MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 15

Section A

Q.1 (a) Solution:

$$H_d(e^{j\omega}) = \begin{cases} 0, & -\pi/4 \le \omega \le \pi/4 \\ e^{-j2\omega}, & \pi/4 \le |\omega| \le \pi \end{cases}$$

Therefore,
$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{-j2\omega}e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} e^{-j2\omega}e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{j\omega(n-2)}d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} e^{j\omega(n-2)}d\omega$$

$$= \frac{1}{\pi(n-2)}\left\{\left[\frac{e^{j(n-2)\pi} - e^{-j(n-2)\pi}}{2j}\right] - \left[\frac{e^{j(n-2)\pi/4} - e^{-j(n-2)\pi/4}}{2j}\right]\right\}$$

$$= \frac{1}{\pi(n-2)}\left[\sin\pi(n-2) - \sin(n-2)\pi/4\right], n \neq 2$$

$$h_d(2) = \frac{2}{2\pi}\int_{\pi/4}^{\pi} e^{-j2\omega}e^{j2\omega}d\omega = \frac{3}{4}$$

The filter coefficients are given by,

$$h_d(2) = \frac{3}{4}, h_d(0) = -\frac{1}{2\pi} = h_d(4) \text{ and } h_d(1) = -\frac{1}{\sqrt{2}\pi} = h_d(3)$$

By applying the window function, the new filter coefficients are obtained as $h(n) = h_d(n)w(n)$ for $0 \le n \le 4$.

$$h(2) = \frac{3}{4}, h(0) = -\frac{1}{2\pi} = h(4) \text{ and } h(1) = -\frac{1}{\sqrt{2}\pi} = h(3)$$

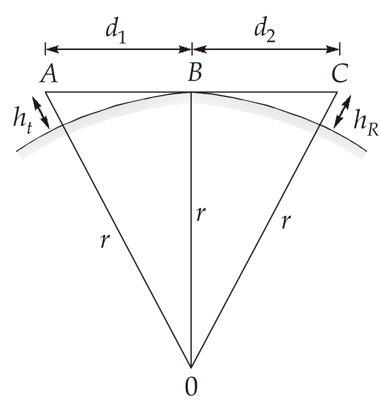
The frequency response $H(e^{j\omega})$, is obtained as

$$H(e^{j\omega}) = \sum_{n=0}^{4} h(n)\cdot e^{-j\omega n}$$

$$H(e^{j\omega}) = e^{-j2\omega}\left[0.75 - \frac{\sqrt{2}}{\pi}\cos\omega - \frac{1}{\pi}\cos 2\omega\right]$$

Q.1 (b) Solution:

In general, space wave communication is possible only upto or slightly beyond line of sight distance and this distance is determined mainly by the heights of transmitting and receiving antenna.



Let d be the distance between transmitter and receiver antenna and height of the transmitting and receiving antenna be h_t and h_r (in meter) respectively above ground.

So, line of sight distance,

$$d = d_1 + d_2 \quad \text{...(i)}$$

If r be the radius of earth (equal to 6370 km) then from ΔABO and ΔCBO,

Similarly,

$$d_1 = \sqrt{(h_t + r)^2 - r^2} = \sqrt{h_t^2 + r^2 + 2h_t \cdot r - r^2}$$

$$\cong \sqrt{2rh_t} \text{ m} \qquad [\because h_t^2 << 2r\,h_t] \quad \text{...(ii)}$$

Similarly,

$$d_2 = \sqrt{(h_r + r)^2 - r^2} = \sqrt{h_r^2 + r^2 + 2h_r r - r^2}$$

$$= \sqrt{2rh_r}\ \text{m} \qquad [\because h_r^2 << 2rh_r] \qquad ...(iii)$$

Putting value of equation (ii) and (iii) in equation (i),

$$d = \left[\sqrt{2rh_t} + \sqrt{2rh_r}\right]\text{m} = \sqrt{2r}\left[\sqrt{h_t} + \sqrt{h_r}\right]\text{m} \qquad ...(iv)$$

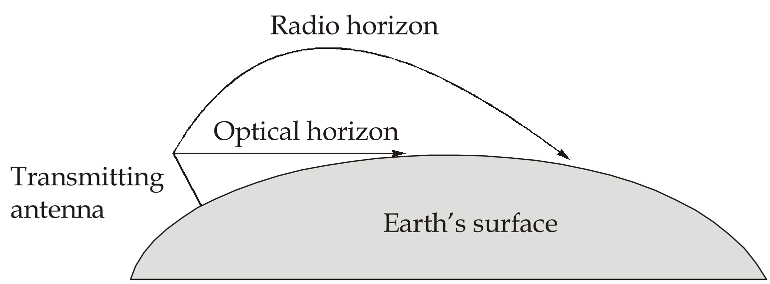
Putting $r = 6370 \times 10^3$ m, we get

$$d = 3.57\left(\sqrt{h_t} + \sqrt{h_r}\right)\text{km};$$

where h_t and h_r are expressed in meters.

This is known as optical horizon.

Radio horizon is the line of sight (LOS) for space waves. It is different from the optical LOS because the earth's atmosphere bends the radio waves which cause the radio horizon to lie beyond the optical horizon as shown below:



When a radio wave travels horizontally, it follows a slightly downwards curvature path due to refraction. This curvature of path tends to overcome the loss of signal due to curvature of earth and permits direct ray to reach point slightly beyond the horizon as found by straight line or line of sight path.

Thus in the calculation, the effective earth radius has to be taken and the effective earth radius is given by

$$R_E = \frac{4}{3}R \qquad ...(v)$$

where, R_E = Effective earth radius

R = Radius of earth (6370 km)

Substituting equation (v) in equation (iv),

$$d = \sqrt{2 \times \frac{4}{3} \times 6370 \times 10^3}\left[\sqrt{h_t} + \sqrt{h_r}\right]\text{m}$$

$$= 4.123 \times 10^3\left[\sqrt{h_t} + \sqrt{h_r}\right]\text{m}$$

$$d = 4.123\left[\sqrt{h_t} + \sqrt{h_r}\right]\text{km} \quad \text{...(vi)}$$

where, h_t = height of transmitting antenna (in meter)

and h_r = height of receiving antenna (in meter).

The equation (vi) corresponds to radio horizon. From equation (iv) and (vi) we can see, radio horizon is greater than optical horizon.

Q.1 (c) Solution:

(i) 1. As we know, array factor for N-element array,

$$AF(\psi) = \left|\frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\frac{\psi}{2}}\right|$$

At maxima, $$\frac{d[AF(\psi)]}{d\psi} = 0$$

$$\Rightarrow \frac{N\sin\left(\frac{\psi}{2}\right)\frac{N}{2}\cdot\cos\left(N\frac{\psi}{2}\right) - \sin\left(N\frac{\psi}{2}\right)\cdot\frac{N}{2}\cos\left(\frac{\psi}{2}\right)}{N^2\sin^2\left(\frac{\psi}{2}\right)} = 0$$

$$\Rightarrow N\sin\left(\frac{\psi}{2}\right)\cos\left(\frac{N\psi}{2}\right) = \sin\left(\frac{N\psi}{2}\right)\cdot\cos\left(\frac{\psi}{2}\right)$$

$$N\tan\left(\frac{\psi}{2}\right) = \tan\left(N\frac{\psi}{2}\right)$$

Hence, the peaks of the array factor are given by solution of the above equation.

2. Consider the transcendental equation obtained above,

$$N\tan\left(\frac{\psi}{2}\right) = \tan\left(N\frac{\psi}{2}\right)$$

For $N = 7$,

$$7\tan\left(\frac{\psi}{2}\right) = \tan\left(7\frac{\psi}{2}\right)$$

This equation is satisfied when $\psi = 0$ which represents main lobe and for all other non zero integral multiple of '2π', then it relate to grating lobe.

$$\psi = \beta d\cos\theta = \frac{2\pi}{\lambda}\times 0.24\lambda\cos 30^\circ$$

$$= 0.415\,\pi$$

$$\psi = 0.415\pi \text{ correspond to peak first lobe.}$$

$$|Af_n| = \left|\frac{\sin\left(\frac{7}{2}\times 0.415\pi\right)}{7\sin\left(\frac{1}{2}\times 0.415\pi\right)}\right| = 0.2328$$

$$|Af_n|_{dB} = 20\log(0.238) = -12.658 \text{ dB}$$

(ii) Power radiated by the antenna

P_{rad} = Power supplied to the antenna × efficiency

= 150 × 0.84 = 126 W

1. Now, we know that radiation intensity in the direction of maximum radiation,

$$U_{max} = U_{av}\cdot D = \frac{P_{rad}}{4\pi}D$$

where Directivity $D = \dfrac{4\pi}{\int_0^{\pi}\int_0^{\pi}\sin^2\theta\sin^2\phi\sin\theta\, d\theta\, d\phi}$

$$= \frac{4\pi}{\int_0^{\pi}\sin^2\phi\, d\phi\int_0^{\pi}\sin^2\theta\sin\theta\, d\theta}$$

$$= \frac{4\pi}{\int_0^{\pi}\left(\frac{1-\cos 2\phi}{2}\right)d\phi\int_0^{\pi}\sin^2\theta\,\sin\theta\, d\theta}$$

$$D = \frac{4\pi}{\left(\frac{\pi}{2}-0\right)\int_0^{\pi}\sin^3\theta\, d\theta}$$

$$D = \frac{4\pi}{\frac{\pi}{2}\int_0^{\pi}\frac{3\sin\theta-\sin 3\theta}{4}d\theta}$$

$$= \frac{4\pi}{\frac{\pi}{2}\left\{\frac{3}{4}[-\cos\theta]_0^{\pi}-\left[\frac{-\cos 3\theta}{3\times 4}\right]_0^{\pi}\right\}} = \frac{4\pi}{\frac{\pi}{2}\left\{\frac{3}{4}\times 2-\frac{1}{6}\right\}}$$

$$= \frac{4\pi}{\frac{\pi}{2}\left\{\frac{3}{2}-\frac{1}{6}\right\}} = \frac{4\pi}{\frac{\pi}{2}\times\frac{4}{3}} = 6$$

The maximum radiation intensity,

$$U_m = \frac{126}{4\pi}\times 6 = \frac{189}{\pi}\text{ W/st rad}$$

2. Power density along the direction of maximum radiation

$$P = \frac{U_{\max}}{r^2} = \frac{189}{\pi(12\times 10^3)^2}$$

$$P = 0.418\ \mu\text{W/m}^2$$

Q.1 (d) Solution:

IC 2764 is selected for EPROM memory and IC 6264 is selected for RAM memory. Both the memory ICs have time compatibility with an 8085 processor.

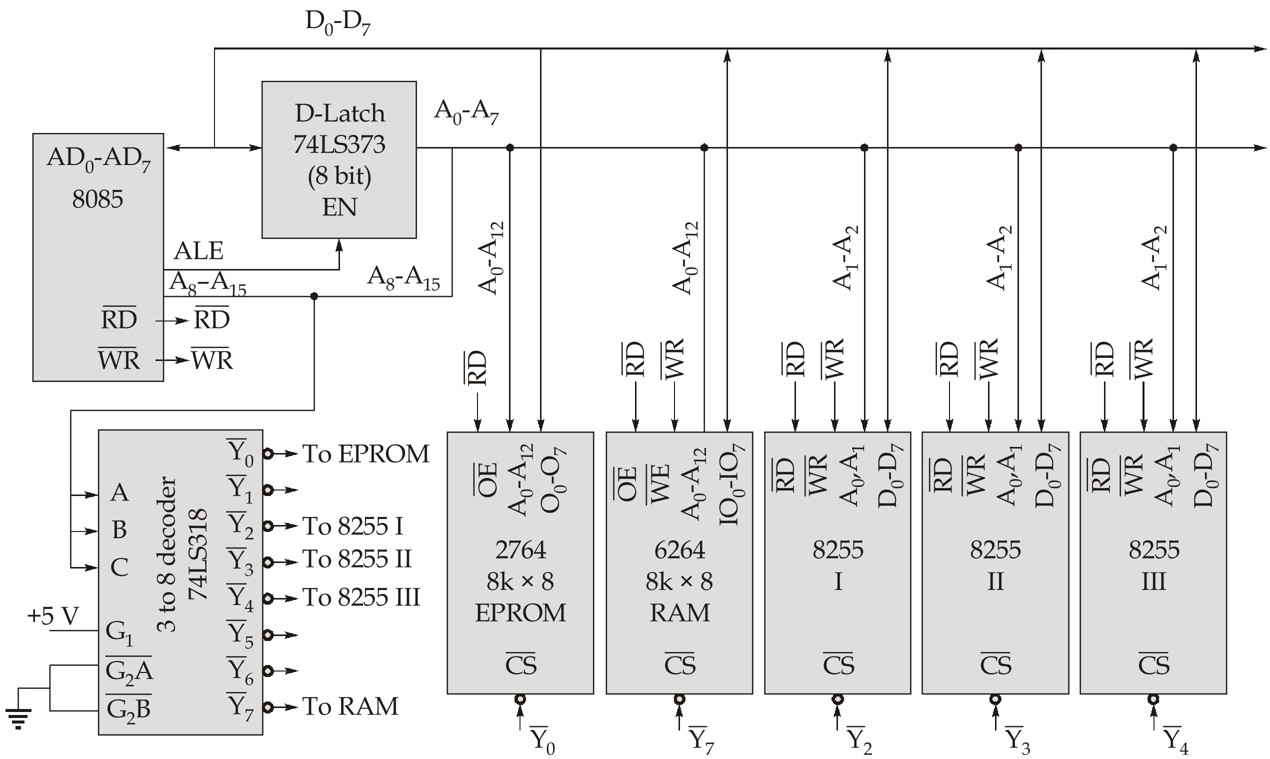
The 8 kB EPROM, 2764 requires 13 address lines ($2^{13} = 8$ k). The 8 kB RAM, 6264 requires 13 address lines ($2^{13} = 8$ k). The address line A_0 to A_{12} are connected to both EPROM and RAM memory ICs. The 8255 requires four internal addresses to access a data register for each port A, B and C or a control word register. Let us connect A_1 of 8085 to A_0 of 8255 and A_2 of 8085 to A_1 of 8255. The 8255 is memory-mapped in the system.

Note: The internal registers of an 8255 can be selected by connecting any two address lines of the processor to A_0 and A_1 address lines of the 8255.

For the memories and 8255's, we require 5 chip select signals. Hence, we can use a 3-to-8 decoder 74 LS138 for generating eight chip select signals by decoding the unused address lines A_{13}, A_{14} and A_{15}. The decoder enable pins are permanently tied to appropriate levels.

In the eight chip select signals generated by decoder, five are used for selecting the memory ICs and 8255, and the remaining three can be used for future expansion.

The memory/8255 interface diagram is shown below:



Memory Interface Diagram

Address allocation table for design:

Binary address

	Decoder Input			Input to address pins of memory/8255													Hexa address
	A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	
2764 EPROM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0 0 0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0 0 1
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	0 0 0 2
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1 F F F
6264 RAM	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	E 0 0 0
	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	E 0 0 1
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	F F F F
8255-I																	
Port A	0	1	0	X	X	X	X	X	X	X	X	X	X	0	0	X	4 0 0 0
Port B	0	1	0	X	X	X	X	X	X	X	X	X	X	0	1	X	4 0 0 2
Port C	0	1	0	X	X	X	X	X	X	X	X	X	X	1	0	X	4 0 0 4
Control Word Register (CWR)	0	1	0	X	X	X	X	X	X	X	X	X	X	1	1	X	4 0 0 6
8255-II																	
Port A	0	1	1	X	X	X	X	X	X	X	X	X	X	0	0	X	6 0 0 0
Port B	0	1	1	X	X	X	X	X	X	X	X	X	X	0	1	X	6 0 0 2
Port C	0	1	1	X	X	X	X	X	X	X	X	X	X	1	0	X	6 0 0 4
CWR	0	1	1	X	X	X	X	X	X	X	X	X	X	1	1	X	6 0 0 6
8255-III																	
Port A	1	0	0	X	X	X	X	X	X	X	X	X	X	0	0	X	8 0 0 0
Port B	1	0	0	X	X	X	X	X	X	X	X	X	X	0	1	X	8 0 0 2
Port C	1	0	0	X	X	X	X	X	X	X	X	X	X	1	0	X	8 0 0 4
CWR	1	0	0	X	X	X	X	X	X	X	X	X	X	1	1	X	8 0 0 6

The EPROM is mapped at the start of the memory space. The RAM is mapped at the end of the memory space. The EPROM is mapped from 0000_H to $1FFF_H$. The RAM is mapped from $E000_H$ to $FFFF_H$, The four internal registers of 8255 are the control word register, port A, port B and port C. A 16 bit address is allotted to each internal register of the 8255 as shown above.

Note: "X" indicate that the address line is not used for the particular device and they are considered as zero.

Q.1 (e) Solution:

(i) Here,
$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{Ke^{-2s}}{s(s^2+8s+12)+Ke^{-2s}}$$

The characteristic equation is

$$s(s^2 + 8s + 12) + Ke^{-2s} = 0 \quad \text{..(i)}$$

For small values of T, we can write from the expansion of the exponential function as

$$e^{-sT} \cong 1 - sT \quad \text{....(ii)}$$

Using the approximated expression as per equation (ii) in equation (i), we get the characteristic equation as

$$s^3 + 8s^2 + 12s + K(1 - 2s) = 0$$
$$s^3 + 8s^2 + (12 - 2K)s + K = 0$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 12-2K \\ s^2 & 8 & K \\ s^1 & \dfrac{96-16K-K}{8} & \\ s^0 & K & \end{array}$$

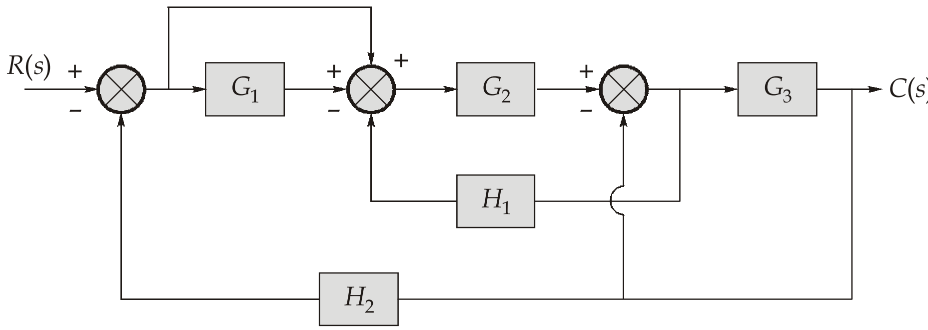
For the system to be stable, there should be no sign change in the first column of the Routh Array. Hence, the stability criteria are:

$$K > 0, \quad 12 - 2K > 0, \quad 6 > K$$

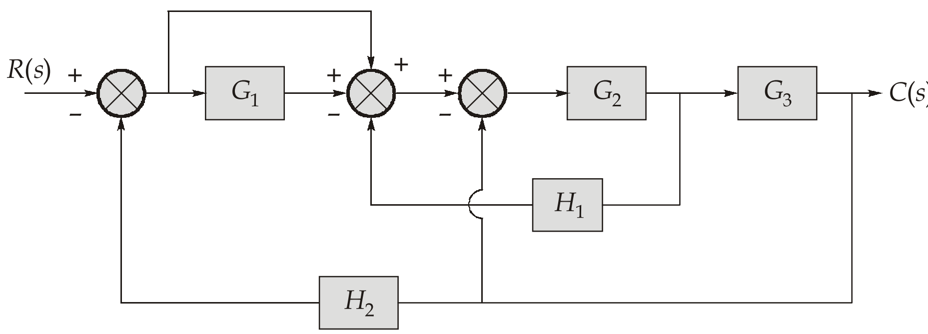
and
$$\frac{96-17K}{8} > 0$$
$$96 - 17K > 0$$
$$K < \frac{96}{17}$$

Therefore, $0 < K < 5.647$

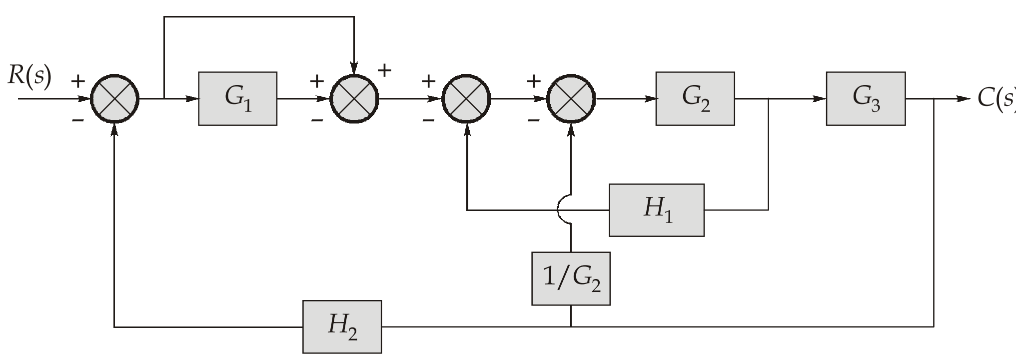
(ii)



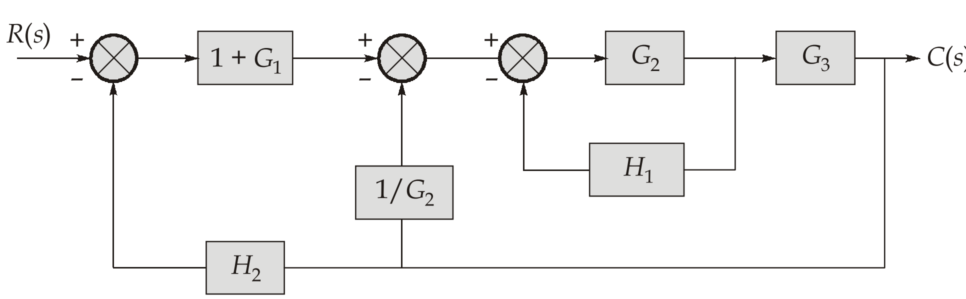
- Shifting the summing point before the block G_2,

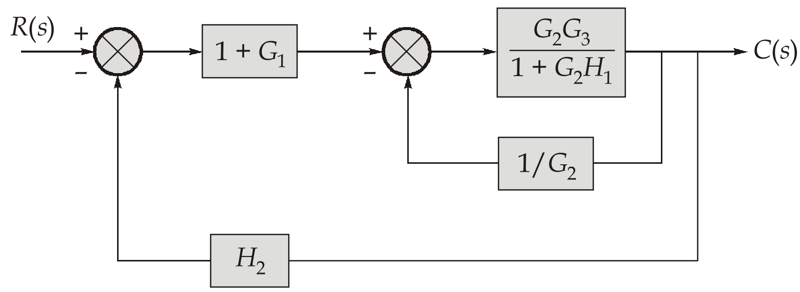


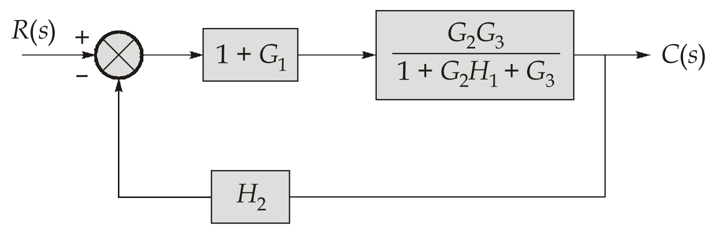
- Splitting 1st summing point,



- Exchanging 2nd and 3rd summing points,



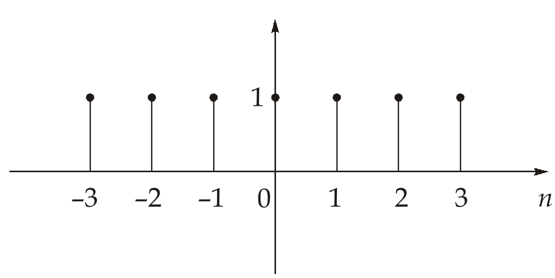




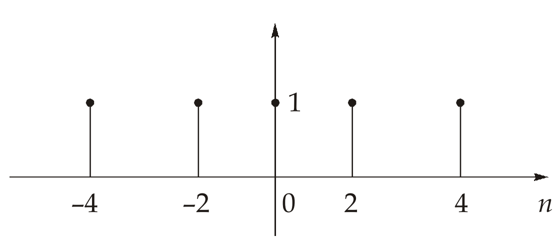
$$\frac{C(s)}{R(s)} = \frac{(1+G_1)G_2G_3}{1+G_2H_1+G_3+G_1G_2G_3H_2+G_2G_3H_2}$$

Q.2 (a) Solution:

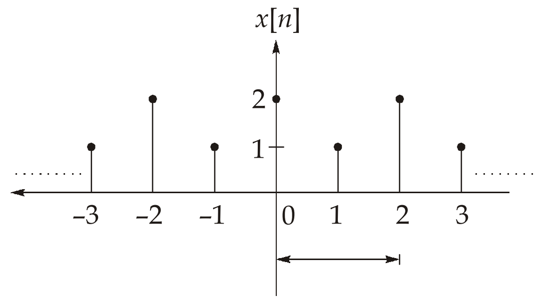
(i) **1.** $\sum_{k=-\infty}^{\infty} \delta[n-k]$ is plotted as below:



$\sum_{k=-\infty}^{\infty} \delta\left[\frac{n}{2}-k\right]$ is plotted as below:



Thus, $x[n] \Rightarrow$



$\therefore$ $x[n]$ is periodic with period, $M = 2$.

2. Power of $x[n]$, $P_x = \frac{1}{M}\sum_{n=0}^{1}|x[n]|^2$

$$= \frac{1}{2}[4+1] = \frac{5}{2} = 2.5 \text{ W}$$

3. $\sum_{n=-4}^{5} x[n] = 2 + 1 + 2 + 1 + 2 + 1 + 2 + 1 + 2 + 1$

$$= 15$$

(ii) Here, we use the frequency shifting property of DFT, $x[n]e^{-j\frac{2\pi}{N}ln} \xleftrightarrow[N]{DFT} X(k+l)$

Here, $N = 8$

We have, $y[n] = x[n]e^{-3j\pi n} = x[n]e^{-j\frac{2\pi}{8}\times\left(\frac{8}{2\pi}\times 3\pi\right)n}$

$$y[n] = x[n]e^{-j\frac{2\pi}{8}(12)n}$$

$$l = 12$$

$$x[n]e^{-j3\pi n} \xleftrightarrow{DFT} X(k+12)$$

$$Y(k) = X(k + 12)$$

For a N-point DFT, $X(k) = X(N + k)$

Hence, $X(k + 8) = X(k)$

$\therefore$ $Y(k) = X(k + 12) = X(k + 8 + 4)$

$$Y(k) = X(k + 4)$$

$$Y(k) = \{\underset{\uparrow}{1}, 2-j, 0, -1-j, 2, -1+j, 0, 2+j\}$$

Q.2 (b) Solution:

(i) At the dielectric-dielectric interface, boundary conditions are as follows:

The tangential components of electric field intensity are continuous.

i.e., $$\bar{E}_{t_1} = \bar{E}_{t_2}$$

From the given expression,

$$\bar{E}_{t_1} = 5\hat{a}_x - 2\hat{a}_y \text{ kV/m}$$

$$\bar{E}_{1n} = 3\hat{a}_z$$

$$\therefore \quad \bar{E}_{2t} = 5\hat{a}_x - 2\hat{a}_y \text{ kV/m}$$

The normal components of electric flux density are continuous.

i.e., $$\bar{D}_{1n} = \bar{D}_{2n} \qquad [\because \text{ Charge density at the interface, } \rho = 0]$$

$$\bar{D}_{1n} = \epsilon_o \epsilon_{r_1} \bar{E}_{1n} = 4\epsilon_o \times (3\hat{a}_z) = 12\epsilon_o \hat{a}_z$$

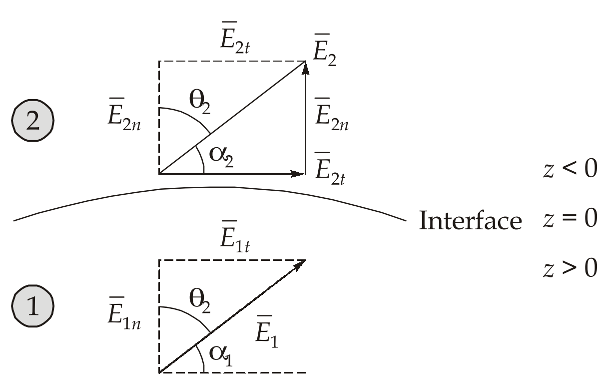
$$\therefore \quad \bar{D}_{2n} = 12\epsilon_o \hat{a}_z$$

$$\epsilon_o \epsilon_{r_2} \bar{E}_{2n} = 12\epsilon_o \hat{a}_z$$

$$\therefore \quad \bar{E}_{2n} = \frac{12}{3}\hat{a}_z = 4\hat{a}_z \text{ kV/m}$$

$$\therefore \quad \bar{E}_2 = 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z \text{ kV/m}$$

(ii) Let α_1, α_2 be the respective angles $\bar{E}_1$ and $\bar{E}_2$ make with the interface while θ_1, θ_2 are the angles with normal to the interface as shown below:



In region (i)

$$\tan\theta_2 = \frac{|\bar{E}_{2t}|}{|\bar{E}_{2n}|}$$

$$\bar{E}_2 = \underbrace{5\hat{a}_x - 2\hat{a}_y}_{\bar{E}_{2n}} + \underbrace{4\hat{a}_z}_{\bar{E}_{2t}}$$

$$\tan\theta_2 = \frac{|5\hat{a}_x - 2\hat{a}_y|}{|4\hat{a}_z|} = \frac{\sqrt{5^2+2^2}}{\sqrt{16}} = \sqrt{\frac{29}{16}}$$

$$\theta_2 = \tan^{-1}\left[\sqrt{\frac{29}{16}}\right]$$

$$\therefore \quad \theta_2 = 53.4°$$

Angle with the dielectric interface,

$$\alpha_2 = 90° - \theta_2 = 90° - 53.4°$$

$$\therefore \quad \alpha_2 = 36.6°$$

In region (i), $\tan\theta_1 = \dfrac{|\bar{E}_{1t}|}{|\bar{E}_{1n}|}$

$$\bar{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$$

$$\tan\theta_1 = \frac{|5\hat{a}_x - 2\hat{a}_y|}{|3\hat{a}_z|} = \frac{\sqrt{25+4}}{\sqrt{9}} = \sqrt{\frac{29}{9}}$$

$$\therefore \quad \theta_1 = \tan^{-1}\left(\sqrt{\frac{29}{9}}\right) = 60.9°$$

$\therefore$ Angle with the dielectric interface,

$$\alpha_1 = 90° - \theta_1 = 90° - 60.9° = 29.1°$$

(iii) The energy density in medium (i), is given by

$$W_{E1} = \frac{1}{2}\epsilon_1|\bar{E}_1|^2$$

where, $\bar{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$

$$\therefore \quad W_{E1} = \frac{1}{2}(4)\times\frac{10^{-9}}{36\pi}(\sqrt{25+4+9})^2$$

$$\therefore \quad W_{E1} = 0.67\times 10^{-9}\ \text{J/m}^3 \ \text{(or)}\ 0.67\ \text{nJ/m}^3$$

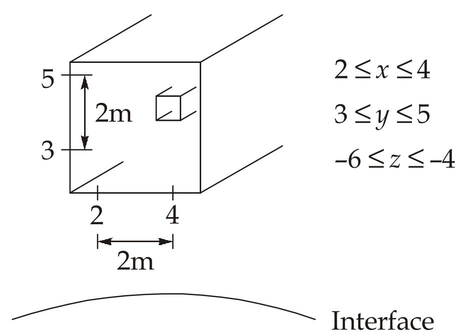
The energy density in medium (ii) is,

$$W_{E2} = \frac{1}{2}\epsilon_2 |\bar{E}_2|^2$$

$$W_{E2} = 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z = \frac{1}{2} \times 3 \times \frac{10^{-9}}{36\pi}(\sqrt{25+4+16})^2$$

$$W_{E2} = 0.597 \text{ nJ/m}^3$$

(iv) The center (3, 4, –5) of the cube of side 2 m is at $z = -5$ i.e. $z < 0$ which lies in second region



Energy within a cube of 2 m in region (2) is

$$E = \int_v W_{E2}\, dv$$

where, $dv = dx\, dy\, dz$ and V is the volume enclosed by the cube.

$$E = \int_x\int_y\int_z 0.597 \times 10^{-9}\, dx\, dy\, dz = \int_2^4\int_3^5\int_{-6}^{-4} 0.597 \times 10^{-9}\, dx\, dy\, dz$$

$$= 0.597 \times 10^{-9}\, (x)_2^4\, (y)_3^5\, (z)_{-6}^{-4}$$

$$= 0.597 \times 10^{-9}\, (4-2)\,(5-3)\,(-4+6)$$

$$E = 4.776 \times 10^{-9} \text{ J}$$

Q.2 (c) Solution:

(i) Advantages

1. Due to constant amplitude of PPM pulses, the information is not contained in the amplitude.
   Hence, the noise added to PPM signal does not distort the information thus, it has good noise immunity.
2. It is possible to reconstruct PPM signal from the noise contaminated PPM signal. This is also possible in PWM but not possible in PAM.
3. Due to constant amplitude of pulses, the transmitted power remains constant. It does not change as it is used, in PWM.

Disadvantages

1. As the position of the PPM pulses is varied with respect to a reference pulse, a transmitter has to send synchronizing pulse to operate the timing circuits in the receiver. Without them, the demodulation would not be possible to achieve.
2. Large bandwidth is required to ensure transmission of undistorted pulses.

(ii) In delta modulator, for proper reconstruction of message signal, the slope of $m(t)$ should be equal to the slope of the message signal i.e.

$$\frac{dm(t)}{dt} = \frac{\Delta}{T_s}$$

For no slope overload distortion,

$$\frac{\Delta}{T_s} > \left|\frac{dm(t)}{dt}\right|_{max}$$

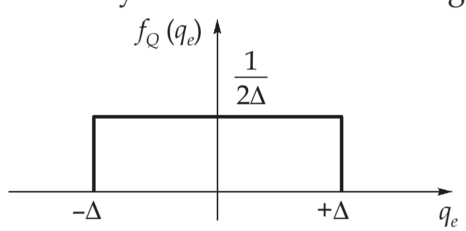
$$\frac{\Delta}{T_s} > A_m 2\pi f_m \qquad [\because m(t) = A_m \sin\omega_m t]$$

$$A_m < \frac{\Delta}{2\pi f_m T_s}; \quad T_s = \frac{1}{f_s}$$

$\therefore$ Maximum signal power,

$$S = \frac{A_m^2}{2} = \frac{\Delta^2}{8\pi^2 f_m^2 T_s^2}$$

Quantization error is uniformly distributed in the range of $-\Delta$ to Δ.



Noise power,
$$N_Q = \int_{-\Delta}^{\Delta} q_e^2 f_Q(q_e)\, dq_e = \frac{\Delta^2}{3}$$

This is the noise power for frequency upto f_s. The LPF will allow only frequencies upto f_c.

$\therefore$ Output noise power $= \frac{f_c}{f_s}\frac{\Delta^2}{3}$

$$\Rightarrow \text{Maximum output SNR} = \frac{\dfrac{\Delta^2}{8\pi^2 f_m^2 T_s^2}}{\dfrac{\Delta^2}{3}\dfrac{f_c}{f_s}} = \frac{3}{8\pi^2}\frac{f_s^3}{f_m^2 \cdot f_c}$$

Given: $f_s = 40$ kHz and $f_c = f_m = 10$ kHz

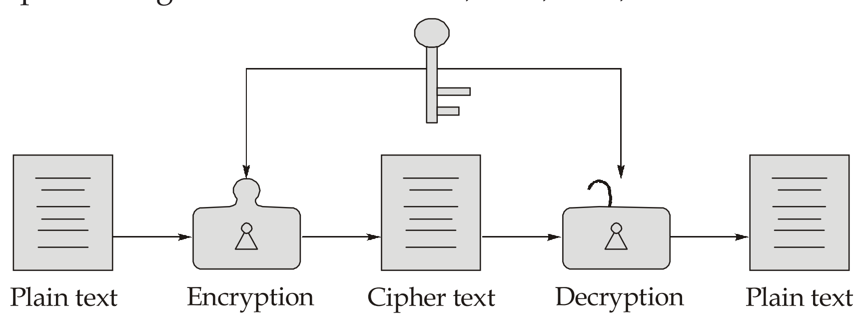
$$\text{SNR} = \frac{3}{8\pi^2}\left(\frac{40\times10^3}{10\times10^3}\right)^3 = 2.43$$

$$\text{SNR (in dB)} = 10\log_{10}(2.43) = 3.86 \text{ dB}$$

Q.3 (a) Solution:

(i) Symmetric Key Cryptography:

- Only one key (symmetric key) is used, and the same key is used to encrypt and decrypt the message.
- It's a simple technique and because of this, the encryption process can be carried out quickly. The resource utilization is low compared to asymmetric key encryption.
- The length of the keys used is typically 128 or 256 bits, based on the security requirements.
- It is mostly used when large chunks of data need to be transferred securely.
- The entities communicating via symmetric key encryption must exchange the key so that it can be used in the decryption process.
- Examples of Algorithms include RC4, AES, DES, 3DES etc.

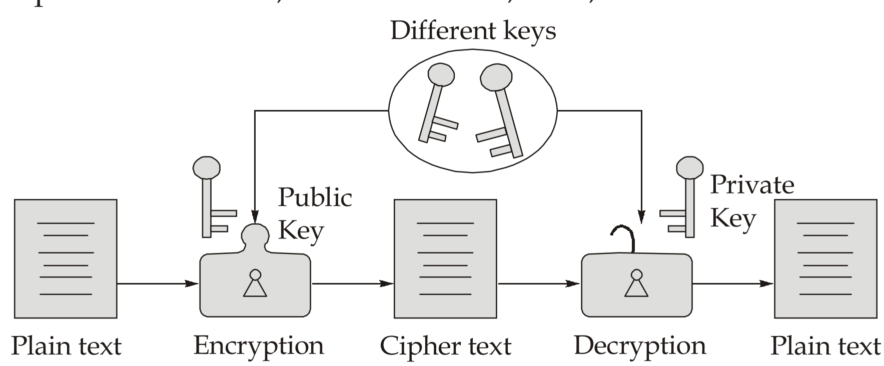


Asymmetric Key Cryptography:

- Two different cryptographic keys (asymmetric keys) called the public and the private keys are used for encryption and decryption. The public key is publicly distributed. Anyone can use this public key to encrypt messages, but only the recipient, who holds the corresponding private key, can decrypt those messages.
- It's a much more complicated process than symmetric key encryption, and the process is slower. The resource utilization is high.
- The length of the keys is much larger, e.g., the recommended RSA key size is 2048 bits or higher.
- It's used in smaller transactions, primarily to authenticate and establish a secure communication channel prior to the actual data transfer.

- Asymmetric cryptography is often used to exchange the secret key to prepare for using symmetric cryptography to encrypt data. In the case of a key exchange, one party creates the secret key and encrypts it with the public key of the recipient. The recipient would then decrypt it with their private key.

  Examples include RSA, Diffie-Hellman, ECC, etc.



(ii) Czochralski is the most widely used process to grow a single silicon crystal from which silicon wafers are cut. The crystal growth involves a phase transition from solid, liquid or gas to crystalline solid phase, i.e., CZ process involves the solidification of a crystal from a melt.

The arrangement for crystal growth in the CZ process is shown in figure below:

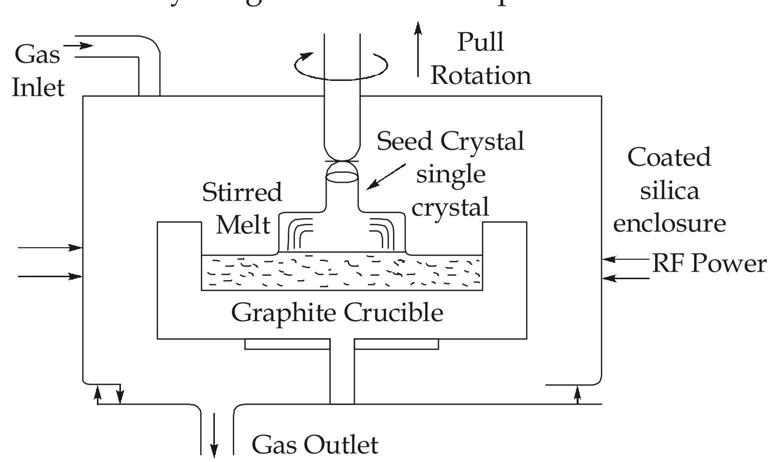


Figure: CZ crystal growth arrangement

The pure polycrystalline electronic grade silicon (EGS) is melted at 1500°C into a quartzite crucible. This crucible is placed in an evacuated chamber and then the chamber is filled with inert gas. A small chemically etched seed crystal (also called puller), containing the desired orientation, is inserted into the molten silicon and slowly pulled out, approximately at the rate of 1 mm/minute. The silicon crystal is manufactured as a cylinder, which is known as ingot or boule.

Q.3 (b) Solution:

(i) Narrow band FM signal is approximately defined by

$$S(t) = A_c \cos 2\pi f_c t - A_c \beta \sin(2\pi f_m t)\sin(2\pi f_c t) \qquad ...(1)$$

We know, $A\cos(2\pi f_o t) \pm B\sin(2\pi f_o t) \xrightarrow{\text{Envelope}} \sqrt{A^2 + B^2}$

Now from equation (i), the envelope of NBFM signal,

$$\alpha(t) = \sqrt{A_c^2 + \left[A_c \beta \sin 2\pi f_m t\right]^2} = A_c\sqrt{1 + \beta^2 \sin^2(2\pi f_m t)}$$

Maximum value of envelope,

$$\alpha_{max} = A_c\sqrt{1+\beta^2}$$

Minimum value of envelope,

$$\alpha_{min} = A_c$$

$$\therefore \qquad \frac{\alpha_{max}}{\alpha_{min}} = \sqrt{1+\beta^2}$$

(ii) $S(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_m t)\sin(2\pi f_c t)$

$S(t) = A_c \cos(2\pi f_c t) + \frac{A_c}{2}\beta\cos(2\pi(f_c + f_m)t) - \frac{A_c\beta}{2}\cos\left[2\pi(f_c - f_m)t\right]$

Average power of $S(t)$

$$P = \frac{A_c^2}{2} + \left(\frac{A_c\beta}{2}\right)^2 \times \frac{1}{2} + \left(\frac{A_c\beta}{2}\right)^2 \times \frac{1}{2}$$

$$P = \frac{A_c^2}{2} + \frac{A_c^2\beta^2}{4} = \frac{A_c^2}{2}\left(1 + \frac{\beta^2}{2}\right)$$

(iii) We know that,

$$A\cos(\omega_o t) - B\sin(\omega_o t) = \sqrt{A^2 + B^2}\cos(\omega_o t + \theta)$$

where, $\qquad \theta = \tan^{-1}\left(\frac{B}{A}\right)$

Here, $\qquad A = A_C, \; B = A_C \beta \sin(2\pi f_m t)$

$$S(t) = (\sqrt{A^2 + B^2})\cos\left(\omega_o t + \tan^{-1}\left(\frac{B}{A}\right)\right)$$

$$S(t) = (\sqrt{A^2 + B^2})\cos\left[\omega_c t + \tan^{-1}[\beta \sin(2\pi f_m t)]\right]$$

Total angle of NBFM signal,

$$\theta_i(t) = 2\pi f_c t + \tan^{-1}[\beta \sin(2\pi f_m t)]$$

Since, $\tan^{-1}(x) = x - \frac{x^3}{3}$

and β = 0.3 i.e., very small so the higher order terms can be neglected. We can write,

$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t) - \frac{\beta^3}{3}\sin^3(2\pi f_m t)$$

We know that, $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$$\sin^3\theta = \frac{1}{4}[3\sin\theta - \sin 3\theta]$$

$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t) - \frac{\beta^3}{4\times 3}[3\sin(2\pi f_m t) - \sin(2\pi(3f_m)t)]$$

$$\theta_i(t) = 2\pi f_c t + \left[\beta - \frac{\beta^3}{4}\right]\sin(2\pi f_m t) + \frac{\beta^3}{12}\sin[2\pi(3f_m)t]$$

Harmonic distortion, $D_h = \frac{\frac{\beta^3}{12}}{\left(\beta - \frac{\beta^3}{4}\right)} = \frac{\beta^2}{3(4-\beta^2)}$

Given β = 0.3

∴ $D_h = \frac{(0.3)^2}{3(4-(0.3)^2)} = 0.00767 = 0.767\%$

Q.3 (c) Solution:

Given open loop transfer function,

$$G(s) = \frac{Ks^3}{(s+1)(s+2)}$$

Put $s = j\omega$, $G(j\omega) = \frac{K(j\omega)^3}{(j\omega+1)(j\omega+2)}$

or $G(j\omega) = \frac{-jK\omega^3}{(2-\omega^2)+3j\omega}$

or $G(j\omega) = \frac{-jK\omega^3((2-\omega^2)-3j\omega)}{(2-\omega^2)^2+(3\omega)^2}$

$$G(j\omega) = \frac{-3K\omega^4}{(2-\omega^2)^2 + (3\omega)^2} - \frac{jK\omega^3(2-\omega^2)}{(2-\omega^2)^2 + (3\omega)^2}$$

$$\phi = \angle G(j\omega) = 270° - \tan^{-1}\omega - \tan\frac{\omega}{2}$$

(i) $\lim_{\omega \to 0} G(j\omega)$: Re$(G(j0)) \to -0$, Img $(Gj0) \to -j0$; $\phi = 270°$

(ii) At the intersection of $G(j\omega)$ with positive real axis,

$$\text{Img}\, G(j\omega) = 0$$

$$\frac{-K\omega^3(2-\omega^2)}{(2-\omega^2)^2 + (3\omega)^2} = 0$$

$$2 - \omega^2 = 0$$

$$\omega = \pm\sqrt{2} \text{ rad/sec}$$

The intersection is obtained by putting $\omega = \sqrt{2}$ in real part of $G(j\omega)$ i.e.,

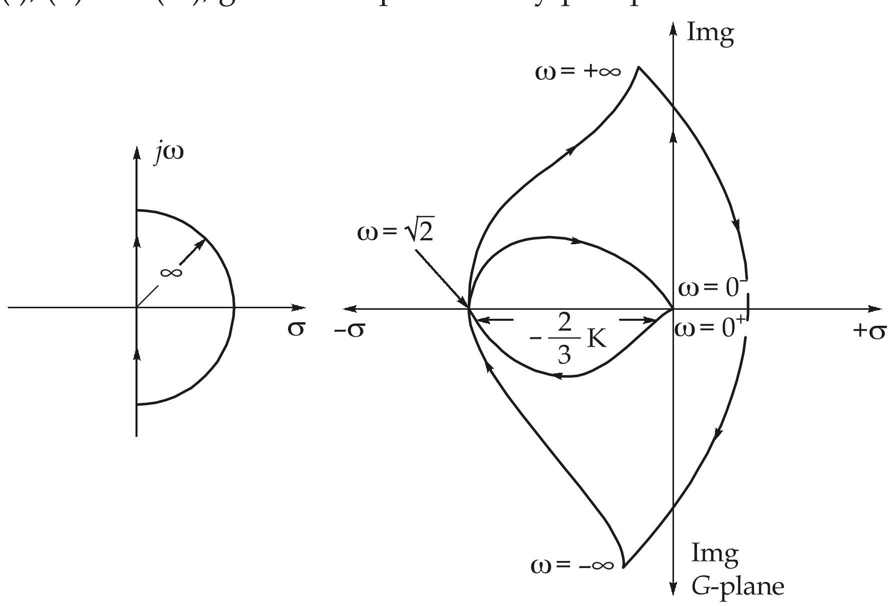
$$G(j\sqrt{2}) = \frac{-3K(\sqrt{2})^4}{(2-2)^2 + (3\sqrt{2})^2} = \frac{-3K \times (2)(2)}{(2-2)^2 + 3\times 3\times 2}$$

or $$G(j\sqrt{2}) = -\frac{2}{3}K$$

(iii) $\lim_{\omega \to \infty} G(j\omega)$: Re $G(j\infty) = -3K$

$$\text{Img}\, G(j\infty) = +j\infty; \quad \phi = 90°$$

As per step (i), (ii) and (iii), general shape of the Nyquist plot is drawn as shown below:



$$\underset{\omega \to \infty}{G(j\omega)} = \frac{K(j\omega)^3}{(j\omega+1)(j\omega+2)} = Kj\omega = Kj(\infty)$$

$$\underset{\omega \to \infty}{G(j\omega)} = |\infty| \angle 90°$$

The closing of Nyquist plot from ω = ∞ to ω = –∞ is as explained below:

In s-plane, the RHS region is closed from $s = j\infty$ to $s = -j\infty$ through a semi circle of infinite radius in clockwise direction, hence the corresponding points in $G(s)$-plane. i.e., $G(j\infty)$ to $G(-j\infty)$ are closed through a semi circle of infinite radius.

It is seen that number of poles of $G(s)$ having positive real part is nil i.e., $P_+ = 0$. The encirclement of critical point $(-1 + j0)$ are determined below:

1. If $K < \frac{3}{2}$

The critical point $(-1 + j0)$ lies outside the Nyquist plot, hence $N = 0$

$$\therefore \quad N = P_+ - Z_+$$

$$0 = 0 - Z_+$$

$$Z_+ = 0 \text{ i.e. no closed loop poles in the RHS of s-plane.}$$

The system is stale for $K < \frac{3}{2}$.

2. $K > \frac{3}{2}$

The critical point $(-1 + j0)$ is encircled twice in clockwise direction by Nyquist plot. Hence,

$$N = -2$$

$$N = P_+ - Z_+$$

$$-2 = 0 - Z_+$$

$$\therefore \quad Z_+ = 2 \text{ i.e. two closed loop poles lie in the RHS of s-plane.}$$

Hence, the system is unstable for $K > \frac{3}{2}$.

For stability, $0 < K \leq \frac{3}{2}$.

Q.4 (a) Solution:

We have,

Hypothetical floating point data format is given as

S	BE	M
1 bit	7 bit	8 bit

We know that,

- sign bit = 0; represent positive number
  1; represent negative number
- Bias Exponent (BE) = Actual Exponent (AE) + Bias factor
  Bias factor = $(2^{n-1} - 1)$ .... for normal bias (by default we use it)
  = (2^{n-1}) ... for excess bias

Here nothing is mention, hence we go for normal bias.

∴ $n = 7 \Rightarrow$ Biasing factor = $(2^{n-1} - 1) = 63$ (0111111)

- The floating point number is represented as : $(1.\underbrace{bbbbb}_{\text{Mantissa}})_2 * 2^{\pm e}$

(i) 1. For 1100001011001000

S	BE	Mantissa
1 bit	7 bit	8 bit
1	1000010	11001000

$S = 1 \Rightarrow$ negative number

$BE = 1000010$

Hence,

$AE = BE - \text{Bias}$

$= (1000010)_2$

$= -(0111111)_2$

$(0000011)_2 \Rightarrow (3)_{10}$

Mantissa, $M = 11001000$

Hence, we get

$N = -(1.11001000)_2 \times 2^{+3}$

$N = -(1110.01000)_2$

$$N = -\left(2^3 + 2^2 + 2^1 + \frac{1}{2^2}\right) = -14.25$$

2. For 0011110101100000

S 0	BE 0111101	01100000

$\Downarrow$ Number is positive

$\Downarrow$ BE = 0111101

$AE = BE$ – Bias

$= (0111101)_2$

$= (0111111)_2$

$(1111110)_2$

As number is negative, take 2's complement of it

0000001

+ 1

$(0000010)_2 \Rightarrow (-2)_{10}$

Hence, number is

$N = +(1.01100000) \times 2^{-2}$

$\Rightarrow \quad N = (0.010110000)_2$

$$\Rightarrow \quad N = 0 + \frac{1}{4} + \frac{1}{16} + \frac{1}{32} = 0.34375$$

Thus, number is +0.34375.

3. $(48B8)_H$ = 0100 1000 1011 1000

Converting the hexadecimal in binary,

0	1001000	1011 1000
S	BE	M

$S = 0 \quad \Rightarrow \quad$ +ve number

$BE = AE$ + Bias

$AE =$ 1001000

–0111111

0001001 $\Rightarrow +9$

Thus, number is $(1.10111000)2^9$

$$\Rightarrow \quad N = \left(1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}\right)2^9$$

$\Rightarrow \quad N = 880$

(ii) **1.** –13.5

+13.5 $\Rightarrow$ $(1101.1)_2$

In normalized form,

$N = (1.1011)_2 \times 2^3$

Actual Exponent, $AE = 3$

$BE = AE + \text{Bias} = (0000011)_2 + (0111111)_2$

$BE = (1000010)_2$

Mantissa = 1011

Sign	BE	Mantissa
1	1000010	10110000

Sign: ⇓ 1 for negative number

BE: ⇓ $BE = AE + \text{Bias}$

$= (0000011)_2$

$= +(0111111)_2$

$(1000010)_2$

Hence, –13.5 can be written as

$(1100001010110000)_2 = (C2B0)_H$

2. 29.5

29.5 $\Rightarrow$ $(11101.1) \times 2^0$

In normalized form,

$N = (1.11011) \times 2^4$

Mantissa = 11011

Actual Exponent = $(4)_{10}$

$= (0000100)_2$

BE = AE + Bias

$= (0000100)_2 + (0111111)_2$

$= (1000011)_2$

Now,

S	BE	M
0	1000011	11011000

As number is positive

Hence, $(29.5)_{10}$ can be represented as $(0100001111011000)_2$ or $(43D8)_H$

Q.4 (b) Solution:

(i) System gain, G_s is the difference between the nominal output power of a transmitter and the minimum RF input power to a receiver necessary to achieve satisfactory performance. It must be greater than or equal to the sum of all gains and losses incurred by a signal as it propagates from a transmitter to a receiver i.e.

$$G_S(\text{dB}) = P_0\,(\text{dBm}) - \text{Min. RF Input (dBm)} \geq \text{Losses} - \text{Gains}$$

$$G_S(\text{dB}) \geq FM(\text{dB}) + FSL(\text{dB}) + L_F(\text{dB}) + L_b(\text{dB}) - G_T(\text{dB}) - G_R(\text{dB})$$

where,

FM = Fade margin

FSL = Free space/Path loss = $92.4 + 20\log_{10}d(\text{km}) + 20\log_{10}[f(\text{GHz})]$

L_F = Feeder loss

L_b = Branching circuit loss

G_T = Gain of transmitting Antenna

G_R = Gain of Receiving Antenna

Therefore, the system gain for microwave link depends on the parameters stated above.

Fade-Margin is given by Barnett-Vigant Reliability formula,

$FM = 30\log_{10} d(\text{km}) + 10\log_{10}[6\,ABf\,(\text{GHz})] - 10\log_{10}(1 - R) - 70$

Where, R = Reliability objective

$$A = \text{Roughness factor} = \begin{cases} 4, \text{ for smooth terrain and water} \\ 1, \text{ for average terrain} \\ 1/4, \text{ for mountains and rough terrain} \end{cases}$$

B = factor to convert the worst month probability to annual probability

$$= \begin{cases} 1/2, \text{ for lakes and humid areas} \\ 1/4, \text{ for inland areas} \\ 1/8, \text{ for mountains and dry areas} \end{cases}$$

Given data; $f = 2.1$ GHz, $d = 50$ km, $R = 0.9896$

Feeder loss = 12.6 dB

Branching loss = 4 dB

$G_T = G_R = 33.6$ dB

For rough terrain and dry areas, $A = \frac{1}{4}$ and $B = \frac{1}{8}$.

So, Fade margin, FM $= 30\log 50 + 10\log\left(6\times\frac{1}{4}\times\frac{1}{8}\times 2.1\right) - 10\log(0.0104) - 70$

$= -3.249$ dB

Free space path loss, FSL $= 92.4 + 20\log_{10}d(\text{km}) + 20\log_{10}f(\text{GHz})$

$= 92.4 + 20\log(50) + 20\log(2.1)$

$= 132.823$ dB

Branching and feeder loss will be present on both transmitting and receiving station.

So,
$$L_b = 5 \text{ dB} \times 2 = 10 \text{ dB}$$
$$L_f = 12.6 \text{ dB} \times 2 = 25.2 \text{ dB}$$

Therefore, system gain,
$$G_s = FM + FSL + L_b + L_f - G_T - G_R$$
$$= -3.249 + 132.823 + 10 + 25.2 - 33.6 - 33.6$$
$$= 95.574 \text{ dB}$$

(ii) Given data,

Frequency, f = 5.65 GHz

Antenna, Diameter, D = 6 m

Transmitter output power, P_T = 10 kW

Height of satellite, d = 39920 km

Efficiency of antenna, η = 0.7

$$\text{Path loss} = F_{SL}(\text{dB}) = 92.4 + 20\log(f(\text{GHz})) + 20\log[d(\text{km})]$$
$$= 92.4 + 20\log(5.65) + 20\log(39920)$$
$$= 199.46 \text{ dB}$$

Transmitting parabolic antenna gain; $G = \eta D$, where D = directivity

$$\text{Directivity of parabolic antenna} = \pi^2\left(\frac{D}{\lambda}\right)^2$$
$$= \frac{\pi^2}{c^2}(Df)^2$$

$$\therefore \quad G_T = \eta\left(\frac{\pi}{c}\right)^2(Df)^2$$
$$G_T(dB) = 20.4 + 20\log[f(\text{GHz})] + 20\log(D) + 10\log(\eta)$$
$$= 20.4 + 20\log(5.65) + 20\log(6) + 10\log(0.7)$$
$$= 49.45 \text{ dB}$$

Transmitting power: $P_T(\text{dB}) = 10\log P_T = 10\log(10 \times 10^3) = 40$ dBW

$$\text{EIRP} = P_T(\text{dB}) + G_T(\text{dB})$$
$$= 40 + 49.45 = 89.45 \text{ dBW}$$

Received power at satellite : P_r

$$P_r(\text{dB}) = \text{EIRP} - \text{FSL}$$
$$= 89.45 - 199.46 = -110.01 \text{ dBW}$$

Improvement in received power by satellite if satellite uses 2.5 m diameter parabolic antenna.

For D' = 2.5 m, Gain of satellite antenna

$$G_T(\text{dB}) = 20.4 + 20\log\,(f\,(\text{GHz})] + 20\log\,(D') + 10\log\eta$$
$$= 20.4 + 20\log(5.65) + 20\log\,(2.5) + 10\log(0.7)$$
$$= 41.85 \text{ dB}$$

Hence, the received power is improved by 41.85 dBW with the use of satellite antenna.

Q.4 (c) Solution:

(i) RIM and SIM instructions are used to control and check the status of interrupts. Their bit pattern for RIM and SIM instruction is as given:

Bit Pattern for RIM instruction							
7	6	5	4	3	2	1	0
SDI	I7.5	I6.5	I5.5	IE	M7.5	M6.5	M5.5

Bit Pattern for SIM instruction							
7	6	5	4	3	2	1	0
SDO	SDE	XXX	R7.5	MSE	M7.5	M6.5	M5.5

	Mnemonics		**Comments**
	RIM	;	Read Interrupt mask
	MOV B, A	;	Save mask information
	ANI 20 H	;	Check whether RST 6.5 is pending
	JNZ NEXT	;	If RST 6.5 is pending, JMP to NEXT
	RET	;	Return to main program
NEXT :	MOV A, B	;	Get bit pattern, RST 6.5 is pending
	ANI 0D H	;	Unmask RST 6.5 by setting $D_1 = 0$
	ORI 08 H	;	Enable MSE "Mask Set Enable" to set the masks by setting $D_3 = 1$
	SIM	;	
	EI	;	Interrupts are enabled
	RET		

The instruction RIM loads the status of pending interrupts in accumulator along with their masking status. ANI 20 H masks all bits except D_5 to check pending RST 6.5. If $D_5 = 0$, the program control is transferred to the main program. $D_5 = 1$ indicates that RST 6.5 is pending. The masking status of the interrupts are taken from the contents obtained from the RIM instruction. Instruction ANI 0D H sets $D_1 = 0$ (RST 6.5 bit for SIM) without affecting other interrupts, instruction ORI 08 H sets $D_3 = 1$

(this is necessary for enabling masking in the SIM instruction) and instruction SIM unmask RST 6.5 without affecting any other interrupts. The interuppts are then enabled bty EI instruction.

(ii) 1. Microprocessor:

(a) It includes ALU, register arrays and control circuits on a single chip.

(b) Addition of external RAM and I/O devices makes the microprocessor based circuitry bulky and costly.

(c) As microprocessor support large amount of external memory these are designed with large number of instructions.

(d) Microprocessor are used for designing general purpose digital computing systems like personal computers.

(e) Microprocessor based system consumes more power.

Microcontroller:

(a) It includes microprocessor, RAM, ROM and I/O signal lines on a single chip.

(b) As the microcontroller includes on-chip memory and I/O peripherals, there is no need of external devices, hence, the microcontroller based circuitry becomes simple and cheap.

(c) As microcontrollers operates with small amount of fixed internal memory, these are designed with small number of instructions.

(d) Microcontrollers are used for designing dedicated (or single purpose) digital computing system like automatic traffic light control system.

(e) Microcontroller based system consumes less power.

2. Block Transfer DMA: In block transfer DMA, DMA controller takes the bus control by CPU. CPU has no access to bus until the transfer is completed. This method is suitable for transferring large amount of data.

Cycle stealing DMA: This is word by word transfer based on CPU cycle stealing. When DMA steals a cycle, CPU is stopped completely for one cycle and data transfer takes place. Cycle stealing is not an interrupt. CPU pauses for just one cycle (machine cycle).

Interleaved DMA: It is similar to block transfer technique, here DMA controller takes the control of system bus only when CPU is not using it, like while performing an ALU operation or incrementing a counter etc.

Section B

Q.5 (a) Solution:

Given, $$G(s)H(s) = \frac{K(s^2+2s+10)}{(s^2+6s+10)}$$

The open-loop poles of the system are given by

$$s^2 + 6s + 10 = 0$$

$$s = \frac{-6 \pm \sqrt{36-40}}{2} = -3 \pm j$$

The open-loop zeros of the system are given by

$$s^2 + 2s + 10 = 0$$

$$s = \frac{-2 \pm \sqrt{4-40}}{2} = -1 \pm j3$$

So, $$G(s)H(s) = \frac{K(s+1+j3)(s+1-j3)}{(s+3+j1)(s+3-j1)}$$

For the given open-loop transfer function $G(s)\,H(s)$:

The open loop poles are at $s = -3 + j1$ and $s = -3 - j1$

Therefore, $n = 2$.

The open loop zeros are at $s = -1 + j3$ and $s = -1 - j3$

Therefore, $m = 2$.

So, the number of branches of root locus $= n = 2$ and number of asymptotes $= n - m = 2 - 2 = 0$.

The complete root locus plot is drawn as shown in figure, as per rules given as follows:

1. Since, the pole-zero configuration of the open-loop transfer function is symmetrical with respect to the real axis, the root locus will be symmetrical with respect to the real axis.
2. The two branches of root locus start at the open-loop poles $s = -3 + j1$ and $s = -3 - j1$, where $K = 0$ and terminate at the open-loop zeroes at $s = -1 + j3$ and $s = -1 - j3$ where $K = \infty$.
3. Since, there are no asymptotes, there is no need to compute the angle of asymptotes.
4. Since, there are no asymptotes, there is no need to compute the centroid.
5. The root locus does not exist on the real axis at all.
6. There are no break points, so no break angles.
7. The angles of departure and the angles of arrival can be computed from the graph by using the formula

For pole at $s = -3 + j$, angle of departure

$$\theta_d = 180° + \phi$$

where, $\phi = \theta_2 + \theta_3 - \theta_1 = 116.56° + 225° - 90° = 251.56°$

For pole at $s = -3 - j$, $\theta_d = -71.56°$

$$\theta_d = 180° + 251.56° = 71.56°$$

For zero at $s = -1 + j3$, angle of arrival

$$\theta_a = 180° - \phi$$

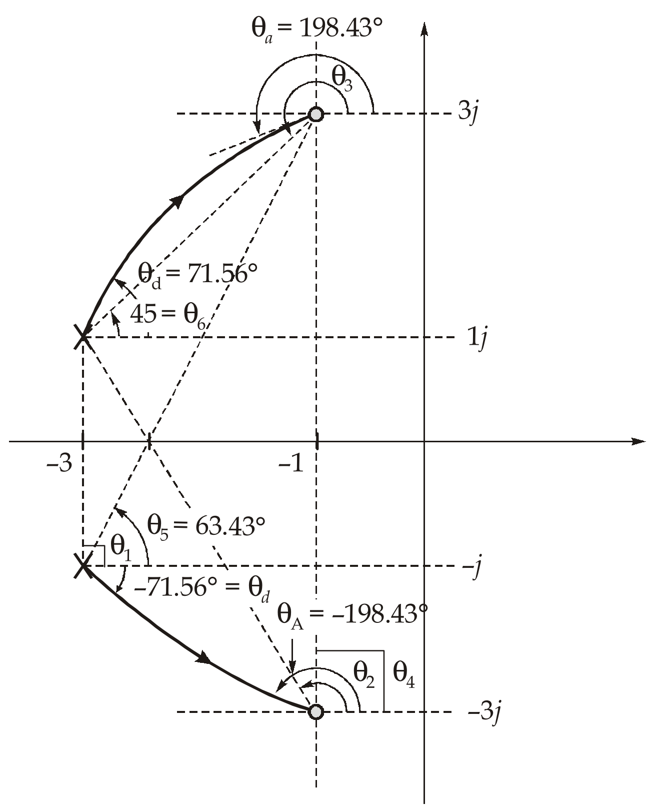
$$\phi = \theta_4 - \theta_5 - \theta_6$$

$$= 90° - 45° - 63.43° = -18.43°$$

$\therefore$ $\theta_a = 180° - (-18.43°) = 198.43°$

For zero at $s = -1 - j3$, $\theta_a = -198.43°$L

The root locus does not intersect the imaginary axis at all. So, there is no need to formulate the Routh table. The complete root locus is shown in figure. There are two branches of the root locus. The root locus starting at the complex pole $s = -3 + j1$ departs at angle of 81.86° travels along an arc of a circle and arrives at an angle of 198.43° and terminates on the complex zero $s = -1 + j3$. The root locus starting at the complex pole $-3 - j1$ depart at angle of $-81.86°$ travels along an arc of a circle and arrives at an angle of $-198.43°$ and terminates at the complex zero $s = -1 - j3$.



The root locus does not lie on the real axis and there are no break points. So, the system cannot be overdamped or critically damped. The range of values of K for which the system is underdamped is $K = 0$ at $(-1 + j3)$ to $K = \infty$ at $(-3 + j1)$. Hence, the system is always under-damped.

To show that the root loci are arcs of a circle with radius = $\sqrt{10}$ and centered at the origin of the s-plane, apply the angle condition i.e.,

$$\angle G(s)H(s) = \angle K\frac{s^2+2s+10}{s^2+6s+10} = \pm(2q+1)\pi$$

Putting $s = \sigma + j\omega$ in the above equation.

$$\angle G(s)H(s) = \left[\angle(\sigma+j\omega)^2+2(\sigma+j\omega)+10\right]-\left[\angle(\sigma+j\omega)^2+6(\sigma+j\omega)+10\right]=\pi$$

$$\text{i.e. } \angle\left[\sigma^2-\omega^2+2\sigma+10)+j\omega(2\sigma+2)\right]-\left[\angle(\sigma^2-\omega^2+6\sigma+10)+j\omega(2\sigma+6)\right]=\pi$$

$$\text{i.e. } \tan^{-1}\left[\frac{\omega(2\sigma+2)}{\sigma^2-\omega^2+2\sigma+10}\right]-\tan^{-1}\left[\frac{\omega(2\sigma+6)}{\sigma^2-\omega^2+6\sigma+10}\right]=\pi$$

$$\frac{\omega(2\sigma+6)}{\sigma^2-\omega^2+6\sigma+10} = \frac{\omega(2\sigma+2)}{\sigma^2-\omega^2+2\sigma+10}$$

$$(\sigma+3)(\sigma^2-\omega^2+2\sigma+10) = (\sigma+1)(\sigma^2-\omega^2+6\sigma+10)$$

$$\sigma^3+5\sigma^2+16\sigma-\sigma\omega^2-3\omega^2+30 = \sigma^3+7\sigma^2+16\sigma-\omega^2-\sigma\omega^2+10$$

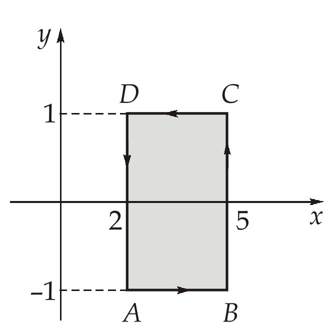
$$2\sigma^2+2\omega^2 = 20$$

$$\sigma^2+\omega^2 = 10$$

This is the equation of a circle centered at the origin of the s-plane with radius equal to $\sqrt{10}$. So, the root loci are arcs of a circle with radius = $\sqrt{10}$ centred at the origin of the s-plane.

Q.5 (b) Solution:

Given, magnetic field, $\bar{H} = 6xy\,\hat{a}_x - 3y^2\hat{a}_y$ A/m



According to Stoke's Theorem

$$\oint_L \bar{H}\cdot\overline{dl} = \int_s (\nabla\times\bar{H})\cdot\overline{ds}$$

L.H.S.

$$\oint_L \bar{H}\cdot\overline{dl} = \oint (6xy\hat{a}_x - 3y^2\hat{a}_y)\,(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$= \oint_L 6xy\,dx - 3y^2dy$$

Along the path AB, $y = -1$ and $dy = 0$

Along the path BC, $x = 5$ and $dx = 0$

Along the path CD, $y = 1$ and $dy = 0$

Along the path DA, $x = 2$ and $dx = 0$

Thus,

$$\oint \bar{H}\cdot\overline{dl} = \int_A^B -6xdx + \int_B^C -3y^2dy + \int_C^D 6xdx - \int_D^A 3y^2dy$$

$$= \left[-3x^2\right]_2^5 + \left[-y^3\right]_1^1 + \left[3x^2\right]_5^2 + \left[-y^3\right]_1^1$$

$$= -3[25-4] + [-1-1] + 3[4-25] + [1+1] = -126\text{ A}$$

R.H.S. $\iint_S (\bar{\nabla}\times\bar{H})\,\overline{ds}$

$$\therefore \quad \bar{\nabla}\times\bar{H} = \frac{1}{1}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & -3y^2 & 0 \end{vmatrix}$$

$$= \hat{a}_x[0-0] - \hat{a}_y\,[0-0] + \hat{a}_z[0-6x] = -6x\,\hat{a}_z$$

$$\therefore \quad \iint_s (\bar{\nabla}\times\bar{H})\,\overline{ds} = \iint_s -6x\,\hat{a}_z\,(dx\,dy\,\hat{a}_z)$$

$$= -6\iint_s x\,\hat{a}_z\cdot dx\,dy\,\hat{a}_z = -6\int_{x=2}^{5} xdx\int_{-1}^{1} dy$$

$$= -6\left[\frac{x^2}{2}\right]_2^5 (y)_{-1}^{1} = -3[25-4][1+1]$$

$$\iint_s (\bar{\nabla}\times\bar{H})\,\overline{ds} = -126\text{ A}$$

Q.5 (c) Solution:

(i) The low power interests are driven both by evolutionary and revolutionary trends. Recently, power dissipation is becoming an important constraint in a design due to following reasons:

1. Portable devices such as laptop/notebook, computers, electronic organizers etc.
2. Low power design is not only needed for portable applications but also to reduce the power of high performance systems. With large integration density and improved speed of operation, systems with high clock frequencies are emerging. It was observed that at higher frequency, power dissipation is too excessive.
3. Another issue related to high power dissipation is reliability because, due to large power dissipation, the temperature of the chip rises which causes more probability of failure like silicon interconnect fatigue, package related failure, electrical parameter shift and electro migration.
4. Since the processor's frequency is increasing which results in increased power, low power design techniques are prerequisites.

Therefore, low power requirements call for global solutions. The solutions can be classified into two categories: Supply and demand.

On the supply side, we need better, denser and smarter batteries, efficient power conversions and regulation, improved heat dissipation, distribution and coding techniques etc.

On the demand side, it is required to reduce the demand for power by better circuit architecture, device technologies, interconnect, wire, etc.

(ii) Dose is the total amount of phosphorous introduced in silicon per unit area after predoposition given as

$$S = 2N_s\sqrt{\frac{Dt}{\pi}} = 2\times10^{19}\sqrt{\frac{10^{-12}\times60\times60}{\pi}}$$

$$S = 6.77 \times 10^{14}/\text{cm}^2$$

The concentration of diffusing atoms after pre-deposit any time t and position 'x' is given by

$$N(x, t) = N_s erfc\left(\frac{x}{2\sqrt{Dt}}\right)$$

At junction depth $x = xj$ and $t = 1$ hr, $N(x, t) = N_B$ i.e. background concentration. We have,

$$x_j = 2\sqrt{Dt}\, erfc^{-1}\left[\frac{N_B}{N_S}\right]$$

$$x_j = 2\sqrt{Dt}\, erfc^{-1}\left[\frac{10^{15}}{10^{19}}\right]$$

$$x_j = 2\sqrt{10^{-12}\times 60\times 60}\times(2.75) \qquad [\because \text{erfc}(2.75) = 10^{-4}]$$

$$x_j = 3.3 \times 10^{-4} \text{ cm}$$

or $$x_j = 3.3\ \mu\text{m}$$

Q.5 (d) Solution:

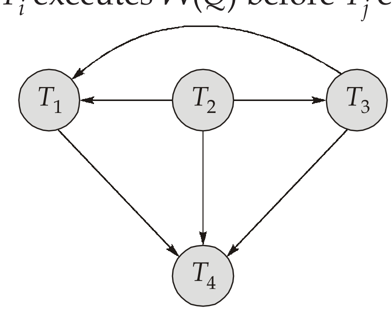
Check for conflict serializable

T_1	T_2	T_3	T_4
	$R(X)$		
		$W(X)$	
		C_3	
$W(X)$			
C_1			
	$W(Y)$		
	$R(Z)$		
	C_2		
			$R(X)$
			$R(Y)$
			C_4

Precedence graph

To draw the precedence graph, we follow the below steps:

1. Create a node $T_i \to Tj$ if T_i executes $W(Q)$ before T_j executes $R(Q)$
2. Create a node $T_i \to Tj$ if T_i executes $R(Q)$ before T_j executes $W(Q)$.
3. Create a node $T_i \to Tj$ if T_i executes $W(Q)$ before T_j executes $R(Q)$.

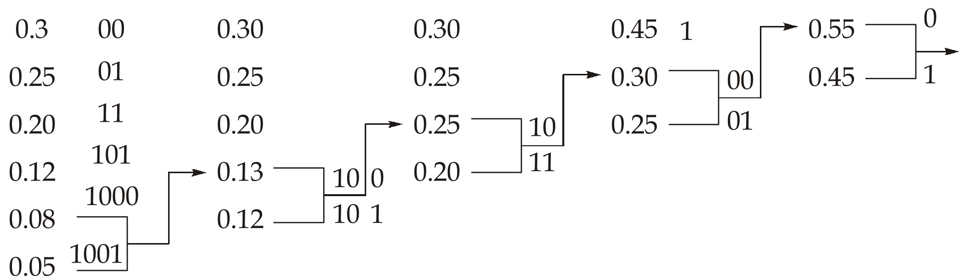


Since, there exist no cycle in the precedence graph. So, the given schedule S is conflict serializable.

Since, conflict serializable schedules are always recoverable. So, the given schedule S is also recoverable.

Q.5 (e) Solution:

Huffman coding:



$$\text{Average code length, } L = \sum_{i=1}^{6} p(x_i)\, n_i$$

$$L = 0.3 \times 2 + 0.25 \times 2 + 0.20 \times 2 + 0.12 \times 3 + 0.08 \times 4 + 0.05 \times 4 = 2.38 \text{ bits/symbol}$$

$$\text{Entropy of the source, } H(x) = -\sum_{i=1}^{6} p(x_i) \log_2 p(x_i)$$

$$= -(0.3\log_2 0.3 + 0.25\log_2 0.25 + 0.2\log_2 0.2 + 0.12\log_2 0.12 + 0.08\log_2 0.08 + 0.05\log_2 0.05)$$

$$= 2.36 \text{ bits/symbol}$$

$$\text{Efficiency,} \quad \eta = \frac{H(x)}{L} = \frac{2.36}{2.38} = 0.9915$$

$$\%\eta = 99.15\%$$

Shannon-Fano coding:

$p(x_i)$		Code
0.3	0 0	0 0
0.25	0 1	0 1
0.20	1 0	1 0
0.12	1 1 0	1 1 0
0.08	1 1 1 0	1 1 1 0
0.05	1 1 1 1	1 1 1 1

$$\text{Average code length, } L = \sum_{i=1}^{6} p(x_i)\, n_i$$

$$L = 0.3 \times 2 + 0.25 \times 2 + 0.20 \times 2 + 0.12 \times 3 + 0.08 \times 4 + 0.05 \times 4 = 2.38 \text{ bits/symbol}$$

$$\text{Entropy of the source, } H(x) = -\sum_{i=1}^{6} p(x_i)\log_2 p(x_i)$$

$$= 2.36 \text{ bits/symbol}$$

$$\text{Efficiency, } \eta = \frac{H(x)}{L} = \frac{2.36}{2.38} = 0.9915 = 99.15\%$$

Q.6 (a) Solution:

(i) 1. The coupling efficiency for a multimode step index fiber with uniform illumination of all propagating modes is given by

$$\eta_{lat} \cong \frac{16(n_1/n)^2}{\left[1+\left(\frac{n_1}{n}\right)\right]^4}\times\frac{1}{\pi}\left\{2\cos^{-1}\left(\frac{y}{2a}\right)-\left(\frac{y}{a}\right)\left[1-\left(\frac{y}{2a}\right)^2\right]^{1/2}\right\}$$

$$= \frac{16(1.5)^2}{[1+(1.5)]^4}\times\frac{1}{\pi}\left\{2\cos^{-1}\left(\frac{6}{60}\right)-\left(\frac{6}{30}\right)\left(1-\left(\frac{6}{60}\right)^2\right)^{1/2}\right\}$$

$$= 0.293\{2.941 - 0.2 \times 0.99\} = 0.804$$

The insertion loss due to lateral misalignment is given by

$$\text{Loss}_{(lat)} = -10\log_{10}\eta_{lat} = -10\log_{10}(0.804) = 0.95 \text{ dB}$$

$$\text{Loss}_{(lat)} = 0.95 \text{ dB}$$

Hence, assuming a small air gap at the joint, the insertion loss is approximately 1 dB when the lateral offset is 10% of the fiber diameter.

2. When the joint is considered index matched (i.e., no air gap), the coupling efficiency may again be obtained as

$$\eta_{lat} \cong \frac{1}{\pi}\left\{2\cos^{-1}\left(\frac{y}{2a}\right)-\left(\frac{y}{a}\right)\left[1-\left(\frac{y}{2a}\right)^2\right]^{1/2}\right\}$$

$$\cong \frac{1}{\pi}\left\{2\cos^{-1}\left(\frac{6}{60}\right)-\left(\frac{6}{30}\right)\left[1-\left(\frac{6}{60}\right)^2\right]^{1/2}\right\}$$

$$\cong \frac{1}{\pi}\left\{2\times(1.471)-0.2(0.99)^{1/2}\right\}$$

$$\cong 0.873$$

Therefore, the insertion loss is:

$$\text{Loss}_{lat} = -10\log_{10}\eta_{lat} = -10\log_{10}(0.873) = 0.59 \text{ dB}$$

(ii) 1. The delay difference between the slowest and fastest modes is given by

$$\delta T_s \cong \frac{Ln_1\Delta}{c} = \frac{6\times10^3\times1.5\times0.001}{3\times10^8} = 30 \text{ ns}$$

2. The rms pulse broadening due to intermodal dispersion may be obtained as

$$\sigma_s = \frac{Ln_1\Delta}{2\sqrt{3}c} = \frac{6\times10^3\times1.5\times0.001}{2\sqrt{3}\times3\times10^8}$$

$$= 8.67 \text{ ns}$$

3. The maximum bit rate may be estimated in two ways. The maximum bit rate assuming no pulse overlap is given by

$$B_{T(\max)} = \frac{1}{2\tau} = \frac{1}{2\delta T_s} = \frac{1}{2\times30\times10^{-9}} = \frac{1}{60\times10^{-9}}$$

$$B_{T(\max)} = 16.67 \text{ M bit s}^{-1}$$

Alternatively, an improved estimate may be obtained using the calculated rms pulse broadening as

$$B_{T(\max)} = \frac{0.2}{\sigma_s} = \frac{0.2}{8.67\times10^{-9}} = 23.068 \text{ M bit s}^{-1}$$

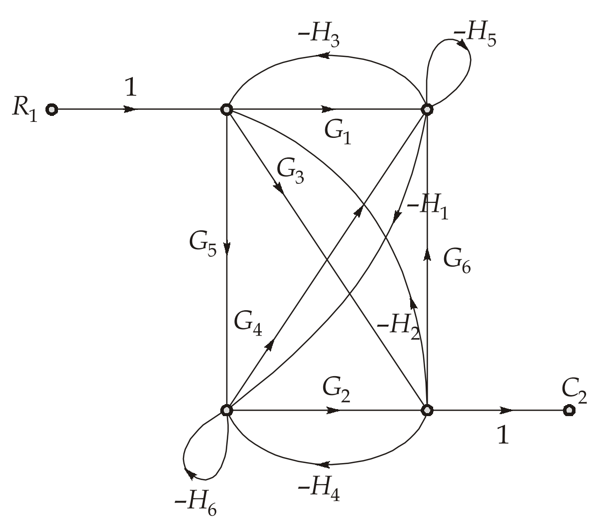
4. Using the most accurate estimate of the maximum bit rate from (c), and assuming return to zero pulses, the bandwidth-length product is:

$$B_{opt} \times L = 23.068 \times 6 \times 10^3 \times 10^6 = 138.408 \text{ MHz km}$$

Q.6 (b) Solution:

To calculate $\frac{C_2}{R_1}$, consider $C_1 = 0$ and $R_2 = 0$.

Redrawing the signal flow graph,



Number of forward paths, $K = 3$

Forward path gains

$$T_1 = G_3$$
$$T_2 = G_2G_5$$
$$T_3 = -G_1G_2H_1$$

Loop gains:

$$L_1 = -G_3H_2$$
$$L_2 = -G_4H_1$$
$$L_3 = -H_5$$
$$L_4 = -H_6$$
$$L_5 = -G_1H_3$$
$$L_6 = -G_2H_4$$
$$L_7 = -G_2G_6H_1$$
$$L_8 = -G_2G_5H_2$$
$$L_9 = -G_4G_5H_3$$
$$L_{10} = -G_3G_6H_3$$
$$L_{11} = -G_2G_5G_6H_3$$
$$L_{12} = G_1G_2H_1H_2$$
$$L_{13} = G_3G_4H_3H_4$$

Non touching loops:

$$L_{34} = L_3L_4 = H_5H_6$$
$$L_{56} = L_5L_6 = G_1G_2H_3H_4$$
$$L_{45} = L_4L_5 = G_1H_3H_6$$
$$L_{36} = L_3L_6 = G_2H_4H_5$$
$$L_{12} = L_1L_2 = G_3G_4H_1H_2$$
$$L_{13} = L_1L_3 = G_3H_2H_5$$
$$L_{14} = L_1L_4 = G_3H_2H_6$$
$$L_{38} = L_3L_8 = G_2G_5H_2H_5$$
$$L_{410} = L_4L_{10} = G_3G_6H_3H_6$$

Three Non touching loops:

$$L_1L_3L_4 = -G_3H_2H_5H_6$$

So, $$\Delta = 1 + G_3H_2 + G_4H_1 + H_5 + H_6 + G_1H_3 + G_2H_4 + G_2G_6H_1 + G_2G_5H_2 + G_4G_5H_3 + G_3G_6H_3 + G_2G_5G_6H_3 + H_5H_6 + G_1G_2H_3H_4 + G_1H_3H_6 + G_2H_4H_5 + G_3G_4H_1H_2 + G_3H_2H_5 + G_2G_5H_2H_5 + G_3G_6H_3H_6 + G_3H_2H_6 + G_3H_2H_5H_6$$

Eliminating the loops that touch the first forward path,

$$\Delta_1 = 1 + H_5 + H_6 + G_4H_1 + H_5H_6$$

Eliminating the loops that touch the second forward path,

$$\Delta_2 = 1 + H_5$$

Eliminating the loops that touch the third forward path,

$$\Delta_3 = 1$$

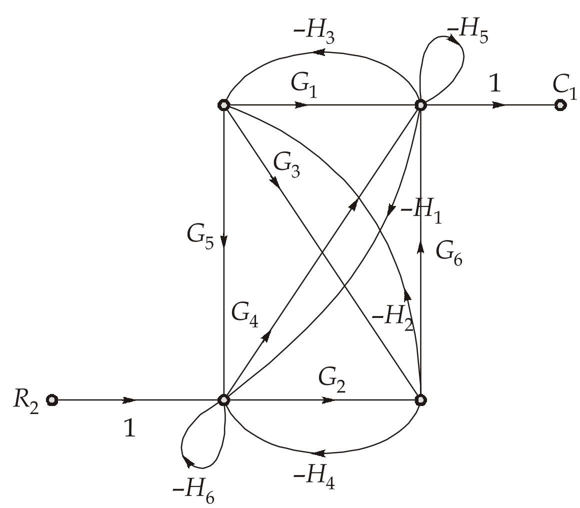
Hence, the overall transfer function, using Mason's Formula is given by,

$$\frac{C_2}{R_1} = \frac{1}{\Delta}\sum_k T_k\Delta_k = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3}{\Delta}$$

$$= \frac{G_3(1+H_5+H_6+G_4H_1+H_5H_6)+G_2G_5(1+H_5)-G_1G_2H_1}{\begin{array}{l}(1+G_3H_2+G_4H_1+H_5+H_6+G_1H_3+G_2H_4+G_2G_6H_1+G_2G_5H_2\\ +G_4G_5H_3+G_3G_6H_3+G_2G_5G_6H_3+G_1G_2H_1H_2+G_3G_4H_3H_4\\ +H_5H_6+G_1G_2H_3H_4+G_1H_3H_6+G_2H_4H_5+G_3G_4H_1H_2\\ +G_3H_2H_5+G_2G_5H_2H_5+G_3G_6H_3H_6+G_3H_2H_6+G_3H_2H_5H_6)\end{array}}$$

To calculate $\frac{C_1}{R_2}$, consider $C_2 = 0$ and $R_1 = 0$

Redrawing the signal flow graph,



Number of forward paths, $K = 3$

$$T_1 = G_4$$
$$T_2 = G_2G_6$$
$$T_3 = -G_1G_2H_2$$

Loop gains:

$$L_1 = -G_3H_2$$
$$L_2 = -G_4H_1$$
$$L_3 = -H_5$$
$$L_4 = -H_6$$
$$L_5 = -G_1H_3$$
$$L_6 = -G_2H_4$$
$$L_7 = -G_2G_6H_1$$
$$L_8 = -G_2G_5H_2$$
$$L_9 = -G_4G_5H_3$$
$$L_{10} = -G_3G_6H_3$$
$$L_{11} = -G_2G_5G_6H_3$$
$$L_{12} = G_1G_2H_1H_2$$
$$L_{13} = G_3G_4H_3H_4$$

Two Non touching loops:

$$L_{34} = L_3L_4 = H_5H_6$$
$$L_{56} = L_5L_6 = G_1G_2H_3H_4$$
$$L_{45} = L_4L_5 = G_1H_3H_6$$
$$L_{36} = L_3L_6 = G_2H_4H_5$$
$$L_{12} = G_3G_4H_1H_2$$
$$L_{13} = G_3H_2H_5$$
$$L_{14} = L_1L_4 = G_3H_2H_6$$
$$L_{38} = L_3L_8 = G_2G_5H_2H_5$$
$$L_{410} = L_4L_{10} = G_3G_6H_3H_6$$

Three Non touching loops:

$$L_1L_3L_4 = -G_3H_2H_5H_6$$

Eliminating the loops that touch the first forward path,

$$\Delta_1 = 1 + G_3H_2$$

Eliminating the loops that touch the second forward path,

$$\Delta_2 = 1$$

Eliminating the loops that touch the third forward path,

$$\Delta_3 = 1$$

Hence, the overall transfer function, using Mason's Formula is given by,

$$\frac{C_1}{R_2} = \frac{1}{\Delta}\sum_k T_k \Delta_k$$

$$= \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3}{\Delta}$$

$$= \frac{G_4(1+G_3H_2)+G_2G_6-G_1G_2H_2}{\begin{gathered}(1+G_3H_2+G_4H_1+H_5+H_6+G_1H_3+G_2H_4+G_2G_6H_1+G_2G_5H_2\\ +G_4G_5H_3+G_3G_6H_3+G_2G_5G_6H_3+G_1G_2H_1H_2+G_3G_4H_3H_4\\ +H_5H_6+G_1G_2H_3H_4+G_1H_3H_6+G_2H_4H_5+G_3G_4H_1H_2\\ +G_3H_2H_5+G_2G_5H_2H_5+G_3G_6H_3H_6+G_3H_2H_6+G_3H_2H_5H_6)\end{gathered}}$$

Q.6 (c) Solution:

(i) Autocorrelation function of $x(t)$ is

$$R_{XX}(\tau) = x(\tau) * x(-\tau)$$

The Fourier Transform of the auto-correlation function gives the power spectral density.

$$S_{XX}(\omega) \longleftrightarrow R_{XX}(\tau)$$

We have, $S_{XX}(\omega) = X(\omega)\, X^*(\omega)$ [For energy signals]

For $x(t) = \sqrt{8}e^{-4t}$,

$$X(\omega) = \frac{\sqrt{8}}{4+j\omega}$$

Thus, $$S_{XX}(\omega) = \frac{\sqrt{8}}{4+j\omega}\times\frac{\sqrt{8}}{4-j\omega}$$

$$S_{XX}(\omega) = \frac{8}{\omega^2+16}$$

Consider $$h(t) \longleftrightarrow H(\omega)$$

$$H(\omega) = \frac{1}{2+j\omega}$$

Output PSD $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$

$$= \left(\frac{1}{4+\omega^2}\right)\left(\frac{8}{\omega^2+16}\right)$$

Thus, $$R_{YY}(\tau) = F^{-1}\left[\frac{8}{(\omega^2+4)(\omega^2+16)}\right]$$

$$= \frac{2}{3}\left[\frac{1}{4}F^{-1}\left(\frac{4}{\omega^2+4}\right) - \frac{1}{8}F^{-1}\left(\frac{8}{\omega^2+16}\right)\right]$$

$$R_{YY}(\tau) = \frac{2}{3}\left[\frac{1}{4}e^{-2|\tau|} - \frac{1}{8}e^{-4|\tau|}\right]$$

(ii) **1.** For symmetrical channel,

Channel capacity, $$C = \log_2 M - H'\left(\frac{Y}{X}\right)$$

$$\left(H'(Y|X) = \text{Entropy of any row of } P\left(\frac{Y}{X}\right)\right)$$

From the channel diagram,

$$P\left(\frac{Y}{X}\right) = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

$$H'\left(\frac{Y}{X}\right) = -\sum_{i=1}^{3} P\left(\frac{y_i}{x_j}\right)\log_2 P\left(\frac{y_i}{x_j}\right)$$

$$\left(\text{Entropy of any row of } P\left(\frac{Y}{X}\right)\right)$$

$$= -(0.5\log_2 0.5 + 0.25\log_2 0.25 + 0.25\log_2 0.25)$$

$$= 1.5 \text{ bits/symbol}$$

M = Number of symbols generated by source = 3

Hence, $C = \log_2 3 - 1.5$

$C = 1.5849 - 1.5 = 0.0849 \simeq 0.085$ bits/symbol

2. Given: $10\log_{10}(S/N) = 39$ dB $\Rightarrow S/N = 10^{3.9} \approx 7943$

Using Shannon's Hartley Theorem,

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

$$C = 3000\log_2(1+7943) \cong 38,867\ \frac{\text{Bits}}{\text{Sec}}$$

Q.7 (a) Solution:

(i)
$$H(\omega) = \left[\frac{\sin^2 4\omega}{\omega^2}\right]\cdot\cos\omega$$

$$H(\omega) = 16\left(\frac{\sin\pi 8f}{\pi 8f}\cdot\frac{\sin\pi 8f}{\pi 8f}\right)\cos\omega$$

$$H(\omega) = 16\,\text{sinc}^2 8f\cdot\left[\frac{e^{j\omega}+e^{-j\omega}}{2}\right]$$

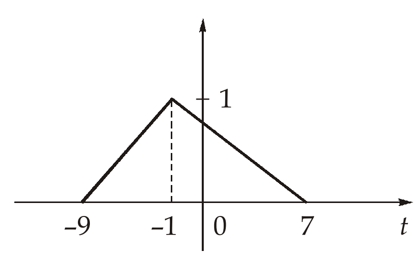
$$H(\omega) = 8\,\text{sinc}^2 8f\,e^{j\omega} + 8\,\text{sinc}^2 8f\,e^{-j\omega}$$

We have,
$$\text{Tri}(t/T) = \begin{cases} 1-|t|/T, & |t|<T \\ 0, & |t|>T \end{cases} \xleftrightarrow{\text{Fourier Transform}} T\,\text{sinc}^2(fT)$$

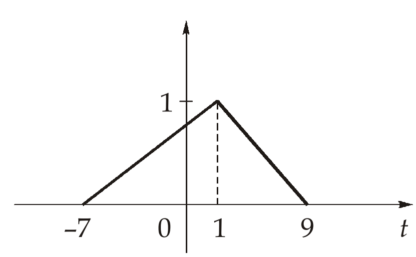
Taking inverse Fourier transform,

$$h(t) = \text{Tri}\left(\frac{t+1}{8}\right) + \text{Tri}\left(\frac{t-1}{8}\right)$$

$$h_1(t) = \text{Tri}\left(\frac{t+1}{8}\right) \rightarrow$$

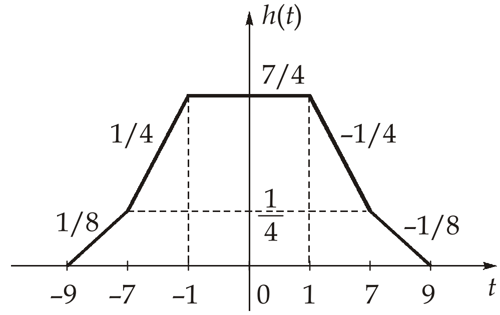


$$h_2(t) = \text{Tri}\left(\frac{t-1}{8}\right) \rightarrow$$



Adding $h_1(t)$ and $h_2(t)$,

$$h(t) = h_1(t) + h_2(t)$$



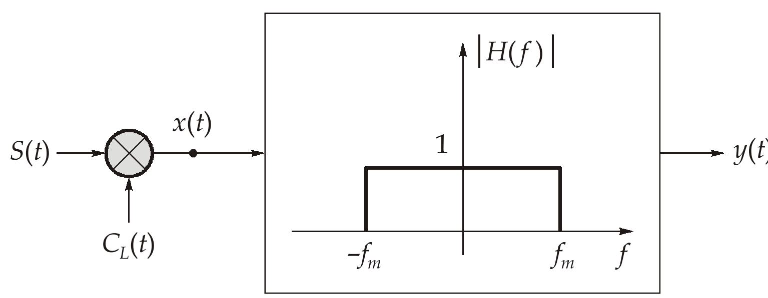
(ii) At $t = -4$ sec,

$$h(-4) = \frac{1}{4} + 3 \times \frac{1}{4} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4}$$

$$h(-4) = 1$$

Q.7 (b) Solution:

(i) The block diagram for the system given in the question can be represented as,



$$x(t) = S(t)\, C_L(t) = Am(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \theta)$$

$$= \frac{A}{2} m(t)\left[\cos(4\pi f_c t + \theta) + \cos\theta\right]$$

$$= \frac{A}{2} m(t)\cos\theta + \frac{A}{2} m(t)\cos(4\pi f_c t + \theta)$$

After passing $x(t)$ through the low pass filter, we get

$$y(t) = \frac{A}{2} m(t)\cos\theta$$

Let us assume that the power of the message signal $m(t)$ is P_m

$$P_{\text{out}} = \frac{A^2}{4}\cos^2\theta P_m$$

The power of the modulated signal can be calculated as below,

$$S(t) = Am(t)\cos(2\pi f_c t) = \frac{A}{2}m(t)\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right]$$

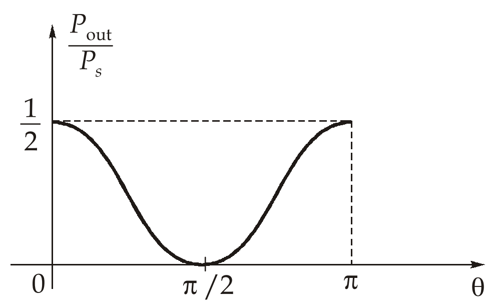
$$= \frac{A}{2}e^{j2\pi f_c t}m(t) + \frac{A}{2}e^{-j2\pi f_c t}m(t)$$

$$P_s = \frac{A^2}{4}P_m + \frac{A^2}{4}P_m = \frac{A^2}{2}P_m$$

The ratio $\frac{P_{out}}{P_s}$ will be,

$$\frac{P_{out}}{P_s} = \frac{\frac{A^2}{4}\cos^2\theta P_m}{\frac{A^2}{2}P_m} = \frac{1}{2}\cos^2\theta$$

The above ratio can be plotted as a function of "θ" as follows:



(ii) The output of the narrowband FM modulator can be given by,

$$x(t) = A\cos[2\pi f_o t + \phi(t)]\ ; \quad |\phi(t)|_{max} = 0.10 \text{ rad}$$

The signal at the output of upper frequency multiplier can be given by,

$$y(t) = A\cos\left[2\pi n_1 f_o t + n_1\phi(t)\right]$$

After mixing $y(t)$ with the output signal of the lower frequency multiplier, we get,

$$Z(t) = A^2\cos\left[2\pi n_1 f_0 t + n_1\phi(t)\right]\cos\left[2\pi n_2 f_0 t\right]$$

$$= \frac{A^2}{2}\cos\left[2\pi(n_1 + n_2)f_0 t + n_1\phi(t)\right] + \frac{A^2}{2}\cos\left[2\pi(n_1 - n_2)f_0 t + n_1\phi(t)\right]$$

It is given that the mixer is designed for up conversion. So, the signal $s(t)$ can be given by,

$$S(t) = \frac{A^2}{2}\cos\left[2\pi(n_1 + n_2)f_0 t + n_1\phi_1(t)\right]$$

It is given that, $f_c = 104$ MHz and $\Delta f_{max} = 75$ kHz for $s(t)$

So, the modulation index of the wideband signal $s(t)$ will be

$$\beta = \frac{\Delta f_{\max}}{f_{m(\max)}} = n_1 |\phi(t)|_{\max}$$

$$n_1(0.10) = \frac{75}{15} = 5$$

$$n_1 = \frac{5}{0.10} = 50$$

We have,
$$f_c = (n_1 + n_2) f_0 = 104 \text{ MHz}$$

$$(n_1 + n_2) \times 100 = 104 \times 1000$$

$$n_2 = 1040 - n_1 = 1040 - 50 = 990$$

Q.7 (c) Solution:

(i) Characteristic equation of the system is

$$1 + G(s) = 0 \quad \text{or} \quad s^2 + 2s + K = 0 \qquad \text{...(i)}$$

Required, $M_p = 0.1 = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$

$$\frac{\xi^2}{(1-\xi^2)} = 0.537 \quad \Rightarrow \xi^2 = \frac{0.537}{1.537} = 0.3495$$

Comparing the characteristic equation with the standard second order characteristic equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$, we get

$$\xi = 0.591$$

$$2\xi\omega_n = 2, \quad \omega_n = \sqrt{K}$$

$$\Rightarrow \quad 0.591\sqrt{K} = 1 \quad \Rightarrow \quad K = \left(\frac{1}{0.591}\right)^2 = 2.86$$

Now,
$$\text{Peak time, } t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\sqrt{2.86}\,\sqrt{1-(0.591)^2}}$$

$$t_p = 2.302 \text{ sec} > 1 \text{ sec (specified)}$$

We conclude that with one adjustable variable (K), the given two specifications can not be met.

Thus, we now modify the specifications

$$t_p = 1\,(1 + x)\,; \quad x < 1 \qquad \text{...(ii)}$$

$$M_p = 0.1\,(1 + x)\,; \quad x < 1 \qquad \text{...(iii)}$$

From equation (i), $\omega_n = \sqrt{K}\ ;\ \xi = \frac{1}{\sqrt{K}}$

Substituting in equation (ii) and (iii),

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\sqrt{K}\sqrt{1-\frac{1}{K}}}$$

or $$t_p = \frac{\pi}{\sqrt{K-1}} = 1\,(1+x) \quad \text{...(iv)}$$

and $$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}, \quad \xi = \frac{1}{\sqrt{K}}$$

$$M_p = e^{-\frac{\pi}{\sqrt{K-1}}} = 0.1\,(1+x) \quad \text{...(v)}$$

From equation (iv) and (v), we get

$$e^{-(1+x)} = 0.1(1+x)$$

Since, these equations are transcendental equations. Thus, we obtain the solution using trial and error method as

$$x = 0.75$$

From equation (iv), we get $K = 4.25$

From these values, we find

$$t_p = 1.75 \text{ sec}; \quad M_p = 0.175$$

$$\% \text{ change in } t_p = \frac{1.75-1}{1} = 0.75 \text{ or } 75\% \text{ increase}$$

$$\% \text{ change in } M_p = \frac{0.175-0.1}{0.1} = 0.75 \text{ or } 75\% \text{ increase}$$

(ii) Characteristic equation of the system with pole at $s = -\alpha$ is given by

$$s^2 + \alpha s + K = 0$$

Comparing with the characteristic equation of the standard second order system,

$$\omega_n = \sqrt{K}, \quad \alpha = 2\xi\omega_n \quad \text{...(vii)}$$

Specifications: $\xi = 0.591\ (M_p = 10\%),\quad t_p = 1$ sec

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega_n\sqrt{1-0.591^2}} = 1 \text{ sec} \quad \text{...(viii)}$$

From equations (viii),

$$\omega_n = \frac{\pi}{0.8} = 3.93 \text{ rad/sec}$$

$\therefore$ $$K = \omega_n^2 = 15.42$$

and $$\alpha = 2 \times 0.591 \times 3.93 = 4.64$$

Hence, $$\alpha = 4.64$$

$$K = 15.42$$

We see that α can be adjusted to $\alpha' > \alpha$ with the use of output rate feedback.

Q.8 (a) Solution:

(i) The fundamental mode in rectangular waveguide is, TE_{10} for $a > b$

The cut-off frequency of TE_{10} mode is,

$$f_{c(TE_{10})} = \frac{1}{2a\sqrt{\mu_o \epsilon_o}} = \frac{c}{2a} = \frac{3\times10^8}{2a} \text{ Hz}$$

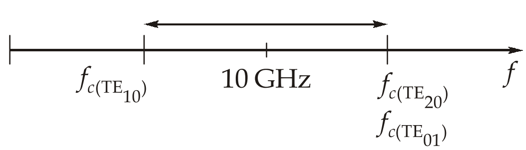
For $b = \frac{a}{2}$, TE_{01} and TE_{20} have same cut-off frequency,

$$f_{c(TE_{01})} = \frac{1}{2b\sqrt{\mu_o \epsilon_o}} = \frac{c}{2b} = \frac{3\times10^8}{2b} \text{ Hz} = \frac{3\times10^8}{a} \text{ Hz} \quad \left[\because\ b = \frac{a}{2}\right]$$

$$f_{c(TE_{20})} = \frac{1}{a\sqrt{\mu_o \epsilon_o}} = \frac{c}{a} = \frac{3\times10^8}{a} \text{ Hz}$$

For single mode operation, $f_{c(TE10)} < f < f_{c(TE20)}$. Hence, the middle of the frequency band for single mode operation can be expressed in terms of the cut-off frequencies as

For single mode



$$\Rightarrow \quad \frac{f_{c(TE_{10})} + f_{c(TE_{01})}}{2} = 10 \text{ GHz}$$

$$10 \times 10^9 = \frac{1}{2}\left[\frac{3\times10^8}{2a} + \frac{3\times10^8}{a}\right]$$

$$20 \times 10^9 = \frac{1}{a}[4.5\times10^8]$$

$$\therefore \qquad a = \frac{4.5\times 10^8}{20\times 10^9} = 0.0225 \text{ m} = 2.25 \text{ cm}$$

$$\therefore \qquad b = \frac{a}{2} = 1.125 \text{ cm}$$

(ii) 1. Given, VSWR = 2

We have, $$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

(or) $$|\Gamma_0| = \frac{\text{VSWR}-1}{\text{VSWR}+1} = \frac{2-1}{2+1} = \frac{1}{3}$$

Reflection coefficient, $\Gamma = \Gamma_L e^{-j2\beta d}$, where $\Gamma_L = |\Gamma_0|\angle\phi$

The distance between successive voltage minima is

$$\frac{\lambda}{2} = 40 \text{ cm}$$

$$\therefore \qquad \lambda = 80 \text{ cm}$$

For the voltage minima

$$2\beta d_{min} = (2n+1)\pi + \phi$$

It is given that distance from load to the first minima is 10 cm.

For the first minima, put $n = 0$, hence,

$$2\times\frac{2\pi}{\lambda}\times 10 = \pi + \phi$$

$$\Rightarrow \qquad 2\times\frac{2\pi}{80}\times 10 = \pi + \phi$$

$$\Rightarrow \qquad \phi = \frac{-\pi}{2}$$

Hence, reflection coefficient,

$$\Gamma = \frac{1}{3}e^{-j\frac{\pi}{2}}e^{-j2\beta d}$$

2. We have, $$\Gamma_L = \frac{1}{3}e^{-j\frac{\pi}{2}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Rightarrow \qquad Z_L\left[1+\frac{j}{3}\right] = Z_0\left[1-\frac{j}{3}\right]$$

$$\Rightarrow \qquad Z_L = \frac{50(3-j)}{3+j}\times\frac{(3-j)}{(3-j)}$$

$$\Rightarrow \qquad Z_L = 5[8-6j] = (40 - j30)\ \Omega$$

Q.8 (b) Solution:

(i) **1. Direct addressing mode:**

Effective address (EA)=Offset = 1000 H

Physical address (PA) =Segment : Offset (EA)

= DS : EA

= 7000 H : 1000 H

= 70000 + 1000 = 71000 H

2. Register indirect addressing mode

In Register indirect addressing mode, registers BX, BP, DI, SI is used to store the effective address (EA). Assuming BX as the register holding the Effective address, i.e.

EA = [BX] = 2000 H

PA = Segment register : EA

= DS : EA

= 7000 H : 2000 H

= 70000 + 2000 = 72000 H

3. Register indirect addressing mode (assuming DI)

EA = [DI] = 4000 H

PA = Segment register : EA = DS : EA

= 7000 H : 4000 H

= 70000 + 4000 = 74000 H

4. Based addressing mode : The effective address is the sum of BX/BP register and 8-bit/16-bit displacement. Assuming the BX register,

EA = [BX] + Displacement

= 2000 H + 1000 H = 3000 H

PA = Segment register : EA

= DS : EA

= 7000 H : 3000 H

= 70,000 + 3000 = 73000 H

5. Based addressing mode : Assuming the BP register,

EA = [BP] + Displacement

= 5000 H + 1000 H = 5000 H

PA = Segment register : EA
= DS : EA
= 7000 H + 5000 H
= 70000 + 5000 = 75000 H

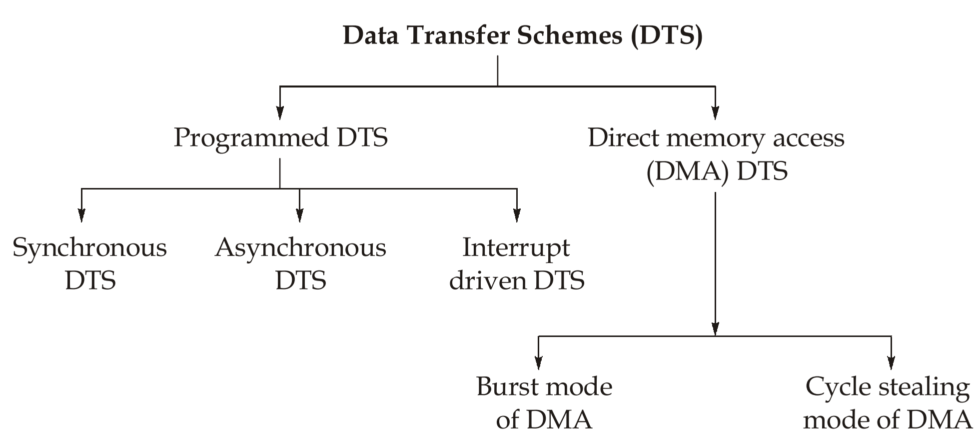
6. Based index addressing mode:

EA = [BX] + [DI]
= 2000H + 4000H = 6000 H
PA = Segment register : EA
= 7000 H : 6000 H
= 70000 + 6000 H
= 76000 H

7. Based Indexed displacement addressing mode: The effective address is sum of the base register and index register (SI/DI). Assuming DI as the index register,

EA = [BX] + [DI] + Displacement
= 2000 H + 4000 H + 1000 H = 7000 H
PA = Segment register : EA = DS : EA
= 7000 H : 7000 H = 77000 H

(ii) A microprocessor based system or computer have several I/O devices of different speed. A slow I/O device cannot transfer data when microprocessor issues instruction for the same because it takes some time to get ready. To solve the problem of speed mismatch between a microprocessor and I/O devices a number of data transfer techniques have been developed. The data transfer techniques are classified as below:



1. Programmed DTS

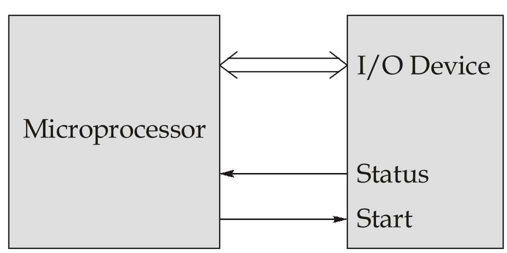
- Programmed data transfer schemes are controlled by the CPU.
- Data is transferred from an I/O device to the CPU or vice versa under the control of programs which reside in the memory. These programs are executed by the CPU when an I/O device is ready to transfer data.
- Programmed DTS are employed when small amount of data are to be transferred.

It is classified into following three categories:

(i) Synchronous DTS

- Synchronous means "at the same time". In this DTS the device which sends the data and the device which receives data are synchronised with the same clock.
- The data transfer with I/O devices is performed executing IN or OUT instructions for I/O mapped I/O devices or using memory read/write instructions for memory mapped I/O devices.
- The status of the I/O device i.e., whether it is ready or not, is not examined before data is transferred. The I/O devices compatible with microprocessors in speed are usually not available. Hence, this technique of data transfer is rarely used for I/O devices.

(ii) Asynchronous DTS



Asynchronous Data Transfer

- Asynchronous means "at irregular intervals". In this DTS, data transfer is not based on pre-determined timing pattern.
- This technique of data transfer is used when the speed of an I/O device does not match the speed of the microprocessor and the timing characteristics of I/O device is not predictable.
- In this technique, the status of the I/O device i.e. whether the device is ready or not, is checked by the microprocessor before the data is transferred. This mode of data transfer is called handshaking mode of transfer.

- It is used for slow I/O devices. This technique is an inefficient technique because the precious time of the microprocessor is wasted in waiting.

(iii) Interrupt Driven DTS:

- In this scheme, the microprocessor initiates an I/O device to get ready and then it executes its main program instead of remaining in a program loop to check the status of the I/O device.
- When the I/O device becomes ready to transfer data, it sends a high signal to the microprocessor through a special input line called an interrupt line. On receiving the microprocessor completes the current instruction at hand and then attends the I/O device.
- It saves the contents of the program counter on the stack first and then takes up a subroutine called Interrupt Service Subroutine (ISS).

2. DMA Data Transfer (DMA-DTS)

- In DMA-DTS, MPU does not participate because data is directly transferred from an I/O device to the memory or vice versa.
- In DMA, the MPU releases the control of the buses to a device called "DMA controller". The controller manages data transfer between memory and a peripheral under its control, thus bypassing the MPU.
- The MPU communicates with the controller by using the chip select line, buses and control signals. Examples, of DMA controller chips are: Intel 8237 A, 8257 etc.
- DMA-DTS is faster scheme as compared to programmed DTS.
- It is used to transfer data from "mass storage devices" (hard disks, floppy disks etc). It is also used for high speed printers.

(i) Burst mode of DMA data transer:

- A scheme of DMA data transfer, in which I/O device withdraws the DMA request only after all the data bytes have been transferred, is called burst mode of data transfer.
- By this technique, a block of data is transferred.
- This technique is employed by magnetic disk drives.

(ii) Cycle stealing mode of DMA data transfer

- In this technique, a long block of data is transferred by a sequence of DMA cycles.
- In this method, after transferring one byte or several bytes, the I/O device withdraws DMA request.

- This method reduces interference in CPU's activities.
- The interference can be eliminated completely by designing an interfacing circuitry which can steal bus cycle for DMA data transfer only when the CPU is not using the system bus.
- This modes of DTS has highest efficiency.

Q.8 (c) Solution:

(i) Average traffic intensity per user, A_t = 0.1 Erlangs (given)

Blocking probability = 0.5% (given)

Number of trunked channels, N = 10 (given)

Corresponding to blocking probability of 0.5% and N = 10, the offered load, A_{av} from the Erlang B Table is found to be 3.96.

Therefore, offered traffic load, A_{av} = 3.96 Erlangs

The total number of mobile users that can be supported in the given system can be computed by dividing the offered traffic load, A_{av} with the given average traffic intensity per user, A_t. That is,

$$\text{Total number of mobile users that can be supported} = \frac{A_{av}}{A_t} = \frac{3.96}{0.1} \approx 39 \text{ users}$$

To find offered traffic Load, A_{av} for increased N:

Number of trunked channels, N = 100, (given)

Corresponding to blocking probability of 0.5% and N = 100, the offered load, A_{av} from Erlang Table is found to be 80.9 Erlangs.

$$\text{Hence, total number of mobile users that can be supported} = \frac{A_{av}}{A_t} = \frac{80.9}{0.1} = 809 \text{ users}$$

(ii) 1. Converting transmitter power in dB m units,

$$P_t = 100 \text{ W or } 100{,}000 \text{ mW}$$

$$P_t(\text{dBm}) = 10\log[P_t(\text{mW})]$$

$$= 10\log(100000) = 50 \text{ dBm}$$

Receiver threshold level, P_r(dBm) = –100 dBm

Maximum possible path loss,

$$L_p(\text{dB}) = P_t(\text{dBm}) - P_r(\text{dBm})$$

$$= 50 \text{ dBm} - (-100 \text{ dBm}) = 150 \text{ dB}$$

To determine the radio coverage range, r

Path loss at first meter, L_o = 30 dB (given)

Propagation path constant, $\gamma = 4$ (given)

We know that path loss,

$$L_p \text{ (dB)} = L_o \text{ (dB)} + 10\log(r)^{\gamma}$$

where r is the radio coverage range of the base station transmitter in meters.

Therefore,
$$L_p \text{ (dB)} = L_o \text{ (dB)} + 10\log(r)^4$$
$$40 \log r = 150 - 30 = 120 \text{ dB}$$
$$r = 10^{120/40} = 10^3 = 1000 \text{ meters}$$

Hence, the radio coverage range, r = 1 km.

2. Given a pure ALOHA system with a channel bit rate = 200 bits per second.

$$\text{Bit rate} = 200 \text{ bits per second}$$

The frame transmission time is 200 bits/200 kbps = 1 ms

If the system creator 500 frames per second, this is (1/2) frame per millisecond.

The load is $\frac{1}{2}$.

In this case, throughput of pure ALOHA system is given by

$$S = Ge^{-2G}$$

where, G = Mean number of transmission attempts per frame time.

$$S = \frac{1}{2}e^{-1} = 0.184 = 18.4\%$$

This means that the throughput is 500 × 0.184 = 92 and that only 92 frames out of 500 will probably survive.

❍❍❍❍