2024]

Solution:

(i)



$t < 0$:

The capacitor reaches to steady state and its voltage is 20 V.

$0 < t < t'$:

Voltage across capacitor

$$V_c(t) = \text{Final value} + (\text{Initial value} - \text{Final value})e^{-t/\tau}$$

Final value = 0; Initial value = 20 V;

$$\tau = RC = 10 \times 10^3 \times 15 \times 10^{-6} = 150 \text{ msec}$$

At $t = t'$, Voltage across capacitor is

$$V_c(t) = 0 + (20 - 0)e^{-t/150\times10^{-3}}$$

$$V_c(t') = 20e^{-6.67t'} \quad (0 < t < t')$$

$t > t'$:

$$V_c(t') = 20e^{-6.67t'}$$

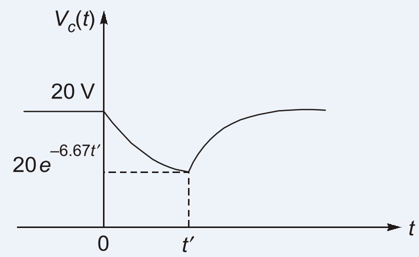
$$V_c(t) = \text{Final value} + (\text{Initial value} - \text{Final value})\, e^{-(t-t')/\tau}$$

$$\text{Final value} = 20 \text{ V}, \quad \text{Initial value} = 20e^{-6.67\tau t'}$$

$$V_c(t) = 20 + \left(20e^{-6.67t'} - 20\right)e^{-(t-t')/150\times10^{-3}}$$

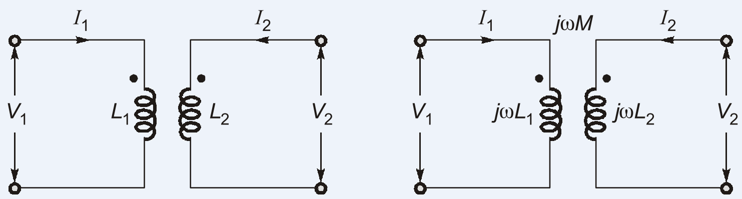
$$V_c(t) = 20 + \left(20e^{-6.67t'} - 20\right)e^{-6.67(t-t')}$$

or use Laplace transform to get a complete response in one equation.



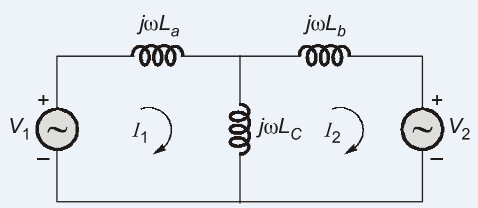
$$V_c(t) = 20 \text{ V}, \qquad (t < 0)$$
$$V_c(t) = 20e^{-6.67t} \qquad (0 < t < t')$$
$$V_c(t) = 20 + (20e^{-6.67t'} - 20)e^{-6.67(t-t')} \qquad (t > t')$$

(ii)



$$V_1 = (j\omega L_1)I_1 + (j\omega M)I_2$$
$$V_2 = (j\omega L_2)I_2 + (j\omega M)I_1$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{...(1)}$$



Loop (1) [KVL]

$$V_1 = j\omega[L_a + L_c]I_1 + (j\omega L_c)I_2$$

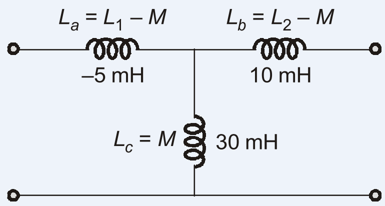
Loop (2) [KVL]

$$V_2 = j\omega L_c I_1 + j\omega[L_b + L_c]I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega[L_a + L_c] & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{...(2)}$$

Comparing eqn. (1) and eqn. (2)

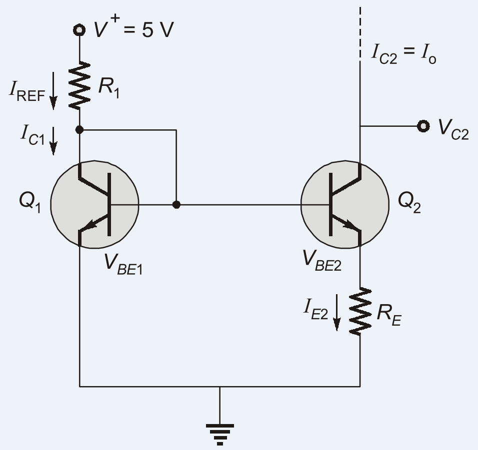
$$L_1 = L_a + L_c$$
$$M = L_c$$
$$L_2 = L_b + L_c$$
$$L_a = L_1 + L_c = L_1 - M$$
$$L_b = L_2 - L_c = L_2 - M$$



$$L_1 = 25 \text{ mH}$$
$$L_2 = 40 \text{ mH}$$
$$M = 30 \text{ mH}$$
$$L_a = 25 - 30 = -5 \text{ mH}$$
$$L_b = 40 - 30 = 10 \text{ mH}$$

End of Solution

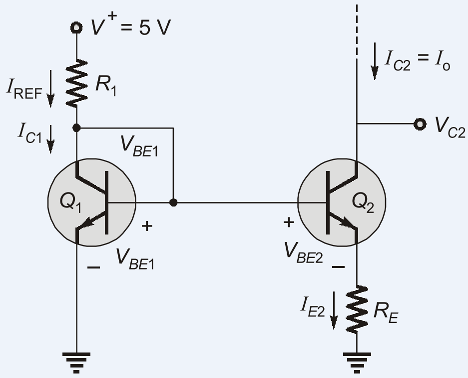
Q.8 (c) The current source circuit shown in the figure below has I_{REF} = 0.7 mA and I_o = 25 μA at V_{C2} = 1 volt. The transistor parameters are β = 150, $V_{BE1(ON)}$ = 0.7 volts and Early voltage V_A = 100 V. Determine R_1, R_E, $V_{BE2(ON)}$ and change in I_o when V_{C2} changes from 1 V to 4 V. Assume that the two transistors are identical and are maintained at same temperature. Derive the equations used.



[20 marks : 2024]

Solution:

Given CKT,



Given data:

$$I_{REF} = 0.7\text{ mA}$$
$$I_0 = 25\text{ μA at } V_{C2} = 1\text{ V}$$
$$\beta = 150$$
$$V_{BE1} = 0.7\text{ V}$$
$$V_A = 100\text{ V}$$

Calculation of R_1 :

$$I_{REF} = \frac{V^+ - V_{BE1}}{R_1}$$

$$R_1 = \frac{V^+ - V_{BE1}}{I_{REF}} = \frac{5 - 0.7}{0.7\text{ mA}} = \frac{4.3}{0.7\text{mA}} = 6.14\text{ k}\Omega$$

$$R_1 = 6.14\text{ k}\Omega$$

Calculation of R_E :

$$R_E = \frac{V_{BE1} - V_{BE2}}{I_{E2}}$$

As
$$\beta = 150\text{ (high)}$$
$$I_{C2} = I_{E2} = I_0 = 25\text{ μA}$$

$$V_{BE1} = V_L \ln\left(\frac{I_0}{I_S}\right)$$

$$R_E = \frac{V_T \ln\left(\frac{I_{REF}}{I_S}\right) - V_T \ln\left(\frac{I_0}{I_S}\right)}{I_0}$$

$$= \frac{V_T \ln\left(\frac{I_{REF}}{I_0}\right)}{I_0}$$

$$R_E = \frac{25\ln\left(\frac{0.7\text{ mA}}{25\ \mu\text{A}}\right)}{25\ \mu\text{A}} = \frac{25\text{ mV}\times 3.33}{25\ \mu\text{A}}$$

$$R_E = 3.33\text{ k}\Omega$$

Calculation of V_{BE2}

$$R_E = \frac{V_{BE1} - V_{BE2}}{I_0}$$

$$3.33\text{ k}\Omega = \frac{0.7\text{ V} - V_{BE2}}{25\ \mu\text{A}}$$

$$0.083\text{ V} = 0.7\text{ V} - V_{BE2}$$

$$V_{BE2} = 0.7\text{ V} - 0.083\text{ V} = 0.617\text{ V}$$

Charge in I_0 when V_{C2} charges from 1 V to 4 V

If $V_A \neq \infty$

$$I_{REF} = I_s e^{V_{BE1}/V_T}\left(1 + \frac{V_{CE1}}{V_A}\right)$$

$$I_0' = I_s e^{V_{BE2}/V_T}\left(1 + \frac{V_{CE2}}{V_A}\right)$$

$$V_{CE1} = V_{BE1} = 0.7\text{ V}$$

$$V_{CE2} = V_{C2} - I_0' R_E$$

$$\frac{I_{REF}}{I_0'} = \frac{I_S e^{V_{BE1}/V_T}}{I_S e^{V_{BE2}/V_T}} \frac{\left(1 + \frac{V_{CE1}}{V_A}\right)}{\left(1 + \frac{V_{CE2}}{V_A}\right)}$$

$$\frac{0.7\text{ mA}}{I_0'} = e^{V_{BE1}-V_{BE2}/V_T} \frac{\left(1 + \frac{0.7}{100}\right)}{\left(1 + \frac{V_{C2} - I_0' R_E}{V_A}\right)}$$

$$\frac{0.7\text{ mA}}{e^{0.083/25\text{ mV}}} = \frac{1.007}{\left(1 + \frac{4 - I_0^2 R_E}{100}\right)} \cdot I_0'$$

$$\frac{0.7\text{ mA}}{27.66 \times 1.007} = \frac{I_0'}{1 + \frac{4 - I_0^1 R_E}{100}}$$

$$0.025\text{ mA}\left(1 + \frac{4 - I_0' R_E}{100}\right) = I_0'$$

$$0.025\text{ mA} + 1\text{ mA} - 0.25\text{ mA}\ I_0' \times 3.33\text{ k} = I_0'$$

$$1.025\text{ mA} = I_0' + 0.08325 I_0'$$

$$1.025\text{ mA} = I_0' \times 1.8325$$

$$I_0' = \frac{1.025\text{ mA}}{1.8325}$$

$$I_0' = 0.559\text{ mA} = 559.34\text{ μA}$$

$$\text{Charge in } I_0 = 559.34\text{ μA} - 25\text{ μA}$$

$$sI_0 = 534.34\text{ μA}$$

End of Solution

■■■■